

**Erdős Renyi Graph**  $G(n, p)$  is constructed in the following manner:

1. Consider  $n$  vertices labeled  $\{1, 2, \dots, n\}$ .
2. Corresponding to each distinct pair  $\{i, j\}$  we perform an independent Bernoulli ( $p$ ) experiment and insert an edge between  $i$  and  $j$  with probability  $p$ . Note that all edges are *undirected* and hence there are total of  $\binom{n}{2}$  possible edges, each occurring with probability  $p$ .

3. In this Homework you will simulate an Erdős Renyi Graph and find the M.L.E for the relevant  $p$ .

1. Choosing  $x$ : Write a simple R-code to generate a number uniformly from  $\{1, 2, 3, 4, 5\}$ . Let  $x$  denote the chosen number. Record  $x$  in the box: .

2. Consider the experiment of rolling a die and (choose) specify an event from that experiment which occurs with probability  $x/6$ . Let it be called  $B$ . Write out the description of the event  $B$  in the box below:

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3. The set of vertices for the graph you are about to construct are  $\{1, 2, \dots, 10\}$ . The graph has no self-edges (i.e Self-loops). What is the total number of possible edges ?

Record answer in the box:

4. Construct the *random* adjacency matrix  $A$  for the graph as follows. For each pair  $1 \leq i < j \leq 10$ : *rolling the die 15 times*(using one at home or at <http://www.randomservices.org/random/apps/Dice.html>) and observe if the event  $B$  has occurred. Designate

$$a_{ij} = \begin{cases} 1 & \text{if } B \text{ occurred.} \\ 0 & \text{if } B \text{ did not occur} \end{cases}$$

Fill in the matrix entries accordingly:

0									
	0								
		0							
			0						
				0					
					0				
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									0

5. Using the **igraph** package draw the random graph , denote by  $G(10, \frac{x}{6})$ , corresponding to the above adjacency matrix (i.e draw an edge between  $i$  and  $j$  if  $a_{ij} = 1$ ). *Send the image of your graph on zoom chat group between 8:15am and 8:30am on May 5th, 2022.*

6. From the graph  $G(10, \frac{x}{6})$  or from  $A$  that you constructed in the worksheet:

(a) fill in the following table from the data in worksheet:

x	# Edges

7. Let  $E$  denote the number of edges in a realisation of  $G(10, \frac{x}{6})$ . Find the likelihood  $L(x; E)$  that  $E$  edges occur in the random Graph  $G(10, \frac{x}{6})$ .

8. Find  $x^*$  that maximizes  $L(x; E)$  with respect to  $x$ . You may assume  $x \in [1, 5]$ .

9. Substitute your value of  $E$  from Question 1, into the expression for  $x^*$ . Is the resulting  $x^*$  close to your chosen  $x$  ?