04/02/2022, Interpolation

. family of functions $\{\Phi_i(x)\}$ of x
single variable x.
$i = 0(1)n$, $\{\Phi_0(x), \Phi_1(x), \ldots, \Phi_n(x)\}$
$\Phi(x) \equiv \Phi(x; \{a_i, i = 0(1)n\})$
. $n+1$, $\{a_0 \ldots a_n\} \to$ unknown.

$(n+1)$ $b_i \equiv (x_i, \Phi_i)$
$i = 0(1)n$, $i \neq k \Rightarrow x_i \neq x_k$

$\Phi(x_i; \{a_j\}) = \Phi_i$
$\forall i = 0(1)n$ ... (**)

. Given $\{(x_i, \Phi_i), i = 0(1)n\}$
. Find $\{a_j\}$
s.t. (**) is true

Examples

① . linear interpolation : $\Phi(x)$ depends linearly on a_i's

$$\Phi(x) = \sum_{i=0}^{n} a_i \Phi_i(x)$$

(1a) Polynomial interpolation

$\Phi_i(x) = x^i$

$$\Phi(x) = \sum_{i=0}^{n} a_i x^i$$

$$\Phi(x) = a_0 + a_1 x + \ldots + a_n x^n$$

(b) $$\Phi(x) = \sum_{p=0}^{n} a_p \exp[i p x] \quad (i = \sqrt{-1})$$

$$= \sum_{p=0}^{n} a_p \cos(px) + i \sum_{p=0}^{n} a_p \sin(px)$$

(c) cubic spline interpolation

. $\Phi(x)$ coincide with a cubic polynomial on every interval $[x_i, x_{i+1}]$, $\forall i \in [0, n-1]$
. Polynomial on different subintervals need not match.

② Non-linear
. Rational

$$\Phi(x) = \left(\sum_{i=0}^{n} a_i x^i\right) \Big/ \left(\sum_{j=0}^{m} b_j x^j\right)$$

$n+m+2 = n+1+m+1 \Rightarrow (\{a_i, i = 0(1)n\}, \{b_j; j = 0(1)m\})$