

Ground rules: Open Class notes from website or notes in own handwriting or typed by one self. No other resources allowed. Individual work.

1. It is assumed that scores of those who took the SIS B.Math.(Hons.) entrance exam follow a Normal distribution with standard deviation is 6.4 and the mean of 25.

- (a) If you choose one student at random what is the probability that the student's score is between 20 and 30 ?
- (b) We sample 25 students. What is the sampling distribution of their average score \bar{X} ?
- (c) What is the probability that the mean score of your sample is between 20 and 30 ?

2. The following R code simulates a random variable X

```
> L = 10
> i = 0
> U = runif(1, min=0, max =1)
> Y = -log(U)/L
> Sum = Y
> while (Sum<1) {
+   U = runif(1, min=0, max =1)
+   Y = -log(U)/L
+   Sum = Sum +Y
+   i = i + 1
+ }
> X = i
```

Find the distribution of X .

3. Gobarkanth collects $X_1, X_2, X_3, \dots, X_n$ of i.i.d measurements of radiation from Canteen's Gobarkanth Gas plant. He assumes that the observations follow a Rayleigh distribution with parameter α , with p.d.f. given by

$$f(x) = \begin{cases} \alpha x \exp(-\frac{1}{2}\alpha x^2) & \text{if } x \geq 0, \\ 0 & \text{otherwise.} \end{cases}$$

Find the maximum likelihood estimate for α , providing appropriate justification.

4. Using R write an R-code to perform the following:

- (a) Using `replicate` generate 100 realisations of Y and Z as prescribed below:
 - (i) Generate 15 samples from Poisson (10) distribution

- (ii) Compute the sample mean \bar{X} and sample variance S^2 of the generated sample.
 - (iii) Compute $Y = \sqrt{15} \frac{\bar{X} - \mu}{\sigma}$ where μ and σ are the mean and variance of the Poisson(10) distribution.
 - (iv) Compute $Z = \sqrt{15} \frac{\bar{X} - \mu}{S}$ where μ is the mean of the Poisson(10) distribution.
- (b) Using $Q - Q$ plot decide if Y or Z is Normally distributed.