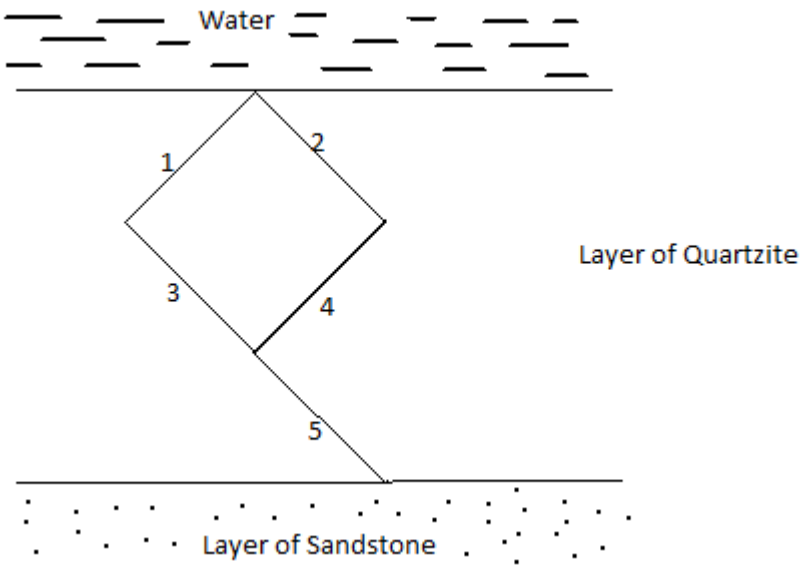
# Probability II: B. Math (Hons.) I
# Academic Year 2021-22, Second Semester
# Midsem Exam: Total Marks = 50

**Note:**

- Please write your name on top of your answer booklet and sign a declaration that says *"I have not taken help from anyone else (either online or in person) for solving the problems given in this exam."*
- This is a take home examination. You are allowed to use your class notes. However, you are not allowed to talk to each other, consult any book or take any form of online help outside the course website.
- It is absolutely important that you follow the rules mentioned above or else, if caught, you will get a zero in the mid-sem exam. The teacher will also report against you to the appropriate authorities.
- Please email the solutions as one pdf file by 2 pm of March 08, 2022. Given the question paper, it is certain that you will be able to email the solutions (as one pdf file) much before the deadline.

1. Consider the following schematic diagram of a drainage network model (as described in class), where each of the five paths is open with probability $p \in (0,1)$ and the paths behave independently of each other.




   (a) (10 marks) Let $X$ be the number of open paths and $Y$ be the indicator that water can pass through the layer of quartzite to the layer of sandstone. Find the joint probability mass function of $X$ and $Y$.

   (b) (5 marks) Find the marginal probability mass functions of $X$ and $Y$.

   (c) (5 marks) If it is given that water has passed through the layer of quartzite to the layer of sandstone, find the conditional probability that exactly 3 paths were open.

2. Suppose $X_1, X_2 \overset{iid}{\sim} N(0,1)$. Define $Y_1 := X_1 + X_2$ and $Y_2 := X_1 - X_2$.

   (a) (10 marks) Show that $Y_1, Y_2 \overset{iid}{\sim} N(0,2)$.

   (b) (8 marks) Without using (a), compute the distribution of $Y_1$ directly using the convolution formula.

   (c) (2 marks) If you are asked to find the distribution of $Y_1$, which method ((a) or (b)) would you prefer to use and why? [As long as you give an answer here, you will get full credit for (c) - after all, your preference depends on your taste.]

   (d) (7 marks) Find the distribution of $Z := X_1/X_2$.

   (e) (3 marks) Does $Z$ have finite mean? Please justify your answer.