Homework-6

1. Let G be a Lie group. A one-parameter subgroup of G is a Lie group homomorphism $: \mathbb{R} \to G$ (definition).

Let $\varphi : \mathbb{R} \to G$ be a 1-parameter subgroup of G. Show that $\exists \, X \in \mathfrak{g}$ such that $\varphi(t) = \operatorname{Exp}_{tX}(e)$.

Conversely, let $X \in \mathfrak{g}$ and $\{\alpha_t\}$ be the associated 1-parameter group of diffeomorphisms. Define $\varphi(t) := \alpha_t(e)$. Show that $\varphi : \mathbb{R} \to G$ is a 1-parameter subgroup of G and $\operatorname{Exp}_{tX} = R_{\varphi(t)}$, the right translation map $R_{\varphi(t)} : G \to G$.

2. Let M be a manifold, $X, Y \in \mathfrak{X}(M)$ and $\{\varphi_s^X\}$, $\{\varphi_t^Y\}$ be the associated (local) families of local diffeomorphisms. Prove that $[X,Y] = 0 \Rightarrow \varphi_s^X \circ \varphi_t^Y = \varphi_t^Y \circ \varphi_t^X$ for s, t in the appropriate domains of definition.