BMath-III, DG2 Mid-semestral exam

Instructions: Total time 2 Hours. All questions are compulsory.

Question 1. Let S^n denote the n-sphere in $\mathbb{R}^{n+1}$. Is it possible to have an atlas for S^n consisting of a single open set, making S^n a (smooth) manifold? Explain. Generalize your answer. (4+4)

Question 2. Recall that for $X \in M_n(\mathbb{R})$ the exponential is defined by $exp(X) = I + X + \frac{X^2}{2!} + \frac{X^3}{3!} + \cdots$ and $SO(n, \mathbb{R}) = \{A \in SL(n, \mathbb{R}) | AA^t = I\}$. For $t \in \mathbb{R}$ compute $exp \begin{bmatrix} 0 & t \\ -t & 0 \end{bmatrix} \in SO(2, \mathbb{R})$. Show by an example that the matrix exponential $exp : M_2(\mathbb{R}) \longrightarrow GL(2, \mathbb{R})$ is not injective. (4+4)