

Microwave Integrated Circuits

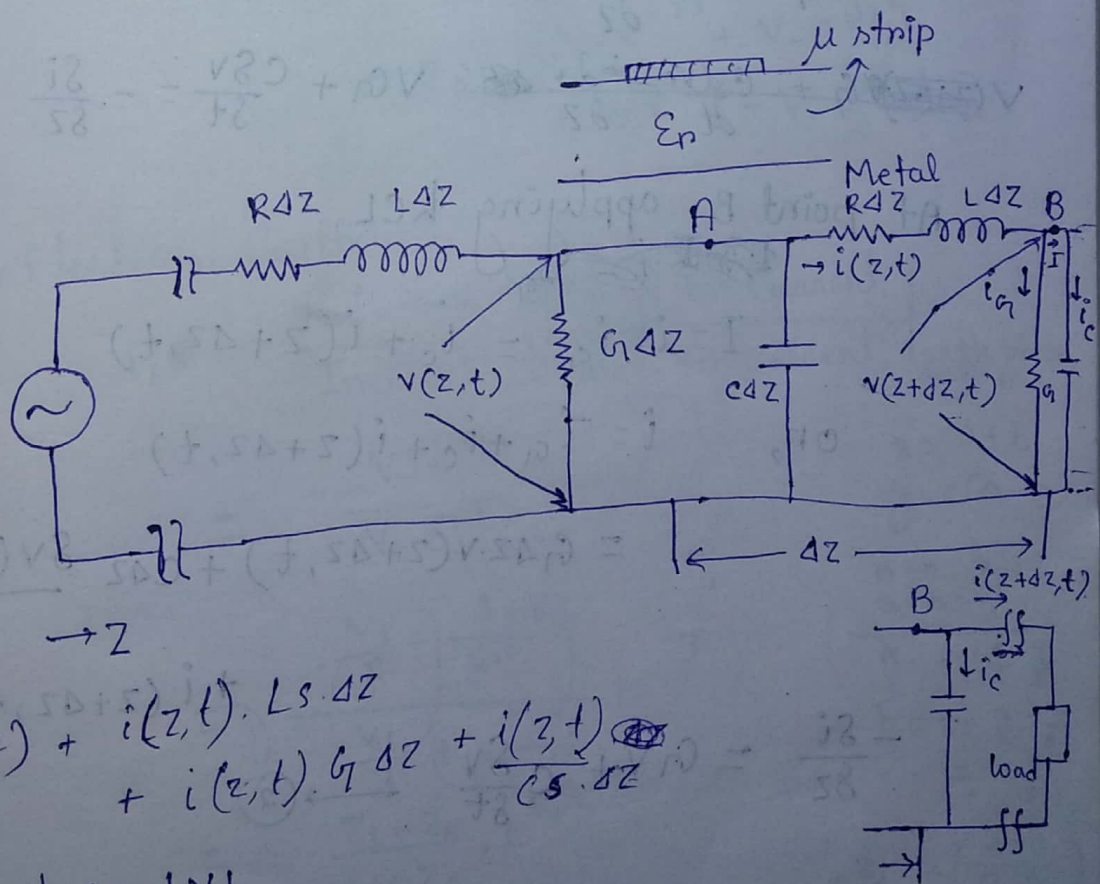
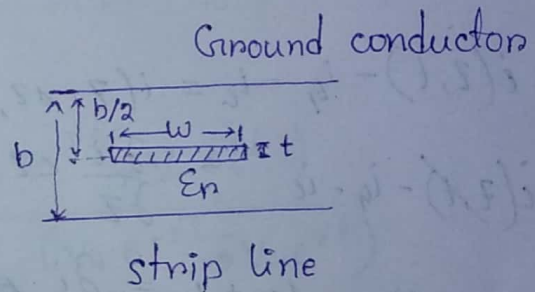
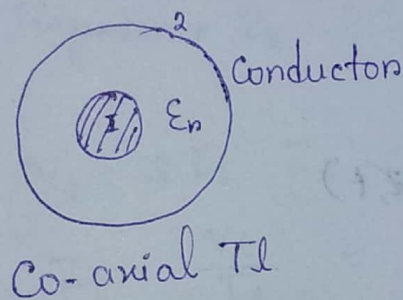
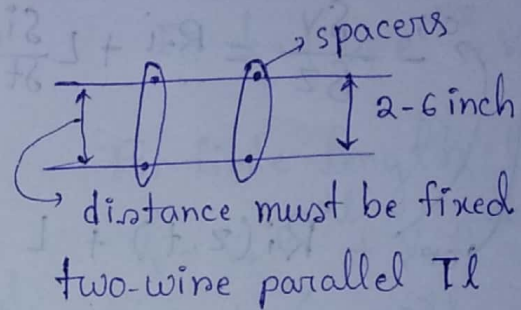
(Sheuli)
(DE-EC804B)

(AKM)

22/3/23

Microwave Transmission lines

- 1) Two-wire parallel Transmission-line
- 2) co-axial
- 3) strip line
- 4) waveguide



$$R\Delta z i(z,t) + i(z,t) \cdot L\Delta z + i(z,t) G\Delta z + \frac{i(z,t)}{C\Delta z}$$

After applying KVL,

$$v(z+\Delta z,t) = v(z,t) + \frac{\delta(z,t)}{\delta(z)} \Delta z$$

$$v(z,t) = R i(z,t) \Delta z + L \frac{\partial i(z,t)}{\partial t} \Delta z + v(z,t) + \frac{\partial v(z,t)}{\partial z} \Delta z$$

or, $-\frac{\partial v}{\partial z} = R \cdot i + L \frac{\partial i}{\partial t} \quad \longrightarrow (1)$

or, $R i(z,t) + L \frac{\partial i(z,t)}{\partial t} + \frac{\partial v(z,t)}{\partial z} = 0$

$$i(z,t) - i_a - i_c = i(z+\Delta z, t)$$

$$i(z,t) - i_a - i_c = \frac{\partial i}{\partial z} \Delta z + i(z,t)$$

or $i_a + i_c = -\frac{\partial i}{\partial z} \Delta z$

$$v(z+\Delta z) + \frac{C dV}{dt} \frac{\partial i}{\partial z} = v(z) + \frac{C \partial v}{\partial t} = -\frac{\partial i}{\partial z}$$

At point B, applying KCL

$$I = i - i_a = i_c + i(z+\Delta z, t)$$

or, $i = i_a + i_c + i(z+\Delta z, t)$

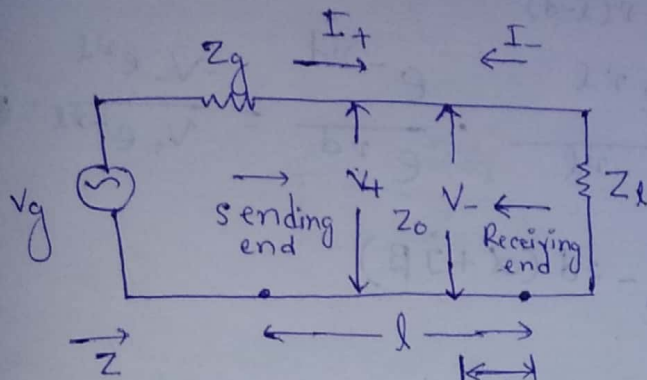
$$= G \Delta z \cdot v(z+\Delta z, t) + C \Delta z \frac{\partial v(z+\Delta z, t)}{\partial t}$$

$$+ i(z+\Delta z, t)$$

$$-\frac{\partial i}{\partial z} = G v + C \frac{\partial v}{\partial t} \quad \longrightarrow (2)$$

Transmission line - Reflection Coefficient

31/3/22



Tr terminated in a load impedance Z_l

$$V = V_+ e^{-\gamma z} + V_- e^{\gamma z}$$

$$I = \frac{1}{Z_0} (V_+ e^{-\gamma z} - V_- e^{\gamma z})$$

For finite length l

$$V_l = V_+ e^{-\gamma l} + V_- e^{\gamma l}$$

$$I_l = \frac{1}{Z_0} (V_+ e^{-\gamma l} - V_- e^{\gamma l})$$

$$V_l = I_l \cdot Z_l$$

$$Z_l = \frac{V_l}{I_l} = \frac{(V_+ e^{-\gamma l} + V_- e^{\gamma l})}{\frac{1}{Z_0} (V_+ e^{-\gamma l} - V_- e^{\gamma l})} = Z_0 \cdot \frac{(V_+ e^{-\gamma l} + V_- e^{\gamma l})}{(V_+ e^{-\gamma l} - V_- e^{\gamma l})}$$

Reflection Coefficient,

$$\Gamma_l = \frac{\text{Reflected Voltage or Current}}{\text{Incident voltage or current}} \quad \left| \begin{array}{l} \text{with phase} \\ \text{angle } \theta \end{array} \right.$$

$$= \frac{V_- e^{\gamma l}}{V_+ e^{-\gamma l}}$$

$$x = a + b$$

$$y = c(a - b)$$

$$a + b = x$$

$$a - b = \frac{y}{c}$$

$$a = \frac{x + \frac{y}{c}}{2}$$

$$b = x - \frac{x + \frac{y}{c}}{2}$$

$$= \frac{x}{2} - \frac{y}{2c}$$

$$Z_l = Z \left(\frac{a + b}{a - b} \right)$$

$$\frac{Z_l}{Z} = \frac{a + b}{a - b}$$

$$\frac{2a}{a - b} = \frac{Z_l + Z}{Z}$$

$$\frac{2b}{a - b} = \frac{Z_l - Z}{Z}$$

$$a : b = \frac{Z_l + Z}{Z_l - Z}$$

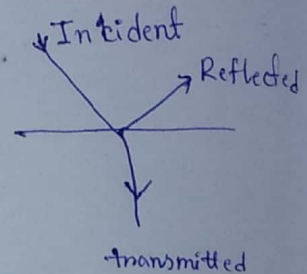
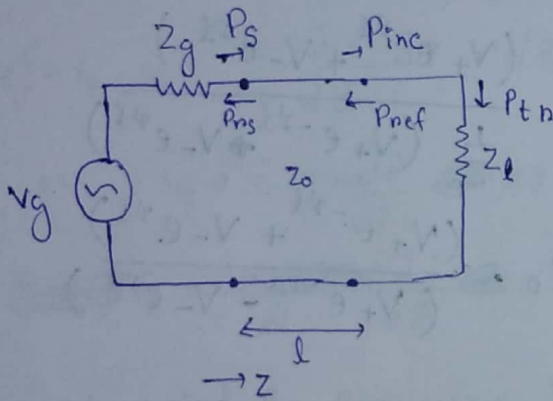
$$V_- e^{\gamma(l-d)}$$

$$\Gamma_{(l-d)} = \frac{V_- e^{\gamma(l-d)}}{V_+ e^{-\gamma(l-d)}}$$

$$= \frac{V_- e^{\gamma l}}{V_+ e^{-\gamma l}} \cdot \frac{e^{-\gamma d}}{e^{\gamma d}} = \frac{V_- e^{\gamma l}}{V_+ e^{-\gamma l}} \cdot e^{-2\gamma d}$$

$$= \Gamma_l \cdot e^{-2d(\alpha + j\beta)}$$

• Transmission line - Transmission coefficient



Transmission Coefficient,

$$= \frac{\text{transmitted voltage or current}}{\text{Incident voltage or current}}$$

$$T = \frac{V_{tn}}{V_{inc}} = \frac{I_{tn}}{I_{inc}}$$

$$V = V_+ e^{-\gamma z} + V_- e^{\gamma z}$$

$$I = \frac{1}{Z_0} (V_+ e^{-\gamma z} - V_- e^{\gamma z})$$

$$V_{tn} e^{-\gamma l} = V_+ e^{-\gamma l} + V_- e^{\gamma l}$$

$$I_{tn} e^{\gamma l} = \frac{1}{Z_0} (V_+ e^{-\gamma l} - V_- e^{\gamma l})$$

$$\frac{V_{tn}}{Z_L} e^{-\gamma l} = \frac{1}{Z_0} (V_+ e^{-\gamma l} - V_- e^{\gamma l}) \quad I_{tn} = \frac{V_{tn}}{Z_L}$$

$$T = \frac{2Z_L}{Z_0 + Z_L}$$

5/4/22

In a transmission line, $Z_0 = 75 + j0.01 \Omega$ & $Z_L = 70 + j50 \Omega$

i) Reflection coefficient

$$\Gamma^2 = \frac{Z_L}{Z_0} (1 - \Gamma^2)$$

(ii) Transmission

(iii) Verify relation b/w Γ & T

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} \quad T = \frac{2Z_L}{Z_L + Z_0}$$

$$= \frac{-5 + j(49.99)}{145 + j(50.01)}$$

• Standing Wave:- $(V_s) = V_0 e^{-j\phi}$

$$a - jb = \sqrt{a^2 + b^2}$$

$$\phi = \tan^{-1}\left(\frac{b}{a}\right)$$

$$= \arctan(b/a)$$

$$V = V_+ e^{-\gamma z} + V_- e^{\gamma z} \quad (\text{voltage expression of } T_n)$$

$$\delta = \alpha + j\beta$$

$$V = V_+ e^{-\alpha z} \cdot e^{-j\beta z} + V_- e^{\alpha z} \cdot e^{j\beta z}$$

$$V = A [\cos(\beta z) - j \sin(\beta z)] + B [\cos(\beta z) + j \sin(\beta z)]$$

$$= (A + B) \cos(\beta z) - j(A - B) \sin(\beta z)$$

$$V_0 = \sqrt{(A+B)^2 \cos^2(\beta z) + (A-B)^2 \sin^2(\beta z)}$$

$$\phi = \arctan \left(\frac{A-B}{A+B} \tan(\beta z) \right)$$

$$\frac{dV_0}{d(\beta z)} = 0$$

$$\text{or, } \frac{dV_0}{d(\beta z)} = \frac{1}{2} \left(-2m \cos \beta z \sin \beta z + 2n \sin \beta z \cos \beta z \right) \left[-m \cos^2(\beta z) + n \sin^2(\beta z) \right]^{-1/2} = 0$$

$$\text{or, } -2m \sin(\beta z) \cos(\beta z) + 2n \sin(\beta z) \cos(\beta z) = 0$$

$$\text{or, } -m \sin 2\theta + n \sin 2\theta = 0$$

$$\text{or, } 2 \sin \theta \cos \theta = 0$$

$$\sin \theta = 0$$

$$\theta = 0, \pm \pi, \pm 2\pi, \pm 3\pi, \dots$$

$$\theta = n\pi, \quad n = 0, 1, 2, 3, \dots$$

$$\cos \theta = 0$$

$$\theta = \frac{n\pi}{2}, \quad n = 1, \pm 3, \pm 5, \dots$$

$$V_{\max} = V_+ e^{-\alpha z} + V_- e^{\alpha z}$$

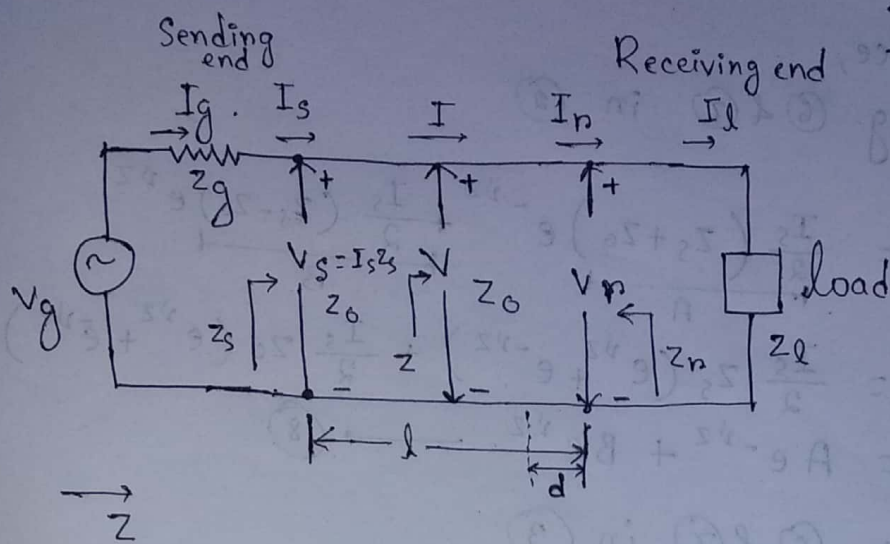
$$= V_+ e^{-\alpha z} (1 + |\Gamma|)$$

$$V_{\min} = V_+ e^{-\alpha z} - V_- e^{\alpha z}$$

$$= V_+ e^{-\alpha z} (1 - |\Gamma|)$$

$$\text{Voltage standing wave ratio} = \frac{|V_{\max}|}{|V_{\min}|}$$

$$V_{\text{SWR}} = \frac{1 + |\Gamma|}{1 - |\Gamma|}$$



Line Impedance

$$Z = \frac{V(z)}{I(z)} \rightarrow (1)$$

V_+ = magnitude of incident vol

V_- = magnitude of

$$V = V_+ e^{-\gamma z} + V_- e^{\gamma z} \rightarrow (2)$$

$$I = \frac{1}{Z_0} (V_+ e^{-\gamma z} - V_- e^{\gamma z}) \rightarrow (3)$$

Y_0 = admittance

$$= Y_0 (V_+ e^{-\gamma z} - V_- e^{\gamma z}) \quad Y_0 = \frac{1}{Z_0}$$

$$Z = \frac{(V_+ e^{-\gamma z} + V_- e^{\gamma z})}{Y_0 (V_+ e^{-\gamma z} - V_- e^{\gamma z})}$$

At source, $z=0$

$$V_s = V_+ + V_- = I_s Z_s \rightarrow (4)$$

$$\begin{aligned} V_+ + V_- &= V_s \\ V_+ - V_- &= \frac{I_s}{Y_0} \end{aligned}$$

$$I_s = Y_0 (V_+ - V_-) \rightarrow (5)$$

$$V_+ = \frac{1}{2} \left(V_s + \frac{I_s}{Y_0} \right)$$

$$V_+ = \frac{1}{2} I_s (Z_s + Z_0) \rightarrow (6)$$

$$V_- = \frac{1}{2} \left(V_s - \frac{I_s}{Y_0} \right)$$

$$V_- = \frac{1}{2} I_s (Z_s - Z_0) \rightarrow (7)$$

$$V_+ = \frac{1}{2} I_s \left(Z_s + \frac{1}{Y_0} \right)$$

$$V_- = \frac{1}{2} I_s \left(Z_s - \frac{1}{Y_0} \right)$$

$$Z_0 = \frac{1}{Y_0}$$

at source,
Putting (6) & (7) in (2)

at source,
Putting (6) & (7) in (2)

$$V_s = \frac{I_s}{2} (z_s + z_0) e^{-\gamma z} + \frac{I_s}{2} (z_s - z_0) e^{\gamma z}$$

$$V = \frac{I_s}{2} z_s (e^{\gamma z} + e^{-\gamma z}) + \frac{I_s}{2} z_0 (e^{\gamma z} - e^{-\gamma z})$$

$$V_s = A e^{-\gamma z} + B e^{\gamma z} \longrightarrow (8)$$

Putting ⑥ & ⑦ in ③

$$I_s = \frac{1}{Z_0} \left[\frac{I_s}{2} (Z_s + Z_0) e^{-\gamma z} - \frac{I_s}{2} (Z_s - Z_0) e^{\gamma z} \right]$$

$$= \frac{1}{z_0} \left[\frac{1}{z_s} \left(e^{-\gamma z} - e^{\gamma z} \right) \right]$$

$$I_s = \frac{1}{Z_0} [A e^{-\gamma z} - B e^{\gamma z}] \longrightarrow (9)$$

line impedance,

at source

$$Z_s = \frac{V_s}{I_s} = Z_0 \cdot \frac{Ae^{-\gamma z} + Be^{\gamma z}}{Ae^{-\gamma z} - Be^{\gamma z}}$$

For total length (L),

$$Z = Z_0 \frac{(Ae^{-\gamma l} + Be^{\gamma l})}{(Ae^{-\gamma l} - Be^{\gamma l})} \leftarrow (\text{line impedance at receiving end})$$

line impedance at any point,

$$Z = Z_0 \frac{(Ae^{-\gamma(l-d)} + Be^{\gamma(l-d)})}{(Ae^{-\gamma(l-d)} - Be^{\gamma(l-d)})}$$

$$= Z_0 \left(\frac{Ae^{-\gamma l} \cdot e^{\gamma d} + Be^{\gamma l} \cdot e^{-\gamma d}}{Ae^{-\gamma l} \cdot e^{\gamma d} - B \cdot e^{\gamma l} \cdot e^{-\gamma d}} \right)$$