image or a graph creates a long lasting impression on mind than text or an equation. A graphical representation of a transmission line helps in pictorial visualization of some of the basic concepts. One can use the graphical means for solving transmission line problems and at times they are also easier as compared to analytical means. However, that is not the whole purpose of graphical representation. A graphical representation provides a good account of tranmission line characteristics in a compact manner. Even while solving transmission line problems analytically, a qualitative cross-check with the graphical model is always helpful in avoiding conceptual mistakes.

2.9 IMPEDANCE SMITH CHART

The graphical representation describes the impedance/admittance characteritics of a transmission line. For the time being let us confine our analysis to passive impedances only. Later we will extend it to the admittances. As we have seen earlier, all impedance expressions can be written in terms of normalized impedances. Let us therefore carry out the analysis in terms of

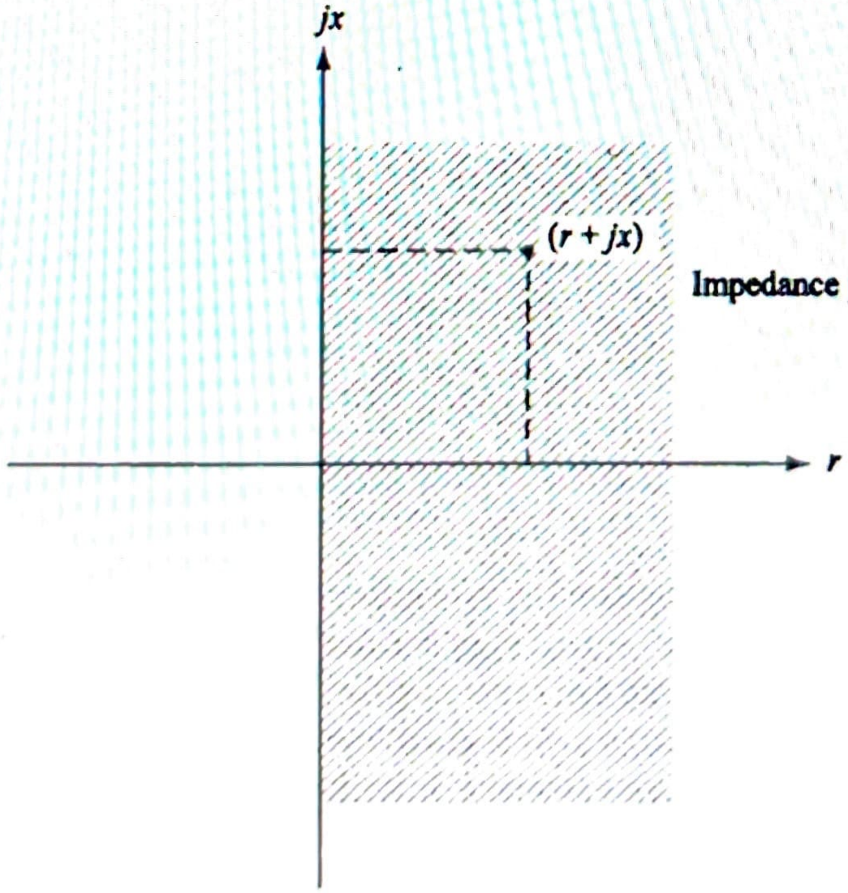

Fig. 2.12 *Complex impedance plane.*

normalized impedances. An impedance $Z = R + jX$ when normalized with the characteristic impedance Z_0, is denoted by $\overline{Z} = r + jx$ where $\overline{Z} = Z/Z_0$, $r = R/Z_0$ and $x = X/Z_0$. The reflection coefficient for a normalized impedance $\overline{Z}$ is then given as

$$\Gamma = \frac{Z - Z_0}{Z + Z_0} = \frac{\overline{Z} - 1}{\overline{Z} + 1} \tag{2.121}$$

$$= \frac{r + jx - 1}{r + jx + 1} = \frac{(r-1) + jx}{(r+1) + jx} \tag{2.122}$$

For passive loads, r lies between 0 and ∞ and x lies between $-\infty$ and $+\infty$. A passive load $r + jx$ therefore can be represented by a point in the right half of the complex plane including the imaginary axis as shown in Fig. 2.12.

The reflection coefficient Γ also is complex in general and can be written as

$$\Gamma \equiv u + jv \equiv Re^{j\theta} \tag{2.123}$$

We can note from Eqn (2.121) that there is one-to-one correspondance between $\overline{Z}$ and Γ, and for a normalized impedance $r + jx$ we obtain a unique complex reflection coefficient Γ. Moreover, the magnitude of the reflection coefficient $|\Gamma|$ is always less than or equal to unity. The possible values of Γ are therefore confined within the unit circle in the complex Γ plane as shown in Fig. 2.13.

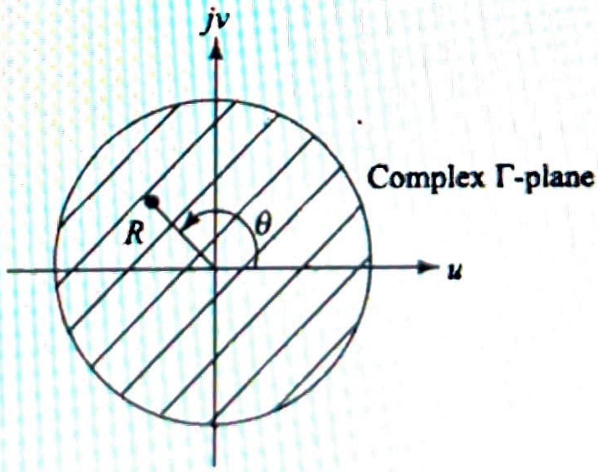

Fig. 2.13 *Complex reflection coefficient plane.*

We, therefore, see that the semi-infinite impedance plane is mapped to the area within the unit circle in the Γ-plane with one to one correspondence between the points in the two planes. One can show that the transformation from $\overline{Z}$-plane to the Γ- plane and vice-versa is a conformal transformation. Let us map a point $\overline{Z} = r + jx$ onto the $\Gamma = u + jv$ plane. Inverting Eqn (2.121) we have

$$\overline{Z} = \frac{1 + \Gamma}{1 - \Gamma} \tag{2.124}$$

$$\Rightarrow \qquad r + jx = \frac{1 + (u + jv)}{1 - (u + jv)} \tag{2.125}$$

Separating the real and the imaginary parts we get two equations of u and v, one in terms of r and other in terms of x, as follows.

$$u^2 - 2\left(\frac{r}{r+1}\right)u + v^2 + \left(\frac{r-1}{r+1}\right) = 0 \tag{2.126}$$

$$u^2 + v^2 - 2u - \left(\frac{2}{x}\right)v + 1 = 0 \tag{2.127}$$

It is interesting to note that both Eqns (2.126) and (2.127) represent circles in the Γ-plane. Since r = constant represents a vertical line in the Z-plane, Eqn (2.126) transforms vertical lines in the Z-plane into circles in the Γ-plane. These circles are called constant resistance circles. Similarly the lines x = constant map into circles in the Γ-plane as given by Eqn (2.127). These are called the constant reactance circles. Remember that only those portions of the circles are of relevance which lie within the unit circle in the Γ-plane. Let us now analyse the characteristics of the transformed circles.

2.9.1 Constant Resistance Circles

The constant resistance circles have their centres at $(\frac{r}{r+1}, 0)$ and radii $(\frac{1}{r+1})$. Figure 2.14 shows the constant resistance circles for different values of r ranging between 0 and ∞.

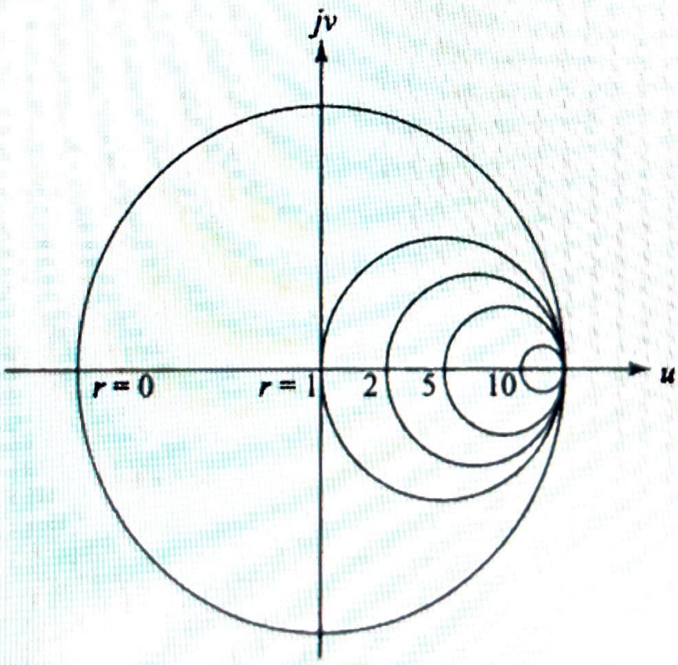

Fig. 2.14 *Constant resistance circles in the complex Γ-plane.*

We can note following things about the constant resistance circles.

(a) The circles always have centres on the real Γ-axis (u-axis).

(b) All circles pass through the point(1, 0) in the complex Γ plane.

(c) For $r = 0$ the center lies at the origin of the Γ plane and it shifts to the right as r increases.

(d) As r increases the radius of the circle goes on reducing and for $r \to \infty$ the radius approaches zero, i.e. the circle reduces to a point.

(e) The outermost circle with center (0, 0) and radius unity, corresponds to $r = 0$ or in other words represents reactive loads only.

(f) The right most point on the unit circle (1, 0) represents $r = 0$ as well as $r = \infty$.

2.9.2 Constant Reactance Circles

The constant reactance circles have their centers at $(1, \frac{1}{x})$ and radii $(\frac{1}{x})$. The centres for these circles lie on a vertical line passing through (1, 0) point in the Γ-plane. The constant reactance circles are shown in Fig. 2.15 for different values of x.

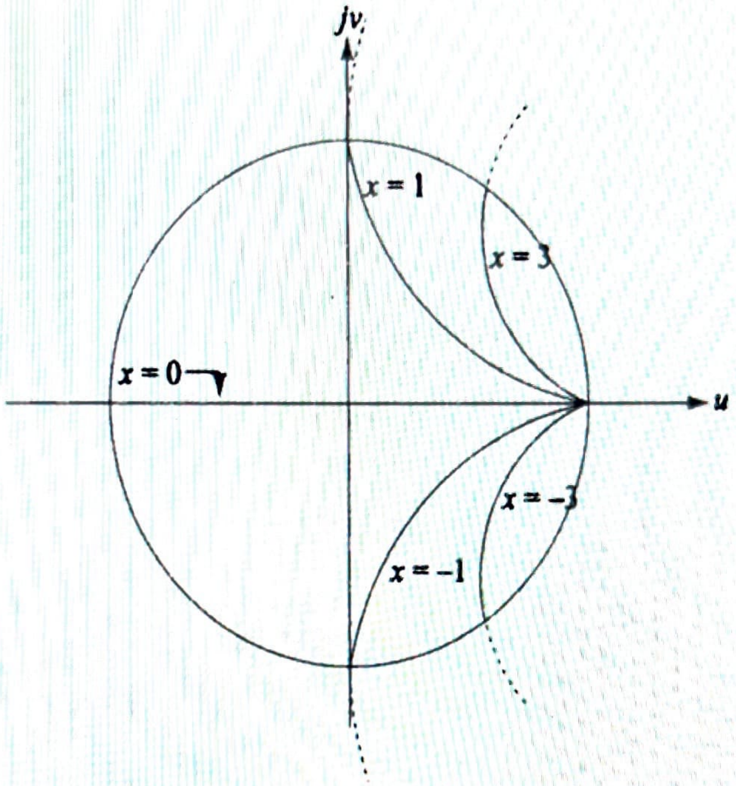

Fig. 2.15 *Constant reactance circles in the complex Γ-plane.*

Note again that only those portions of the circles are of significance which lie within the unit circle in the Γ-plane. The curves shown dotted do not correspond to any passive load impedance. We can note the following points about constant reactance circles:

(a) These circles have their centers on a vertical line passing through point (1, 0).

(b) For positive x the center lies above the real Γ-axis and for negative x, the center lies below the real Γ-axis.

(c) For $x = 0$, the center is at $(1, \pm\infty)$ and radius is ∞. This circle therefore represents a straight line.

(d) As the magnitude of the reactance increases the center moves towards the real Γ-axis and it lies on the real Γ-axis at (1,0) for $x = \pm\infty$.

(e) As the magnitude of the reactance increases, the radius of the circle $(\frac{1}{x})$ decreases and it approaches zero as $x \to \pm\infty$.

(f) All circles pass through the point (1, 0).

(g) The real Γ-axis (u-axis) corresponds to $x = 0$ and therefore represents real load impedances, i.e. purely resistive impedances.

(h) The right most point on the unit circle, $(1, 0)$, corresponds to $x = 0$ as well as $x = \pm\infty$.

2.9.3 The Smith Chart

The Smith chart is a graphical figure which is obtained by superposing the constant resistance and the constant reactance circles within the unit circle in the complex Γ-plane. A Smith chart is shown in Fig. 2.16. Since we have mapped here the impedances to the Γ-plane, let us call this the Impedance Smith chart.

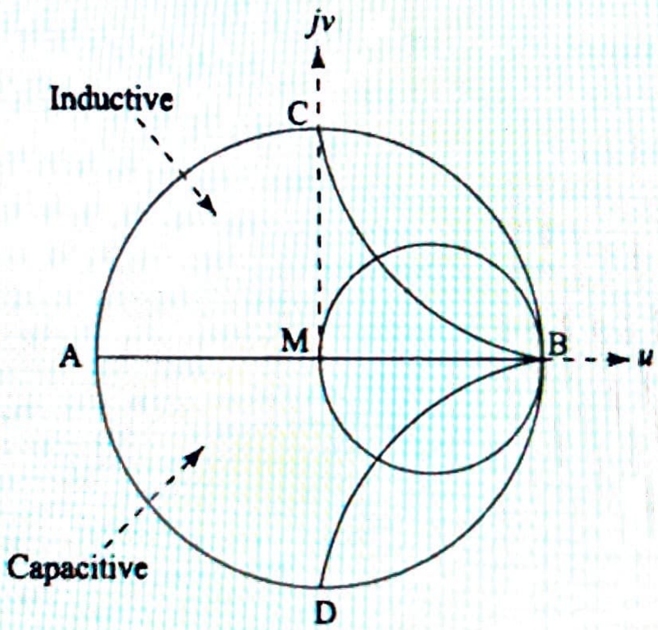

Fig. 2.16 *Smith chart: Superposition of constant resistance and constant reactance circles in the complex Γ-plane.*

Generally the u, v axes are not drawn on the Smith chart. However one should not forget that the Smith chart is a figure which is drawn on the complex Γ-plane with its center as origin. The intersection of constant resistance and constant reactance circles uniquely defines a complex load impedance in the Γ-plane. Let us identify some special points on the Smith Chart.

(a) The left most point A on the Smith chart corresponds to $r = 0, x = 0$ and therefore represents ideal short-circuit load.

(b) The right most point B on the Smith chart corresponds to $r = \infty, x = \infty$ and therefore represents ideal open circuit load.

(c) The center of the Smith chart M , corresponds to $r = 1, x = 0$ and hence represents the matched load.

(d) Line AB represents pure resistive loads and the outermost circle passing through A and B represents pure reactive loads.

(e) The upper most point C represents a pure inductive load of unity reactance and the lower most point D represents a pure capacitive load of unity reactance.

(f) In general the upper half of the Impedance Smith chart represents the complex inductive loads and the lower half represents the complex capacitive loads.

2.9.4 Constant VSWR Circles

We have seen earlier that the voltage reflection coefficient at any location l from the load is given as (see Eqn (2.63))

$$\Gamma(l) = \Gamma_L e^{-j2\beta l} \tag{2.128}$$

where Γ_L is the complex reflection coefficient at the load and is given by Eqn (2.37). Let the Γ_L be represented in the polar form as

$$\Gamma_L = |\Gamma_L| e^{j\theta_L} \tag{2.129}$$

Then the reflection coefficient $\Gamma(l)$ will be

$$\Gamma(l) \equiv Re^{j\theta} = |\Gamma_L| e^{j(\theta_L - 2\beta l)} \tag{2.130}$$

The magnitude of the reflection coefficient is

$$R \equiv |\Gamma(l)| = |\Gamma_L| \tag{2.131}$$

and the phase of the reflection coefficient is

$$\theta \equiv \theta_L - 2\beta l \tag{2.132}$$

As we change the value of l, i.e. as we move along the tranmission line the magnitude of $\Gamma(l)$ remains same (R = constant) but its phase varies linearly. As l increases, i.e. as we move towards the generator, the phase of the reflection coefficient $\Gamma(l)$ becomes more negative. In the complex Γ-plane therefore a point $Re^{j\theta}$ moves clockwise on a circle with center at (0,0) and radius $|\Gamma_L|$ as shown in Fig. 2.17.

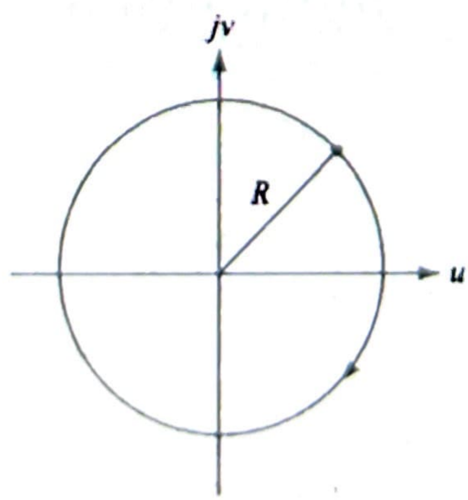

Fig. 2.17

All points on this circle have same $|\Gamma| = |\Gamma_L|$. Now since the VSWR on the line is

$$\rho = \frac{1 + |\Gamma_L|}{1 - |\Gamma_L|} \tag{2.133}$$

all the points on this circle have same VSWR. Hence, these circles are called the 'constant VSWR circles'. The constant VSWR circles are shown in Fig 2.18.

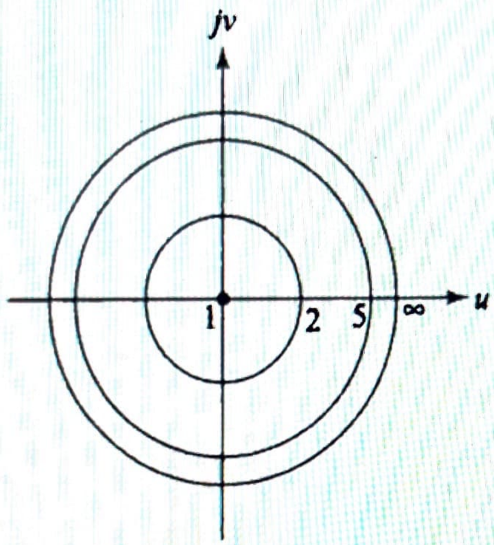

Fig. 2.18 *Constant VSWR circles drawn in the complex Γ-plane.*

We can make following observations about the constant VSWR circles:

(a) All the circles have same center, the origin of the complex Γ-plane.

(b) The origin in the Γ-plane represents $|\Gamma_L| = 0$ or $\rho = 1$. As we move radially outwards the $|\Gamma_L|$ and hence ρ increases monotonically and for the outermost unity circle, $|\Gamma_L| = 1$ and $\rho = \infty$.

(c) The origin corresponds to the condition $|\Gamma_L| = 0$, i.e. no reflection on the line. This point represents the best matching of the load as there is no reflected power on the line. For the outer most circle, since $|\Gamma_L| = 1$, we get the worst impedance matching as the entire power is reflected on the transmission line. We can therefore make a general statement that closer is the point to the origin of the Γ-plane, (i.e. the center of the Smith chart) better is the impedance matching.

(d) As we have defined earlier, the $+l$ indicates a distance towards the generator. As l becomes more positive, θ decreases and the point moves clockwise on the constant VSWR circle. If we move away from the generator, l becomes negative and then the point on the circle moves in the anticlockwise direction.

For analysing a transmission line problem graphically, the constant VSWR circles are to be superimposed or drawn on the Smith chart. For the sake of visual clarity the constant VSWR circles are not permanently drawn on the Smith chart. As and when required the user draws an appropriate constant VSWR circle on the Smith chart.