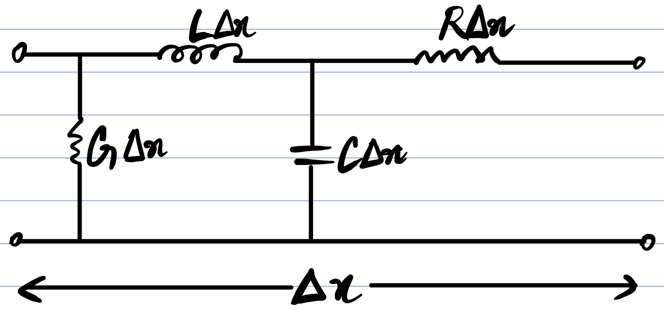
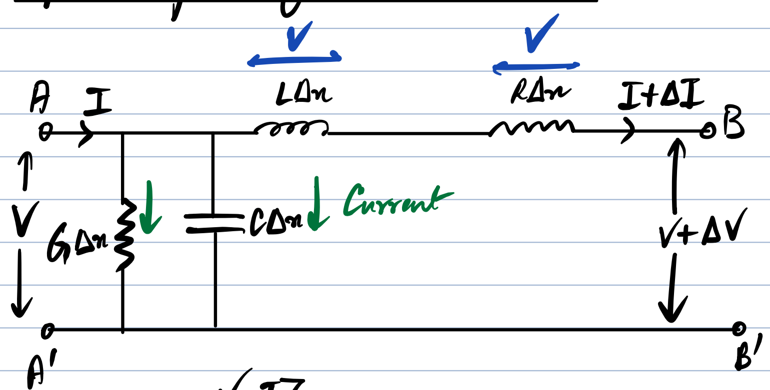


All possible combinations are possible.

i) C (Parallel)
ii) R (Series)
iii) L (Series)
iv) G (Parallel)

'Primary Constants of the line'

Equations of Voltage and Current



$V = IZ$

$$\Delta V = -(R\Delta x + j\omega L\Delta x)I \quad \text{--- (1)}$$

$$\Delta I = -(G\Delta x + j\omega C\Delta x)V \quad \text{--- (2)}$$

$$\frac{\Delta V}{\Delta x} = -(R+j\omega L)I \quad \text{--- (3)}$$

$$\frac{\Delta I}{\Delta x} = -(G+j\omega C)V \quad \text{--- (4)}$$

$\because \Delta x \to 0 \Rightarrow \dfrac{\Delta V}{\Delta x} = \dfrac{dV}{dx}$

$\dfrac{\Delta I}{\Delta x} = \dfrac{dI}{dx}$

$$\frac{dV}{dx} = -(R+j\omega L)I \quad \text{--- (5)}$$

$$\frac{dI}{dx} = -(G+j\omega C)V \quad \text{--- (6)}$$

$\dfrac{d}{dx}$

$$\frac{d^2V}{dx^2} = -(R+j\omega L)\frac{dI}{dx} \quad \text{--- (7)}$$

$$\Rightarrow \boxed{\frac{d^2V}{dx^2} = (R+j\omega L)(G+j\omega C)V} \quad \text{--- (8)}$$

$$\boxed{\frac{d^2V}{dx^2} = \gamma^2 V} \quad (9)$$

$\gamma \rightarrow$ Propagation Constant

Similarly, $\boxed{\frac{d^2I}{dx^2} = \gamma^2 I}$ (10)

(9) & (10) $\rightarrow$ Homogeneous equs. with constant coeff.

$$V = V^+e^{-\gamma x} + V^-e^{\gamma x} \quad (12)$$

$$\& \quad I = I^+e^{-\gamma x} + I^-e^{\gamma x} \quad (13)$$

$$V(t) = \left(V^+e^{-\gamma x} + V^-e^{\gamma x}\right)e^{j\omega t}$$

$$\Rightarrow V(t) = \left(V^+e^{-\gamma x + j\omega t} + V^-e^{\gamma x + j\omega t}\right)$$

$$\& \quad I(t) = \left(I^+e^{-\gamma x + j\omega t} + I^-e^{\gamma x + j\omega t}\right)$$

$\because \gamma$ is a complex no. so $\gamma = \alpha + j\beta$

$$V(t) = \left(V^+e^{-(\alpha + j\beta)x + j\omega t} + V^-e^{(\alpha + j\beta)x + j\omega t}\right)$$

$$\Rightarrow V(t) = \left[V^+e^{-\alpha x}e^{j(-\beta x + \omega t)} + V^-e^{\alpha x}e^{j(\beta x + \omega t)}\right] \quad (16)$$

$$\& \quad I(t) = \left[I^+e^{-\alpha x}e^{j(-\beta x + \omega t)} + I^-e^{\alpha x}e^{j(\beta x + \omega t)}\right] \quad (17)$$

$\alpha \rightarrow$ Attenuation const.

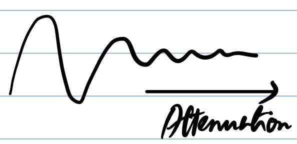
$\beta \rightarrow$ Phase const.

$\beta = \frac{2\pi}{\lambda}$

$\angle$	dist
2π	$\rightarrow \lambda$
$\lambda \rightarrow 2\pi$	

$\boxed{\beta = 2\pi/\lambda}$

$V^+e^{-\alpha x} \rightarrow (+)$ve x dirn.



$$\frac{dV}{dx} = -(R+j\omega L)I \quad \text{——} \; (5)$$

$$\frac{dI}{dx} = -(G+j\omega C)V \quad \text{——} \; (6)$$

$$V = V^+e^{-\gamma x} + V^-e^{\gamma x} \quad \text{——} \; (12)$$

$$I = I^+e^{-\gamma x} + I^-e^{\gamma x} \quad \text{——} \; (13)$$

$$\frac{d}{dx}\left(V^+e^{-\gamma x} + V^-e^{\gamma x}\right) = -(R+j\omega L)\left(I^+e^{-\gamma x} + I^-e^{\gamma x}\right)$$

$$\Rightarrow -\gamma V^+e^{-\gamma x} + \gamma V^-e^{\gamma x} = -(R+j\omega L)I^+e^{-\gamma x} - (R+j\omega L)I^-e^{\gamma x}$$

$e^{-\gamma x}$

$$\cancel{-}\gamma V^+\cancel{e^{-\gamma x}} = \cancel{-}(R+j\omega L)I^+\cancel{e^{-\gamma x}}$$

$$\Rightarrow \frac{V^+}{I^+} = \frac{(R+j\omega L)}{\gamma} = Z_0$$

$$\Rightarrow Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}}$$

$e^{\gamma x}$

$$\gamma V^-\cancel{e^{\gamma x}} = -(R+j\omega L)I^-\cancel{e^{\gamma x}}$$

$$\Rightarrow \frac{V^-}{I^-} = \frac{-(R+j\omega L)}{\gamma} = -Z_0$$

$$\Rightarrow \frac{V^-}{I^-} = -Z_0 = -\sqrt{\frac{R+j\omega L}{G+j\omega C}}$$

$$\gamma^2 = (G+j\omega C)(R+j\omega L) \Rightarrow \gamma = \sqrt{(G+j\omega C)(R+j\omega L)}$$

$$Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}} \quad //$$

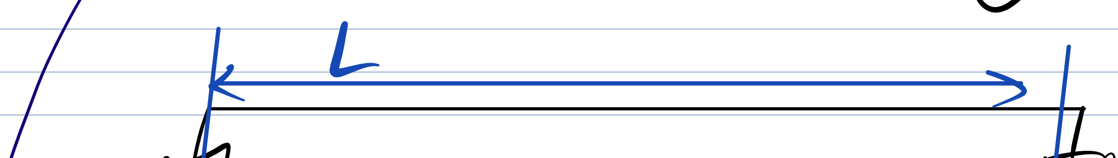
Characteristic Impedance

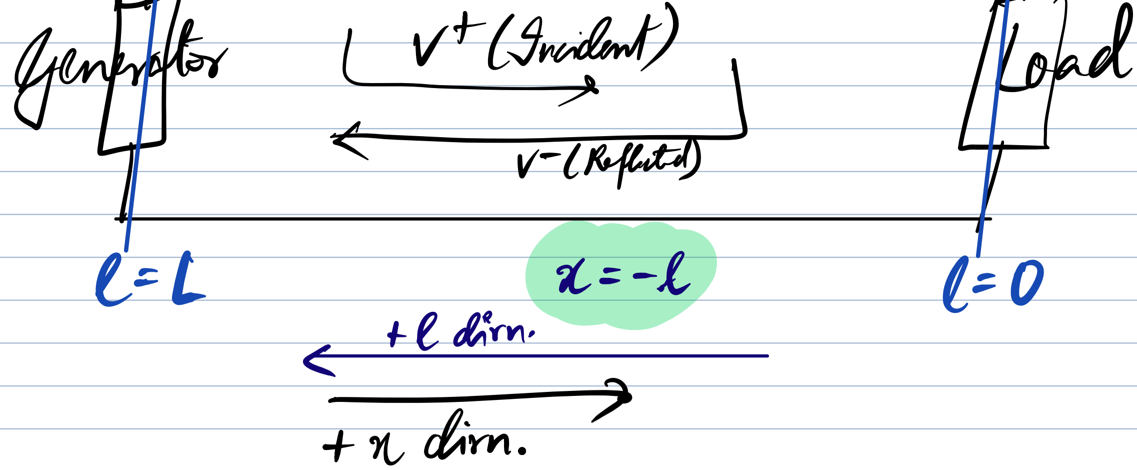
$$\boxed{\frac{V^+}{I^+} = Z_0 \quad \& \quad \frac{V^-}{I^-} = -Z_0} \rightarrow I^+ = \frac{V^+}{Z_0} \quad \& \quad I^- = \frac{-V^-}{Z_0}$$

Now,

$$V = V^+e^{-\gamma x} + V^-e^{\gamma x} \quad \text{——} \; (27)$$

$$I = I^+e^{-\gamma x} + I^-e^{\gamma x} = \frac{V^+}{Z_0}e^{-\gamma x} - \frac{V^-}{Z_0}e^{\gamma x} \quad \text{——} \; (28)$$





$$V(l) = V^+ e^{\gamma l} + V^- e^{-\gamma l} \quad \text{—— (29)}$$

$$I(l) = I^+ e^{\gamma l} + I^- e^{-\gamma l} \quad \text{—— (30)}$$

Reflection Coefficient, $\Gamma(l) = \dfrac{V^- e^{-\gamma l}}{V^+ e^{\gamma l}} = \left(\dfrac{V^-}{V^+}\right) e^{-2\gamma l}$ —— (31)

$$V(l) = V^+ e^{\gamma l}\left[1 + \frac{V^- e^{-\gamma l}}{V^+ e^{\gamma l}}\right]$$

$$V(l) = V^+ e^{\gamma l}(1 + \Gamma(l))$$

$$I(l) = \frac{V^+}{Z_0} e^{\gamma l} - \frac{V^-}{Z_0} e^{-\gamma l} = \frac{V^+}{Z_0} e^{\gamma l}\left[1 - \frac{V^-/Z_0 \, e^{-\gamma l}}{V^+/Z_0 \, e^{\gamma l}}\right]$$

$$I(l) = \frac{V^+}{Z_0} e^{\gamma l}\left[1 - \Gamma(l)\right]$$

$$V(l) = V^+ e^{\gamma l}(1 + \Gamma(l)) \quad \text{—— (32)}$$

$$I(l) = \frac{V^+}{Z_0} e^{\gamma l}(1 - \Gamma(l)) \quad \text{—— (33)}$$

$$\boxed{Z(l) = \frac{V(l)}{I(l)} = Z_0\left(\frac{1+\Gamma(l)}{1-\Gamma(l)}\right)}$$

Impedance at any point 'l' on the Tx Line starting from the Load End.

Mathematics yeilds

$$\boxed{\Gamma(l) = \frac{Z(l) - Z_0}{Z(l) + Z_0}}$$

Reflection Coeff. at any pt. 'l'

$l = 0 \longrightarrow$ Load End $\rightarrow \Gamma(l=0) \equiv \Gamma_L$

$$\boxed{\Gamma(0) = \Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}}$$

$$\Gamma(l) = \left(\frac{V^- e^{-\gamma l}}{V^+ e^{\gamma l}}\right) \Rightarrow \boxed{\Gamma_L = \Gamma(0) = \frac{V^-}{V^+}}$$

$$\Rightarrow \Gamma(l) = \Gamma_L \frac{e^{-\gamma l}}{e^{\gamma l}} = \Gamma_L e^{-2\gamma l}$$

$$V(l) = V^+ e^{\gamma l}[1 + \Gamma(l)]$$

$$\Rightarrow V(l) = V^+ e^{\gamma l}[1 + \Gamma_L e^{-2\gamma l}]$$

$$I(l) = \frac{V^+}{Z_0} e^{\gamma l}[1 - \Gamma(l)]$$

$$= \frac{V^+}{Z_0} e^{\gamma l}[1 - \Gamma_L e^{-2\gamma l}]$$

Impedance Transformation Equation

$$Z(l) = Z_0\left[\frac{1 + \Gamma_L e^{-2\gamma l}}{1 - \Gamma_L e^{-2\gamma l}}\right]\left(\frac{\times e^{\gamma l}}{\times e^{\gamma l}}\right) \Rightarrow \boxed{Z(l) = Z_0\left[\frac{Z_L \cosh\gamma l + Z_0 \sinh\gamma l}{Z_L \sinh\gamma l + Z_0 \cosh\gamma l}\right]}$$

Reflection Coefficient, $\Gamma(l) = \dfrac{V^- e^{-\gamma l}}{V^+ e^{\gamma l}} = \left(\dfrac{V^-}{V^+}\right) e^{-2\gamma l}$ —— (31)

$$\Gamma(l) = \frac{Z(l) - Z_0}{Z(l) + Z_0} \quad \text{—— (32)}$$

Reflection Coeff. at $l=0$ (i.e Load End), $\boxed{\Gamma(0) = \Gamma_L = \dfrac{Z_L - Z_0}{Z_L + Z_0}}$

Transmission Coefficient, $\boxed{T = \dfrac{2Z_L}{Z_L + Z_0}}$

$\boxed{T = 1 + \Gamma}$

$Z_L \rightarrow$ Load Impedance

$Z_0 \rightarrow$ Characteristic Impedance

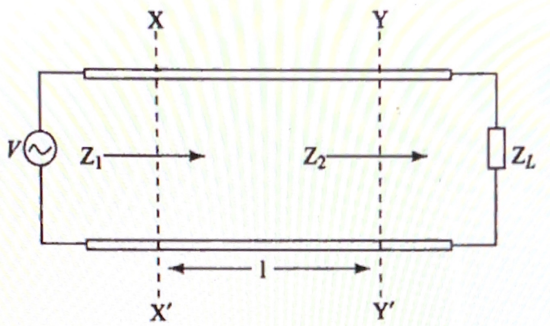


Fig. 2.8 *Impedance transformation on a transmission line.*

It should be noted here that although Eqn (2.45) gives the transformation of the load impedance, there is nothing special about the load impedance. We obtained transformation of load impedance because we defined $l = 0$ at the load end. If we define $l = 0$ at some other location on the line we get transformation of the impedance from that point. We can therefore generalise the impedance transformation equation for any two points on the transmission line.

Let the impedances measured at two locations on the line be Z_1 and Z_2 respectively as shown in Fig. 2.8.

Then from Eqn (2.45) we get

$$Z_1 = Z_0 \left[\frac{Z_2 \cosh \gamma l + Z_0 \sinh \gamma l}{Z_2 \sinh \gamma l + Z_0 \cosh \gamma l}\right] \quad (2.46)$$

If we invert Eqn (2.46) we get

$$Z_2 = Z_0 \left[\frac{Z_1 \cosh \gamma l - Z_0 \sinh \gamma l}{-Z_1 \sinh \gamma l + Z_0 \cosh \gamma l}\right] \quad (2.47)$$

Since, $\sinh(-\gamma l) = -\sinh \gamma l$ and $\cosh(-\gamma l) = \cosh \gamma l$, Eqn (2.47) can be re-written as

$$Z_2 = Z_0 \left[\frac{Z_1 \cosh(-\gamma l) + Z_0 \sinh(-\gamma l)}{Z_1 \sinh(-\gamma l) + Z_0 \cosh(-\gamma l)}\right] \quad (2.48)$$

From Eqns (2.46) and (2.48) it is evident that the two expressions are identical

Loss-Less And Low-Loss Transmission Lines

We know, propagation constant, $\gamma = \sqrt{(R+j\omega L)(G+j\omega C)}$

and characteristic impedance, $Z_0 = \sqrt{\dfrac{R+j\omega L}{G+j\omega C}}$

For Loss-Less Tx line ⇒ No Resistance & Conductance

So, $R=0$ & $G=0$

(i) $\gamma = \sqrt{(j\omega L).(j\omega C)} = j\omega\sqrt{LC}$

(ii) $Z_0 = \sqrt{\dfrac{j\omega L}{j\omega C}} = \sqrt{\dfrac{L}{C}}$

Characteristics of Loss-less

i) $Z(l+\lambda/2) = Z(l)$

ii) $Z(l+\lambda/4) = \dfrac{1}{Z(l)}$

iii) If $Z_L = Z_0 \Rightarrow Z(l) = Z_0$

For Low-Loss Transmission Line

$$\gamma = \alpha + j\beta = \sqrt{(R+j\omega L)(G+j\omega C)}$$

$$= \sqrt{j\omega L\left\{1-j\frac{R}{\omega L}\right\} j\omega C\left\{1-j\frac{G}{\omega C}\right\}}$$

$$\gamma = \alpha + j\beta = j\omega\sqrt{LC}\left\{1-j\frac{R}{2\omega L}\right\}\left\{1-j\frac{G}{2\omega C}\right\}$$

On further simplification

$$\gamma = \alpha + j\beta = j\omega\sqrt{LC}\left\{1-j\frac{R}{2\omega L}-j\frac{G}{2\omega C}\right\}$$

So, $\boxed{\alpha = \dfrac{R}{2}\sqrt{\dfrac{C}{L}} + \dfrac{G}{2}\sqrt{\dfrac{L}{C}}}$ for Low-Loss Tx Line

$\boxed{\beta = \omega\sqrt{LC}}$

Voltage Standing Wave Ratio (VSWR)

Now, $V(l) = V^+ e^{\gamma l}\left(1+\Gamma(l)\right)$

$I(l) = \dfrac{V^+}{Z_0} e^{\gamma l}\left(1-\Gamma(l)\right)$

and $\Gamma(l) = \dfrac{V^-}{V^+} e^{-2\gamma l} = \dfrac{Z(l)-Z_0}{Z(l)+Z_0}$

$\therefore \Gamma(l) = \Gamma_L e^{-2\gamma l}$

Now, $\Gamma_L = \Gamma(0) = \dfrac{Z_L - Z_0}{Z_L + Z_0} = \dfrac{V^-}{V^+}$

$\therefore V(l) = V^+ e^{\gamma l}\left(1+\Gamma_L e^{-2\gamma l}\right)$ → $|V|_{max} = |V^+|\left(1+|\Gamma_L|\right)$; $|V|_{min} = |V^+|\left(1-|\Gamma_L|\right)$

$I(l) = \dfrac{V^+}{Z_0} e^{\gamma l}\left(1-\Gamma_L e^{-2\gamma l}\right)$ → $|I|_{max} = \left|\dfrac{V^+}{Z_0}\right|\left(1+|\Gamma_L|\right) = \dfrac{|V|_{max}}{Z_0}$; $|I|_{min} = \left|\dfrac{V^+}{Z_0}\right|\left(1-|\Gamma_L|\right) = \dfrac{|V|_{min}}{Z_0}$

Voltage Standing Wave Ratio (VSWR), $\rho = \dfrac{|V|_{max}}{|V|_{min}}$ (ρ is always > 1 $\because |V|_{max} > |V|_{min}$)

$$\Rightarrow \boxed{\rho = \frac{1+|\Gamma_L|}{1-|\Gamma_L|}}$$

when $|\Gamma_L| = 0 \Rightarrow \rho = 1$

when $|\Gamma_L| = 1 \Rightarrow \rho = \infty$

$$\Rightarrow \boxed{|\Gamma_L| = \frac{\rho-1}{\rho+1}}$$

Power Transfer of a Transmission Line, $\boxed{P_L = \dfrac{|V^+|^2}{2Z_0}\left\{1-|\Gamma_L|^2\right\}}$