

Information bits = K

$$i = [i_1, i_2, i_3, \dots, i_K]$$

Parity bits / Redundant bits = R

$$P = [P_1, P_2, P_3, \dots, P_R]$$

Total no. of bits of code = $n = K + R$

$$m = [i_1, i_2, i_3, \dots, i_K, P_1, P_2, P_3, \dots, P_R]$$

(n, K) is block code representation.

Total code words required as per n Block codes = 2^n

Total code words required as per K information = 2^K

Total redundant code words required as per R parity bits = $2^R - 2^K$

$$\therefore \text{Code rate} = R = \frac{K}{n}$$

Code word, $C = [i, P]$

$$C = [i_1, i_2, i_3, \dots, i_K, P_1, P_2, P_3, \dots, P_R]$$

Error code word:

$$e = [e_1, e_2, e_3, \dots, e_n]$$

where $e_j = 1$ means error, $e_j = 0$ means no error.

Valid data = received codeword + error code word.

Q1) Given the (5,4) even parity block code, find codewords corresponding to $I = (1011)$ and (1010)

$$k=4, n=k+4=5$$

For Information $I = 1011 \rightarrow I_1 I_2 I_3 I_4$

For even parity, $P = I_1 + I_2 + I_3 + I_4$

$$P = 1 + 0 + 1 + 1 = 1$$

$$P = 1$$

So codeword = [Information, parity]

$$C = [i_1, i_2, i_3, i_4, P]$$

$$C = [1, 0, 1, 1, 1]$$

For Information $I = 1010 \rightarrow I_1 I_2 I_3 I_4$

For even parity, $P = I_1 + I_2 + I_3 + I_4$

$$P = 1 + 0 + 1 + 0$$

$$P = 0$$

$$\text{Codeword } C = [1, 0, 1, 0, 0]$$

Q2) Given the (8,7), even parity block code, determine whether $v_1 = (10110110)$ and $v_2 = (01101001)$ gives parity failure.

$$v_1 = (10110110)$$

$$\text{check sum } S = 1+0+1+1+0+1+0+0 = 6$$

For Even received codeword, it is ~~the~~ the parity failure.

Means there is error in received data.

$$v_2 = (01101001)$$

$$\text{check sum } S = 0+1+1+0+1+0+0+1 \\ S = 6$$

For Even received codeword, it is parity success

Means there is no error in received data.

Definition and Basics for Block codes for Product.

- As we have seen single parity check codes have no error correction capability.
- However error correction can be achieved by combining two single parity check codes in the form of rectangular array.

Example of Block Code for Product code

Data bits				Row parity check	:
↑	1	1	0	1	+
6	0	1	1	0	0
	1	0	0	0	1
	0	0	0	0	0
	1	1	1	0	1
	1	0	0	1	0
Column parity check				Overall parity check	
	0	1	0	0	1

out of 35 bits 24 are information bits

so given block code is $(35, 24)$

It is used to detect & correct one bit error

Example of Block Code for Repetition code

Let's have an example of $(3,1)$ repetition code.

$$(3,1) = (n, k)$$

Info. bits = $K = 1$

$$\text{Parity bits} = r = n - K = 3 - 1 = 2$$

- Encoding process

Info bits	Parity bits		Codeword		
0	0	0	0	0	0
1	1	1	1	1	1

- Decoding process, it is done based on majority vote decoding.

Received data	Decoding decision	Output Data	Info.
0 0 0	No error	0 0 0	0
0 0 1	One Bit Error	0 0 0	0
0 1 0	One Bit Error	0 0 0	0
1 0 0	One Bit Error	0 0 0	0
1 1 1	No error	1 1 1	1
1 1 0	One bit error	1 1 1	1
1 0 1	One bit error	1 1 1	1
0 1 1		1 1 1	1

Majority of vote for (V_1, V_2, V_3) is taken as $i = V_1 V_2 + V_3 V_4$

Hamming Code

- It is used to detect and correct errors.
- In Hamming code, we send data along with parity bits or Redundant bits.
- It is represented by (m, k) code

$\downarrow \quad \downarrow$
 Total bits message bits.
- Parity bits $P = m - k$
- To identify parity bits, H should satisfy given condition

$$\Rightarrow 2^P \geq P + k + 1$$

∴ for $k = 4$ message bits.

$$\Rightarrow 2^P \geq P + 4 + 1$$

$$\Rightarrow 2^P \geq P + 5$$

For $P = 1$

$$2^1 \geq 5$$

~~X~~

For $P = 2$

$$2^2 \geq 7$$

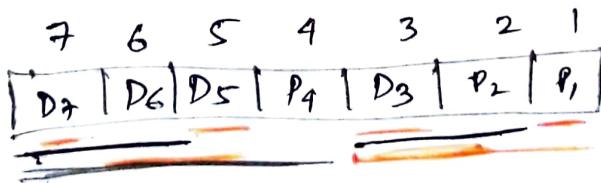
~~X~~

for $P = 3$

$$2^3 \geq 8$$

✓

$\therefore P = 3$ for $k = 4$ bits.
 $m = 3 + 4 = 7$ bits
 \therefore This code is $(7, 3)$ code



$$P_1 \rightarrow D_3 D_5 D_7 \quad (\oplus \oplus R)$$

$$P_2 \rightarrow D_3 D_6 D_7$$

$$P_4 \rightarrow D_5 D_6 D_7$$

$$P_1 = 2^0 = 1$$

$$P_2 = 2^1 = 2$$

$$P_4 = 2^2 = 4$$

Generation Hamming code.

- 5 bit data (01101) is given, represent given data in Hamming code.

→ $K=5$ bits

→ Identify parity bits

$$\Rightarrow 2^P \geq P+K+1$$

$$\Rightarrow 2^P \geq P+6$$

For $P=1, P=2, P=3, P=4$

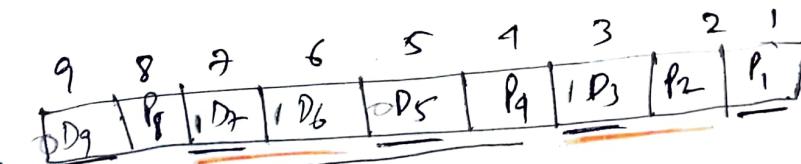
$$\begin{array}{c} 2^1 > 2 \\ \cancel{2^1 > 2} \\ \cancel{2^2 > 8} \\ \cancel{2^3 > 9} \\ \cancel{2^4 > 16} \end{array}$$

$$\therefore P=4,$$

so for $P=4, K=5, m=9$

→ This is $(9, 5)$ Hamming code.

Ans



Position of parity bit

val. of parity bit.

$$P_1 = 2^0 = 1$$

$$P_1 \rightarrow D_3 D_5 D_7 D_9$$

$$P_2 = 2^1 = 2$$

$$P_2 \rightarrow D_3 D_6 D_7$$

$$P_4 = 2^2 = 4$$

$$P_4 \rightarrow D_5 D_6 D_7$$

$$P_8 = 2^3 = 8$$

$$P_8 \rightarrow D_9$$

$$P_1 = 1 + 0 + 1 + 0 = 0$$

$$P_2 = 1 + 1 + 1 = 1$$

$$P_4 = 0 + 1 + 1 = 0$$

$$P_8 = D_9 = 0$$

$\therefore \text{code} = \boxed{001100110}$
for 5 bit data

Hamming code Error detection & correction

If Received Hamming code is 1110101 with even parity
then detect & correct error

In 7bit Hamming code

7	6	5	4	3	2	1
D ₇	D ₆	D ₅	P ₄	D ₃	P ₂	P ₁
1	1	0	0	1	0	1

$$P_1 = D_3 + D_5 + D_7 = 1 + 1 + 1 \Rightarrow P_1 = 0 \quad \checkmark$$

$$P_2 = D_3 + D_6 + D_7 = 0 + 1 + 1 \Rightarrow P_2 = 1 \times$$

$$P_4 = D_2 + D_6 + D_5 = 0 + 1 + 1 \Rightarrow P_4 = 1 \times$$

$$P_4 P_2 P_1 = 110 = 6$$

∴ On 6th bit there is error

Error syndrome : $E = [0 | 1 | 1 | 0 | 1 | 0 | 0]$

Received $R = [1 | 1 | 1 | 0 | 1 | 0 | 1]$

Corrected data = $R \oplus E$

$$\therefore = [1 | 1 | 0 | 1 | 0 | 1 | 1]$$

Linear codes bases & property with example.

Definition - A block code is said to be linear code if its codewords satisfy the condition that the sum of any two codewords gives another codeword. i.e. $C_p = C_i + C_K$

- Property
- i) The all-zero words $[0, 0, 0 \dots 0]$ is always a code word.
 - ii) Given any three codewords C_i, C_j and C_K such that $C_p = C_i + C_K$, then $d(C_i, C_j) = w(C_p)$
 - iii) minimum distance of the code
 $d_{min} \geq W_{min}$

eg: (7,4) Hamming code

$$\begin{array}{r} C_1 = 0001011 \\ C_{10} = 1010011 \\ \hline C_{11} = 1011000 \end{array} \quad \begin{matrix} \text{weight} \\ 3 \\ 4 \\ 3 \end{matrix} \quad C_{11} = C_1 + C_{10}$$

$$\rightarrow C_0 = [0000000]$$

$$d(C_1, C_0) = 3 \quad | \quad d(C_1, C_{10}) = 3 = w(C_{11})$$
$$w(C_{11}) = 3 \quad |$$

$$\rightarrow C_{15} = [1111111], w=7$$

other than C_{15} codewords having weight 3 &

$$d_{min} = W_{min}$$

Show that (7,3) Even parity code is a linear and (7,3) odd parity is not linear

$\rightarrow (7,3)$ even parity code

C	d ₁	d ₂	d ₃	P
c ₀	0	0	0	0
c ₁	0	0	1	1
c ₂	0	1	0	1
c ₃	0	1	1	0
c ₄	1	0	0	1
c ₅	1	0	1	0
c ₆	1	1	0	0
c ₇	1	1	1	1

\rightarrow

0 0 1 1	c ₁
0 1 0 1	+ c ₂
0 1 1 0	c ₃

$d_{\min}(c_1, c_2) = 2$

$w(c_3) = 2$

$- d_{\min} = w$

\rightarrow odd parity (7,3) code

C	d ₁	d ₂	d ₃	P
c ₀	0	0	0	1
c ₁	0	0	1	0
c ₂	0	1	0	0
c ₃	0	1	1	1
c ₄	1	0	0	0
c ₅	1	0	1	1
c ₆	1	1	0	1
c ₇	1	1	1	0

$\rightarrow c_1 + c_2$ is not present in odd parity (7,3) code.

\rightarrow It is not linear but code

Generator matrix to generate code words in linear code.

→ Using generator matrix to generate code words in a better approach

$$\begin{aligned} [c] &= [i][G] & [G] &= [I : P] \\ [c] &= \text{Code word} & [c] &\Rightarrow [m : p_c] \text{ //in case of systematic} \\ [i] &= \text{Information words} & [P_c] &= [m][P] \\ [G] &= \text{Generator matrix} \end{aligned}$$

→ The Generator matrix of an (n, k) linear code has ' k ' rows and ' n ' columns.

→ Generator matrix for $(7, 4)$ code is given by

$$\begin{aligned} [G] &= [I : P] \\ &= \left[\underbrace{\begin{matrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{matrix}}_{\text{Identity matrix } I_4} \quad \underbrace{\begin{matrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{matrix}}_{\text{Parity matrix}} \right] \end{aligned}$$

$$\rightarrow [c] = [i][G]$$

Example: Generate code words for $i = (1110)$ with $(7, 4)$ generator matrix code.

$$G = \left[\begin{matrix} 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{matrix} \right]$$

$$\text{message } [i] = [1110]$$

$$c = [i][G]$$

$$\begin{aligned} &= [1110] \left[\begin{matrix} 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{matrix} \right] \\ &= [1110100] \end{aligned}$$

Example: Determine the set of codewords for the $(6,3)$ code with generator matrix.

$$G_1 = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix} \quad m = 6, k = 3$$

message bits = 3

Possible combinations
of message bits

m_0	m_1	m_2
0	0	0
0	0	1
0	1	0
0	1	1
1	0	0
1	0	1
1	1	0
1	1	1

$$\rightarrow C = [i] [G_1]$$

$$\rightarrow c_0 = [0 \ 0 \ 0] \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix}$$

$$= [0 \ 0 \ 0 \ 0 \ 0 \ 0]$$

$$\rightarrow c_1 = [0 \ 0 \ 1] \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix}$$

$$= [0 \ 0 \ 1 \ 1 \ 1 \ 0]$$

$$\left. \begin{array}{l} [C] = [m : k] \\ [P_c] = [m][k] \end{array} \right\}$$

	m_0	m_1	m_2	p_0	p_1	p_2
c_0	0	0	0	0	0	0
c_1	0	0	1	1	1	0
c_2	0	1	0	1	0	1
c_3	0	1	1	0	1	1
c_4	1	0	0	0	1	1
c_5	1	0	1	1	0	1
c_6	1	1	0	1	1	0
c_7	1	1	1	0	0	0

$$C = [m : k]$$

Systematic generator matrix in linear block codes

- A generator matrix $[G] = [I_k : P]$ is said to be in a systematic form if it generates the systematic code words.
 $[C] = [I][G] = [m, p]$

→ here, $[I_k] \rightarrow K \times K$ matrix

$[P] \rightarrow K \times (m-K)$ matrix

$[G] \rightarrow K \times m$ matrix

- In these matrix information bits are placed together

→ Codeword

$$\begin{aligned}[C] &= [I][G] \\ &= \begin{bmatrix} i_1, i_2, i_3, i_4 \end{bmatrix} \begin{bmatrix} i_1 & i_2 & i_3 & i_4 & P_1 P_2 P_3 \\ 1 & 0 & 0 & 0 & 101 \\ 0 & 1 & 0 & 0 & 111 \\ 0 & 0 & 1 & 0 & 110 \\ 0 & 0 & 0 & 1 & 011 \end{bmatrix} \end{aligned}$$

→ So when you get codeword

$$c = (i_1, i_2, i_3, i_4, P_1, P_2, P_3)$$

→ Identity matrix keeps information together

→ Parity matrix generates parity bits.

$$P_1 = i_1 + i_2 + i_3$$

$$P_2 = i_2 + i_3 + i_4$$

$$P_3 = i_1 + i_2 + i_4$$

The $(5,3)$ linear code has the generator matrix

$$G_1 = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{bmatrix} = [I_3 : P]$$

→ Determine systematic form of G_1

→ Generate codeword for information (011) with
systematic G_1 and Non-systematic G_1

$$[G_1] = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{bmatrix}$$

$$[G_1] = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix} \quad [R_3 = R_2 + R_1]$$

$$[G_1] = \begin{bmatrix} 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix} \quad [R_1 = R_1 + R_3]$$

$$G_1 = [I_3 : P]$$

→ For non systematic $[G_1]$

$$c = [i] [G_1]$$

$$= [0 1 1] \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{bmatrix}$$

$$= [0 0 1 1 1]$$

→ For systematic $[G_1]$

$$c = [i] [G_1]$$

$$= [0 1 1] \begin{bmatrix} 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix}$$

$$= [0 1 1 0]$$

$\underbrace{\text{Info}}_{\text{Info}} \quad \underbrace{\text{Parity}}_{\text{Parity}}$

Symbol is somewhere between. Hence
called non systematic

Parity check Matrices in Linear block codes with example,

→ From generator matrix $[G] = [I_{k \times k} | P]$ we can identify parity matrix.

→ By taking P^T we can make parity check Matrix $[H]$

$$[H] = [P^T : I_{n-k}]$$

→ Parity check matrix is used at Rx to decode data.

Example - Generate Parity check matrix for (7,4) code

$$[G] = \left[\begin{array}{cccc|ccc} 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{array} \right]$$

$\underbrace{\hspace{2cm}}_{\text{Identity matrix I}}$ $\underbrace{\hspace{2cm}}_{\text{Parity matrix } P}$

$$\rightarrow P = \left[\begin{array}{cccc} 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 \end{array} \right] \Rightarrow P^T = \left[\begin{array}{cccc} 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{array} \right]$$

$$\Rightarrow I_{n-k} = I_{7-4} = I_3 \\ = \left[\begin{array}{ccc} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array} \right]$$

$$\rightarrow [H] = [P^T : I_{n-k}]$$

$$= \left[\begin{array}{cccc|ccc} 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 \end{array} \right]$$

④ Prove that $G_i H^T$ and $C_i H^T = 0$

$$\begin{aligned} \rightarrow G_i H^T &= [I_k | P] [P^T | I_{n-k}]^T \\ &= [I_k | P] \left[\frac{P}{I_{n-k}} \right] \\ &= I_k P + P I_{n-k} \\ &= P + P \\ \text{P } \xrightarrow{\substack{\circ \\ \times 1}} \quad \text{P is mod-2 odd, } &\quad 0+0=0 \\ &\quad 1+1=0 \end{aligned}$$

$$\therefore \underline{G_i H^T = 0} :$$

$$\begin{aligned} C_i H^T &= [i] [G_i] [H^T] \\ &= [i] 0 \end{aligned}$$

$$\therefore \underline{C_i H^T = 0}$$

Error syndromes in linear block codes with Examples.

→ If received codeword is $[Y]$

→ Then error syndromes

$$[S] = [Y] [H^T]$$

Here, Received codeword $[Y]$

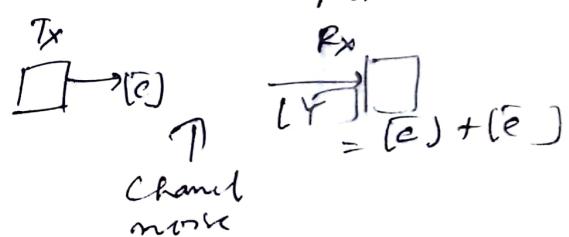
$$[Y] = [c] + [e]$$

where, $[c] = \text{codeword}$

$[e]$ = error

→ If error $[e] = 0$, $[Y] = [c]$

$$\text{So, } [S] = [c] [H^T] = 0$$



Q) Find the error syndromes of $V_1 = (1101101)$, for $H^T = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
 where V is received codeword, also calculate
 errored bit.

→ Received codeword $= V_1 = (1101101)$

$$\rightarrow [S] = [V_1][H^T]$$

$$= \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 2 & 3 & 4 & 5 \end{bmatrix} \left[\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 2 \\ 1 & 1 & 0 & 3 \\ 0 & 1 & 1 & 4 \\ \hline 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & \\ 0 & 0 & 1 & \end{array} \right]$$

$$= [1 \ 0 \ 0] \quad ; \text{error bit is at 5th position.}$$

Q) $Y = [1101101]$

$$e = [0 \ 0 \ 0 \ 0 \ 1 \ 0 \ 0]$$

$$x = Y + e$$

$$= [1101001]$$

Error detection and correction capacity of linear block code

Step 1 - Identify d_{min} [minimum Hamming distance]

Error detection capacity of linear block code

$$\Rightarrow d_{min} \geq s+1$$

→ where, s = error detection capacity

Error correction capacity of linear block code

$$\Rightarrow d_{min} \geq 2t+1$$

→ where, t = error correction capacity

Q) If minimum Hamming dist⁻ of linear block code is 3.
Find Linear block code error correction & detection capability.

$$\rightarrow d_{\min} = 3$$

\rightarrow for error detection

$$\rightarrow d_{\min} \geq 2t + 1$$

$$\Rightarrow 3 \geq 2t + 1$$

$$\Rightarrow 2 \geq t$$

$$\Rightarrow t \leq 1$$

For error correction.

$$\Rightarrow d_{\min} \geq 2t + 1$$

$$\Rightarrow 3 \geq 2t + 1$$

$$\Rightarrow 2 \geq 2t$$

$$\Rightarrow t \leq 1$$

This code can correct 1 bit error

\rightarrow This code can detect 2 bit error

Linear block codes complete examples

— For a $(6, 3)$ code, the generator matrix G is

$$G = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix}$$

Find Q) All corresponding code vectors.

B) Minimum Hamming dist d_{\min}

C) Error detection & Error correction capability

D) Parity check matrix

E) Find error if received code is (100011)

$$\rightarrow G = [I_3 : P]$$

$$I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$P = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

$$\rightarrow [C] = [I_3][G]$$

$$= [m : P_c]$$

$$\rightarrow [P_c] = [I_m][P]$$

$$[P_0, P_1, P_2] = [i_0, i_1, i_2] \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

$$P_0 = (i_0 \oplus i_2)$$

codewords:

$$P_1 = (i_1 \oplus i_2)$$

$$P_2 = (i_0 \oplus i_1)$$

	i_0	i_1	i_2	P_0	P_1	P_2	N
c_1	0	0	0	0	0	0	—
c_2	0	0	1	1	1	0	3
c_3	0	1	0	0	1	1	3
c_4	0	1	1	1	0	1	4
c_5	1	0	0	1	1	1	3
c_6	1	0	1	1	1	0	4
c_7	1	1	0	0	0	0	3
c_8	1	1	1	0	0	1	3

d_{\min}

$$d_{\min} = 3$$

Error detection capability $s \Rightarrow$

$$d_{\min} \geq s+1$$

$$\Rightarrow s \geq d_{\min} - 1$$

$$\Rightarrow s \leq 2$$

It can detect 2-bit error correction

$$d_{\min} \geq 2t + 1$$

$$3 \geq 2t + 1$$

$\Rightarrow t \leq 1 \quad \therefore$ It can correct 1-bit error

① Parity check matrix.

$$H = [P^T : I_{n-k}]$$

$$\rightarrow P^T = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix} \quad I_{6-3} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\rightarrow H = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\rightarrow [S] = [Y][H^T]$$

$$\rightarrow [Y] = [1 \ 0 \ 0 \ 0 \ 1 \ 1]$$

$$\rightarrow H^T = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$$S = TH^T$$

$$= [1 \ 0 \ 0 \ 0 \ 0 \ 1] \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = [1 \ 1 \ 0]$$

\therefore error is at 3rd position

$$e = [0 \ 0 \ 1 \ 0 \ 0 \ 0]$$

$$Y = [1 \ 0 \ 0 \ 0 \ 1 \ 1]$$

$$\Rightarrow X = O + Y$$
$$= [1 \ 0 \ 1 \ 0 \ 1 \ 1] \rightarrow \text{Actual Transmitted Signal.}$$

Cyclic codes basics & properties with example.

- Cyclic codes are ~~subset~~ subpart of linear block codes.

- It follows following properties.

① Linearity property:

- If we have two code words c_i & c_j then

$$c_p = c_i + c_j$$

where c_p should be a code word.

② By cyclic shifting

$$\text{Code word} = (c_1, c_2, c_3, \dots, c_n)$$

- After shifting left or right by any no. of bits, resultant code should be a code word.

- If code words follow above two property then only their code will be cyclic codes.

eg: $\{0000, 0110, 1001, 1111\}$ Is it cyclic code?

→ Check property of linearity:

$$\begin{array}{r} 0110 \\ 1001 \\ \hline 1111 \end{array}$$

$$\begin{array}{r} 0110 \\ 1111 \\ \hline 1001 \end{array}$$

$$\begin{array}{r} 1001 \\ 1111 \\ \hline 0110 \end{array}$$

→ So it follows property of linearity

→ Check property of shifting

$$\begin{array}{r} 0110 \\ \rightarrow \rightarrow \rightarrow \\ 0011 \end{array}$$

It is not a codeword

∴ It is not a cyclic code.

eg: $\{0000, 0101, 1010, 1111\}$, Is it a cyclic code

→ Check property of linearity

$$\begin{array}{r} 0101 \\ 1010 \\ \hline 1111 \end{array}$$

$$\begin{array}{r} 0101 \\ 1010 \\ \hline 1010 \end{array}$$

$$\begin{array}{r} 1010 \\ 1111 \\ \hline 0101 \end{array}$$

→ It follows property of linearity

→ Check shifting property

$$\begin{array}{r} 0101 \\ \rightarrow \rightarrow \rightarrow \\ 1010 \end{array}$$

1010 it is a codeword

$$\begin{array}{r} 1010 \\ \rightarrow \rightarrow \\ 0101 \end{array}$$

It follows shifting property.

∴ Hence it is a cyclic code.

Cyclic codes for non-systematic code word.

* Code word for non-systematic code word is given by

$$c(x) = m(x) \cdot g(x)$$

where, $c(x)$ = code word polynomial

$m(x)$ = message polynomial

$g(x)$ = generator polynomial

$$\rightarrow \left[\begin{array}{l} c = [\text{message, parity}] \\ \text{for systematic codeword.} \end{array} \right] \rightarrow \begin{aligned} & 1011 \\ & x^3 x^2 x^1 x^0 \\ & = x^3 \cdot 1 + x^2 \cdot 0 + x^1 \cdot 1 + x^0 \\ & = x^3 + x + 1 \end{aligned}$$

* Construct

* Construct Non-systmatic cyclic codes ($7,4$), using generator polynomial $g(x) = x^3 + x^2 + 1$, with message (1010)

$$m = [1010]$$
$$\downarrow \quad \downarrow \quad \downarrow$$
$$x^3 \quad x^2 \quad x^1 \quad x^0$$

$$\rightarrow m(x) = x^3 + x$$

$$g(x) = x^3 + x^2 + 1$$

$$\rightarrow c(x) = m(x)g(x)$$

$$\begin{aligned} &= (x^3 + x)(x^3 + x^2 + 1) \\ &= x^6 + x^5 + x^3 + x^4 + x^3 + x \\ &= x^6 + x^5 + x^4 + x \end{aligned}$$

$$\therefore c = [1110010]$$

Cyclic codes for systematic codeword

→ Codeword for systematic codeword is given by

$$c(x) = x^{n-k} m(x) + p(x)$$

where, $p(x) = \text{Rem} \left[\frac{x^{n-k} m(x)}{g(x)} \right]$

✓ $c(x)$ = codeword polynomial

$m(x)$ = message polynomial

$g(x)$ = generator polynomial

$$\boxed{c = [m(x), p(x)]}$$

* Construct a ~~non~~ systematic cyclic code $(7,4)$, using generator polynomial $g(x) = x^3 + x^2 + 1$, with message (1010)

$$m = 7, k = 4$$

$$m = (1010) \Rightarrow m(x) = x^3 + x$$

$$\begin{matrix} | & \downarrow & \downarrow & \downarrow \\ x^3 & x^2 & x^1 & x^0 \end{matrix}$$

$$c(x) = x^{n-k} m(x) + p(x)$$

$$p(x) = \text{Rem} \left[\frac{x^{n-k} m(x)}{g(x)} \right]$$

$$\text{Rem} \left[\frac{x^3(1010)}{x^3 + x^2 + 1} \right] = \text{Rem} \left[\frac{x^6 + x^4}{x^3 + x^2 + 1} \right]$$

$$= x^3 + x^2 + 1 \left[\frac{x^6 + x^4}{x^6 + x^5 + x^3} \right]$$

$$\frac{x^6 + x^4}{x^6 + x^5 + x^3}$$

$$\frac{x^5 + x^4 + x^2}{x^5 + x^4 + x^2}$$

$$\frac{x^3 + x^2}{x^3 + x^2}$$

$$\frac{x^3 + x^2 + 1}{x^3 + x^2 + 1}$$

$$= 1$$

$$\boxed{1}$$

$$c(x) = x^{n-k} m(x) + p(x)$$

$$= x^3(x^3+x) + 1$$

$$= x^6 + x^4 + 1$$

$$C = \left[\begin{array}{ccccc} & & & & \\ & 1 & 0 & 1 & 0 & 0 & 1 \\ & & \downarrow & & & & \\ \text{message} & & \text{parity} & & & & \end{array} \right]$$

Generator matrix of cyclic code

→ Generator matrix

$$[G] = [I: P]$$

↗ ↘

identity Parity
matrix matrix

→ If cyclic code is (m, k) , then generator matrix G is having m columns of k rows.

→ Puncturing matrix is known by m .

→ So row of parity matrix based on generator polynomial $R(x)$ is given by.

$$1^{\text{st}} \text{ row : } \text{Rem} \left[\begin{array}{c} x^{m-1} \\ \hline g(x) \end{array} \right]$$

$$2^{\text{nd}} \text{ row : } \text{Rem} \left[\begin{array}{c} x^{m-2} \\ \hline g(x) \end{array} \right]$$

$$\vdots$$

$$k^{\text{th}} \text{ row : } \text{Rem} \left[\begin{array}{c} x^{m-k} \\ \hline g(x) \end{array} \right]$$

* If generator polynomial of cyclic code $(7,4)$ is given by $g(x) = x^3 + x + 1$, then construct generator matrix.

→ $(7,4)$ code

$$\rightarrow \boxed{7 = n}, \boxed{k = 4}$$

$$\rightarrow G = \left[\begin{array}{cccc|ccc} 1 & 0 & 0 & 0 & - & - & - \\ 0 & 1 & 0 & 0 & - & - & - \\ 0 & 0 & 1 & 0 & - & - & - \\ 0 & 0 & 0 & 1 & - & - & - \end{array} \right]$$

1st row of parity matrix

$$\Rightarrow \text{Rem} \left[\frac{x^{7-1}}{g(x)} \right] = \text{Rem} \left[\frac{x^6}{x^3+x+1} \right] = x^2+1 = [101]$$

2nd Row of parity matrix

$$\Rightarrow \text{Rem} \left[\frac{x^{7-2}}{g(x)} \right] = \text{Rem} \left[\frac{x^5}{x^3+x+1} \right] = x^2+x+1 = [111]$$

3rd Row of parity matrix

$$\Rightarrow \text{Rem} \left[\frac{x^{7-3}}{g(x)} \right] = \text{Rem} \left[\frac{x^4}{x^3+x+1} \right] = x^2+1 = [101]$$

4th Row of parity Matrix

$$\Rightarrow \text{Rem} \left[\frac{x^{7-4}}{g(x)} \right] = \text{Rem} \left[\frac{x^3}{x^3+x+1} \right] = mH = [011]$$

$$\begin{array}{c} \begin{array}{c} \frac{x^3+x+1}{x^3+x+1} \\ \hline \frac{x^6}{x^6+x^4+x^3} \\ \hline \frac{x^6}{x^4+x^3} \\ \hline \frac{x^4}{x^4+x^2+x} \\ \hline \frac{x^4}{x^2+x} \\ \hline \frac{x^2}{x^2+x} \\ \hline \frac{x^2}{x^2+1} \end{array} & \left| \begin{array}{c} \frac{x^2+1}{x^3+x+1} \\ \hline \frac{x^5}{x^5+x^3+x^2} \\ \hline \frac{x^5}{x^3+x^2} \\ \hline \frac{x^3}{x^3+x+1} \\ \hline \frac{x^3}{x^2+x} \end{array} \right| & \left| \begin{array}{c} \frac{x}{x^3+x+1} \\ \hline \frac{x^4}{x^4+x^2+x} \\ \hline \frac{x^4}{x^2+x} \end{array} \right| \end{array}$$

$$\therefore C_1 = \begin{bmatrix} 1 & 0 & 0 & 0 & ; & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & ; & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & ; & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & ; & 0 & 1 & 1 \end{bmatrix}$$

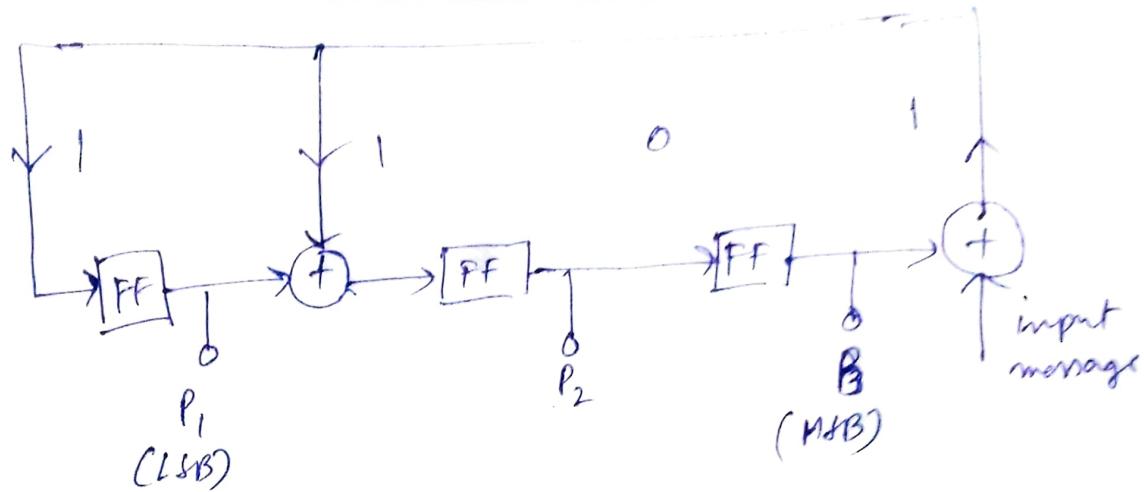
Cyclic encoder designing

- If generator polynomial is goes = $x^3 + x + 1$

Calculate ① Cyclic encoder

② codeword if message is (1110)

$$\begin{aligned} g(x) &= 1 + x + x^3 \\ &= 1 + x + \underline{\quad} + x^3 \end{aligned}$$



m	P_1	P_2	P_3
	0	0	0
1	1	1	0
1	1	0	1
1	0	1	0
0	0	0	1

Legend: LSB (Low-Significance Bit) and MSB (Most-Significant Bit) are indicated by arrows pointing to the corresponding columns in the table.

\therefore codeword = [message, parity]
 $= [1110, 100]$