

Precedence and Associativity



December
Saturday

Precedence : $10 + 20 * 20 \Rightarrow 10 + 400 \Rightarrow 50$

Associativity (L to R) : $10 + 20 - 3 \Rightarrow 12 - 3 \Rightarrow 9$

Associativity (R to L) : $2 \wedge 1 \wedge 2 \Rightarrow 2 \wedge 1 \Rightarrow 2$

operators	Associativity
\wedge	R to L
$*, /$	L to R
$+, -$	L to R

()

Prefix and Postfix

Infix	Prefix	Postfix
$x + y * z$	$+ x * y z$	$xyz * +$
$(x + y) * z$	$* + xy z$	$xy + z *$

Steps for Postfix Conversion:

$$\begin{array}{lcl}
 x + y * z & \Rightarrow & (x + (y * z)) \\
 & \Rightarrow & (x + (yz *)) \\
 & \Rightarrow & xyz * +
 \end{array}
 \quad
 \begin{array}{lcl}
 (x + y) * z & \Rightarrow & ((xy) * z) \\
 & \Rightarrow & (xy +) * z \\
 & = & xy + z *
 \end{array}$$

Infix to postfix Conversion:

Q/p: infix = "a + b * c"

a/p: postfix = "bc * a +"

Q/p: infix = "(a + b) * c"

a/p: postfix = "ab + c *"

Q/p: infix = "a ^ b ^ c"

a/p: postfix = "abc ^ ^"

Q/p: infix = "(a + b) * (c + d)"

a/p: postfix = "ab + cd + *"

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Naive Approach:

$$\text{infix} = a + b * c \Rightarrow (a + (b * c)) \Rightarrow (a + (bc *)) \Rightarrow abc * +$$

$$\text{infix} = (a + b) * c \Rightarrow ((a + b) * c) \Rightarrow ((ab +) * c) \Rightarrow ab + c *$$

$$\text{infix} = a \wedge b \wedge c \Rightarrow (a \wedge (b \wedge c)) \Rightarrow (a \wedge (bc \wedge)) \Rightarrow abc \wedge \wedge$$

$$\text{infix} = (a + b) * (c + d) \Rightarrow ((a + b) * (c + d)) \Rightarrow ((ab +) * (cd +)) \Rightarrow a + cd + *$$

Infix to Postfix using Stack: Efficient Approach

- ① Create an empty stack (st)
- ② Do following for every character x from left to right.
- ③ If x is :
 - a) Operand: Output it
 - b) Left parenthesis: Push to st
 - c) Right parenthesis: Pop from st until left parenthesis is found. Output the popped operators.
- d) Operator: If st is empty, push x to st.
Else compare with st top.
 - (i) Higher precedence (than st top), push to st.
 - (ii) Lower precedence, pop st top and output until a higher precedence operator is found. Then push ^{current} x to st.
 - (iii) Equal precedence, use associativity.
- ④ Pop and output everything from st.

Infix to postfix using stack

Input: $a + b * c$



December
Thursday

Week-52

360-005

1st
Month
'03

1 We

2 Th

3 Fr

4 Sa

5 Su

6 Mo

7 Tu

8 We

9 Th

10 Fr

11 Sa

12 Su

13 Mo

14 Tu

15 We

16 Th

17 Fr

18 Sa

19 Su

20 Mo

21 Tu

22 We

23 Th

24 Fr

25 Sa

26 Su

27 Mo

28 Tu

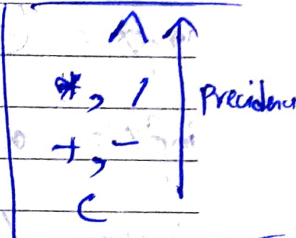
29 We

30 Th

31 Fr

JAN

Input Symbol	Stack	Result (Postfix)
a		a
+	+	a
b	+	ab
*	+	ab
c	+	abc
Pop everything one by one and print		abc*



Input Symbol	Stack	Result (Postfix)
c	c	
a	c	a
+	+	a
b	+	ab
)		ab+
*	*	ab+
c	*	ab+c
Pop everything		ab+c*

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December

Wednesday

Algorithm for evaluation of postfix

- 1) Create an empty stack st.
- 2) Traverse through every symbol x of given postfix
 - 1) If x is operand, push to st.
 - 2) Else (x is an ~~operand~~ operator)
 - (i) $op1 = \text{st.pop}()$;
 - (ii) $op2 = \text{st.pop}()$;
 - (iii) Compute $op2 \ x \ op1$ and push the result to st.
- 3) Return st.top

Postfix evaluation example Input: 10 2 3 ^ ^

Input Symbol(x)

stack (st)

10

10

2

2
10

3

3
2
10

^

8
10

^

108

Infix to Prefix Conversion (Naive Approach)



Two steps:

- ① Fully parenthesize
- ② Start converting from innermost to outermost.

$$x + y * z \Rightarrow (x + (y * z)) \Rightarrow (x + (*yz)) \Rightarrow +x * yz$$

$$(x + y) * z \Rightarrow ((x + y) * z) \Rightarrow ((+xy) * z) \Rightarrow * + xy z$$

$$x \wedge y \wedge z \Rightarrow (x \wedge (y \wedge z)) \Rightarrow (x \wedge (\wedge yz)) \Rightarrow \wedge x \wedge yz$$

$$(x + y) * (z + w) \Rightarrow ((x + y) * (z + w)) \Rightarrow ((+xy) * (+zw)) \Rightarrow * + xy + zw$$

Infix to ~~prefix~~ prefix (efficient) Example: $x + y / z - w * u$

Input Symbol(c)	Stack	Prefix
u		u
*		u
w		uw
-		uw*
z		uw*z
/		uw*z
y		uw*z y
+		uw*z y /
x		uw + z y / x
		uw + z y / x + -

output: $- + x / y z * w u$

Infix to Prefix (efficient)

- ① Create an empty stack st.
- ② Create an empty string, prefix
- ③ Do the following for every character x from right to left
- ④ If x is:
 - a) Operand: Push it to prefix
 - b) Right parenthesis: Push to st.
 - c) Left parenthesis: Pop from st until right parenthesis is found. Append the popped character to prefix.
 - d) Operator: If st is empty, push x to st.
Else compare with st top
 - (i) Higher Precedence (than st top): Push x to st.
 - (ii) Lower precedence: Pop st top and append the popped item to prefix until a higher precedence operator is found (or st becomes empty). Push x to st.
 - (iii) Equal precedence: Use Associativity.
- ⑤ Pop everything from st and append to prefix
- ⑥ Return reverse of prefix.

Evaluation of Prefix

Exp: $+ * 10 2 3$

10
2
3

 \rightarrow

20
3

 \rightarrow

23

 o/p: 23

Exp: $* + 10 2 3$

10
2
3

 \rightarrow

12
3

 \rightarrow

36

 o/p: 36

Exp: $\wedge 10 \wedge 2 3$

2
3

 \rightarrow

10
8

 \rightarrow

10000000

 o/p: 100000000