Toy example 1

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Abstract

Here we provide a brief documentation of the first toy example supplied with our code. In section 1 we illustrate the input Clifford+T quantum circuit, present its output distribution, and convert it to the appropriate adaptive Clifford circuit. Then, in section 2, we carry out the corresponding Pauli-based computation (PBC), explicitly computing all the possible PBC paths and demonstrating that the output distribution obtained from this procedure is the same as that of the original input circuit.

The information in this document can be compared against the results obtained when running this example with our code. The user will see that everything is consistent and works according to what is expected.

Note that since this is a very small example, from the point of view of the quantum resources used, there is no actual advantage in using PBC. Nevertheless, it is a good example to illustrate how the procedure works.

1 The input Clifford+T circuit

The input unitary Clifford+T quantum circuit is illustrated in figure 1. Its output distribution can be easily computed and is given by:

$$p(\sigma = 0) = \frac{1}{2} + \frac{\sqrt{2}}{4} \quad \text{and} \quad p(\sigma = 1) = \frac{1}{2} - \frac{\sqrt{2}}{4}. \quad (1)$$

To apply the PBC procedure we need to start by transforming this circuit into an adaptive Clifford circuit, as depicted in figure 2.

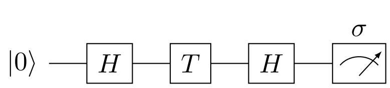

Figure 1: Depiction of the unitary Clifford+T quantum circuit that we are going to simulate using PBC.

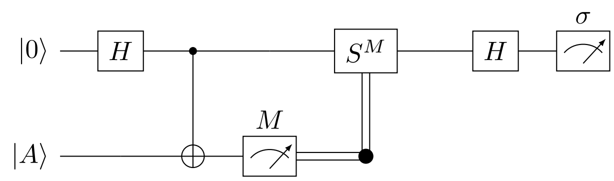

Figure 2: The adaptive Clifford circuit, obtained by replacing the T gate with the T-gadget.

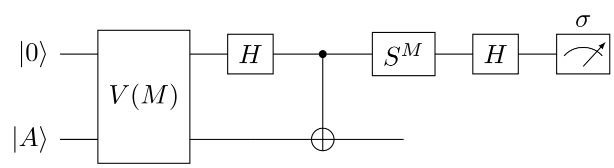

Figure 3: Quantum circuit after the first Pauli P_1 has been dealt with. Its outcome M, obtained from a coin toss (using the classical computer), determines $V(M)$ as well as the presence or absence of the S gate.

2 Compilation and weak simulation with Pauli-based computation

We start by propagating the Z-measurement of the auxiliary (magic) qubit to the very beginning of the computation:

$$(I \otimes Z) \xrightarrow{CX_{12}H_1} P_1 = (X \otimes Z). \quad (2)$$

Since this operator anti-commutes with $(Z \otimes I)$, its output M can be obtained classically by making a random draw from the uniform distribution $\{0, 1\}$. Additionally, the corresponding Clifford unitary

$$V(M) = \frac{(Z \otimes I) + (-1)^M (X \otimes Z)}{\sqrt{2}}, \quad (3)$$

needs to be prefixed before any other gates in the quantum circuit. This is illustrated in figure 3.

2.1 Path 1

If the result of the draw is $M = 0$, then the S gate is not applied and we need to propagate the Z-measurement of the main qubit through the unitary: $U_a = H_1 CX_{12} H_1 V(M = 0)$. Doing so yields:

$$(Z \otimes I) \xrightarrow{U_a} P_2 = U_a^\dagger (Z \otimes I) U_a = (Z \otimes X). \quad (4)$$

To obtain the outcome of this operator we need to measure it. However, as it is known from the PBC procedure, it suffices to measure X on the auxiliary qubit register in the magic state $|A\rangle$. This will yield the outcome $\sigma = 0$ with probability $p(\sigma = 0) = \frac{1}{2} + \frac{\sqrt{2}}{4}$, and the outcome $\sigma = 1$ with probability $p(\sigma = 1) = \frac{1}{2} - \frac{\sqrt{2}}{4}$.

2.2 Path 2

If instead the outcome of the draw is $M = 1$, then the S gate needs to be present in the circuit, in which case the Z-measurement of the main qubit has to be propagated through: $U_b = H_1 S_1 CX_{12} H_1 V(M = 1)$. Doing so yields:

$$(Z \otimes I) \xrightarrow{U_b} P_2 = U_b^\dagger (Z \otimes I) U_b = (I \otimes Y). \quad (5)$$

To obtain the outcome of this operator we need to measure it, but it suffices to measure Y on the auxiliary qubit register in the magic state $|A\rangle$. This will yield the outcome $\sigma = 0$ with probability $p(\sigma = 0) = \frac{1}{2} + \frac{\sqrt{2}}{4}$, and the outcome $\sigma = 1$ with probability $p(\sigma = 1) = \frac{1}{2} - \frac{\sqrt{2}}{4}$.

2.3 Code output

It is clear that the procedure described above (and implemented in our code) yields exactly the same output distribution as the original quantum circuit. This can be seen from the "Output_distribution-Toy1-input.pdf" output file which presents the histogram corresponding to 1024 shots (see figure 4). Additionally, it is also possible to check the different paths explored during the computation in the file "Compilation_data.txt". These paths are consistent with the calculation above:

- either $(X \otimes Z)$ followed by $(Z \otimes X)$ (when $M = 0$);
- or $(X \otimes Z)$ followed by $(I \otimes Y)$ (when $M = 1$).

For more details, the interested user can explore the additional information accessible in the other output files.

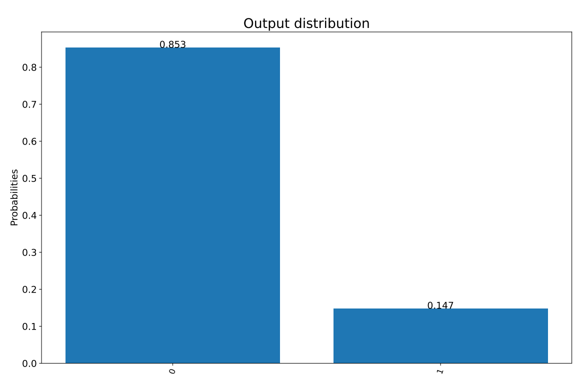

Figure 4: Output distribution obtained by compiling and weakly simulating the circuit depicted in figure 1 a total of 1024 shots.