Result_analysis

March 1, 2022

1 Analysis the results of hybrid PBC

In this notebook, we will analyse the results obtained for the **Toffoli toy example** using our hybrid PBC script.

We run the script simulating **2 virtual qubits** and demanding a precision $\epsilon_{desired} = 0.01$.

The number of samples is being determined as:

$$N = \frac{20 \cdot \sum_{i=1}^{\chi} \alpha_i^2}{\epsilon_{desired}^2},$$

and the actual relative error ϵ_{actual} as:

$$\epsilon_{actual} = \sqrt{20}\frac{\sigma_{samples}}{\sqrt{N}}.$$

Note that since necessarily $\sigma_{samples}^2 \leq \sum_{i=1}^{\chi} \alpha_i^2$ then $\epsilon_{actual} \leq \epsilon_{desired}$.

As a result, we will have 95%-confidence intervals given by $\mathbb{E}(\xi) \pm \epsilon_{actual}$.

[1]:
```
import math
from math import sqrt

import numpy as np
import json
from matplotlib import pyplot as plt
```

[2]:
```
def nr_samples(precision, virtual_qubits):
    weights_1vq = [1 / 2, (1 - sqrt(2)) / 2, 1 / sqrt(2)]

    sum_squares = 0
    for i in range(3**virtual_qubits):
        label = np.base_repr(i, base=3)
        label = str(label)
        if not len(label) == virtual_qubits:
            label = '0' * (virtual_qubits - len(label)) + label

        weight = 1
        for s in label:
            weight = weight * weights_1vq[int(s)]
```

```
        sum_squares += abs(weight)**2

    N = int(math.ceil(20 * sum_squares / precision**2))

    return N
```

[3]:
```
precision = [0.1, 0.09, 0.07, 0.05, 0.03, 0.02, 0.01]
vq = 2  # choosing 2 virtual qubits
samples = []

for p in precision:
    samples.append(nr_samples(p, vq))

print(samples)
```

```
[1258, 1553, 2567, 5030, 13971, 31434, 125736]
```

Because we run the code for the precision $\epsilon = 0.01$, we should have a total of 125 736 values to use for the computation of the probability p. We can use those values to estimate the expected value of this probability and determine how close it is to the exact value $p = 0$.

We can also determine the standard deviation of the sample, and use it to compute the appropriate confidence interval.

Since we have access to smaller sets of values as well, we can use the first M values to estimate the results that would have been obtained if we have requested a different precision (e.g., $\epsilon = \{0.10, 0.09, 0.07, 0.05, 0.03, 0.02\}$).

[4]:
```
exact_value = 0
```

1.1 $\epsilon = 0.01 \Rightarrow N = 125\,736$

[5]:
```
with open('Resources_data--virt2--probabilities.txt', 'r') as file_object:
    probabilities1 = json.load(file_object)
```

[6]:
```
probabilities1 = np.array(probabilities1)
prec = precision[-1]
print(len(probabilities1) == samples[-1])

mean1 = np.mean(probabilities1)
std1 = np.std(probabilities1)
error1 = sqrt(20) * std1 / sqrt(len(probabilities1))

print(f'p= {round(mean1, 4)} +/- {round(error1, 4)}')
print(error1 < prec)
```

```
True
p= 0.0015 +/- 0.0043
```

```
True
```

1.2 $\epsilon = 0.02 \Rightarrow N = 31\,434$

[7]:
```python
probabilities2 = probabilities1[:samples[-2]]
```

[8]:
```python
prec = precision[-2]
print(len(probabilities2) == samples[-2])

mean2 = np.mean(probabilities2)
std2 = np.std(probabilities2)
error2 = sqrt(20) * std2 / sqrt(len(probabilities2))

print(f'p= {round(mean2, 4)} +/- {round(error2, 4)}')
print(error2 < prec)
```

```
True
p= 0.0015 +/- 0.0087
True
```

1.3 $\epsilon = 0.03 \Rightarrow N = 13\,971$

[9]:
```python
probabilities3 = probabilities1[:samples[-3]]
```

[10]:
```python
prec = precision[-3]
print(len(probabilities3) == samples[-3])

mean3 = np.mean(probabilities3)
std3 = np.std(probabilities3)
error3 = sqrt(20) * std3 / sqrt(len(probabilities3))

print(f'p= {round(mean3, 4)} +/- {round(error3, 4)}')
print(error3 < prec)
```

```
True
p= 0.0012 +/- 0.0129
True
```

1.4 $\epsilon = 0.05 \Rightarrow N = 5\,030$

[11]:
```python
probabilities4 = probabilities1[:samples[-4]]
```

[12]:
```python
prec = precision[-4]
print(len(probabilities4) == samples[-4])

mean4 = np.mean(probabilities4)
std4 = np.std(probabilities4)
error4 = sqrt(20) * std4 / sqrt(len(probabilities4))
```

```python
print(f'p= {round(mean4, 4)} +/- {round(error4, 4)}')
print(error4 < prec)
```

```
True
p= 0.0044 +/- 0.0215
True
```

1.5 $\epsilon = 0.07 \Rightarrow N = 2\,567$

[13]:
```python
probabilities5 = probabilities1[:samples[-5]]
```

[14]:
```python
prec = precision[-5]
print(len(probabilities5) == samples[-5])

mean5 = np.mean(probabilities5)
std5 = np.std(probabilities5)
error5 = sqrt(20) * std5 / sqrt(len(probabilities5))

print(f'p= {round(mean5, 4)} +/- {round(error5, 4)}')
print(error5 < prec)
```

```
True
p= 0.0078 +/- 0.0304
True
```

1.6 $\epsilon = 0.09 \Rightarrow N = 1\,553$

[15]:
```python
probabilities6 = probabilities1[:samples[-6]]
```

[16]:
```python
prec = precision[-6]
print(len(probabilities6) == samples[-6])

mean6 = np.mean(probabilities6)
std6 = np.std(probabilities6)
error6 = sqrt(20) * std6 / sqrt(len(probabilities6))

print(f'p= {round(mean6, 4)} +/- {round(error6, 4)}')
print(error6 < prec)
```

```
True
p= 0.0058 +/- 0.0389
True
```

1.7 $\epsilon = 0.100 \Rightarrow N = 1\,258$

[17]:
```
probabilities7 = probabilities1[:samples[-7]]
```

[18]:
```
prec = precision[-7]
print(len(probabilities7) == samples[-7])

mean7 = np.mean(probabilities7)
std7 = np.std(probabilities7)
error7 = sqrt(20) * std7 / sqrt(len(probabilities7))

print(f'p= {round(mean7, 4)} +/- {round(error7, 4)}')
print(error7 < prec)
```

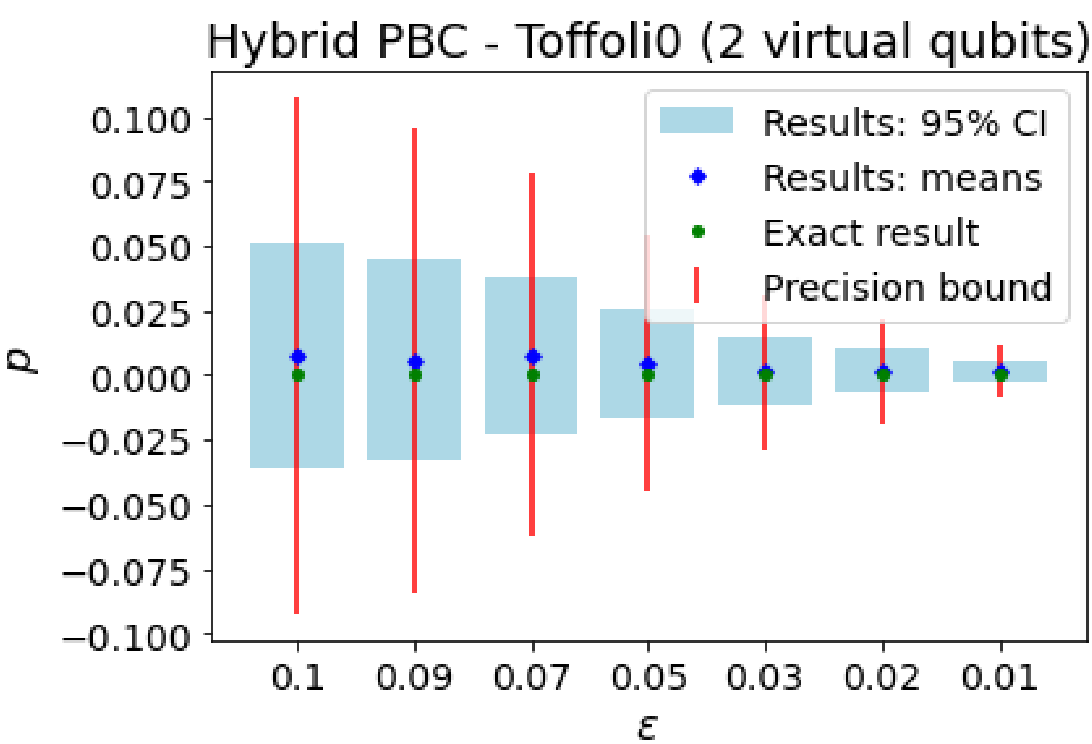
```
True
p= 0.0074 +/- 0.0431
True
```

1.8 Plotting the data:

[19]:
```
means = [mean7, mean6, mean5, mean4, mean3, mean2, mean1]
errors = [error7, error6, error5, error4, error3, error2, error1]

bot_bar = [means[i] - errors[i] for i in range(len(means))]
heights = [2*errors[i] for i in range(len(means))]
```

[20]:
```
fig = plt.figure()

plt.bar([i for i in range(len(means))],
        heights,
        width=0.8,
        bottom=bot_bar,
        align='center',
        color='lightblue',
        label='Results: 95% CI')

plt.errorbar([i for i in range(len(means))],
             means,
             color='blue',
             marker='D',
             markersize=4,
             linestyle='None',
             label='Results: means')

plt.errorbar([i for i in range(len(means))],
             [exact_value for _ in range(len(means))],
```

```
             color='green',
             marker='o',
             markersize=4,
             linestyle='None',
             label='Exact result')

plt.errorbar([i for i in range(len(means))],
             means,
             yerr=precision,
             color='red',
             linestyle='None',
             label='Precision bound')

plt.xticks([i for i in range(len(means))], precision, size=14)
plt.xlabel(r'$\epsilon$', fontsize=16)

#plt.ylim([0.05, 0.25])
plt.yticks(size=14)
plt.ylabel(r'$p$', fontsize=16)

plt.legend(fontsize=14)

plt.title('Hybrid PBC - Toffoli0 (2 virtual qubits)', fontsize=18)

plt.show()
```



Note that since the probability cannot be negative, whenever the expectation value is lower than zero we can replace its outcome by zero instead. Additionally, we can also stop the confidence interval (and precision bound) at zero.

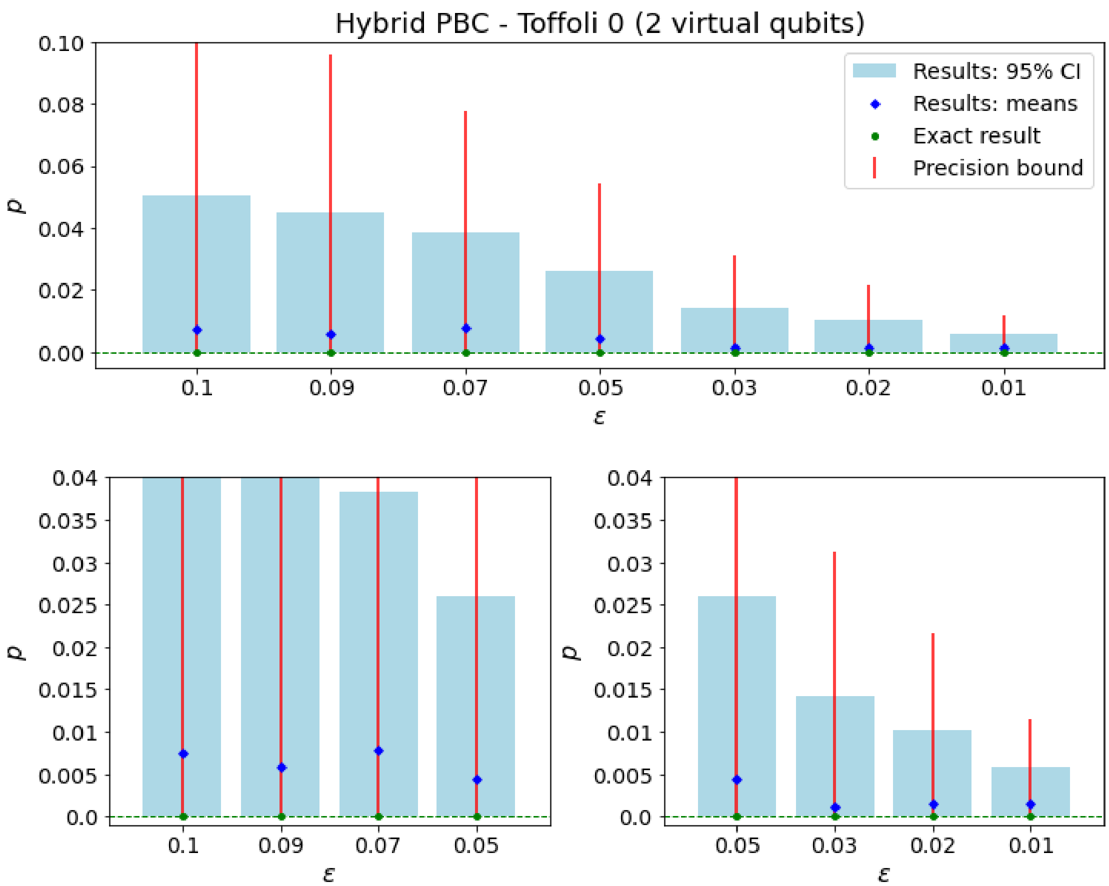
[21]:
```python
new_heights = []
lower_error = []

for i in range(len(means)):
    if means[i] < 0:
        means[i] = 0
    new_heights.append(errors[i] + means[i])
    lower_error.append(means[i])
```

[22]:
```python
fig = plt.figure(constrained_layout=True, figsize=(10, 8))

subfigs = fig.subfigures(2, 1, hspace=0.07)

# Upper subfigure
subplot = subfigs[0].add_subplot(111)
plt.bar([i for i in range(len(means))],
        new_heights,
        width=0.8,
        align='center',
        color='lightblue',
        label='Results: 95% CI')

plt.errorbar([i for i in range(len(means))],
             means,
             color='blue',
             marker='D',
             markersize=4,
             linestyle='None',
             label='Results: means')

plt.errorbar([i for i in range(len(means))],
             [exact_value for _ in range(len(means))],
             color='green',
             marker='o',
             markersize=4,
             linestyle='None',
             label='Exact result')

plt.errorbar([i for i in range(len(means))],
             means,
             yerr=[lower_error, precision],
```

```
            color='red',
            linestyle='None',
            label='Precision bound')

plt.plot(np.arange(-1, 8, 1), [exact_value for _ in np.arange(-1, 8, 1)],
         '--',
         color='green',
         linewidth=1)

plt.xlim([-0.75, 6.75])
plt.xticks([i for i in range(len(means))], precision, size=14)
plt.xlabel(r'$\epsilon$', fontsize=16)

plt.ylim([-0.005, 0.10])
plt.yticks(size=14)
plt.ylabel(r'$p$', fontsize=16)

plt.legend(fontsize=14)

plt.title('Hybrid PBC - Toffoli 0 (2 virtual qubits)', fontsize=18)

# Lower subfigure (left)
subplot = subfigs[1].add_subplot(121)
plt.bar([i for i in range(4)],
        new_heights[:4],
        width=0.8,
        align='center',
        color='lightblue',
        label='Results: 95% CI')

plt.errorbar([i for i in range(4)],
             means[:4],
             color='blue',
             marker='D',
             markersize=4,
             linestyle='None',
             label='Results: means')

plt.errorbar([i for i in range(4)], [exact_value for _ in range(4)],
             color='green',
             marker='o',
             markersize=4,
             linestyle='None',
             linewidth=1,
             label='Exact result')

plt.errorbar([i for i in range(4)],
```

```
             means[:4],
             yerr=[lower_error[:4], precision[:4]],
             color='red',
             linestyle='None',
             label='Precision bound')

plt.plot(np.arange(-1, 8, 1), [exact_value for _ in np.arange(-1, 8, 1)],
         '--',
         color='green',
         linewidth=1)

plt.xlim([-0.75, 3.75])
plt.xticks([i for i in range(4)], precision[:4], size=14)
plt.xlabel(r'$\epsilon$', fontsize=16)

plt.ylim([-0.001, 0.04])
plt.yticks(np.arange(0, 0.045, 0.005),
           [round(i, 3) for i in np.arange(0, 0.045, 0.005)],
           size=14)
plt.ylabel(r'$p$', fontsize=16)

# Lower subfigure (right)
subplot = subfigs[1].add_subplot(122)
plt.bar([i for i in range(4)],
        new_heights[3:],
        width=0.8,
        align='center',
        color='lightblue',
        label='Results: 95% CI')

plt.errorbar([i for i in range(4)],
             means[3:],
             color='blue',
             marker='D',
             markersize=4,
             linestyle='None',
             label='Results: means')

plt.errorbar([i for i in range(4)], [exact_value for _ in range(4)],
             color='green',
             marker='o',
             markersize=4,
             linestyle='None',
             label='Exact result')

plt.errorbar([i for i in range(4)],
             means[3:],
```

```
             yerr=[lower_error[3:], precision[3:]],
             color='red',
             linestyle='None',
             label='Precision bound')

plt.plot(np.arange(-1, 8, 1), [exact_value for _ in np.arange(-1, 8, 1)],
         '--',
         color='green',
         linewidth=1)

plt.xlim([-0.75, 3.75])
plt.xticks([i for i in range(4)], precision[3:], size=14)
plt.xlabel(r'$\epsilon$', fontsize=16)

plt.ylim([-0.001, 0.04])
plt.yticks(np.arange(0, 0.045, 0.005),
           [round(i, 3) for i in np.arange(0, 0.045, 0.005)],
           size=14)
plt.ylabel(r'$p$', fontsize=16)

fig.savefig("Probability_vs_precision.pdf", bbox_inches='tight')
plt.show()
```



Hybrid PBC - Toffoli 0 (2 virtual qubits)
Results: 95% CI
Results: means
Exact result
Precision bound
p
ε
0.10
0.08
0.06
0.04
0.02
0.00
0.1
0.09
0.07
0.05
0.03
0.02
0.01
0.035
0.03
0.025
0.015
0.005
0.0

[ ]: