Toy example 3

F.C.R. Peres
International Iberian Nanotechnology Laboratory (INL),
Av. Mestre José Veiga, 4715-330 Braga, Portugal.
Departamento de Física e Astronomia, Faculdade de Ciências, Universidade do Porto,
R. do Campo Alegre, 4169-007 Porto, Portugal.
filipa.peres@inl.int

Abstract

This document provides a brief description of the third (and final) toy example supplied with our code. We illustrate the input Clifford+T quantum circuit under consideration, and discuss the expected output for each of the 8 different input states considered.

These expected outputs can be compared against the results obtained with our Python code; the reader will verify that everything is consistent.

The input Clifford+T circuit

In this final toy example we consider the case of the Toffoli gate, which can be written in terms of Clifford+T gates as depicted in figure 1. There are 8 possible input states for this quantum circuit, each of which has been suitably encoded in the input files so that:

- Toffoli0.qasm: $|\Psi_{input}\rangle = |000\rangle$, and $|\Psi_{output}\rangle = |000\rangle$;
- Toffoli1.qasm: $|\Psi_{input}\rangle = |001\rangle$, and $|\Psi_{output}\rangle = |001\rangle$;
- Toffoli2.qasm: $|\Psi_{input}\rangle = |010\rangle$, and $|\Psi_{output}\rangle = |010\rangle$;
- Toffoli3.qasm: $|\Psi_{input}\rangle = |100\rangle$, and $|\Psi_{output}\rangle = |100\rangle$;
- Toffoli4.qasm: $|\Psi_{input}\rangle = |011\rangle$, and $|\Psi_{output}\rangle = |011\rangle$;
- Toffoli5.qasm: $|\Psi_{input}\rangle = |101\rangle$, and $|\Psi_{output}\rangle = |101\rangle$;
- Toffoli6.qasm: $|\Psi_{input}\rangle = |110\rangle$, and $|\Psi_{output}\rangle = |111\rangle$;
- Toffoli7.qasm: $|\Psi_{input}\rangle = |111\rangle$, and $|\Psi_{output}\rangle = |110\rangle$.

Therefore, for each of the 8 input circuits, the output is deterministic. The output of our compilation code can be compared against the known results presented above. The reader will note that the histograms found in the folder "output-compilation" exhibit a single column, corresponding to the correct output state.

Evidently, the probability that each qubit returns the outcome 1 is either 0 or 1, depending on the case. Using the hybrid PBC set-up we can estimate the probability p that the third qubit of the circuit yields 1, using a varying number of virtual qubits and for different input states. The results can be found in the folders: "output-hybrid0_1vq", "output-hybrid0_2vq", "output-hybrid0_3vq", and "output-hybrid6_1vq".

References

[1] Vivek V. Shende and Igor L. Markov. "On the CNOT-Cost of TOFFOLI Gates". In: *Quantum Information and Computation* 9 (2009). eprint: https://arxiv.org/abs/0803.2316.

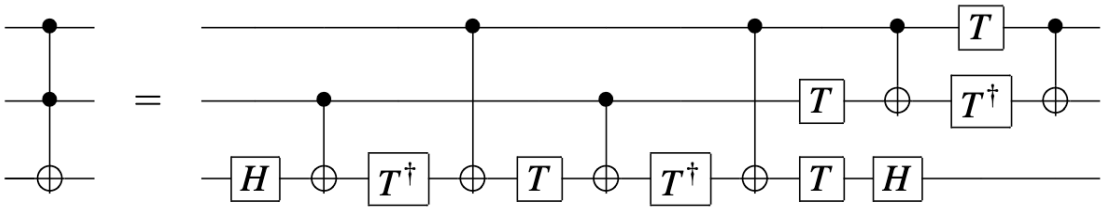

Figure 1: Decomposition of the Toffoli gate in terms of Clifford+T unitary gates following [1].