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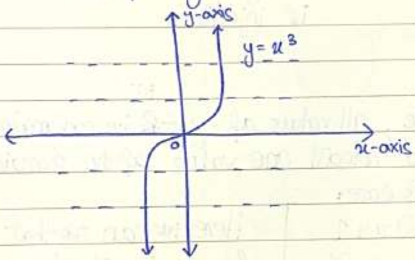
*) Introduction to Computer Science: Sheet 04

*) Problem 4.1:

a) $f: R \longrightarrow R$ with $f(x) = x^3$

For $f: R \rightarrow R$ with $f(x) = x^3$ to be injective, all values of $x \in R$ or the domain have to be mapped to a unique element of the $y \in R$ or codomain of the function.
In this case, we can prove this by the horizontal line test on a graph as the following:



As the line test only intersects the graph at one point for each value of $f(x)$ or $y \in R$, this function is injective.

For $f(x) = x^3$ to be surjective, all elements of $f(x)$ i.e $y \in R$ must be mapped onto atleast one element of the $x \in R$ i.e domain of $f(x)$.
The graph above shows an infinite graph that has corresponding elements in the domain for all values of the codomain. Therefore, this function is also surjective.

As $f(x) = x^3$ is injective and surjective, we can say that it is also a bijective function.

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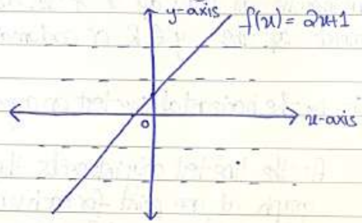
b) $g: \mathbb{N} \to \mathbb{N}$ with $f(x) = 2x+1$.

for $f(x) = 2x+1$ to be injective, all elements of the domain i.e $x \in \mathbb{N}$ must be mapped onto a unique element from the codomain i.e $y \in \mathbb{N}$. This can be proven using the horizontal line test as following:



Therefore, this shows that for each $x \in \mathbb{N}$ there exists a unique element in the codomain i.e $y \in \mathbb{N}$. So, $f(x) = 2x+1$ is injective.

For $f(x) = 2x+1$ to be surjective, all values of $y \in \mathbb{N}$ i.e codomain of $f(x)$ must be mapped onto atleast one value of the domain of $f(x)$. However in this case:

$$\left.\begin{aligned} y &= 2x+1 \\ \frac{y-1}{2} &= x \\ x &= \frac{y-1}{2} \end{aligned}\right\}$$

Here we can see that for an element of $y \in \mathbb{N}$ the statement $x \in \mathbb{N}$ is not true. as the answer could be a fraction and not natural number.

Therefore the function $f(x) = 2x+1$ is injective, but not surjective and henceforth also not bijective.

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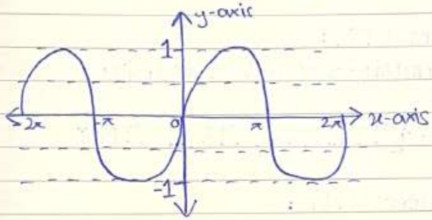
c) $h: R \to R$ with $f(x) = \sin(x)$

For $f(x) = \sin(x)$ to be injective, the elements of the domain of $f(x)$ i.e $x \in R$ must be mapped onto a unique element of the codomain of $f(x)$ i.e: $y \in R$. However, the horizontal line test on the graph of $y = \sin(x)$ proves otherwise and shows that there are more than one values of $x \in R$ that map onto the same element of $y \in R$ i.e the codomain of $f(x)$.



Therefore, the function is not injective.

The $f(x) = \sin(x)$ is also not surjective because when graphed we see that $\forall x \in R : \sin(x) \in (-1, 1)$ meaning that the function fails to map $y \in R$ onto $x \in R$ at least once for each value.

Finally, as it is neither an injective or surjective function, we can say that it also is not a bijective function.//

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Problem 4.2:

$$f : X \to Y \text{ and } g : Y \to Z$$

a) f and g are injective, then $g \circ f$ is injective.

Suppose that f, g are both injective and also that their Compositions are equal as the following:

$$(g \circ f)(x) = (g \circ f)(y)$$

This would mean that

$$g(f(x)) = g(f(y))$$

Now, because g is injective we can show that

$$f(x) = f(y)$$

And because f is also injective the following is true:

$$x = y.$$

Henceforth, proving that $g \circ f$ is injective

b) f and g are surjective, then $g \circ f$ is surjective.

Suppose that f, g are surjective and lets assume that $z \in Z$. As we already assumed that g is surjective, we know that there exists some

$$y \in Y \text{ with } g(y) = z.$$

Also, as we assumed that f is surjective, we know that there exists some

$$x \in X \text{ with } f(x) = y.$$

Hence, we can say that $z = g(f(x)) = (g \circ f)(x)$ and that $z \in rng(g \circ f)$

Thus, this proves that $(g \circ f)$ is also surjective

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c) f, g are bijective, then g∘f is bijective.

As we have already proven that if f, g are both injective and surjective then their composition i.e g∘f would also be injective and surjective.
Hence, for a function to be bijective it has to be both injective and surjective. Therefore, g∘f would also be ~~surjective~~ bijective if it has f, g being bijective. //

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Problem 4.3:

a) Entities:

1. Customers (C):
   Set of all customers at the cinema
   Notation:
   $$C = \{C1, C2, C3 \ldots CN\}$$

2. Ticket Takers (TT):
   Set of cinema staff that check ticket validity
   Notation:
   $$TT = \{TT1, TT2, TT3, \ldots TTN\}$$

3. Movie Theaters (MT):
   Set of all theatres at the cinema
   Notation:
   $$MT = \{MT1, MT2, MT3 \ldots MTN\}$$

4. Coffee Bar Staff (CS):
   Set of cinema employees serving drinks at the bar.
   Notation:
   $$CS = \{CS1, CS2, CS3 \ldots CSN\}$$

5. Cashier (CA):
   Set of the cinema's cashier selling tickets.
   Notation:
   $$CA = \{CA1, CA2, CA3 \ldots CAn\}$$

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b) 1. Purchasing_Tickets $\subseteq (C \times CA)$
→ A set relation of customers purchasing tickets from the cashier.

2. Drinks_Order $\subseteq (C \times CS)$
→ A relation that matches the customer to a caffee bar staff for their order.

3. Validate_Ticket $\subseteq (C \times TT)$
→ A relation describing how many customers have their tickets validated by the Ticket Taker.

4. Ticket_Taker_at $\subseteq$ (~~R_4~~ $TT \times MT$)
→ A relation describing which movie theater of the cinema the ticket taker is infront of for.

5. Which_movie $\subseteq (C \times MT)$
→ A relation to check which movie ~~has~~ is been watched by the Customer at the cinema.

c) 1. Same_movie_watched $\subseteq (C \times C)$
→ An equivalence set relation comparing two customers (C1, C2) that have watched the same movie at the cinema. This is transitive, reflexive and symmetric.

2. Chose_which_theatre $\subseteq (C \times MT)$
→ A strict partial order relation comparing a customer and their selected movie theater. This is ireflexive, asymmetric and transitive, as (C1, MT1) cannot be compared in the set with ~~C1, MT~~ (C1, C1), neither does (MT1, C1) exist in the set and that two customers can choose the same theater.

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3. Ticket_taker_experience $\subseteq$ (TT x TT)
(TT $\leqslant$ TT)

→ A partial order relation where TT1's experience is compared to TT2's experience. This is reflexive, antisymmetric and transitive as if TT1 has less experience than TT2 then there cannot be a tuple describing TT1 with more experience than TT2 in the set.

4. Price_of_Drinks $\subseteq$ (D x D)

→ An equivalence set relation comparing the prices of drinks served at the coffee bar. This is reflexive, symmetric and transitive.

5. Price_of_Ticket $\subseteq$ (D x D)

→ An equivalence set relation describing the comparison of ticket prices. This is reflexive, symmetric and transitive.

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Problem 4.4:

a) `ord '='`

The result of the expression is
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b) `putStrLn [chr 128119]`

is the expression used to print the character. This results in a construction worker emoji being outputted on mac.

c) `:type zipWith`

zipWith :: (a -> b -> c) -> [a] -> [b] -> [c]

The above is the type signature of zipWith. This shows that the function combines two lists [a],[b] of different argument of types and combines them for the result [c].

d) `import Data.List`
`:type isPrefixOf`

isPrefixOf :: Eq a => [a] -> [a] -> Bool

Here Eq a is an equality comparison of type a in the list [a] where we are checking for the prefix, and the output is a boolean to show if the prefix is present in the list or not.