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*) Introduction to Computer Science: (Sheet # 02)

Problem 2.1:

$a \in \mathbb{Z}$, if $\underbrace{a^{32} \text{ is odd}}_{P}$, then $\underbrace{a^4 \text{ is odd}}_{Q}$ as well.

$$P \rightarrow Q$$
$$\neg Q \rightarrow \neg P \quad \} \text{ Contrapositive}$$

So, $P = a^{32}$ is odd
$\neg P = a^{32}$ is even.

$Q = a^4$ is odd
$\neg Q = a^4$ is even

To prove $\neg Q$
Assume that a^4 is even, then there would be an integer ~~k~~ k such that,
$$a^4 = 2k$$
if we raise the power of both sides by the power of 8,

$$(a^4)^8 = (2k)^8$$
$$a^{32} = 2^8 k^8$$
$$a^{32} = 256 k^8 \quad ; \text{Now lets factor this by } 2.$$
$$a^{32} = 2(128 k^8)$$

let $128k^8 = m$,
So, $a^{32} = 2m$

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Since 2 being multiplied by m, means that m will be even, we can show that a^{32} is even.
Hence, proving $\neg P$ is true.

This proves $P \rightarrow Q$ by the contrapositive proof being true. i.e $(\neg Q \rightarrow \neg P)$.

Problem 2.2:

Prove by Induction,

$n \in \mathbb{N}$, $n \geq 1$, $n^3 + (n+1)^3 + (n+2)^3$ is divisible by 9.

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*) Problem 2.2:

$n \in \mathbb{N}$, $n \geq 1$, $n^3 + (n+1)^3 + (n+2)^3$, divisible by 9.

So, To show that it is divisible by 9
we can equate it by $9L$, where L is a variable
and 9 would divide the left-hand-side of the equation.
This is as such:

$$n^3 + (n+1)^3 + (n+2)^3 = 9L \quad \} \; L \in \mathbb{N}$$

So, to prove the basis =
let $n = 1$.

So,

$$n^3 + (n+1)^3 + (n+2)^3 = 9L$$
$$(1)^3 + (1+1)^3 + (1+2)^3 = 9L$$
$$1 + (2)^3 + (3)^3 = 9L$$
$$1 + 8 + 27 = 9L$$
$$36 = 9L$$
$$\frac{36}{9} = L$$
$$L = 4$$

Hence, proving that it is divisible by 9 with $n=1$, proving the base case.

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Now, to prove it with induction:

let, $n=k$.

Through this we'll substitute as such:

$$(k)^3 + (k+1)^3 + (k+2)^3 = 9L$$

Lets assume this inductive hypothesis is true. & divisible by 9.

Now, let $n = k+1$ and substitute these values:

$$n^3 + (n+1)^3 + (n+2)^3 = 9L$$

$$(k+1)^3 + (k+1+1)^3 + (k+1+2)^3$$

$$(k+1)^3 + (k+2)^3 + (k+3)^3$$

lets only expand $(k+3)^3$,

$$\left[(k+1)^3 + (k+2)^3\right] + (k^2 + 6k + 9)(k+3)$$

$$\left[(k+1)^3 + (k+2)^3\right] + (k^3 + 6k^2 + 9k + 3k^2 + 18k + 27)$$

$$\left[(k+1)^3 + (k+2)^3\right] + (k^3 + 9k^2 + 27k + 27)$$

$$\left[k^3 + (k+1)^3 + (k+2)^3\right] + 9k^2 + 27k + 27$$

let $9L = k^3 + (k+1)^3 + (k+2)^3$ from inductive hypothesis.

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hence,

$$9L + 9k^2 + 27k + 27$$

$$9(L + k^2 + 3k + 3)$$

Since $(L + k^2 + 3k + 3) \in \mathbb{N}$, we can say that

~~9~~ the statement is true for $n = k+1$ as it is a natural number divisible by 9.

Henceforth, the statement has been proven by mathematical Induction.

*) Problem 2.3:

a)

```
-- Return the list of positive divisors of an integer n
divisors :: Int -> [Int]
divisors n = [x | x <- [1..n], n `mod` x == 0]
```

b)

```
-- Return the sum of divisors of n taken to the power of z
sigma :: Int -> Int -> Int
sigma z n = sum [x^z | x <- [1..n], n `mod` x == 0]
```