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*) Introduction to Computer Science: (Sheet # 03)

Problem 3.1:

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

let $x \in A \cap (B \cup C)$,

This means that

$= x \in A$ and $x \in (B \cup C)$

$= x \in A$ and $(x \in B$ or $x \in C)$

$= (x \in A$ and $x \in B)$ or $(x \in A$ and $x \in C)$

Hence, $= x \in (A \cap B) \cup x \in (A \cap C)$

Thus, $x \in (A \cap B) \cup (A \cap C)$

Since we let $x \in A \cap (B \cup C)$

we can prove that

$$A \cap (B \cup C) \subseteq (A \cap B) \cup (A \cap C) \; \square$$

As left hand side has been proven, we can show that the distributivity law for sets has been proven.

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Problem 3.2:

$$(A \cap B) \times (C \cap D) = (A \times C) \cap (B \times D)$$

a) $(x, y) \in ((A \cap B) \times (C \cap D))$

can be written as:

$$x\,(A \cap B) \times (C \cap D)\, y \quad \Big\} \wedge = \text{and}$$

for cartesian product $A \times B$:

$$A \times B := \{ (a,b) \mid a \in A \wedge b \in B \}$$

So, $x \in (A \cap B) \times y \in (C \cap D)$

$= x \in (A \cap B) \wedge y \in (C \cap D)$

$= x \in A \wedge x \in B \wedge y \in C \wedge y \in D$

$= x\,(A \times C)\,y \wedge x\,(B \times D)\,y$

$= x\,(A \times C)\,y \cap x\,(B \times D)\,y$

$= (x, y) \in (A \times C) \cap (B \times D)$ $\square$

Thus,

$\therefore (A \cap B) \times (C \cap D) \subseteq (A \times C) \cap (B \times D)$ $\square$

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$$b)\ (A \cup B) \times (C \cup D) = (A \times C) \cup (B \times D)$$

$$= x\,(A \cup B) \times (C \cup D)\,y$$

$$= x \in (A \cup B) \wedge y \in (C \cup D)$$

$$= (x \in A \overset{\vee}{\cancel{\text{or}}} x \in B) \wedge (y \in C \overset{\vee}{\cancel{\text{or}}} y \in D)$$

$$= \left[(x \in A \wedge y \in C) \overset{\vee}{\cancel{\text{or}}} (x \in A \wedge y \in D)\right] \vee \left[(x \in B \wedge y \in C) \vee (x \in B \wedge y \in D)\right]$$

$$= \left[(x\,(A \times C)\,y) \vee (x\,(A \times D)\,y)\right] \vee \left[(x\,(B \times C)\,y) \vee (x\,(B \times D)\,y)\right]$$

$$\therefore\ (x, y) \in (A \times C) \cup (A \times D) \cup (B \times C) \cup (B \times D)$$

Looking at this, the statement given misses $(B \times C) \cup (A \times D)$

So, we can say that it doesnt equal to the right hand side of the statement and hence disproves the proposition $\square$

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<u>Problem 3.3:</u>

a) $R = \{ (a,b) \mid a,b \in \mathbb{Z} \wedge |a-b| \leq 3 \}$

let $a = 4$, $b = 2$ for proving these cases

for reflexive:

we need to prove $(a,a) \in R \; \forall x \in A$

So, $|a - a|$

$= |4 - 4|$

$= 0$

as $0 \leq 3$ and $0 \in \mathbb{Z}$ we can say that it is reflexive.

for symmetric:

we need to prove $(aRb \wedge bRa) \; \forall (a,b) \in A$

$$\left.\begin{aligned} \text{So, } &|a-b| \\ &= |4-2| \\ &= |2| \\ &= 2 \end{aligned}\right\} \quad \begin{aligned} \text{and } &|b-a| \\ &= |2-4| \\ &= |-2| \\ &= 2 \end{aligned}$$

Since both are ≤ 3, we can prove that the R is symmetric.

for transitive:

~~The set only includes (a,b), hence it cannot be transitive as there is no 'c' to prove the following:~~

~~$\{ (aRb)$ and (bRc) and $(aRc) \; \forall a,b,c \in R$~~

this relation is not transitive

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~~b) $R = \{ (a,b) \mid a,b \in \mathbb{Z} \wedge (a \bmod 10) = (b \bmod 10) \}$~~

for transitive:

we need to prove $(aRb) \wedge (bRc) \wedge (aRc) \ \forall a,b,c \in R$

However,

If a set A has $a = 6$, $b = 4$, $c = 2$,

we can see that

$$|a-b| = |6-4| = 2 \text{ i.e } \leqslant 3 \quad \square$$

$$|b-c| = |4-2| = 2 \text{ i.e } \leqslant 3 \quad \square$$

But, $|a-c|$

$= |6-2|$

$= |4|$

$= 4$ i.e ~~it~~ does not stand true for $\leqslant 3$

Hence, the relationship is not transitive. However, it is reflexive and symmetric.

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b) $R = \{(a,b) \mid a,b \in \mathbb{Z} \wedge (a \bmod 10) = (b \bmod 10)\}$

for reflexive:

By the properties of the equality sign we can prove that
$(a \bmod 10) = (a \bmod 10)$
As, $(a,a) \in R \ \forall\ x \in A$ we can show that
the relationship is reflexive.

for symmetric:

for this case too, by the properties of the equality sign we can prove that if $a \bmod 10 = b \bmod 10$ then
$b \bmod 10 = a \bmod 10$.
Hence, we can say that
$(a\ R\ b) \wedge (b\ R\ a) \ \forall\ (a,b) \in A$

and that this relationship is symmetric.

for transitive:

If a set has $(a \bmod 10) = (b \bmod 10)$ and
$(b \bmod 10) = (c \bmod 10)$,
we can prove that by the equality sign's property $(a \bmod 10) = (c \bmod 10)$

Hence, $(a R b) \wedge (b R c) \wedge (a R c) \ \forall\ a,b,c \in A$ $\square$

So, the relationship is reflexive, symmetric and transitive.
This is an equivalence relation

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Problem 3.4:

a) By using ':type zip' on haskell we can determine that the type signature of the zip function is

$$zip :: [a] \rightarrow [b] \rightarrow [(a,b)]$$

This shows that only two type variables appear in the type signature of zip. It cannot be more or less as the function only allows for two type variables. We cannot change its type to accommodate more or less types as that modifies the behaviour of the function.

b) → $2 + 3 :: Num\ a \Rightarrow a$

This involves the sum of two integers so the type of the output is also an Integer.

→ $2 + 9$ 'div' $3 :: Integral\ a \Rightarrow a$

This expression involves an integer division and is then added to 2 which is Int. Hence, resulting in the integral typeclass where the elements are two types: Int and Integer.

→ $2 + 9 / 3 :: Fractional\ a \Rightarrow a$

This expression uses fractional division and results in a fractional number being added to 2. So, because of the division operator this is fractional type class. that involves operations like fractions.

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→ 2 + sqrt 9 :: Floating a => a

Due tothe sqrt operator the expression is of type floating as a floating point number isthe output thatis added to '2' and still remains floating.