Assignment 1

Claudia Feochari (40000060)
Cynthia Cherrier (40005808)
Pierre Watine (40027675)
Samantha Kerr (40007328)

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Problem 1

a) Breadth-First Search

nth State Visited	State Visited	Closed List	Open List
1			S
2	S	S	A, B, C
3	A	S, A	B, C, D
4	B	S, A, B	C, D, E, F
5	C	S, A, B, C	D, E, F
6	D	S, A, B, C, D	E, F
7	E	S, A, B, C, D, E	F
8	F	S, A, B, C, D, E, F	

b) Depth-First Search

nth State Visited	State Visited	Closed List	Open List
1			S
2	S	S	A, B, C
3	A	S, A	D, B, C
4	D	S, A, D	F, B, C
5	F	S, A, D, F	B, C

c) Iterative Deepening

Depth 1			
nth State Visited	State Visited	Closed List	Open List
1			S(0)
2	S	S	A(1), B(1), C(1)
3	A	S, A	B(1), C(1)
4	B	S, A, B	C(1)
5	C	S, A, B, C	

Depth 2			
nth State Visited	State Visited	Closed List	Open List
1			S(0)
2	S	S	A(1), B(1), C(1)
3	A	S, A	D(2), B(1), C(1)
4	D	S, A, D	B(1), C(1)
5	B	S, A, D, B	E(2), F(2), C(1)
6	E	S, A, D, B, E	F(2), C(1)
7	F	S, A, D, B, E, F	C(1)

d) Uniform Cost Search

nth State Visited	State Visited	Closed List	Open List
1			S(0)
2	S	S	C(3), A(4), B(7)
3	C	S, C	A(4), B(7)
4	A	S, C, A	B(7), D(9)
5	B	S, C, A, B	D(9), E(11), F(13)
6	D	S, C, A, B, D	E(11), F(13)
7	E	S, C, A, B, D, E	F(13)
8	F	S, C, A, B, D, E, F	

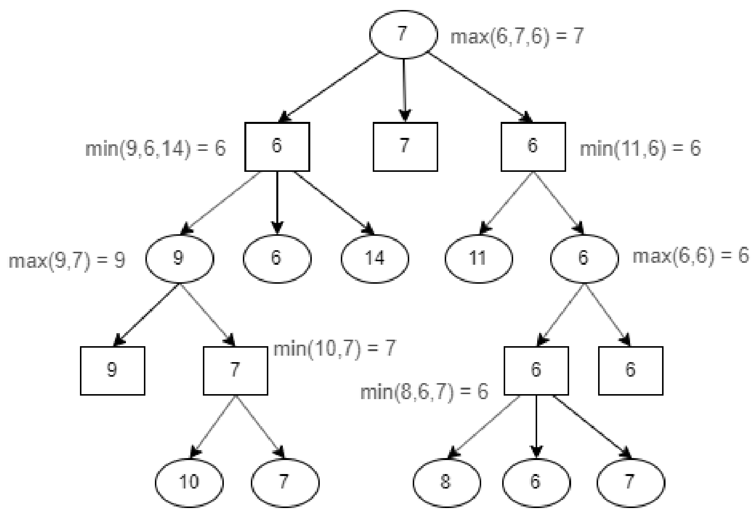
e) A* Search

nth State Visited	State Visited	Closed List	Open List
1			S(4)
2	S	S	C(3+5=8), B(7+4=11), A(4+9=13)
3	C	S, C	B(11), A(13)
4	B	S, C, B	A(13), F(7+6+0=13), D(7+3+4=14), E(7+4+3=14)
5	A	S, C, B, A	D(4+5+4=13), F(13), E(14)
6	D	S, C, B, A, D	F(13), E(14)
7	F	S, C, B, A, D, F	E(14)

Problem 2

1)

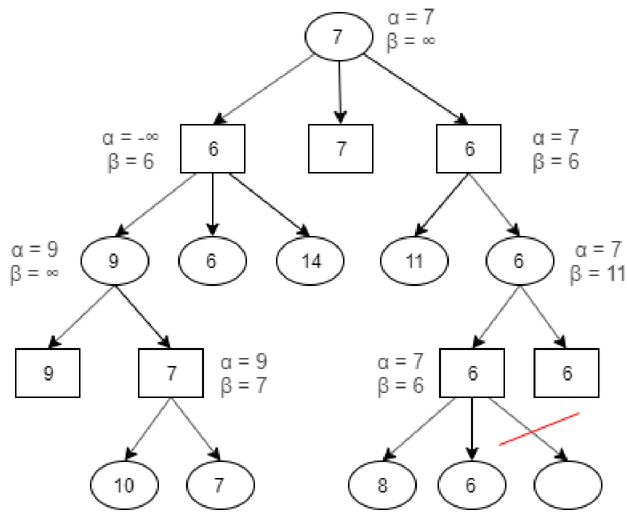
Node	Value
F	7
D	9
B	6
G	6
E	6
C	6
A	7



2)

Step	Node	α	β	Note	Value
1	A	$-\infty$	∞		
2	B	$-\infty$	∞		
3	D	$-\infty$, MAX($-\infty$,9)=9	∞		
4	F	9	∞, MIN(∞,10)=10, MIN(10,7)=7	Since $\beta(10)<=\alpha(9)$ is false, continue to next branch	7
5	D	MAX(9,7)=9	∞		9
6	B	$-\infty$	MIN(∞,9)=9, MIN(9,6)=6, MIN(6,14)=6		6
7	A	MAX($-\infty$,6)=6, MAX(6,7)=7	∞		
8	C	7	∞, MIN(∞,11)=11		
9	E	7	11		
10	G	7	11, MIN(11,8)=8, MIN(8,6)=6	Since $\beta(6)<=\alpha(7)$ is true, then the value at G is at most 6 which will not be selected at the next MAX node. Thus, we can prune branch G-7 since it will not change MIN node G.	<=6
11	E	MAX(7,6)=7, MAX(7,6)=7	11		6
12	C	7	MIN(11,6)=6		6
13	A	MAX(7,6)=7	∞		7

	Final Values		
Node	α	β	Value at Node
F	9	7	7
D	9	∞	9
B	$-\infty$	6	6
G	7	6	6
E	7	11	6
C	7	6	6
A	7	∞	7



The branch from G to 7 is pruned.

3)

No, there is no game tree that exists where the Minimax and Alpha-Beta Pruning algorithms would produce different results. Alpha-Beta is simply an algorithm that optimizes Minimax by pruning branches of the tree that do not contribute to the solution. The pruning occurs where the best MAX or MIN value has already been achieved/guaranteed. Therefore, both algorithms find the best solution, which is why given any tree, the solution will be the same.

Problem 3

Abbreviations
G = good
A = average
B = bad
ACC = accident

Probabilities	
P(G) = 25% = 0.25	P(ACC|G) = 5% = 0.05
P(A) = 50% = 0.50	P(ACC|A) = 15% = 0.15
P(B) = 25% = 0.25	P(ACC|B) = 25% = 0.25

$$\begin{aligned} P(ACC) &= P(ACC|G)xP(G) + P(ACC|A)xP(A) + P(ACC|B)xP(B) \\ &= (0.05x0.25) + (0.15x0.50) + (0.25x0.25) \\ &= 0.15 \end{aligned}$$

$$\begin{aligned} P(G|ACC) &= \frac{P(ACC|G)xP(G)}{P(ACC)} \\ &= \frac{0.05x0.25}{0.15} \\ &= 0.0833 = 8.33\% \end{aligned}$$

Problem 4

Abbreviations
L = light
B = boost
N = no name
H = heart attack

Probabilities	
P(L) = 30% = 0.30	P(H|L) = 5% = 0.05
P(B) = 40% = 0.40	P(H|B) = 20% = 0.20
P(N) = 30% = 0.30	P(H|N) = 30% = 0.30

$$\begin{aligned} P(H) &= P(H|L)xP(L) + P(H|B)xP(B) + P(H|N)xP(N) \\ &= (0.05x0.30) + (0.20x0.40) + (0.30x0.30) \\ &= 0.185 \end{aligned}$$

a)

$$\begin{aligned} P(L|H) &= \frac{P(H|L)xP(L)}{P(H)} \\ &= \frac{0.05x0.30}{0.185} \\ &= 0.0811 = 8.11\% \end{aligned}$$

b)

$$\begin{aligned} P(B|H) &= \frac{P(H|B)xP(B)}{P(H)} \\ &= \frac{0.20x0.40}{0.185} \\ &= 0.4324 = 43.24\% \end{aligned}$$

c)

$$\begin{aligned} P(N|H) &= \frac{P(H|N)xP(N)}{P(H)} \\ &= \frac{0.30x0.30}{0.185} \\ &= 0.4865 = 48.65\% \end{aligned}$$

Problem 5

1)

Abbreviations	
C = Chest Pain	E = Exercises
M = Male	H = Heart Attack
S = Smokes	

Level 0 (Root)

Heart Attack:

$$H(H) = H(\frac{4}{6}, \frac{2}{6}) = -(\frac{4}{6}\log_2\frac{4}{6} + \frac{2}{6}\log_2\frac{2}{6}) = 0.918$$

Chest Pain:

$$H(H|C = yes) = H(\frac{3}{3}, \frac{0}{3}) = -(0 + 0) = 0$$

$$H(H|C = no) = H(\frac{1}{3}, \frac{2}{3}) = -(\frac{1}{3}\log_2\frac{1}{3} + \frac{2}{3}\log_2\frac{2}{3}) = 0.918$$

$$\begin{aligned} H(H|C) &= P(C = yes) * H(H|C = yes) + P(C = no) * H(H|C = no) \\ &= \frac{3}{6}(0) + \frac{3}{6}(0.918) = 0.459 \\ gain(C) &= H(H) - H(H|C) = 0.918 - 0.459 = 0.459 \end{aligned}$$

Male

$$H(H|M = yes) = H(\frac{2}{4}, \frac{2}{4}) = -(\frac{2}{4}\log_2\frac{2}{4} + \frac{2}{4}\log_2\frac{2}{4}) = 1$$

$$H(H|M = no) = H(\frac{2}{2}, \frac{0}{2}) = -(0 + 0) = 0$$

$$\begin{aligned} H(H|M) &= P(M = yes) * H(H|M = yes) + P(M = no) * H(H|M = no) \\ &= \frac{4}{6}(1) + \frac{2}{6}(0) = 0.667 \\ gain(M) &= H(H) - H(H|M) = 0.918 - 0.667 = 0.251 \end{aligned}$$

Smokes:

$$H(H|S = yes) = H(\frac{3}{4}, \frac{1}{4}) = -(\frac{3}{4}\log_2\frac{3}{4} + \frac{1}{4}\log_2\frac{1}{4}) = 0.811$$

$$H(H|S = no) = H(\frac{1}{2}, \frac{1}{2}) = -(\frac{1}{2}\log_2\frac{1}{2} + \frac{1}{2}\log_2\frac{1}{2}) = 1$$

$$\begin{aligned} H(H|S) &= P(S = yes) * H(H|S = yes) + P(S = no) * H(H|S = no) \\ &= \frac{4}{6}(0.811) + \frac{2}{6}(1) = 0.874 \\ gain(S) &= H(H) - H(H|S) = 0.918 - 0.874 = 0.044 \end{aligned}$$

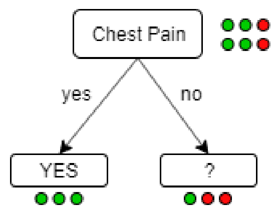
Exercise:

$$H(H|E=yes) = H(\frac{2}{4}, \frac{2}{4}) = -(\frac{2}{4}\log_2\frac{2}{4} + \frac{2}{4}\log_2\frac{2}{4}) = 1$$

$$H(H|E=no) = H(\frac{2}{2}, \frac{0}{2}) = -(0+0) = 0$$

$$\begin{aligned} H(H|E) &= P(E=yes)*H(H|E=yes) + P(E=no)*H(H|E=no) \\ &= \frac{4}{6}(1) + \frac{2}{6}(0) = 0.667 \\ gain(E) &= H(H) - H(H|E) = 0.918 - 0.667 = 0.251 \end{aligned}$$

Decision Tree:
For the root, we select the the attribute with the highest information gain, which is Chest Pain.



Level 1

Heart Attack:

$$H(H) = H(\frac{1}{3}, \frac{2}{3}) = -(\frac{1}{3}\log_2\frac{1}{3} + \frac{2}{3}\log_2\frac{2}{3}) = 0.918$$

Male:

$$H(H|M=yes) = H(\frac{0}{2}, \frac{2}{2}) = -(0+0) = 0$$

$$H(H|M=no) = H(\frac{1}{1}, \frac{0}{1}) = -(0+0) = 0$$

$$\begin{aligned} H(H|M) &= P(M=yes)*H(H|M=yes) + P(M=no)*H(H|M=no) \\ &= \frac{2}{3}(0) + \frac{1}{3}(0) = 0 \\ gain(M) &= H(H) - H(H|M) = 0.918 - 0 = 0.918 \end{aligned}$$

Smokes:

$$H(H|S=yes) = H(\frac{1}{2}, \frac{1}{2}) = -(\frac{1}{2}\log_2\frac{1}{2} + \frac{1}{2}\log_2\frac{1}{2}) = 1$$

$$H(H|S=no) = H(\frac{0}{1}, \frac{1}{1}) = -(0+0) = 0$$

$$\begin{aligned} H(H|S) &= P(S=yes)*H(H|S=yes) + P(S=no)*H(H|S=no) \\ &= \frac{2}{3}(1) + \frac{1}{3}(0) = 0.667 \\ gain(S) &= H(H) - H(H|S) = 0.918 - 0.667 = 0.251 \end{aligned}$$

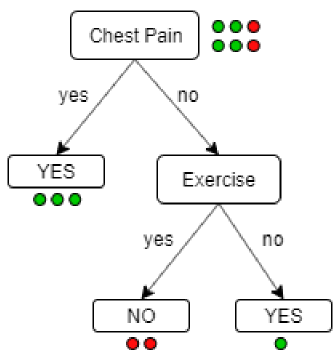
Exercise:

$$H(H|E=yes) = H(\frac{0}{2}, \frac{2}{2}) = -(0+0) = 0$$

$$H(H|E=no) = H(\frac{1}{1}, \frac{0}{1}) = -(0+0) = 0$$

$$H(H|E) = P(E=yes) * H(H|E=yes) + P(E=no) * H(H|E=no)$$
$$= \frac{2}{3}(0) + \frac{1}{3}(0) = 0$$
$$gain(E) = H(H) - H(H|E) = 0.918 - 0 = 0.918$$

Decision Tree:
For the next attribute, we can select either Exercise or Male since they both have the highest (and equal) information gain. We select Exercise.



2)

Rules:
1. IF Chest Pain = Yes THEN Heart Attack = Yes
2. IF Chest Pain = No AND Exercise = Yes THEN Heart Attack = No
3. IF Chest Pain = No AND Exercise = No THEN Heart Attack = Yes

Problem 6

Abbreviations	
H = Huge	L = Language
G = Good Looking	T = Threat

a)

$$H(T) = H(\frac{5}{10}, \frac{5}{10}) = H(\frac{1}{2}, \frac{1}{2}) = -(\frac{1}{2}\log_2\frac{1}{2} + \frac{1}{2}\log_2\frac{1}{2}) = 1$$

b)

Huge:

$$H(T|H = yes) = H(\frac{2}{5}, \frac{3}{5}) = -(\frac{2}{5}\log_2\frac{2}{5} + \frac{3}{5}\log_2\frac{3}{5}) = 0.97$$

$$H(T|H = no) = H(\frac{3}{5}, \frac{2}{5}) = -(\frac{3}{5}\log_2\frac{3}{5} + \frac{2}{5}\log_2\frac{2}{5}) = 0.97$$

$$H(T|H) = P(H = yes) * H(T|H = yes) + P(H = no) * H(T|H = no)$$

$$= \frac{5}{10}(0.97) + \frac{5}{10}(0.97) = 0.97$$

$$gain(H) = H(T) - H(T|H) = 1 - 0.97 = 0.03$$

Good Looking:

$$H(T|G = yes) = H(\frac{4}{6}, \frac{2}{6}) = -(\frac{4}{6}\log_2\frac{4}{6} + \frac{2}{6}\log_2\frac{2}{6}) = 0.918$$

$$H(T|G = no) = H(\frac{1}{4}, \frac{3}{4}) = -(\frac{1}{4}\log_2\frac{1}{4} + \frac{3}{4}\log_2\frac{3}{4}) = 0.811$$

$$H(T|G) = P(G = yes) * H(T|G = yes) + P(G = no) * H(T|G = no)$$

$$= \frac{6}{10}(0.918) + \frac{4}{10}(0.811) = 0.875$$

$$gain(G) = H(T) - H(T|G) = 1 - 0.875 = 0.125$$

Language:

$$H(T|L = French) = H(\frac{1}{5}, \frac{4}{5}) = -(\frac{1}{5}\log_2\frac{1}{5} + \frac{4}{5}\log_2\frac{4}{5}) = 0.72$$

$$H(T|L = English) = H(\frac{3}{3}, \frac{0}{3}) = -(0 + 0) = 0$$

$$H(T|L = None) = H(\frac{0}{1}, \frac{1}{1}) = -(0 + 0) = 0$$

$$H(T|L = Other) = H(\frac{1}{1}, \frac{0}{1}) = -(0 + 0) = 0$$

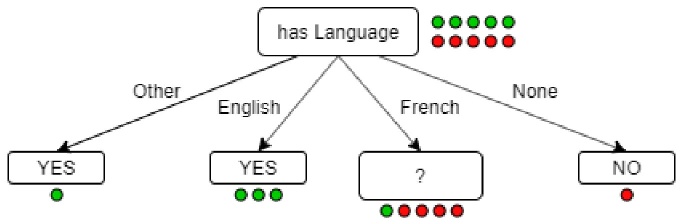
$$H(T|L) = P(L = French) * H(T|L = French) + P(L = English) * H(T|L = English)$$

$$+ P(L = None) * H(T|L = None) + P(L = Other) * H(T|L = Other)$$

$$= \frac{5}{10}(0.72) + \frac{3}{10}(0) + \frac{1}{10}(0) + \frac{1}{10}(0) = 0.36$$

$$gain(L) = H(T) - H(T|L) = 1 - 0.36 = 0.64$$

Decision Tree:
For the root, we select the attribute with the highest information gain, which is Language. Doing so would classify 5 of the table entries, eliminating them from the rest of the decision tree.



c)

Threat:

$$H(T) = H(\frac{1}{5}, \frac{4}{5}) = -(\frac{1}{5}\log_2\frac{1}{5} + \frac{4}{5}\log_2\frac{4}{5}) = 0.72$$

Huge:

$$H(T|H = yes) = H(\frac{1}{4}, \frac{3}{4}) = -(\frac{1}{4}\log_2\frac{1}{4} + \frac{3}{4}\log_2\frac{3}{4}) = 0.811$$

$$H(T|H = no) = H(\frac{0}{1}, \frac{1}{1}) = -(0 + 0) = 0$$

$$H(T|H) = P(H = yes) * H(T|H = yes) + P(H = no) * H(T|H = no)$$
$$= \frac{4}{5}(0.811) + \frac{1}{5}(0) = 0.649$$
$$gain(H) = H(T) - H(T|H) = 0.72 - 0.649 = 0.071$$

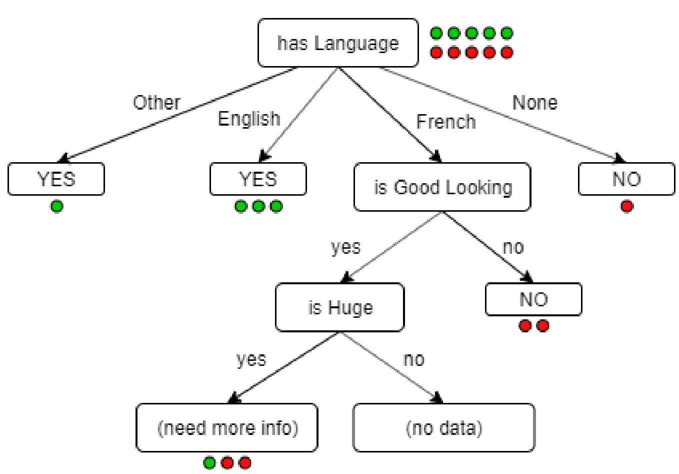
Good Looking:

$$H(T|G = yes) = H(\frac{1}{3}, \frac{2}{3}) = -(\frac{1}{3}\log_2\frac{1}{3} + \frac{2}{3}\log_2\frac{2}{3}) = 0.918$$

$$H(T|G = no) = H(\frac{0}{2}, \frac{2}{2}) = -(0 + 0) = 0$$

$$H(T|G) = P(G = yes) * H(T|G = yes) + P(G = no) * H(T|G = no)$$
$$= \frac{3}{5}(0.918) + \frac{2}{5}(0) = 0.55$$
$$gain(G) = H(T) - H(T|G) = 0.72 - 0.55 = 0.17$$

Decision Tree:
For the next level branching from L=French, we select the attribute with the highest information gain, which is Good Looking. Doing so would classify 2 of the table entries, eliminating them from the rest of the decision tree. The next level branching from G=yes would be the Huge attribute, which requires more information for its branching.



d)

person is Huge	person is good looking	person has language	person is Threat
Yes	No	English	Yes
No	No	French	No
Yes	No	French	No