DIVERGENCE

Definition: The **divergence of a vector field** is a scalar quantity that measures the rate at which the vector field spreads out (diverges) or converges (contracts) at a given point.

Key Characteristics:

1. **Positive divergence**: Indicates a "source," where the field vectors spread out.
2. **Negative divergence**: Indicates a "sink," where the field vectors converge.
3. **Zero divergence**: Indicates the field is neither spreading nor converging, such as in an incompressible fluid.

2. Divergence

- **Definition:**
  Divergence measures how much a vector field spreads out (or converges) at a point.
- **Example:**
  Imagine water flowing out of a hose. If water spreads out in all directions, the divergence is positive. If it flows inward, the divergence is negative.
- **Mathematical Expression:**
  For a vector field $\mathbf{F} = (F_x, F_y, F_z)$:

$$\text{Divergence: } \nabla \cdot \mathbf{F} = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z}$$

- **Derivation:**
  Divergence is derived by summing the partial derivatives of the vector field components with respect to their respective directions. It tells us the "outflow" rate at a point.

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Question 2: Divergence

Find the divergence of $\mathbf{F} = (xy, z, x^2)$.

Solution:

1. The formula for divergence is:

$$\nabla \cdot \mathbf{F} = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z}$$

2. Compute partial derivatives:
   - $\frac{\partial F_x}{\partial x} = \frac{\partial (xy)}{\partial x} = y$
   - $\frac{\partial F_y}{\partial y} = \frac{\partial z}{\partial y} = 0$
   - $\frac{\partial F_z}{\partial z} = \frac{\partial (x^2)}{\partial z} = 0$

3. Add them together:

$$\nabla \cdot \mathbf{F} = y + 0 + 0 = y$$

Final Answer: y

PARTIAL DERIVATIVES

What are Partial Derivatives?

Partial derivatives involve taking the derivative of a function with respect to **one variable**, treating all other variables as constants. For example:

1. $\frac{\partial}{\partial x}$: Differentiate with respect to x, treat y and z as constants.
2. $\frac{\partial}{\partial y}$: Differentiate with respect to y, treat x and z as constants.
3. $\frac{\partial}{\partial z}$: Differentiate with respect to z, treat x and y as constants.

HOW TO CALCULATE DIVERGENCE

For Example, You Want To calculate the Divergence of a vector field F(x,y,z) = x^2 +2z - y

Divergence Formula

The divergence of a vector field $\mathbf{F} = F_x\mathbf{i} + F_y\mathbf{j} + F_z\mathbf{k}$ is:

$$\nabla \cdot \mathbf{F} = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z}.$$

Here:

- $F_x = x^2$,
- $F_y = 2z$,
- $F_z = -y$.

Step-by-Step Partial Derivatives

1. Compute $\frac{\partial F_x}{\partial x}$:

$$F_x = x^2 \quad \Longrightarrow \quad \frac{\partial F_x}{\partial x} = 2x.$$

Explanation:

- x^2 is a function of x, so we differentiate it normally: $\frac{d}{dx}(x^2) = 2x$.
- y and z are constants here and do not appear in F_x.

2. Compute $\frac{\partial F_y}{\partial y}$:

$$F_y = 2z \quad \Longrightarrow \quad \frac{\partial F_y}{\partial y} = 0.$$

Explanation:

- $2z$ does not involve y (it's constant with respect to y), so its derivative is 0.

3. Compute $\frac{\partial F_z}{\partial z}$:

$$F_z = -y \quad \Longrightarrow \quad \frac{\partial F_z}{\partial z} = 0.$$

Explanation:

- $-y$ does not involve z (it's constant with respect to z), so its derivative is 0.

Step 4: Add the Partial Derivatives

Substitute into the divergence formula:

$$\nabla \cdot \mathbf{F} = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z}.$$

$$\nabla \cdot \mathbf{F} = 2x + 0 + 0 = 2x.$$

Final Answer

The divergence of $\mathbf{F}(x, y, z) = x^2\mathbf{i} + 2z\mathbf{j} - y\mathbf{k}$ is:

$$\nabla \cdot \mathbf{F} = 2x.$$

Key Notes on Partial Derivatives

1. When differentiating with respect to one variable, treat all other variables as constants.
2. Always carefully substitute into the divergence formula. Each term corresponds to a specific component of the vector field.

Key Points: Gradient vs Divergence

Gradient ($\nabla\phi$):

- **Purpose**: Shows the direction and rate of steepest increase in a scalar field.
- **Input**: Scalar field ($\phi(x, y, z)$).
- **Output**: Vector field ($\nabla\phi$).
- **Formula**:

$$\nabla\phi = \left(\frac{\partial \phi}{\partial x}, \frac{\partial \phi}{\partial y}, \frac{\partial \phi}{\partial z}\right)$$

- **Real-life analogy**: Direction of steepest climb on a hill.

Divergence ($\nabla \cdot \mathbf{F}$):

- **Purpose**: Measures the rate at which a vector field spreads out (source) or converges (sink) at a point.
- **Input**: Vector field ($\mathbf{F}(x, y, z) = F_x\mathbf{i} + F_y\mathbf{j} + F_z\mathbf{k}$).
- **Output**: Scalar field ($\nabla \cdot \mathbf{F}$).
- **Formula**:

$$\nabla \cdot \mathbf{F} = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z}$$

- **Real-life analogy:** Water spreading out from or converging into a point.