CURL

Definition: The **curl** of a vector field measures the tendency of the field to rotate or "circulate" around a point.

The result of the curl is another vector field that describes the rotation of **F** each point:

- **Direction**: Points along the axis of rotation.
- **Magnitude**: Indicates the strength of the rotation around that axis.

Real-Life Analogy

1. Imagine placing a paddle wheel in a flowing river:
    - If the water pushes the wheel to spin, the vector field has a curl at that location.
    - The curl's direction aligns with the axis around which the wheel spins.
    - The curl's magnitude tells how fast it spins.
2. **Cyclones or Tornadoes**:
   The curl measures the swirling of wind in the atmosphere. A large curl indicates strong circulation.

3. Curl

- **Definition:**
  Curl measures the rotation or twisting of a vector field around a point.
- **Example:**
  Imagine water swirling in a whirlpool. The curl of the velocity field describes the intensity and direction of the swirl.
- **Mathematical Expression:**
  For a vector field $\mathbf{F} = (F_x, F_y, F_z)$:

$$\text{Curl: } \nabla \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_x & F_y & F_z \end{vmatrix}$$

This is the determinant of a matrix and expands to:

$$\text{Curl: } \left(\frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z}\right)\mathbf{i} - \left(\frac{\partial F_z}{\partial x} - \frac{\partial F_x}{\partial z}\right)\mathbf{j} + \left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y}\right)\mathbf{k}$$

If you notice any errors, please report them to Bilal Ahmad Khan, also known as Mr. BILRED, at your earliest convenience.

Visual Summary

Concept	What It Measures	Physical Example
Gradient	Steepness and direction of a slope	Steepest path uphill on a hill
Divergence	Spread-out or convergence of a field	Water flowing out/in of a pipe
Curl	Rotation or swirl in a field	Swirling water in a whirlpool

---

- **Mathematical Expression**:
  For a vector field $\mathbf{F} = (F_x, F_y, F_z)$:

$$\text{Curl: } \nabla \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_x & F_y & F_z \end{vmatrix}$$

This is the determinant of a matrix and expands to:

$$\text{Curl: } \left(\frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z}\right)\mathbf{i} - \left(\frac{\partial F_z}{\partial x} - \frac{\partial F_x}{\partial z}\right)\mathbf{j} + \left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y}\right)\mathbf{k}$$

- **Derivation**:
  Curl is derived from the cross product of the del operator ∇ and the vector field $\mathbf{F}$. It shows how much the vector field "twists" at a point.

Question 3: Curl

Find the curl of $\mathbf{F} = (-y, x, 0)$.

Solution:

1. The formula for curl is:

$$\nabla \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ -y & x & 0 \end{vmatrix}$$

2. Expand the determinant:

$$\nabla \times \mathbf{F} = \mathbf{i}\left(\frac{\partial 0}{\partial y} - \frac{\partial x}{\partial z}\right) - \mathbf{j}\left(\frac{\partial 0}{\partial x} - \frac{\partial(-y)}{\partial z}\right) + \mathbf{k}\left(\frac{\partial x}{\partial x} - \frac{\partial(-y)}{\partial y}\right)$$

3. Simplify terms:

$$\nabla \times \mathbf{F} = \mathbf{i}(0-0) - \mathbf{j}(0-0) + \mathbf{k}(1-(-1))$$

$$\nabla \times \mathbf{F} = \mathbf{k}(2)$$

Final Answer: $2\mathbf{k}$

Question 3: Curl Revisited

We are tasked with finding the **curl** of $\mathbf{F} = (-y, x, 0)$.

---

1. Formula for Curl

$$\nabla \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_x & F_y & F_z \end{vmatrix}$$

Here:

$$F_x = -y, \quad F_y = x, \quad F_z = 0$$

2. Expand the Determinant

$$\nabla \times \mathbf{F} = \mathbf{i} \begin{vmatrix} \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x & 0 \end{vmatrix} - \mathbf{j} \begin{vmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial z} \\ -y & 0 \end{vmatrix} + \mathbf{k} \begin{vmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} \\ -y & x \end{vmatrix}$$

---

3. Compute Each Minor Determinant

First Term (Coefficient of $\mathbf{i}$):

$$\begin{vmatrix} \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x & 0 \end{vmatrix} = \left(\frac{\partial 0}{\partial y} - \frac{\partial x}{\partial z} \right) = 0 - 0 = 0$$

Second Term (Coefficient of $\mathbf{j}$):

$$\begin{vmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial z} \\ -y & 0 \end{vmatrix} = \left(\frac{\partial 0}{\partial x} - \frac{\partial (-y)}{\partial z} \right) = 0 - 0 = 0$$

Third Term (Coefficient of $\mathbf{k}$):

$$\begin{vmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} \\ -y & x \end{vmatrix} = \left(\frac{\partial x}{\partial x} - \frac{\partial (-y)}{\partial y} \right) = 1 - (-1) = 1 + 1 = 2$$

4. Combine the Results

$$\nabla \times \mathbf{F} = \mathbf{i}(0) - \mathbf{j}(0) + \mathbf{k}(2)$$

$$\nabla \times \mathbf{F} = 2\mathbf{k}$$

Final Answer

The curl is:

$$\nabla \times \mathbf{F} = 2\mathbf{k}$$