GRADIENT

Definition: The **gradient of a scalar field** is a “vector field” that points in the direction of the steepest rate of increase of the scalar field, and its magnitude represents the rate of change in that direction.

In easy words, Gradient Converts A Scalar Field Into Vector Field

Simplified Explanation: The gradient converts a scalar field (a single value at each point) into a vector field (direction + magnitude) by telling us where and how quickly the scalar value changes.

1. Gradient

- **Definition:**
  The gradient of a scalar function points in the direction of the greatest rate of increase of the function and tells you how fast the function is changing in that direction.
- **Example:**
  Imagine a hill where the height is given by $h(x, y)$. The gradient tells you the steepest path uphill from any point on the hill.
- **Mathematical Expression:**
  For a scalar function $f(x, y, z)$:

$$\text{Gradient: } \nabla f = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} \right)$$

- **Derivation:**
  Gradient comes from the partial derivatives of f with respect to its variables, measuring the rate of change along x, y, and z axes.

MORE EXPLANATION

1. Gradient (∇f)

Intuitive Explanation:

- Imagine you are on a mountain, and the height of the mountain is a function $h(x, y)$.
  The gradient tells you:
  - **Direction**: Where to walk to reach the steepest ascent.
  - **Magnitude**: How steep the ascent is.

Mathematical Definition:

For a scalar field $f(x, y, z)$, the gradient is:

$$\text{Gradient: } \nabla f = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} \right)$$

This is a vector field, meaning it assigns a vector to every point in the domain.

Geometric Insight:

- The gradient is always **perpendicular** to the level curves (contours) of f. For example, in a height map, it's perpendicular to lines of constant height.

Physical Example:

- **Heat distribution**: Imagine a heated metal plate where $T(x, y)$ represents the temperature at point (x, y).
  The gradient ∇T points towards the hottest spot on the plate and tells how fast the temperature increases in that direction.

Example Calculation:

Let $f(x, y) = x^2 + y^2$.

$$\nabla f = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right) = (2x, 2y)$$

At $(x, y) = (1, 2)$,

$$\nabla f = (2(1), 2(2)) = (2, 4)$$

This means the steepest ascent is in the direct $(2, 4)$, and the steepness is proportional to the vector's magnitude.

Question 1: Gradient

Find the gradient of $f(x, y, z) = x^2 + y^2 + z^2$ and evaluate it at $(1, 1, 1)$.

Solution:

1. The gradient is:

$$\nabla f = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} \right)$$

2. Compute partial derivatives:

$$\frac{\partial f}{\partial x} = 2x, \quad \frac{\partial f}{\partial y} = 2y, \quad \frac{\partial f}{\partial z} = 2z$$

3. So, $\nabla f = (2x, 2y, 2z)$.
4. At $(1, 1, 1)$, substitute $x = 1, y = 1, z = 1$:

$$\nabla f = (2(1), 2(1), 2(1)) = (2, 2, 2)$$

Final Answer: $(2, 2, 2)$

"Knowledge Should Be Shared,
Only With The One
Who Knows Its True Worth!
Not Everyone Deserves It"
\- **Bilal Ahmad Khan Aka Mr BILRED**