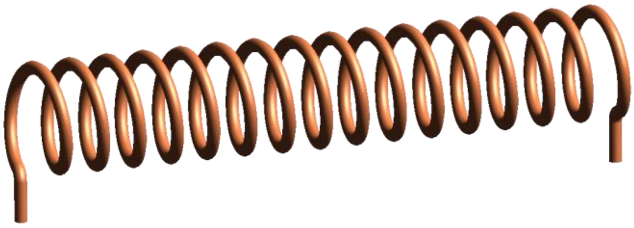
SOLENOID NUMERICALS

What is a Solenoid?

A **solenoid** is a coil of wire wound in a cylindrical shape that generates a magnetic field when an electric current passes through it. It acts like an electromagnet and is commonly used in motors, relays, and inductors.



Solenoid-1.png (1060×376)

What is Induced EMF?

Electromotive Force (EMF) is the voltage generated when the magnetic flux through a coil changes. According to **Faraday's Law**, a changing magnetic field induces an EMF, which is the principle behind transformers and electric generators.

QUESTIONS (Try to solve these at your own first):

1. A solenoid **1.30 m** long and **2.60 cm** in diameter carries a current of **18.0 A**. The magnetic field inside the solenoid is **23.0 mT**.
   Find the total length of the wire used to form the solenoid.
2. A solenoid has **2000 turns** uniformly wound over a length of **50 cm**. If it carries a current of **5 A**, find the magnetic field inside the solenoid
3. A solenoid is **1.5 m** long and has **6000 turns**. The magnetic field inside it is measured to be **15 mT**. Find the current flowing through the solenoid.
4. A solenoid with **500 turns per meter** carries a current of **3 A**. What should be its **length** if the required magnetic field inside the solenoid is **6 mT**?
5. A solenoid has **1500 turns** over a **50 cm** length. If the **current** changes from **2 A to 5 A** in **0.02 s**, find the **induced EMF** in the solenoid(Assume the diameter of the solenoid is 2cm)

Formulas for Reference

Formula Name	Formula	Description
Magnetic Field in a Solenoid	$B = \mu_0 n I$	$n = \frac{N}{L}$, where N is total turns, L is solenoid length
Number of Turns per Unit Length	$n = \frac{N}{L}$	Defines turns per meter
Total Wire Length in Solenoid	$L_{\text{wire}} = N \times \text{Circumference}$	Total length of wire used
Circumference of a Coil Turn	$C = 2\pi r$	Used for calculating total wire length
Induced EMF (Faraday's Law)	$\mathcal{E} = -N\frac{d\Phi}{dt}$	$\Phi = BA$, Magnetic flux
Area of Circular Cross-section	$A = \pi r^2$	Used in flux calculation

Question 1:

A solenoid **1.30 m** long and **2.60 cm** in diameter carries a current of **18.0 A**. The magnetic field inside the solenoid is **23.0 mT**.
Find the total length of the wire used to form the solenoid.

Answer:

Definition:

A **solenoid** is a coil of wire wound in a helical shape, often used to generate a uniform magnetic field when current flows through it. The magnetic field inside a long solenoid is given by:

$$B = \mu_0 n I$$

where:

- B = Magnetic field inside the solenoid (T)
- μ_0 = Permeability of free space ($4\pi \times 10^{-7}$ T·m/A)
- n = Number of turns per unit length ($n = \frac{N}{L}$)
- I = Current (A)
- N = Total number of turns
- L = Length of the solenoid (m)

To find the total wire length, we multiply the number of turns by the circumference of each loop.

Solution:

Step 1: Find the Number of Turns per Unit Length (n)

Using the formula for the magnetic field:

$$n = \frac{B}{\mu_0 I}$$

$$n = \frac{0.023}{(4\pi \times 10^{-7})(18.0)}$$

$$n = \frac{0.023}{2.261 \times 10^{-5}}$$

$$n \approx 1017 \text{ turns per meter}$$

Step 2: Find the Total Number of Turns (N)

$$N = n \times L$$

$$N = (1017) \times (1.30)$$

$$N \approx 1322 \text{ turns}$$

Step 3: Find the Total Length of the Wire

Each turn forms a circular loop, so the length of one turn is the **circumference** of the solenoid:

$$\text{Circumference} = 2\pi r = 2\pi(0.0130)$$

$$\text{Circumference} \approx 0.08168 \text{ m}$$

Total wire length:

$$L_{\text{wire}} = N \times \text{Circumference}$$

$$L_{\text{wire}} = (1322) \times (0.08168)$$

$$L_{\text{wire}} \approx 108 \text{ m}$$

Final Answer:

The total length of the wire used to form the solenoid is **approximately 108 meters.** ✅

Question 2:

A solenoid has **2000 turns** uniformly wound over a length of **50 cm**. If it carries a current of **5 A**, find the magnetic field inside the solenoid

Answer:

Definition:

The **magnetic field inside a solenoid** is given by:

$$B = \mu_0 n I$$

where:

- B = Magnetic field (T)
- μ_0 = Permeability of free space $4\pi \times 10^{-7}$ T·m/A
- n = Turns per unit length $n = \frac{N}{L}$
- I = Current (A)
- N = Total number of turns
- L = Length of solenoid (m)

Solution:

Step 1: Find Turns per Unit Length n

$$n = \frac{N}{L} = \frac{2000}{0.50}$$

$$n = 4000 \text{ turns per meter}$$

Step 2: Find the Magnetic Field B

$$B = \mu_0 n I$$

$$B = (4\pi \times 10^{-7})(4000)(5)$$

$$B = (4\pi \times 10^{-7}) \times 20000$$

$$B = 0.0251 \text{ T} = 25.1 \text{ mT}$$

Final Answer:

The magnetic field inside the solenoid is **25.1 mT**. ✅

Question 3:

A solenoid is **1.5 m** long and has **6000 turns**. The magnetic field inside it is measured to be **15 mT**. Find the current flowing through the solenoid.

Answer:

Definition:

Using:

$$B = \mu_0 n I$$

Solving for **current** I:

$$I = \frac{B}{\mu_0 n}$$

Solution:

Step 1: Find n

$$n = \frac{N}{L} = \frac{6000}{1.5}$$

$$n = 4000 \text{ turns per meter}$$

Step 2: Find I

$$I = \frac{15 \times 10^{-3}}{(4\pi \times 10^{-7})(4000)}$$

$$I = \frac{15 \times 10^{-3}}{5.027 \times 10^{-3}}$$

$$I \approx 2.98 \text{ A}$$

Final Answer:

The current flowing through the solenoid is **2.98 A.** ✅

Question 4:

A solenoid with **500 turns per meter** carries a current of **3 A**. What should be its **length** if the required magnetic field inside the solenoid is **6 mT**?

Answer:

Definition:

Using the **solenoid magnetic field formula:**

$$B = \mu_0 n I$$

Solving for L:

$$L = \frac{N}{n}$$

Solution:

Step 1: Find n

We already know:

$$n = 500 \text{ turns per meter}$$

Step 2: Find the Required Number of Turns

Using $B = \mu_0 n I$:

$$6 \times 10^{-3} = (4\pi \times 10^{-7})(500)(3)$$

$$6 \times 10^{-3} = 1.884 \times 10^{-3}$$

$$n = \frac{6 \times 10^{-3}}{1.884 \times 10^{-3}}$$

$$n \approx 3.18$$

$$L = \frac{N}{n} = \frac{500}{3.18}$$

$$L \approx 157.2 \text{ cm} = 1.57 \text{ m}$$

Final Answer:

The required solenoid length is **1.57 meters.** ✅

Question 5:

A solenoid has **1500 turns** over a **50 cm** length. If the **current** changes from **2 A to 5 A** in **0.02 s**, find the **induced EMF** in the solenoid(Assume the diameter of the solenoid is 2cm)

Answer:

Definition:

Using **Faraday's Law:**

$$\mathcal{E} = -N\frac{d\Phi}{dt}$$

Magnetic flux (Φ) is:

$$\Phi = BA$$

Since $B = \mu_0 nI$,

$$\mathcal{E} = -NA\frac{dB}{dt}$$

Or:

$$\mathcal{E} = -\mu_0 NnA\frac{dI}{dt}$$

Solution:

Step 1: Find n and dI/dt

$$n = \frac{N}{L} = \frac{1500}{0.50} = 3000 \text{ turns per meter}$$

Change in current:

$$\frac{dI}{dt} = \frac{5-2}{0.02} = \frac{3}{0.02} = 150 \text{ A/s}$$

Step 2: Calculate $\mathcal{E}$

$$\mathcal{E} = -\mu_0 NnA\frac{dI}{dt}$$

Assuming the solenoid has a **diameter of 2 cm** ($r = 0.01$ m):

$$A = \pi r^2 = \pi(0.01)^2 = 3.14 \times 10^{-4}\ \mathrm{m}^2$$

$$\mathcal{E} = -(4\pi \times 10^{-7})(1500)(3000)(3.14 \times 10^{-4})(150)$$

$$\mathcal{E} = -0.848\ \mathrm{V}$$

Ignoring the negative sign (which just indicates direction),

Final Answer:

The induced EMF in the solenoid is **0.85 V**. ✅