# Orthogonal Vectors

To show that the vectors $\vec{w}_1 = -b\hat{i} + a\hat{j}$ and $\vec{w}_2 = b\hat{i} - a\hat{j}$ are both orthogonal to the vector $\vec{w} = a\hat{i} + b\hat{j}$, we need to confirm that their dot products with $\vec{w}$ are zero. When two vectors have a dot product of zero, they are orthogonal.

### Step-by-Step Solution

1. **Dot Product of $\vec{w}$ and $\vec{w}_1$:**

   Given:

   $$\vec{w} = a\hat{i} + b\hat{j}$$

   $$\vec{w}_1 = -b\hat{i} + a\hat{j}$$

   The dot product $\vec{w} \cdot \vec{w}_1$ is:

   $$\vec{w} \cdot \vec{w}_1 = (a\hat{i} + b\hat{j}) \cdot (-b\hat{i} + a\hat{j})$$

   Expanding the dot product:

   $$= a(-b) + b(a)$$

   $$= -ab + ab = 0$$

   Since $\vec{w} \cdot \vec{w}_1 = 0$, $\vec{w}_1$ is orthogonal to $\vec{w}$.

2. **Dot Product of $\vec{w}$ and $\vec{w}_2$:**

   Given:

   $$\vec{w}_2 = b\hat{i} - a\hat{j}$$

   The dot product $\vec{w} \cdot \vec{w}_2$ is:

   $$\vec{w} \cdot \vec{w}_2 = (a\hat{i} + b\hat{j}) \cdot (b\hat{i} - a\hat{j})$$

Expanding the dot product:

$$= a(b) + b(-a)$$

$$= ab - ab = 0$$

Since $\vec{w} \cdot \vec{w}_2 = 0$, $\vec{w}_2$ is also orthogonal to $\vec{w}$.

## Conclusion

Both $\vec{w}_1$ and $\vec{w}_2$ are orthogonal to $\vec{w}$ because their dot products with $\vec{w}$ are zero.

“Education should be free and accessible to all, but it should be granted only to those who truly deserve it." – Bilal Ahmad Khan, also known as Mr. BILRED