Proof Of The Identity:

To prove the identity:

$$|\vec{A} \times \vec{B}|^2 + |\vec{A} \cdot \vec{B}|^2 = |\vec{A}|^2|\vec{B}|^2$$

we'll use properties of the dot product and cross product, along with trigonometric identities. Here's the step-by-step derivation:

Step 1: Define Terms

Let:

- $\vec{A}$ and $\vec{B}$ be two vectors.
- θ be the angle between $\vec{A}$ and $\vec{B}$.

Step 2: Compute $|A \cdot B|^2$

The dot product $\vec{A} \cdot \vec{B}$ is given by:

$$\vec{A} \cdot \vec{B} = |\vec{A}||\vec{B}| \cos\theta$$

So, the square of the magnitude of the dot product is:

$$|\vec{A} \cdot \vec{B}|^2 = (|\vec{A}||\vec{B}| \cos\theta)^2 = |\vec{A}|^2|\vec{B}|^2 \cos^2\theta$$

Step 3: Compute $|\vec{A} \times \vec{B}|^2$

The cross product $\vec{A} \times \vec{B}$ has a magnitude given by:

$$|\vec{A} \times \vec{B}| = |\vec{A}||\vec{B}| \sin\theta$$

The square of the magnitude of the cross product is:

$$|\vec{A} \times \vec{B}|^2 = (|\vec{A}||\vec{B}| \sin\theta)^2 = |\vec{A}|^2|\vec{B}|^2 \sin^2\theta$$

Step 4: Add the Two Expressions

Now we add $|\vec{A} \times \vec{B}|^2$ and $|\vec{A} \cdot \vec{B}|^2$:

$$|\vec{A} \times \vec{B}|^2 + |\vec{A} \cdot \vec{B}|^2 = |\vec{A}|^2|\vec{B}|^2 \sin^2\theta + |\vec{A}|^2|\vec{B}|^2 \cos^2\theta$$

Step 5: Factor Out Common Terms

Factor out $|\vec{A}|^2|\vec{B}|^2$ from the right-hand side:

$$= |\vec{A}|^2|\vec{B}|^2(\sin^2\theta + \cos^2\theta)$$

Step 6: Simplify Using the Pythagorean Identity

Since $\sin^2\theta + \cos^2\theta = 1$, we get:

$$|\vec{A} \times \vec{B}|^2 + |\vec{A} \cdot \vec{B}|^2 = |\vec{A}|^2|\vec{B}|^2 \cdot 1 = |\vec{A}|^2|\vec{B}|^2$$

Conclusion

This proves that:

$$|\vec{A} \times \vec{B}|^2 + |\vec{A} \cdot \vec{B}|^2 = |\vec{A}|^2|\vec{B}|^2$$

KNOWLEDGE MUST ONLY BE SHARED WITH THE ONE

WHO KNOWS ITS TRUE WORTH

NOT EVERYONE DESERVES IT - BILRED