FINDING THE EQUATION OF LINE A(1,1,-1), B(2,0,2), C(0,-2,1)

To find the equation of a plane passing through three points $A(1, 1, -1)$, $B(2, 0, 2)$, and $C(0, -2, 1)$, we can use the following steps:

Step 1: Determine Two Vectors in the Plane

To compute the plane, we need two vectors that lie on the plane. We can obtain these by finding the vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$.

1. **Vector $\overrightarrow{AB}$**:

$$\overrightarrow{AB} = B - A = (2 - 1, 0 - 1, 2 - (-1)) = (1, -1, 3)$$

2. **Vector $\overrightarrow{AC}$**:

$$\overrightarrow{AC} = C - A = (0 - 1, -2 - 1, 1 - (-1)) = (-1, -3, 2)$$

Step 2: Find the Normal Vector to the Plane

The normal vector $\vec{n}$ of the plane can be found by taking the cross product of $\overrightarrow{AB}$ and $\overrightarrow{AC}$:

$$\vec{n} = \overrightarrow{AB} \times \overrightarrow{AC}$$

Calculate the cross product:

$$\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 3 \\ -1 & -3 & 2 \end{vmatrix}$$

Expanding this determinant:

$$\vec{n} = \hat{i}((-1)(2) - (3)(-3)) - \hat{j}((1)(2) - (3)(-1)) + \hat{k}((1)(-3) - (-1)(-1))$$

Simplify each component:

- $\hat{i}$ component: $(-1)(2)-(3)(-3)=-2+9=7$
- $\hat{j}$ component: $-((1)(2)-(3)(-1))=-(2+3)=-5$
- $\hat{k}$ component: $(1)(-3)-(-1)(-1)=-3-1=-4$

So, $\vec{n}=7\hat{i}-5\hat{j}-4\hat{k}$, or in component form:

$$\vec{n}=(7,-5,-4)$$

Step 3: Write the Equation of the Plane

The equation of a plane with normal vector $\vec{n}=(A,B,C)$ passing through a point (x_0,y_0,z_0) is:

$$A(x-x_0)+B(y-y_0)+C(z-z_0)=0$$

Using point $A(1,1,-1)$ and $\vec{n}=(7,-5,-4)$, we substitute into the formula:

$$7(x-1)-5(y-1)-4(z+1)=0$$

Expanding and simplifying:

$$7x-7-5y+5-4z-4=0$$

$$7x-5y-4z-6=0$$

Final Answer

The equation of the plane is:

$$7x-5y-4z=6$$