Trigonometric Formulas

Basic Trigonometric Functions:

- **Sine:**

$$\sin(\theta)$$

- **Cosine:**

$$\cos(\theta)$$

- **Tangent:**

$$\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)}$$

- **Cotangent:**

$$\cot(\theta) = \frac{1}{\tan(\theta)} = \frac{\cos(\theta)}{\sin(\theta)}$$

- **Secant:**

$$\sec(\theta) = \frac{1}{\cos(\theta)}$$

- **Cosecant:**

$$\csc(\theta) = \frac{1}{\sin(\theta)}$$

Pythagorean Identities:

1. $\sin^2(\theta) + \cos^2(\theta) = 1$
2. $1 + \tan^2(\theta) = \sec^2(\theta)$
3. $1 + \cot^2(\theta) = \csc^2(\theta)$

Sum and Difference Formulas:

1. **Sine:**

$$\sin(A+B) = \sin(A)\cos(B) + \cos(A)\sin(B)$$

$$\sin(A-B) = \sin(A)\cos(B) - \cos(A)\sin(B)$$

2. **Cosine:**

$$\cos(A+B) = \cos(A)\cos(B) - \sin(A)\sin(B)$$

$$\cos(A-B) = \cos(A)\cos(B) + \sin(A)\sin(B)$$

3. **Tangent:**

$$\tan(A+B) = \frac{\tan(A)+\tan(B)}{1-\tan(A)\tan(B)}$$

$$\tan(A-B) = \frac{\tan(A)-\tan(B)}{1+\tan(A)\tan(B)}$$

Double Angle Formulas:

1. **Sine:**

$$\sin(2A) = 2\sin(A)\cos(A)$$

2. **Cosine:**

$$\cos(2A) = \cos^2(A) - \sin^2(A) = 2\cos^2(A) - 1 = 1 - 2\sin^2(A)$$

3. **Tangent:**

$$\tan(2A) = \frac{2\tan(A)}{1-\tan^2(A)}$$

Half Angle Formulas:

1. **Sine:**

$$\sin\left(\frac{A}{2}\right) = \pm\sqrt{\frac{1-\cos(A)}{2}}$$

2. **Cosine:**

$$\cos\left(\frac{A}{2}\right) = \pm\sqrt{\frac{1+\cos(A)}{2}}$$

3. **Tangent:**

$$\tan\left(\frac{A}{2}\right) = \pm\sqrt{\frac{1-\cos(A)}{1+\cos(A)}} = \frac{\sin(A)}{1+\cos(A)} = \frac{1-\cos(A)}{\sin(A)}$$

Product-to-Sum and Sum-to-Product Formulas:

1. **Product-to-Sum:**

$$\sin(A)\sin(B) = \frac{1}{2}[\cos(A-B) - \cos(A+B)]$$

$$\cos(A)\cos(B) = \frac{1}{2}[\cos(A-B) + \cos(A+B)]$$

$$\sin(A)\cos(B) = \frac{1}{2}[\sin(A+B) + \sin(A-B)]$$

2. **Sum-to-Product:**

$$\sin(A) + \sin(B) = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

$$\sin(A) - \sin(B) = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$

$$\cos(A) + \cos(B) = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

$$\cos(A) - \cos(B) = -2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$

Inverse Trigonometric Functions:

1. Arcsine (Inverse of Sine):

$$\sin^{-1}(x) \quad \text{or} \quad \arcsin(x)$$

2. Arccosine (Inverse of Cosine):

$$\cos^{-1}(x) \quad \text{or} \quad \arccos(x)$$

3. Arctangent (Inverse of Tangent):

$$\tan^{-1}(x) \quad \text{or} \quad \arctan(x)$$

4. Arccotangent (Inverse of Cotangent):

$$\cot^{-1}(x) \quad \text{or} \quad \operatorname{arccot}(x)$$

5. Arcsecant (Inverse of Secant):

$$\sec^{-1}(x) \quad \text{or} \quad \operatorname{arcsec}(x)$$

6. Arccosecant (Inverse of Cosecant):

$$\csc^{-1}(x) \quad \text{or} \quad \operatorname{arccsc}(x)$$

Trigonometric Limits:

1. Limit of Sine:

$$\lim_{x \to 0} \frac{\sin(x)}{x} = 1$$

2. Limit of Tangent:

$$\lim_{x \to 0} \frac{\tan(x)}{x} = 1$$

Derivatives of Trigonometric Functions:

1. Derivative of Sine:

$$\frac{d}{dx} \sin(x) = \cos(x)$$

2. Derivative of Cosine:

$$\frac{d}{dx} \cos(x) = -\sin(x)$$

3. Derivative of Tangent:

$$\frac{d}{dx} \tan(x) = \sec^2(x)$$

4. **Derivative of Cotangent:**

$$\frac{d}{dx}\cot(x) = -\csc^2(x)$$

5. **Derivative of Secant:**

$$\frac{d}{dx}\sec(x) = \sec(x)\tan(x)$$

6. **Derivative of Cosecant:**

$$\frac{d}{dx}\csc(x) = -\csc(x)\cot(x)$$

Integrals of Trigonometric Functions:

1. **Integral of Sine:**

$$\int \sin(x)\,dx = -\cos(x) + C$$

2. **Integral of Cosine:**

$$\int \cos(x)\,dx = \sin(x) + C$$

3. **Integral of Tangent:**

$$\int \tan(x)\,dx = \ln|\sec(x)| + C$$

4. **Integral of Cotangent:**

$$\int \cot(x)\,dx = \ln|\sin(x)| + C$$

5. **Integral of Secant:**

$$\int \sec(x)\,dx = \ln|\sec(x) + \tan(x)| + C$$

6. **Integral of Cosecant:**

$$\int \csc(x)\,dx = \ln|\csc(x) + \cot(x)| + C$$

SUMMARY:

1. Inverse Trigonometric Functions:

The **range** of inverse trigonometric functions is important when using them in calculus problems:

- $\sin^{-1}(x)$ (arcsin) has a range of $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.
- $\cos^{-1}(x)$ (arccos) has a range of $[0, \pi]$.
- $\tan^{-1}(x)$ (arctan) has a range of $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

These ranges are essential when solving problems, as the inverse functions only give results within specific intervals.

2. Trigonometric Identities:

All **Pythagorean**, **Sum-to-Product**, and **Product-to-Sum** formulas are correct. Just remember that some are applicable only in specific scenarios (such as specific angles or limits).

3. Derivative and Integral of Tangent:

The formulas for the **derivatives** and **integrals** of the tangent function are accurate:

- **Derivative of** $\tan(x)$:
  $\frac{d}{dx}\tan(x) = \sec^2(x)$
- **Integral of** $\tan(x)$:
  $\int \tan(x)\,dx = \ln|\sec(x)| + C$

However, while calculating the **integral of** $\tan(x)$, some prefer writing it as:

$$\int \tan(x)\,dx = -\ln|\cos(x)| + C$$

This is an alternative form and is mathematically equivalent because:

$$\ln|\sec(x)| = -\ln|\cos(x)|$$

4. Derivative of Cotangent:

The **derivative of** $\cot(x)$ is correct:

- $\frac{d}{dx}\cot(x) = -\csc^2(x)$.

5. Integral of Cotangent:

The **integral of** $\cot(x)$ is also correctly given:

- $\int \cot(x)\,dx = \ln|\sin(x)| + C$