~1800 hrs 04/12/2024 CAFÉ MCS

BILRED

End ~2005 hrs

Explain the SANDWICH Theorem

(1)

Definition:

Sandwich Theorem is a way to find the limits of a func by "trapping" it between two other functions.
If the ~~oth~~ other two functions have the same limit at a certain Point. The middle function will have ~~the~~ that same limit

Asaan Alfaaz main
Sandwich jese hota hai

- Top bread = $h(u)$ an upper function.
- Filling = $g(u)$ the function you're trying to find the limit of
- Bottom bread = $f(u)$ (a lower function)

$$f(u) \leq g(u) \leq h(u)$$

Basic SANDWICH

$$\lim_{u \to 0} u^2 \sin(u)$$

→ **Step 1 - Bound the function**

We know, for all values of u, $\sin(u)$ is always between -1 and $+1$. This is the property of sine function, so we can write

$$-1 \leq \sin(u) \leq 1$$

→ **STEP 2 - Multiply the inequality by u^2**

BILREC

STEP 2 -Multiply the inequality by x^2

Since x^2 is always positive (whether x is positive or negative), multiplying all parts of the inequality by x^2 doesn't change the direction of the inequalities.

$$-x^2 \leq x^2 \sin(x) \leq x^2$$

STEP 3: Take the limits of the bound.

Now, lets find the limits of the expressions on the left and right sides of the inequality as $x \to 0$

For the left side $\lim\limits_{x \to 0} -x^2 = 0$

For the right side $\lim\limits_{x \to 0} x^2 = 0$

STEP 4: Apply SANDWICH THEOREM

WE have our SANDWICH THERE,

(lower bound) $\leq$ $g(x)$ $\leq$ (upper bound)
(lower bread) $\leq$ filling $\leq$ (upper bread)

we have

$$\lim_{x \to 0} (-x^2) = \lim_{x \to 0} (x^2) = 0$$

By SANDWICH THEOREM

$$\lim_{x \to 0} x^2 \sin(x) = 0$$

Summary

1) Find the bounds for $\sin(u)$
2) Multiply the entire inequality by u^2
3) Take the limits of the bounds as $u \to 0$
4) Use the sandwich theorem to conclude that the limit of $u^2\sin(u)$ as $u \to 0$ is 0.

Where The Heck is SANDWICH Theorem USED IRL?

Physics - Particle Speed and Energy.
Computer Graphics - Antialiasing, you might've heard this one if you're a gamer.
This smooths the edges of objects in a digital image.
Economics - Cost/Profit limits in business models.

BILREE

EXERCISE - SANDWICH THEOREM

1) if $\sqrt{5-2x^2} \le f(x) \le \sqrt{5-x^2}$ for $-1 \le x \le 1$, find $\lim_{x \to 0} f(x)$

We are given

$$g(x) \le f(x) \le h(x)$$

$$\lim_{x \to a} g(x) = \lim_{x \to a} h(x) = L$$

(i) lower Bound (lower bread)

$$\lim_{x \to 0} \sqrt{5-2x^2} = \sqrt{5-2(0)} = \sqrt{5}$$

(ii) Upper Bound (upper bread)

$$\lim_{x \to 0} \sqrt{5-x^2} = \sqrt{5-0} = \sqrt{5}$$

Apply Sandwich

$$\sqrt{5-2x^2} \le f(x) \le \sqrt{5-x^2}$$

$$\lim_{x \to 0} \sqrt{5-2x^2} = \lim_{x \to 0} \sqrt{5-x^2} = \sqrt{5}$$

So, $\lim_{x \to 0} f(x) = \sqrt{5}$

(ii) if $2-x^2 \le g(x) \le 2\cos x$ for all x, find $\lim\limits_{x \to 0} g(x)$.

Solution

→ lower bound $\le g(x) \le$ upper bound

→ lower bound $= 2-x^2$

$\lim\limits_{x\to 0} 2-x^2 \Rightarrow 2-0 \Rightarrow 2$

→ Upper bound $= 2\cos x$

$= \lim\limits_{x\to 0} 2\cos x \Rightarrow 2\cos(0) = 2$

→ So,

→ $\lim\limits_{x\to 0} 2-x^2 = \lim\limits_{x\to 0} 2\cos x = 2$

→ $\boxed{g(x) = 2}$

(iii) If $1 \leq f(x) \leq x^2 + 2x + 2$ for $\forall x$
find $\lim_{x \to -1} f(x)$

Solution

lets just keep things simple, because from previous questions you might be ~~not~~ understood what I really wanna tell you.

$\Rightarrow \lim_{x \to -1} x^2 + 2x + 2$

$\Rightarrow (-1)^2 + 2(-1) + 2$

$\Rightarrow 1 + (-2) + 2$

$\Rightarrow 1.$

$\Rightarrow \lim_{x \to -1} 1$

It remains 1.

So we have.

$$\lim_{x \to -1} 1 \doteq \lim_{x \to -1} x^2 + 2x + 2 = 1$$

So for ~~gfu~~ $f(x)$, we get

$$\boxed{\lim_{x \to -1} f(x) = +1}$$

(iv) Using squeeze theorem, show that

$$\lim_{u\to 0} u^2\cos(20\bar{\pi}u) = 0$$

Solution:

⇒ look step ① is that, dont panic if you saw something like this. They term $20\bar{\pi}u$ is oscillatory but it always lies between -1 and 1

$$-1 \leq \cos(20\pi u) \leq 1$$

Multiply by u^2 (which is always non-negative)

$$-u^2 \leq u^2\cos(20\pi u) \leq u^2$$

⇒ Applying limits to the bounds (breads)

$$\lim_{u\to 0}(-u^2) = 0 \qquad \text{(lower bound)}$$

$$\lim_{u\to 0}(u^2) = 0 \qquad \text{(upper bound)}$$

So, $\lim_{u\to 0} u^2\cos(20\bar{\pi}u) = 0.$

⟷

For hint, Sine and cosine functions always oscillate between -1 and $+1$, no matter the input. $-1 \leq \sin(u) \leq 1$

$-1 \leq \cos(u) \leq 1$

BILRED

Why do we multiply by x^2?

→ x^2 represents the "amplitude" of the oscillation caused by $\cos(20\pi x)$

→ As $x \to 0$, x^2 becomes smaller and smaller, effectively "squeezing" the oscillations of $\cos(20\pi x)$ towards 0.

→ Multiplying $\cos(20\pi x)$ with x^2 dampen the oscillation

You must have asked why x^2? why not x

If $x \to 0$, this would still squeeze to 0, and the Squeeze Theorem would work fine. In fact, this approach is valid for such a limit.

x approaches 0 more slowly than x^2
x^2 dampens the oscillations of $\cos(2x)$ more quickly making it decay faster $x \to 0$.

(v) If $3x \leq f(x) \leq x^3+2$ for $0 \leq x \leq 2$
evaluate $\lim_{x \to 1} f(x)$.

Solution

→ $\lim_{x \to 1} 3x \Rightarrow 3(1) = 3$ (upper bound)

→ $\lim_{x \to 1} x^3+2 \Rightarrow 1^3+2 = 3$ (lower bound)

So, $\lim_{x \to 1} 3x = \lim_{x \to 1} x^3+2 = 3$

$\lim_{x \to 1} f(x) = 3$ Answer