Coplanarity In Vectors

To determine the value of x that makes vectors $\vec{a} = \hat{i} + 5\hat{j} + 4\hat{k}$, $\vec{b} = \hat{i} + 2\hat{j} - 3\hat{k}$, and $\vec{c} = 3\hat{i} + x\hat{j} + 5\hat{k}$ coplanar, we can use the condition for coplanarity:

Coplanarity Condition

Three vectors $\vec{a}$, $\vec{b}$, and $\vec{c}$ are coplanar if their **scalar triple product** is zero:

$$\vec{a} \cdot (\vec{b} \times \vec{c}) = 0$$

The scalar triple product can be evaluated using the determinant:

$$\vec{a} \cdot (\vec{b} \times \vec{c}) = \begin{vmatrix} 1 & 5 & 4 \\ 1 & 2 & -3 \\ 3 & x & 5 \end{vmatrix}$$

Solving the Determinant

Expanding this determinant:

$$\begin{vmatrix} 1 & 5 & 4 \\ 1 & 2 & -3 \\ 3 & x & 5 \end{vmatrix} = 1 \begin{vmatrix} 2 & -3 \\ x & 5 \end{vmatrix} - 5 \begin{vmatrix} 1 & -3 \\ 3 & 5 \end{vmatrix} + 4 \begin{vmatrix} 1 & 2 \\ 3 & x \end{vmatrix}$$

Step-by-Step Expansion

1. Expanding the first term:

$$1 \cdot ((2 \cdot 5) - (-3 \cdot x)) = 1 \cdot (10 + 3x) = 10 + 3x$$

2. Expanding the second term:

$$-5 \cdot ((1 \cdot 5) - (-3 \cdot 3)) = -5 \cdot (5 + 9) = -5 \cdot 14 = -70$$

3. Expanding the third term:

$$4 \cdot ((1 \cdot x) - (2 \cdot 3)) = 4 \cdot (x - 6) = 4x - 24$$

Putting it all together:

$$10 + 3x - 70 + 4x - 24 = 0$$

Combine like terms:

$$7x - 84 = 0$$

Solving for x:

$$7x = 84$$

$$x = 12$$

Final Answer

The value of x that makes the vectors $\vec{a}$, $\vec{b}$, and $\vec{c}$ coplanar is:

$$x = 12$$

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