Asymptotes

1. Vertical Asymptotes

- **Condition**: Vertical asymptotes occur where the function approaches infinity as x approaches a specific value.
  - Typically, this happens when the **denominator of a rational function** equals zero (while the numerator is non-zero at that value).
- **Mathematical Expression**:
  - For a rational function $\frac{f(x)}{g(x)}$, if $g(x) = 0$ and $f(x) \neq 0$ at some $x = a$, then $x = a$ is a vertical asymptote.

Example:

- $f(x) = \frac{1}{x-3}$, vertical asymptote at $x = 3$.

2. Horizontal Asymptotes

- **Condition**: Horizontal asymptotes describe the behavior of the function as $x \to \infty$ or $x \to -\infty$. It is the **limit of the function** as x becomes very large.
  - It occurs when the **function approaches a constant** as $x \to \infty$ or $x \to -\infty$.
- **Conditions for Horizontal Asymptotes**:
  1. **If degree of numerator < degree of denominator**: Horizontal asymptote is at $y = 0$.
  2. **If degree of numerator = degree of denominator**: Horizontal asymptote is at $y = \frac{\text{leading coefficient of numerator}}{\text{leading coefficient of denominator}}$.
  3. **If degree of numerator > degree of denominator**: **No horizontal asymptote**, but you may have an oblique asymptote instead.

Example:

- For $f(x) = \frac{2x^2+3}{x^2+1}$, degree of numerator = degree of denominator, so horizontal asymptote is $y = \frac{2}{1} = 2$.

3. Oblique (Slant) Asymptotes

- **Condition**: Oblique asymptotes occur when the **degree of the numerator** is **one greater** than the degree of the denominator.
  - In this case, there is no horizontal asymptote, but the function will approach a line that is neither horizontal nor vertical.
- **How to Find**:
  - Perform **polynomial division** to find the equation of the slant asymptote.

  Example:
  - For $f(x) = \frac{x^2+2x+1}{x+1}$, degree of numerator is one greater than the degree of denominator. Perform division to get the slant asymptote $y = x + 1$.

Summary of Asymptote Conditions:

1. **Vertical Asymptote**:
   - Occurs where the denominator is zero and the numerator is non-zero at the same point.
2. **Horizontal Asymptote**:
   - Degree of numerator < degree of denominator: Horizontal asymptote at $y = 0$.
   - Degree of numerator = degree of denominator: Horizontal asymptote at $y = \frac{\text{leading coefficient of numerator}}{\text{leading coefficient of denominator}}$.
   - Degree of numerator > degree of denominator: **No horizontal asymptote**, but check for oblique asymptote.
3. **Oblique Asymptote**:
   - Occurs if the degree of the numerator is one greater than the degree of the denominator.

“Education should be free and accessible to all, but it should be granted only to those who truly deserve it." – Bilal Ahmad Khan, also known as Mr. BILRED