1002 hrs

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0859 hrs COMPUTER LAB 2, FRONT BENCH. MCS 05/12/2024

LIMITS.

A LIMIT DESCRIBES WHAT A FUNCTION'S VALUE APPROACHES AS THE INPUT (x) gets close to a certain point

→ **Left hand limit:**

The value of $f(x)$ as x approaches a certain point from the left (smaller values of x)

Notation: $\lim_{x \to c^-} f(x)$ ∴ $x \to c^-$ means x approaches c from values less than c

• **How to find L.H.L**

→ Look at the part of the func where $x < c$
→ Plug in values "slightly smaller than" c into the func to see the behaviour

→ **Right hand limit**

The value of $f(x)$ as x approaches a certain point from the right (greater values of x)

Notation: $\lim_{x \to c^+} f(x)$ ∴ $x \to c^+$ means x approaches c from values greater than c

• **How to find R.H.L:**

→ look at the part of the func where $x > c$
→ Plug in the values slightly larger than c into the func to see the behaviour.

If R.HL = L.H.L, the limit exists at that point

$$\lim_{x \to c} f(x) = LHL = RHL$$

Otherwise the limit does not exist.

Quick Tips for RHL and L.H.L

→ L.H.L look at $x < c$

→ R.H.L look at ~~$x \geq c$~~ $x > c$

→ L.H.L = R.H.L limit exists

→ L.H.L = R.H.L limit does not exists

Domain

Example:

Got it?

$$f(x) = \begin{cases} x^2+3 & \text{if } x < 1 \\ x+1 & \text{if } x \geq 1 \end{cases}$$

This sign means its L.H.L

- Left hand limit $x \to 1^-$

We'll use $f(x) = x^2+3$

$$\lim_{x \to 1^-} f(x) = 1^2 + 3 = 4$$

- Right hand limit $x \to 1^+$

As I've mentioned it before, look for $x \geq c$

This means its R.H.L

we'll see $x \to 1^+$

$$\lim_{x \to 1^+} f(x) = x + 1$$

$$= 1 + 1 = 2$$

So, Limit does not exists at $x = 1$ because L.H.L ≠ R.H.L

EXERCISE

L.H.L $x < 2$

if $f(x) = \begin{cases} 3 & \text{if } x \leq -2 \\ -1/2\, x^2 & \text{if } -2 < x\ 2 \\ 3 & \text{if } x \geq 2 \end{cases}$

$x > 2$ RHL

Find

(i) $\lim\limits_{x \to 2^-} f(x)$

As I told ya before that this sign means L.H.L. So, look for value $x < C$. ~~So~~. what I see, $x \leq -2$ at 3.

& $\lim\limits_{x \to 2^-} f(x) = 3$

This is a constant, and limit of this constant remains same.

(ii) $\lim\limits_{x \to 2^+} f(x)$

Here, we need to find the Right hand limit. We need to look for $x > c$.

$\lim\limits_{x \to 2^+} f(x) = 3$ (Again, its at $x \geq 2$)

R.H.limit is at 3!

(iii) $\lim_{x \to -2^+} f(x)$

Again, we need to find R.H.L but its -2 this time so $x > -2$. Got it?

And from there I can see $x > -2$ at

$\frac{-1}{2} x^2$

$$\lim_{x \to -2^+} f(x) = \cancel{\lim} \frac{-1 x^2}{2}$$

Applying limits

$$\lim_{x \to -2^+} f(x) = \frac{-1(-2)^2}{2}$$

$$= \frac{-4^{2}}{\cancel{2}}$$

$$= -2 \quad \text{ANSWER}$$

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Question

Let $f(x) = (x+3)\dfrac{|x+2|}{(x+2)}$

Find

(i) $\lim\limits_{x \to -2^+} (x+3)\dfrac{|x+2|}{(x+2)}$

(ii) $\lim\limits_{x \to -2^-} (x+3)\dfrac{|x+2|}{(x+2)}$

Solution:

Before proceeding, lets understand what the modulus Does.
The modulus $|x+3|$ behaves differently depending on the value of x:

(1) When $x+2 \geq 0$, $|x+2|$ it will be $x+2$ (R.H.L)

(2) When $x+2 \leq 0$, $|x+2|$ it will be $-(x+2)$ (L.H.L)

This gives us two cases

$f(x) =$

Solve for R.H.L $\lim\limits_{x \to -2^+}$

* For $x \to -2^+$, means $x > c$, $x > -2$.

This means $|x+2| = x+2$

$$f(x) = (x+3)\frac{\cancel{(x+2)}}{\cancel{x+2}}$$

$$= -2+3$$

$$= 1 \quad \text{R.H.L}$$

→ Solve for L.H.L:

$x \to -2^-$, $x < -2$ Means $|x+2| = -(x+2)$

Substitute into the func

$$\lim_{x \to -2^-} f(x) = \frac{(x+3)\cancel{-(x+2)}}{\cancel{(x+2)}}$$

$$= -(x+3)$$

$$= -(-2+3) \quad \text{Applying limits}$$

$$= -1 \quad \text{L.H.L}$$

Conclusion:

$$\lim_{x \to -2^+} f(x) = 1$$

$$\lim_{x \to -2^-} f(x) = -1$$

The left hand limit and right hand limit is not equal, the two sided limit does not exists

Key-points

(i) **Left hand limit ($x \to c^-$):**
This happen when x is smaller than constant
$x < c$

(ii) **Right hand limit ($x \to c^+$):**
This happens when x ~~is~~ is greater than constant $x > c$

Regardless of sign - - - - - - - - - - -

< I guess, you've learnt something with this ONE! >

What the heck is PIECE-WISE FUNCTION?

Fortunately it's not heck, but in easy words its defined as,

A Piecewise function is a function that is made up of different formulas or rules for different parts of its domain (the input values).

Example:

$$f(x) = \begin{cases} x^2, & \text{if } x < 1 \\ x+2 & \text{if } x \geq 1 \end{cases}$$

This means.
- if $x < 1$, use the rule $f(x) = x^2$
- if $x \geq 1$, use the rule $f(x) = x+2$