



## LIMITS.

A LIMIT DESCRIBES WHAT A FUNCTION'S VALUE APPROACHES AS THE INPUT ( $u$ ) gets close to a certain point

### → Left hand limit:

The value of  $f(u)$  as  $u$  approaches a certain point from the left (smaller values of  $u$ )

Notation:  $\lim_{u \rightarrow c^-} f(u)$   $\because u \rightarrow c^-$  means  $u$  approaches  $c$  from values less than  $c$

### • How to find L.H.L

- Look at the part of the func where  $u < c$
- Plug in values "slightly smaller than"  $c$  into the func to see the behaviour

### → Right hand limit

The value of  $f(u)$  as  $u$  approaches a certain point from the right (greater values of  $u$ )

Notation:  $\lim_{u \rightarrow c^+} f(u)$   $\because u \rightarrow c^+$  means  $u$  approaches  $c$  from values greater than  $c$

### • How to find R.H.L:

- Look at the part of the func where  $u > c$
- Plug in the values slightly larger than  $c$  into the func to see the behaviour.

If  $R.H.L = L.H.L$ , the limit exists at that point

$$\lim_{u \rightarrow c} f(u) = L.H.L = R.H.L$$

Otherwise the limit does not exist.





## Quick Tips for RHL and L.H.L

- L.H.L look at  $u < c$
- R.H.L look at  $u > c$
- L.H.L = R.H.L limit exists
- L.H.L  $\neq$  R.H.L limit does not exist

Example:

$$f(u) = \begin{cases} u^2 + 3 & \text{if } u < 1 \\ u + 1 & \text{if } u \geq 1 \end{cases}$$

Domain

Got it?

- Left hand limit  $u \rightarrow 1^-$

We'll use  $f(u) = u^2 + 3$

$$\lim_{u \rightarrow 1^-} f(u) = 1^2 + 3 = 4$$

- Right hand limit  $u \rightarrow 1^+$

As I've mentioned it before,

look for  $u \geq c$

we'll see

$$u + 1$$

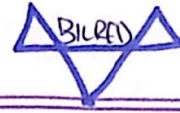
This means its R.H.L

$$\lim_{u \rightarrow 1^+} f(u) = u + 1$$

$$= 1 + 1 = 2$$

So, Limit does not exist at  $u = 1$  because  
L.H.L  $\neq$  R.H.L





## EXERCISE

$$\text{if } f(u) = \begin{cases} 3 & \text{if } u \leq -2 \\ -1/2 u^2 & \text{if } -2 < u < 2 \\ 3 & \text{if } u \geq 2 \end{cases}$$

$$\begin{aligned} &\text{if } u \leq -2 \\ &\text{if } -2 < u < 2 \\ &\text{if } u \geq 2 \end{aligned}$$

L.H.L.  
 $u < 2$

$u > 2$   
R.H.L.

Find

(i)  $\lim_{u \rightarrow 2^-} f(u)$

As I told ya before that this sign means L.H.L.  
So, look for value  $u < c$ . So, what I see,  
 $u \leq -2$  at 3.

$$\lim_{u \rightarrow 2^-} f(u) = 3$$

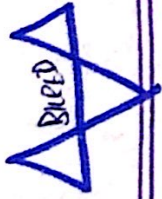
This is a constant, and limit of this constant remains same.

(ii)  $\lim_{u \rightarrow 2^+} f(u)$

Here, we need to find the Right hand limit. We need to look for  $u > c$ .

$$\lim_{u \rightarrow 2^+} f(u) = 3 \quad (\text{Again, it's at } u \geq 2)$$

R.H.Limit is at 3!



(iii)  $\lim_{u \rightarrow -2^+} f(u)$

Again, we need to find R.H.L but its  $-2$  this time so  $u > -2$ . Got it?

And from there I can see  $u > -2$  at

$$\frac{-1 \cdot u^2}{2}$$

$$\lim_{u \rightarrow -2^+} f(u) = \frac{-1 \cdot u^2}{2}$$

Applying limits

$$\lim_{u \rightarrow -2^+} f(u) = \frac{-1(-2)^2}{2}$$

$$= \frac{-4}{2}$$

$$= -2 \quad \text{ANSWER}$$





## Question

$$\text{Let } f(x) = \frac{(x+3) |x+2|}{(x+2)}$$

Find

$$(i) \lim_{x \rightarrow -2^+} \frac{(x+3) |x+2|}{(x+2)}$$

$$(ii) \lim_{x \rightarrow -2^-} \frac{(x+3) |x+2|}{(x+2)}$$

## Solution:

Before proceeding, let's understand what the modulus does. The modulus  $|x+2|$  behaves differently depending on the value of  $x$ :

(1) When  $x+2 \geq 0$ ,  $|x+2|$  it will be  $x+2$  (R.H.L)

(2) When  $x+2 < 0$ ,  $|x+2|$  it will be  $-(x+2)$  (L.H.L)

This gives us two cases

$$f(x) =$$

Solve for R.H.L  $\lim_{x \rightarrow -2^+}$

\* For  $x \rightarrow -2^+$ , means  $x > -2$ ,  $x > -2$

This means  $|x+2| = x+2$

$$f(x) = \frac{(x+3) \cancel{(x+2)}}{\cancel{x+2}}$$

$$= -2+3$$

$$= 1 \quad \text{R.H.L}$$





→ Solve for L.H.L:

$$u \rightarrow -2^-, u < -2 \text{ Means } |u+2| = -(u+2)$$

Substitute into the func

$$\lim_{u \rightarrow -2^-} f(u) = \frac{(u+3) - (u+2)}{(u+2)}$$

$$= -(u+3)$$

$$= -(-2+3)$$

$$= -1$$

Applying limits

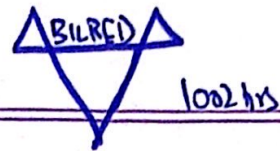
L.H.L

Conclusion:

$$\lim_{u \rightarrow -2^+} f(u) = 1$$

$$\lim_{u \rightarrow -2^-} f(u) = -1$$

The left hand limit and right hand limit is not equal, the two sided limit does not exist



## Key-points

(i) Left hand limit ( $x \rightarrow c^-$ ):

This happens when  $x$  is smaller than constant  
 $x < c$

(ii) Right hand limit ( $x \rightarrow c^+$ ):

This happens when  $x$  is greater than  
constant  $x > c$

Regardless of sign - - - - -

< I guess, you've learnt something with this ONE! >

## What the heck is Piece-wise Function?

Fortunately it's not heck, but in easy words  
its defined as,

A Piecewise function is a function  
that is made up of different formulas  
or rules for different parts of its  
domain (the input values).

Example:

$$f(x) = \begin{cases} x^2 & \text{if } x < 1 \\ x+2 & \text{if } x \geq 1 \end{cases}$$

This means.

- if  $x < 1$ , use the rule  $f(x) = x^2$
- if  $x \geq 1$ , use the rule  $f(x) = x+2$