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Linear Algebra

Assignment 4

Ex 5.2

Q8.

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

$$\det(A) = 1\begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} + 0 - 0$$

$$= 1(1-1) = 0$$

$$\det(\lambda I - A) = 0$$

$$\begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \lambda-1 & 0 & 0 \\ 0 & \lambda-1 & -1 \\ 0 & -1 & \lambda-1 \end{bmatrix} = 0$$

$$\det = (\lambda-1)\begin{vmatrix} \lambda-1 & -1 \\ -1 & \lambda-1 \end{vmatrix}$$

$$= (\lambda-1)\left[(\lambda-1)(\lambda-1)-1\right] = 0$$

$$= (\lambda-1)\left[(\lambda-1)^2-1\right] = 0$$

$$= (\lambda-1)(\lambda^2-2\lambda+1-1) = 0$$

$$= (\lambda-1)(\lambda^2-2\lambda) = 0$$

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Dated:

$(\lambda-1)(\lambda(\lambda-2)) = 0$

$\lambda = 0 \quad , \quad \lambda = 1 \quad , \quad \lambda = 2$

For $\lambda = 0$

$$\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & -1 \\ 0 & -1 & -1 \end{bmatrix} \begin{matrix} -R_1 \\ -R_2 \\ -R_3 \end{matrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix} \quad R_3 - R_2$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$x_1 = 0$

$x_2 + x_3 = 0$

$x_3 = t$

Rank = 2

$x_1 = 0$

$x_2 + t = 0$

$x_2 = -t$

$x_3 = t$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ -t \\ t \end{bmatrix} = t \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}$$

For $\lambda = 1$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{bmatrix} \quad R_1 \leftrightarrow R_3$$

$$\begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \end{bmatrix} \begin{matrix} -R_1 \\ -R_2 \\ \end{matrix}$$

Dated:

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \quad \begin{aligned} x_1 &= t \\ x_2 &= 0 \\ x_3 &= 0 \end{aligned} \qquad \text{Rank} = 2$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} t \\ 0 \\ 0 \end{bmatrix} = t\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

For $\lambda = 2$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{bmatrix} \quad R_3 + R_2$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix} \quad \begin{aligned} x_1 &= 0 \\ x_2 - x_3 &= 0 \\ x_3 &= t \end{aligned} \qquad \text{Rank} = 2$$

$$\begin{aligned} x_1 &= 0 \\ x_2 &= t \\ x_3 &= t \end{aligned} \qquad \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ t \\ t \end{bmatrix} = t\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

$$P = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$P^{-1} = \begin{bmatrix} 0 & -1/2 & 1/2 \\ 1 & 0 & 0 \\ 0 & 1/2 & 1/2 \end{bmatrix}$$

Dated:

$B = P^{-1} A P$

$$B = \begin{bmatrix} 0 & -1/2 & 1/2 \\ 1 & 0 & 0 \\ 0 & 1/2 & 1/2 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 0(1) - 1/2(0) + 1/2(0) & 0(0) - 1/2(1) + 1/2(1) & 0(0) - 1/2(1) + 1/2(1) \\ 1(1) + 0 + 0 & 1(0) + 0 + 0 & 1(0) + 0 + 0 \\ 0(1) + 1/2(0) + 1/2(0) & 0(0) + 1/2(1) + 1/2(1) & 1/2(1) + 1/2(1) \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 \\ 1(0) & 1(1) & 1(0) \\ 0(0) + 1(-1) + 1(1) & 0 & 0(0) + 1(1) + 1(1) \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

Eigenvalues = 0, 1, 2

Determinent = 0, so diagonalizes

Dated:

Q10.

a. $A = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 1 & 2 \end{bmatrix}$

$\det(A) = 3 \begin{vmatrix} 2 & 0 \\ 1 & 2 \end{vmatrix}$

$= 3(4)$

$= 12$

$\det(\lambda I - A) = 0$

$\begin{bmatrix} \lambda-3 & 0 & 0 \\ 0 & \lambda-2 & 0 \\ 0 & -1 & \lambda-2 \end{bmatrix}$

$\lambda - 3 \begin{vmatrix} \lambda-2 & 0 \\ -1 & \lambda-2 \end{vmatrix}$

$\cancel{(\lambda-3) \times (}\ (\lambda-3)\left[(\lambda-2)^2\right] = 0$

$\lambda = 3, \lambda = 2$

Dated:

b.

For $\lambda = 3$
$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \quad \begin{matrix} R_1 \leftrightarrow R_2 \\ R_2 \leftrightarrow R_3 \end{matrix}$$

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \quad R_2 - R_1$$

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \quad \text{Rank} = 2$$

For $\lambda = 2$

$$\begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix} \quad R_2 \leftrightarrow R_3$$

$$\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \begin{matrix} -R_1 \\ -R_2 \end{matrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \text{Rank} = 2$$

c. Not diagonalizable as the geometric multiplicity of $\lambda = 2$ is is 1 while algebraic multiplicity of $\lambda = 2$ is 2.

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Dated:

Q15

$(\lambda-1)(\lambda+3)(\lambda-5)=0$

For Dimension

$\lambda = 1 \rightarrow 1$

$\lambda = -3 \rightarrow 1$

$\lambda = 5 \rightarrow 1$

For size

$\lambda = 3$

Size $= 3 \times 3$

6. $\lambda^2(\lambda-1)(\lambda-2)^3 = 0$

$\lambda^2(\lambda-1)^1(\lambda-2)^3 = 0$

For Dimension

$\lambda^2 \rightarrow \dim(1, 2)$

$(\lambda-1) \rightarrow \dim(1)$

$(\lambda-2)^3 \rightarrow \dim(1, 2, 3)$

For size

$= 6 \times 6$ matrix

Dated:

Q20.

$$A = \begin{bmatrix} +1 & -2 & 8 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad P = \begin{bmatrix} 1 & -4 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$P^{-1} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & -1 \\ 1 & -1 & 4 \end{bmatrix}$$

$$B = P^{-1} A P$$

$$\begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 1 & -1 & 4 \end{bmatrix} \begin{bmatrix} 1 & -2 & 8 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & -4 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0(1)+0+0 & 0(-2)-1(-1)+0(0) & 0(8)-1(0)+0(-1) \\ 0(1)+0(0)-1(0) & 0(-2)+0(0)-1(0) & 0(8)+0(0)-1(-1) \\ 1(1)-1(0)+4(0) & 1(-2)-1(0)+4(0) & 1(8)-1(0)+4(0) \end{bmatrix}$$

$$\begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 1 & -1 & 4 \end{bmatrix} \begin{bmatrix} 1 & -4 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Dated:

trace (A) = 1 - 1 - 1 => -1

trace (B) = -1 - 1 + 1 = -1

tr(A) = tr(B)

P diagonalizes A

a. A^{1000}

$$= \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -1 & 4 \end{bmatrix} \begin{bmatrix} (-1)^{1000} & 0 & 0 \\ 0 & (-1)^{1000} & 0 \\ 0 & 0 & (1)^{1000} \end{bmatrix} \begin{bmatrix} 1 & -4 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -1 & 4 \end{bmatrix} \begin{bmatrix} 1 & -4 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

b. A^{-1000}

$$= \begin{bmatrix} 1 & -4 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} (-1)^{-1000} & 0 & 0 \\ 0 & (-1)^{-1000} & 0 \\ 0 & 0 & (1)^{-1000} \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -1 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -4 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -1 & -4 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Dated:

c. A^{2301}

$$\begin{bmatrix} 1 & 4 & 1 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} (-1)^{2301} & 0 & 0 \\ 0 & (-1)^{2301} & 0 \\ 0 & 0 & (1)^{2301} \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -1 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 4 & 1 \\ -1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -1 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -2 & 8 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

d. A^{-2301}

$$= \begin{bmatrix} 1 & -4 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} (-1)^{-2301} & 0 & 0 \\ 0 & (-1)^{-2301} & 0 \\ 0 & 0 & (1)^{-2301} \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -1 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 4 & 1 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -2 & 8 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

Dated:

Ex 6.1

Q2. $\langle u, v \rangle = \frac{1}{2} u_1 v_1 + 5 u_2 v_2$

$u = (1, 1)$
$v = (3, 2)$
$w = (0, -1)$
$k = 3$

a. $\langle u, v \rangle$

$= \frac{1}{2}(1)(3) + 5(1)(2)$

$= \frac{23}{2}$

b. $\langle kv, w \rangle$

$kv = (9, 6)$

$\langle kv, w \rangle = \frac{1}{2}(9)(0) + 5(6)(-1)$

$= -30$

c. $\langle u+v, w \rangle$

$u+v = (4, 3)$

$\langle u+v, w \rangle = \frac{1}{2}(4)(0) + 5(3)(-1)$

$= -15$

d. $\|v\| = \sqrt{\frac{1}{2}(3)(3) + 5(2)(2)}$

$= \frac{7\sqrt{2}}{2}$

Dated:

e. $d(u,v) = \|u-v\|$ $\quad u-v = (-2,-1)$

$= \sqrt{\frac{1}{2}(-2)(-2) + 5(-1)(-1)}$

$= \sqrt{7}$

f. $\|u-kv\|$

$kv = (9,6)$

$u-kv = (-8,-5)$

$\|u-kv\| = \sqrt{\frac{1}{2}(-8)(-8) + 5(-5)(-5)}$

$= \sqrt{157}$

Q13. $\langle u,v \rangle = 3u_1v_1 + 5u_2v_2$

$$\begin{bmatrix} \sqrt{3} & 0 \\ 0 & \sqrt{5} \end{bmatrix}$$

Q15. $p = x + x^3$, $q = 1 + x^2$

$x_0 = -2$, $x_1 = -1$, $x_2 = 0$, $x_3 = 1$

$p(-2) = -10$	$q(-2) = 5$
$p(-1) = -2$	$q(-1) = 2$
$p(0) = 0$	$q(0) = 1$
$p(1) = 2$	$q(1) = 2$

$= -10(5) + (-2)(2) + 0(1) + 2(2)$

$= -50$

Dated:

Q22. $U = \begin{bmatrix} 1 & 2 \\ -3 & 5 \end{bmatrix}$, $V = \begin{bmatrix} 4 & 6 \\ 0 & 8 \end{bmatrix}$

$\|U\| = \sqrt{(1)^2 + (2)^2 + (-3)^2 + (5)^2}$
$\sqrt{39}$

$d(U, V) = \|U - V\|$

$U - V = \begin{bmatrix} -3 & -4 \\ -3 & -3 \end{bmatrix}$

$(U-V)^T = \begin{bmatrix} -3 & -3 \\ -4 & -3 \end{bmatrix}$

$= \sqrt{tr\left[(U-V)^T (U-V)\right]}$

$(U-V)^T(U-V) = \begin{bmatrix} -3 & -3 \\ -4 & -3 \end{bmatrix} \begin{bmatrix} -3 & -4 \\ -3 & -3 \end{bmatrix}$

$= \begin{bmatrix} 18 & 21 \\ 21 & 25 \end{bmatrix}$

$= \sqrt{18 + 25}$
$= \sqrt{43}$

Dated:

Q26.

$u = (-1, 2)$, $v = (2, 5)$

$$A = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$$

$$\|u\| = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix} \begin{bmatrix} -1 \\ 2 \end{bmatrix}$$

$$= \begin{bmatrix} -1+4 \\ 1+6 \end{bmatrix} \Rightarrow \begin{bmatrix} 3 \\ 7 \end{bmatrix}$$

$$\sqrt{\left(\begin{bmatrix} 3 \\ 7 \end{bmatrix} \begin{bmatrix} 3 \\ 7 \end{bmatrix} \right)}$$

$$= \sqrt{58}$$

$d(u, v) = \|u - v\|$

$u - v = (-3, -3)$

$$= \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix} \begin{bmatrix} -3 \\ -3 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix} \begin{bmatrix} -3 \\ -3 \end{bmatrix}$$

$$= \sqrt{\begin{bmatrix} -9 \\ -6 \end{bmatrix} \begin{bmatrix} -9 \\ -6 \end{bmatrix}}$$

$$= \sqrt{117}$$

$$= 3\sqrt{13}$$

Dated:

Q28.

~~a =~~ a. $\langle u - v - 2w, 4u + v \rangle$

$\langle u, v \rangle = 2$ $\quad ||u|| = 1$

$\langle v, w \rangle = -6$ $\quad ||v|| = 2$

$\langle u, w \rangle = -3$ $\quad ||w|| = 7$

$= \langle u, 4u + v \rangle - \langle v, 4u + v \rangle - \langle 2w, 4u + v \rangle$

$= \langle u, 4u \rangle + \langle u, v \rangle - \langle v, 4u \rangle - \langle v, v \rangle - \langle 2w, 4u \rangle - \langle 2w, v \rangle$

$= 4 \langle u, u \rangle + \langle u, v \rangle - 4\langle v, u \rangle - \langle v, v \rangle - 8\langle w, u \rangle - 2\langle w, v$

$= 4(1) - 3(2) - 4 - 8(-3) - 2(-6)$

$= 4 - 6 - 4 + 24 + 12$

$= 30$

b $= ||2w - v||$

$= \sqrt{\langle 2w, v, 2w - v \rangle}$

$= \sqrt{\langle 2w, 2w - v \rangle - \langle v, 2w - v \rangle}$

$= \sqrt{\langle 2w, 2w \rangle - \langle 2w, v \rangle - \langle v, 2w \rangle + \langle v, v \rangle}$

$= \sqrt{4 \langle w, w \rangle - 2\langle w, v \rangle - 2\langle v, w \rangle + \langle v, v \rangle}$

$= \sqrt{4(49) - 4(-6) + 4}$

$= \sqrt{224}$

Dated:

Ex 6.2

Q2. $u = (-1, 0)$, $v = (3, 8)$

$$\cos\theta = \frac{\langle u, v\rangle}{\|u\|\|v\|} = \frac{(-1)(3) + (0)(8)}{\sqrt{1} \cdot \sqrt{9+64}}$$

$$= \frac{-3}{\sqrt{1} \cdot \sqrt{73}} \Rightarrow \frac{-3}{\sqrt{73}}$$

b. $u = (4, 1, 8)$, $v = (1, 0, -3)$

$$\cos\theta = \frac{\langle u, v\rangle}{\|u\|\|v\|} = \frac{4(1) + 1(0) + 8(-3)}{\sqrt{81} \cdot \sqrt{10}}$$

$$= \frac{-20}{9\sqrt{10}}$$

c. $u = (2, 1, 7, -1)$, $v = (4, 0, 0, 0)$

$$\cos\theta = \frac{\langle u, v\rangle}{\|u\|\|v\|} = \frac{2(4)}{\sqrt{55} \cdot \sqrt{16}}$$

$$= \frac{8}{4\sqrt{55}}$$

Dated:

Q4.

$$p = x - x^2 \quad , \quad q = 7 + 3x + 3x^2$$

$$\cos\theta = \frac{\langle p, q\rangle}{\|p\|\,\|q\|} = \frac{0(7) + 1(3) - 1(3)}{\sqrt{2} \cdot \sqrt{67}}$$

$$\cos\theta = 0$$

Q6.

$$A = \begin{bmatrix} 2 & 4 \\ -1 & 3 \end{bmatrix} \quad , \quad B = \begin{bmatrix} -3 & 1 \\ 4 & 2 \end{bmatrix}$$

$$\cos\theta = \frac{\langle U, V\rangle}{\|U\|\,\|V\|} = \frac{2(-3) + 4(1) + (-1)(4) + 3(2)}{\sqrt{30} \cdot \sqrt{30}}$$

$$= \frac{0}{\sqrt{30} \cdot \sqrt{30}}$$

$$= 0$$

$$A^T = \begin{bmatrix} 2 & -1 \\ 4 & 3 \end{bmatrix}$$

$$A^T B = \begin{bmatrix} 2 & -1 \\ 4 & 3 \end{bmatrix} \begin{bmatrix} -3 & 1 \\ 4 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} -8 & 0 \\ 0 & \end{bmatrix}$$

Dated:

Q8.

a. $u = (u_1, u_2, u_3)$, $v = (0, 0, 0)$

$\langle u, v \rangle = u_1(0) + u_2(0) + u_3(0)$

$= 0$

is Orthogonal

b. $u = (-4, 6, -10, 1)$, $v = (2, 1, -2, 9)$

$\langle u, v \rangle = -4(2) + 6(1) + (-10)(-2) + 1(9)$

$= 27$

Not orthogonal

c. $u = (a, b, c)$, $v = (-c, 0, a)$

$\langle u, v \rangle = a(-c) + 0 + ac$

$= 0$

Orthogonal

Q12. $U = \begin{bmatrix} 5 & -1 \\ 2 & -2 \end{bmatrix}$, $V = \begin{bmatrix} 1 & 3 \\ -1 & 0 \end{bmatrix}$

$\langle U, V \rangle = 5(1) - 1(3) + 2(-1) - 2(0)$

$= 0$

Orthogonal

Dated:

Q18. $u = \begin{bmatrix} 3 \\ 3 \end{bmatrix}$, $v = \begin{bmatrix} 5 \\ -8 \end{bmatrix}$

$$A = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$$

$$\langle u, v \rangle = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 3 \end{bmatrix} \cdot \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 5 \\ -8 \end{bmatrix}$$

$$= \begin{bmatrix} 9 \\ 6 \end{bmatrix} \begin{bmatrix} 2 \\ -3 \end{bmatrix}$$

$$= 0$$

$\therefore$ Orthogonal