National University of Computer & Emerging Sciences, Karachi
Fall 2021 CS-Department
Quiz-01-Solution
8th Oct 2021, 9:40 AM – 09:55 AM

Course Code: CS 1005	Course Name: Discrete Structure
Instructor Name / Names: Safia Baloch	
Student Roll No:	Section:

Instructions:

- Return the question paper in the end.
- Read the question completely before answering it.

Time: 15 minutes. **Max Marks:20**

---

1. **Prove that: [(p → q) ∧ (q → r)] → [p → r] is a tautology.**
   a. **By using the truth table.**

(0 points) (b)

p	q	r	$p \to q$	$q \to r$	$(p \to q) \wedge (q \to r)$	$p \to r$	$[(p \to q) \wedge (q \to r)] \to (p \to r)$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	F	T	T
T	F	F	F	T	F	F	T
F	T	T	T	T	T	T	T
F	T	F	T	F	F	T	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

Since $[(p \to q) \wedge (q \to r)] \to (p \to r)$ is always T, it is a tautology.

b. By using logic equivalence laws

7. (0 points), page 35, problem 30. Show that $(p \vee q) \wedge (\neg p \vee r) \rightarrow (q \vee r)$ is a tautology.
sol:

$$
\begin{aligned}
& (p \vee q) \wedge (\neg p \vee r) \rightarrow (q \vee r) & \\
\equiv & \neg[(p \vee q) \wedge (\neg p \vee r)] \vee (q \vee r) & \text{by implication law} \\
\equiv & [\neg(p \vee q) \vee \neg(\neg p \vee r)] \vee (q \vee r) & \text{by de Morgan's law} \\
\equiv & [(\neg p \wedge \neg q) \vee (p \wedge \neg r)] \vee (q \vee r) & \text{by de Morgan's law} \\
\equiv & [(\neg p \wedge \neg q) \vee q] \vee [(p \wedge \neg r) \vee r] & \text{by commutative and associative laws} \\
\equiv & [(\neg p \vee q) \wedge (\neg q \vee q)] \vee [(p \vee r) \wedge (\neg r \vee r)] & \text{by distributive laws} \\
\equiv & (\neg p \vee q) \vee (p \vee r) & \text{by negation and identity laws} \\
\equiv & (\neg p \vee p) \vee (q \vee r) & \text{by communicative and associative laws} \\
\equiv & T & \text{by negation and domination laws}
\end{aligned}
$$