LA-ASSIGNMENT 04

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EXERCISE 5.2

Q8:

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

$$\det(A) = 1 \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} = 1(1-1) = 0$$

$$\boxed{|A| = 0}$$

$$\det(\lambda I - A) = 0$$

$$\begin{vmatrix} \lambda-1 & 0 & 0 \\ 0 & \lambda-1 & -1 \\ 0 & -1 & \lambda-1 \end{vmatrix} = 0$$

$$(\lambda-1)\left[(\lambda-1)^2 - 1\right] = 0$$

$$(\lambda-1)(\lambda^2 - 2\lambda + \cancel{1-1}) = 0$$

$$\lambda(\lambda-1)(\lambda-2) = 0$$

$\boxed{\lambda = 0}$ $\boxed{\lambda = 1}$ $\boxed{\lambda = 2}$

FOR $\lambda = 0$

$$\lambda I - A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & -1 \\ 0 & -1 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

$R_2 - R_3$

$$\lambda I - A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$x_1 = 0$

$x_2 + x_3 = 0$

$x_3 = t$

$x_1 = 0$

$x_2 = -t$

$x_3 = t$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = t \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}$$

$(x_1, x_2, x_3) = (0, -1, 1)$

Ar

FOR $\lambda = 1$

$$\lambda I - A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{bmatrix}$$ (R_1 swaps R_3)

$$= \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$x_1 = t$

$x_2 = 0$

$x_3 = 0$

$(x_1, x_2, x_3) = (1, 0, 0)$

Ar

FOR $\lambda = 2$

$$\lambda I - A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{bmatrix}$$

In augmented form

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & -1 & 1 & 0 \end{bmatrix}$$

$R_2 + R_3$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$x_1 = 0$

$x_2 = t$

$x_3 = t$

$$\boxed{(x_1, x_2, x_3) = (0, 1, 1)}$$

$$P = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$P^{-1} = \begin{bmatrix} 0 & -1/2 & 1/2 \\ 1 & 0 & 0 \\ 0 & 1/2 & 1/2 \end{bmatrix}$$

$B = P^{-1}AP$

$$= \begin{bmatrix} 0 & -1/2 & 1/2 \\ 1 & 0 & 0 \\ 0 & 1/2 & 1/2 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

$$\boxed{B = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}}$$

Eigen values : 0, 1, 2

Determinant = 0

Thus the matrix digonalises.

Q15

a) $(\lambda-1)(\lambda+3)(\lambda-5)=0$

For dimensions:

$\lambda=1 \rightarrow 1$

$\lambda=-3 \rightarrow 1$

$\lambda=5 \rightarrow 1$

For size

$\lambda=3$

size $= 3\times 3$

B) $\lambda^2(\lambda-1)(\lambda-2)^3=0$

Dimension

$\lambda^2 \rightarrow$ dim $(1,2)$

$(\lambda-1) \rightarrow$ dim (1)

$(\lambda-2)^3 \rightarrow$ dim $(1,2,3)$

For size

$=6\times 6$ matrix

Q10

$$A=\begin{bmatrix}3 & 0 & 0\\ 0 & 2 & 0\\ 0 & 1 & 2\end{bmatrix}$$

$\boxed{\det(A)=3(4)=12}$

$\det(\lambda I-A)=0$

$$\begin{vmatrix}\lambda-3 & 0 & 0\\ 0 & \lambda-2 & 0\\ 0 & -1 & \lambda-2\end{vmatrix}=0$$

$(\lambda-3)\left[(\lambda-2)^2-0\right]=0$

$(\lambda-3)(\lambda-2)^2=0$

$\boxed{\lambda=3}$ $\boxed{\lambda=2}$

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FOR $\lambda = 3$

$$\lambda I - A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$$

Converting in augmented form

$$= \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \text{Rank} = 2$$

FOR $\lambda = 2$

$$\lambda I - A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix}$$ (Converting in augmented form)

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Since the geometric multiplicity of $\lambda = 2$ is 1 while algebraic multiplicity of $\lambda = 2$ is 2.

Q20)

$$A = \begin{bmatrix} 1 & -2 & 8 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \quad P = \begin{bmatrix} 1 & -4 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

a) A^{1000}

$B = P^{-1}AP$

$$= \begin{bmatrix} 1 & -4 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 1 \end{bmatrix}^{-1} \times \begin{bmatrix} 1 & -2 & 8 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \times \begin{bmatrix} 1 & -4 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$tr(A) = -1$

$tr(B) = -1$

$tr(A) = tr(B)$

Thus P digonalizes A

$$A^{1000} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -1 & 4 \end{bmatrix} \begin{bmatrix} -1^{1000} & 0 & 0 \\ 0 & -1^{1000} & 0 \\ 0 & 0 & -1^{1000} \end{bmatrix} \begin{bmatrix} 1 & -4 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -1 & 4 \end{bmatrix} \begin{bmatrix} 1 & -4 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$\boxed{A^{1000} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}$$

b) A^{-1000}

$$A^{-1000} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -1 & 4 \end{bmatrix} \begin{bmatrix} (-1)^{-1000} & 0 & 0 \\ 0 & (-1)^{-1000} & 0 \\ 0 & 0 & (-1)^{-1000} \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -1 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -1 & 4 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -1 & 4 \end{bmatrix}$$

$$\boxed{A^{-1000} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}$$

c) A^{2301}

$$A^{2301} = \begin{bmatrix} 1 & 4 & 1 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} (-1)^{2301} & 0 & 0 \\ 0 & (-1)^{2301} & 0 \\ 0 & 0 & (-1)^{2301} \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -1 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 4 & 1 \\ -1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -1 & 4 \end{bmatrix}$$

$$\boxed{A^{2301} = \begin{bmatrix} 1 & -2 & 8 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}}$$

d) A^{-2301}

$$A^{-2301} = \begin{bmatrix} 1 & -4 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} (-1)^{-2301} & 0 & 0 \\ 0 & (-1)^{-2301} & 0 \\ 0 & 0 & (-1)^{-2301} \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -1 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 4 & -1 \\ -1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -1 & 4 \end{bmatrix}$$

$$\boxed{A^{-2301} = \begin{bmatrix} 1 & -2 & 8 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}}$$

x —— x —— x

Ex 6.1

Q2 $\langle u, v\rangle = \frac{1}{2}u_1v_1 + 5u_2v_2$

$u = (1,1)$
$v = (3,2)$
$w = (0,-1)$
$k = 3$

a) $= \frac{1}{2}(1)(3) + 5(1)(2)$

$= \frac{23}{2}$

b) $\langle kv, w\rangle$

$kv = (9,6)$

$\langle kv, w\rangle = \frac{1}{2}(9)(0) + 5(6)(-1)$

$\boxed{\langle kv, w\rangle = -30}$

c) $\langle u+v, w\rangle$

$u+v = (4,3)$

$\langle u+v, w\rangle = \frac{1}{2}(4)(0) + 5(3)(-1)$

$\boxed{\langle u+v, w\rangle = -15}$

d) $\|v\| = \sqrt{\frac{1}{2}(3)(3) + 5(2)(2)}$

$\boxed{\|v\| = \frac{7\sqrt{2}}{2}}$

e) $d(u,v)$ $\qquad u-v=(-2,-1)$

$= \|u-v\|$

$= \sqrt{\frac{1}{2}(-2)(-2)+5(-1)(-1)}$

$\boxed{d(u,v)=\sqrt{7}}$

f) $\|u-kv\|$

$kv=(9,6)$

$u-kv=(-8,-5)$

$\|u-kv\|=\sqrt{\frac{1}{2}(-8)(-8)+5(-5)(-5)}$

$\boxed{\|u-kv\|=\sqrt{157}}$

Q13

$\langle u,v\rangle = 3u_1v_1 + 5u_2v_2$

$$=\begin{bmatrix}\sqrt{3} & 0\\ 0 & \sqrt{5}\end{bmatrix}$$

Q15

$p=x+x^3$ & $q=1+x^2$

$x_0=-2$; $x_1=-1$; $x_2=0$; $x_3=1$

$p(-2)=-10$	$q(-2)=5$
$p(-1)=-2$	$q(-1)=2$
$p(0)=0$	$q(0)=1$
$p(1)=2$	$q(1)=2$

$=-10(5)+(-2)(2)+0(1)+2(2)$

$=-50$

Q22

$u=\begin{bmatrix}1 & 2\\ -3 & 5\end{bmatrix}$; $v=\begin{bmatrix}4 & 6\\ 0 & 8\end{bmatrix}$

$\|u\|=\sqrt{(1)^2+(2)^2+(-3)^2+(5)^2}$

$\boxed{\|u\|=\sqrt{39}}$

$d(u,v)=\|u-v\|$

$u-v=\begin{bmatrix}-3 & -4\\ -3 & -3\end{bmatrix}$

$d(u,v)=\sqrt{tr\left[(u-v)^T(u-v)\right]}$

$$(u-v)^T(u-v) = \begin{bmatrix} -3 & -3 \\ -4 & -3 \end{bmatrix}\begin{bmatrix} -3 & -4 \\ -3 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 18 & 21 \\ 21 & 25 \end{bmatrix}$$

$$= \sqrt{18+25}$$

$$\boxed{d(u,v) = \sqrt{43}}$$

Q26

$u = (-1, 2)$, $v = (2, 5)$

$$A = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$$

$$\|u\| = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}\begin{bmatrix} -1 \\ 2 \end{bmatrix}$$

$$= \begin{bmatrix} -1+4 \\ 1+6 \end{bmatrix} = \begin{bmatrix} 3 \\ 7 \end{bmatrix}$$

$$= \sqrt{3^2+7^2}$$

$$\boxed{\|u\| = \sqrt{58}}$$

$$d(u,v) = \|u-v\|$$

$u - v = (-3, -3)$

$$= \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}\begin{bmatrix} -3 \\ -3 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}\begin{bmatrix} -3 \\ -3 \end{bmatrix}$$

$$= \sqrt{\begin{bmatrix} -9 \\ -6 \end{bmatrix}\begin{bmatrix} -9 \\ -6 \end{bmatrix}}$$

$$= \sqrt{9^2+6^2}$$

$$= \sqrt{117}$$

$$\boxed{d(u,v) = 3\sqrt{13}}$$

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Q20

a)

$= \langle u - v - 2w, 4u + v \rangle$

$\langle u, v \rangle = 2$ $\quad \|u\| = 1$

$\langle v, w \rangle = -6$ $\quad \|v\| = 2$

$\langle u, w \rangle = -3$ $\quad \|w\| = 7$

$= \langle u, 4u + v \rangle - \langle v, 4u + v \rangle - \langle 2w, 4u + v \rangle$

$= \langle u, 4u \rangle + \langle u, v \rangle - \langle v, 4u \rangle - \langle u, v \rangle - \langle 2w, 4u \rangle - \langle 2w, v \rangle$

$= 4 \langle u, u \rangle + \langle u, v \rangle - 4 \langle v, u \rangle - \langle u, v \rangle - 8 \langle w, v \rangle - 2 \langle w, v \rangle$

$= 4(1) - 3(2) - 4 - 8(-3) - 2(-6)$

$= 4 - 6 - 4 + 24 + 12$

$\boxed{= 30}$

b) $\|2w - v\|$

$= \sqrt{\langle 2w, v, 2w - v \rangle}$

$= \sqrt{\langle 2w, 2w - v \rangle - \langle v, 2w - v \rangle}$

$= \sqrt{\langle 2w, 2w \rangle - \langle 2w, v \rangle - \langle v, 2w \rangle + \langle v, v \rangle}$

$= \sqrt{4\langle w, w \rangle - 2\langle w, v \rangle - 2\langle v, w \rangle + \langle v, v \rangle}$

$= \sqrt{4(49) - 4(-6) + 4}$

$\boxed{= \sqrt{224}}$

Ex 6.2

Q2

a) $u = (-1, 0)$ $\quad v = (3, 8)$

$$\cos\theta = \frac{\langle u, v\rangle}{\|u\|\|v\|} = \frac{(-1)(3) + (0)(8)}{\sqrt{1} \cdot \sqrt{9+64}}$$

$$\boxed{\cos\theta = \frac{-3}{\sqrt{73}}}$$

b) $u = (4, 1, 8)$, $v = (1, 0, -3)$

$$\cos\theta = \frac{(4)(1) + (0)(1) + 8(-3)}{\sqrt{81} \cdot \sqrt{10}}$$

$$\boxed{\cos\theta = \frac{-20}{9\sqrt{10}}}$$

c) $u = (2, 1, 7, -1)$; $v = (4, 0, 0, 0)$

$$\cos\theta = \frac{2(4) + 0 + 0 + 0}{\sqrt{55} \cdot \sqrt{16}}$$

$$= \frac{2\cancel{8}}{\cancel{4}\sqrt{55}}$$

$$\boxed{\cos\theta = \frac{2}{\sqrt{55}}}$$

Q4

$p = x - x^2$; $q = 7 + 3x + 3x^2$

$$\cos\theta = \frac{\langle p, q\rangle}{\|p\|\|q\|} = \frac{(0)(7) + 1(3) - 1(3)}{\sqrt{2} \cdot \sqrt{67}}$$

$$\boxed{\cos\theta = 0}$$

Q6

$A = \begin{bmatrix} 2 & 4 \\ -1 & 3 \end{bmatrix}$ $\quad B = \begin{bmatrix} -3 & 1 \\ 4 & 2 \end{bmatrix}$

$$\cos\theta = \frac{\langle u, v\rangle}{\|u\|\|v\|}$$

$$= \frac{2(-3) + 4(1) + 4(-1) + 3(2)}{\sqrt{30}\sqrt{30}}$$

$$\boxed{\cos\theta = 0}$$

Q8

a) $U = (U_1, U_2, U_3)$; $V = (0,0,0)$

$\boxed{\langle U, V\rangle = 0}$

orthogonal

b) $U = (-4, 6, -10, 1)$; $V = (2, 1, -2, 9)$

$\langle U,V\rangle = -4(2) + 6\cdot(1) + (-10)\cdot(-2) + 1(9)$

$\boxed{\langle U,V\rangle = 27}$

Not orthogonal

c) $U = (a, b, c)$; $V = (-c, 0, a)$

$\langle U,V\rangle = -c(a) + 0 + a(c)$

$\boxed{\langle U,V\rangle = 0}$

orthogonal

Q12 $U = \begin{bmatrix} 5 & -1 \\ 2 & -2 \end{bmatrix}$; $V = \begin{bmatrix} 1 & 3 \\ -1 & 0 \end{bmatrix}$

$\langle U,V\rangle = 5(1) - 1(3) + 2(-1) + 0$

$\boxed{\langle U,V\rangle = 0}$

orthogonal

Q10 $U = \begin{bmatrix} 3 \\ 3 \end{bmatrix}$; $V = \begin{bmatrix} 5 \\ -8 \end{bmatrix}$

$A = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$

$\langle U,V\rangle = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 3 \\ 3 \end{bmatrix} \cdot \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 5 \\ -8 \end{bmatrix}$

$= \begin{bmatrix} 9 \\ 6 \end{bmatrix}\begin{bmatrix} 2 \\ -3 \end{bmatrix}$

$\boxed{\langle U,V\rangle = 0}$

orthogonal

x ——— x ——— x

End of Assignment