Chapter 1, Part I: Propositional Logic

The Foundations: Logic and Proofs

Propositional Logic Summary

- The Language of Propositions
  - Connectives
  - Truth Values
  - Truth Tables
- Applications
  - Translating English Sentences
  - System Specifications
  - Logic Puzzles
  - Logic Circuits
- Logical Equivalences
  - Important Equivalences
  - Showing Equivalence
  - Satisfiability

Propositional Logic

Section 1.1

Chapter Summary

- Propositional Logic
  - The Language of Propositions
  - Applications
  - Logical Equivalences
- Predicate Logic
  - The Language of Quantifiers
  - Logical Equivalences
  - Nested Quantifiers
- Proofs
  - Rules of Inference
  - Proof Methods
  - Proof Strategy

Section Summary

- Propositions
- Connectives
  - Negation
  - Conjunction
  - Disjunction
  - Implication; contrapositive, inverse, converse
  - Biconditional
- Truth Tables

Propositions

- A *proposition* is a declarative sentence that is either true or false.
- Examples of propositions:

  a) The Moon is made of green cheese.

  b) Toronto is the capital of Canada.

  c) $1 + 0 = 1$

  d) $0 + 0 = 2$

- Examples that are not propositions.

  a) Sit down!

  b) What time is it?

  c) $x + 1 = 2$

  d) $x + y = z$

Propositional Logic

- Constructing Propositions
  - Propositional Variables(sentential variables):[*p*, *q*, *r*, *s*,..]
  - true is denoted by **T**
  - false is denoted by **F**.
  - Compound Propositions; constructed from logical connectives and other propositions
    - Negation $\neg$ $(p, \neg p)$
    - Conjunction $\wedge$ $(p \wedge q)$
    - Disjunction $\vee$ $(p \vee q)$
    - Implication $\rightarrow$ $(p \rightarrow q)$
    - Biconditional $\leftrightarrow$ $(p \leftrightarrow q)$

Compound Propositions: Negation

- The *negation* of a proposition p is denoted by $\neg p$
- **Example**:If p denotes "Michael's PC runs Linux",then

  $\neg p$ denotes " ________________________________"

p	$\neg p$
T	
F	

Conjunction

- The *conjunction* of propositions p **and** q is denoted by $p \wedge q$
- **Example**: If p denotes "I am at home." and q denotes "It is raining." then $p \wedge q$ denotes "I am at home and it is raining."

p	q	$p \wedge q$
T	T	
T	F	
F	T	
F	F	

Disjunction

- The *disjunction* of propositions p and q is denoted by $p \vee q$
- **Example**: If p denotes "I am at home." and q denotes "It is raining." then $p \vee q$ denotes "I am at home or it is raining."

p	q	$p \vee q$
T	T	
T	F	
F	T	
F	F	

The Connective Or in English

- In English "or" has two distinct meanings.
  - "**Inclusive Or**" - For $p \lor q$ to be true, either one or both of p and q must be true.
  -
  - "**Exclusive Or**" - When reading the sentence "I will use all my savings to travel to Europe or to buy an electric car".
  - This is the meaning of Exclusive Or (Xor). In $p \oplus q$, one of p and q must be true, but not both. The truth table for $\oplus$ is:

p	q	$p \oplus q$
T	T	
T	F	
F	T	
F	F	

Implication

- $p \rightarrow q$ is a ***conditional statement*** or *implication*
  - which is read as "if p, then q".
- In $p \rightarrow q$, p is the *hypothesis* (*antecedent* or *premise*) and q is the *conclusion* (or *consequence*).
  - "If I am elected, then I will lower taxes."
- **Example**: If p denotes "I am at home." and q denotes "It is raining." then $p \rightarrow q$ denotes "If I am at home then it is raining."

p	q	$p \rightarrow q$
T	T	
T	F	
F	T	
F	F	

Different Ways of Expressing $p \rightarrow q$

p: Maria learns discrete mathematics
q: Maria will find a good job

- q **unless** $\neg p$
- q **when** p
- q **is necessary for** p
- q **if** p
- q **whenever** p
- q **follows from** p

- **if** p, **then** q
- **if** p, q
- p **is sufficient for** q
- p **only if** q
- p **implies** q

Understanding Implication

- In $p \rightarrow q$ there does not need to be any connection between the antecedent or the consequent. The "meaning" of $p \rightarrow q$ depends only on the truth values of p and q.
- These implications are perfectly fine, but would not be used in ordinary English.
  - "If the moon is made of green cheese, then I have more money than Bill Gates. "
  - "If the moon is made of green cheese then I'm on welfare."
  - "If 1 + 1 = 3, then your grandma wears combat boots."

Understanding Implication (cont)

- One way to view the logical conditional is to think of an obligation or contract.
  - “If I am elected, then I will lower taxes.”
  - “If you get 100% on the final, then you will get an A.”
- If the politician is elected and does not lower taxes, then the voters can say that he or she has broken the campaign pledge. Something similar holds for the professor. This corresponds to the case where p is true and q is false.

Inverse

- From $p \rightarrow q$ we can form new conditional statements .
  - $q \rightarrow p$ is the **converse** of $p \rightarrow q$
  - $\neg q \rightarrow \neg p$ is the **contrapositive** of $p \rightarrow q$
  - $\neg p \rightarrow \neg q$ is the **inverse** of $p \rightarrow q$

Which one would be equivalent to whom? or have same truth values?

Inverse

Example

Example: Find the converse, inverse, and contrapositive of

“*The home team wins* **whenever** *it is raining*”

Solution: “if it is raining, then the home team wins.”

converse: “if the home team wins , then it is raining”

contrapositive: “if the home team does not win, then it is not raining.”

inverse: “if it is not raining, then the home team does not win”

CONNECTIVE	MEANINGS	SYMBOLS	CALLED
Negation	not	$\sim$	Tilde
Conjunction	and	$\wedge$	Hat
Disjunction	or	$\vee$	Vel
Conditional	if…then…	$\rightarrow$	Arrow
Biconditional	if and only if	$\leftrightarrow$	Double arrow

Biconditional/Bi-Implication

- If p and q are propositions, then we can form the *biconditional* proposition $p \leftrightarrow q$, read as "p ***if and only if*** q."
  Equivalence (↔) corresponds to the use of "if and only if" in natural language and related terms, such as "just in case" and "if … , then … , and vice versa".
- ***p*** **is necessary and sufficient for** ***q***
- **if** ***p*** **then** ***q*** **, and conversely**
- ***p*** **iff** ***q*** ***(if and only if)***
- If p denotes "you can take the flight " and q denotes "You buy a ticket." then $p \leftrightarrow q$ denotes "You

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Truth Tables For Compound Propositions

- Construction of a truth table:
- We can use connectives(Conjunction, disjunction, exclusive or,implication, bidirectional operator and negation) to build up ***complicated compound propositions*** involving any number of propositional variables. We can use ***truth tables*** to determine the ***truth values*** of these compound propositions,

Precedence of Logical Operators

Operator	Precedence
$\neg$	1
$\wedge$ $\vee$	2 3
$\rightarrow$ $\leftrightarrow$	4 5

- $p \vee q \rightarrow \neg r$ is equivalent to $(p \vee q) \rightarrow \neg r$
- $(p \vee q) \wedge (\neg r)$ *is the conjunction of* $p \vee q$ *and* $\neg r$
- $\neg p \vee q$ *is means that* $\neg p \vee q$ *is the disjunction of* $\neg p$ ***and q but not, the negation of disjunction of p and q*** $\neg(p \vee q)$
-

Example Truth Table

- Construct a truth table for $p \vee q \to \neg r$

p	q	r	¬r	p ∨ q	p ∨ q → ¬r
T	T	T			
T	T	F			
T	F	T			
T	F	F			
F	T	T			
F	T	F			
F	F	T			
F	F	F			

Equivalent Propositions

- Two propositions are *equivalent* if they always have the same truth value.
- **Example**: Show using a truth table that the biconditional is equivalent to the contrapositive.

Solution:

p	q	$\neg p$	$\neg q$	$p \rightarrow q$	$\neg q \rightarrow \neg p$
T	T	F	F		
T	F	F	T		
F	T	T	F		
F	F	T	T		

Using a Truth Table to Show Non-Equivalence

Example: Show using truth tables that neither the **converse nor inverse** of an implication are not equivalent to the implication.

Solution:

p	q	$\neg p$	$\neg q$	$p \rightarrow q$	$\neg p \rightarrow \neg q$	$q \rightarrow p$
T	T	F	F			
T	F	F	T			
F	T	T	F			
F	F	T	T			

- $p \leftrightarrow q$ is equivalent to $(p \rightarrow q) \land (q \rightarrow p)$

$$\neg(p \land q) \equiv \neg p \lor \neg q$$

- $\neg(p \lor q)$ is equivalent with $\neg p \land \neg q$

- show $\neg p \lor q$ is equivalent to $p \to q$.

Translating Sentences into Propositional logic

- It is raining (p)
- Jimin is a loud-mouth (q)
- Suga stayed up late last night (r)
- It is not the case that Mary is sick (s)

Translating Sentences into Propositional logic

- **It is not raining**
- **Jimin is in no way a loud-mouth**
- **Suga did not stay up late last night**
- **It is not the case that Mary is not sick.**

Translating Sentences into Propositional logic

- **Suga stayed up late last night and Jimin is a loud-mouth**
- **It is raining and Mary is sick**
- **Jimin is a loud-mouth but Mary is not sick**
- **It is not the case that it is raining and Mary is sick**

Translating Sentences into Propositional logic

- **Mary is sick or mary is not sick.**
- **Jimin is a loud-mouth or Mary is sick or it is raining.**
- **It is not the case that Mary is sick or Suga stayed up late last night**

Translating Sentences into Propositional logic

- **It is raining , when Jimin is a loud-mouth.**
- **Mary is sick and it is raining implies that Suga stayed up late last night**
- **It is not the case that if it is raining then Jimin is not a loud-mouth.**

Translating Sentences into Propositional logic

- **It is raining if and only if Mary is sick.**
- **If Mary is sick then it is raining , and vice versa**
- **It is raining is equivalent to Jimin is a loud-mouth**
- **It is raining is not equivalent to Jimin is a loud-mouth**

Translating Sentences into Propositional logic

- **Neither the storm blast not the flood did any damage to the house**
- **If Global warming is not controlled, more forests will die in Amazon.**
- **Drivers should neither drive over 65 miles per hour nor cross the red light, or they will get a ticket.**

Applications of Propositional Logic

- **Translating English Sentences**

You cannot ride the roller coaster if you are under 4 feet tall unless you are older than 16 years old.”

$(r \land \neg s) \to \neg q$.

- **System Specifications(requirements)**

“The automated reply cannot be sent when the file system is full”

$q \to \neg p$

- **Boolean Searches**

Web pages with search techniques from propositional logic, are called Boolean searches.

NEW AND MEXICO AND UNIVERSITIES. The results of this search will include those pages that contain the three words NEW, MEXICO, and UNIVERSITIES. This will include all of the pages of interest, together with others such as a page about new universities in Mexico.

Applications of Propositional Logic (Reading Home Task)

- **Logic Puzzles**
- **Logic Circuits**