FAST- National University of Computer and Emerging Sciences, Karachi.

FAST School of Computing

Assignment # 3 -- Solution, Fall 2021.

CS1005-Discrete Structures

Instructions: **Max. Points: 100**

1- This is hand written assignment.
2- Just write the question number instead of writing the whole question.
3- You can only use A4 size paper for solving the assignment.

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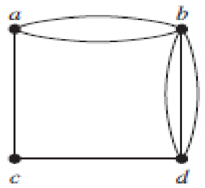
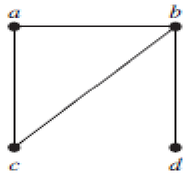
1. Determine whether the graph shown in figure i to iv has directed or undirected edges, whether it has multiple edges, and whether it has one or more loops. Use your answers to determine the type of graph.

Solution:

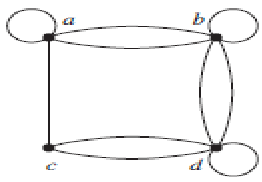
i) It has undirected edges.
It has multiple edges.
It has no loops.
It is undirected Multigraph.



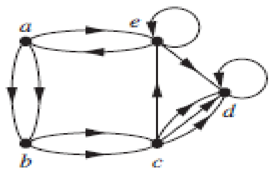
ii) It has undirected edges.
It has no multiple edges.
It has no loops.
It is undirected simple graph.



iii) It has undirected edges.
It has multiple edges.
It has three loops.
It is undirected Pseudo graph.



iv)It has directed edges.
It has multiple edges.
It has two loops.
It is directed Multi graph.



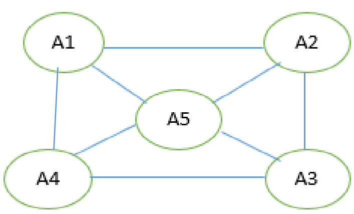
2. The intersection graph of a collection of sets A1, A2,..., An is the graph that has a vertex for each of these sets and has an edge connecting the vertices representing two sets if these sets have a nonempty intersection. Construct the intersection graph of these collections of sets.

i) A1 = {0, 2, 4, 6, 8}, A2 = {0, 1, 2, 3, 4}, A3 = {1, 3, 5, 7, 9}, A4 = {5, 6, 7, 8, 9}, A5 = {0, 1, 8, 9}

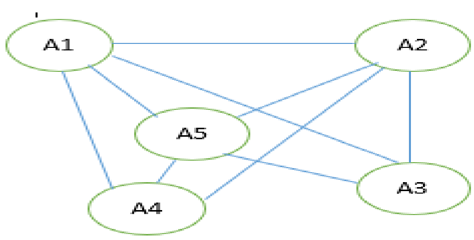
ii) A1 = {…,−4,−3,−2,−1, 0}, A2 = {…,−2,−1, 0, 1, 2,…}, A3 = {…,−6,−4,−2, 0, 2, 4, 6,…}, A4 = {…,−5,−3,−1, 1, 3, 5,…}, A5 = {…,−6,−3, 0, 3, 6,…}

Solution:

i)



ii)



3. (a) Find the number of vertices, the number of edges, and the degree of each vertex in the given undirected graph. Also find the neighborhood vertices of each vertex in given graphs.

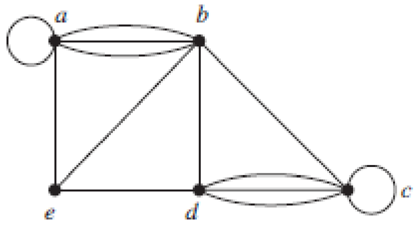
i) Number of Vertices: 5 Number of edges:13

Degree of vertices:

deg(a) = deg(b) = deg(c) = 6, deg(d) = 5, deg(e) = 3.

Neighborhood Vertices:

$N(a)$ = {a, b, e}, $N(b)$ = {a, c, d, e}, $N(c)$ = {b, c, d}, $N(d)$ = {b, c, e}, $N(e)$ = a, b, d}



ii) Number of Vertices: 9 Number of edges:12

Degree of vertices:

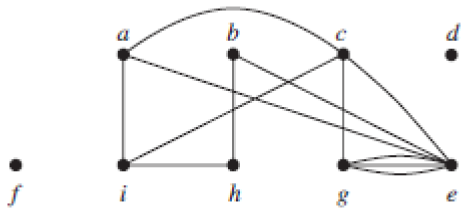
deg(a) =3, deg(b) =2, deg(c) = 4, deg(d) = 0, deg(e) = 6.

deg(f) =0, deg(g) =4, deg(h) = 2, deg(i) = 3.

Neighborhood Vertices:

$N(a)$ = {c, e, i}, $N(b)$ = {e, h}, $N(c)$ = {a, e, g, i}, $N(d)$ = $\varnothing$,

$N(e)$ = {a, b, c, g}, $N(f)$ = $\varnothing$, $N(g)$ = {c, e}, $N(h)$ = {b, i}, $N(i)$ = {a, c, h}.



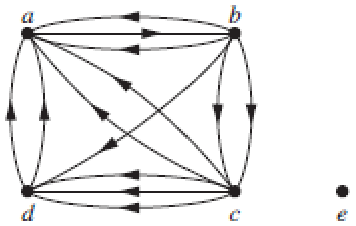
(b) Determine the number of vertices and edges and find the in-degree and out-degree of each vertex for the given directed multigraph.

i) In-degree of a vertices

$deg^-(a)$ = 6, $deg^-(b)$ = 1, $deg^-(c)$ = 2, $deg^-(d)$ = 4, $deg^-(e)$ = 0.

Out-degree of a vertices

$deg^+(a)$ = 1, $deg^+(b)$ = 5, $deg^+(c)$ = 5, $deg^+(d)$ = 2, $deg^+(e)$ = 0.

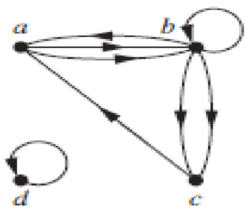


ii) In-degree of a vertices

$deg^-(a)$ = 2, $deg^-(b)$ = 3, $deg^-(c)$ = 2, $deg^-(d)$ = 1.

Out-degree of a vertices

$deg^+(a)$ = 2, $deg^+(b)$ = 4, $deg^+(c)$ = 1, $deg^+(d)$ = 1.

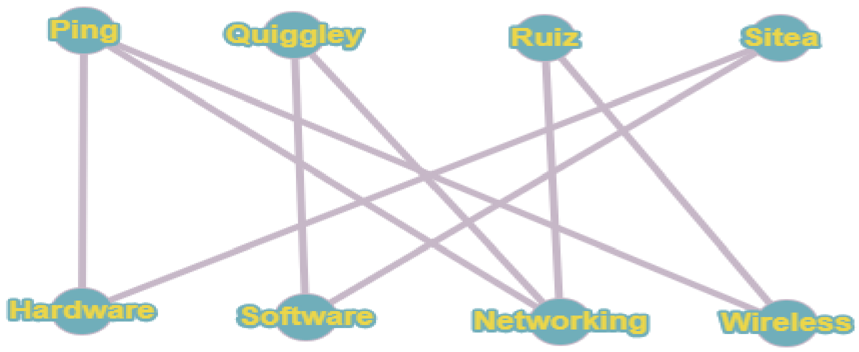


4. (a) Suppose that a new company has five employees: Zamora, Agraharam, Smith, Chou, and Macintyre. Each employee will assume one of six responsibilities: planning, publicity, sales, marketing, development, and industry relations. Each employee is capable of doing one or more of these jobs: Zamora could do planning, sales, marketing, or industry relations; Agraharam could do planning or development; Smith could do publicity, sales, or industry relations; Chou could do planning, sales, or industry relations; and Macintyre could do planning, publicity, sales, or industry relations. Model the capabilities of these employees using appropriate graph.

Solution: Bipartite Graph

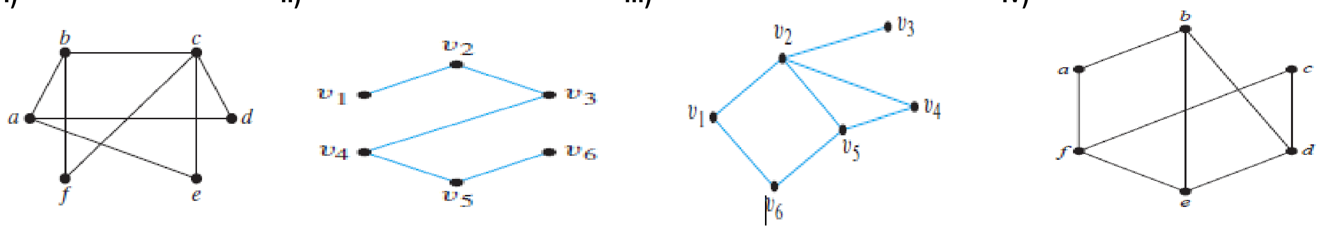
(b) Suppose that there are four employees in the computer support group of the School of Engineering of a large university. Each employee will be assigned to support one of four different areas: hardware, software, networking, and wireless. Suppose that Ping is qualified to support hardware, networking, and wireless; Quiggley is qualified to support software and networking; Ruiz is qualified to support networking and wireless, and Sitea is qualified to support hardware and software. Use appropriate graph to model the four employees and their qualifications.

Solution: Bipartite Graph



5. Find which of the following graphs are bipartite. Redraw the bipartite graphs so that their bipartite nature is evident. Also write the disjoint set of vertices.

i) ii) iii) iv)



Solution:

(a) Not bipartite (since a is adjacent to b & f vertices)

(b) Bipartite (A (V_1, V_3, V_5) & B (V_2, V_4, V_6))

(c) Not bipartite (since V_4 & V_5 are adjacent vertices)

(d) Not Bipartite (since b is adjacent to d & e vertices)

6. Draw a graph with the specified properties or show that no such graph exists.

a) A graph with four vertices of degrees 1, 1, 2, and 3

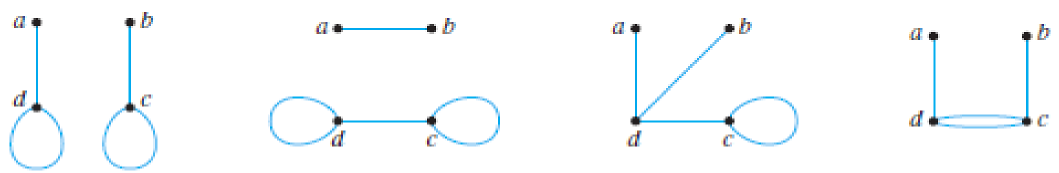
b) A graph with four vertices of degrees 1, 1, 3, and 3

c) A simple graph with four vertices of degrees 1, 1, 3, and 3

Solution:

a) No such graph is possible. By Handshaking theorem, the total degree of a graph is even. But a graph with four vertices of degrees 1, 1, 2, and 3 would have a total degree of 1 + 1 + 2 + 3 = 7, which is odd.

b) Let G be any of the graphs shown below.



In each case, no matter how the edges are labeled, deg(a) = 1, deg(b) = 1, deg(c) =3, and deg(d) = 3.

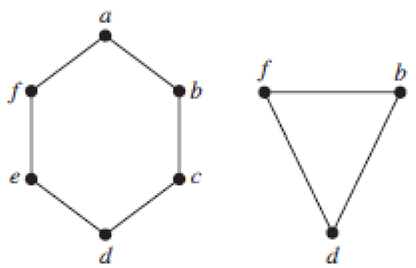
c) There is no simple graph with four vertices of degrees 1, 1, 3, and 3.

7. a) In a group of 15 people, is it possible for each person to have exactly 3 friends? Explain. (Assume that friendship is a symmetric relationship: If x is a friend of y, then y is a friend of x.)
Solution:
By using Handshaking theorem.
No! there is no graph possible, such that 15 vertices have degree 3. Since (15 * 3) ≠ 2e.

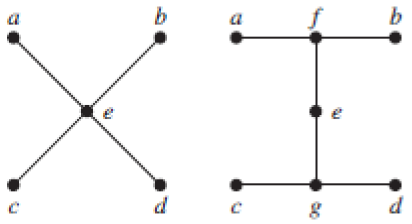
b) In a group of 4 people, is it possible for each person to have exactly 3 friends? Why?
Solution:
By using Handshaking theorem.
Yes! there is graph possible, such that 4 vertices have degree 3. Since (4 * 3) = 2e.

8. (a) Find the union of the given pair of simple graphs. (Assume edges with the same endpoints are the same.)

i)

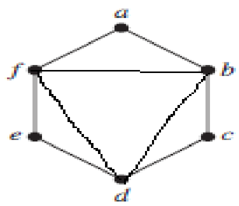


ii)

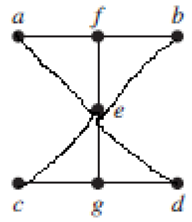


Solution:

i)



ii)



b) How many vertices does a regular graph of degree four with 10 edges have?
Solution:

We want to determine a regular graph of degree four with $m = 10$ edges.

Let the graph contain n vertices $v_1, v_2, ..., v_n$, then each of these n vertices have degree 4.

$$\deg(v_i) = 4$$
$$i = 1, 2, ..., n$$

By the Handshaking theorem, the sum of degrees of all vertices is equal to twice the number of edges:

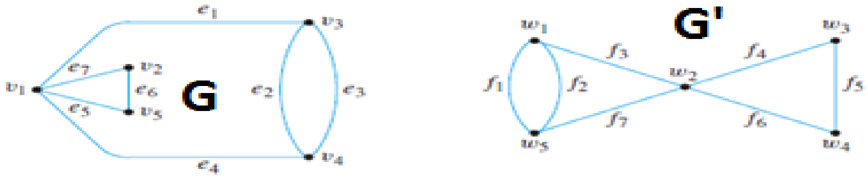
$$20 = 2(10) = 2m = \sum_{v=1}^{n} \deg(v_i) = \sum_{v=1}^{n} 4 = 4n$$

We then obtained the equation $20 = 4n$. Divide each side of the equation by 4:

$$n = \frac{20}{4} = 5$$

9. **For given pair (G, G') of graphs. Determine whether they are isomorphic. If they are, give function g: V (G) →V (G') that define the isomorphism. If they are not, give an invariant for graph isomorphism that they do not share.**

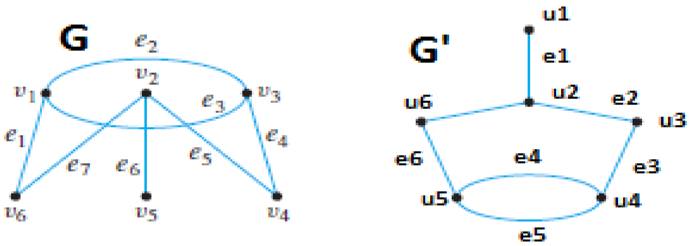
i)



Solution: Both graph G and G' are satisfying all the invariant. Hence, they are isomorphic.

Function: ***$g(V_1) = W_2$,*** ***$g(V_2) = W_3$*** ***$g(V_3) = W_1$*** ***$g(V_4) = W_5$*** ***$g(V_5) = W_4$***

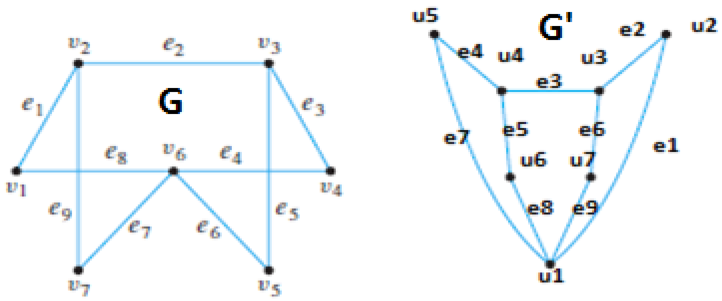
ii)



Solution: Both graph G and G' are satisfying all the invariant. Hence, they are isomorphic.

Function: ***$g(V_1) = U_5$,*** ***$g(V_2) = U_2$*** ***$g(V_3) = U_4$*** ***$g(V_4) = U_3$*** ***$g(V_5) = U_1$*** ***$g(V_6) = U_6$***

iii)

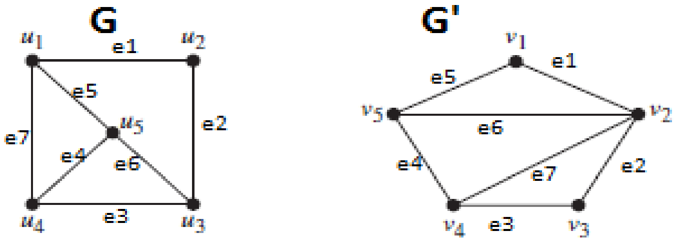


Solution: Both graph G and G' are satisfying all the invariant. Hence, they are isomorphic.

Function: ***$g(V_1) = U_5$,*** ***$g(V_2) = U_4$*** ***$g(V_3) = U_3$*** ***$g(V_4) = U_2$***

$g(V_5) = U_7$ ***$g(V_6) = U_1$*** ***$g(V_7) = U_6$***

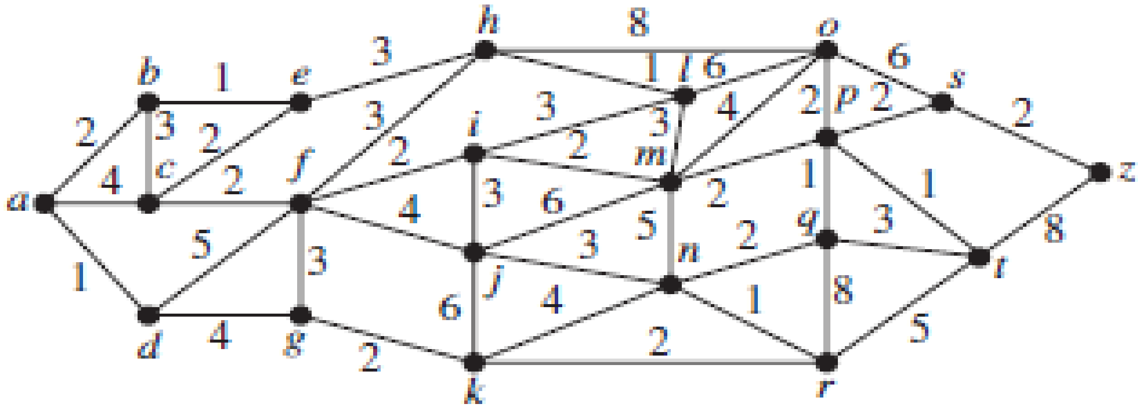
iv)



Solution: Graph G has no vertex of degree 4 where G' has vertex V_2 with degree 4. Hence, they are not isomorphic.

10. **Find the length of a shortest path between *a* and *z* in the given weighted graph by using Dijkstra's algorithm.**

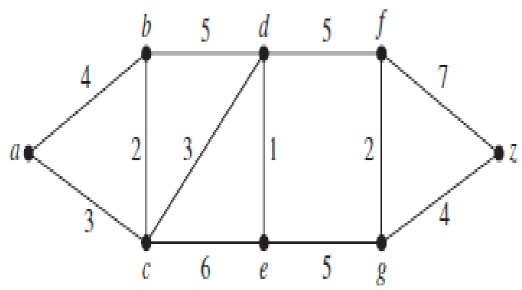
i)



N	D(b)	D (c)	D(d)	D(e)	D(f)	D(g)	D(h)	D(i)	D(j)	D(k)	D(l)	D(m)	D(n)	D(o)	D(p)	D(q)	D(r)	D(s)	D(t)	D(z)
a	2,a	4,a	1,a																	
ad	2,a	4,a			6,d	5,d														
adb		4,a		3,b	6,d	5,d														
adbe		4,a			6,d	5,d	6,e													
adbec					6,c	5,d	6,e													
adbeg					6,c		6,e			7,g										
adbegf							6,e	8,f	10,f	7,g										
adbegfh								8,f	10,f	7,g	7,h			14,h						
adbegfhk								8,f	10,f		7,h		11,k	14,h			9,k			
adbegfhkl								8,f	10,f			10,l	11,k	13,l			9,k			
adbegfhkli									10,f			10,l	11,k	13,l			9,k			
adbegfhklir									10,f			10,l	10,r	13,l		17,r			14,r	
adbegfhklirj												10,l	10,r	13,l		17,r			14,r	
adbegfhklirjm													10,r	13,l	12,m	17,r			14,r	
adbegfhklirjmn														13,l	12,m	12,n			14,r	
adbegfhklirjmnp														13,l		12,n		14,p	13,p	
adbegfhklirjmnpq														13,l				14,p	13,p	
adbegfhklirjmnpqo																		14,p	13,p	
adbegfhklirjmnpqot																		14,p		21,t
adbegfhklirjmnpqots																				16,s
adbegfhklirjmnpqotsz																				

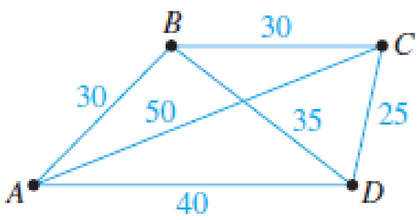
ii)

N	D(b)	D(c)	D(d)	D(e)	D(f)	D(g)	D(z)
a	4,a	3,a	∞	∞	∞	∞	∞
ac			6,c	9,c	∞	∞	∞
acb			6,c	9,c	∞	∞	∞
acbd				7,d	11,d	∞	∞
acbde					11,d	12,e	∞
acbdef						12,e	18,f
acbdefg							16,g
acbdefgz	4,a	3,a	6,c	7,d	11,d	12,e	16,g

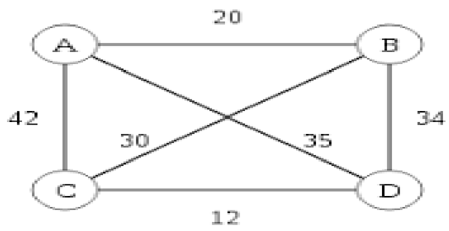


11. Imagine that the drawing below is a map showing four cities and the distances in kilometers between them. Suppose that a salesman must travel to each city exactly once, starting and ending in city A. Which route from city to city will minimize the total distance that must be traveled?

i)



ii)



i) Solution:

Hamiltonian Circuit are: ABCDA = 125; ABDCA = 140; ACBDA = 155.

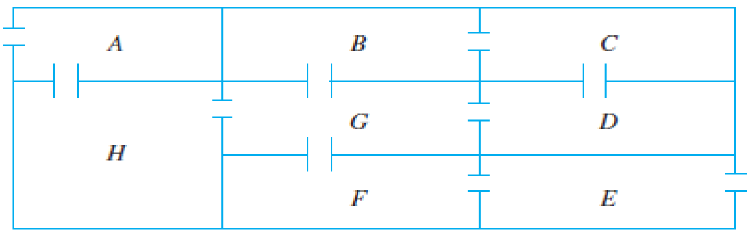
Hence ABCDA = 125 is the minimum distance travelled.

ii) Solution:

Hamiltonian Circuit are: ABCDA = 97; ABDCA = 108; ACBDA = 141.

Hence ABCDA = 97 is the minimum distance travelled.

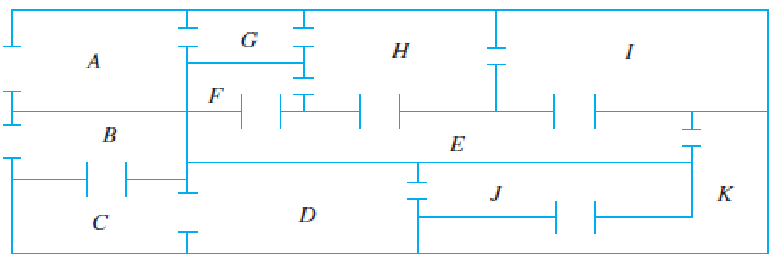
12. (a) The following is a floor plan of a house. Is it possible to enter the house in room A, travel through every interior doorway of the house exactly once, and exit out of room E? If so, how can this be done?



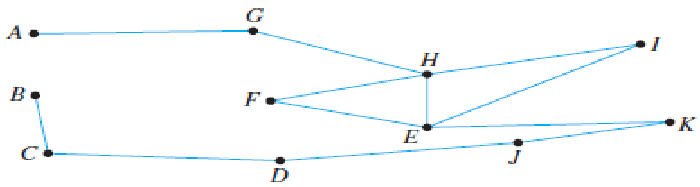
Solution:

Yes! Path: A →H→G→B→C→D→G→F→E

(b) The floor plan shown below is for a house that is open for public viewing. Is it possible to find a trail that starts in room A, ends in room B, and passes through every interior doorway of the house exactly once? If so, find such a trail.



Solution Let the floor plan of the house be represented by the graph below.

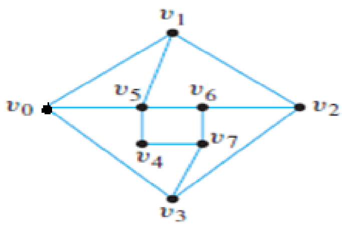


Each vertex of this graph has even degree except for A and B, each of which has degree 1. Hence by Corollary 10.2.5, there is an Euler path from A to B. One such trail is

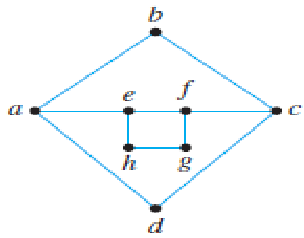
$$AGHFEIHEKJDCB.$$

13. Find Hamiltonian circuits AND Path for those graphs that have them. Explain why the other graphs do not.

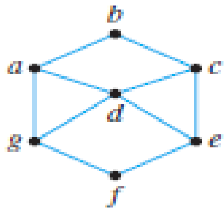
i)



ii)



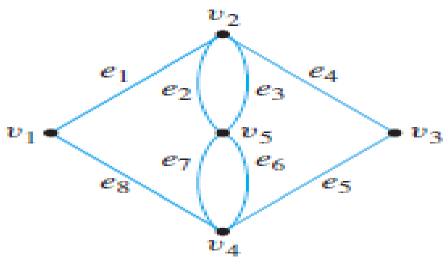
iii)



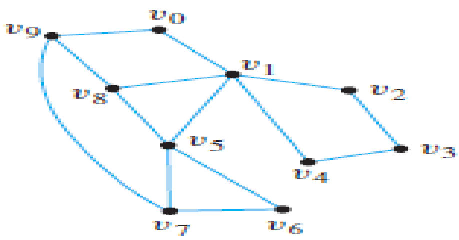
i) Solution:
Hamiltonian Circuit: V_0, V_1, V_2, V_6, V_5, V_4, V_7, V_3, V_0
Hamiltonian Path: V_0, V_1, V_2, V_6, V_5, V_4, V_7, V_3
ii) Solution:
Hamiltonian Circuit: doesn't exist
Hamiltonian Path: b, c, f, g, h, e, a, d
iii) Solution:
Hamiltonian Circuit: d, c, b, a, g, f, e, d
Hamiltonian Path: d, c, b, a, g, f, e

14. a) Determine which of the graphs have Euler circuits. If the graph does not have an Euler circuit, explain why not. If it does have an Euler circuit, describe one.

i)



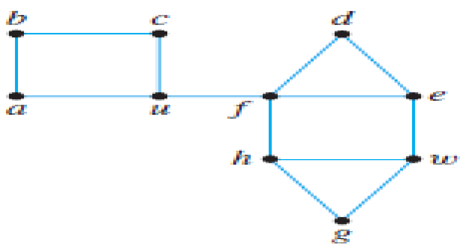
ii)



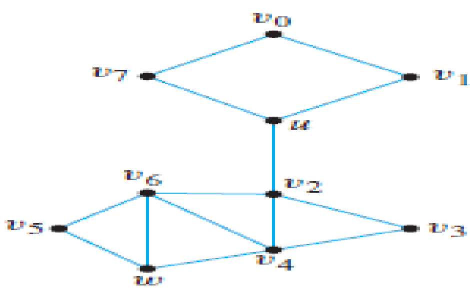
i) Solution: All vertices have even degree so circuit exists.
Euler Circuit: V_1, V_2, V_5, V_4, V_5, V_2, V_3, V_4, V_1
ii) Solution:
Euler Circuit do not exist because all vertices don't have even degree.

b) Determine whether there is an Euler path from u to w. If the graph does not have an Euler path, explain why not. If it does have an Euler path, describe one.

i)



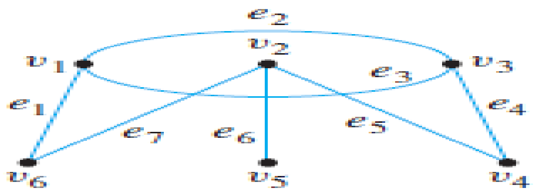
ii)



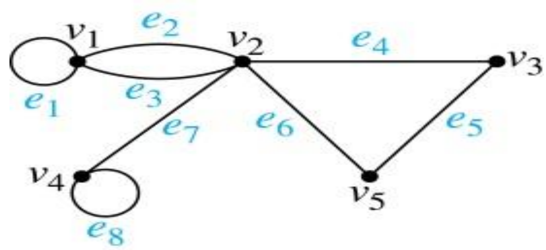
i) Solution:
Euler Path doesn't exist because four vertices have odd degree.
ii) Solution: Euler Path exists because exact two vertices have odd degree.
Euler path: U, V_1, V_0, V_7, U, V_2, V_3, V_4, V_2, V_6, V_5, W, V_6, V_4, W

15. (a) Use an incidence matrix to represent the graph shown below.

i)



ii)



Solution: i)

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

ii)

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \end{bmatrix}$$

(b) Draw a graph using below given incidence matrix.

i)

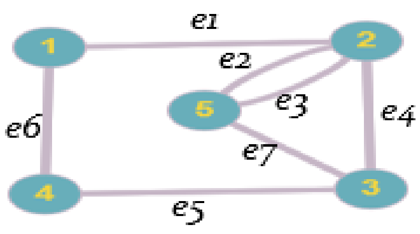
$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 1 \end{pmatrix}$$

ii)

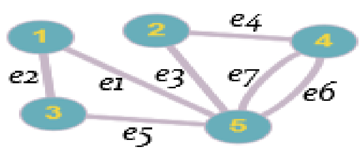
$$\begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 1 & 1 \end{pmatrix}$$

Solution:

i)

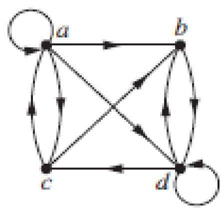


ii)



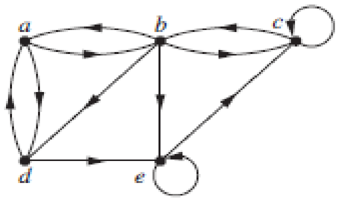
16. Use an adjacency list and adjacency matrix to represent the given graph.

i)



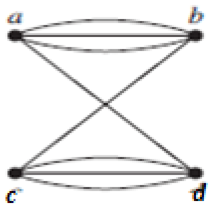
Initial Vertex	Terminal Vertices
a	a, b, c, d
b	d
c	a, b
d	b, c, d

(ii)



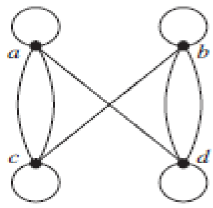
Initial Vertex	Terminal Vertices
a	b, d
b	a, c, d, e
c	b, c,
d	a, e
e	c, e

iii)



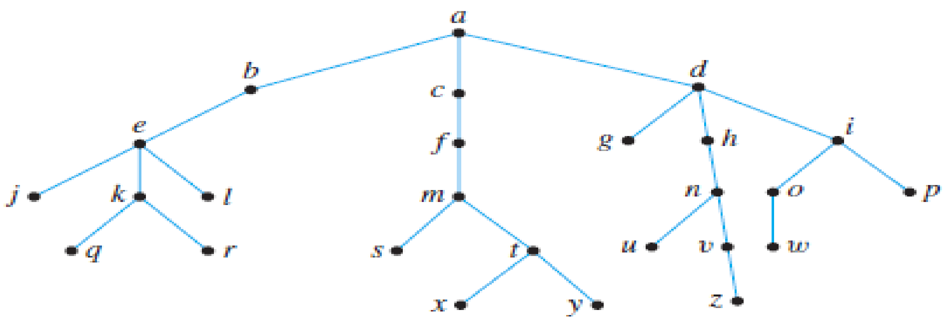
Vertex	Adjacent Vertices
a	b, d
b	a, c
c	b, d
d	a, c

iv)



Vertex	Adjacent Vertices
a	a, c, d
b	b, c, d
c	a, b, c
d	a, c, d

17. Consider the tree shown at right with root a.



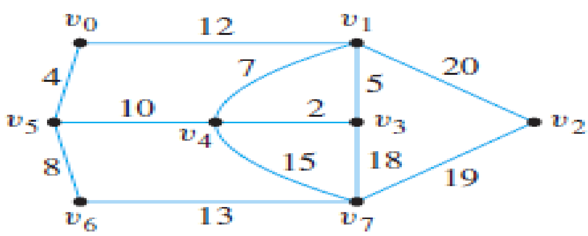
Solution:

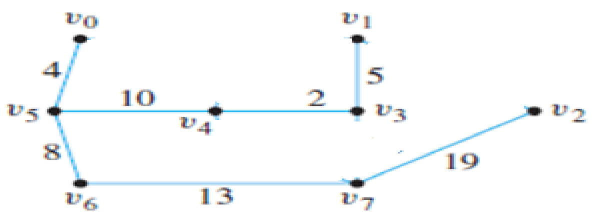
i) What is the level of n?	Level of n is 3.
ii) What is the level of a?	Level of a is 0.
iii) What is the height of this rooted tree?	Height of this rooted tree is 5
iv) What are the children of n?	u & v are the children of n.
v) What is the parent of g?	d is the parent of g.
vi) What are the siblings of j?	k & l are the siblings of j.
vii) What are the descendants of f?	m, s, t, x & y are the descendants of f.
viii) What are the internal nodes?	a, b, e, k, c, f, m, t, d, h, i, n, o & v are the internal nodes.
ix) What are the ancestors of z?	v, n, h, d & a are the ancestors of z.
x) What are the leaves?	j, l, q, r, s, x, y, g, p, u, w & z are the leaves.

18. Use Prim's algorithm to find a minimum spanning tree starting from V_0 for given graphs. Indicate the order in which edges are added to form each tree.

i) Solution: MST Cost = 61

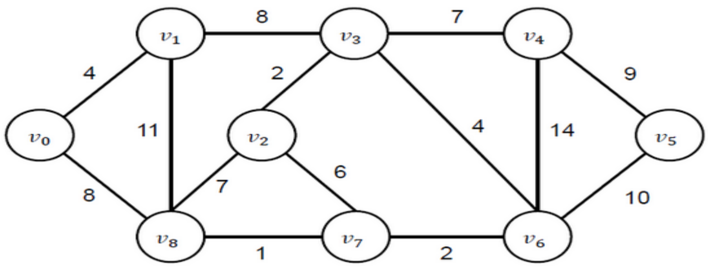
$(V_0,V_5) = 4$, $(V_5,V_6) = 8$, $(V_4,V_5) = 10$,
$(V_3,V_4) = 2$, $(V_1,V_3) = 5$, $(V_6,V_7) = 13$,
$(V_2,V_7) = 19$.

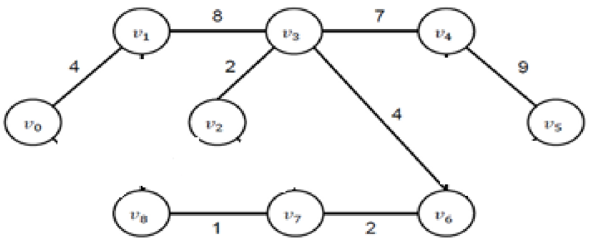


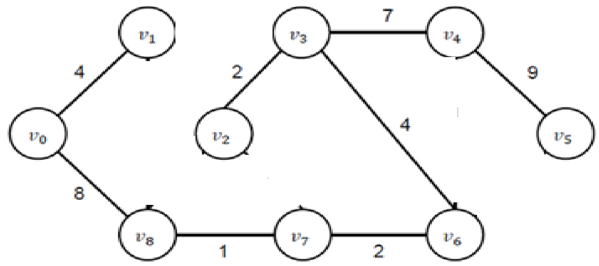


ii) Solution: MST Cost = 37

$(V_0,V_1) = 4$, $(V_0,V_8) = 8$, $(V_7,V_8) = 1$,
$(V_6,V_7) = 2$, $(V_3,V_6) = 4$, $(V_2,V_3) = 2$,
$(V_3,V_4) = 7$, $(V_4,V_5) = 9$,

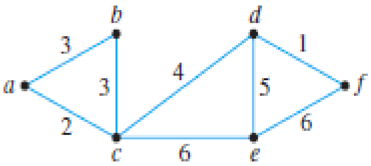




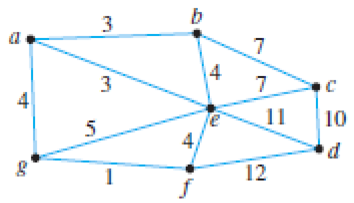


19. Use Kruskal's algorithm to find a minimum spanning tree for given graphs. Indicate the order in which edges are added to form each tree.

i)

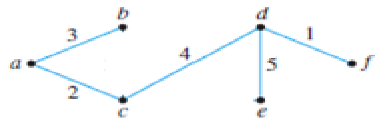


ii)



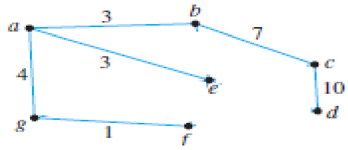
i) Solution: MST cost = 15
Order of edges added is:

(d, f) =1,	(a, c) =2,	(a, b) =3,	~~(b, c) =3,~~
(c, d) =4,	(d, e) =5,	~~(c, e) =6,~~	~~(e, f) =6~~



ii) Solution: MST cost = 28
Order of edges added is:

(g, f) =1,	(a, b) =3,	(a, e) =3,	(a, g) =4,
~~(b, e) =4,~~	~~(e, f) =4,~~	~~(g, e) =5,~~	(b, c) =7
~~(c, e) =7,~~	(c, d) =10,	~~(d, e) =11,~~	~~(d, f) =12~~

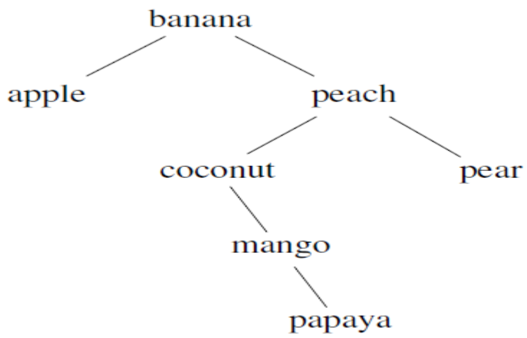


20. (a) i) Build a binary search tree for the word's banana, peach, apple, pear, coconut, mango, and papaya using alphabetical order.

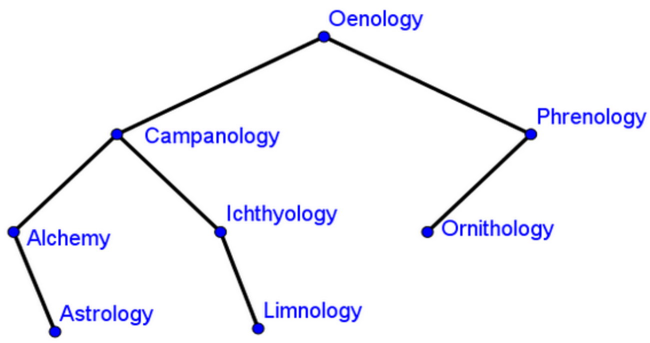
ii) Build a binary search tree for the word's oenology, phrenology, campanology, ornithology, ichthyology, limnology, alchemy, and astrology using alphabetical order.

Solution:

i)

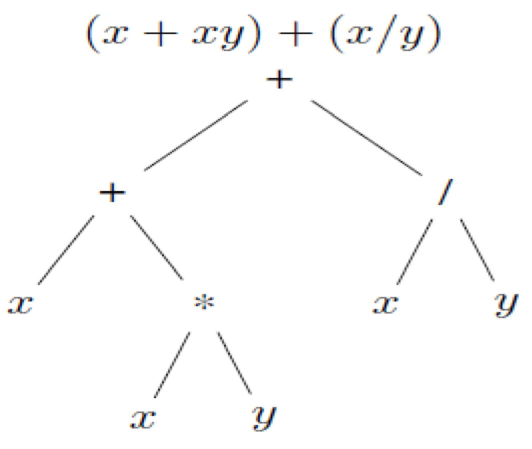


ii)

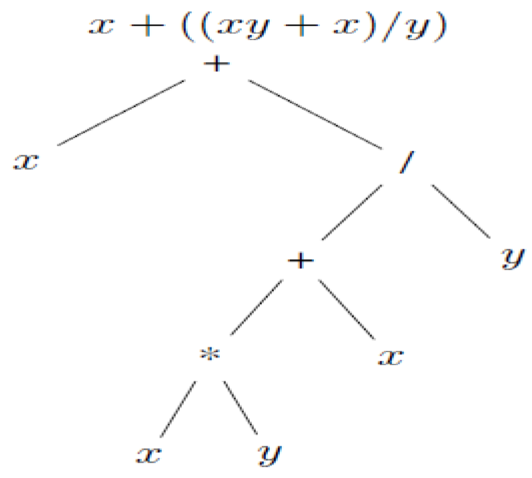


(b) Represent these expressions using binary trees.
(i) (x + xy) + (x / y)
(ii) x + ((xy + x) / y)
Solution:

i)

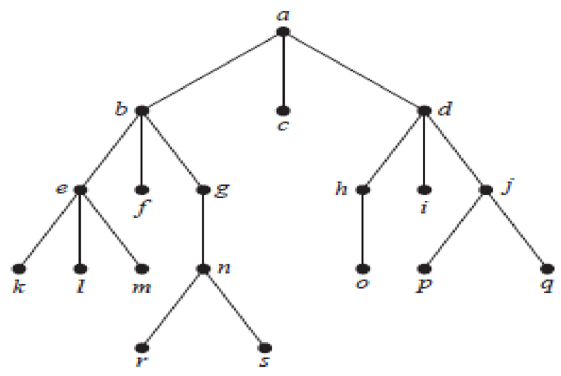


ii)

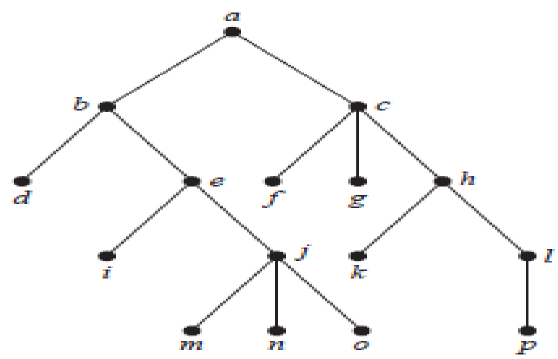


21. Determine the order in which preorder, Inorder and Postorder traversal visits the vertices of the given ordered rooted tree.

i)



ii)



Solution:

i)

Preorder: a b e k l m f g n r s c d h o l j p q
Inorder: k e l m b f r n s g a c o h d i p j q
Postorder: k l m e f r s n g b c o h l p q j d a

ii)

Preorder: a b d e i j m n o c f g h k l p
Inorder: d b i e m j n o a f c g k h p l
Postorder: d i m n o j e b f g k p l h c a

22. (a) How many edges does a tree with 10000 vertices have?
Solution:
A tree with n vertices has n – 1 edge. Hence 10000 – 1 = 9999 edges.
(b) How many edges does a full binary tree with 1000 internal vertices have?
Solution:
A full binary tree has two edges for each internal vertex. So, we'll just multiply the number of internal vertices by the number of edges. Hence 1000 * 2 = 2000 edges.

(c) How many vertices does a full 5-ary tree with 100 internal vertices have?
Solution:

A full $m - ary$ tree with I internal vertices has $n = mi + 1$ vertices.
From the given information, we have $m = 5$, $i = 100$
So $n = 5 \times 100 + 1 = 501$
Therefore a full 5-ary tree with 100 internal vertices has 501 vertices.

23. a) Write these expressions in Prefix and Postfix notation:

i) (x + xy) + (x / y)

Solution:

Prefix: + + x * x y / x y

Postfix: x x y * + x y / +

ii) x + ((xy + x) / y)

Solution:

Prefix: + x / + * x y x y

Postfix: x x y * x + y / +

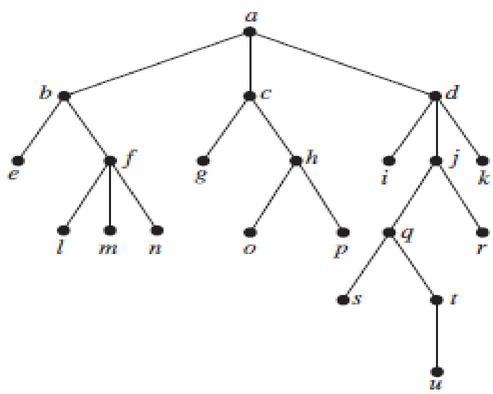
b) i) What is the value of this prefix expression + – ↑ 3 2 ↑ 2 3 / 6 – 4 2

Solution: 4

ii) What is the value of this postfix expression 4 8 + 6 5 – * 3 2 – 2 2 + * /

Solution: 3

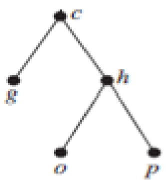
24. Answer these questions about the rooted tree illustrated.



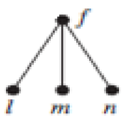
i) Is the rooted tree a full m-ary tree?
Solution: It is not a full m-ary tree for any m because some of its internal vertices have two children and others have three children.
ii) Is the rooted tree a balanced m-ary tree?
Solution: It is not balanced m-ary tree because it has leaves at levels 2, 3, 4 and 5.

iii) Draw the subtree of the tree that is rooted at
a) c. b) f. c) q.
Solution:

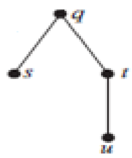
a)



b)

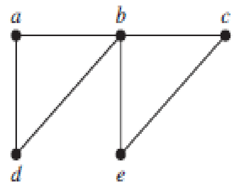


c)

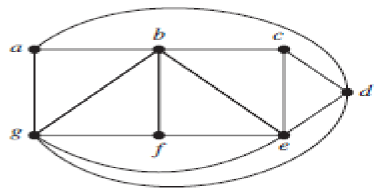


25. Find a spanning tree for the graph shown by removing edges in simple circuits. Write down the removed edges.

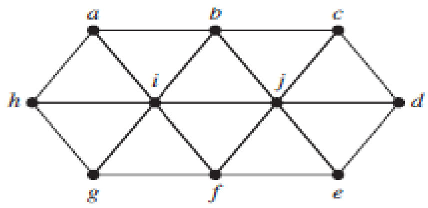
(i)



ii)

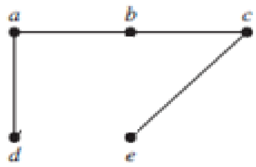


iii)

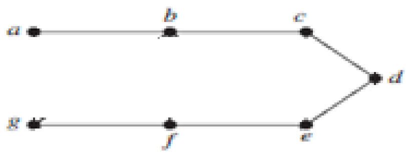


Solution:

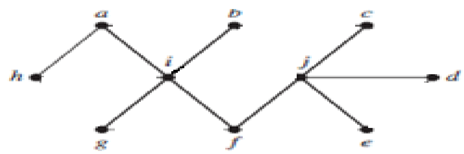
i)



ii)



iii)



26. (a) An office building contains 27 floors and has 37 offices on each floor. How many offices are in the building?
Solution: There are 27 * 37 = 99 offices in the building.

(b) A particular brand of shirt comes in 12 colors, has a male version and a female version, and comes in three sizes for each sex. How many different types of this shirt are made?
Solution: 12 * 2 * 3 shirts are required.

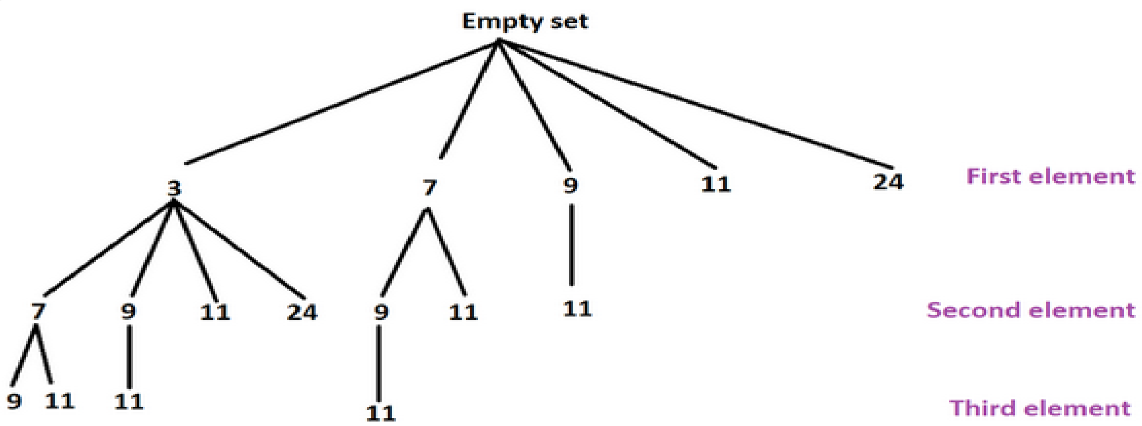
27. (a) How many different three-letter initials can people have?
Solution:
People can have 26 *26 * 26 = 26^3 different three-letter initials.
(b) How many different three-letter initials with none of the letters repeated can people have?
Solution:
People can have 26 *25 * 24 = 15,600 different three-letter initials with none of the letters repeated.

28. (a) A wired equivalent privacy (WEP) key for a wireless fidelity (WiFi) network is a string of either 10, 26, or 58 hexadecimal digits. How many different WEP keys are there?
Solution:
There are16 place values for hexadecimal numbers: 0 to 9, A, B, C, D, E and F.
So, $16^{10} + 16^{28} + 16^{58}$ different WEP keys are possible.
(b) How many strings are there of four lowercase letters that have the letter x in them?
Solution:
There would be $26^4 – 25^4$ = 66,351 strings.

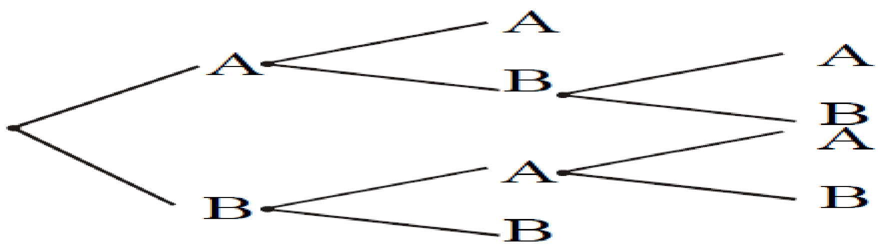
29. (a) How many functions are there from the set {1, 2, . . . , *m*}, where *m* is a positive integer, to the set {0, 1}?
Solution:
Since each value of the domain can be mapped to one of two values. Number of functions are:
= 2 * 2 * 2 * 2 * ….. * m = 2^m .

(b) How many one-to-one functions are there from a set with five elements to sets with five elements?
Solution:
Each successive element from the domain will have one option than its predecessor as it is one-to-one function. So, number of functions are 5 * 4 * 3 * 2 * 1 = 120.

30. (a) Use a tree diagram to determine the number of subsets of {3, 7, 9, 11, 24} with the property that the sum of the elements in the subset is less than 28.
Solution:



(b) Teams A and B play in a tournament. The team that wins first two games wins the tournament. Use a tree diagram to find the number of possible ways in which the tournament can occur.
Solution:



31. (a) Eight members of a school marching band are auditioning for 3 drum major positions. In how many ways can students be chosen to be drum majors?
Solution:
There are ${}^{8}C_{3}$ = 56 ways to choose the students.

(b) You must take 6 CS elective courses to meet your graduation requirements at FAST-NUCES. There are 12 CS courses you are interested in. In how many ways can you select your elective Courses?
Solution:
There are ${}^{12}C_{6}$ = 924 ways to select the elective courses.

(c) Nine people in our class want to be on a 5-person basketball team to represent the class. How many different teams can be chosen?
Solution:
${}^{9}C_{5}$= 126 different teams can be selected.

32. (a) A committee of five people is to be chosen from a group of 20 people. How many different ways can a chairperson, assistant chairperson, treasurer, community advisor, and record keeper be chosen?
Solution:
There are ${}^{20}P_{5}$= 1,860,480 ways to choose a chairperson, assistant chairperson, treasurer, community advisor, and record keeper.

(b) A relay race has 4 runners who run different legs of the race. There are 16 students on your track team. In how many ways can your coach select students to compete in the race? Assume that the order in which the students run matters.
Solution:
There are ${}^{16}P_{4}$= 43,680 ways coach can select students to compete in the race.

(c) Your school yearbook has an editor in chief and an assistant editor in chief. The staff of the yearbook has 15 students. In how many ways can a student be chosen for these 2 positions?
Solution:
There are ${}^{15}P_{2}$= 210 ways student can be chosen for these 2 positions.

33. (a) A deli offers 5 different types of meat, 3 types of breads, 4 types of cheeses and 6 condiments. How many different types of sandwiches can be made of 1 meat, 2 bread, 1 cheese, and 3 condiments?
Solution:
${}^{5}C_{1}$ * ${}^{3}C_{2}$ * ${}^{4}C_{1}$ * ${}^{6}C_{3}$ = 1200 Sandwiches can be made of 1 meat, 2 bread, 1 cheese, and 3 condiments.

(b) Police use photographs of various facial features to help eyewitnesses identify suspects. One basic identification kit contains 15 hairlines, 48 eyes and eyebrows, 24 noses, 34 mouths, and 28 chins and 28 cheeks. Find the total number of different faces.
Solution:
There are 15 * 48 * 24 * 34 * 28 * 28 = 460,615,680 different faces.

34. (a) How many bit strings of length 10 either begin with three 0s or end with two 0s?
Solution:
A = Strings begins with three 0s = 2^7 = 128 **B = Strings end with two 0s = 2^8 = 256**
A∩B = 2^5 = 32
AUB = A + B – A∩B = 128 + 256 – 32 = 352.

(b) How many bit strings of length 5 either begin with 0 or end with two 1s?
A = Strings begins with 0s = 2^4 = 16 **B = Strings end with two 1s = 2^3= 8**
A∩B = 2^2 = 4
AUB = A + B – A∩B = 16 + 8 – 4 = 20.

35. (a) Show that if there are 30 students in a class, then at least two have last names that begin with the same letter.
Solution:
The first letter of each last name are the pigeonholes, and the letters of the alphabet are pigeons. By the generalized pigeonhole principle, $\left\lceil \frac{30}{26} \right\rceil$ = 2. So there are at least two students, have last names that begin with the same letter.

(b) Assuming that no one has more than 1,000,000 hairs on the head of any person and that the population of New York City was 8,008,278 in 2010, show there had to be at least nine people in New York City in 2010 with the same number of hairs on their heads.
Solution:
By the generalized pigeonhole principle, $\left\lceil \frac{8008278}{1000000} \right\rceil$ = 9.

(c) There are 38 different time periods during which classes at a university can be scheduled. If there are 677 different classes, how many different rooms will be needed?
Solution:
The 38 time periods are the pigeonholes, and the 677 classes are the pigeons. By the generalized pigeonhole principle there is at least one time period in which at least $\left\lceil \frac{677}{38} \right\rceil$ = 18 classes are meeting. Since each class must meet in a different room, we need 18 rooms.

36. (a) What is the coefficient of x^5 in $(1 + x)^{11}$?
Solution:
From binomial theorem, it follows that coefficient is:
$^{n}C_{r} = {}^{11}C_{5}$= 462.

(b) What is the coefficient of a^7b^{17} in $(2a – b)^{24}$?
Solution:
From binomial theorem, it follows that coefficient is:
$^{n}C_{r} = {}^{24}C_{17}\,(2)^7\,(-1)^{17}$ = - 44,301,312.

37. A class has 20 women and 16 men. In how many ways can you

(a) put all the students in a row?

Solution:

There are 36 students. They can be put in a row in 36! ways.

(b) put 7 of the students in a row?

Solution:

You need to have an ordered arrangement of 7 out of 36 students. The number of such arrangements is *P* (36,7).

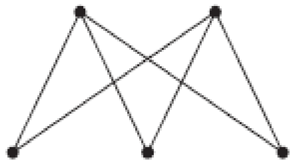
(c) put all the students in a row if all the women are on the left and all the men are on the right?

Solution:

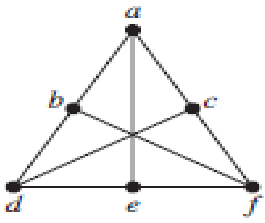
You need to have an ordered arrangement of all 20 women and ordered arrangement of all 16 men. By the product rule, this can be done in 20! * 16! ways.

38. Determine whether the given graph is planar. If so, draw it so that no edges cross.

(a)



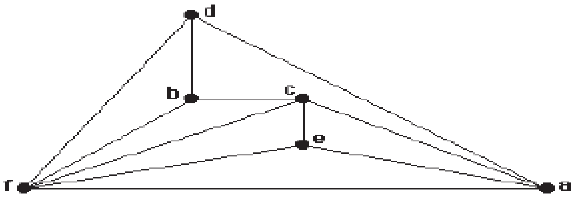
(b)



Solution:

(a) This is $K_{3,3}$, with parts {a, d, f} and {b, c, e}. Therefore it is not planar.

(b) This graph can be untangled if we play with it long enough. The following picture gives a planar representation of it.



39. As we have discussed, the practical application of all the topics in the class. Now you are required to submit at least one real world applications of the following topics.

(a) Combination
(b) Permutations
(c) Binomial Theorem
(d) Pascal Triangle
(e) Directed and Undirected Graphs
(f) Isomorphic Graphs
(g) Planar Graphs
(h) Trees Traversal
(i) Tree Structure
(j) Spanning Trees
(k) Shortest Path Algorithms
(l) Bipartite Graphs