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Sec: B

Discrete Assignment

Q1

p	q	$\neg p$	$(\neg p \wedge q)$	$\neg(p \vee q)$
T	T	F	F	F
T	F	F	F	F
F	T	T	T	F
F	F	T	F	T

Hence proved.

Q2

p	q	$\neg p$	$\neg q$	$\neg(p \to \neg q)$	$p \wedge q$	$x \vee y$
T	T	F	F	T	T	T
T	F	F	T	F	F	F
F	T	T	F	T	F	T
F	F	T	T	F	F	F

Hence proved.

Q3

values for which $x = (p \to \neg q)$ is true

p	q	$\neg q$	$p \to \neg q$	v	x	
T	T	F	F	T	F	
T	F	T	T	T	F	
F	T	F	T	T	T	
F	F	T	T	T	T	True
T	T	F	F	F	T	
T	F	T	T	F	F	
F	T	F	T	F	F	
F	F	T	T	F	F	

Q4

$= q \to (p \to q)$

$= q \to (\neg p \vee q)$

$=$

$= (q \to \neg p) \vee (q \to q)$

$= (\neg q \vee \neg p) \vee (\neg q \vee q)$

$= (\neg q \vee \neg p) \vee T$

$= F \vee T$

$= T$

Thus proved that $q \to (p \to q)$ is a tautology.

Q5

p	r	q	$\neg q$	$\neg p$	$p \leftrightarrow \neg q$	$\neg p \vee \neg q$	x
T	T	T	F	F	F	F	T
T	F	T	F	F	F	F	T
T	T	F	T	F	T	T	T
T	F	F	T	F	T	T	T
F	T	T	F	T	T	T	T
F	F	T	F	T	T	T	T
F	T	F	T	T	F	T	T
F	F	F	T	T	F	T	T

Hence

$x = (p \leftrightarrow \neg q) \to (\neg p \vee \neg q)$

is a tautology

Q8

let $A = \{\}$, $B = \{2\}$; $C = \{2, 3\}$

$A \cup C = \{\} \cup \{2, 3\} = \{2, 3\}$

$B \cup C = \{2\} \cup \{2, 3\} = \{2, 3\}$

$A \cup C = B \cup C$ but $A \neq B$

Q7

$a \to$ wearing a pink tie

$b \to$ wearing a red shirt

$c \to$ It is saturday

$$\begin{array}{l} \neg a \vee b \\ \neg c \to a \\ \underline{\neg b} \\ \therefore c \end{array}$$

$$\begin{array}{l} \neg b \vee \neg c \\ \underline{\neg b} \\ \therefore c \end{array}$$

$$\begin{array}{l} \neg b \to c \\ \underline{\neg b} \\ \therefore c \end{array}$$

Q11

$(\overline{A \cap B}) = \bar{A} \cup \bar{B}$

$\bar{A} \cup \bar{B} = (\overline{A \cap B})$

$= (\bar{A} \cup \bar{B})$

$\boxed{\bar{A} \cup \bar{B} = \bar{A} \cup \bar{B}}$

Q12

let

$A =$ no. of triumphs $= 30$

$B =$ no. of Honda $= 32$

who owns a neither $= 15$

Total no. of motorcycles $= 50$

Since

$(A \cup B)_n = |A|_n + |B|_n - (A \cap B)$

$35 = 30 + 32 - (A \cap B)$

$35 = 62 - (A \cap B)$

$\boxed{(A \cap B) = 27}$

27 of each of them own each bike

Q14 gcd (3142, 900)

$3142 = 900.3 + 442$

$900 = 422.2 + 16$

$422 = 16.27 + 10$

$16 = 10.1 + 6$

$10 = 4.1 + 2$

$4 = 2.2 + 0$

gcd $(3142, 900) = 2$

~~$2 = 6 - 4$~~

~~$2 = 6 - (10 - 6)$~~

$2 = 1.6 - 1.4$

$2 = 1.6 - 1.(1.10 - 1.6)$

$2 = 1.6 - (1.10 + 1.6)$

$2 = 2.6 - 1.10$

$2 = 2(1.16 - 1.10) - 1.10$

$2 = 2.16 - 2.10 - 1.10$

$2 = 2.16 - 3.10$

$2 = 2.16 - 3.(1.442 - 27.16)$

$2 = 2.16 - 3.442 + 81.16$

$2 = 83.16 - 3.442$

$2 = 83.(1.900 - 2.442) - 3.442$

2 = 83.900 - 166.442 - 3.442

2 = 83.900 - 169.442

2 = 83.900 - 169.(1.3142 - 3.900)

2 = 83.900 - 169.3142 + 507.900

2 = 590.900 - 169.3142

$\boxed{x = -169, \ y = 590}$

Q15

gcd(2017, 122)

2017 = 16.122 + 65

122 = 1.65 + 57

65 = 1.57 + 8

57 = 7.8 + 1

gcd = 1

1 = 1.57 - 7.8

1 = 1.57 - 7(1.65 - 1.57)

1 = 1.57 - 7.65 + 7.57

1 = 8.57 - 7.65

1 = 8(1.122 - 1.65) - 7.65

1 = 8.122 - 8.65 - 7.65

1 = 8.122 - 15.65

1 = 8.122 - 15(1.2017 - 16.122)

1 = 8.122 - 15.2017 + 240.122

1 = 248.122 - 15.2017

$\boxed{x = -15, \ y = 248}$

Q16

gcd(578, 442)

578 = 1.442 + 116

442 = 3.116 + 94

116 = 1.94 + 22

94 = 4.22 + 6

22 = 3.6 + 4

6 = 1.4 + 2

4 = 2.2 + 0

$\boxed{gcd = 2}$

2 = 1.6 - 1.4

2 = 1.6 - 1.(1.22 - 3.6)

2 = 1.6 - 1.22 + 3.6

2 = 4.6 - 1.22

2 = 4.(1.94 - 4.22) - 1.22

$2 = 4.94 - 16.22 - 1.22$

$2 = 4.94 - 17.22$

$2 = 4.94 - 17(1.116 - 1.94)$

$2 = 4.94 - 17.116 + 17.94$

$2 = 21.94 - 17.116$

$2 = 21.(1.442 - 3.116) - 17.116$

$2 = 21.442 - 63.116 - 17.116$

$2 = 21.442 - 80.116$

$2 = 21.442 - 80(1.578 - 1.442)$

$2 = 21.442 - 80.578 + 80.442$

$2 = 101.442 - 80.578$

$$\boxed{\begin{array}{l} x = -80 \\ y = 101 \end{array}}$$

Q17

a) $2^2 \times 3^2 \times 5^2$

b) $\gcd(m,n)$, $\text{lcm}(m,n) = 2 \times 3^3 \times 5^2 \times 7$

c) smallest multiple of k of 189 such that $\gcd(m,k) = 45$ is 5×189

Q17

$a = 4$

$a_n = 7a_{n-1} + 4$

$a_1 = 7a_0 + 4$

$4 = 7a_0 + 4$

$a_0 = 0$

$a_2 = 7a_1 + 4$

$a_2 = 28 + 4$

$\boxed{a_2 = 32}$

$a_3 = 7(a_2) + 4 = 7(32) + 4$

$\boxed{a_3 = 228}$

$a_4 = 7a_3 + 4$

$= 7(228) + 4$

$\boxed{a_4 = 1600}$

Q20

Total = 200

Coffee = 78 (C)

tea = 70 (T)

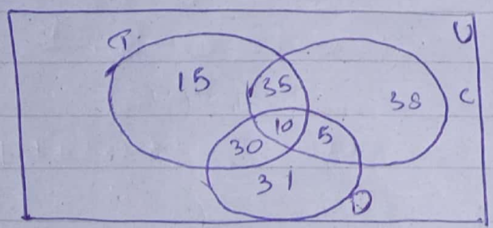
orange = 66 (O)

$C \cap T = 35$

$T \cap O = 30$

$C \cap O = 15$

$C \cap T \cap O = 10$



people who like

↳ orange juice = 31

↳ don't like drink = 56

Q21

Total customer = 50

$|M \cup R| = |M| + |R| + |M \cup R|' - |M \cap R|$

$50 = 30 + 35 - 15 - x$

$\boxed{x = 30}$

30 people will have both a mountain and road bike.

Q22

Total = 348

122 Maths = 321

101 Maths = 286

16 like neither of them

a)

$(A \cup B)_n = |A|_n + |B|_n - (A \cap B)_n$

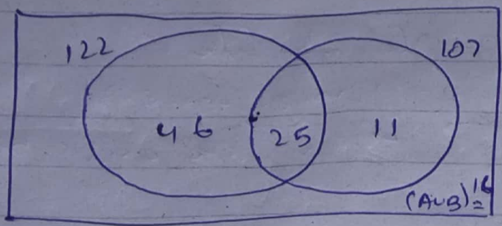
$332 = 321 + 286 - (A \cap B)_n$

$332 - 607 = -(A \cap B)_n$

$\boxed{(A \cap B)_n = 275}$

275 people like both courses

b)



— 46 student doesn't like math 101

Q27

$P(d) = 3/20$
$P(d)' = 17/20$

8 is the probability of not choosing defective balls.

Q26 ISBN-10 125973128θ

$x_{10} = 1\times1 + 2\times2 + 5\times3 + 9\times4 + 7\times5 + 3\times6 + 1\times7 + 2\times8 + 9.8 \bmod_{11}$

$x_{10} = 204 \bmod_{11}$

$\boxed{x_{10} = 4}$ (check digit)

Q28

lets suppose $f(x) = f(y)$

$g(f(x)) = g\{f(y)\}$
$g \circ f(x) = g \circ f(y)$
$x = y$

This is one to one function.

Q30 reflexive

$\frac{n}{n+2} \geq \frac{n}{n+2}$ $\therefore n \sim n$

$\therefore$ It is reflexive

anti symmetric suppose $a \sim b \wedge b \sim a$
want $a = b$ since $a \sim b$, $\frac{a}{b+2} \geq \frac{b}{a+2}$

since $b \sim a$, $\frac{b}{a+2} \geq \frac{a}{b+2}$

$\therefore \frac{a}{b+2} = \frac{b}{a+2}$ so, $a(a+2) = b(b+2)$
$a^2 + 2a + 1 = b^2 + 2b + 1$
$a^2 + 2a + 1 = b^2 + 2b + 1$
$(a+1)^2 = (b+1)^2$

Since $a, b \in N$, $a+1 = b+1$ $\therefore a = b$ it is anti-symmetric

Q31

* reflexive:

For any $x \subseteq \{1, 2, 3, 4\}$. the smallest element of x equals

$\{$ even at $x = \phi \}$

$\therefore \sim$ is reflexive

* symmetric

Suppose $x \sim y$

- The smallest element of x equals to the smallest of y
- The smallest element of y equals to the smallest of x

$y \sim x \Rightarrow \sim$ is symmetric

* Transitive:

Suppose $x \sim y$ & $y \sim z$

The smallest element of x & smallest element of y equals the smallest element of x

Hence the smallest element of x equals to x

$x \sim z \therefore \sim$ is transitive

So $\sim$ is an equivalence relation.

$[\{2,4\}] = [\{2,3\}, \{4,2,3\}, \{2,4\}, \{2,3\}]$

Q32

let $R = \{(1,2)\}$ & $S = \{(2,1)\}$

Then R & S are anti symmetric

But $R \cup S = \{(1,2), (2,1)\}$ is not anti-symmetric b/c $1 \neq 2$.

Binomial Theorem:

Q36] $1.1! + 2.2! + 3.3! + \cdots + n.n! = (n+1)! - 1$

let $n = k$

$1.1! + 2.2! + 3.3! + \cdots + k.k! = (k+1)! - 1$

let $n = k+1$

$1.1! + 2.2! + \cdots + (k+1)(k+1)! = (k+1)! - 1 + (k+1)(k+1)!$

$(k+1)(k+1)! = (k+1)!(1+k+1)$

$(k+1)(k+1)! + 1 = (k+1)!(k+2)$

$= (k+2)! - (k+2)(k+1)!$

$(k+1)(k+1)! = (k+2)! - 1$

$$\boxed{= (k+1+1)! - 1}$$

Hence proved

Q38 $\left[\frac{2}{3}x+1\right]^3 = (1)\left(\frac{2}{3}x\right)^3(1)^0 + 1(3)\left(\frac{2}{3}x\right)^2 + 3\left(\frac{2}{3}x\right)(1)^2 + \left(\frac{2}{3}x\right)^0(1)$

$$\boxed{= \frac{8}{27}x^3 + \frac{4}{3}x^2 + 2x + 1}$$

Ans

Q39 Coefficient of $\frac{1}{y^{10}}$ in

$\left(y^3 + \frac{a^7}{y^5}\right)^{10}$

at term $= 5$

$T_5 = (y^3)^5 \times \left(\frac{a^7}{y^5}\right)^5 \times {}^{10}C_5$

$= \frac{a^{35}(252)}{y^{10}}$

Q40 $(x-y)^{15}$; $(x^3y^{12}) = ?$; $y^{13}x^2 = ?$

$T_{12} = {}^{15}C_{12}\, x^{15-12}(-y)^{12}$

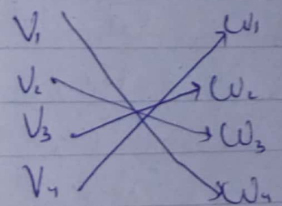
$= 455x^3y^{12}$ Coefficient $= 455$

$T_{13} = {}^{15}C_{13}\,(x)^{15-13}(-y)^{13} = -105x^2y^{13}$

Coefficient $= -105$

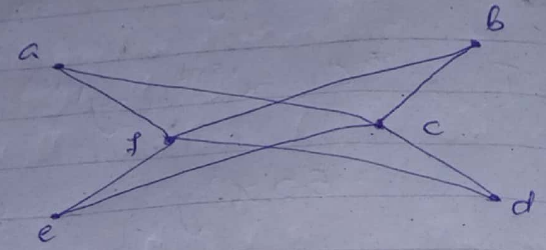
Q43

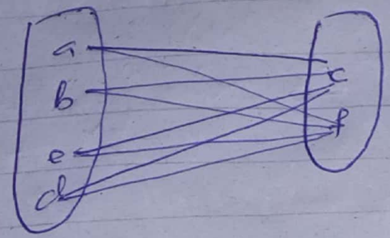
G (vertex) = G′ (vertex)
G (edge) = G′ (edge)
G (degree) = G′ (degree)



hence isomorphic

Q44.





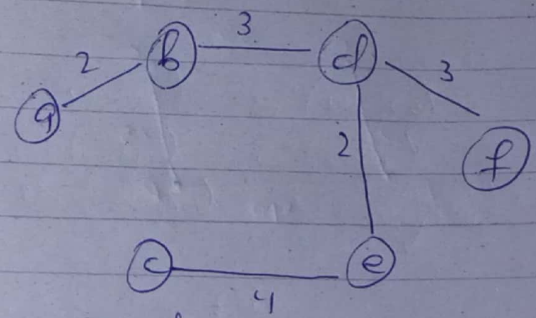
V_1 vertices are not connected with each other so the graph is bipartite the same stands for V_2

Q45

Incidence matrix

	e_1	e_2	e_3	e_4	e_5	e_6	e_7
V_1	1	0	0	0	0	1	0
V_2	1	1	1	1	0	0	0
V_3	0	0	0	1	1	0	1
V_4	0	0	0	0	1	1	0
V_5	0	1	1	0	0	0	1

~~Q46~~ Q47



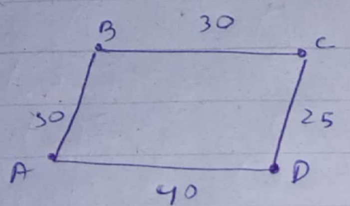
Total lowest weight = 2+3+3+4+2

$\boxed{\text{Total lowest weight} = 14}$

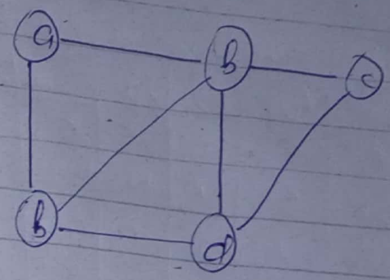
Q48

Hamilton circuit
says that,
"All nodes should be covered"

ABCDA = 125 is the minimum distance
as this traversal doesn't have any repitition.



Q49

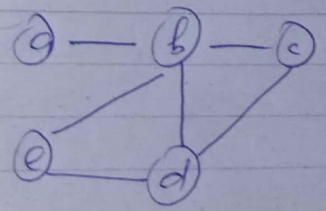


Here traversal goes like
a → b → d → b → a → b → c → d
The edge between a & b is repeated which means that it's not a Euler's path.

Euler's path.

deg (a) 2
deg (b) = 4
deg(c = 2
deg (d) = 2
deg (e) = 3

Here odd degrees ≤ 2
Thus euler path exists



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