National University of Computer & Emerging Sciences, Karachi

Fall-2020 - Department of Computer Science

Bachelor of Science (Computer Science)

Midterm 1 Examination -- Solution

October 20, 2020, 01:15 pm – 02:15 pm

Course Code: CS211	**Course Name: Discrete Structures**
Instructor Names: Dr. Fahad Samad, Mr. Shoaib Raza, Ms. Bakhtawer	
Student Roll No:	**Section No:**

Instructions:

- Return the question paper together with the answer script. Read each question completely before answering it. There are **3 questions and 2 pages.**
- In case of any ambiguity, you may make assumption. But your assumption should not contradict any statement in the question paper.
- Attempt all the questions in given sequence of the question paper.

Total Time: 60 minutes **Maximum Marks**: 26

Question # 1 (Propositional Logic and Rules of Inference) **[5x2=10 points]**

(i) Let p and q be the propositions.

p: Swimming at the New Jersey shore is allowed. q: Sharks have been spotted near the shore.

Write these propositions using p and q and logical connectives (including negations):

a) Swimming at the New Jersey shore is not allowed and either Swimming at the New Jersey shore is allowed or sharks have not been spotted near the shore.

Solution: $\neg p \wedge (p \vee \neg q)$

b) Swimming at the New Jersey shore is allowed iff sharks have not been spotted near the shore.

Solution: $p \leftrightarrow \neg q$

(ii) Prove the following logical equivalence using the laws of logic:

$$\neg [c \vee (b \wedge (\neg c \rightarrow \neg a))] \cong \neg c \wedge (a \vee \neg b)$$

Solution:

$\equiv \neg c \wedge \neg(b \wedge (\neg c \rightarrow \neg a))$	**De Morgan's law**
$\equiv \neg c \wedge \neg(b \wedge (\neg\neg c \vee \neg a))$	**conditional rewritten as disjunction**
$\equiv \neg c \wedge \neg(b \wedge (c \vee \neg a))$	**double negation law**
$\equiv \neg c \wedge (\neg b \vee \neg(c \vee \neg a))$	**De Morgan's law**
$\equiv \neg c \wedge (\neg b \vee (\neg c \wedge a))$	**De Morgan's law and double negation**
$\equiv (\neg c \wedge \neg b) \vee (\neg c \wedge (\neg c \wedge a))$	**distributive law**
$\equiv (\neg c \wedge \neg b) \vee ((\neg c \wedge \neg c) \wedge a)$	**associative law**
$\equiv (\neg c \wedge \neg b) \vee (\neg c \wedge a)$	**idempotent law**
$\equiv \neg c \wedge (\neg b \vee a)$	**distributive law**
$\equiv \neg c \wedge (a \vee \neg b)$	**commutative law**

Hence Proved.

(iii) Determine using truth table that whether the following is a tautology, contradiction or a contingency.

$[(p \to q) \wedge (q \to r)] \to (p \to r)$

Solution:

P	q	r	$p \to q$	$(q \to r)$	$(p \to q) \wedge (q \to r)$	$(p \to r)$	$[(p \to q) \wedge (q \to r)] \to (p \to r)$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	F	T	T
T	F	F	F	T	F	F	T
F	T	T	T	T	T	T	T
F	T	F	T	F	F	T	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

Hence, it's a Tautology.

(iv) What relevant conclusion or conclusions can be drawn from the following premises? Also, explain the rules of inference used to obtain each conclusion from the premises.

"The file is either a binary file or a text file." "My program won't accept the file if it's a binary file." "My program will accept the file."

Assume,

$p =$ "The file is a binary file." $q =$ "The file is a text file." $r =$ "My program will accept the file."

Solution:

Let p denote the proposition 'The file is a binary file', let q denote 'The file is a text file', and let r denote 'My program will accept the file'. Then:

$$P_1 \equiv p \vee q$$
$$P_2 \equiv p \to \neg r$$
$$P_3 \equiv r$$
$$Q \equiv q$$

The argument now takes the form:

$$[(p \vee q) \wedge (p \to \neg r) \wedge r] \to q$$

(v) Write the negation of the following sentences in English.

a) If Jaffar lives in Pakistan, then he lives in Karachi.

Solution: Jaffer lives in Pakistan and he does not live in Karachi ($p \wedge \neg q$).

b) If n is divisible by 6, then n is divisible by 2 and n is divisible by 3.

Solution: n is divisible by 6 but n is not divisible by 2 or by 3 ($p \wedge \neg(q \vee r)$).

Question # 2 (Predicate and Quantifiers) **[3x2=6 points]**

(i) Let $F(a, b)$ means "$a + 3b = ab$", where a and b are Positive integers. Determine the truth value of the statement.

a) $\forall a\, \exists b\, F(a, b)$. **Solution: False**

b) $\forall b\, \exists a\, \neg F(a, b)$. **Solution: True**

(ii) Let $B(x)$ be the statement **"x has an Internet connection"** and $C(x, y)$ be the statement **"x and y have chatted over the Internet,"** where the domain for the variables x and y consists of all students in your class. Write the statement in good English without using variables in your answers.

a) $\neg \forall x\, B(x)$. **Solution: Not everyone in your class has an Internet connection.**

b) $\exists x\, B(x) \wedge \forall y\, \neg C(x, y)$

Solution: Someone in your class has an Internet connection but has not chatted with anyone else in your class.

(iii) Express the following sentences using logical expression with nested quantifiers:

a) The Sum of two negative integers numbers is negative.

Solution: $\forall x \forall y ((x < 0) \wedge (y < 0) \rightarrow (x + y < 0))$

b) The difference of two positive integers is not necessarily positive.

Solution: $\neg\forall x \forall y ((x > 0) \wedge (y > 0) \rightarrow (x - y > 0))$ or

(b) A value is positive, when it is larger than 0. A value is positive, when it is smaller than or equal to 0.

We can rewrite the given sentence as: "There exists an integer x and an integer y, such that x is positive and y is positive and their difference is not positive"

$$\exists x \exists y((x > 0) \wedge (y > 0) \rightarrow (x - y \leq 0))$$

<u>Question # 3 (Functions and Set theory)</u> **[5x2=10 points]**

(i) Let ***f***: R → R be defined by the formula ***f(x) = 4x-1 ∀x ∈ R.*** Is ***f*** a bijective function? If no, give reason why? If yes, find its inverse.

Solution:

Then f is bijective, therefore f^{-1} exists. By definition of f^{-1},

$$f^{-1}(y) = x \Leftrightarrow f(x)=y$$

Now solving $f(x) = y$ for x

$$\Leftrightarrow 4x-1 = y \quad \text{(by definition of f)}$$

$$\Leftrightarrow 4x = y + 1$$

$$\Leftrightarrow x = \frac{y+1}{4}$$

Hence, $f^{-1}(y) = \dfrac{y+1}{4}$ is the inverse of $f(x)=4x-1$ which defines $f^{-1}: R \rightarrow R$.

(ii) Let ***f : Z →Z*** and ***g: Z → Z*** be defined by ***f(n) = n+1*** for ***n ∈ Z*** and ***g(n) = n²*** for ***n ∈ Z.***

a) Find the compositions ***gof*** and ***fog***.

b) Is ***gof = fog***?

Solution:

<u>SOLUTION:</u>

a. By definition of the composition of functions

$(gof)(n) = g(f(n)) = g(n+1) = (n+1)^2$ for all $n \in Z$ and

$(fog)(n) = f(g(n)) = f(n^2) = n^2+1$ for all $n \in Z$

b. Two functions from one set to another are equal if, and only if, they take the same values.

In this case,

$(gof)(1) = g(f(1)) = (1 + 1)^2 = 4$ where as

$(fog)(1) = f(g(1)) = 1^2 + 1 = 2$

Thus $fog \neq gof$

REMARK: The composition of functions is not a commutative operation.

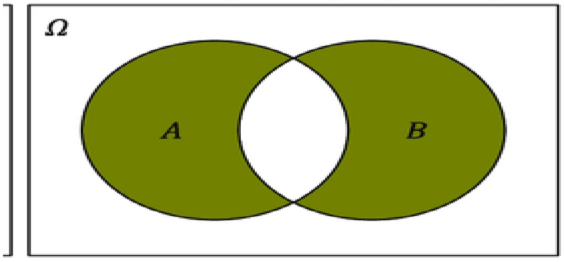
(iii) Draw Venn Diagram of the following relationships between the sets:

a) (A U B) – (A ∩ B)

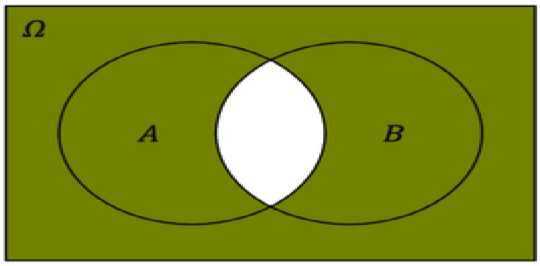
b) $\bar{A} \cup \bar{B}$

Solution:

$A \,\Delta\, B = (A \cup B) - (A \cap B)$



$A' \cup B' = (A \cap B)'$



(iv) Using set-builder notation, prove or disprove the following set operations:

$A - (B \cap C) = (A - B) \cap (A - C).$

Solution:

$A - (B \cap C) = (A - B) \cap (A - C)$	
$A \cap \overline{B \cap C} = (A \cap \overline{B}) \cap (A \cap \overline{C})$	$A \cap \overline{B} = (A - B)$
$A \cap \overline{B} \cup \overline{C} = (A \cap A) \cap (\overline{B} \cap \overline{C})$	De-Morgan and Associative Law
$A \cap \overline{B} \cup \overline{C} \neq A \cap \overline{B} \cap \overline{C}$	Idempotent Law

Hence, it a disproof.

(v) Among a group of 165 students, 8 are taking calculus, psychology, and computer science; 33 are taking calculus and computer science; 20 are taking calculus and psychology; 24 are taking psychology and computer science; 79 are taking calculus; 83 are taking psychology; and 63 are taking computer science. How many are taking none of the three subjects?

Solution:

SOLUTION Let CALC, PSYCH, and COMPSCI denote the sets of students taking calculus, psychology, and computer science, respectively. Let U denote the set of all 165 students (see Figure 1.1.6). Since 8 students are taking calculus, psychology, and computer science, we write 8 in the region representing CALC ∩ PSYCH ∩ COMPSCI. Of the 33 students taking calculus and computer science, 8 are also taking psychology; thus 25 are taking calculus and computer science but not psychology. We write 25 in the region representing CALC ∩ $\overline{\text{PSYCH}}$ ∩ COMPSCI. Similarly, we write 12 in the region representing CALC ∩ PSYCH ∩ $\overline{\text{COMPSCI}}$ and 16 in the region representing $\overline{\text{CALC}}$ ∩ PSYCH ∩ COMPSCI. Of the 79 students taking calculus, 45 have now been accounted for. This leaves 34 students taking only calculus. We write 34 in the region representing CALC ∩ $\overline{\text{PSYCH}}$ ∩ $\overline{\text{COMPSCI}}$. Similarly, we write 47 in the region representing $\overline{\text{CALC}}$ ∩ PSYCH ∩ $\overline{\text{COMPSCI}}$ and 14 in the region representing $\overline{\text{CALC}}$ ∩ $\overline{\text{PSYCH}}$ ∩ COMPSCI. At this point, 156 students have been accounted for. This leaves 9 students taking none of the three subjects. ◀

ALL THE BEST 😊