Assignment #3 Linear Aljebra

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Ex 5.2 Q 8, 10, 15, 20

Ex 6.1 Q 2, 13, 15, 22, 26 & 26

Ex# 6.2 Q 2, 4, 6, 8, 12 & 18

Ex # 5.2

Q.8) $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$

$\det(\lambda I - A) = 0$

$$\begin{bmatrix} \lambda-1 & 0 & 0 \\ 0 & \lambda-1 & -1 \\ 0 & -1 & \lambda-1 \end{bmatrix}$$

$(\lambda-1)[(\lambda-1)^2 - 1] = 0$

$\cancel{(\lambda-1)^3 - (\lambda-1)}$

$(\lambda-1)[\lambda^2-2\lambda+1-1] = 0$

$(\lambda-1)[\lambda(\lambda-2)] = 0$

$\lambda(\lambda-1)(\lambda-2)=0$

$\lambda=0,1,2$

for

$\lambda=0$

$$\begin{bmatrix} +1 & 0 & 0 \\ 0 & +1 & +1 \\ 0 & +1 & +1 \end{bmatrix}$$

$$R_3-R_2 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

let $x_3=t$

$(x_1, x_2, x_3)=(0,-t,t)=t(0,-1,1)$

$$P_1=\begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}$$

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For $\lambda = 1$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & +1 \\ 0 & +1 & 0 \end{bmatrix}$$

let $x_1 = s$

$$(x_1, x_2, x_3) = (s, 0, 0) = s(1, 0, 0)$$

$$P_2 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

For $\lambda = 2$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{bmatrix}$$

$$R_3 + R_2 \quad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

let $x_3 = t$

$(x_1, x_2, x_3) = (0, t, t) = t(0, 1, 1)$

$$P_3 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

So $P = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix}$

$D = PAP^{-1}$

$$D = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1/2 & 1/2 \\ 1 & 0 & 0 \\ 0 & 1/2 & 1/2 \end{bmatrix}$$

$$D = \begin{bmatrix} 0 & \cancel{0}1 & 1 \\ -1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1/2 & 1/2 \\ 1 & 0 & 0 \\ 0 & 1/2 & 1/2 \end{bmatrix}$$

$$D = \begin{bmatrix} \quad \end{bmatrix}$$

$D = P^{-1} A P$

$$D = \begin{bmatrix} 0 & -1/2 & 1/2 \\ 1 & 0 & 0 \\ 0 & 1/2 & 1/2 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

$$D = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

$$D = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$ So $tr(A) = tr(D)$

Ans 10)
$$\begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 1 & 2 \end{bmatrix}$$

a) $\det(\lambda I - A) = 0$

$$\begin{bmatrix} \lambda - 3 & 0 & 0 \\ 0 & \lambda - 2 & 0 \\ 0 & -1 & \lambda - 2 \end{bmatrix}$$

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$(\lambda-3)[\lambda-2]^2 = 0$

So $\lambda = 3$ aljebraic multiplicity $= 1$

$\lambda = 2$ aljebraic multiplicity $= 2$

b) for $\lambda = 2$

$$\begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

let $x_3 = t$

$(x_1, x_2, x_3) = (0, 0, t)$

$P_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ rank$(2I-A) = 2$

For $\lambda = 3$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$ rank $(3I - A) = 2$

c) Since the Geometric multiplicity of $\lambda = 2$ is 1 & aljebraic multiplicity is 2 so the A is not diagonalizable

Ans15)

a) $(\lambda-1)(\lambda+3)(\lambda-5) = 0$

Since highest power is 3 so 3x3 matrix:

All the eigen spaces $\lambda = 1$ & $\lambda = -3$ & $\lambda = 5$ have dimensions $= 1$

Ans 15.b) $x^2(\lambda-1)(\lambda-2)^3 = 0$

Since the highest power is 6 so 6×6 matrix.

possible dimension for $\lambda = 0$ can be 0, 1, 2

possible dimension for $\lambda = 1$ must be 1

possible dimension for $\lambda = 2$ must be 1, 2, 3

Ans 20) $D = P^{-1}AP$

$$D = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -1 & 4 \end{bmatrix} \begin{bmatrix} 1 & -2 & 8 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & -4 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$D = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 1 & -1 & 4 \end{bmatrix} \begin{bmatrix} 1 & -4 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$D = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

& $tr(A) = 1 - 1 - 1 = -1$

$tr(D) = -1 - 1 + 1 = -1$

Since $tr(A) = tr(D)$

So P diagonalizes A.

a) A^{1000}

$A^{1000} = P^{-1} D^{1000} P$

$$A^{1000} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -1 & 4 \end{bmatrix} \begin{bmatrix} (-1)^{1000} & 0 & 0 \\ 0 & (-1)^{1000} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -4 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$A^{1000} = \begin{bmatrix} 0 & +1 & 0 \\ 0 & 0 & 1 \\ +1 & -1 & 4 \end{bmatrix} \begin{bmatrix} 1 & -4 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} +1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

b) $A^{-1000} = P^{-1} D^{-1000} P$

$$\begin{bmatrix} 1 & -4 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} (-1)^{-1000} & 0 & 0 \\ 0 & (-1)^{-1000} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -1 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -4 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -1 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

c) $A^{2301} = P D^{2301} P^{-1}$

$$\begin{bmatrix} 1 & -4 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} (-1)^{2301} & 0 & 0 \\ 0 & (-1)^{2301} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -1 & 4 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 4 & 1 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -1 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 & 8 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

d) $A^{-2301} = P D^{-2301} P^{-1}$

$$\begin{bmatrix} 1 & -4 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} (-1)^{-2301} & 0 & 0 \\ 0 & (-1)^{-2301} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -1 & 4 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 4 & 1 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -1 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -2 & 8 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

Ex#6.1

Ans2)

a) $\langle u, v\rangle = \frac{1}{2}(3) + 5(2) = \frac{3}{2} + 10 = \frac{23}{2}$

b) $\langle kv, w\rangle = \frac{1}{2}(9)(0) + 5(6)(-1) = -30$

c) $\langle u+v, w\rangle = \frac{1}{2}(4)(0) + 5(3)(-1) = -15$

d) $\|v\| = \sqrt{\frac{1}{2}(3)^2 + 5(2)^2} = \sqrt{\frac{9}{2} + 20} = \sqrt{\frac{49}{2}}$

e) $d(u,v) = \|u - v\| = \sqrt{\frac{1}{2}(-2)^2 + 5(-1)^2}$

$= \sqrt{\frac{1}{2}(4) + 5} = \sqrt{7}$

f) $\|u - kv\| = \sqrt{\frac{1}{2}(-8)^2 + 5(-5)^2}$

$= \sqrt{32 + 125} = \sqrt{157}$

Ans13) $\begin{bmatrix} \sqrt{3} & 0 \\ 0 & \sqrt{5} \end{bmatrix}$

Ans15) $p = x + x^3$

$q = 1 + x^2$

$x_0 = -2, \; x_1 = -1; \; x_2 = 0, \; x_3 = 1$

$\langle p, q \rangle =$

$p(-2)q(-2) + p(-1)q(-1) + p(0)q(0) + p(1)q(1)$

$(-10)(5) + (-2)(2) + 0 + 4$

$-50 - 4 + 4$

$\boxed{\langle p, q \rangle = -50}$

Ans22)

$\|U\| = \sqrt{1+4+9+25} = \sqrt{39}$

$d(U-V) = \sqrt{(-3)^2 + (-4)^2 + (-3)^2 + (-3)^2}$

$d(U-V) = \sqrt{9+16+9+9}$

$= \sqrt{43}$

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Ans26)

$\|U\| = \sqrt{(3)^2 + 7^2} = \sqrt{658} = \sqrt{58}$

$d(U,V) = \sqrt{(-9)^2 + (-6)^2} = \sqrt{81+36} = \sqrt{117} = 3\sqrt{13}$

Ans28.a)

$\langle U - V - 2W, 4U + V \rangle$

$= \langle U, 4U + V \rangle - \langle V, 4U + V \rangle - \langle 2W, 4U + V \rangle$

$= \langle U, 4U \rangle + \langle U, V \rangle - \langle V, 4U \rangle - \langle V, V \rangle - \langle 2W, 4U \rangle - \langle 2W, V \rangle$

$= 4\langle U, U \rangle + \langle U, V \rangle - 4\langle V, U \rangle - \langle V, V \rangle - 8\langle W, U \rangle - 2\langle W, V \rangle$

$= 4\|U\|^2 - 3\langle V, U \rangle - \|V\|^2 - 8\langle U, W \rangle - 2\langle V, W \rangle$

$= 4(1) - 3(2) - (4) - 8(-3) - 2(-6)$

$= 4 - 6 - 4 + 24 + 12$

$= 30$

b) $\|2W - V\|$

$= \sqrt{\langle 2W - V, 2W - V \rangle} = \sqrt{\langle 2W, 2W - V \rangle - \langle V, 2W - V \rangle}$

$= \sqrt{\langle 2W, 2W \rangle - \langle 2W, V \rangle - \langle V, 2W \rangle + \langle V, V \rangle}$

$= \sqrt{4\langle W, W \rangle - 2\langle W, V \rangle - 2\langle V, W \rangle + \langle V, V \rangle}$

$= \sqrt{4\|W\|^2 - 4\langle V, W \rangle + \|V\|^2}$

$= \sqrt{4(49) - 4(-6) + 4} = \sqrt{224}$

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Ex # 6.2

Ans2.a) $\cos\theta = \dfrac{\langle u, v \rangle}{\|u\| \|v\|} = \dfrac{-3}{\sqrt{73}}$

b) $\cos\theta = \dfrac{\langle u, v \rangle}{\|u\| \|v\|} = \dfrac{4(1) + 1(0) + 8(-3)}{\sqrt{4^2+1^2+8^2}\sqrt{1+0+9}}$

$\cos\theta = \dfrac{-20}{\sqrt{81}\sqrt{10}}$

c) $\cos\theta = \dfrac{\langle u, v \rangle}{\|u\| \|v\|} = \dfrac{(2)(4) + (1)(0) + 7(0) + (-1)(0)}{\sqrt{2^2+1+7^2+1}\ \sqrt{4^2}}$

$\cos\theta = \dfrac{8}{4\sqrt{55}}$

Ans4) $\cos\theta = \dfrac{(0)(7) + (1)(3) + (-1)(3)}{\sqrt{2}\ \sqrt{49+9+9}}$

$\cos\theta = \dfrac{0}{\sqrt{2}\sqrt{67}} = 0$

Ans6) $\cos\theta = \frac{-6+4-4+6}{\sqrt{30}\sqrt{30}} = 0$

Ans8.a) $\langle u, v\rangle = u_1(0) + u_2(0) + u_3(0)$
$= 0$ So orthogonal

b) $\langle u, v\rangle = (-4)(2) + (6)(1) + (-10)(-2) + (1)(9)$

$\langle u, v\rangle = -8 + 6 + 20 + 9 = 27 \neq 0.$
So not orthogonal

c) $\langle u, v\rangle = -ac + b(0) + c(a)$
$= -ac + ac = 0$
So orthogonal.

Ans12) $\langle u, v\rangle = (5)(1) - (1)(3) + 2(-1) - 2(0)$
$= 5 - 3 - 2 = 0$
So orthogonal.

Ans18) $\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 3 \\ 3 \end{bmatrix} \cdot \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 5 \\ -8 \end{bmatrix}$

$= \begin{bmatrix} 9 \\ 6 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ -3 \end{bmatrix} = 18 - 18 = 0$