Total Time: 60 Minutes **Maximum Points**: 26

Question # 1 (Propositional Logic and Rules of Inference) **[5x2=10 points]**

(i) Consider the following propositions **i, j, k & l**:

i: Hashim is a cricketer. **j:** Hashim is determined.

k: Hashim is an early riser. **l:** Hashim likes banana.

Express these statements using the propositions **i**, **j**, **k,** and **l** together with logical connectives (including negations).

(a) "If Hashim is a cricketer, then he is determined." Solution: $i \rightarrow j$

(b) "If Hashim is an early riser then he does not like banana." Solution: $k \rightarrow \neg l$

(c) "If Hashim is determined then he is an early riser." Solution: $j \rightarrow k$

(ii) Write in English, the Converse of statement **(a)**, Contrapositive of statement **(b)** & Inverse of statement **(c)** of part **(i)** above.

Solution:

Converse(a): If Hashim is determined then he is a cricketer.

Contrapositive(b): If Hashim like banana then he is not an early riser.

Inverse(c): If Hashim is not determined then he is not an early riser.

(iii) Using the premises(statements) from **part(i)**, apply rules of inference to obtain conclusion(s) from these premises.

The premises are: (a) $i \rightarrow j$ (b) $k \rightarrow \neg l$ (c) $j \rightarrow k$

(i) $i \rightarrow j$ premise (a)

(ii) $j \rightarrow k$ premise (c)

(iii) $i \rightarrow k$ Hypothetical Syllogism (i) & (ii)

(iv) $k \rightarrow \neg l$ premise (b)

(v) $i \rightarrow \neg l$ Hypothetical Syllogism (iii) & (iv)

Conclusion: $i \rightarrow \neg l$: If Hashim is a cricketer then he does not like banana.

(iv) Prove or disprove the following logical equivalence using the laws of logic:

$\neg (a \leftrightarrow b)$ and $a \leftrightarrow \neg b$

SOLUTION: Replace "P" with "a" and "Q" with "b"

Solution:

$\neg(P \leftrightarrow Q) \equiv P \leftrightarrow \neg Q$

** $P \to Q \equiv \neg P \vee Q$
$P \leftrightarrow Q \equiv (P \to Q) \wedge (Q \to P)$
$\equiv (\neg P \vee Q) \wedge (\neg Q \vee P)$

$\neg(P \leftrightarrow Q)$
$\equiv \neg((\neg P \vee Q) \wedge (\neg Q \vee P))$ ##
$\equiv \neg(\neg P \vee Q) \vee \neg(\neg Q \vee P)$ De Morgan's Laws
$\equiv (P \wedge \neg Q) \vee (Q \wedge \neg P)$ De Morgan's Laws
$\equiv ((P \wedge \neg Q) \vee Q) \wedge ((P \wedge \neg Q) \vee \neg P)$ Distributive Laws
$\equiv ((P \vee Q) \wedge (\neg Q \vee Q)) \wedge ((P \vee \neg P) \wedge (\neg Q \vee \neg P))$ Distributive Laws
$\equiv (P \vee Q) \wedge T \wedge T \wedge (\neg Q \vee \neg P)$ Negation Laws
$\equiv (P \vee Q) \wedge (\neg Q \vee \neg P)$ Identify Laws
$\equiv P \leftrightarrow \neg Q$ ##

(v) Let **a**, **b** and **c** be statements. Determine, using a truth table, whether **(a v b) ∧ (b v c) ∧ (c v a)** and **(a ∧ b) v (b ∧ c) v (c ∧ a)** is logically equivalent or not.
Solution:

a	b	c	a V b	b V c	c V a	(a v b) ∧ (b v c) ∧ (c v a)	a ∧ b	b ∧ c	c ∧ a	(a ∧ b) v (b ∧ c) v (c ∧ a)
T	T	T	T	T	T	T	T	T	T	T
T	T	F	T	T	T	T	T	F	F	T
T	F	T	T	T	T	T	F	F	T	T
T	F	F	T	F	T	F	F	F	F	F
F	T	T	T	T	T	T	F	T	F	T
F	T	F	T	T	F	F	F	F	F	F
F	F	T	F	T	T	F	F	F	F	F
F	F	F	F	F	F	F	F	F	F	F

Question # 2 (Predicate, Quantifiers and Set theory) **[4x2= 08 points]**

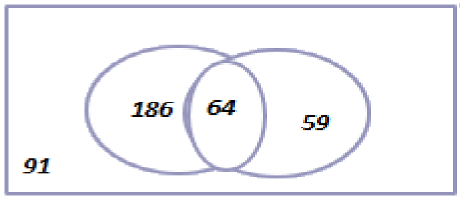
(i) Transform the following sentence into an expression using predicate logic and quantifiers, where:
C(x,y) = **x** can do **y**. **D(x,y)** = **x** does **y**. **T(x,y)** = **x** disturbs **y**.
"He who can, does. He who cannot, disturbs all others".
Solution:
$\forall x \forall y[C(x,y) \to D(x,y)] \wedge \forall x \forall y [\neg C(x,y) \to T(x,y)]$

(ii) Suppose **R (x, y)** is the predicate "**x** plays **y**", the universe of discourse for **x** is "the set of students in your discrete class", and the universe of discourse for **y** is "the set of all sports." Write the following predicate expressions in good English without using variables in your answers:
(a) **∃x ∀y R (x, y)** Solution: There exist some students in discrete class who play all sports.

(b) **∀x ∃y R (x, y)** Solution: All the students in the class play some sports (or at least one sport).

(iii) FAST-NUCES, Department of Computer Science received **400** applications. Suppose that **250** majored in Computer Science, **123** majored in Software Engineering, and **64** majored in both. How many of these applicants majored either in Computer Science or in Software Engineering? Draw the Venn diagram.

Solution:
Let, A: Applicants majored in Computer Science
B: Applicants majored in Software Engineering
Now,
$|A \cup B| = |A| + |B| - |A \cap B| = 250 + 123 - 64 = 309$



(iv) Let **A** and **B** be two sets. Prove or disprove using set identities that **A − (A ∩ B) = (A − B).**
Solution:

$= A \cap (A \cap B)^c$	$A - B = A \cap B^c$
$= A \cap (A^c \cup B^c)$	De-Morgan Law
$= (A \cap A^c) \cup (A \cap B^c)$	Distributive Law
$= \emptyset \cup (A \cap B^c)$	Complement Law
$= A - B$ Hence Proved	$A - B = A \cap B^c$

<u>Question # 3 (Functions and Relations)</u> **[4x2=08 points]**

(i) Abdul Karim is teaching Relational Algebra to Database Systems students. Before he starts teaching, he wants to check the basic concepts of relation of his students. He has designed a simple quiz as shown below: Find a matrix that represents the given relation. Use elements in the order given to determine rows and columns of the matrix.
(a) **R1** on {−2, −1, 0, 1, 2} where a R b means $a^2 = b^2$
(b) **R2** on {1, 2, 3, 4, 6} where a R b means a | b.

Solution:

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Solution:

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Abdullah is a 2nd semester student. He missed the lectures of the topic "Functions". He asked his friend to help him in understanding the topic and asked the questions given below **(ii-iv)**:

(ii) Define **a: R→Z** by the rule **a(x) = [2x – 1].** Is "**a"** one-to-one function (injective) **OR** onto function (surjective)?
Solution:
Yes, Function a(x) is Surjective. Because g (0.3) = g (0.4) = 1.

(iii) Suppose **b:** P → Q and **a:** Q → R where P = {1, 2, 3, 4}, Q = {f, g, h}, R = {2, 7, 10}. "**a"** and "**b"** are defined by **a** = {(f, 10), (g, 7), (h, 2)} and **b** = {(1, g), (2, f), (3, f), (4, g)}. Find **a ° b** and **b ° a** (if the composition does not exist, give reason).
Solution:
a ° b: a(1) = 7, a(2) = 10, a(3) = 10, a(4) = 7.
b ∘ a is not defined, because the range of a is not a subset of the domain of b.

(iv) Are the functions **a** and **b** in part (iii) invertible? If yes find $\mathbf{a^{-1}}$ and $\mathbf{b^{-1}}$ or if not, give reason in one sentence.
Solution:
a is invertible since it's a bijective function. $a^{-1}(10) = f$, $a^{-1}(7) = g$, $a^{-1}(2) = h$.
b is not invertible since it's not a bijective function.

ALL THE BEST