LA-ASSIGNMENT - 02

Bilal Ahmed Khan 20K-0183 "B"

EXERCISE 4.1

QUESTION 02:

~~Axiom 01.~~

a)

FOR $u+v$

$u+v = (0+1+1, 4-3+1)$

$\boxed{u+v = (2,2)}$

FOR ku

$ku = 2(0,4)$

$\boxed{ku = (0,8)}$

b)

let $0 = (0,0)$ & $u = (u_1, u_2)$

$u+0 = (u_1+0+1, u_2+0+1)$

$\boxed{u+0 = (u_1+1, u_2+1) \neq u}$

$\boxed{\text{Thus} \quad (0,0) \neq 0}$

c) let $O = (-1,-1)$ & $U = (U_1, U_2)$

$U + O = (-1 + U_1 + 1, -1 + U_2 + 1)$

$\boxed{U + O = (U_1, U_2) = U}$

$\boxed{\text{Thus } (-1,-1) = O}$

d) let $U = (U_1, U_2)$

let $-U = (-U_1 - 2, -U_2 - 2)$

Now,

$U + (-U) = (\cancel{U_1} - \cancel{U_1} - 2 + 1, \cancel{U_2} - \cancel{U_2} - 2 + 1)$

$\boxed{U + (-U) = (-1, -1) = O}$

e) Testing Axiom 07

let $U = (U_1, U_2)$; $V = (V_1, V_2)$

$= c(U+V)$

$= c(U_1 + V_1 + 1, U_2 + V_2 + 1)$

$c(U+V) = (cU_1 + cV_1 + c, cU_2 + cV_2 + c)$

$= cU + cV$

$= c(U_1, U_2) + c(V_1, V_2)$

$= (cU_1, cU_2) + (cV_1, cV_2)$

$cU + cV = (cU_1 + cV_1 + 1, cU_2 + cV_2 + 1)$

$\boxed{c(U+V) \neq c(U) + c(V)}$

Testing Axiom#08:

$(K_1 + K_2)U = ((K_1 + K_2)U_1, (K_1 + K_2)U_2)$

$\boxed{(K_1 + K_2)U = (K_1U_1 + K_2U_1, K_1U_2 + K_2U_2)}$

$K_1U + K_2U = K_1(U_1, U_2) + K_2(U_1, U_2)$

$= (K_1U_1, K_1U_2) + (K_2U_1, K_2U_2)$

$\boxed{K_1U + K_2U = (K_1U_1 + K_2U_1 + 1, K_1U_2 + K_2U_2 + 1)}$

$\boxed{(K_1 + K_2)U \neq K_1U + K_2U}$

Therefore Axiom # 7 & 8 don't hold.

QUESTION #09

$$A = \begin{bmatrix} a_1 & 0 \\ 0 & b_1 \end{bmatrix} ; \quad B = \begin{bmatrix} a_2 & 0 \\ 0 & b_2 \end{bmatrix} ; \quad C = \begin{bmatrix} a_3 & 0 \\ 0 & b_3 \end{bmatrix}$$

Axiom #01

$$A = \begin{bmatrix} a_1 & 0 \\ 0 & b_1 \end{bmatrix} ; \quad B = \begin{bmatrix} a_2 & 0 \\ 0 & b_2 \end{bmatrix}$$

$$\boxed{A+B = \begin{bmatrix} a_1+a_2 & 0 \\ 0 & b_1+b_2 \end{bmatrix}}$$

The sum of two 2x2 matrices is also a 2x2 matrix.

Axiom #02

$$A+B = \begin{bmatrix} a_1+a_2 & 0 \\ 0 & b_1+b_2 \end{bmatrix}$$

$$B+A = \begin{bmatrix} a_1+a_2 & 0 \\ 0 & b_1+b_2 \end{bmatrix}$$

$$\boxed{A+B = B+A}$$

Axiom #03

$$A+(B+C) = \begin{bmatrix} a_1 & 0 \\ 0 & b_1 \end{bmatrix} + \begin{bmatrix} a_2+a_3 & 0 \\ 0 & b_2+b_3 \end{bmatrix}$$

$$A+(B+C) = \begin{bmatrix} a_1+a_2+a_3 & 0 \\ 0 & b_1+b_2+b_3 \end{bmatrix}$$

$$(A+B)+C = \begin{bmatrix} a_1+a_2 & 0 \\ 0 & b_1+b_2 \end{bmatrix} + \begin{bmatrix} a_3 & 0 \\ 0 & b_3 \end{bmatrix}$$

$$(A+B)+C = \begin{bmatrix} a_1+a_2+a_3 & 0 \\ 0 & b_1+b_2+b_3 \end{bmatrix}$$

$$\boxed{A+(B+C) = (A+B)+C}$$

Axiom #04

$$O = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

is the zero vector for the given vector space.

Axiom #05

$$A = \begin{bmatrix} a_1 & 0 \\ 0 & b_1 \end{bmatrix} ; \quad -A = \begin{bmatrix} -a_1 & 0 \\ 0 & -b_1 \end{bmatrix}$$

$$A+(-A) = \begin{bmatrix} a_1-a_1 & 0 \\ 0 & b_1-b_1 \end{bmatrix}$$

$$\boxed{A+(-A) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}}$$

Axiom #06

$$KA = K\begin{bmatrix} a_1 & 0 \\ 0 & b_1 \end{bmatrix}$$

$$\boxed{KA = \begin{bmatrix} Ka_1 & 0 \\ 0 & Kb_1 \end{bmatrix}}$$

KA is also a 2x2 matrix

Axiom 07:

$$K(A+B) = K\begin{bmatrix} a_1+a_2 & 0 \\ 0 & b_1+b_2 \end{bmatrix} = \begin{bmatrix} Ka_1+Ka_2 & 0 \\ 0 & Kb_1+Kb_2 \end{bmatrix}$$

$$KA+KB = \begin{bmatrix} Ka_1 & 0 \\ 0 & Kb_1 \end{bmatrix} + \begin{bmatrix} Ka_2 & 0 \\ 0 & Kb_2 \end{bmatrix}$$

$$KA+KB = \begin{bmatrix} Ka_1+Ka_2 & 0 \\ 0 & Kb_1+Kb_2 \end{bmatrix}$$

$$\boxed{K(A+B) = KA+KB}$$

Axiom 08:

$$(K_1+K_2)A = \begin{bmatrix} K_1a+K_2a & 0 \\ 0 & K_2b+K_1b_1 \end{bmatrix}$$

$$K_1A+K_2A = \begin{bmatrix} K_1a_1 & 0 \\ 0 & K_1b_1 \end{bmatrix} + \begin{bmatrix} K_2a_1 & 0 \\ 0 & K_2b_1 \end{bmatrix}$$

$$K_1A+K_2A = \begin{bmatrix} K_1a_1+K_2a_1 & 0 \\ 0 & K_1b_1+K_2b_1 \end{bmatrix}$$

$$\boxed{(K_1+K_2)A = K_1A+K_2A}$$

AXIOM 9

$$K_1(K_2A) = K_1\begin{bmatrix} K_2a_1 & 0 \\ 0 & K_2b_1 \end{bmatrix} = \begin{bmatrix} K_1K_2a_1 & 0 \\ 0 & K_1K_2b_1 \end{bmatrix}$$

$$K_1K_1A = \begin{bmatrix} K_1K_2a_1 & 0 \\ 0 & K_1K_2b_1 \end{bmatrix}$$

$$\boxed{K_1(K_2A) = K_1K_2A}$$

AXIOM 10:

$$IA = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} a_1 & 0 \\ 0 & b_1 \end{bmatrix}$$
$$= \begin{bmatrix} a_1+0 & 0+0 \\ 0+0 & 0+b_1 \end{bmatrix}$$
$$= \begin{bmatrix} a_1 & 0 \\ 0 & b_1 \end{bmatrix} = A$$

$$\boxed{IA = A}$$

CONCLUSION: This is a vector space since all axioms hold.

QUESTION #11

Given that
$$(1,y)+(1,y') = (1, y+y') \;\&$$
$$k(1,y) = (1, ky)$$

Axiom 01:

let $u = (1,y)$; $v = (1,y')$; $w = (1,y'')$

$$u+v = (1,y)+(1,y') = (1, y+y')$$
is within V

Axiom 02:

$$u+v = (1,y)+(1,y') = (1, y+y')$$

$$v+u = (1,y')+(1,y) = (1, y'+y)$$

$$\boxed{u+v = v+u}$$

Axiom 03:

$$u+(v+w) = (1,y)+(1,y'+y'')$$
$$u+(v+w) = (1, y+y'+y'')$$

$$(u+v)+w = (1, y+y')+(1,y'')$$
$$(u+v)+w = (1, y+y'+y'')$$

$$\boxed{u+(v+w) = (u+v)+w}$$

Axiom 04:

taking $O = (1,0)$

$U + O = (1,y) + (1,0)$

$\boxed{U + O = (1,y) = U}$

Axiom 05:

let $-U = (1,-y)$

$U + (-U) = (1,+y) + (1,-y)$

$\boxed{U + (-U) = (1,0) = O}$

Axiom 06:

let K be a scalar

$KU = K(1,y)$

$\boxed{KU = (1,Ky)}$ is in V.

Axiom 07:

$K(U+V) = K((1,y) + (1,y'))$

$= K(1, y+y')$

$\boxed{K(U+V) = (1, Ky+Ky')}$

$KU + KV = K(1,y) + K(1,y')$

$= (1,Ky) + (1,Ky')$

$\boxed{KU + KV = (1, Ky+Ky')}$

$\boxed{K(U+V) = KU + KV}$

Axiom 08:

$(K_1+K_2)U = (K_1+K_2)(1,y)$

$\boxed{(K_1+K_2)U = (1, K_1y + K_2y)}$

$K_1U + K_2U = K_1(1,y) + K_2(1,y)$

$= (1,K_1y) + (1,K_2y)$

$\boxed{K_1U + K_2U = (1, K_1y + K_2y)}$

$\boxed{(K_1+K_2)U = K_1U + K_2U}$

Axiom 09:

$K_1(K_2U) = K_1(1,K_2y) = (1, K_1K_2y)$

$K_1K_2U = K_1K_2(1,y) = (1, K_1K_2y)$

$\boxed{K_1(K_2U) = K_1K_2U}$

Axiom 10:

$1U = 1(1,y) = (1,y) = U$

$\boxed{1 \cdot U = U}$

CONCLUSION: All 10 axioms hold thus its a vector space.

x—x —x

QUESTION 12

Axiom 01:

let $U = a_0 + a_1 x$, $V = b_0 + b_1 x$, $W = c_0 + c_1 x$

$U+V = a_0 + a_1 x + b_0 + b_1 x$

$\boxed{U+V = (a_0+b_0) + (a_1+b_1)x}$

is in V

Axiom 02:

$U+V = a_0 + a_1 x + b_0 + b_1 x$

$\boxed{U+V = a_0 + b_0 + (a_1+b_1)x}$

$V+U = b_0 + b_1 x + a_0 + a_1 x$

$\boxed{V+U = a_0 + b_0 + (b_1+a_1)x}$

$\boxed{U+V = V+U}$

Axiom 03:

$U+(V+W) = a_0 + a_1 x + (b_0+c_0) + (b_1+c_1)x$

$\boxed{U+(V+W) = (a_0+b_0+c_0) + (b_1+c_1+a_1)x}$

$(U+V)+W = (a_0+b_0) + (a_1+b_1)x + c_0 + c_1 x$

$\boxed{(U+V)+W = (a_0+b_0+c_0) + (a_1+b_1+c_1)x}$

$\boxed{U+(V+W) = (U+V)+W}$

Axiom 04:

let $0 = 0 + 0x$

$u + 0 = a_0 + a_1 x + 0 + 0x$

$\boxed{u + 0 = a_0 + a_1 x = u}$

Axiom 05:

let $-u = -a_0 + (-a_1)x$

$u + (-u) = a_0 + a_1 x - a_0 - a_1 x$

$\boxed{u + (-u) = 0 - 0x = 0}$

Axiom 06:

$Ku = K(a_0 + a_1 x)$

$\boxed{Ku = Ka_0 + (Ka_1)x}$ is in V

Axiom 07:

$K(u+v) = K(a_0 + a_1 x + b_0 + b_1 x)$

$= K(a_0 + b_0 + (a_1 + b_1)x)$

$\boxed{K(u+v) = Ka_0 + Kb_0 + Kb_1 x + Ka_1 x}$

$Ku + Kv = K(a_0 + a_1 x) + K(b_0 + b_1 x)$

$= Ka_0 + Ka_1 x + Kb_0 + Kb_1 x$

$\boxed{Ku + Kv = Ka_0 + Kb_0 + (Ka_1 + Kb_1)x}$

$\boxed{K(u+v) = Ku + Kv}$

Axiom 08:

$(K_1 + K_2)u = (K_1 + K_2)(a_0 + a_1 x)$

$\boxed{(K_1 + K_2)u = K_1 a_0 + K_2 a_0 + (K_1 a_1 + K_2 a_1)x}$

$K_1 u + K_2 u = K_1(a_0 + a_1 x) + K_2(a_0 + a_2 x)$

$= K_1 a_0 + K_1 a_1 x + K_2 a_0 + K_2 a_1 x$

$\boxed{K_1 u + K_2 u = K_1 a_0 + K_2 a_0 + (K_1 a_1 + K_2 a_1)x}$

$\boxed{(K_1 + K_2)u = K_1 u + K_2 u}$

Axiom 09:

$K_1(K_2 u) = K_1(K_2 a_0 + K_2 a_1 x)$

$\boxed{K_1(K_2 u) = K_1 K_2 a_0 + (K_1 K_2 a_1)x}$

$K_1 K_2 u = K_1 K_2 (a_0 + a_1 x)$

$\boxed{K_1 K_2 u = K_1 K_2 a_0 + (K_1 K_2 a_1)x}$

Axiom 10:

$1u = 1(a_0 + a_1 x) = a_0 + a_1 x = u$

Conclusion: Since all 10 axioms hold it is a vector space

x ——— x ——— x

EXERCISE 4.2

QUESTION 04

a) let W be the subset of all n×n matrices such that $A^T = -A$

let us suppose two set $A \& B$ in the subset W. now.

$$(A+B)^T = A^T + B^T = -A - B$$
$$(A+B)^T = -(A+B)$$

Thus it's closed under scalar addition

Now,

$$(KA^T) = KA^T = K(-A) = -KA$$

This makes it closed under scalar multiplication.

Thus we can say that the subset W is the subspace of M_{nn}.

B)

let W Be the set of all $n\times n$ matrices A for which $Ax=0$ has only the trivial solution.
This means that every member of subset W must Be an invertible matrix. Now consider
matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and scalar $K=0$

For scalar multiplication:

$$KA = 0\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

The result of KA is not an invertible matrix thus the set W is not closed under scalar multiplication hence its not a subspace.

c) let W Be the set such that all $n\times n$ matrices A such that $AB=BA$ for some fixed $n\times n$ matrix B.

For vector Addition:

let us assume two vectors A & C in space W. Now Ale to the condition
$AB=BA$ so, $CB=BC$
Now, since
$$(A+C)B = AB+CB = BA+BC = B(A+C)$$
Thus it hold vector addition property

For scalar multiplication:

we have
$$(kA)B = k(AB) = k(BA) = B(kA)$$

Since the set holds both scalar multiplication & vector addition we can say that its a subspace.

d) let W be the set of all invertible matrices

For Scalar Multiplication:

The set is not closed for scalar multiplication when scalar is equal to 0

Thus the set W is not a subspace

QUESTION NO 12

a) Let W be the set of all 2×2 matrices A such that $A\begin{bmatrix}1\\-1\end{bmatrix}=\begin{bmatrix}0\\0\end{bmatrix}$

For vector Addition:

$$(A+B)\begin{bmatrix}1\\-1\end{bmatrix}=A\begin{bmatrix}1\\-1\end{bmatrix}+B\begin{bmatrix}1\\-1\end{bmatrix}$$

$$=\begin{bmatrix}0\\0\end{bmatrix}+\begin{bmatrix}0\\0\end{bmatrix}$$

$$=\begin{bmatrix}0\\0\end{bmatrix}$$

FOR ~~vector~~ scalar Multiplication:

$$(KA)\begin{bmatrix}1\\-1\end{bmatrix}=K\left(A\begin{bmatrix}1\\-1\end{bmatrix}\right)=K\begin{bmatrix}0\\0\end{bmatrix}$$

$$=\begin{bmatrix}0\\0\end{bmatrix}$$

Thus the set is a subspace of M_{22} since it hold vector addition & scalar multiplication b)

let W be the set of all 2×2 matrices A such that

$$A\begin{bmatrix} 0 & 2 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 2 \\ -2 & 1 \end{bmatrix} A$$

For vector addition:

$$(A+B)\begin{bmatrix} 0 & 2 \\ -2 & 1 \end{bmatrix} = A\begin{bmatrix} 0 & 2 \\ -2 & 1 \end{bmatrix} + B\begin{bmatrix} 0 & 2 \\ -2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 2 \\ -2 & 1 \end{bmatrix} A + \begin{bmatrix} 0 & 2 \\ -2 & 1 \end{bmatrix} B$$

$$= \begin{bmatrix} 0 & 2 \\ -2 & 1 \end{bmatrix} (A+B)$$

For ~~scl~~ scalar multiplication

$$(KA)\begin{bmatrix} 0 & 2 \\ -2 & 1 \end{bmatrix} = K\left(A\begin{bmatrix} 0 & 2 \\ -2 & 1 \end{bmatrix}\right)$$

$$= K\begin{bmatrix} 0 & 2 \\ -2 & 1 \end{bmatrix} A$$

$$= \begin{bmatrix} 0 & 2 \\ -2 & 1 \end{bmatrix} (KA)$$

Thus W is a subspace of M_{22}

c)

let W be the set for all 2×2 matrices A such that for which $\det(A) = 0$

For vector addition

$$\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

The result is not a singular matrix thus vector addition doesn't hold & W is not a subspace.

x —— x —— x

QUESTION 14

a) let w be the set of All vectors x in R^4 such that $Ax = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, where

$$A = \begin{bmatrix} 0 & -1 & 0 & 2 \\ -1 & 1 & 0 & 1 \end{bmatrix}$$

For scalar multiplication:

let $k = 0$

$$k(Ax) \stackrel{?}{=} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \qquad \because Ax = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$0 \begin{bmatrix} 0 \\ 1 \end{bmatrix} \stackrel{?}{=} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} \neq \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Thus the set W is not a subspace for all vectors in R^4

b) let W be the set of All vectors x is R^4 such that $Ax = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ where A is as in part A

For scalar multiplication:

let $K = 0$

$$K(Ax) \stackrel{?}{=} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \qquad \because Ax = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$0 \begin{bmatrix} 0 \\ 1 \end{bmatrix} \stackrel{?}{=} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} \neq \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Thus W is not a subspace for all vectors in R^4.

QUESTION 16

a) let W be the set of all polynomials with even coefficients.

For vector addition:

The sum of even coefficients will always be even thus vector addition holds.

For scalar multiplication:

The multiplication of a scalar with an even co-efficient will always be an even coefficient.

Thus we can say that W is a ~~sub~~ subspace of P_∞

b) let W be the set of polynomials whose co-efficients sum to 0.

For vector addition

If you add two polynomials whose individual sum of co-efficients is 0 then the collective sum will also be 0.

For scalar multiplication:

If you multiply a polynomial with a scalar. then after multiplication. The result ~~of~~ of the sum of co-efficients will still be zero.

Thus W is the ~~subset~~ subspace of P_∞

c)

let w be the set which contains All polynomials of even degree.

For vector addition:

If you add 2 polynomials of even degrees the sum will also be a polynomial of ~~...~~ even degree.

For scalar multiplication:

If you multiply a polynomial of even degree by a scalar The result will still be a polynomial of even degree.

Thus we can say that w is a subset of W_o

EXERCISE 4.3

QUESTION #15

a)

$U = (1,0,-1,0)$; $V_2(0,1,0,-1)$

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

$R_2 - R_1$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & -1 & 0 & -1 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

$R_1 + R_2$

$$= \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & -1 & 0 & -1 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

i) $R_3 + R_2$
ii) $-R_3$

$$A = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$(x,y,z,w) = (s,t,-s,-t) = s(1,0,-1,0) + t(0,1,0,1)$

Thus we can say that U & V span the space

b) In part (a) I have already proven that $U = V_1 = (1,0,-1,0)$ & $V = V_1 + V_2$ Thus we can say that the vectors ~~$U_2(0,$~~ $U = (1,0,-1,0)$ & $V_2 = (1,1,-1,-1)$ span the vector space.

× —— × —— ×

QUESTION 16

a) $U = (1,1,1,0)$; $V = (0,-1,0,1)$

$$A = \begin{bmatrix} 0 & 1 & -1 & 1 \\ 0 & 2 & -2 & 2 \\ 0 & 3 & -3 & 3 \end{bmatrix}$$

$R_3 - 3R_1$; $R_2 - 2R_1$

$$A = \begin{bmatrix} 0 & 1 & -1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$(x, y, z, w) = (r, s-t, s, t)$

$= r(1,0,0,0) + s(0,1,1,0) + t(0,-1,0,1)$

Thus the solution is spanned by

$V_1 = (1,0,0,0)$, $V_2 = (0,1,1,0)$; $V_3 = (0,-1,0,1)$

Thus we can say that $U = (1,1,1,0)$ & $V = (0,-1,0,1)$ donot span the vector space.

b) Using part (a)

The solution is spanned by

$V_1 = (1,0,0,0)$; $V_2 = (0,1,1,0)$; $V_3 = (0,-1,0,1)$

Thus we can say that $U = (0,1,1,0)$ & $V = (1,0,1,1)$ donot span the space

Question 18

a) The vectors

$T_A(0,1,1) = (1,0)$, $T_A(2,-1,1) = (1,1)$, $T_A(1,1,-2) = (0,-1)$ span R^2 if

$b = (b_1, b_2)$ can be a linear combination of

$$(b_1, b_2) = K_1(1,0) + K_2(1,-2) + K_3(2,3)$$

writing in equation form.

$$K_1 + K_2 + 2K_3 = b_1$$
$$0K_1 - 2K_2 + 3K_3 = b_2$$

writing in matrix form.

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 0 & -2 & 3 \end{bmatrix}$$

$R_2 = R_2 \div -2$

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & -3/2 \end{bmatrix}$$

$R_1 = R_1 - R_2$

$$A = \begin{bmatrix} 1 & 0 & 7/2 \\ 0 & 1 & -3/2 \end{bmatrix}$$

Thus the system is consistent & we can say that

$T_A(u_1), T_A(u_2), T_A(u_3)$ span R^2.

x ——x—— x

b) The vectors

$T_A(0,1,1) = (1,4)$, $T_A(2,-1,1) = (-1,4)$ & $T_A(1,1,-2) = (1,-4)$ span R^2 if an arbitrary vector $b = (b_1, b_2)$ can be a linear combination of

$$(b_1, b_2) = K_1(1,4) + K_2(-1,4) + K_3(1,4)$$

Equating corresponding components on both sides yields linear combination

$$1K_1 - 1K_2 + 1K_3 = b_1$$
$$4K_1 + 4K_2 - 4K_3 = b_2$$

writing in matrix form

$$A = \begin{bmatrix} 1 & -1 & 1 \\ 4 & 4 & -4 \end{bmatrix}$$

$R_1 \to 4R_1 + R_2$

$$A = \begin{bmatrix} 8 & 0 & 0 \\ 4 & 4 & -4 \end{bmatrix}$$

$R_2 \to R_2 - R_1$

$$= \begin{bmatrix} 8 & 0 & 0 \\ -4 & 4 & -4 \end{bmatrix}$$

$R_1 \to R_1 \div 8$; $R_2 \to R_2/4$

$$= \begin{bmatrix} 1 & 0 & 0 \\ -1 & 01 & -1 \end{bmatrix}$$

$R_2 \to R_2 + R_1$

$$\boxed{A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \end{bmatrix}}$$

The system is consistent & we can say that $T_A(u_1), T_A(u_2)$ & $T_A(u_3)$ span R^2.

EXERCISE 4.4

QUESTION 11:

The vector equation can be written in matrix form as

$$\begin{bmatrix} \lambda & -1/2 & -1/2 \\ -1/2 & \lambda & -1/2 \\ -1/2 & -1/2 & \lambda \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Now

$$\begin{vmatrix} \lambda & -1/2 & -1/2 \\ -1/2 & \lambda & -1/2 \\ -1/2 & -1/2 & \lambda \end{vmatrix} = \lambda^3 - 3/4\,\lambda - 1/4$$

By factorizing.

$$\lambda^3 - 3/4\,\lambda - 1/4 = (\lambda + 1/2)(\lambda^2 - 1/2\,\lambda - 1/2)$$
$$= (\lambda + 1/2)(\lambda + 1/2)(\lambda - 1)$$

Thus the values of λ that make the vectors linearly dependant are

$\lambda = -1/2$ & $\boxed{\lambda = 1}$

QUESTION 14

a) For $T_A(1,0,0) = (1,1,2)$,
$T_A(2,-1,1) = (3,-1,2)$, $T_A(0,1,1) = (3,-3,2)$

The vector equation

$$K_1(1,1,2) + K_2(3,-1,2) + K_3(3,-3,2) = (0,0,0)$$

can be written equation form

$$1K_1 + 3K_2 + 3K_3 = 0$$
$$1K_1 - 1K_2 - 3K_3 = 0$$
$$2K_1 + 2K_2 + 2K_3 = 0$$

Finding determinant of A

$$\begin{vmatrix} 1 & 3 & 3 \\ 1 & -1 & -3 \\ 2 & 2 & 2 \end{vmatrix} = 1\begin{vmatrix} -1 & -3 \\ 2 & 2 \end{vmatrix} - 3\begin{vmatrix} 1 & -3 \\ 2 & 2 \end{vmatrix} + 3\begin{vmatrix} 1 & -1 \\ 2 & 2 \end{vmatrix}$$

$$= 1(-2+6) - 3(2+6) + 3(2+2)$$
$$= 4 - 3(8) + 3(4)$$
$$= 16 - 24$$
$$= -8 \neq 0$$

Thus we can say that the system has only trivial solution & the ~~system~~ set is linearly independant.

b)

~~Since $T_A(H_1) =$~~

We calculate $T_A(1,0,0) = (1,1,1)$,
$T_A(2,-1,1) = (1,1,-3)$; $T_A(0,1,1) = (2,2,0)$.

The vector equation

$$K_1(1,1,1) + K_2(1,1,-3) + K_3(2,2,0) = (0,0,0)$$

can be written in equation form.

$$K_1 + K_2 + 2K_3 = 0$$
$$K_1 + K_2 + 2K_3 = 0$$
$$K_1 - 3K_2 + 0K_3 = 0$$

For det (A)

$$= \begin{vmatrix} 1 & 1 & 2 \\ 1 & 1 & 2 \\ 1 & -3 & 0 \end{vmatrix}$$ (Two rows are same thus determinant is 0)

$$= 0$$

Therefore the sets are linearly dependant