National University of Computer & Emerging Sciences, Karachi
Spring-2021 CS-Department
Mid-term II Examination
19th April 2021, 11:15 pm to 12:15 pm

Course Code: MT205	Course Name: Probability & Statistics
Instructor Names: Osama Bin Ajaz, Dr Fahad Riaz, Miss Amber Shaikh, and Miss Javeria Iftikhar	
Student Roll No:	**Section No:**

Instructions:

- Return the question paper.
- Read each question completely before answering it. There are 5 **questions and 2 pages**
- In case of any ambiguity, you may make the assumption. But your assumption should not contradict any statement in the question paper.
- All the answers must be solved according to the sequence given in the question paper.
- **Show detailed steps in the solution to each question.**

Time: 1 hr. **Max Points:** 30

Q1) From a sack of fruit containing 3 oranges, 2 apples, and 3 bananas, a random sample of 4 pieces of fruit is selected. If X is the number of oranges and Y is the number of apples in the sample, find

(i) Find the joint probability distribution of X and Y [2]
(ii) Find $\mu_x - \mu_y$. [2]
(iii) Find Coefficient of Correlation of X and Y. [4]

Q2) The time to failure in hours of an important piece of electric equipment used in a manufactured DVD player has the density function:

$$f(x) = \frac{1}{2000}e^{\left(-\frac{x}{2000}\right)}, \quad 0 \le x \le 3000 \quad \& \quad zero\ elsewhere$$

(i) Find $F(x)$. [2]
(ii) Determine the probability that the component will last more than 1000 hours. [1.5]
(iii) Determine the probability that the component fails before 2000 hours. [1.5]

Q3) Given the joint density function

$$f(x,y) = \frac{x(1+3y^2)}{4}, \quad 0 < x < 2, \quad 0 < y < 1, \quad \& \quad zero\ elsewhere$$

(i) Find $g(x)$ and $h(y)$ and use them to evaluate conditional distribution $f(x|y)$. [3]
(ii) Use marginal distributions and check whether the random variables X and Y are statistically independent. [1]

Q4) **(a)** The ministry of defence installed three radar systems to detect incoming drone missiles. The probability that it will detect an incoming drone is 0.75.

(i) what is the probability that an incoming drone will not be detected by any of the 3 systems? [1]
(ii) what is the probability that the drone will be detected by only one system? [1]
(iii) what is the probability that it will be detected by at least two out of three systems? [1]
(iv) what is the average number of detection for the three systems? [1]
(v) what is the standard deviation? [1]

(b) An electronic switching device occasionally malfunctions, but the device is considered satisfactory if it makes, on average, no more than 0.20 error per hour. A particular 5-hour period is chosen for testing the device. If no more than 1 error occurs during the time period, the device will be considered satisfactory.

(i) What is the probability that a satisfactory device will be considered unsatisfactory on the basis of the test? **[1.5]**

(ii) What is the probability that a device will be accepted as satisfactory when, in fact, the mean number of errors is 0.25? **[1.5]**

Q5) The finished inside diameter of a piston ring is normally distributed with a mean of 10 centimetres and a standard deviation of 0.03 centimetre.

(i) What proportion of rings will have inside diameters exceeding 10.075 centimetres? **[1]**

(ii) What is the probability that a piston ring will have an inside diameter between 9.97 and 10.03 centimetres? **[2]**

(iii) Below what value of inside diameter will 15% of the piston rings fall? **[2]**

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
−3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
−3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
−3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
−3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
−2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
−2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
−2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
−2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
−2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
−2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
−2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
−2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
−2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
−2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
−1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
−1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
−1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
−1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
−1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
−1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
−1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
−1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
−1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
−1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

MID - II Solution

Q # 01

3O → X
2A → Y
3B → n = 4
Total = 8

i) JOINT PROBABILITY DISTN:-

$f(x, y)$

Formula:-

$$f(x,y) = \frac{{}^{3}C_{x} \cdot {}^{2}C_{y} \cdot {}^{3}C_{4-x-y}}{{}^{8}C_{4}}$$

$f(x,y)$		y			
		0	1	2	Total
x	0	0	1/35	3/70	1/14
	1	3/70	9/35	9/70	3/7
	2	9/70	9/35	3/70	3/7
	3	3/70	1/35	0	1/14
Total		3/14	4/7	3/14	1

(ii) $\mu_x - \mu_y = ?$

1) $\mu_x = E(X) = \sum_{x=0}^{3} x \cdot g(x)$

$\Rightarrow \mu_x = (0)\,g(0) + (1)(g(1)) + (2)(g(2)) + (3)(g(3))$

$= \frac{3}{7} + 2\left(\frac{3}{7}\right) + 3\left(\frac{1}{14}\right)$

$\Rightarrow \boxed{\mu_x = \frac{3}{2}}$

2) $\mu_y = E(Y) = \sum_{y=0}^{2} y \cdot h(y)$

$\Rightarrow \mu_y = (0)(h(0)) + (1)(h(1)) + (2)(h(2))$

$\Rightarrow \mu_y = \frac{4}{7} + 2\left(\frac{3}{14}\right)$

$\Rightarrow \boxed{\mu_y = 1}$

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$$\Rightarrow \mu_x - \mu_y = \frac{3}{2} - 1$$

$$\Rightarrow \boxed{\mu_x - \mu_y = \frac{1}{2}}$$

(iii) $$\rho_{XY} = \frac{\sigma_{XY}}{\sigma_X \sigma_Y} \quad \text{--- (1)}$$

1) $$\sigma_{XY} = E(XY) - \mu_x \cdot \mu_y$$

$$\Rightarrow E(XY) = \sum_{y=0}^{2} \sum_{x=0}^{3} f(x,y)$$

$$= \frac{9}{35} + 2\left(\frac{9}{70}\right) + 2\left(\frac{9}{35}\right) + 4\left(\frac{3}{70}\right) + 3\left(\frac{1}{35}\right).$$

$$\Rightarrow \boxed{E(XY) = \frac{9}{7}}$$

$$\Rightarrow \boxed{\mu_x = \frac{3}{2}}$$

$$\Rightarrow \boxed{\mu_y = 1}$$

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$\Rightarrow \sigma_{XY} = \frac{9}{7} - \frac{3}{2}$

$\Rightarrow \boxed{\sigma_{XY} = \frac{-3}{14}}$

Now; $\sigma_X^2 = E(X^2) - [E(X)]^2$.

1) $E(X^2) = \sum_{x=0}^{3} x^2 \cdot g(x)$

$= 0^2 g(0) + 1^2 g(1) + 2^2 g(2) + 3^2 g(3)$

$= \frac{3}{7} + 4\left(\frac{3}{7}\right) + 9\left(\frac{1}{14}\right)$.

$\Rightarrow E(X^2) = \frac{39}{14}$

2) $[E(X)]^2 = [\mu_X]^2 = \left[\frac{3}{2}\right]^2 = \frac{9}{4}$.

$\Rightarrow \sigma_X^2 = \frac{39}{14} - \frac{9}{4} = \frac{15}{25}$

$$\Rightarrow \boxed{\sigma_x = \frac{\sqrt{105}}{14}}$$

2) $\sigma_Y^2 = E(Y^2) - [E(Y)]^2$

1) $E(Y^2) = \sum_{y=0}^{2} y^2 \cdot h(y)$

$= 0^2 h(0) + 1^2 h(1) + 2^2 h(2)$

$= \frac{4}{7} + 4\left(\frac{3}{14}\right)$

$\Rightarrow E(Y^2) = \frac{10}{7}$

Also; $[E(Y)]^2 = [\mu_Y]^2 = (1)^2 = 1$

$\Rightarrow \sigma_Y^2 = \frac{10}{7} - 1 = \frac{3}{7}$

$$\Rightarrow \boxed{\sigma_Y = \frac{\sqrt{21}}{7}}$$

$$\Rightarrow f_{xy} = \frac{\frac{-3}{14}}{\left[\frac{\sqrt{105}}{14}\right]\left[\frac{\sqrt{21}}{7}\right]}$$

$$\Rightarrow f_{xy} = \left(\frac{-3}{\cancel{14}}\right)\left(\frac{\cancel{14}}{\sqrt{105}}\right)\left(\frac{7}{\sqrt{21}}\right)$$

$$\Rightarrow f_{xy} = \frac{-21}{21\sqrt{5}}$$

$$\Rightarrow \boxed{f_{xy} = \frac{-1}{\sqrt{5}}}$$

Ans

Q # 02

$$f(x) = \frac{1}{2000} e^{\frac{-x}{2000}} \; ; \; 0 < x < 3000$$

(i) $F(x) = ?$

$$\Rightarrow F(x) = \int_{-\infty}^{x} f(x)\, dx.$$

$$\Rightarrow F(x) = \int_{0}^{x} \left[\frac{1}{2000} e^{\frac{-x}{2000}}\right] dx$$

$$\Rightarrow F(x) = \frac{1}{\cancel{2000}} \left[\frac{e^{\frac{-x}{2000}}}{\frac{-1}{\cancel{2000}}} \Bigg|_{0}^{x} \right].$$

$$\Rightarrow F(x) = -\left\{ e^{\frac{-x}{2000}} - 1 \right\}.$$

$$\Rightarrow \boxed{F(x) = 1 - e^{\frac{-x}{2000}}}$$

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ii

$$P(X > 1000) = \int_{1000}^{3000} \left(\frac{e^{-\frac{x}{2000}}}{2000} \right) dx$$

$$= -\left\{ e^{-\frac{x}{2000}} \Big|_{1000}^{3000} \right\}$$

$$= -e^{-\frac{3}{2}} + e^{-\frac{1}{2}}$$

$$\Rightarrow \boxed{P(X > 1000) = 0.3834}$$

iii

$$P(X < 2000) = \int_{0}^{2000} \left(\frac{e^{-\frac{x}{2000}}}{2000} \right) dx$$

$$\Rightarrow \quad = -\left\{ e^{-\frac{x}{2000}} \Big|_{0}^{2000} \right\}$$

$$= -e^{-1} + 1$$

$$\Rightarrow \boxed{P(X < 2000) = 0.63212}$$

Q #03

$$f(x, y) = \frac{x(1+3y^2)}{4}$$

1) $$g(x) = \int_{y=0}^{1} \left[\frac{x(1+3y^2)}{4} \right] dy$$

$$\cancel{= \frac{1+3y^2}{4} \left[\frac{x^2}{2} \Big|_0^1 \right]}$$

$\cancel{g(x)}$

$$= \frac{x}{4} \left[y \Big|_0^1 + y^3 \Big|_0^1 \right]$$

$$\Rightarrow g(x) = \frac{x}{4}(1+1)$$

$$\Rightarrow \boxed{g(x) = \frac{x}{2}}$$

2) $h(y) = \int_{x=0}^{2} \left(\frac{x(1+3y^2)}{4} \right) dx$

$\Rightarrow h(y) = \frac{1+3y^2}{4} \left[\frac{x^2}{2} \Big|_0^2 \right]$

$\Rightarrow h(y) = \frac{1+3y^2}{4} \left[2 - 0 \right]$

$\Rightarrow \boxed{h(y) = \frac{1+3y^2}{2}}$

3) $f(x|y) = \frac{f(x,y)}{h(y)}$

$= \left[\frac{x\cancel{(1+3y^2)}}{\cancel{4}\,2} \right] \left[\frac{\cancel{2}}{\cancel{1+3y^2}} \right]$

$\Rightarrow \boxed{f(x|y) = \frac{x}{2}}$

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4) for Statisticall Independant X & Y :-

$$f(x, y) = g(x) \cdot h(y)$$

$$\frac{x(1+3y^2)}{4} = \left(\frac{x}{2}\right)\left(\frac{1+3y^2}{2}\right)$$

$$\Rightarrow \boxed{\frac{x(1+3y^2)}{4} = \frac{x(1+3y^2)}{4}}$$

proved

X & Y are Statistically Independant.

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Q # 04 (a)

DATA:-

$n = 3$; $p = 0.75$

$q = 0.25$

use Binomial Distn.

(i) $P(X=0) = {}^{3}C_{0}(0.75)^{0}(1-0.75)^{3}$

$\Rightarrow \boxed{P(X=0) = 0.015}$

(ii) $P(X=1) = {}^{3}C_{1}(0.75)^{1}(0.25)^{2}$

$\Rightarrow \boxed{P(X=1) = 0.140}$

(iii) $P(X \geq 2) = 1 - P(X<2)$

$= 1 - \{P(X=0) + P(X=1)\}$

$$\Rightarrow P(X \geq 2) = 1 - \left\{ 0.015625 + 0.140625 \right\}$$

$$\Rightarrow \boxed{P(X \geq 2) = 0.84375}$$

(iv)

$$\mu = np = (3)(0.75)$$

$$\Rightarrow \boxed{\mu = 2.25}$$

(v)

$$\sigma = \sqrt{npq} = \sqrt{(3)(0.75)(0.25)}$$

$$\Rightarrow \boxed{\sigma = \sqrt{0.562} = 0.75}$$

(b)

DATA: $\lambda = 0.20$, $t = 5$

$\Rightarrow \lambda t = (0.20)(5)$

$\Rightarrow \boxed{\lambda t = 1}$

use poisson Distn.

$\Rightarrow$ i) $P(X > 1) = 1 - P(X \leq 1)$

$= 1 - [P(X=0) + P(X=1)]$

$\Rightarrow P(X > 1) = 1 - \left[\sum_{x=0}^{1} \frac{e^{-\lambda t} (\lambda t)^x}{x!} \right]$

$\Rightarrow \boxed{P(X > 1) = 0.264}$

ii) $P(X \leq 1) = P(X=0) + P(X=1)$

$\Rightarrow P(X \leq 1) = \frac{e^{-1.25}(1.25)^0}{0!} + \frac{e^{-1.25}(1.25)^1}{1!}$

$\Rightarrow \boxed{P(X \leq 1) = 0.644}$

Q # 05

DATA:-

$\mu = 10$; $\sigma = 0.03$

$x_1 = 10.075$

$P(X > 10.075) = ?$

Use Standard Normal Dist.

$$\Rightarrow Z = \frac{10.075 - 10}{0.03}$$

$$\Rightarrow \boxed{Z = 2.5}$$

$$\Rightarrow P(Z > 2.5) = 1 - P(Z < 2.5)$$

$$= 1 - 0.9938$$

$$\Rightarrow \boxed{P(Z > 2.5) = 0.0062}$$

(ii)

$$P(9.97 < X < 10.03) = ?$$

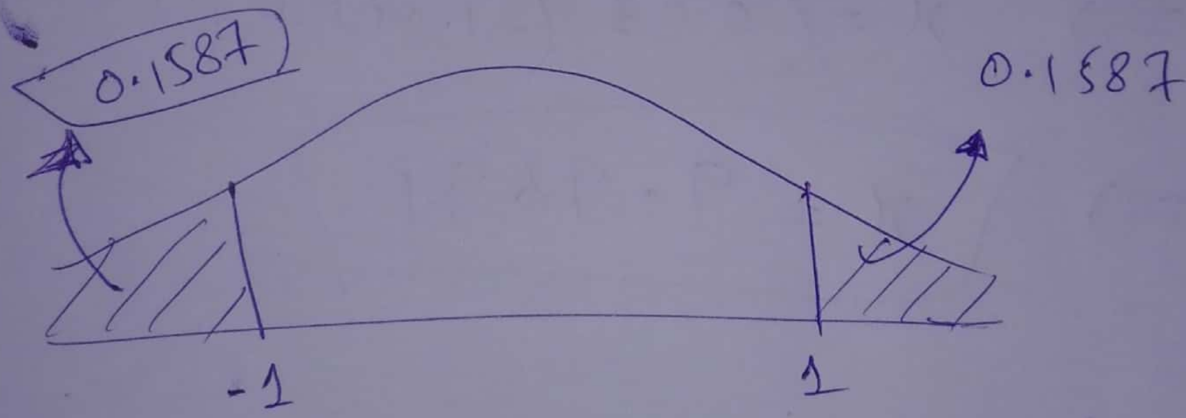
$$Z_1 = \frac{9.97 - 10}{0.03} = -1$$

$$Z_2 = \frac{10.03 - 10}{0.03} = 1$$

$$\Rightarrow P(-1 < Z < 1) = P(Z < 1) - P(Z < -1)$$

$$\boxed{\text{if } P(Z < -1) = 0.1587}$$

then;



$$\boxed{\begin{aligned} P(Z < 1) &= 1 - 0.1587 \\ &= 0.8413 \end{aligned}}$$

$$\Rightarrow P(-1 < Z < 1) = 0.8413 - 0.1587$$
$$= 0.6826 \quad \underline{\text{Ans}}$$

iii

$$P(Z < z) = 0.15$$

$$\Rightarrow \boxed{z = -1.03}$$

$$\Rightarrow x = \sigma z + \mu.$$

$$\Rightarrow x = (0.03)(-1.03) + 10$$

$$\Rightarrow \boxed{x = 9.9691}$$