PROB & STATS ASSIGNMENT 02

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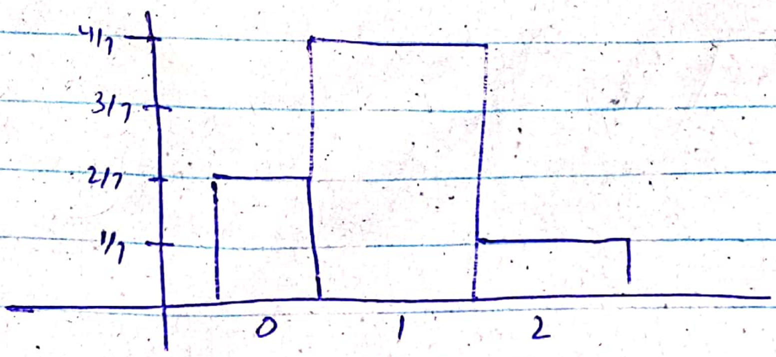
Ex 3.1

X = Discrete
Y = Continuous
M = Continuous
N = Discrete
P = Discrete
Q = Continuous

Ex 3.11

$$f(x) = \frac{\binom{2}{x}\binom{5}{3-x}}{\binom{7}{3}} \quad x = 0, 1, 2$$

x	0	1	2
$f(x)$	2/7	4/7	1/7



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Ex 3.20

$$F(x) = \frac{2}{27}\int_2^x (1+t)\,dt = \frac{2}{27}\left(t + \frac{t^2}{2}\right)\Big|_2^x$$

$$= \frac{(x+4)(x-2)}{27}$$

$$P(3 \leq X \leq 4) = F(4) - F(3) = \frac{8(2)}{27} - \frac{(7)(1)}{27} = \boxed{\frac{1}{3}}$$

Ex 3.13

The CDF of X is

$$F(x) = \begin{cases} 0 & \text{for } x < 0 \\ 0.41 & \text{for } 0 \leq x < 1 \\ 0.75 & \text{for } 1 \leq x < 2 \\ 0.94 & \text{for } 2 \leq x < 3 \\ 0.99 & \text{for } 3 \leq x < 4 \\ 1 & \text{for } x \geq 4 \end{cases}$$

Ex 3.39

a) $f(x,y) = \dfrac{\binom{3}{x}\binom{2}{y}\binom{3}{4-x-y}}{\binom{8}{4}}$ $\quad x = 0,1,2,3;\ y = 0,1,2$

$1 \leq x + y \leq 4$

b) $P[(X,Y) \in A] = P(X+Y \leq 2) = f(1,0) + f(2,0) + f(0,1) + f(1,1) + f(0,2)$

$$= \frac{3}{70} + \frac{9}{70} + \frac{2}{70} + \frac{18}{70} + \frac{3}{70} = \boxed{\frac{1}{2}}$$

Ex ~~5~~ 3.57

a) $1 = k\int_0^2\int_0^1\int_0^1 xy^2z\,dx\,dy\,dz = 2k\int_0^1\int_0^1 y^2 z\,dy\,dz$

$$= \frac{2k}{3}\int_0^1 z\,dz = \frac{k}{3}$$

$\boxed{k = 3}$

b) $P(X < \frac{1}{4}, Y > \frac{1}{2}, 1 < Z < 2) = 3\int_1^2\int_{1/2}^1\int_0^{1/4} xy^2z\,dx\,dy\,dz$

$$= \frac{9}{2}\int_0^{1/4}\int_{1/2}^1 y^2\,dy\,dz = \frac{21}{16}\int_1^2 z\,dz = \boxed{\frac{21}{512}}$$

Ex 3.62

a)

x	1	3	5	7
$f(x)$	0.4	0.2	0.2	0.2

b) $P(4 < x \leq 7) = P(x \leq 7) - P(\leq 4) = F(7) - F(4) = 1 - $

$\boxed{P(4 < x \leq 7) = 0.4}$ A.

x —— x —— x

End of CH03

Chapter 04:

Ex 4.1

$\mu = E(x) = 0(0.41) + (1)(0.37) + 2(0.16) + 3(0.05) + 4(0.01)$

$\boxed{\mu = 0.88}$ A.

Ex 4.7

Expected gain $= 4000(0.3) + (-1000)(0.7)$

$\boxed{\text{Expected gain} = \$500}$ A.

Ex 4.12

$E(x) = \int_0^1 2x(1-x)\,dx = 1/3$

Avg. profit $= \frac{1}{3}(\$5000)$.

$\boxed{\text{Avg. profit per automobile} = \$1,667.67}$

Ex 4.20

$E[g(x)] = E(e^{2x/3}) = \int_0^{\infty} e^{2x/3} e^{-x} dx = \int_0^{\infty} e^{-x/3} dx$

$\boxed{E[g(x)] = 3}$ Ans.

Ex 4.34

$\mu = (-2)(0.3) + (3)(0.2) + 5(0.5)$

$\boxed{\mu = 2.5}$

$E(X^2) = (-2)^2(0.3) + (3)^2(0.2) + 5^2(0.5) = 15.5$

$\boxed{E(x^2) = 15.5}$

$\sigma^2 = E(x^2) - \mu^2 = 15.5 - (2.5)^2 = 9.25$

$\boxed{\sigma^2 = 9.25}$

$\boxed{\sigma = 3.041}$

Ex 4.45

$\mu_x = \sum_x x g(x) = 2.45, \quad \mu_y = \sum_y y h(y) = 3.20$

$\Sigma(XY) = \sum_x \sum_y xy f(x,y) = 1(0.05) + 2(0.05) + 3(0.10)$
$+ 2(0.05) + 4(0.10) + 6(0.35)$
$+ 3(0) + 6(0.20) + 9(0.10) = 7.85$

$\boxed{\Sigma(XY) = 7.85}$

$\sigma_{XY} = 7.85 - (2.45)(3.20)$

$\boxed{\sigma_{XY} = 0.01}$

Ex 4.48

$\sigma_{XY} = Cov(a + bX, X) = b\sigma_X^2 \;\&\; \sigma_Y^2 = b^2\sigma_X^2$

$\rho = \frac{\sigma_{XY}}{\sigma_X \sigma_Y} = \frac{b\sigma_x^2}{\sqrt{\sigma_x^2 b^2 \sigma_x^2}} = \frac{b}{|b|} = \text{sign of } b$

Thus we can say that

$\rho = 1$ if $b > 0$ & $\rho = -1$ if $b < 0$

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Ex 4.58

$$E(x) = \int_0^1 x^2 dx + \int_1^2 x(2-x)dx = 1$$

$$E(x^2) = \int_0^1 x^3 2dx + \int_1^2 x^2(2-x)dx = 7/6$$

$$E(Y) = 60E(x^2) + 39E(x) = 60(7/6) + 39(1)$$

$$\boxed{E(Y) = 109 \text{ KWhrs}}$$

x —— x —— x

END OF CHAP 04

CHAPTER 05

Ex 5.3

Uniform distribution

$f(x) = 1/10$ for $x = 1, 2, \ldots, 10$

$f(x) = 0$ elsewhere

Therefore $P(X<4) = \sum_{x=1}^{3} f(x) = 3/10$

Ex 5.12

$n = 9$ $p = 0.25$

$$P(X<4) = \cancel{\binom{9}{0}(0.25)^0(0.75)^9 + \binom{9}{1}(0.25)^1(0.75)^8 + \binom{9}{2}(0.25)^2(0.75)^7 + \binom{9}{3}(0.25)^3(0.75)^5}$$

$$P(x<4) = \sum_{x=0}^{3} \binom{9}{x}(0.25)^x(0.75)^{9-x}$$

where $x = 0, 1, 2, 3$

$$\boxed{P(X<4) = 0.834}$$

Ex 5.26

$n = 8$; $p = 0.60$

a) $P(X=6) = \binom{8}{6}(0.6)^6(0.4)^2$

$\boxed{P(X=6) = 0.2090}$

b) $P(X=6) = P(X \leq 6) - P(X \leq 5)$

$= 0.8936 - 0.6846 = 0.2090$

$\boxed{P(X=6) = 0.2090}$

Ex 5.30

$P(X \geq 1) = 1 - P(X=0)$

$= 1 - \frac{\binom{6}{0}\binom{9}{3}}{\binom{15}{3}} = \frac{53}{65}$

$\boxed{P(X \geq 1) = \frac{53}{65}}$

Ex 5.33

a) $= \frac{\binom{12}{2}\binom{40}{5}}{\binom{52}{7}} = \boxed{0.3246}$ A

b) $= 1 - \frac{\binom{40}{7}}{\binom{52}{7}} = \boxed{0.4496}$ A

Ex 5.34

$= \frac{\binom{4}{2}\binom{5}{3}}{\binom{9}{5}} = \frac{10}{21} = \boxed{0.476}$

Ex 5.50

a) $= \binom{6}{2}(1/2)^7$

$\boxed{b(7, 3, 1/2) = 0.1172}$

b) $= (1/2)(1/2)^3 = (1/16)$

$\boxed{g(4, 1/2) = 0.0625}$

Ex 5.55

a) $P(X=3) = g(3; 0.7) = (0.7)(0.3)^2 = 0.0630$

b) $P(X<4) = \sum_{x=1}^{3} g(x, 0.7) = \sum_{x=1}^{3} (0.7)(0.3)^{x-1} = 0.97$

$P(X<4) = 0.9730$

Ex 5.57

a) $P(X \geq 4) = 1 - P(X \leq 3) = 0.1429$

b) $P(X=0) = p(0; 2) = 0.1353$

x —— x —— x

END OF CHAP. 05

CHAPTER 06

Ex 6.8

a)

$z = \frac{(17-30)}{6} = -2.17$

Area $= 1 - 0.01510$

Area $= 0.9850$

b) $z = \frac{(22-30)}{6} = -1.33$

Area $= 0.0918$

c) $z_1 = \frac{(32-30)}{6} = 0.33$; $z_2 = \frac{(41-30)}{6} = 1.83$

Area $= 0.9664 - 0.6293$

Area $= 0.3371$

Ex 6.11

a) $z = \frac{224-200}{15} = 1.6$

$P(z>1.6) = 0.0548$

b) $z_1 = (191-200)/15 = -0.6$; $z_2 = \frac{(209-200)}{15} = 0.6$

$P(191<x<209) = P(-0.6<z<0.6) =$

$= 0.7257 - 0.2743$

$\boxed{P(191<x<209) = 0.4514}$

c) $z = \frac{(230-200)}{15} = 2.0$

$P(x>230) = P(z>2.0) = 0.0228$

Thus $1000(0.0228) = 22.8 \approx$ 23 cups will overflow

d) $z = -0.67$, $x = 15(-0.67)+200 = 189.95$ ml

Ex 6.19

$\mu = \$15.90$ & $\sigma = \$1.50$

a) $P(13.75<x<16.22) = P\left(\frac{13.745-15.9}{1.5} < z < \frac{16.225-15.9}{1.5}\right)$

$= P(-1.437 < z < 0.217) = 0.5871 - 0.0749 = \boxed{0.5122}$

Thus 51% of workers

b) $P(z>1.645) = 0.05$; $x = (1.645)(1.50)+15.90+0.005$

$\boxed{= 18.37}$

$18.37 is the highest 5% of the employee hourly wages.

Ex 6.21

a) $z = \frac{10,175 - 10,000}{100} = 1.75$

$P(X > 10,175) = P(1.75) = 0.0401 = \boxed{4.01\% \text{ of compone... exceed tensile stren...}}$

b) $z_1 = \frac{(9775 - 10000)}{100} = -2.25$

$z_2 = \frac{(10,225 - 10000)}{100} = 2.25$

$P(X < 9775) + P(X > 10,225) = P(Z < -2.25) + P(Z > 2...$

$= 2P(Z < -2.25) = 0.0244$

$\boxed{= 2.44\% \text{ of pieces would be expected to scrap}}$

X —— X —— X

END OF CHAP 06

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END OF ASSIGNME...

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