Testing the Difference Between Two Means, Two Proportions, and Two Variances

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1. Test the difference between sample means, using the *z* test.
2. Test the difference between two means for independent samples, using the *t* test.
3. Test the difference between two means for dependent samples.
4. Test the difference between two proportions.
5. Test the difference between two variances or standard deviations.

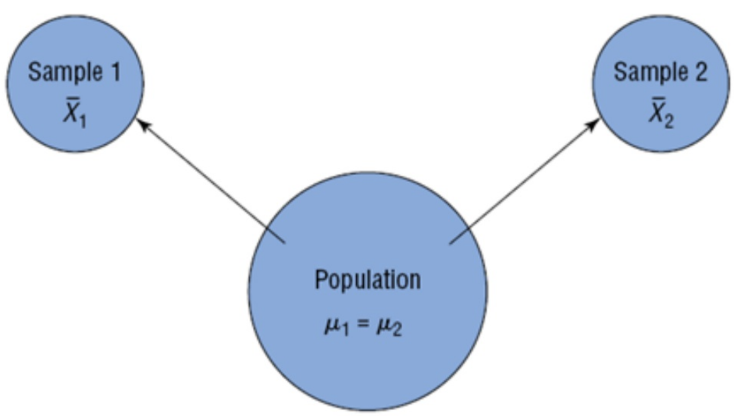
9.1 Testing the Difference Between Two Means: Using the *z* Test

Assumptions:

The samples must be independent of each other. That is, there can be no relationship between the subjects in each sample.

The standard deviations of both populations must be known, and if the sample sizes are less than 30, the populations must be normally or approximately normally distributed.

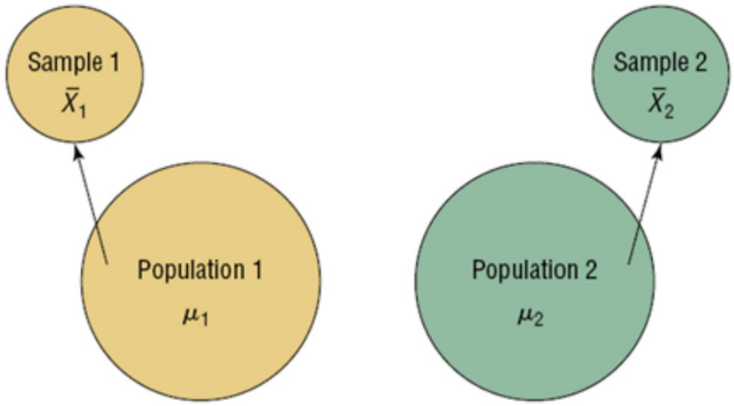
Hypothesis Testing Situations in the Comparison of Means



(a) Difference is not significant

Do not reject H_0: $\mu_1 = \mu_2$ since $\bar{X}_1 - \bar{X}_2$ is not significant.

Hypothesis Testing Situations in the Comparison of Means



(b) Difference is significant

Reject H_0: $\mu_1 = \mu_2$ since $\bar{X}_1 - \bar{X}_2$ is significant.

Testing the Difference Between Two Means: Large Samples

Formula for the z test for comparing two means from independent populations

$$z = \frac{\left(\bar{X}_1 - \bar{X}_2\right) - \left(\mu_1 - \mu_2\right)}{\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}}$$

σ = std dev

$\sigma^2 \rightarrow$ variance

n = sample size

Example 9-1: Hotel Room Cost

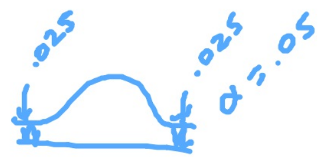
A survey found that the average hotel room rate in New Orleans is \$88.42 and the average room rate in Phoenix is \$80.61. Assume that the data were obtained from two samples of 50 hotels each and that the standard deviations of the populations are \$5.62 and \$4.83, respectively. At α = 0.05, can it be concluded that there is a significant difference in the rates?

Step 1: State the hypotheses and identify the claim.

H_0: $\mu_1 = \mu_2$ and H_1: $\mu_1 \neq \mu_2$ (claim)

NO Ph.

Step 2: Find the critical value.



The critical value is $z = \pm 1.96$.

Example 9-1: Hotel Room Cost

A survey found that the average hotel room rate in New Orleans is \$88.42 $\bar{X}_1$ and the average room rate in Phoenix is \$80.61 $\bar{X}_2$. Assume that the data were obtained from two samples of 50 n_1, n_2 hotels each and that the standard deviations of the populations are \$5.62 σ_1 and \$4.83 σ_2, respectively. At α = 0.05, can it be concluded that there is a significant difference in the rates?

Step 3: Compute the test value.

$$z = \frac{\left(\bar{X}_1 - \bar{X}_2\right) - \overset{0}{\cancel{(\mu_1 - \mu_2)}}}{\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}}$$

Example 9-1: Hotel Room Cost

A survey found that the average hotel room rate in New Orleans is $88.42 and the average room rate in Phoenix is $80.61. Assume that the data were obtained from two samples of 50 hotels each and that the standard deviations of the populations are $5.62 and $4.83, respectively. At α = 0.05, can it be concluded that there is a significant difference in the rates?

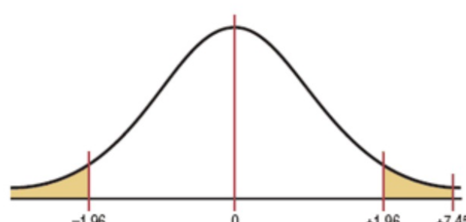
Step 3: Compute the test value.

$$z = \frac{(88.42 - 80.61) - (0)}{\sqrt{\frac{5.62^2}{50} + \frac{4.83^2}{50}}} = 7.45$$

Example 9-1: Hotel Room Cost

Step 4: Make the decision.

Reject the null hypothesis at α = 0.05, since 7.45 > 1.96.



Step 5: Summarize the results.

There is enough evidence to support the claim that the means are not equal. Hence, there is a significant difference in the rates.

Example 9-2: College Sports Offerings

A researcher hypothesizes that the average number of sports that colleges offer for males is greater than the average number of sports that colleges offer for females. A sample of the number of sports offered by colleges is shown. At $\alpha = 0.10$, is there enough evidence to support the claim? Assume σ_1 and $\sigma_2 = 3.3$.

$n_2 = 50$

Males $n_1 = 50$

Males					Females				
6	11	11	8	15	6	8	11	13	8
6	14	8	12	18	7	5	13	14	6
6	9	5	6	9	6	5	5	7	6
6	9	18	7	6	10	7	6	5	5
15	6	11	5	5	16	10	7	8	5
9	9	5	5	8	7	5	5	6	5
8	9	6	11	6	9	18	13	7	10
9	5	11	5	8	7	8	5	7	6
7	7	5	10	7	11	4	6	8	7
10	7	10	8	11	14	12	5	8	5

Example 9-2: College Sports Offerings

Step 1: State the hypotheses and identify the claim.

H_0: $\mu_1 = \mu_2$ and H_1: $\mu_1 > \mu_2$ (claim)

Step 2: Compute the test value.

For the males: $\bar{X}_1 = 8.6$ and $\sigma_1 = 3.3$

For the females: $\bar{X}_2 = 7.9$ and $\sigma_2 = 3.3$

Substitute in the formula

$$z = \frac{\left(\bar{X}_1 - \bar{X}_2\right) - \left(\mu_1 - \mu_2\right)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} = \frac{(8.6 - 7.9) - (0)}{\sqrt{\frac{3.3^2}{50} + \frac{3.3^2}{50}}} = 1.06$$

Example 9-2: College Sports Offerings

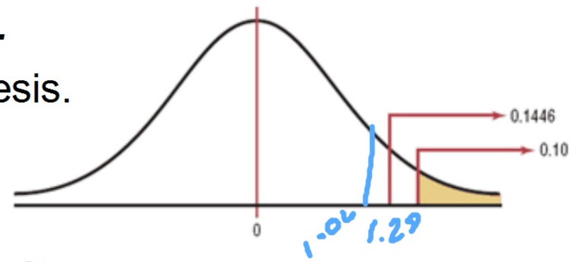
~~**Step 3: Find the *P*-value.**~~ C.V. = 1.28

For z = 1.06, the area is (Table E).

The *P*-value is

Step 4: Make the decision.

Do not reject the null hypothesis.



Step 5: Summarize the results.

There is not enough evidence to support the claim that colleges offer more sports for males than they do for females.

Confidence Intervals for the Difference Between Two Means

Formula for the z confidence interval for the difference between two means from independent populations

$$(\bar{X}_1 - \bar{X}_2) - z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} < (\mu_1 - \mu_2)$$

$$< (\bar{X}_1 - \bar{X}_2) + z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

Example 9-3: Confidence Intervals

Find the 95% confidence interval for the difference between the means for the data in Example 9–1.

$$\left(\bar{X}_1 - \bar{X}_2\right) - z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} < \mu_1 - \mu_2$$

$$< \left(\bar{X}_1 - \bar{X}_2\right) + z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

$$\left(88.42 - 80.61\right) - 1.96\sqrt{\frac{5.62^2}{50} + \frac{4.83^2}{50}} < \mu_1 - \mu_2$$

$$< \left(88.42 - 80.61\right) + 1.96\sqrt{\frac{5.62^2}{50} + \frac{4.83^2}{50}}$$

$$7.81 - 2.05 < \mu_1 - \mu_2 < 7.81 + 2.05$$

$$5.76 < \mu_1 - \mu_2 < 9.86$$