Probability

Topics

→ Sample space, event

→ Tree diagram

→ Set theory

→ Venn diagram

→ Multiplicative & Additive rules for prob.

→ Conditional Prob.

→ Bayes Theorem.

- — Measurement of chances is probability

- — Experiment: Any process or activity that generates data

  eg: Playing cards
  Rolling dice
  Tossing coins.

- — Sample space: list of all possible outcomes of statistical experiment. (S)

eg:-

* Toss a
* Toss 2
* Toss 3

* Tree

* T

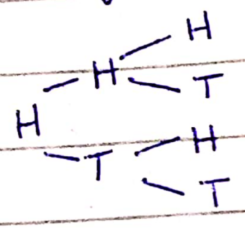
*

*

eg:-

* Toss a coin $S = 2^1 = \{H, T\}$
* Toss 2 coins together $S = 2^2 = \{HH, HT, TH, TT\}$.
* Toss 3 coins together $S = 2^3 = \{HHH, HTH, HHT, HTT, THH, THT, TTH, TTT\}$

* Tree diagram : Toss a coin twice.



* Tossing a dice $S = \{1, 2, 3, 4, 5, 6\} = 6^1$

* Tossing a dice & coin together

* Tossing a dice twice $= 6^2 = 36$

$\{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (2,1), (2,2), (2,3), (2,4), ($
$(2,6), (3,1), (3,2), (3,3), (3,4), (3,5), (3,6), (4,1), (4,2), (4,3), (4,4), ($
$(4,6), (5,1), (5,2), (5,3), (5,4), (5,5), (5,6), (6,1), (6,2),$
$(6,3), (6,4), (6,5), (6,6)\}$.

* Suppose 3 items are selected as defected and non-defected (N, D)
What are the possible outcomes.

$= 2^3 = 8$

$\Rightarrow$ {(DDD), (DDN), (DND), (DNN), (NNN), (NND), (NDN), (NDD)}

- Event: Subset of a sample space.

eg:

$S = \{1, 2, 3, 4, 5, 6\}$

Event = A = No. less than 4 occur = $\{1, 2, 3\}$.

- Mutually exclusive events / disjoint

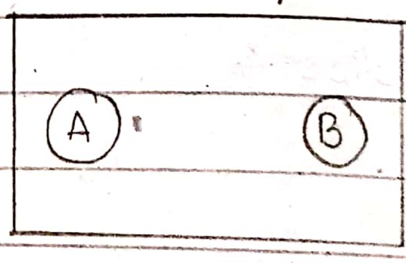
$S = \{1, 2, 3, 4, 5, 6\}$

$A = \{1, 3, 5\}$

$B = \{2, 4, 6\}$

$\boxed{A \cap B = \phi}$

$A \cap B = \phi$



2.1

* Set of integers between 1 to 50 divisible by 8

$S = \{8, 16, 24, 32, 40, 48\}$

* $\{x \mid x$ is a Continent$\}$

S = {Asia, Antarctica, Europe, Africa, North America, South America, Australia}

* $\{x \mid x^2 + 4x - 5 = 0\}$

  $S = \{1, -5\}$.

* Fundamental rule of counting:

  $n_1, n_2, \ldots\ldots, n_k$ ways

  $= n_1 * n_2 * n_3 * \ldots\ldots * n_k$

Q(ii) How many three digit numbers can formed from 2, 4, 6, 8

(i) If repititions not allowed

$4 * 3 * 2 = 24$

(ii) If repititions allowed

$4 * 4 * 4 = 64$

Q(iii)

T < Ranch, Two-story, Split-level floor

R < Ranch, Two-story, Split-level floor

C < Ranch, Two-story, Split-level floor

T < Ranch, Two-story, Split-level floor.

= 12 ways

Q(iv) Members = 22 , Elect Treasurer & a chair.
Total ways = 22 × 21 = 462 ways.

Q(v) Brands = 2 , Memory = 3
Hard disk = 4 , Accessories = 5

Total ways = 2 * 3 * 4 * 5
= 120 ways.

Q(vi) Even - four digit numbers from 0, 1, 2, 5, 6, 9 if each digit can be used only one

(0 at even place) 5 × 4 × 3 × 1 = 60
(2) → 4 × 4 × 3 × 1 = 48
(6) → 4 × 4 × 3 × 1 = 48
Total ways = 156

Q(vii) Vowels occupy even places "FAVOUR"

= 3 × 3 × 2 × 2 × 1 × 1
Total ways = 36 ways.

* **Permutation:** Permutation is an arrangement of parts of set of objects

- The number of permutation of n objects is $n!$

eg: Arrange 3 books in a shelf.
{Stats, Maths, Physics}

$3! = 3 \cdot 2 \cdot 1 = 6$ ways

$S = \{SMP, SPM, PSM, MSP, MPS, PMS\}$

eg: Arrange 3 books taking 2 at a time in shelf.

$$^3P_2 = \frac{3!}{(3-2)!} = 6 \text{ ways.}$$

- Permutation of n-objects taking r at a time

$$^nP_r = \frac{n!}{(n-r)!}$$

eg: 25 students and 3 awards $25 \times 24 \times 23$

$$^{25}P_3 = \frac{25!}{(25-3)!} = 13800 \text{ ways}$$

eg 10 (a) $^{50}P_2$ (OR) $50 \times 49 = 2450$ ways

(b) $49 + {}^{49}P_2 = 2401$ ways

(c) $2 + {}^{48}P_2 = 2258$ ways.

Tuesday
12/Feb/19

- Circular permutations:

$$(n-1)!$$

- Permutations of n object when they are not all different

$$\frac{n!}{n_1! \cdot n_2! \cdots n_k!}$$

eg: No. of permutations of 9995

$$= \frac{4!}{3!\,1!} = 4 \text{ ways.}$$

eg: In how many ways word STATISTICS can be arranged?

$$= \frac{10!}{3!\,3!\,1!\,2!\,1!} = 50400 \text{ ways}$$

eg: 2 red, 3 blue, and 4 ~~red~~ green chips

$$= \frac{9!}{2!\,3!\,4!} = 1260 \text{ ways}$$

eg: 10 players, 2 Senior, 1 freshman, 4 junior, 3 senior

$$= \frac{10!}{2!\,1!\,4!\,3!} = 12600 \text{ ways}$$

* **Combinations:**

Selection of 'r' objects from 'n' different objects and when the order is not important.

$$\binom{n}{r} = \frac{n!}{r!\,(n-r)!}$$

eg: Committee of 3 students from 4 students.

$$= {}^{4}C_3 = \frac{4!}{3!\,1!} = 4 \text{ ways}$$

eg: 10 boys & 6 girls → Comittee of 3 boys and 2 girls.

$$= {}^{10}C_3 * {}^{6}C_2 = 1800 \text{ ways.}$$

2.22

2.33 5 questions, each with 4 possible answers of which only 1 is correct.

(a) $4 \times 4 \times 4 \times 4 \times 4 = 1024$ ways

(b) $3 \times 3 \times 3 \times 3 \times 3 = 243$ ways

2.37. 5 girls and 4 boys sit alternatively.

Total ways $= 5 \times 4 \times 4 \times 3 \times 3 \times 2 \times 2 \times 1 \times 1$

$= 2880$ ways.

2.45 Distinct permutations from word "INFINITY".

Total ways $= \dfrac{8!}{3! \times 2!}$

$= 3360$ ways.

2.47 3 candidates from 8 equally qualified graduates...?

Total ways $= {}^{8}C_{3} = 56$ ways

2.48 ${}^{365}P_{60}$ ways.

Probability: Measure of the chance that an uncertain event will occur.

* Subjective
  - Personal experiences

* Objective
  - Classical approach
  - Relative frequency approach
  - Axiomatic approach.

→ Classical Approach:

$$P(A) = \text{No. of outcomes in } A / \text{No. of outcomes in } S$$

→ Relative frequency approach:

$$P(A) = n/N \quad \text{(long run)}$$

→ Axiomatic approach:

Assumptions.

→ Mutually exclusive events:

$P_1 + P_2 + P_3 \dots$ $\quad P(A \cup B) = P(A) + P(B)$

eg:1. A coin is tossed twice. What is the probablity that atleast one head occurs

$$S = \{HH, HT, TH, TT\}$$

$$P = 3/4$$

eg2:

(a) $P(\text{even}) = 3/6$

(b) $P(\text{No.} < 3) = 2/6 = 1/3$

(c) $P(\text{No} > 4) = 3/6 = 1/2$

(d) $P(7) = 0$

(e) $P(S) = 1$

eg 3:

eg 4: (a) $P = 2/36 = 1/18$

(b) $P = 6/36$

(c) $P = 0$

eg5:

1(a) Card is a jack:

$P(A) = 4/52 = 1/13$.

(b) Card is not a jack:

$P(A) = 48/52 = 12/13$.

Deck of 52 cards

Spades | hearts | diamond | Clubs

13 cards in each

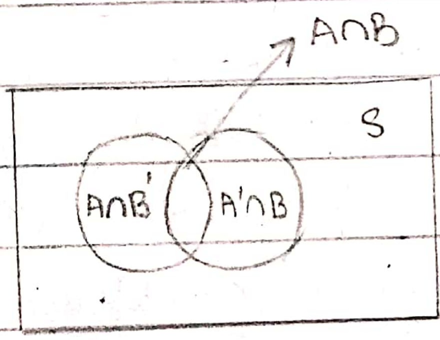
eg 6:

eg 7: (a) $25/53$ (b) $18/53$

→ Non-Mutually Exclusive Events : (More than one event occur).

Additive Rule.

$$P(A\cup B) = P(A) + P(B) - P(A\cap B).$$



$$A\cup B = A\cup(A'\cap B)$$

$$P(A\cup B) = P(A) + P(A'\cap B) \rightarrow (1)$$

$$B = (A\cap B)\cup(A'\cap B)$$

$$P(B) = P(A\cap B) + P(A'\cap B)$$

$$(1) \Rightarrow P(A\cup B) = P(A) + P(B) - P(A\cap B)$$

if A & B are disjoint $P(A\cap B) = 0$ (Mutually Exclusive

$$P(A\cup B\cup C) = P(A) + P(B) + P(C) - P(A\cap B) - P(B\cap C) - P(A\cap C) + P(A\cap B\cap C).$$

eg 8: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 0.8 + 0.6 - 0.5$

$P(A \cup B) = 0.9.$

eg 9: $P(A) = \underbrace{6/36}_{\text{sum}=7} + \underbrace{2/36}_{\text{sum}=11}.$

eg 10: $P(G \cup W \cup U \cup R \cup B) = 0.09 + 0.15 + 0.21 + 0.23.$

eg 11: $P(x \geq 5) = 1 - P(x < 5)$

$= 1 - [0.12 + 0.19]$

eg 12: $1990 \leq \text{length}$ $\quad$ $2010 \geq \text{length}$

(a) $P(\text{too large}) = (1 - 0.99)/2 = 0.005$

(b) $P(x > 1990) = 1 - P(S) = 1 - 0.005 = 0.995$

Exercise :-

2.19 (a) $P(\text{sales}) = 0.19 + 0.38 + 0.29 + 0.15 = 1.01 > 1$

(b) Probability < 1

(c) Negative probability.

(d)

2.51 $S = \{100, 25, 10\}$

$P(x < 100) = (150+275)/500 = 425/500$

$= 0.85$.

2.53 $P(S) = 0.7$

$P(B) = 0.4$

$P(S \cup B) = 0.8$

(a) $P(S \cup B) = P(S) + P(B) - P(S \cap B)$

$0.8 = 0.7 + 0.4 - P(S \cap B)$

$P(S \cap B) = 0.3$

(b) $P(S \cap B)' = 1 - P(S \cup B)$

$= 1 - 0.8$

$= 0.2$

Q $P(B) = 0.25$, $P(T) = 0.18$, $P(F) = 0.17$, $P(O) = 0.40$

(a) $P(B \cup F) = P(B) + P(F) - P(B \cap F)$

$= 0.25 + 0.17 - 0.15$

$= 0.27$

(b) $P(B \cap F)' = 1 - (B \cup F)$

$= 1 - 0.27$

$= 0.73$.

→ Product Rule : Independent Events.

$$P(A \cap B) = P(A) \cdot P(B)$$

→ Independent Events :-

(i) A' and B $= P(A' \cap B) = P(A') . P(B)$
(ii) A and B' $= P(A \cap B') = P(A) . P(B')$
(iii) A' and B' $= P(A' \cap B') = P(A') . P(B')$

- A and B independent ⇒ not mutually exclusive.
- A, B, C independent ⇒ $P(A \cap B \cap C)' = 1 - P(A) \cdot P(B) \cdot P(C)$
- A, B, C independent ⇒ $P(A \cup B \cup C) = 1 - P(A' \cap B' \cap C')$.

eg#13 $P(\text{Fire Engine}) \cdot P(\text{Ambulance}) = (0.92)(0.98) = 0.90$.

eg#14 $P(\text{red}) = 5$, $P(\text{black}) = 7$, Total $= 12$
$P(\text{red} \cap \text{black}) = 5/12 \times 7/12 = 35/144$

eg#15 $P(A) = 6/36 \times 18/36 = 1/12$

eg#16 $P(A) = 0.4$, $P(B) = 0.8$
(a) $P(A \cap B) = P(A) \cdot P(B) = 0.4 \times 0.8 = 0.32$
(b) $P(A') \cdot P(B') = (1 - 0.4)(1 - 0.8) = 0.12$
(c) $P(A) \cdot P(B') = 0.4(1 - 0.8) = 0.08$

$\because P(B|A) = P(B)$
B & A are independent.

eg# 17

(a) $(1-0.96)^2 = 0.0016$

(b) $(1-0.0016)^2 = 0.9968$

→ Product Rule: Dependent Events.

$P(A \cap B) = P(A) \cdot P(B|A)$ (Conditional probabil

$\because P(A) > 0$

$\because P(B|A) = \dfrac{P(B \cap A)}{P(A)}$ $\because P(A|B) = \dfrac{P(B \cap A)}{P(B)}$

eg#18 $P(F_1 \cap F_2) = P(F_1) \cdot P(F_2|F_1)$

$= P(F_1) \cdot \dfrac{P(F_2 \cap F_1)}{P(F_1)}$

$= 5/20 \cdot 4/19$

$=$

eg#19 $P(S_1 \cap S_2) = 13/52 \cdot 12/51$

eg# 20 $= 5/8 \cdot 4/7$

eg# 21

18/Feb/19

(52)

eg # 22

$$P(6|3) = \frac{P(6 \cap 3)}{P(3)}$$

$$= \frac{2/36}{11/36} = \frac{2}{3}$$

2.76

	Non-smokers	Moderate	Heavy	Total
H	21	36	30	87
NH	48	26	19	93
Total	69	62	49	180

(a)

$$P(H|HS) = P(H \cap HS)/P(HS)$$

$$= \frac{30/18}{49/18}$$

$$= 30/49$$

(b) $P(NS|NH) = P(NS \cap NH)/P(NH)$

$$= \frac{48/180}{93/180}$$

$$= 48/93.$$

q1 (a) $P(Q_1 \cap Q_2 \cap Q_3 \cap Q_4) = P(Q)_1 \, P(Q_2 | Q_1) \cdot P(Q_3 | Q_1 \cap Q_2)$
$P(Q_4 | Q_1 \cap Q_2 \cap Q_3)$.

$$= \frac{15}{20} \cdot \frac{14}{19} \cdot \frac{13}{18} \cdot \frac{12}{17}.$$

$$= 91/323.$$

b)

$$\frac{^{15}C_4}{^{20}C_4} = \frac{91}{323}.$$

$n = {^{15}C_4}$ $\quad N = {^{20}C_4}$.

$$P(A) = n/N = 91/323.$$

q#23 $P(d \cup f \cup K) = P(d) + P(f) + P(K) - P(d \cap f)$
$- P(d \cap K) - P(f \cap k) + P(d \cap f \cap$

$$= \frac{13}{52} + \frac{12}{52} + \frac{4}{52} - \frac{3}{52} - \frac{1}{52} - \frac{4}{52} + \frac{1}{52}.$$

q#24 (i) the sum is odd.

$$P(A | B) = P(A \cap B)/P(B)$$

$$= 6|36 / 18|36$$

$$= 6/18.$$

(ii) the sum is greater than 6.

$\Rightarrow P(A|B) = 6/36 / 21/36$

$= 6/21$

(iii) two dice had same outcome.

$\Rightarrow P(A|D) = 0$.

eg#25 $S = \{HH, HT, TH, TT\}$

(i) $P(H|A) = \frac{P(H \cap A)}{P(A)} = \frac{1/4}{3/4} = 1/3$.

(ii)

(iii) $P(A \cap B) = P(A) \cdot P(B|A)$

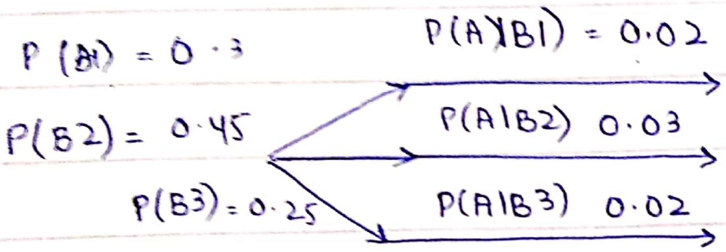
g#x

$P(A) = P(A\cap B_1) + P(A\cap B_2) + P(A\cap B_3)$

$= P(A)P(B_1|A) + P(A)\cdot P(B_2|A) + P(A)\cdot P(B_3|A).$

$= (0.02)(0.30) + (0.03)(0.45) + (0.02)(0.25).$

Total Prob $= P(A) = 0.0245.$



Law of Total Probability:-

$S = A_1 \cup A_2 \cup \cdots \cup A_n.$

$P(B) = P(B\cap A_1) + P(B\cap A_2) + \cdots + P(B\cap A_i)$

$= \sum_{i=1}^{n} P(A_i \cap B)$

Baye Rule:

$P(A_i|B) = \dfrac{P(A_i\cap B)}{\sum_{i=1} P(A_i\cap B)}$ (Expansion of conditional prob eg# 27

$$P(B_3|d) = \frac{P(B_3 \cap d)}{P(d)} \qquad \because d = A$$

$$= \frac{0.005}{0.0245}$$

$$= 0.2040 \text{ Ans.}$$

eg# 28

$$P(\text{Male}|\text{Diploma}) = \frac{P(M \cap D)}{P(D)} = \frac{2/20}{5/20} = \frac{2}{5} \text{ Ans.}$$

$$P(D) = P(M \cap D) + P(F \cap D)$$
$$= 2/20 + 3/20$$
$$= 5/20$$

eg# 29 25 coloured balls, 3 bags.

(a) Ball is yellow

$$P(Y) = P(B_1 \cap Y) + P(B_2 \cap Y) + P(B_3 \cap Y)$$
$$= P(B_1)P(Y|B_1) + P(B_2)P(Y|B_2) + P(B_3)P(Y|B_3).$$
$$= \frac{1}{3} \cdot \frac{2}{7} + \frac{1}{3} \cdot \frac{2}{10} + \frac{1}{3} \cdot \frac{3}{8}$$
$$P(Y) = 0.286.$$

(b) Ball is yellow, bag 2 selected.

$$P(B_2|Y) = \frac{P(B_2 \cap Y)}{P(Y)} = \frac{2/30}{0.286} = 0.233$$

(i) $P(C|H) = P(C \cap H)/P(H)$

$$= \frac{5/43}{19/43} = 5/19$$