$K_3 = 0.5 \times f(0.75, 1.35365) = \underline{0.27370}$

$K_4 = 0.5 \times f(1, 1.21411 + 0.27370)$
$= 0.5 \times (1, 1.48781) = \underline{0.30708}$

$\boxed{w_2} = 1.21411 + \frac{1}{6}(\,\,) = \boxed{1.49044}$

• Solve System of Linear Equations.

Monday • 9/5/22

L.U decomposition Method

$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = 0.$
$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = 0$
$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = 0,$

1) $Ax = b$ (2) $A = LU$

$L = \begin{pmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{pmatrix}, \quad U = \begin{pmatrix} 1 & u_{12} & u_{13} \\ 0 & 1 & u_{23} \\ 0 & 0 & 1 \end{pmatrix}$

2) $y = Ux$ 1) $Ly = b.$

Q1) Solve

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & -4 & 6 \\ 3 & -9 & -3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 5 \\ 18 \\ 6 \end{bmatrix}. \qquad A \cdot x = b.$$

(2) $A = LU.$

$$\begin{pmatrix} 1 & 2 & 3 \\ 2 & -4 & 6 \\ 3 & -9 & -3 \end{pmatrix} = \begin{pmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{pmatrix}\begin{pmatrix} 1 & u_{12} & u_{13} \\ 0 & 1 & u_{23} \\ 0 & 0 & 1 \end{pmatrix}$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & -4 & 6 \\ 3 & -9 & -3 \end{bmatrix} = \begin{bmatrix} l_{11}+0+0 & l_{11}u_{12}+0+0 & l_{11}u_{13}+0+0 \\ l_{21}+0+0 & l_{21}u_{12}+l_{22}+0 & l_{21}u_{13}+l_{22}u_{23}+0 \\ l_{31}+0+0 & l_{31}u_{12}+l_{32}+0 & l_{31}u_{13}+l_{32}u_{23}+l_{33} \end{bmatrix}$$

1) $1 = l_{11}$, $2 = l_{11}u_{12}$, $\boxed{u_{12}=2}$.

$3 = l_{11}u_{13} \rightarrow \boxed{u_{13}=3}$.

2) $\boxed{l_{21}=2}$ $l_{21}u_{12}+l_{22}=-4$

$2\times 2 + l_{22} = -4$, $\boxed{l_{22}=-8}$.

3) $l_{21}u_{13}+l_{22}u_{23}=6$ $\rightarrow 2\times 3+(-8)u_{23}=6$.

$6+u_{23}(-8)=6$.

$\boxed{u_{23}=0}$

$\boxed{l_{31}=3}$

$l_{31}u_{12}+l_{32}=-9$

$3\times 2+l_{32}=-9 \rightarrow \boxed{l_{32}=-15}$

$l_{31}u_{13}+l_{32}u_{23}+l_{33}=-3$

$3\times 3+0+l_{33}=-3 \rightarrow \boxed{l_{33}=-12}$

- $L=\begin{bmatrix} 1 & 0 & 0 \\ 2 & -8 & 0 \\ 3 & -15 & -12 \end{bmatrix}$ $U=\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(3) $Ly=b$. $\begin{bmatrix} 1 & 0 & 0 \\ 2 & -8 & 0 \\ 3 & -15 & -12 \end{bmatrix}\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}=\begin{bmatrix} 5 \\ 18 \\ 6 \end{bmatrix}$

$y_1=5$

$y_2=-1$, $\boxed{y_3=2.}$

(4) $y=Ux$ $\begin{bmatrix} 5 \\ -1 \\ 2 \end{bmatrix}=\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\cdot\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$.

S.S $x_1=1$

$x_2=-1$, $x_3=2$.

Q#2

$$x - 3y + z = 4$$
$$2x - 8y + 8z = -2$$
$$-6x + 3y - 15z = 9.$$

$$\begin{pmatrix} 1 & -3 & 1 \\ 2 & -8 & 8 \\ -6 & 3 & -15 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 4 \\ -2 \\ 9 \end{pmatrix}$$

$$A \cdot x = b.$$

$$\begin{bmatrix} 1 & -3 & 1 \\ 2 & -8 & 8 \\ -6 & 3 & -15 \end{bmatrix} = \begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix} \begin{bmatrix} 1 & u_{12} & u_{13} \\ 0 & 1 & u_{23} \\ 0 & 0 & 1 \end{bmatrix}.$$

$\boxed{l_{11} = 1}$. $l_{11} u_{12} = -3$, $\boxed{u_{12} = -3.}$ $l_{11} u_{13} = 1.$

$\boxed{u_{13} = 1}$

$\boxed{l_{21} = 2}$. $l_{21} u_{12} + l_{22} = -8.$

$2 \times (-3) + l_{22} = -8$, $\boxed{l_{22} = -2.}$

$l_{21} u_{13} + l_{22} u_{23} = 8 \rightarrow 2 \times 1 + (-2)(u_{23}) = 8.$

$2 - 2 \times u_{23} = 8.$

$\boxed{u_{23} = -3}$

$\boxed{l_{31} = -6}$

$l_{31} u_{12} + l_{32} = 3.$

$-6 \times (-3) + l_{32} = 3.$, $\boxed{l_{32} = -15}$

$l_{31} u_{13} + l_{32} u_{23} + l_{33} = -15.$

$-6 \times 1 + \underset{-15}{\cancel{21}} \times (-3) + l_{33} = -15$

$-6 + 45 + l_{33} = -15 \rightarrow \boxed{l_{33} = -54}$

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10/May/22 **Q1 solve** Tuesday

$$\begin{bmatrix} 2 & 0 & 0 \\ -1 & 1 & 0 \\ 3 & 2 & -1 \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -1 \\ 3 \\ 0 \end{bmatrix}$$

$L.U. x = b$

(3) $Ly = b$. (4) $y = U.x$.

$$\begin{bmatrix} 2+0+0 & 2+0+0 & 2+0+0 \\ -1+0+0 & -1+1+0 & -1+2+0-1 \\ 3+0+0 & 3+2+0 & 3+4-1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -1 \\ 3 \\ 0 \end{bmatrix}.$$

$$\begin{bmatrix} 2 & 2 & 2 \\ -1 & 0 & 0 \\ 3 & 5 & 6 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$ x not required

- 1st 2 steps are not required.

- (3) $$\begin{bmatrix} 2 & 0 & 0 \\ -1 & 1 & 0 \\ 3 & 2 & -1 \end{bmatrix}\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} -1 \\ 3 \\ 0 \end{bmatrix}.$$

$$\begin{bmatrix} 2y_1 + 0 + 0 \\ -y_1 + y_2 + 0 \\ 3y_1 + 2y_2 - 0 - y_3 \end{bmatrix} = \begin{bmatrix} -1 \\ 3 \\ 0 \end{bmatrix}$$

$2y_1 = -1$

$\boxed{y_1 = -1/2}$

$y_2 - y_1 = 3$

$y_2 = 3 - 1/2 = \boxed{5/2}$

$3 \times \left(\frac{-1}{2}\right) + 2 \times \frac{5}{2} - y_3 = 0$

$-3/2 + 5 - y_3 = 0$ $\boxed{y_3 = 7/2}$

$\frac{1}{16} \cdot 4 + \left(\frac{1}{11/2}\right) \cdot \frac{11}{4} + d_3 = 2$ for d_3.

$\boxed{d_3 = 2 - \frac{3}{11}} = \boxed{19/11}$

$$\begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

$$\begin{pmatrix} x_1 + x_2 + x_3 \\ 0 + x_2 + x_3 \\ 0 + 0 + x_3 \end{pmatrix} = \begin{pmatrix} -1/2 \\ 5/2 \\ 7/2 \end{pmatrix}$$

$\boxed{x_3 = 7/2}$

$x_2 = -2 + \frac{1}{2} + 7/2$

$\boxed{x_2}$

$x_2 = -1 + \frac{5}{2}$

$\boxed{x_2 = -3/2}$

$x_1 = -\frac{9}{2} + \frac{7}{2} = -\frac{1}{2}$

$\boxed{\cancel{x_3 = 7/2}}$

$x_1 = -\frac{1}{2} = -1/2$, $x_1 = -\frac{1}{2} + 1$

$\boxed{x_1 = \frac{1}{2}}$

Q2) Cholesky LDL^t factorization

$$L = \begin{pmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{pmatrix}, \quad D = \begin{pmatrix} d_1 & 0 & 0 \\ 0 & d_2 & 0 \\ 0 & 0 & d_3 \end{pmatrix}$$

$A = LDL^t$

$$\begin{pmatrix} A \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{pmatrix} \begin{pmatrix} d_1 & 0 & 0 \\ 0 & d_2 & 0 \\ 0 & 0 & d_3 \end{pmatrix} \begin{pmatrix} 1 & l_{21} & l_{31} \\ 0 & 1 & l_{32} \\ 0 & 0 & 1 \end{pmatrix}$$

$$A = \begin{pmatrix} d_1 & 0 & 0 \\ d_1 l_{21} & d_2 & 0 \\ l_{31} d_1 & l_{32} d_2 & d_3 \end{pmatrix} \begin{pmatrix} 1 & l_{21} & l_{31} \\ 0 & 1 & l_{32} \\ 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 4 & -1 & 1 \\ -1 & 3 & 0 \\ 1 & 0 & 2 \end{pmatrix} = \begin{pmatrix} d_1 & d_1 l_{21} & d_1 l_{31} \\ d_1 l_{21} & d_1 l_{21}^2 + d_2 & d_1 l_{21} l_{31} + d_2 l_{32} \\ l_{31} d_1 & l_{31} d_1 l_{21} + l_{32} d_2 & l_{31}^2 d_1 + l_{32}^2 d_2 + d_3 \end{pmatrix}$$

$l_{32} = \frac{1}{4} \cdot \frac{4}{11}$

$\boxed{l_{32} = 1/11}$

$\underline{d_1 = 4}$, $d_1 l_{21} = -1$, $\boxed{l_{21} = -1/4}$

$d_1 l_{21} = -1$

$\boxed{l_{21} = -1/4}$

$d_1 l_{31} = 1$, $\boxed{l_{31} = 1/4}$

$l_{31} d_1 l_{21} + l_{32} d_2 = 0$

$d_1 l_{21}^2 + d_2 = 3$

$\frac{1}{4} \cdot 4 \cdot (-1/4) + l_{32} \cdot \frac{11}{4} = 0$

$4 \times \frac{1}{16} + d_2 = 3 \rightarrow d_2 = 3 - \frac{1}{4}$

$-\frac{1}{4} + l_{32} \cdot \frac{11}{4} = 0$

$\boxed{d_2 = 11/4}$

Monday. **Jacobi's Method** 16/may/22

Q1) Solve eq.

(1) $10x_1 - x_2 + 2x_3 = 6$ — (i)

(2) $-x_1 + 11x_2 - x_3 + 3x_4 = 25$ — (2)

(3) $2x_1 - x_2 + 10x_3 - x_4 = -11$

(4) $3x_2 - x_3 + 8x_4 = 15$.

(1) $\left(x_1 = \dfrac{6 + x_2 - 2x_3}{10}\right) = 0.6.$

; $x_1 = x_2 = x_3 = x_4 = 0$. initial subject

(2) $\left(x_2 = \dfrac{+25 + x_1 + x_3 - 3x_4}{11}\right)$

(3) $x_3 = \dfrac{-11 + x_2 - 2x_1 + x_4}{10}$.

(4) $x_4 = \dfrac{15 - 3x_2 + x_3}{8}$.

$x_2 = 2.2727$, $x_3 = -1.1$, $x_4 = 1.875$.

for 2nd iteration

$\boxed{x_1 = 1.04727}$ $\boxed{x_2 = 1.7159}$

$\boxed{x_3 = -0.80523}$ $\boxed{x_4 = 0.88}$

3rd it
0.93
2.05
-1.04
1.13

find relative error.

	error 2nd-1st iter	3rd-2nd
x_1	0.44727	-0.11727
x_2	-0.55679	0.33409
x_3	0.29477	-0.23477
x_4	-0.99500	0.25

Solve $AX=b$ for system of linear eq.

$$\begin{bmatrix} 1 & 1 & 5 \\ -3 & -6 & 2 \\ 10 & 2 & -1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -21.5 \\ -61.5 \\ 27 \end{bmatrix}.$$

initial value.

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

∴ Sol:-

$x_1 + x_2 + 5x_3 = -21.5$ — (1

$-3x_1 - 6x_2 + 2x_3 = -61.5$ — (2

$10x_1 + 2x_2 - x_3 = 27$ — (3.

Diagonal Matrix

$$A = \begin{bmatrix} 10 & 2 & -1 \\ -3 & -6 & 2 \\ 1 & 1 & 5 \end{bmatrix}$$

$10 > |2| + |-1| = 10 > 3$.

$~~|-3| + |-6|~~ +$

$|-6| > |-3| + |2| = 5$.

$6 > 5$.

$|5| > |1| + |1| = 5 > 2$.

- Take subject x_1 from eq 1.
- " " x_2 " 2.
- " " x_3 " 3.

- It is Seidel's Method.

Sol,

$x_1 + x_2 + 5x_3 = -21.5. \rightarrow x_3$

$-3x_1 - 6x_2 + 2x_3 = -61.5 \rightarrow x_2$

$10x_1 + 2x_2 - x_3 = 27. \rightarrow x_1$

$x_1 = -21.5 - x_2 - 5x_3$. , $\boxed{x_1 = -21.5}$

$x_2 = -\left(\dfrac{-61.5 + 3x_1 - 2x_3}{6}\right)$

$\boxed{x_2 = +21.0}$

$x_3 = 10x_1 + 2x_2 - 27$.

$\boxed{x_3 = -200}$

1st iterative tech

2nd $x_1 = -21.5 - 21 - 5(-284)$

$\boxed{x_1 = ~~1149.5~~}$ 957.5 $x_2 =$

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Tuesday. Eigenvalues & vector. 17/May/22.
by power method.

Q1) eigen values & vector

$A = \begin{bmatrix} 2 & -12 \\ 1 & -5 \end{bmatrix}$. $\underline{\underline{x_n = A x_{n-1}}}$ $x_0 = \begin{bmatrix} 1 \\ +1 \end{bmatrix}$.

↓
power method

for $n=1$

$x_1 = A x_0$.

$$\begin{bmatrix} 2 & -12 \\ 1 & -5 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -10 \\ -4 \end{bmatrix} = -4\begin{bmatrix} +5/2 \\ 1 \end{bmatrix}.$$

eigen value $= -4$, vector $\begin{bmatrix} 2.5 \\ 1 \end{bmatrix}$.

$$X_2 = \begin{bmatrix} 2 & -12 \\ 1 & -5 \end{bmatrix}\begin{bmatrix} \cancel{-10} \\ -4 \end{bmatrix} = \begin{bmatrix} \cancel{-8} \\ \cancel{-23} \end{bmatrix} \begin{bmatrix} -20+48 \\ -10+20 \end{bmatrix}$$

$$\begin{bmatrix} 28 \\ 10 \end{bmatrix} \rightarrow 2\begin{bmatrix} 14 \\ 5 \end{bmatrix} \rightarrow \text{vector}.$$

↓
value

$$X3 = \begin{bmatrix} 2 & -12 \\ 1 & -5 \end{bmatrix}\begin{bmatrix} 28 \\ 10 \end{bmatrix} \rightarrow \begin{bmatrix} 56 - 120 \\ 28 - 50 \end{bmatrix} = \begin{bmatrix} -64 \\ -22 \end{bmatrix}$$

2x1

Q2. $A = \begin{bmatrix} +1 & 2 & 0 \\ -2 & 1 & 2 \\ 1 & 3 & 1 \end{bmatrix}$ $x_0 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$.

$$X_1 = \begin{bmatrix} 1 & 2 & 0 \\ -2 & 1 & 2 \\ 1 & 3 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1+2+0 \\ -2+1+2 \\ 1+3+1 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 5 \end{bmatrix}.$$

$$X_2 = \begin{bmatrix} 1 & 2 & 0 \\ -2 & 1 & 2 \\ 1 & 3 & 1 \end{bmatrix}\begin{bmatrix} 3 \\ 1 \\ 5 \end{bmatrix} \rightarrow \begin{bmatrix} 3+2+0 \\ -6+1+10 \\ 3+3+5 \end{bmatrix} = \begin{bmatrix} 5 \\ 5 \\ 11 \end{bmatrix} \rightarrow \begin{bmatrix} 0.45 \\ 0.45 \\ 1 \end{bmatrix}$$

$$X_3 = \begin{bmatrix} 1 & 2 & 0 \\ -2 & 1 & 2 \\ 1 & 3 & 1 \end{bmatrix} \begin{bmatrix} 5 \\ 5 \\ 11 \end{bmatrix} = \begin{bmatrix} 5+10+0 \\ -10+5+22 \\ 5+15+11 \end{bmatrix} = \begin{bmatrix} 15 \\ 17 \\ 21 \end{bmatrix}$$

$$= 21 \begin{bmatrix} 0.71 \\ 0.80 \\ \cancel{0.21} \; 01 \end{bmatrix}$$

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