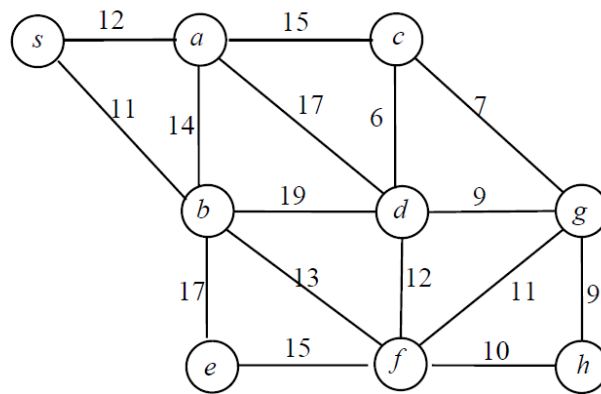
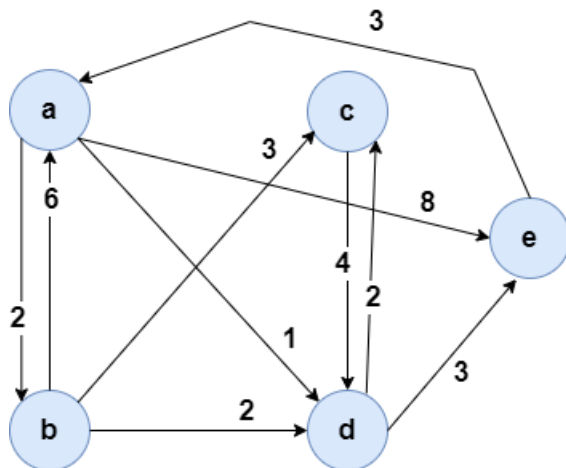


- 1 of 3

3. Use Prim's algorithm to compute a minimum spanning tree for the following graph. Use node  $s$  as the root. You need to show each step. [10 Points]

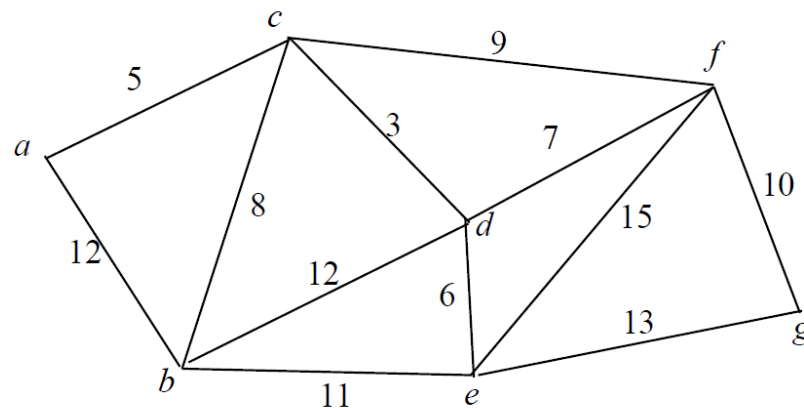


4. Using Floyd-Warshall, find all pairs shortest path following Figure ( $D^0$  weight matrix is also provided). Discuss the its complexity as well [10 Points]



$$\begin{bmatrix} 0 & 2 & \infty & 1 & 8 \\ 6 & 0 & 3 & 2 & \infty \\ \infty & \infty & 0 & 4 & \infty \\ \infty & \infty & 2 & 0 & 3 \\ 3 & \infty & \infty & \infty & 0 \end{bmatrix}$$

5. Go through the lecture <https://www.youtube.com/watch?v=2E7MmKv0Y24> and write summary with focus on proof of Dijkstra Algorithm [10 Points]
6. (Widest path problem) Given a weighted (undirected) graph  $G(V, E)$ , the weight of an edge is called the width of the edge. The width of a path is defined to be the smallest weight among all edges on the path. (An edge with the smallest weight is called the bottle neck edge.) For example, in the following graph, the path  $\langle a, c, f, g \rangle$  has width 5.



A path  $P(u, v)$  is called the widest if the width of the path is the largest among all paths from  $u$  to  $v$ . For example, in the above figure, the widest path between vertex  $a$  and vertex  $g$  is  $\langle a, b, e, g \rangle$  whose width is 11.

- Modify the Dijkstra's algorithm to compute the widest path from a given vertex  $s \in V$  to every other vertex. The pseudo code is required. [10 Points]
  - For the above graph, use the algorithm in part (a) to compute the widest path from source node  $a$  to each and every other node. You need to show each step, including the initialization step. Also show the final widest path tree. [10 Points]
7. Read Book, Lectures or Search through Internet to explain the worst time complexities of BFS, DFS, Kruskal's and Prims in your own words using (i) Adjacent Matrix (ii) Adjacent List [10 Points]