Design and Analysis of Algorithms Introduction to graphs, representations of a graph

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From
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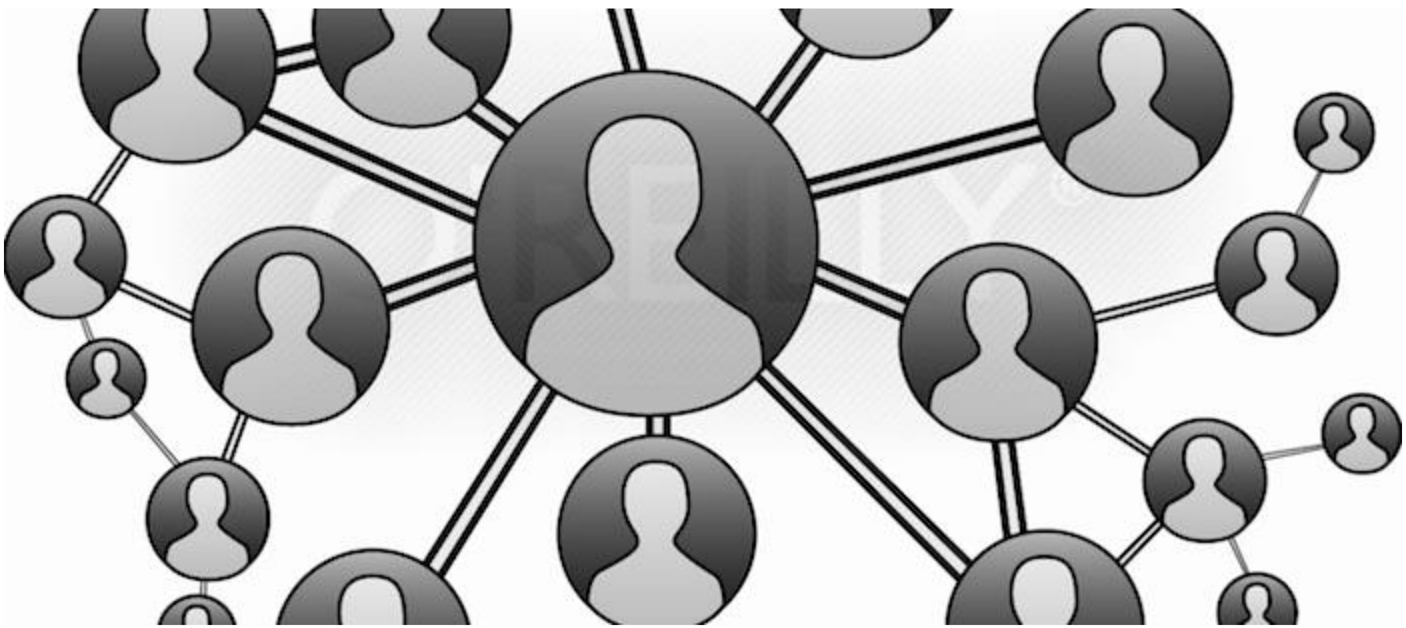
Why graphs are important

- Graphs are used a lot in computer science
  - Social network (face book, linked in)

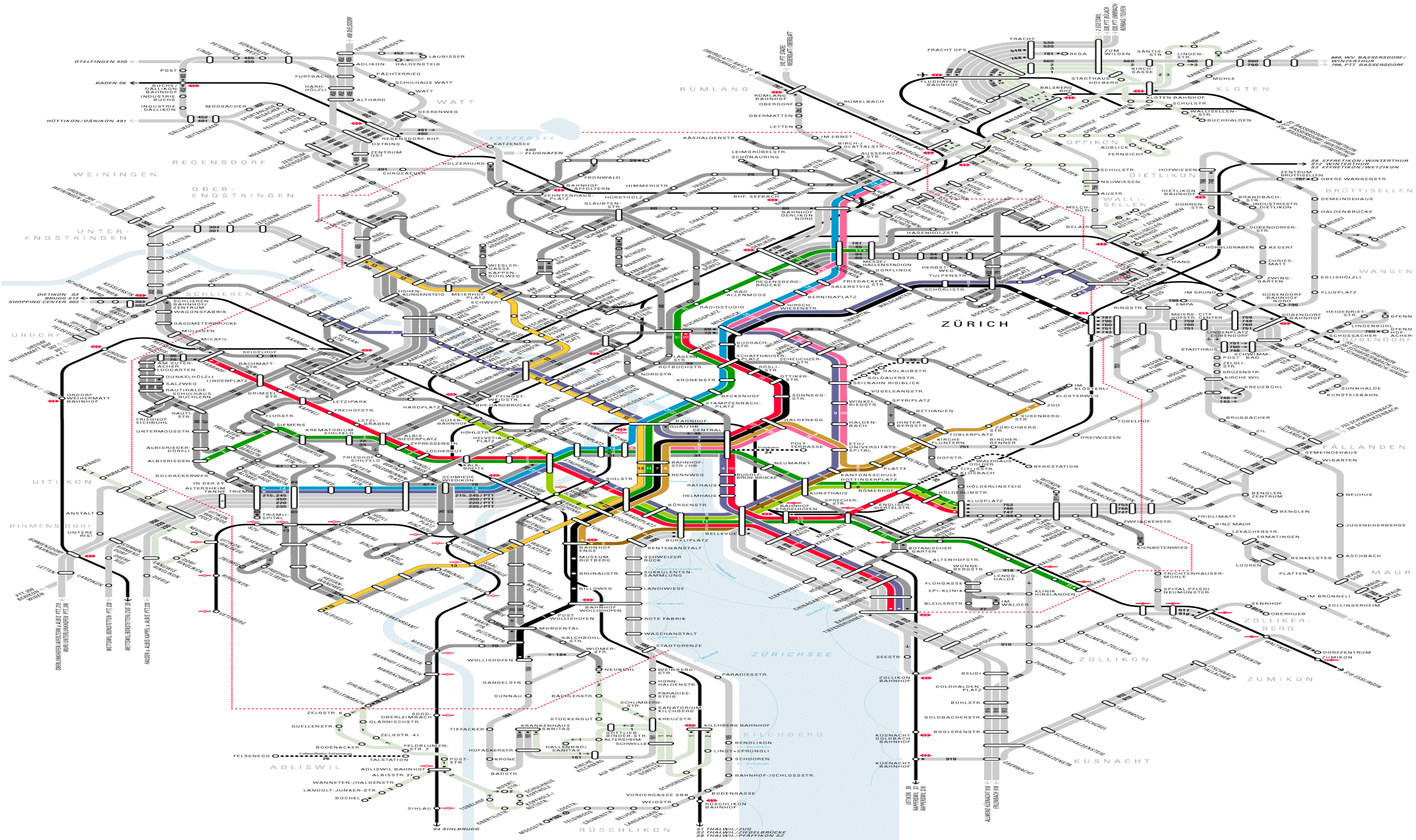


Why graphs are important

- Graphs are used a lot in computer science
  - Computer networks

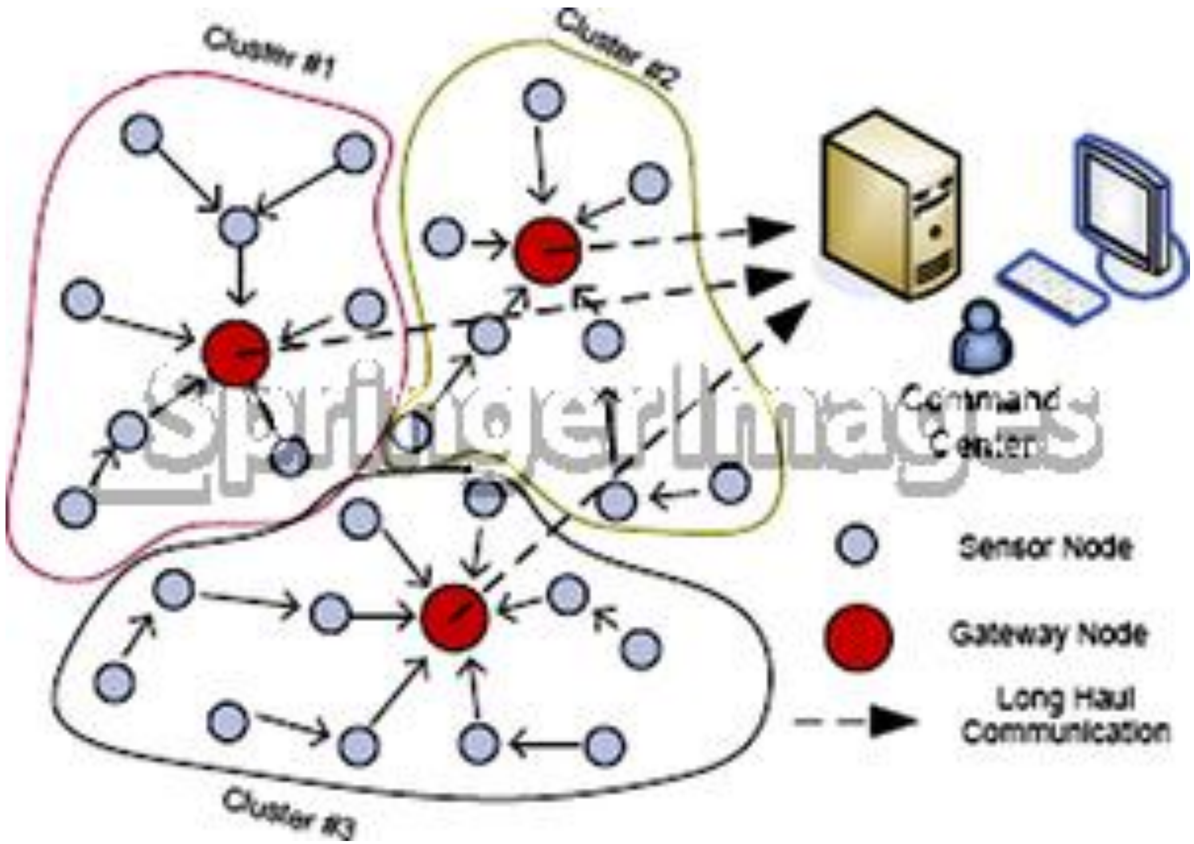
Why graphs are important

- Graphs are used a lot in computer science
  - Transportation network



Why graphs are important

- Graphs are used a lot in computer science
  - Wireless sensors



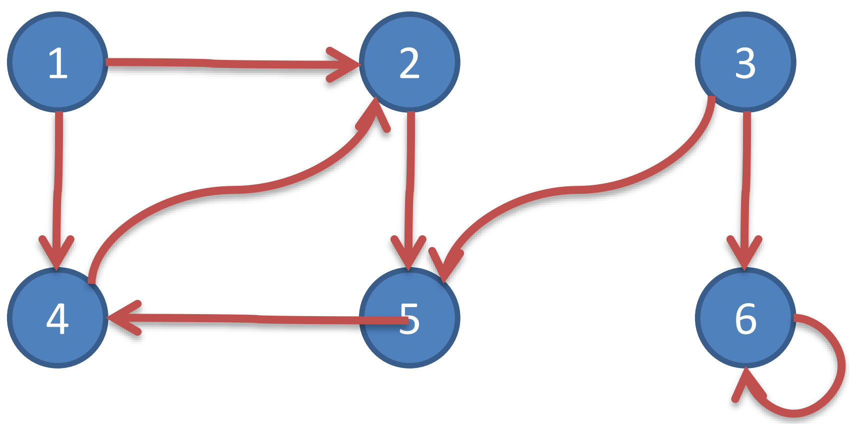
What are graphs?

- A set of vertices V and a set of edges E
- Each edge is a ordered pair of two vertices
- G = (V, E)
- E.g.
  - V = {1, 2, 3, 4, 5, 6}
  - E = {<1, 2>, <1, 4>, <2, 5>, <3, 6>, <3, 5>, <4, 2>, <5, 4>, <6, 6>}

What are graphs?

V = {1, 2, 3, 4, 5, 6}

E = {<1, 2>, <1, 4>, <2, 5>, <3, 6>, <3, 5>, <4, 2>, <5, 4>, <6, 6>}



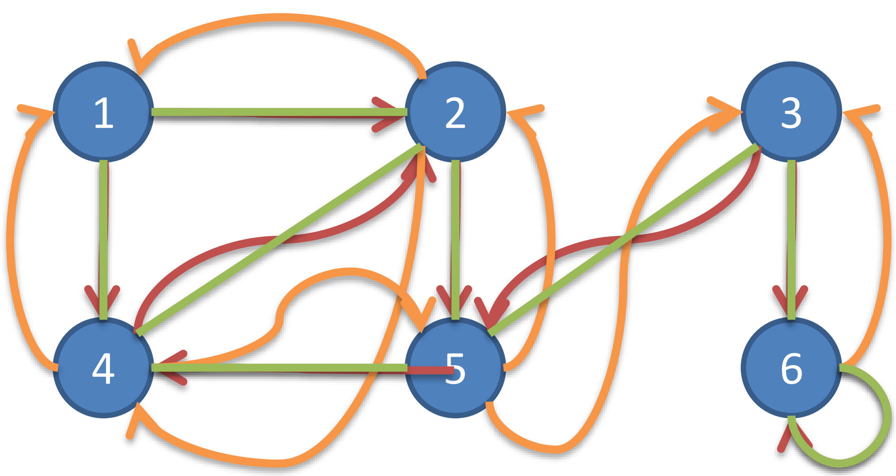
What are graphs?

- Undirected graph
  - A special graph
  - If $v_i \neq v_j < v_i, v_j > \in \mathrm{E}, < v_j, v_i > \in \mathrm{E}$
  - E.g.

V = {1, 2, 3, 4, 5, 6}

E = {<1, 2>, <1, 4>, <2, 5>, <3, 6>, <3, 5>, <4, 2>, <5, 4>, <6, 6>}

U {<2, 1>, <4, 1>, <5, 2>, <6, 3>, <5, 3>, <2, 4>, <4, 5>}



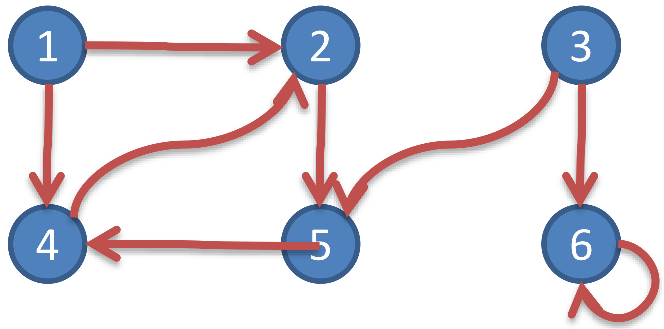
Use a undirected line to indicate a pair of edges or a self edge

How to represent a graph?

- Adjacency-matrix
  - A $|V| \times |V|$ matrix
  - if $< v_i, v_j > \in \mathrm{E}$, the matrix element $a_{ij} = 1$
  - if$< v_i, v_j > \notin \mathrm{E}$, the matrix element $a_{ij} = 0$
- E.g. …

How to represent a graph?

- E.g.



	1	2	3	4	5	6
1	0	1	0	1	0	0
2	0	0	0	0	1	0
3	0	0	0	0	1	1
4	0	1	0	0	0	0
5	0	0	0	1	0	0
6	0	0	0	0	0	1

What is the space complexity? S(n) = $\Theta(|V|^2)$

How to represent a graph?

- Adjacency-matrix
  - Advantage
    - Simple
    - For some operations, it is efficient, e.g.: isConnected($< v_i, v_j >$)
  - Disadvantage
    - When |E| is small, $\Theta(|V|^2)$ is a waste of space

Can we improve the space complexity?

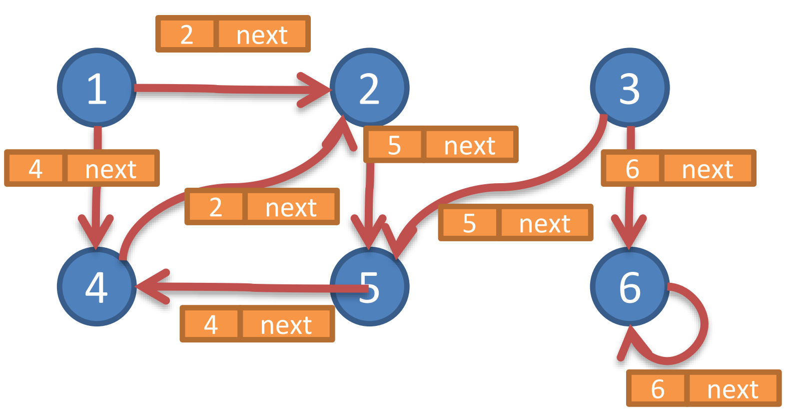
	1	2	3	4	5	6
1	0	1	0	1	0	0
2	0	0	0	0	1	0
3	0	0	0	0	1	1
4	0	1	0	0	0	0
5	0	0	0	1	0	0
6	0	0	0	0	0	1

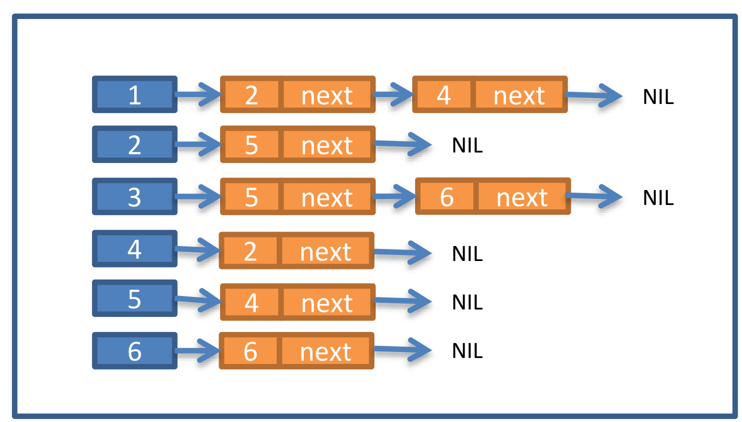
How to represent a graph?

- Adjacency-list
  - A arc adjacency list(Adj[]) with a size of |V|
  - After each element in the adjacency list, there is a arc node list
  - A arc node is:
    - {Destination vertex;
    - Next arc node}
  - For each $< v_i, v_j > \in \mathrm{E}$, there is a arc node:
    - In the arc node list of Adj[i]
    - With the destination vertex j

How to represent a graph?

- Adjacency-list
  - An adjacency list with size of |V|
  - After each arc head, there is a arc listFor each $< v_i, v_j > \in \mathrm{E}$, there is a arc node.





Adjacency-list