MULTIVARIABLE

CALCULUS (MT2008)

ASSIGNMENT # 01

"PARTIAL DERIVATIVES"

ASHMAL ANIS

19K-0305

SEC:- H

DATE OF SUBMISSION:-

EXERCISE # 13.2

Question # 33 :-

$$f(x,y) = \frac{x^2y}{x^4+y^2}$$

(a) Does $f(x,y)$ has a limit $(x,y) \to (0,0)$?

Limit does not exist because there seem to be points near with $z=0$ and other points near $(0,0)$ with $z=\frac{1}{2}$

(b) $f(x,y) \to 0$ along line $y=mx$.

$$\lim_{(x,y)\to(0,0)} f(x,y)$$

$$\lim_{(x,y)\to(0,0)} \frac{x^2y}{y^2+x^4}$$

$$y = mx$$

$$\lim_{(x,y)\to(0,0)} \frac{x^2(mx)}{m^2x^2+x^4}$$

$$\cancel{\lim_{x\to0} \frac{m(0)^3}{m^2(0)+}}$$

$$\lim_{x\to0} \frac{m(x^3)}{x^2(m^2+x^2)}$$

$$\lim_{x\to0} \frac{mx}{m^2+x^2}$$

$$\lim_{x\to0} \frac{m(0)}{m^2+x^2}$$

$$=0$$

(c) $f(x,y) \to \frac{1}{2}$, $y = x^2$

$$\lim_{(x,y)\to(0,0)} \frac{x^2 y}{y^2 + x^4}$$

substitute $y = x^2$

$$\lim_{x\to 0} \frac{x^2(x^2)}{x^4 + x^4}$$

$$\lim_{x\to 0} \frac{\cancel{x^4}}{2x^4}$$

$$= \frac{1}{2}$$

(d) does limit exists?

Limits must be unique and values must be same, thus limit does not exists.

Question #35)

(a) $\frac{xyz}{x^2 + y^4 + z^4}$ $(x,y,z) \to (0,0,0)$ $x = at$ $y = bt$ $z = ct$

$$\lim_{(x,y,z)\to(0,0,0)} \frac{xyz}{x^2 + y^4 + z^4} = \lim_{t\to 0} \frac{(at)(bt)(ct)}{(at)^2 + (bt)^4 + (ct)^4}$$

$$\lim_{t\to 0} \frac{abct^3}{a^2t^2 + b^4t^4 + c^4t^4} = abc$$

$$\lim_{t\to 0} \frac{\cancel{t^2}(abct)}{\cancel{t^2}(a^2 + b^4t^2 + c^4t^2)} = \lim_{t\to 0} \frac{abct}{a^2 + b^4t^2 + c^4t^2}$$

$$\frac{abc(0)}{a^2 + 0 + 0} = 0$$

(b) $\lim\limits_{(x,y,z)\to(0,0,0)} \dfrac{xyz}{x^2+y^4+z^4}$ $\quad x=t^2, y=t, z=t$

$\lim\limits_{(x,y,z)\to(0,0,0)} \dfrac{xyz}{x^2+y^4+z^4} = \lim\limits_{t\to 0} \dfrac{(t^2)(t)(t)}{t^2+t^4+t^4}$

$\lim\limits_{t\to 0} \dfrac{t^4}{t^2(1+t^2+t^2)} = \lim\limits_{t\to 0} \dfrac{t^2}{1+t^2+t^2}$

$= \dfrac{0}{1+0+0} = 0$

Question #40

$f(x) = \begin{cases} x^2+7y^2, & \text{if } (x,y) \neq (0,0) \\ -4 & \text{if } (x,y) = (0,0) \end{cases}$

$\lim\limits_{(x,y)\to(0,0)} f(x,y)$ must exist

(1) condition #1:

Along x-axis:- $y=0$

$\lim\limits_{(x,y)\to(x,0)} = \lim\limits_{x\to 0} x^2 = (0)^2 = 0$

Along y-axis:- $x=0$

$\lim\limits_{(x,y)\to(0,y)} = \lim\limits_{y\to 0} 7y^2 = 7(0)^2 = 0$

Along line $y=x$:

$\lim\limits_{(x,y)\to(x,x)} x^2+7x^2 = \lim\limits_{(x,y)\to(x,x)} 8(x^2)$

or $\lim\limits_{(x,x)\to(0,0)} 8x^2 = 8(0)^2 = 0$

$\lim\limits_{(x,y)\to(0,0)} f(x,y)$ exists

Condition #02:-

$f(x,y)$ exist

$f(0,0)=0$

condition #03:-

$\lim_{(x,y)\to(0,0)} f(x,y) = f(x,y)$

$0 \neq -4$

Removable discontinuity , function is not continous.

EXERCISE # 13.3 :-

Question # 18:-

(a) $\frac{\partial r}{\partial v}$ $\quad v = 85\,ft \, , \, \theta = 45^\circ$

$$\frac{\partial r}{\partial v} = \lim_{\Delta v \to 0} \frac{r(v+\Delta v, \theta) - r(v, \theta)}{\Delta v} = \lim_{\Delta v \to 0} \frac{r(85+\Delta v, 45) - r(85, 45)}{\Delta v}$$

· at $\Delta v = 5$

$$\lim_{\Delta v \to 0} \frac{r(90, 45) - r(85, 40)}{5} = \frac{253 - 226}{5} = 5.4$$

· at $\Delta v = -5$

$$\lim_{\Delta v \to 0} \frac{r(80, 45) - r(85, 45)}{-5} = \frac{200 - 226}{-5} = 5.2$$

$$Avg = \frac{5.4 + 5.2}{2} = 8.3$$

(b) $\frac{\partial r}{\partial v}$ $\quad v = 85\,ft/s \, , \, \theta = 45^\circ$

$$\cancel{\frac{\partial r}{\partial \theta} = \lim_{\Delta v \to 0} \frac{r(v+\Delta v, \theta) - r(v, \theta)}{\Delta v} = \lim_{\Delta v \to 0} r(}$$

$$\frac{\partial r}{\partial \theta} = \lim_{\Delta \theta \to 0} \frac{r(v, \theta + \Delta\theta) - r(v, \theta)}{\Delta \theta} = \lim_{\Delta \theta \to 0} \frac{r(85, 45 + \Delta\theta) - r(85, 40)}{\Delta \theta}$$

at $\Delta\theta = 5$

$$\lim_{\Delta \theta \to 0} \frac{r(85, 50) - r(85, 45)}{5} = \frac{222 - 226}{5} = -0.8$$

at $\Delta\theta = -5$

$$\lim_{\Delta \theta \to 0} \frac{r(85, 40) - r(85, 45)}{-5} = \frac{197 - 226}{-5} = 0.8$$

$$Avg = \frac{-0.8 + 0.8}{2} = 0$$

~~Ques~~ Question #59:-

$V = \pi r^2 h$

(a) $\frac{\partial V}{\partial r}$ h=constant

$$\frac{\partial V}{\partial r} = 2\pi r h$$

(b) $\frac{\partial V}{\partial h}$, r=constant

$$\frac{\partial V}{\partial h} = \pi r^2$$

(c) $\frac{\partial V}{\partial r}$ r=6 h=4

$$\left.\frac{\partial V}{\partial r}\right|_{r=6} = 2\pi r h = 2\pi(6)h = 12\pi h = 12\pi(4) = 48\pi$$

(d) r=8, $\left.\frac{\partial V}{\partial h}\right|_{h=10}$

$$\frac{\partial V}{\partial h} = \pi r^2 = \pi(8)^2 = 64\pi$$

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Question # 99:-

$f(x,y,z) = x^3y^5z^7 + xy^2 + y^3z$

(a) $f_{xy} = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial}{\partial y}\left(3x^2y^5z^7 + y^2\right) = 15x^2y^4z^7 + 2y$

(b) $f_{yz} = \frac{\partial}{\partial z}\left(\frac{\partial f}{\partial y}\right) = \frac{\partial}{\partial z}\left(5x^3y^4z^7 + 2xy + 3y^2z\right) = 35x^3y^4z^6 + 3y^2$

(c) $f_{xz} = \frac{\partial}{\partial z}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial}{\partial z}\left(3x^2y^5z^7 + y^2\right) = 21x^2y^5z^6$

(d) $f_{zz} = \frac{\partial}{\partial z}\left(\frac{\partial}{\partial z}\right) = \frac{\partial}{\partial z}\left(7x^3y^5z^6 + y^3\right) = 42x^3y^5z^5$.

(e) $f_{zyy} = \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial}{\partial z}\right)\right) = \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(7x^3y^5z^6 + y^3\right)\right) = \frac{\partial}{\partial y}\left(35x^3y^4z^6 + 3y^2\right)$

$= 140x^3y^3z^6 + 6y$

(f) $f_{xxy} = \frac{\partial}{\partial y}\left[\frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right)\right] = \frac{\partial}{\partial y}\left[\frac{\partial}{\partial x}\left(3x^2y^5z^7 + y^2\right)\right] = \frac{\partial}{\partial y}\left(6xy^5z^7\right) = 30xy^4z^7$

(g) $f_{zyx} = \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial f}{\partial z}\right)\right) = \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(7x^3y^5z^6 + y^3\right)\right) = \frac{\partial}{\partial x}\left(35x^3y^4z^6 + 3y^2\right)$

$= 105x^2y^4z^6$

(h) $f_{xxyz} = \frac{\partial}{\partial z}\left(\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right)\right)\right) = \frac{\partial}{\partial z}\left(\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(3x^2y^5z^7 + y^2\right)\right)\right) = \frac{\partial}{\partial z}\left(\frac{\partial}{\partial y}\left(6xy^5z^7\right)\right)$

$\frac{\partial}{\partial z}\left(30xy^4z^7\right) = 210xy^4z^6$

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EXERCISE # 13.4

Question # 32:-

$V = \frac{1}{3}\pi r^2 h$ $\quad \overset{h\ \ r}{(20, 4)}$ $\quad (19.95, 4.05)$

$\partial z = f_x dx + f_y dy$

or

$\partial V = V_h dh + V_r dr$

$\left(\frac{1}{3}\pi r^2\right)(-0.05) + \left(\frac{2}{3}\pi r h\right)(0.05)$

$\left(\frac{1}{3}\pi (4)^2\right)(-0.05) + \left(\frac{2}{3}\pi (4)(20)\right)(0.05)$

$\frac{-4}{15}\pi + \frac{8}{3}\pi \quad = \frac{12}{5}\pi = 2.4\pi$

$|\Delta V| = V(h+\Delta h, r+\Delta r) - V(h, r)$

$V(19.95, 4.05) - V(20, 4)$

$\frac{1}{3}\pi(4.05)^2(19.95) - \frac{1}{3}\pi(4)^2(20)$

$\Delta V = 7.571107$

$|\partial z - \Delta z| = 0.03128$

$|dV - \Delta V|$

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Question #54, r 1% h 4%

$$\frac{\pi(2rh\,dr)}{3} + \frac{(\pi r^2)\,dh}{3} = dV$$

$$\frac{2\,dr}{r} + \frac{dh}{h} = \frac{dV}{V}$$

$$0.01 = \frac{dr}{r}$$

$$0.04 = \frac{dh}{h}$$

$$\frac{dV}{V} = 0.01 \times 2 + 0.04$$

$$0.06 = \frac{dV}{V}$$

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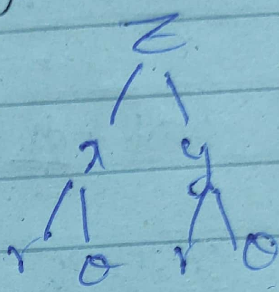
EXERCISE # 13.5

Question # 33

$$\left.\frac{\partial z}{\partial r}\right|_{r=2,\ \theta=\pi/6} \qquad \left.\frac{\partial z}{\partial \theta}\right|_{r=2,\ \theta=\pi/6}$$

$$z = xye^{x/y} \ ; \ x = r\cos\theta \ , \ y = r\sin\theta$$



$$\frac{\partial z}{\partial r} = \frac{\partial z}{\partial x}\cdot\frac{\partial x}{\partial r} + \frac{\partial z}{\partial y}\cdot\frac{\partial y}{\partial r}$$

$$\frac{\partial z}{\partial x} = y\left[\frac{xe^{x/y}}{y} + e^{x/y}\right] = xe^{x/y} + ye^{x/y}$$

$$\frac{\partial z}{\partial y} = x\left[ye^{x/y}\left(\frac{-x}{y^2}\right) + e^{x/y}\right] = \left(\frac{-x^2e^{x/y}}{y} + xe^{x/y}\right)$$

$$\frac{\partial z}{\partial r} = \left(xe^{x/y} + ye^{x/y}\right)(\cos\theta) + \left(\frac{-x^2}{y}e^{x/y} + xe^{x/y}\right)(\sin\theta)$$

$$\cos\theta\left(r\cos\theta e^{\frac{r\cos\theta}{r\sin\theta}} + r\sin\theta e^{\frac{r\cos\theta}{r\sin\theta}}\right) + \left(\frac{-r^2\cos^2\theta}{r\sin\theta}e^{r\cos\theta/r\sin\theta} + r\cos\theta e^{r\cos\theta/r\sin\theta}\right)(\sin\theta)$$

$$\left(2\cos\pi/6\, e^{\frac{\cos\pi/6}{\sin\pi/6}} + 2\sin\frac{\pi}{6}e^{\frac{\cos\pi/6}{\sin\pi/6}}\right)\cos\frac{\pi}{6} + \left(\frac{-2\cos^2(\pi/6)e^{\frac{\cos\pi/6}{\sin\pi/6}}}{\sin\pi/6} + 2\cos\pi/6\, e^{\frac{\cos\pi/6}{\sin\pi/6}}\right)\left(\sin\frac{\pi}{6}\right)$$

$$\left(\sqrt{3}e^{\sqrt{3}} + e^{\sqrt{3}}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{-3e^{\sqrt{3}}}{\cancel{2\times 1/2}} + \sqrt{3}e^{\sqrt{3}}\right)\left(\frac{1}{2}\right)$$

$$\frac{\cancel{3}}{2}e^{\sqrt{3}} + \frac{\sqrt{3}e^{\sqrt{3}}}{2} - \frac{3}{\cancel{2}}e^{\sqrt{3}} + \frac{\sqrt{3}e^{\sqrt{3}}}{2}$$

$$\sqrt{3}e^{\sqrt{3}}$$

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$$\frac{\partial z}{\partial \theta} = \frac{\partial z}{\partial x}\cdot\frac{\partial x}{\partial \theta} + \frac{\partial z}{\partial y}\cdot\frac{\partial y}{\partial \theta}$$

$$= \left(xe^{x/y} + ye^{x/y}\right)(-r\sin\theta) + \left(-\frac{x^2}{y}e^{x/y} + xe^{x/y}\right)(r\cos\theta)$$

$$= \left(r\cos\theta\, e^{\frac{r\cos\theta}{r\sin\theta}} + r\sin\theta\, e^{\frac{r\cos\theta}{r\sin\theta}}\right)(-r\sin\theta) + \left(\frac{-r^2\cos^2\theta}{r\sin\theta} e^{\frac{r\cos\theta}{r\sin\theta}} + r\cos\theta\, e^{\frac{r\cos\theta}{r\sin\theta}}\right)(r\cos\theta)$$

$$\left(2\cos\frac{\pi}{6} e^{\frac{\cos\pi/6}{\sin\pi/6}} + 2\sin\pi/6\, e^{\frac{\cos\pi/6}{\sin\pi/6}}\right)\left(-2\sin\frac{\pi}{6}\right) + \left(\frac{-2\cos^2(\pi/6)}{\sin(\pi/6)} e^{\frac{\cos\pi/6}{\sin\pi/6}} + 2\cos\frac{\pi}{6} e^{\frac{\cos\pi/6}{\sin\pi/6}}\right)\left(2\cos\frac{\pi}{6}\right)$$

$$\left(\sqrt{3}e^{\sqrt{3}} + e^{\sqrt{3}}\right)(-1) + \left(-3e^{\sqrt{3}} + \sqrt{3}e^{\sqrt{3}}\right)(\sqrt{3})$$

$$-\sqrt{3}e^{\sqrt{3}} - e^{\sqrt{3}} - 3\sqrt{3}e^{\sqrt{3}} + 3e^{\sqrt{3}}$$

$$2e^{\sqrt{3}} - 4\sqrt{3}e^{\sqrt{3}} = \left(2-4\sqrt{3}\right)e^{\sqrt{3}}$$

Question #58: $z = \tan^{-1}\frac{2xy}{x^2-y^2}$ $\quad x = r\cos\theta,\ y = r\sin\theta$

$$\cancel{z_x = \frac{1}{1+\left(\frac{2xy}{x^2-y^2}\right)^2}\cdot\frac{\partial}{\partial x}\left(\frac{2xy}{x^2-y^2}\right) = \frac{(x^2-y^2)^2}{(x^2-y^2)^2+(2xy)^2}\left[(x^2-y^2)(2y)-(2xy)(2x)\right]}$$

$$\cancel{\frac{(x^2-y^2)^2}{x^4-2x^2y^2+y^4+2x^2y^2}\cdot\left(2x^2y-2y^3-4x^2y\right) = \frac{(x^4-2x^2y^2+y^4)(2x^2y-2y^3-4x^2y)}{x^4+y^4}}$$

$$\cancel{= \frac{2x^6y-2x^4y^3-4x^6y-4x^4y^3-4x^2y^5+8x^4y^3+2x^2y^5-2y^7-4x^2y^5}{x^4+y^4}}$$

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Question # 8 $z = \tan^{-1}\dfrac{2xy}{x^2-y^2}$ $x = r\cos\theta,\ y = r\sin\theta$

$$Z_x = \frac{1}{1+\left(\frac{2xy}{x^2-y^2}\right)^2}\frac{\partial}{\partial x}\left(\frac{2xy}{x^2-y^2}\right) = \frac{\cancel{(x^2-y^2)^2}}{(x^2-y^2)^2+(2xy)^2}\cdot\frac{(x^2-y^2)(2y)-(2xy)(2x)}{\cancel{(x^2-y^2)^2}}$$

$$\frac{2x^2y-2y^3-4x^2y}{x^4-2x^2y^2+y^4+4x^2y^2} = \frac{-2y\cancel{(x^2+y^2)}}{(x^2+y^2)^{\cancel{2}}}$$

$$Z_x = \frac{-2y}{x^2+y^2}$$

$$Z_{xx} = -2y\left(\frac{-1}{(x^2+y^2)^2}\right)(2x) = \frac{+4xy}{(x^2+y^2)^2}$$

$$Z_y = \frac{1}{1+\left(\frac{2xy}{x^2-y^2}\right)^2}\frac{\partial}{\partial y}\left(\frac{2xy}{x^2-y^3}\right) = \frac{\cancel{(x^2-y^2)^2}}{(x^2+y^2)^2}\cdot\frac{(x^2-y^2)(2x)-(2xy)(-2y)}{\cancel{(x^2-y^2)^2}}$$

$$= \frac{2x^3-2xy^2+4xy^2}{(x^2+y^2)^2} = \frac{2x^3+2xy^2}{(x^2+y^2)^2}$$

$$Z_y = \frac{2x\cancel{(x^2+y^2)}}{(x^2+y^2)^{\cancel{2}}} = \frac{2x}{x^2+y^2}$$

$$Z_{yy} = 2x\left(\frac{-2y}{(x^2+y^2)^2}\right) = \frac{-4xy}{(x^2+y^2)^2}$$

$$Z_{xx}+Z_{yy} = 0$$

$$\frac{4xy}{(x^2+y^2)^2} - \frac{4xy}{(x^2+y^2)^2} = 0$$

$$0 = 0$$

$$z = \tan^{-1} \frac{2(r\cos\theta)(r\sin\theta)}{(r\cos\theta)^2 - (r\sin\theta)^2} = \tan^{-1} \frac{2r^2\cos\theta\sin\theta}{r^2\cos^2\theta - r^2\sin^2\theta}$$

$$= \tan^{-1} \frac{2r^2\cos\theta\sin\theta}{\cos^2\theta - \sin^2\theta} = \tan^{-1}\tan 2\theta$$

$= 2\theta + k\pi$ for some fixed k; $z_r = 0$

$z_{\theta\theta} = 0.$

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EXERCISE # 13.6

Question # 16: $f(x,y,z) = y - \sqrt{x^2+z^2}$; $P(-3,1,4)$;
$a = 2i - 2j + k$

$f_x = \frac{-x}{\sqrt{x^2+z^2}}$, $f_x(-3,1,4) = \frac{3}{\sqrt{9+16}} = \frac{3}{5}$

$f_y = 1$, $f_y(-3,1,4) = 1$

$f_z = \frac{-z}{\sqrt{x^2+z^2}}$, $f_z(-3,1,4) = \frac{-4}{5}$

$u = \frac{2i - 2j + k}{\sqrt{4+4+1}} = \frac{2}{3}i - \frac{2}{3}j - \frac{1}{3}k$

$D_u f(x,y,z) = f_x u_1 + f_y u_2 + f_z u_3$

$= \left(\frac{3}{5}\right)\left(\frac{2}{3}\right) + (1)\left(\frac{-2}{3}\right) + \left(\frac{-4}{5}\right)\left(\frac{-1}{3}\right)$

$D_u f = 0$

Question # 57: $f(x,y,z) = x^3z^2 + y^3z + z - 1$; $P(1,1,-1)$

$f_x = 3x^2z^2$, $f_x(1,1,-1) = 3(1)(-1)^2 = 3$

$f_y = 3y^2z$, $f_y(1,1,-1) = 3(1)^2(-1) = -3$

$f_z = 2x^3z + y^3 + 1$, $f_z(1,1,-1) = 2(1)^3(-1) + (1)^3 + 1 = -2 + 2 = 0$

$\nabla f(x,y,z) = f_x i + f_y j + f_z k$

$= 3i - 3j$

$\|\nabla f(x,y,z)\| = \sqrt{9+9} = 3\sqrt{2}$

$u = \frac{\nabla f(x,y,z)}{\|\nabla f(x,y,z)\|} = \frac{3i - 3j}{3\sqrt{2}} = \frac{1}{\sqrt{2}}i - \frac{1}{\sqrt{2}}j$

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Question # 78: $T(x,y) = \dfrac{xy}{1+x^2+y^2}$

(a). Rate of change at (1,1) $a = 2i - j$

$$f_x = \frac{(1+x^2+y^2)(y) - (xy)(2x)}{(1+x^2+y^2)^2} = \frac{y+x^2y+y^3-2x^2y}{(1+x^2+y^2)^2} = \frac{y+y^3-x^2y}{(1+x^2+y^2)^2}$$

$$f_y = \frac{(1+x^2+y^2)(x) - (xy)(2y)}{(1+x^2+y^2)^2} = \frac{x+x^3+xy^2-2xy^2}{(1+x^2+y^2)^2} = \frac{x+x^3-\cancel{2}xy^2}{(1+x^2+y^2)^2}$$

$$\nabla f(x,y) = \frac{y(1+y^2-x^2)}{(1+x^2+y^2)^2} i + \frac{x(1+x^2-y^2)}{(1+x^2+y^2)^2} j$$

$$\nabla T(1,1) = \frac{1(1+\cancel{1}-\cancel{1})}{(1+1+1)^2} i + \frac{1(1+\cancel{1}-\cancel{1})}{(1+1+1)^2} j$$

$$\nabla T(1,1) = \frac{1}{9} i + \frac{1}{9} j$$

$$U = \frac{2i-j}{\sqrt{2^2+(-1)^2}} = \frac{2i-j}{\sqrt{5}} = \frac{2}{\sqrt{5}} i - \frac{j}{\sqrt{5}}$$

$$D_U T = \left(\frac{1}{9}\right)\left(\frac{2}{\sqrt{5}}\right) + \left(\frac{1}{9}\right)\left(-\frac{1}{\sqrt{5}}\right) = \frac{1}{9\sqrt{5}}$$

(b) ~~Show that electric potential decreases most rapidly.~~ ~~value of E~~

(b) in direction where temperature drops most rapidly.

$U = \dfrac{-(i+j)}{\sqrt{2}}$ opposite to $\nabla T(1,1)$

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EXERCISE # 13.7:

Question # 02:- $xz - yz^3 + yz^2 = 2$

(a) equation of tangent plane to the surface at point (2, -1, 1)

$f(x,y,z) = xz - yz^3 + yz^2 - 2$

$f_x = z$, $f_x(2,-1,1) = 1$

$f_y = -z^3 + z^2$, $f_y(2,-1,1) = -1+1 = 0$

$f_z = x - 3yz^2 + 2yz$, $f_z(2,-1,1) = (2) - 3(-1)(1) + 2(-1)(1) = 3$

$n = \langle 1, 0, 3 \rangle$

$f_x(x-x_0) + f_y(y-y_0) + f_z(z-z_0) = 0$

$(1)(x-2) + 0 + 3(z-1) = 0$

$x - 2 + 3z - 3 = 0$

$x + 3z - 5 = 0$ or $\boxed{x + 3z = 5}$

(b) parametric equations

$x = x_0 + f_x(x,y,z)t = 2 + t$

$y = y_0 + f_y(x,y,z)t = -1$

$z = z_0 + f_z(x,y,z)t = 1 + 3t$

(c) acute angle at the point (2, -1, 1) makes with xy-plane.

$\cos\theta = \dfrac{n \cdot k}{\|n\|}$

$\theta = \cos^{-1} \dfrac{\langle 1,0,3 \rangle \cdot \langle 0,0,1 \rangle}{\sqrt{1^2 + 3^2}} = \cos^{-1} \dfrac{3}{\sqrt{10}}$

$\boxed{\theta = 18.43^\circ}$

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Question#29:- $z = x^2 + y^2$ (curve of paraboloid)

$x^2 + 4y^2 + z^2 = 9$ (ellipsoid) at point $(1, -1, 2)$

paraboloid

$f_x = 2x$, $f_x(1,-1,2) = 2$

$f_y = 2y$, $f_y(1,-1,2) = -2$ $\quad f_z = 1$.

$n = \langle 2, -2, 1 \rangle$

$f_x(x - x_0) + f_y(y - y_0) + f_z(z - z_0)$

$2(x-1) + -2(y+1) + 1(z-2)$

$2x - 2 - 2y - 2 + z - 2$

$2x - 2y - 4 - 2 + z = 0$

$2x - 2y + z = 6$

Ellipsoid

$f_x = 2x = 2(1) = 2$

$f_y = 8y = 8(-1) = -8$

$f_z = 2z = 2(2) = 4$

$f_x(x - x_0) + f_y(y - y_0) + f_z(z - z_0) = 0$

$2(x-1) + -8(y+1) + 4(z-2) = 0$

$2x - 2 - 8y - 8 + 4z - 8 = 0$

$2x - 8y + 4z - 18 = 0$

$n_1 = 2i - 2j + k$

$n_2 = 2i - 8j + 4k$

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$n_1 \times n_2 = (2i-2j-k)\ (2i-8j+4k)$

$$\begin{vmatrix} i & j & k \\ 2 & -2 & -1 \\ 2 & -8 & 4 \end{vmatrix} = i\begin{vmatrix} -2 & -1 \\ -8 & 4 \end{vmatrix} - j\begin{vmatrix} 2 & -1 \\ 2 & 4 \end{vmatrix} + k\begin{vmatrix} 2 & -2 \\ 2 & -8 \end{vmatrix}$$

$i(-8-8) - j(8+2) + k(-16+4) = 0$

$-16i - 10j - 12k = 0$

$x(t) = x_0 + f_x(x,y,z)t = 1+8t$

$y(t) = y_0 + f_y(x,y,z)t = -1+5t$

$z(t) = z_0 + f_z(x,y,z)t = 2+6t$

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EXERCISE # 13.8

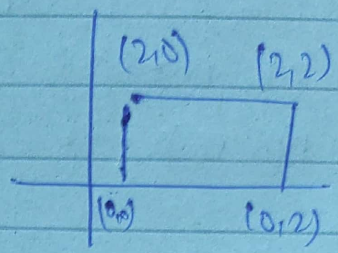
Question # 33. $f(x,y) = x^2 - 3y^2 - 2x + 6y$; $(0,0), (0,2), (2,2), (2,0)$

$f(x) = 2x - 2$

$f_y = -6y + 6$

$f_x = 0$, $2x - 2 = 0$, $\boxed{x = 1}$

$f_y = 0$, $-6y + 6 = 0$, $\boxed{y = 1}$



Along line u: (0,0) & (0,2) $x = 0$, $0 < y < 2$

$f(0,y) = -3y^2 + 6y$

$u'(0,y) = -6y + 6$

$\boxed{y = 1}$ $cp = (0,1)$

Along line (0,0) and (2,0)

$y = 0$, $0 \le x \le 2$

$f(x,0) = x^2 - 2x$

$f_x(x,0) = 2x - 2 = 0$ cp at $(1,0)$

Along line $y = 2$ $0 \le x \le 2$

$f(x,2) = x^2 - 3(4) - 2x + 6(2)$

$x^2 - 2x$

$f(x,2) = 2x - 2 = 0$ cp at $(1,2)$

Along line $x = 2$, $0 \le y \le 2$

$f(2,y) = 4 - 3y^2 - 4 + 6y$

$f(2,y) = -3y^2 + 6y$

$f_y(2,y) = -6y + 6 = 0$

cp at $(2,1)$

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(x,y)	$(1,1)$	$(1,0)$	$(1,2)$	$(0,1)$	$(2,2)$	$(0,0)$	$(2,1)$	$(0,2)$	$(2,0)$
$f(x,y)$	2	-1	-1	3	0	0	3	0	0

Abs max = 3

Abs min = -1

Question#45:- $f(x,y)=4x^2-3y^2+2xy$, $0\le x\le 1$, $0\le y\le 1$ $n=5$

coeff m of regression line can be obtained by the following equations.

$$m=\frac{n\sum_{i=1}^{n}x_iy_i-\sum_{i=1}^{n}x_i\sum_{i=1}^{n}y_i}{n\sum_{i=1}^{n}x_i^2-\left(\sum_{i=1}^{n}x_i\right)^2}=\frac{5\sum_{i=1}^{5}x_iy_i-\sum_{i=1}^{5}x_i\sum_{i=1}^{5}y_i}{5\sum_{i=1}^{5}x_i^2-\left(\sum_{i=1}^{5}x_i\right)^2}$$

$$\sum_{i=1}^{5}x_iy_i=(1)(4.2)+(2)(3.5)+(3)(3)+(4)(2.4)+(5)(2)=39.8$$

$$\sum_{i=1}^{5}x_i\sum_{i=1}^{5}y_i=(1+2+3+4+5)(4.2+3.5+3+2.4+2)=226.5$$

$$\left(\sum_{i=1}^{5}x_i\right)^2=(1+2+3+4+5)^2=225$$

$$\sum_{i=1}^{5}x_i^2=1^2+2^2+3^2+4^2+5^2=55\ ,\quad m=\frac{5\times 39.8-226.5}{5\times 55-225}\quad \boxed{m=-0.55}$$

coeff b can be obtained by

$$b=\frac{1}{n}\left(\sum_{i=1}^{n}y_i-m\sum_{i=1}^{n}x_i\right)=\frac{1}{5}\left(\sum_{i=1}^{5}y_i-m\sum_{i=1}^{5}x_i\right)$$

$$\sum_{i=1}^{5}y_i=4.2+3.5+3+2.4+2=15.1$$

$$\sum_{i=1}^{5}x_i=1+2+3+4+5=15$$

$b=\frac{1}{5}(15.1-(-0.55)\times 15)=4.67$. Line is $y=mx+b$

$\boxed{y=-0.55x+4.67}$

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Question # 45: $f(x,y) = 4x^2 - 3y^2 + 2xy$, $0 \le x \le 1$, $0 \le y \le 1$

(a) on edge of the square:-

for $x=0$, $f(0,y)$

$f(0,y) = 4(0)^2 - 3y^2 + 2(0)y$

$= -3y^2 \quad 0 \le y \le 1$

So, $-3 \le f \le 0$

for $y=0$

$f(x,0) = 4x^2 - 3(0)^2 + 2x(0)$

$= 4x^2 \quad 0 \le x \le 1$

$0 \le f \le 4$

for $x=1$

$f(1,y) = 4 - 3y^2 + 2y$

$-6y + 2$

$\boxed{y = 1/3}$

~~$0 \le x \le 1$~~

~~$0 \le f \le 4$~~

$f(1,\frac{1}{3}) = 1 - \frac{3}{9} + \frac{2}{3}$

$= \frac{13}{3} > 4$

So, $3 \le f \le \frac{13}{3}$

for $y=1$

$f(x,1) = 4x^2 - 3(1) + 2x$

$= 8x + 2$

$x = -\frac{1}{4} < 0$

$\boxed{min = -3}$, $\boxed{max = 3}$

(b) on each diagnal:

At $y = -x+1$

$f(x, -x+1) = 4x^2 - 3(1-x^2) + 2x(-x+1)$

$= -x^2 + 8x - 3$

$2x = 8$

$\boxed{x = 4}$

$-3 \le f \le 4$

$\boxed{max = 4}$, $\boxed{min = -3}$

(c) on the entire square:-

$f_x = 8x + 2y = 0$

$y = -4x \longrightarrow (1)$

$f_y = -6y + 2x = 0$

$-6(-4x) + 2x = 0$

$\boxed{x = 0}$

putting in (1)

$\boxed{y = 0}$

$f(0,0) = (0,0) = 0$ This is not the interior point of square

$-3 \le f(x,y) \le \frac{13}{3}$

$\boxed{min = \frac{13}{3}}$, $\boxed{max = -3}$

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EXERCISE #13.9

Question # 07: $f(x,y) = 4x^3 + y^2$, $2x^2 + y^2 = 1$

$\nabla f = \lambda \nabla g$

$\nabla f = 12x^2 i + 2y j$

$\nabla g = (4x) i + (2y) j$

$\nabla f = \lambda \nabla g$

$(12x^2 i + 2y j) = \lambda (4x i + 2y j)$

$= \lambda (4x) = 12x^2$ $\quad\quad$ $\lambda (2y) = 2y$

If $y \neq 0$, then $\lambda = 1$

$12x^2 - 4x = 0$

$\boxed{x = \frac{1}{3}}, \boxed{x = 0}$

at $x = 0$ $\quad 0 + y^2 = 1$, $\boxed{y = \pm 1}$

at $x = \frac{1}{3}$, $2\left(\frac{1}{9}\right) + y^2 = 1$

$\boxed{y = \pm \frac{\sqrt{7}}{3}}$

let $y = 0$

$2x^2 + 0 = 1$

$x = \pm \frac{1}{\sqrt{2}}$

$cp = (0, \pm 1), \left(\frac{1}{3}, \pm \frac{\sqrt{7}}{3}\right), \pm\left(\frac{1}{\sqrt{2}}, 0\right)$

$f(x,y) = 4x^3 + y^2 = f(0, \pm 1) = 1$

$f(x,y) = 4x^3 + y^2 = f\left(\frac{1}{3}, \pm \frac{\sqrt{7}}{3}\right) = 4\left(\frac{1}{3}\right)^3 + \frac{7}{9} = \frac{25}{27}$

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$f(x,y) = 4x^3 + y^2 \Rightarrow f\left(\frac{1}{\sqrt{2}}, 0\right) = \sqrt{2}$

$f(x,y) = 4x^3 + y^2 \Rightarrow f\left(\frac{-1}{\sqrt{2}}, 0\right) = -\sqrt{2}$

Max value $= \sqrt{2}$ at $\left(\frac{1}{\sqrt{2}}, 0\right)$

Min value $= -\sqrt{2}$ at $\left(\frac{-1}{\sqrt{2}}, 0\right)$