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Sec: H

66 Algorithms Assignment Date: D D M M Y Y

Q1:

→ $\boxed{15}$, 6, 12, 8, 7, 1, 9, 3, 5, 23 key = 6
→ 15, 6, $\boxed{12}$, 8, 7, 1, 9, 3, 5, 23 key = 12
→ 15, 12, 6, $\boxed{8}$, 7, 1, 9, 3, 5, 23 key = 8
→ 15, 12, 8, 6, $\boxed{7}$, 1, 9, 3, 5, 23 key = 7
→ 15, 12, 8, 7, 6, $\boxed{1}$, 9, 3, 5, 23 key = 1
→ 15, 12, 8, 7, 6, 1, $\boxed{9}$, 3, 5, 23 key = 9
→ 15, 12, 9, 8, 7, 6, 1, $\boxed{3}$, 5, 23 key = 3
→ 15, 12, 8, 7, 9, 6, 3, 1, $\boxed{5}$, 23 key = 5
→ 15, 12, 9, 8, 7, 6, 5, 3, 1, $\boxed{23}$ key = 23
→ 23, 15, 12, 9, 8, 7, 6, 5, 3, 1 Sorted

Time Complexity:

$$T(n) = an + C_2 \cdot (n-1) + C_4 (n-1) + C_5 \sum_{j=2}^{n} (t_j) + C_6 \sum_{j=2}^{n} \cancel{(t_j - 1)} (t_j - 1) + C_7 \sum_{j=2}^{n} (t_j - 1) + C_8 (n-1)$$

$$= C_1 n + C_2 (n-1) + C_4 (n-1) + C_5 \left( \frac{n(n+1)}{2} - 1 \right) + C_6 \left( \frac{n(n+1)}{2} - 1 \right) + C_7 \left( \frac{n(n+1)}{2} - 1 \right) + C_8 (n-1)$$

Hence, $T(n) = O(n^2)$

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Loop Invariant: Condition: $a[1 \ldots j-1]$ has all sorted elements

Initialization: Before the beginning of first iteration, we consider the array in the form $a[1 \ldots 1]$ consist of only one element which is the first element, and it is already sorted

Maintenance: After the first iteration $j=3$ Hence, we take the array $a[1 \ldots 2]$ and this array is already sorted.

Termination: After the last iteration $j=n+1$ Therefore $a[1 \ldots n+1-1] = a[1 \ldots n]$ which is completely sorted.

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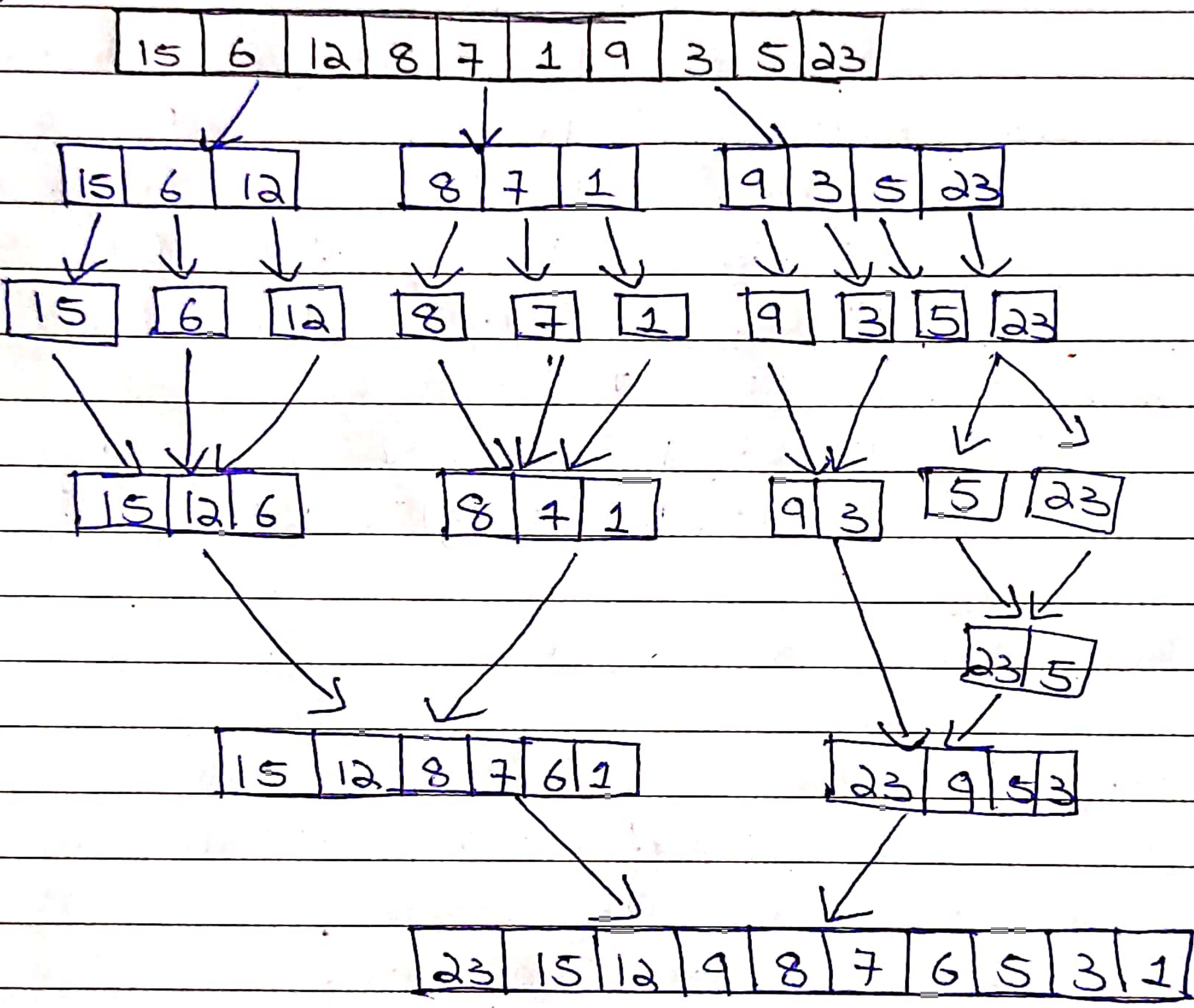
Q2 :




Time Complexity :

$$T(n) = a.T\left(\frac{n}{b}\right) + f(n)$$

$$a = 3, \; b = 3, \; d = 1$$

$$b^d = 3^1 = 3$$

$$a = b^d$$

Hence, $T(n) = \Theta(n^d \log n)$

$$T(n) = \Theta(n \log_3 n)$$

Date:

Q3:

→ 15, 6, 12, 8, 7, 1, 9, 3, 5, 23 pivot element (P.E) = 23
→ 23, 6, 12, 8, 7, 1, 9, 3, 5, 15 (P.E) = 15
→ 23, 15, 12, 8, 7, 1, 9, 3, 5, 6 P.E = 6
→ 23, 15, 12, 8, 7, 9, 6, 3, 5, 1 P.E = 9
→ 23, 15, 12, 9, 7, 8, 6, 3, 5, 1 P.E = 8
→ 23, 15, 12, 9, 7, 8, 6, 3, 5, 1 P.E = 1
→ 23, 15, 12, 9, 7, 8, 6, 3, 5, 1 P.E = 5
→ 23, 15, 12, 9, 8, 7, 6, 5, 3, 1 Sorted.

~~Loop~~ ~~Find~~ ~~Ter~~

Loop Invariant:

Initialization: Before the first iteration begins j=p and i<p. There exist no values between i+1 and j-1. Hence initialization is satisfied.

Maintenance: After first iteration i=p and all elements between i+1 & j-1 are sorted correctly. Hence maintenance is satisfied.

Termination: After the last iteration j=r Hence, all of the elements in the array are sorted. Therefore, termination is satisfied.

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Q4:

We apply merge sort to find the majority element that is majority in either a1, a2 or both.
This takes $O(\log n)$ time. Now we count the repetition which takes $O(n)$ time complexity. Hence, total time complexity to solve the problem is

$$T(n) = O(n\log n)$$

To solve the same problem in time complexity $O(n)$ we use linear praversal to find the element with maximum frequency using any accurative loop and a counter variable

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Q5:

Q3

1) We apply any sort that uses time complexity of $O(\log n)$

2) After the rating is merged we apply any sort (prefered: linear sort) that takes time complexity $O(n)$

Hence, the rating has been merged and divided into even & odd.

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Q7:

An algorithm is efficient if it is efficient in terms of time and space complexity as well as it gives an output on every input

Asymptotic bounds:

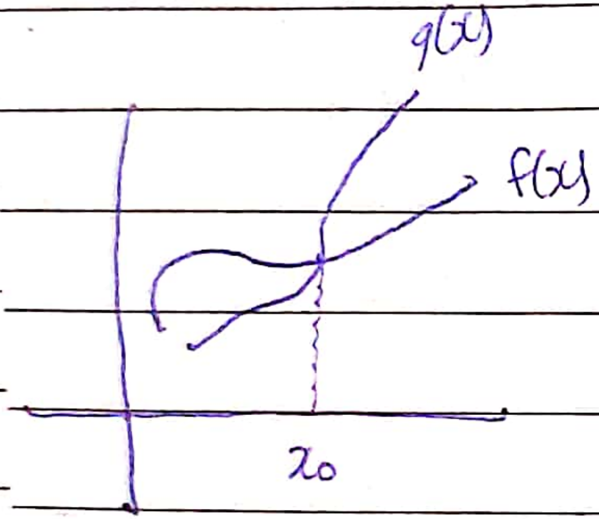
1) $f(x) \in O(g(x))$ is true if $x_0 > 0$ such that there is a constant c and $f(x) \leq cg(x)$ for all values of $x \geq x_0$.

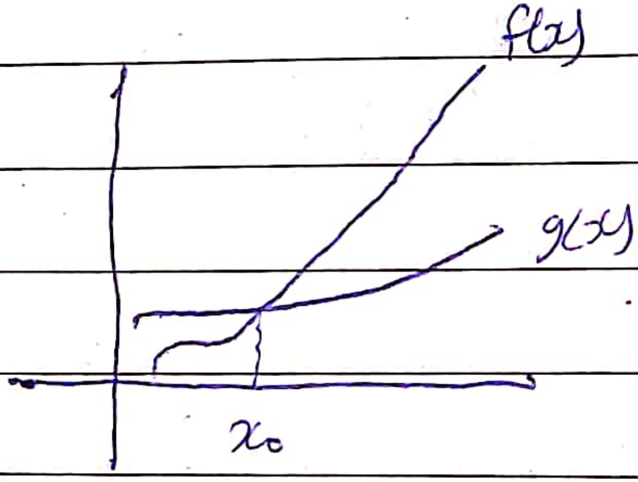
It might be possible that function f(x) exceeds g(x) at some constant value of c before $x = x_0$. After the point $x_0$ $f(x) \leq cg(x)$

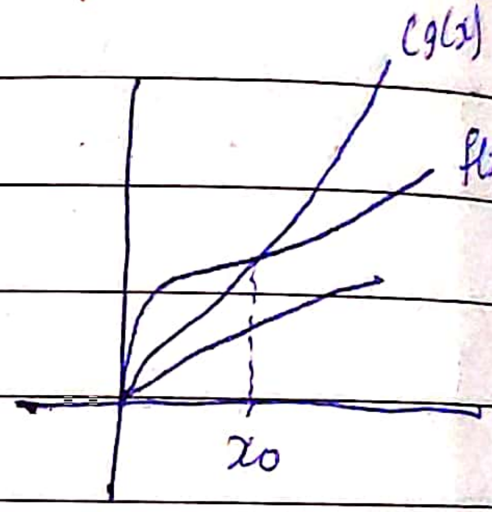
2) $f(x) \in \Omega (g(x))$ is true if $x_0 > 0$ such that there is a constant c and $f(x) \geq cg(x)$ for all values of $x > x_0$. there is a constant c at which it might be possible that g(x) is above f(x)

3) $f(x) \in \Theta(g(x))$ if $x_0 \geq 0$ and $x \geq x_0$ and $cg(x) \geq f(x) \geq c_2 g(x)$ - there might be a point that ~~f(x)~~ the condition is not satisfied

**Brilliant**




$\downarrow$ $O(n)$ $\qquad$ $\downarrow$ $\Omega(n)$ $\qquad$ $\downarrow$ $\theta(n)$

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Q8:

a) $T(n) = 4T\left(\frac{n}{2}\right) + n^3$

$a = 4,\ b = 2,\ d = 3$

$b^d = 2^3 = 8$

$a = 4,\ b^d = 8$

Hence, $a < b^d$

$T(n) = \Theta(n^d)$

$T(n) = \Theta(n^3)$

b) $T(n) = 3T\left(\frac{n}{2}\right) + n^2$

$a = 3,\ b = 2,\ d = 2$

$b^d = 2^2 = 4$

Hence, $a < b^d$

$T(n) = \Theta(n^d)$

$T(n) = \Theta(n^2)$

c) $T(n) = 9T\left(\frac{n}{2}\right) + n$

$a = 9,\ b = 2,\ d = 1$

$b^d = 2^1 = 2$

Hence, $a > b^d$

$T(n) = \Theta(n^{\log_b a})$

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$T(n) = \Theta(n^{\log_2 9})$

Q9:

a) $T(n) = 6T\left(\frac{n}{2}\right) + n, \quad T(1) = 2$

$T\left(\frac{n}{2}\right) = 6T\left(\frac{n/2}{2}\right) + \frac{n}{5} = 6T\left(\frac{n}{4}\right) + \frac{n}{2}$

$T(n) = 6\left[6T\left(\frac{n}{4}\right) + \frac{n}{2}\right] + n$

$T(n) = 6^2T\left(\frac{n}{2^2}\right) + \frac{6n}{2} + n$

$T(n/4) = 6T\left(\frac{n/4}{2}\right) + \frac{n}{4} = 6T\left(\frac{n}{8}\right) + \frac{n}{4}$

$T(n) = 6^2\left[6T\left(\frac{n}{8}\right) + \frac{n}{4}\right] + \frac{6n}{2} + n$

$T(n) = 6^3T\left(\frac{n}{2^3}\right) + \frac{6^2n}{2^2} + \frac{6n}{2} + n$

$T(n) = 6^kT\left(\frac{n}{2^k}\right) + \frac{6^{k-1}n}{2^{k-1}} + \frac{6^{k-2}n}{2^{k-2}} + \dots \frac{6n}{2} + n$

Base case: $T(1) = 1$

$\frac{n}{2^k} = 1 \qquad n = 2^k, \quad k = \log_2 n$

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$$T(n) = 6^{\log_2 n} T\left(\frac{n}{n}\right) + n\left(\frac{6^{k-1}}{2^{k-1}} + \frac{6^{k-2}}{2^{k-2}} + \dots \frac{6}{2} + 1\right)$$

$$6^{\log_2 n} T(1) + n(1+1)$$

$$n^{\log_2 6} + 6n$$

$$T(n) = O(n^{\log_2 6})$$

b) $3 + T\left(\frac{n}{2}\right)$ , $T(1) = 1$

$$T(n) = T\left(\frac{n}{2}\right) + 3$$

$$T\left(\frac{n}{2}\right) = 3T\left(\frac{n/2}{2}\right) + 3 = 3T\left(\frac{n}{4}\right) + 3$$

$$\Rightarrow T(n) = \left[3T\left(\frac{n}{4}\right) + 3\right] + 3 = 3T\left(\frac{n}{4}\right) + 6$$

$$T\left(\frac{n}{4}\right) = T\left(\frac{n/4}{2}\right) + 3 = T\left(\frac{n}{8}\right) + 3$$

$$\Rightarrow T(n) = 3T\left(\frac{n}{8}\right) + 9$$

$$T\left(\frac{n}{8}\right) = T\left(\frac{n}{16}\right) + 3$$

$$T(n) = 3T\left(\frac{n}{16}\right) + 9 + 9$$

$$3T\left(\frac{n}{2^k}\right) + 3 + 3 + \dots + k$$

$$\sum_{T=0}^{6} 3 = \log n$$

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$T\left(\frac{n}{n}\right) + \log n$

$T(1) + \log n$

$1 + \log n$

$T(n) = \Theta(\log n)$

Q10:

a) $f(n) = \cancel{h(n)} = \frac{n}{2} + 5$ $\quad h(n) = -\frac{n}{2} + 5$

$g(n) = n$

$f(n) = O(n)$

$\frac{n}{2} + 5 = O(n)$

$\frac{n}{2} + 5 = n - \left(-\frac{n}{2} + 5\right)$

$n - \frac{n}{2} + 5 = \frac{n}{2} + 5$

Hence, true

b) $f(n) = \frac{n}{2} + 5$

$g(n) = n$

$h(n) = -\frac{n}{2} + 5$

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$f(n) = O(n)$

$\frac{n}{2}+5 = \Omega\left(\frac{n}{2}+5\right)$

$f(n) = \Omega(g(n))$

$\left(\frac{n}{2}+5\right)^3 = O(n)^3$

$\frac{n^3}{8} + 3\left(\frac{n}{2}\right)(5)\left(\frac{n}{2}+3\right) + 125 = O(n)^3$

$\left(\frac{n}{2}+5\right)^3 = O(n^3)$

Hence, true

c) $f(n) = \frac{n}{2} + 5$

$g(n)^2 = n$

$\left(\frac{n}{2}+5\right)^2 = \frac{n^2}{4} + 5n + 25$

False as $\frac{n^2}{4} + 5n + 25$ $<$ $n^2$