

# ALGO ASSIGNMENT 03

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QUESTION 1-5 (Bilal Ahmed)

QUESTION 6-10 (Ahmed Ahsan)

## QUESTION 01

\* Largest Common Sequence:

X: { B, B, C, A, B, A }

Y: { A, B, C, B, D, A, D }

$$c[i,j] = \begin{cases} c[i-1,j-1] + 1 & \text{if } i,j > 0 \text{ \& } x_i = y_j \\ \max(c[i,j-1], c[i-1,j]) & \text{if } i,j > 0 \text{ \& } x_i \neq y_j \end{cases}$$

	$y_j$	A	B	C	B	D	A	D
$x_i$	0	0	0	0	0	0	0	0
B	0	0	1	1	1	1	1	1
B	0	0	1	1	2	2	2	2
C	0	0	1	2	2	2	2	2
A	0	1	1	2	2	2	3	3
B	0	1	2	2	3	3	3	3
A	0	1	2	2	3	3	4	4

Largest common subsequence of "BBCABA" & "ABCBDAD" has length 4.



## QUESTION 02

X: {B, I, L, A, U}; Y: {A, H, M, E, D}

Shortest Common Supersequence.

	$x_i$	B	I	L	A	U
y <sub>j</sub>						
A	0	0	0	0	1	0
H	0	0	0	0	0	0
M	0	0	0	0	0	0
E	0	0	0	0	0	0
D	0	0	0	0	0	0

LCS = 1

Thus shortest common supersequence will be.

$$= \text{len}(X) + \text{len}(Y) - \text{len}(\text{LCS})$$

$$= 5 + 5 - 1$$

Shortest  
common  
supersequence 9

## QUESTION 03

Largest increasing subsequence:

Array: {5, 1, 10, 2, 1, 2, 20}

t = take, nt = not take

Not take condition

$$\text{len} = 0 + f(\text{ind} + 1, \text{prev} - \text{ind})$$

take case:

$$\text{len} = \max(\text{len}, f(\text{ind} + 1, \text{ind}) + 1)$$

1) len = 0

$$\begin{array}{c} 5, 1, 10, 2, 1, 2, 20 \\ \underline{+} \\ \text{len} = 1 \end{array}$$

2) len = 1

$$\begin{array}{c} 5, 1, 10, 2, 1, 2, 20 \\ \underline{+} \quad \underline{+} \\ \text{len} = 1 \end{array}$$

len = 1

3) len = 1

$$\begin{array}{c} 5, 1, 10, 2, 1, 2, 20 \\ \underline{+} \quad \underline{+} \\ \text{len} = 2 \end{array}$$

4)  $len = 2$

5, 1, 10, 2, 1, 2, 20  
 $\begin{array}{ccccccc} \underline{5} & 1 & \underline{10} & 2 & 1 & 2 & 20 \\ + & & + & & + & & \end{array}$

5)  $len = 2$

5, 1, 10, 2, 1, 2, 20  
 $\begin{array}{ccccccc} \underline{5} & 1 & \underline{10} & 2 & \underline{1} & 2 & 20 \\ + & & + & & + & & \end{array}$

$len = 2$

6)  $len = 2$

5, 1, 10, 2, 1, 2, 20  
 $\begin{array}{ccccccc} \underline{5} & 1 & \underline{10} & 2 & \underline{1} & 2 & 20 \\ + & & + & & + & & \end{array}$

$len = 2$

7)  $len = 2$

5, 1, 10, 2, 1, 2, 20  
 $\begin{array}{ccccccc} \underline{5} & 1 & \underline{10} & 2 & 1 & 2 & \underline{20} \\ + & & + & & & & + \end{array}$

$len = 3$

8)  $len = 3$

ind = 2n, array complete

Answer length of the longest increasing subsequence is 3.

## QUESTION 04

$S_1 = \text{"PLASMA"}$

$S_2 = \text{"ALTRUISM"}$

character match:

$\neq(i-1, j-1)$

character not match:

$1 + \min(\neq(i-1, j), \neq(i, j-1), \neq(i-1, j-1))$

base case:

if  $(i < 0)$  return  $j+1$

if  $(j < 0)$  return  $i+1$

Using dynamic programming

1) PLASMA (Removing A)  
 ALTRUISM

2) P L A S M (Adding u btw index 2 & 3)  
 A L T R U I S M  
 P L A U S M



3)

0 1 2 3 4 5  
PLAUSM  
ALTRUISM

Adding I b/w  
index 3 & 4

PLAUISM  
ALTRUISM

04)

Replacing P with A

ALAUISM  
ALTRUISM

05)

Replacing A with T

0 1 2 3 4 5 6  
ALTUISM  
ALTRUISM

06)

Adding R b/w ind 2 & 3

ALTRUISM  
ALTRUISM

Thus it will take minimum  
3 steps to change PLASMA  
ATRUISM

		P	L	A	S	M	A
	0	1	2	3	4	5	6
A	1	1	2	2	3	4	5
L	2	2	1	2	3	4	5
T	3	3	2	2	3	4	5
R	4	4	3	3	3	4	5
U	5	5	4	4	4	4	5
I	6	6	5	5	5	5	5
S	7	7	6	6	5	6	6
M	8	8	7	7	6	5	6

Table for Edit-Distance  
problem



# QUESTION 05

## Matrix chain multiplication

$$p_0 = 2, p_1 = 25, p_2 = 3, p_3 = 16, p_4 = 1, p_5 = 5$$

	2	3	4	5
2	0	150	246	101
3		0	120	51
4			0	48
5				0

## Matrix m

$$m[1,2] = m[1,1] + m[2,2] + p_0 \cdot p_1 \cdot p_2 = 150$$

$$m[2,3] = m[2,2] + m[3,3] + p_1 \cdot p_2 \cdot p_3 = 1200$$

$$m[3,4] = m[3,3] + m[4,4] + p_2 \cdot p_3 \cdot p_4 = 48$$

$$m[4,5] = m[4,4] + m[5,5] + p_3 \cdot p_4 \cdot p_5 = 80$$

~~$$m[1,3] = m[1,1] + m[2,3] + p_0 \cdot p_1 \cdot p_3 = 1850$$

$$m[1,3] = m[1,2] + m[3,3] + p_0 \cdot p_2 \cdot p_3 = 246$$~~

$$m[1,3] = m[1,1] + m[2,3] + p_0 \cdot p_1 \cdot p_3 = 2000$$

$$m[1,3] = m[1,2] + m[3,3] + p_0 \cdot p_2 \cdot p_3 = 246$$

K=2 for m[1,3]

$$m[2,4] = m[2,2] + m[3,4] + p_1 \cdot p_2 \cdot p_4 = 51$$

$$m[2,4] = m[2,3] + m[4,4] + p_1 \cdot p_3 \cdot p_4 = 1216$$

K=2 for m[2,4]

$$m[3,5] = m[3,3] + m[4,5] + p_2 \cdot p_3 \cdot p_5 = 320$$

$$m[3,5] = m[3,4] + m[5,5] + p_2 \cdot p_4 \cdot p_5 = 63$$

K=4 for m[3,5]

$$m[1,4] = m[1,1] + m[2,4] + p_0 \cdot p_1 \cdot p_4 = 101$$

$$= m[1,2] + m[3,4] + p_0 \cdot p_2 \cdot p_4 = 204$$

$$= m[1,3] + m[4,4] + p_0 \cdot p_3 \cdot p_4 = 278$$

K=1 for m[1,4]



$$\begin{aligned}
 m[2,5] &= m[2,2] + m[3,5] + p_1 p_2 p_5 = 436 \\
 &= m[2,3] + m[4,5] + p_1 p_3 p_5 = 3280 \\
 &= m[2,4] + m[5,5] + p_1 p_4 p_5 = 176 \\
 K=4 \text{ for } m[2,5]
 \end{aligned}$$

$$\begin{aligned}
 m[1,5] &= m[1,1] + m[2,5] + p_0 p_1 p_5 = 426 \\
 &= m[1,2] + m[3,5] + p_0 p_2 p_5 = 243 \\
 &= m[1,3] + m[4,5] + p_0 p_3 p_5 = 486 \\
 &= m[1,4] + m[5,5] + p_0 p_4 p_5 = 111 \\
 K=4 \text{ for } m[1,5]
 \end{aligned}$$

Matrix 3

0	1	2	1	4
	0	2	2	4
		0	3	4
			0	4
				0

Parenthesis

$$(A_1)(A_2 A_3 A_4)(A_5)$$