Design and Analysis of Algorithms
BFS, DFS, and topological sort

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Graph Traversal

Two techniques of graph traversing for both directed or undirected graphs:

- Breadth First Search (BFS) – Uses Queue
- Depth First Search (DFS) – Uses Stack

What are BFS and DFS?

- Two ambiguous terms: search, traversal
- Visit each of vertices once
  - E.g.: tree walks of a binary search tree
  - Traversal
  - Search
- Start from a vertex, visit all the **reachable** vertices
  - Search

What are BFS and DFS?

- To eliminate the ambiguity, in my class
- Search indicates
  - Start from a vertex, visit all the **reachable** vertices
- Traversal indicates
  - Visit each of vertices once
- However, in other materials, you may see some time “search” is considered as “traversal”

What are BFS and DFS?

- BFS
  - Start from a vertex, visit all the reachable vertices in a breadth first manner
- DFS
  - Start from a vertex, visit all the reachable vertices in a depth first manner
- BFS or DFS based traversal
  - Repeat BFS or DFS for unreachable vertices

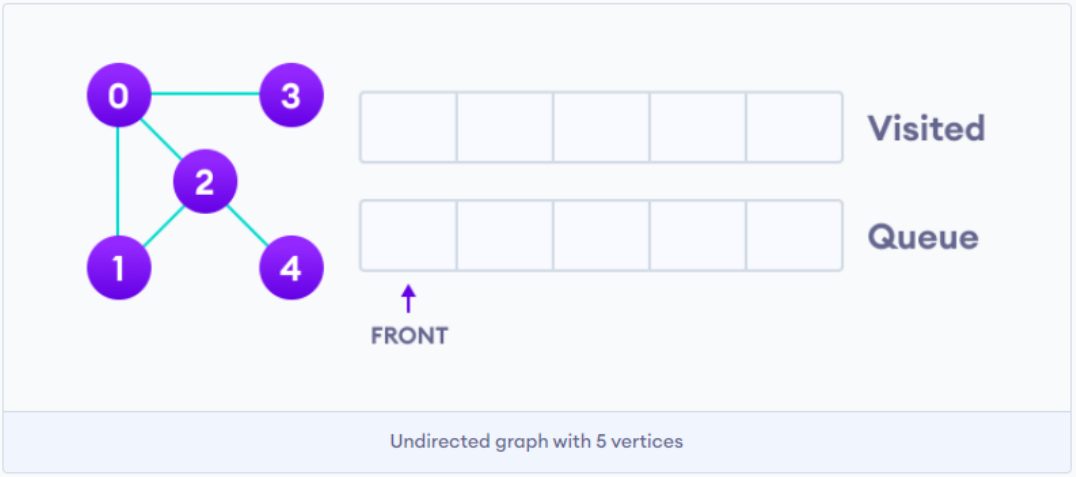
BFS, BF tree and shortest path

- Breadth-first search
  - From a source vertex s
  - Breadth-firstly explores the edges to discover every vertex that is reachable from s
- **BFS**(s)

```
visit(s);
queue.insert(s);
while( queue is not empty ){
    u = queue.extractHead();
    for each edge <u, d>{
        if(d has not been visited)
            visit(d);
            queue.insert(d);
    }
}
```

Graph Traversal : BFS

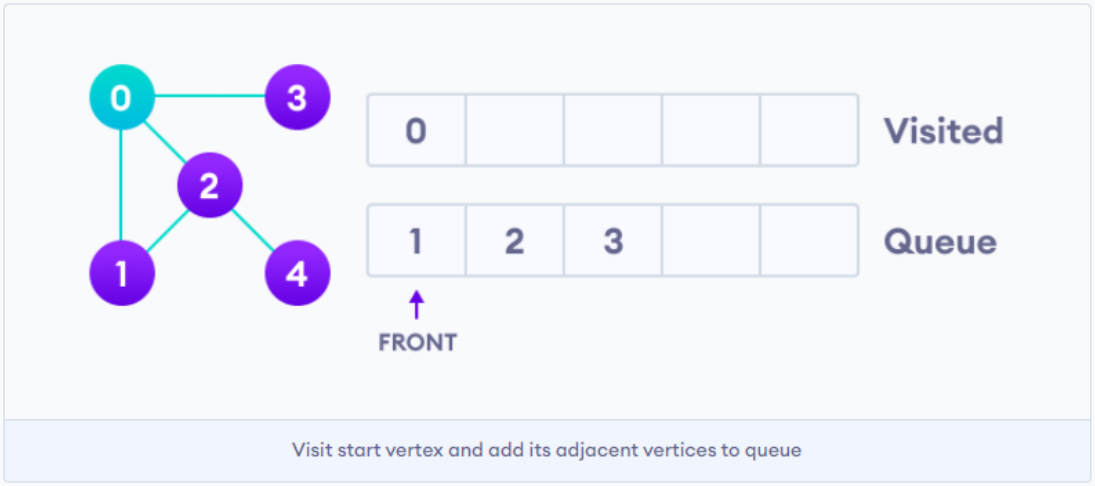
Let's see how the Breadth First Search algorithm works with an example. We use an undirected graph with 5 vertices.



Undirected graph with 5 vertices

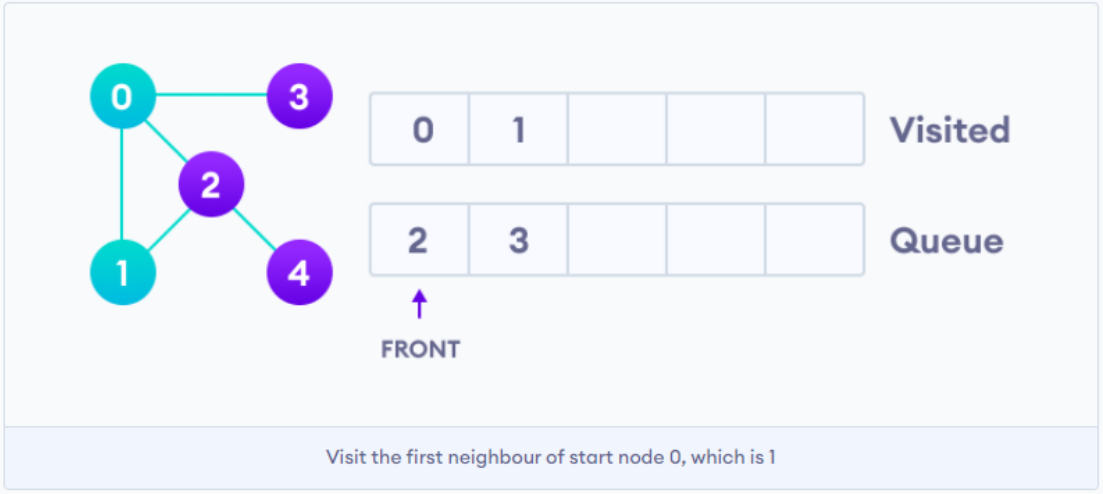
Graph Traversal : BFS

We start from vertex 0, the BFS algorithm starts by putting it in the Visited list and putting all its adjacent vertices in the stack.



Graph Traversal : BFS

Next, we visit the element at the front of queue i.e. 1 and go to its adjacent nodes. Since 0 has already been visited, we visit 2 instead.



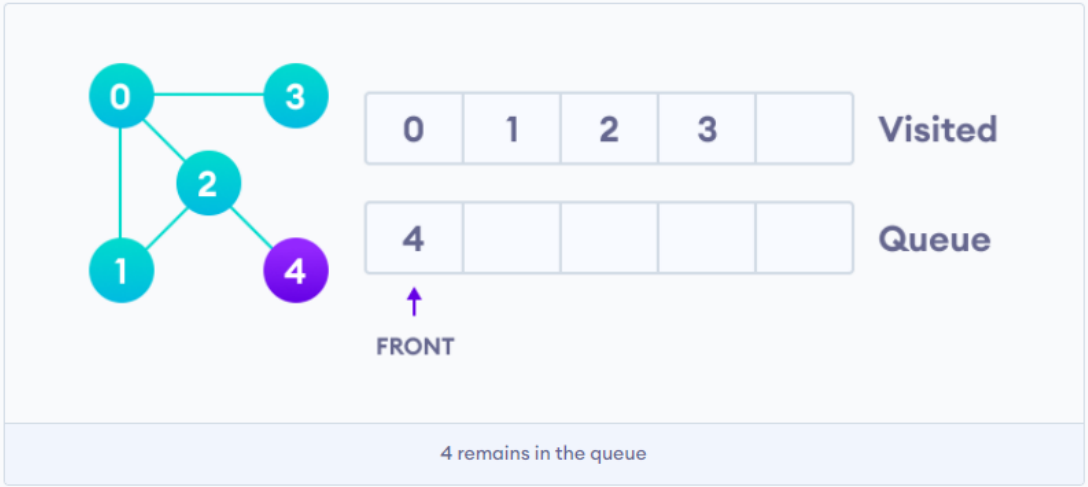
Visit the first neighbour of start node 0, which is 1

Graph Traversal : BFS

Vertex 2 has an unvisited adjacent vertex in 4, so we add that to the back of the queue and visit 3, which is at the front of the queue.

Visit 2 which was added to queue earlier to add its neighbours

Graph Traversal : BFS



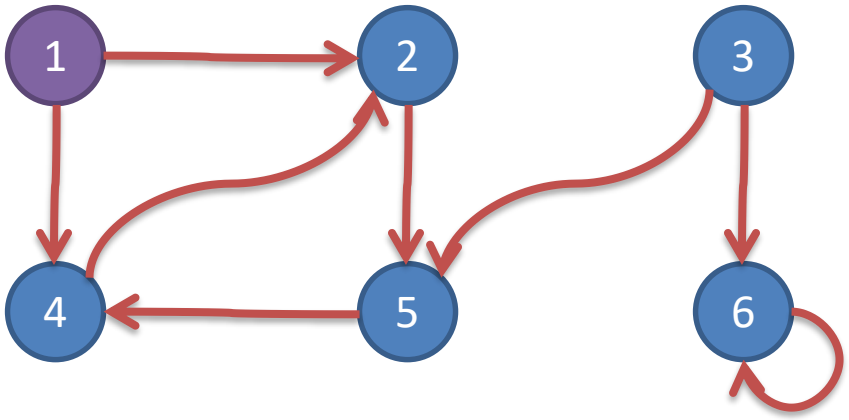
Only 4 remains in the queue since the only adjacent node of 3 i.e. 0 is already visited. We visit it.

Graph Traversal : BFS

Since the queue is empty, we have completed the Breadth First Traversal of the graph.

BFS, BF tree and shortest path

BFS(1)



Queue: 1 4 2 5

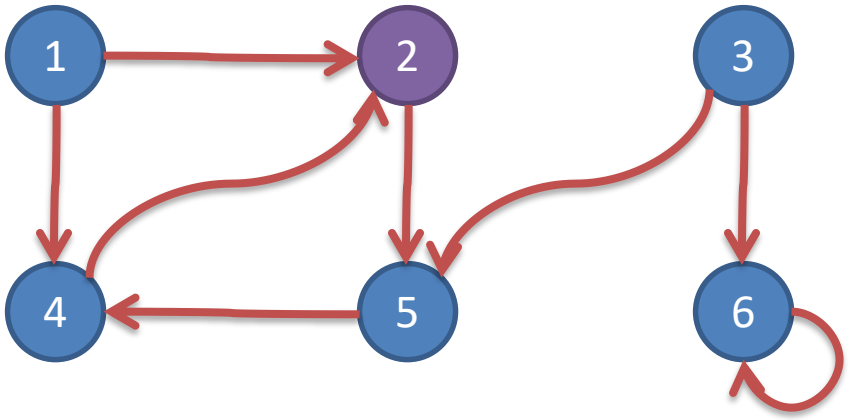
Visit order: 1 4 2 5

BFS(s)

```
visit(s);
queue.insert(s);
while( queue is not empty ){
    u = queue.extractHead();
    for each edge <u, d>{
        if(d has not been
        visited)
            visit(d);
            queue.insert(d);
    }
}
```

BFS, BF tree and shortest path

BFS(2)



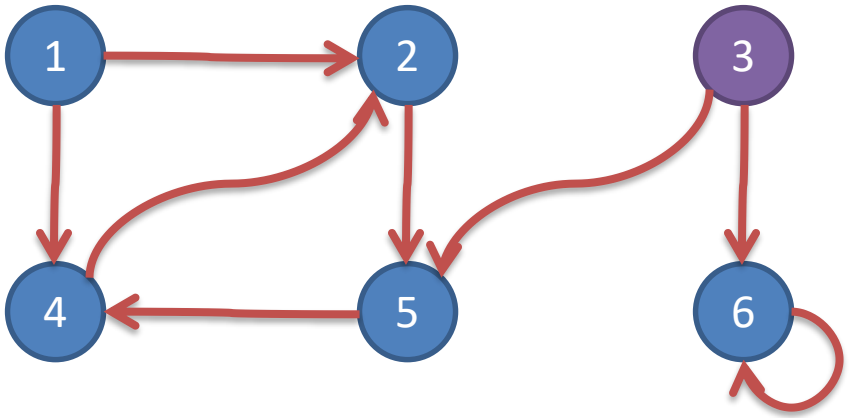
Queue: 2 5 4

Visit order: 2 5 4

```
BFS(s)
    visit(s);
    queue.insert(s);
    while( queue is not empty ){
        u = queue.extractHead();
        for each edge <u, d>{
            if(d has not been
            visited)
                visit(d);
                queue.insert(d);
        }
    }
```

BFS, BF tree and shortest path

BFS(3)



Queue: 3 6 5 4 2

Visit order: 3 6 5 4 2

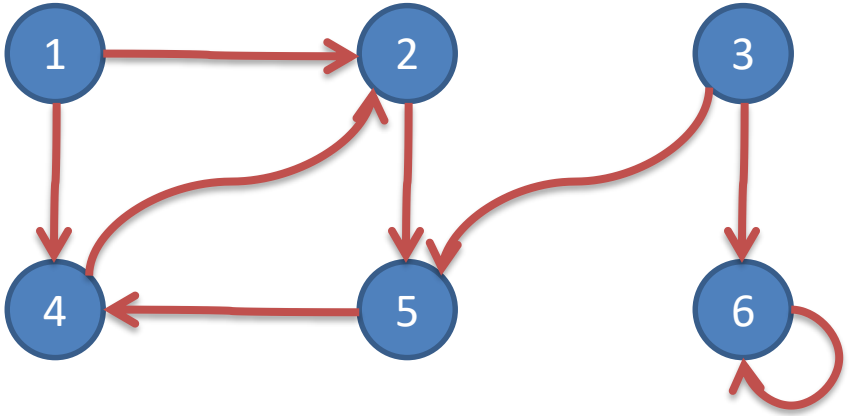
```
BFS(s)
    visit(s);
    queue.insert(s);
    while( queue is not empty ){
        u = queue.extractHead();
        for each edge <u, d>{
            if(d has not been
            visited)
                visit(d);
                queue.insert(d);
        }
    }
```

Note that: no matter visit 5 first or visit 6 first, they are BFS

BFS, BF tree and shortest path

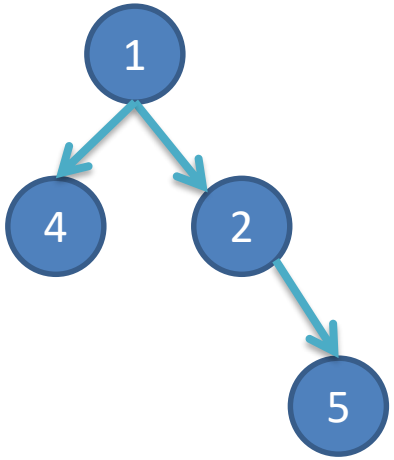
- Byproducts of BFS(s)
  - Breadth first tree
    - The tree constructed when a BFS is done
  - Shortest path
    - A path with minimum number of edges from one vertex to another
    - BFS(s) find out all the shortest paths from s to all its reachable vertices

BFS, BF tree and shortest path



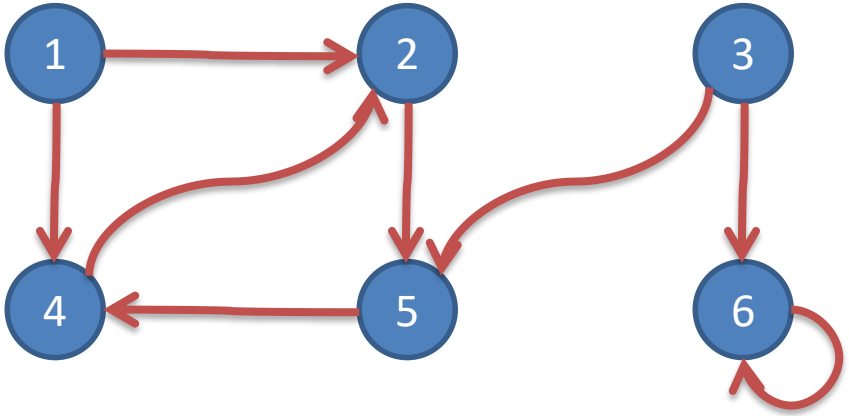
BFS(1): 1 4 2 5

BF Tree:



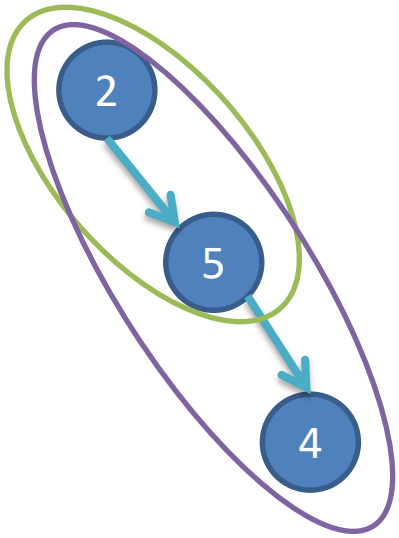
All shortest paths started from vertex 1 are found
e.g. 1 to 5

BFS, BF tree and shortest path



BFS(2): 2 5 4

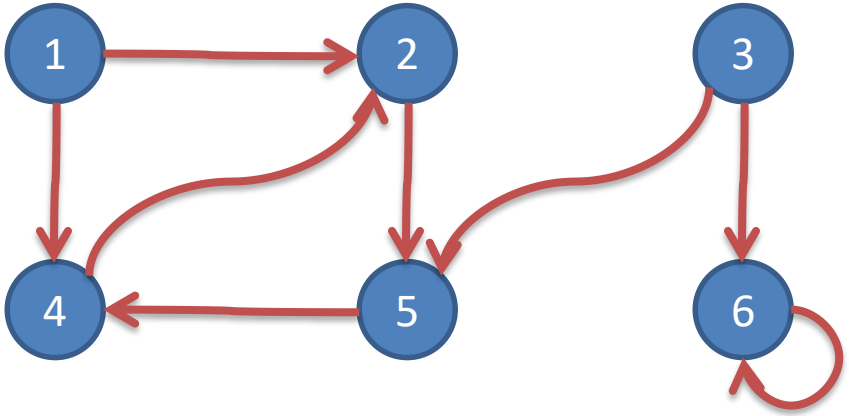
BF Tree:



Shortest 2 to 5

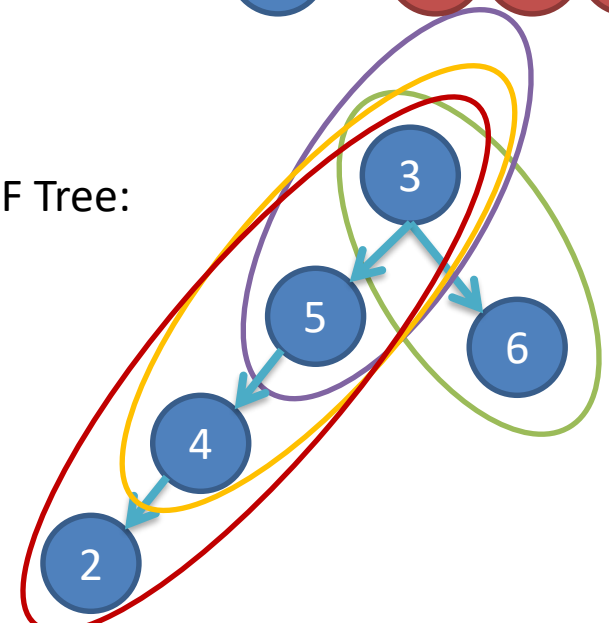
Shortest 2 to 4

BFS, BF tree and shortest path



BFS(3): 3 6 5 4 2

BF Tree:



Shortest 3 to 6

Shortest 3 to 5

Shortest 3 to 4

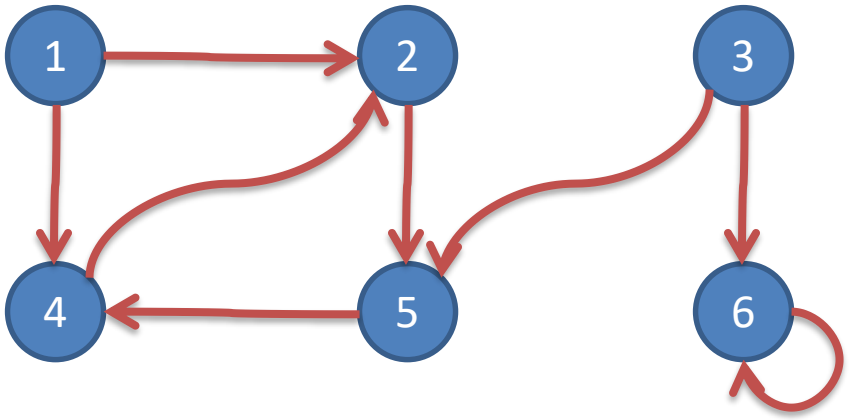
Shortest 3 to 2

BFS, BF tree and shortest path

- BFS Traversal
- **BFS_Traversal**(G)

```
for each v in G{
    if (v has not been visited)
        BFS(v);
}
```

BFS, BF tree and shortest path




```
BFS_Traversal(G)
for each v in G{
        if (v has not been visited)
                BFS(v);
}
```

Queue: 1 4 2 5 3 6

Visit order: 1 4 2 5 3 6

DFS, DF tree

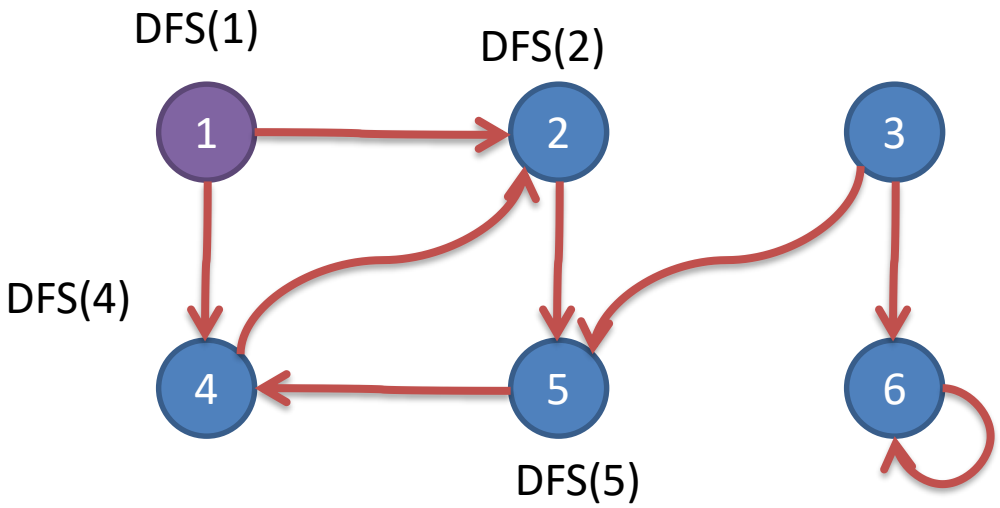
- Depth-first search
  - From a source vertex s
  - Depth-firstly search explores the edges to discover every vertex that is reachable from s
- **DFS**(s):

```
s.underDFS = true;    // grey
for each edge <s, d>{
    if(! d.underDFS and d has not been visited)
        DFS(d);
}
Visit(s); // black
```

From the deepest one to the current one

DFS, DF tree

DFS(1)

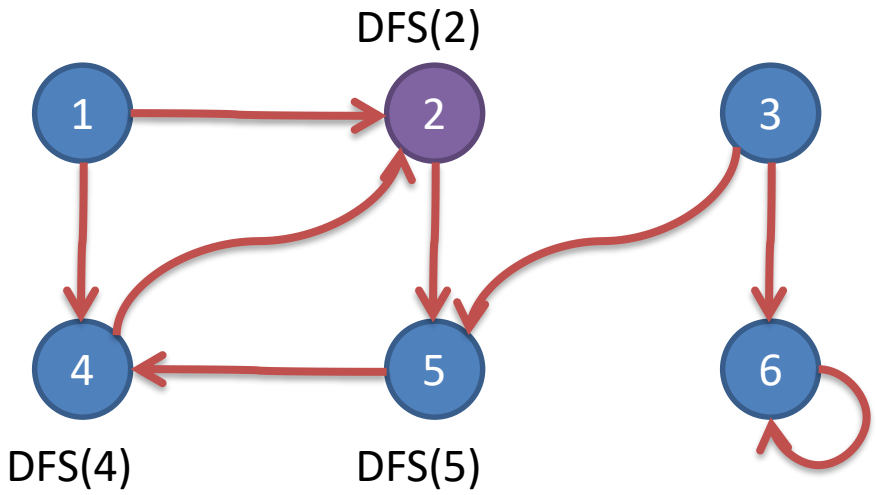



```
DFS(s):
    s.underDFS = true;
     for each edge <s, d>{
          if((! d.underDFS and d has
          not been visited)
           DFS(d);
    }
    Visit(s);
```

Visit order:

DFS, DF tree

DFS(2)

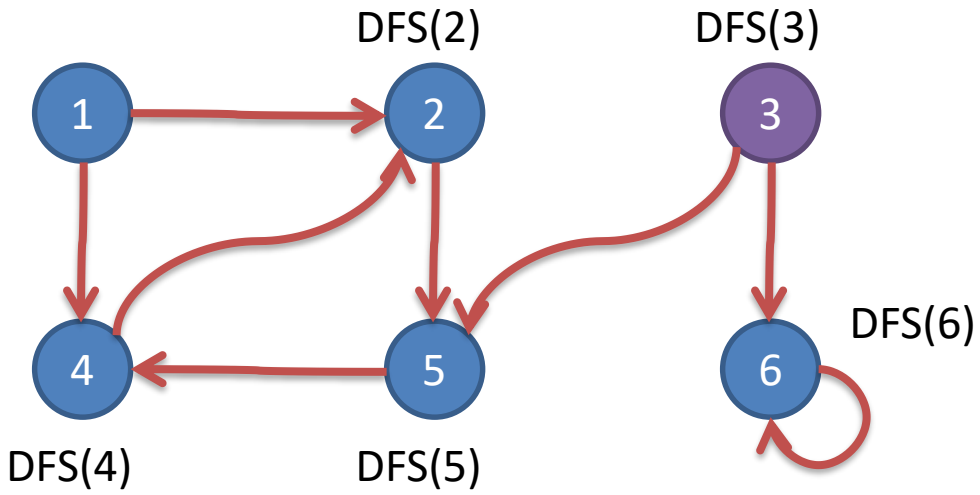



```
DFS(s):
    s.underDFS = true;
     for each edge <s, d>{
          if((! d.underDFS and d has
          not been visited)
               DFS(d);
     }
     Visit(s);
```

Visit order:

DFS, DF tree

DFS(3)




```
DFS(s):
    s.underDFS = true;
    for each edge <s, d>{
        if((!d.underDFS and d has
        not been visited)
            DFS(d);
    }
    Visit(s);
```

Visit order: 6 2 4 5 3

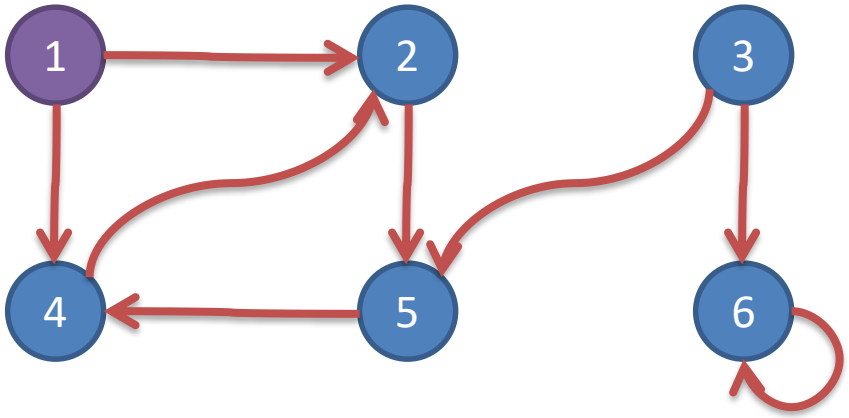
The reachable vertices are exactly the same with BFS, but with a different order

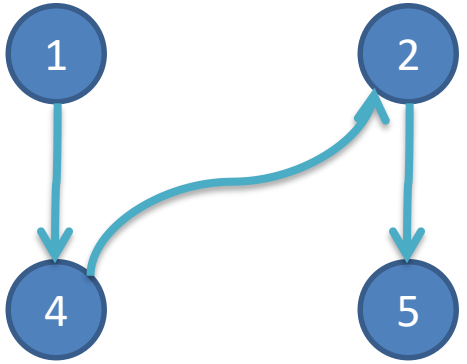
DFS, DF tree

- Depth first tree
  - The tree constructed when a DFS is done

DFS, DF tree

DF Tree **of DFS**(1)



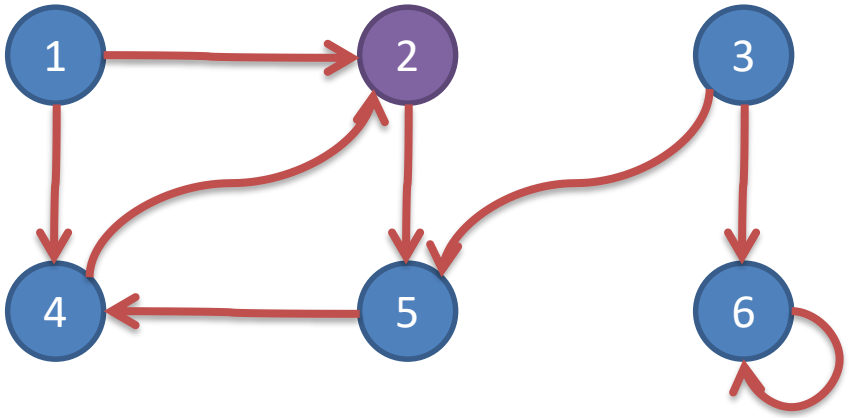


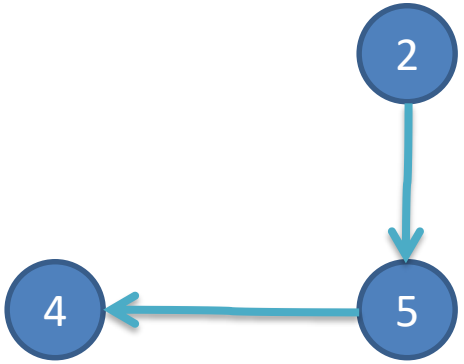
DF Tree

Visit order:

DFS, DF tree

DF Tree of **DFS**(2)



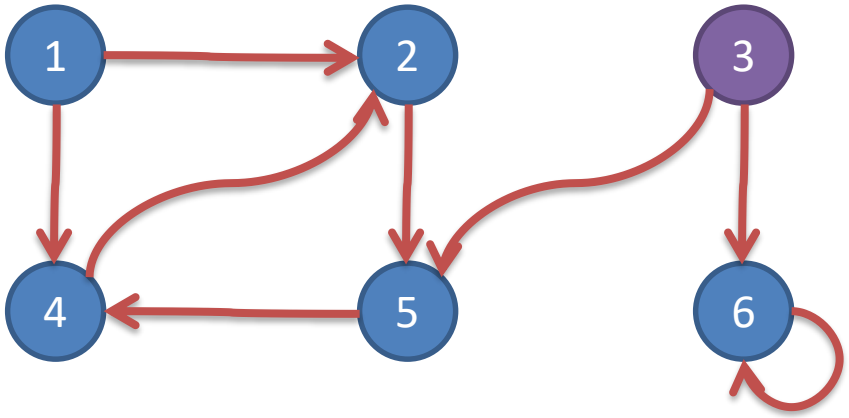


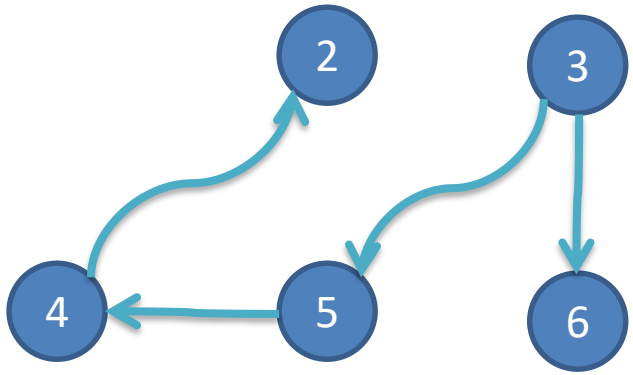
DF Tree

Visit order:

DFS, DF tree

DF Tree **of DFS**(3)





DF Tree

Visit order:

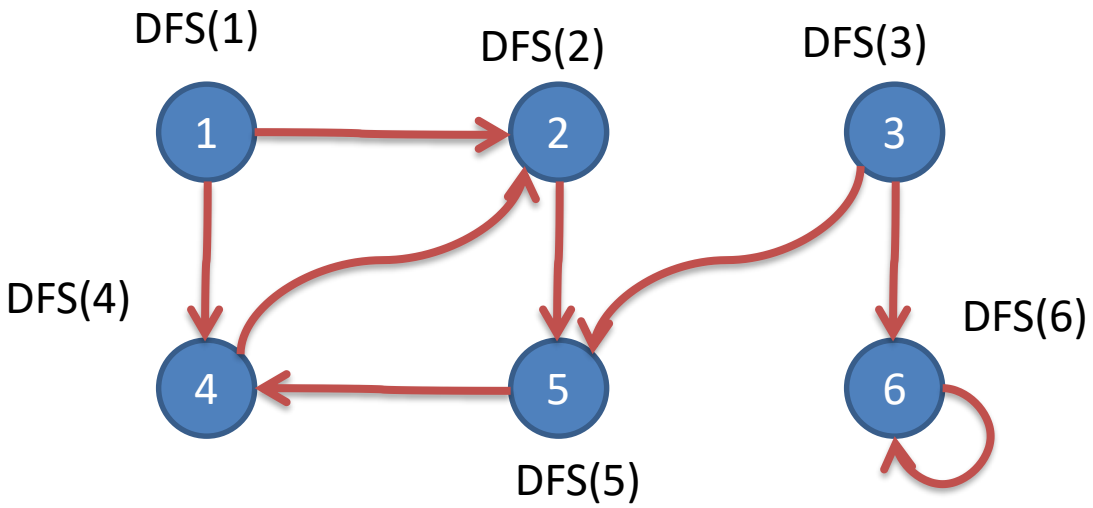
DFS, DF tree

- DFS Traversal // (The DFS in the textbook)
- **DFS_Traversal**(G)

```
for each v in G{
    if (v has not been visited)
        DFS(v);
}
```

DFS, DF tree

DFS_Traversal(G)

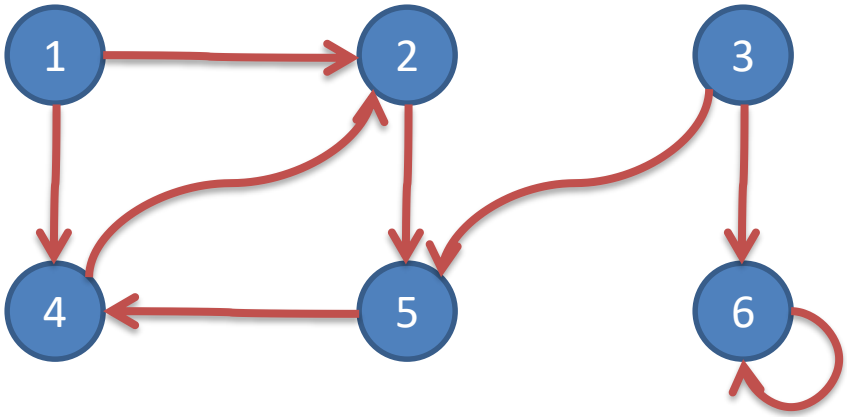



```
DFS_Traversal(G)
for each v in G{
        if (v has not been visited)
                DFS(v);
}
```

Visit order: 5 2 4 1 6 3

Topological sort

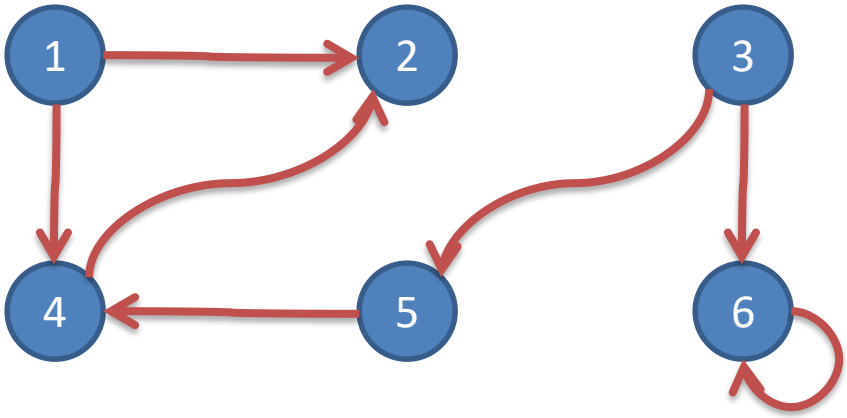
- DAG: directed acyclic graph
  - A graph without cycles



Dag? No

Topological sort

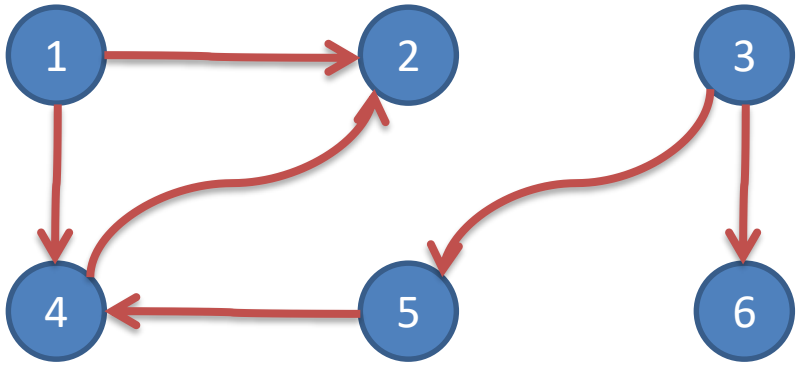
- DAG: directed acyclic graph
  - A graph without cycles



Dag? No

Topological sort

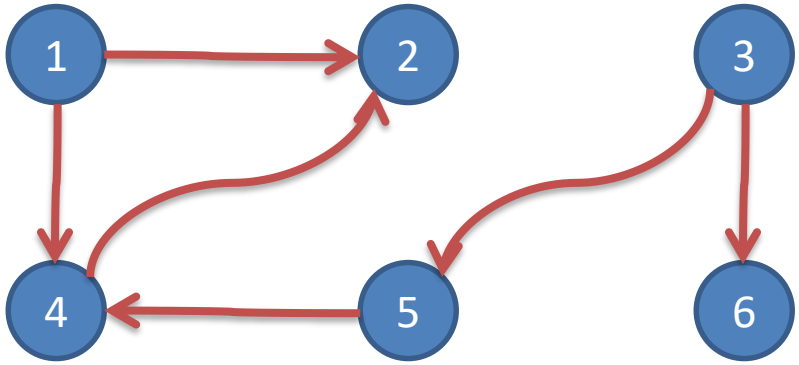
- Ordering in DAGs
  - If there is an edge <u, v>, then u appears before v in the ordering

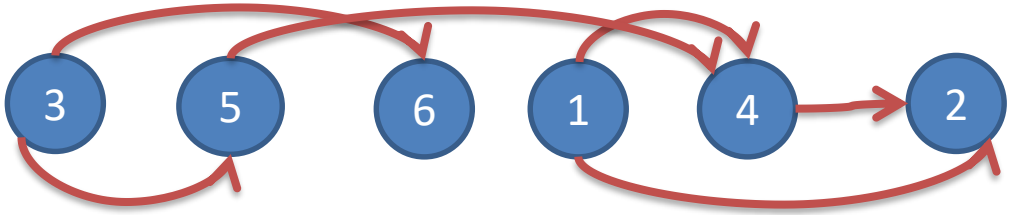


Dag? Yes

Topological sort

- Example



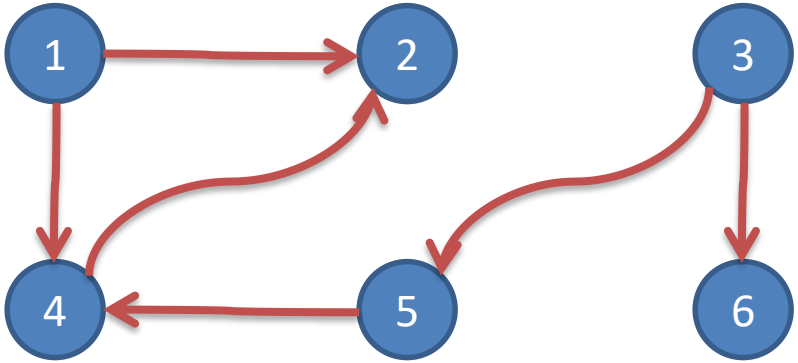


Put all the topological sorted vertices in a line, all edges go from left to right

Topological sort

- How to topological sort a dag?
- Just use **DFS_Traversal**
- The reverse order of DFS_Traversal is a topological sorted order

Topological sort



DFS_Traversal(G): 2 4 1 6 5 3