National University of Computer & Emerging Sciences, Karachi
Fall-2021 Department of Computer Science
Mid Term-2
22 November 2021, 10:30 AM – 12:30 PM

Course Code: CS2009	**Course Name:** Design and Analysis of Algorithm
Instructor Name / Names: Dr. Muhammad Atif Tahir, Dr. Fahad Sherwani, Dr. Farrukh Saleem, Waheed Ahmed, Waqas Sheikh, Sohail Afzal	
Student Roll No:	**Section:**

Instructions:

- Return the question paper.
- Read each question completely before answering it. There are **6 questions** on **2 pages.**
- In case of any ambiguity, you may make assumption. But your assumption should not contradict any statement in the question paper.

Time: 120 minutes. **Max Marks: 17.5**

Question # 1 **[0.5*4 = 2 marks]**

Are these following statements True or False? Prove your answer by computing the values of n_0, c_1, c_2 or by contradiction.

a) $2^{2n} = O\ (2^n)$
b) $n^2\ (2n - 5) = \Theta(\ n^3)$
c) $n^2 + 2n + 10\ = \Omega\ (n)$
d) $\sqrt{1000n^2 + 100n} = \Theta(\ n^2)$

Question # 2 **[1 + 1 = 2 marks]**

Compute the time complexity for both below mentioned algorithms separately. Show all the steps.

(a)

```
 public static void main(String[ ] args) {

for (int i=1 ; i<=n ; i++) {
          for (int j=1; j<n; j=j+i) {
                    System.out.println("*");
                    break;
                    }
          }
}
```

(b)

```
 public static void main(String[ ] args) {
 int n=100;
for (int i=n/2 ; i<=n ; i++) {
          for (int j=1; j<n; j=j*2) {
                    for (int k=1; k<=n; k=k*2) {
                        System.out.println("*");
                    }
          }
}}
```

Question # 3 **[2.5+1.5 = 4 marks]**

a) Prove by contradiction below given Lemma. Also give a small example

Given: Let **G** be a weighted graph, and **S** be a subset of its edges. Let **T** be the *MST* of **G**, when the edges of **S** are given weights lower than those of any other edge in the graph.

Prove that No matter how the edge weights in **S** are changed, the *MST* of **G** will always contain the edges in **T-S**

b) An edge in an undirected graph G is a bridge if removing it disconnects the graph. Design algorithm to find all the bridge edges.

Question # 4 **[3 marks]**

Consider the following instance of the 0/1 knapsack problem

Item	1	2	3	4	5
Benefit	15	35	10	9	9
Weight	4	12	4	4	5

The maximum allowable total weight in the knapsack is W = 9.

Find an optimal solution for the above problem with the weights and benefits above using Dynamic Programming. Be sure to state both the value of the maximum benefit that you obtain as well as the item(s) that you need to obtain this benefit. Show all steps.

Question # 5 **[4 marks]**

Suppose we have an m x n grid ("m" rows and "n" columns), where each cell has a "reward" associated with it. Let's also assume that there's a robot placed at the starting location, and that it has to find its way to a "goal cell". While doing this, it will be judged by the path it chooses. We want to get to the "goal" via a path that collects the maximum reward. The only moves allowed are "up" and "right". Give an algorithm that utilizes dynamic programming to find the required path.

Collect Maximum Reward				
	Column 0	Column 1	Column 2	Column 3
Row 3	4	2	1	5
Row 2	4	8	3	3
Row 1	3	5	6	2
Row 0	1	7	2	1
Start Cell: (0, 0) Goal Cell: (3, 3) Possible Moves: U, R				

Question # 6 **[1+1.5=2.5 marks]**

You are given a list of ***n-1*** unsorted integers and these integers are in the range of ***1 to n***. There are no duplicates in the list. One of the integers is missing in the list.

a) Design algorithm to find the missing integer in **O(n)** time using linear sorting

b) Design algorithm to find the missing integer in **O(n)** time without using linear sorting

Example **Input:** arr[] = {1, 2, 4, 6, 3, 7, 8} **Output:** 5