GT ASSIGNMENT 02

Bilal Ahmed Khan

20K0183 **Sec: B**

QUESTION 01

i) The total degree of a tree on n vertices $2n-2$.

We prove that a tree of n vertices has $n-1$ edges.

1) For $n=1$ the graph will have 0 edge
2) For $n=k$, the graph will have $k-1$ edges
3) For $n=k+1$ the edge will be k, removing one vertex from the tree will prove point 02
   i) Removing any leaf from the graph The vertices are now k.
   ii) Since leaves only contribute 1 edge in trees, the total no. of edges is reduced to $k-1$
   iii) Thus The total no. of edges in a tree of n vertices is $n-1$

From Handshaking Lemma: Total degree is twice the no. of edges.

Degree of tree $= 2$(No of edges)
$= 2(n-1)$

$\boxed{\text{Degree of a tree} = 2n-2}$

ii) Let G be a graph with n vertices, the $\kappa(G) = \delta(G)$ if $\delta(G) \geq n-2$

We have to prove that the size of cut vertex set is equal to the minimum degree of the graph only when the minimum degree is atleast 2.

(Continue from next page)

Suppose we have a graph of n vertices where $n \geq 2$, for every induced graph of order 3 or more must be connected since if a & b are non adjacent, they are adjacent to every other vertex in the subgraph.

So deleting $n-3$ will be never sufficient to disconnect the graph

Hence Proved.

iii) Let G Be a graph with adjacent vertices x & y. Prove $e(x)$ & $e(y)$ differ by at most 1

Let max distance i.e. eccentricity of x from all vertices Be a and eccentricity of y Be b.

Since x and y are adjacent, they are at a distance of 1 from each other

Thus, all distances from x will be separated by a path of length of one from all distances from y. Hence proved that $a = b \pm 1$

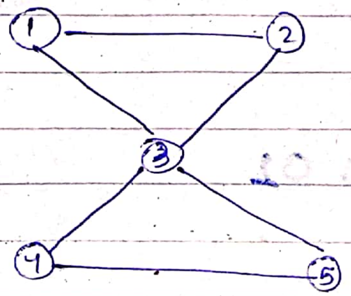
QUESTION 02

02(a)

Since all paths between graph components include the bridge, they will also include both of its endpoints. Therefore, by definition of the cut-vertex i.e. the vertex that included in all paths, both end-points of the cut vertex will be cut-vertices.

02:b)

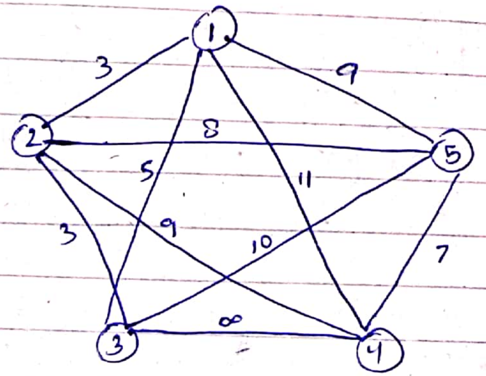
The converse is not true, consider the following example.



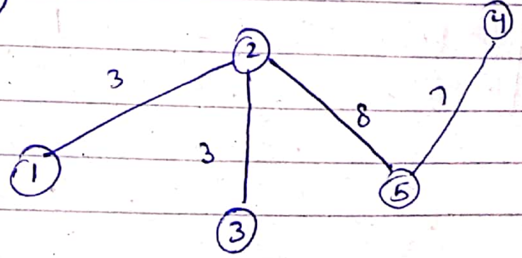
The graph has 5 vertices and a cut-vertex but lacks any Bridge Because all edges are contained in a cycle and bridges are never included in any cycle or circuit.

QUESTION 03

To calculate the minimum spanning tree we will use Kruskal algorithm.



The MST with the cost of 21 is given below



QUESTION 04

Using Flyod-warshall Algorithm

Travel Time

Immediate vertex (- means directly connected)

A^0

	1	2	3	4	5
1	0	20	20	∞	17
2	7	0	5	22	13
3	14	13	0	15	17
4	30	∞	17	0	10
5	∞	15	12	8	0

	1	2	3	4	5
1	-	-	-	-	-
2	-	-	-	-	-
3	-	-	-	-	-
4	-	-	-	-	-
5	-	-	-	-	-

A^1

	1	2	3	4	5
1	0	10	20	∞	17
2	7	0	5	22	24
3	14	13	0	15	27
4	30	40	17	0	10
5	∞	15	12	8	0

	1	2	3	4	5
1	-	-	-	-	-
2	-	-	-	-	1
3	-	-	-	-	-
4	-	1	-	-	-
5	-	-	-	-	-

A^2

	1	2	3	4	5
1	0	10	15	32	17
2	7	0	5	22	24
3	14	13	0	15	27
4	30	40	17	0	10
5	22	15	12	18	0

	1	2	3	4	5
1	-	-	2	2	-
2	-	-	-	-	1
3	-	-	-	-	-
4	-	1	-	-	-
5	2	-	-	-	-

A^3

	1	2	3	4	5
1	0	10	15	30	17
2	7	0	5	20	24
3	14	13	0	15	27
4	30	30	17	0	10
5	22	15	12	8	0

	1	2	3	4	5
1	-	-	2	3	-
2	-	-	-	3	1
3	-	-	-	-	-
4	-	3	-	-	-
5	2	-	-	-	-

A^4

	1	2	3	4	5
1	0	10	15	30	17
2	7	0	5	20	24
3	14	13	0	15	25
4	30	30	17	0	10
5	22	15	12	8	0

	1	2	3	4	5
1	-	-	2	3	-
2	-	-	-	3	1
3	-	-	-	-	4
4	-	3	-	-	-
5	2	-	-	-	-

A^5

	1	2	3	4	5
1	0	10	13	25	17
2	7	0	5	20	24
3	14	13	0	15	25
4	30	25	17	0	10
5	22	15	12	8	0

*	1	2	3	4	5
1	-	-	2	5	-
2	-	-	-	3	1
3	-	-	-	-	4
4	-	5	-	-	-
5	2	-	-	-	-

QUESTION 05

For calculating total no. of spanning trees we use the method from geeksforgeeks.com/total-number-spanning-trees-graph/

The matrix formed is.

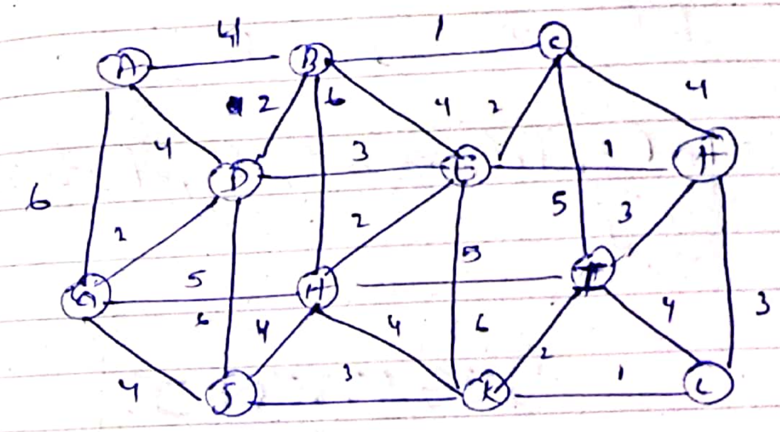
2	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	-1
-1	2	-1	0	0	0	0	0	0	0	0	0	0	0	0	0
0	-1	2	-1	0	0	0	0	0	0	0	0	0	0	0	0
0	0	-1	4	-1	0	0	-1	0	0	0	0	0	0	0	-1
0	0	0	-1	2	-1	0	0	0	0	0	0	0	0	0	0
0	0	0	0	-1	2	-1	0	0	0	0	0	0	0	0	0
0	0	0	0	0	-1	2	-1	0	0	0	0	0	0	0	0
0	0	0	-1	0	0	-1	4	-1	0	0	-1	0	0	0	0
0	0	0	0	0	0	0	-1	2	-1	0	0	0	0	0	0
0	0	0	0	0	0	0	0	-1	2	-1	0	0	0	0	0
0	0	0	0	0	0	0	0	0	-1	2	-1	0	0	0	0
0	0	0	0	0	0	0	-1	0	0	-1	4	-1	0	0	-1
0	0	0	0	0	0	0	0	0	0	0	-1	2	-1	0	0
0	0	0	0	0	0	0	0	0	0	0	0	-1	2	-1	0
0	0	0	0	0	0	0	0	0	0	0	0	0	-1	2	-1
-1	0	0	-1	0	0	0	0	0	0	0	-1	0	0	-1	4

The cofactor matrix is calculated to be.

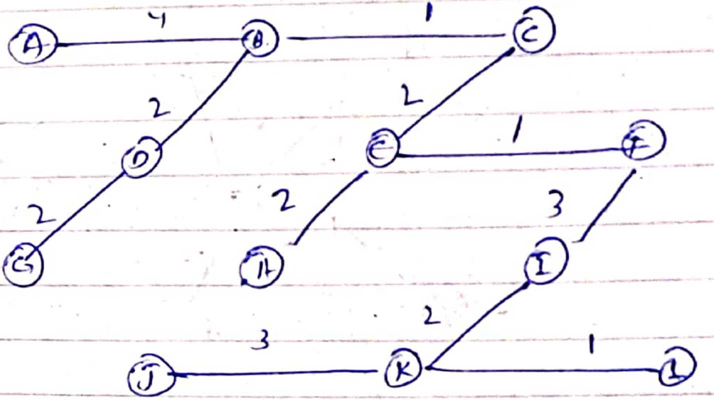
2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000
2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000
2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000
2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000
2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000
2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000
2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000
2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000
2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000
2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000
2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000
2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000
2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000
2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000
2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000
2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000
2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000

Therefore the number of spanning trees is 2000.

QUESTION 06



06: a)



Order: bc, ef, kl, bd, dg, ce, eh, ki,
kj, fi, ab.

Cost: 23.

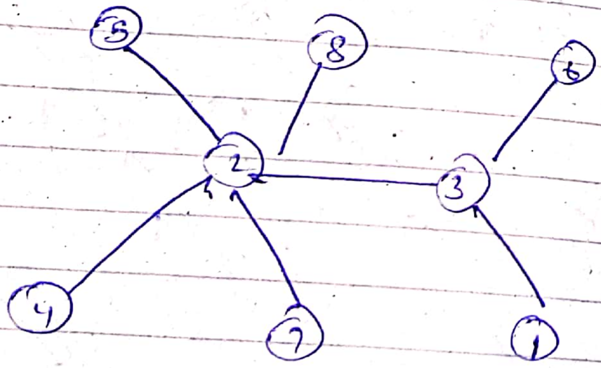
06:b)

MST will be same
but order will be

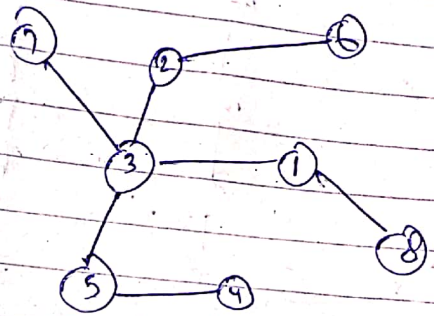
ab, bc, bd, ce, ef, dg, eh, fi, ik,
kl, kj

QUESTION 07

07:a)

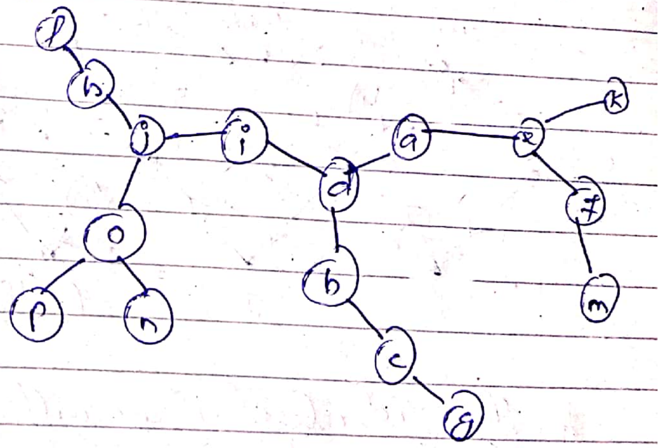


07:b)

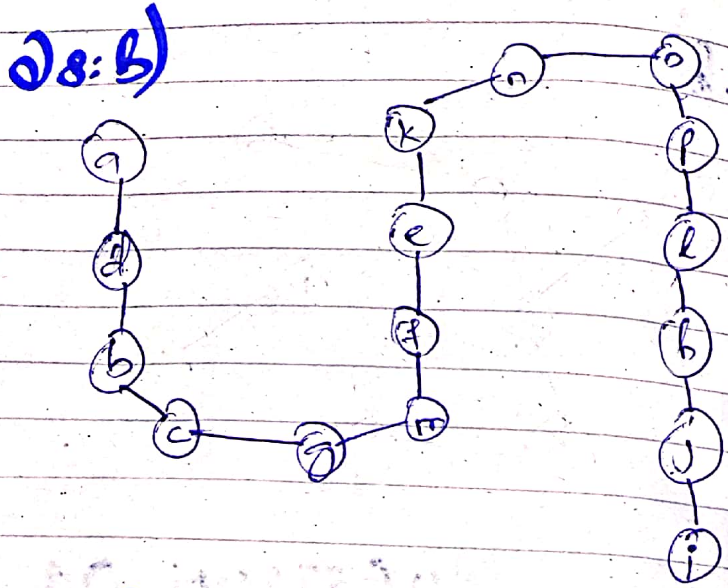


QUESTION 08

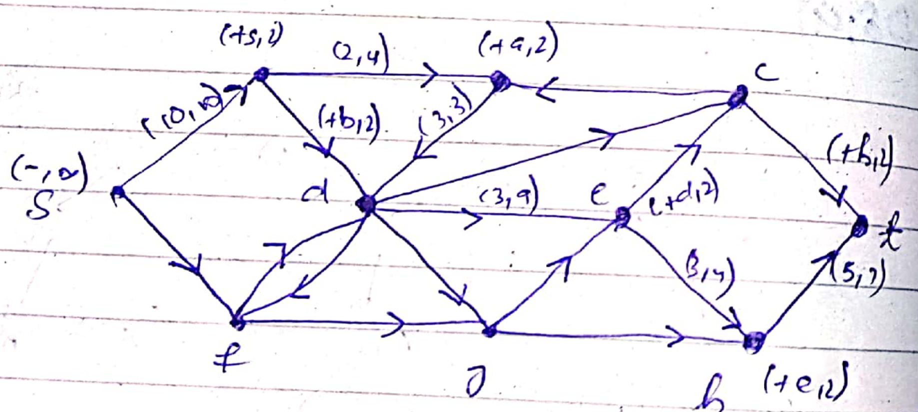
08:a)



Q8: B)



QUESTION 07



In the 2nd iteration we will be blocked at s. Hence only s will be labeled and a flow of 2 is pumped

Total flow is 14.

MIN-CUT METHOD:

Partitions are: P = {s} & $\overline{P}$ = {a, B, c, d, e, f, g, h, t}

Flow from P to $\overline{P}$: 10 + 4 = 14.

x ——— x ———

End of Assignment.