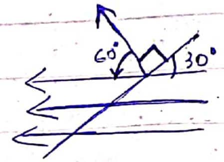
Gauss's Law Problems

Question 01:-

Data



$r = 12\,cm = 0.12\,m$

$\theta' = 30°$; $\theta = 90 - 30 = 60°$

$E = 450\,N/C$

$\phi = ?$

Solution

$$\phi = \oint \vec{E} \cdot d\vec{A}$$

$$\phi = E \cdot \pi r^2 \cos\theta$$

$$\phi = 450 \times 3.142 \times (0.12)^2 \times \cos 60°$$

$$\boxed{\phi = 10.18\ Nm^2/C}$$

Question 02

Data

$q_1 = 6\,\mu C$

$q_2 = -8\,\mu C$

$q_{net} = 6 - 8 = -2\,\mu C$

$r = 5\,cm = 0.05\,m$

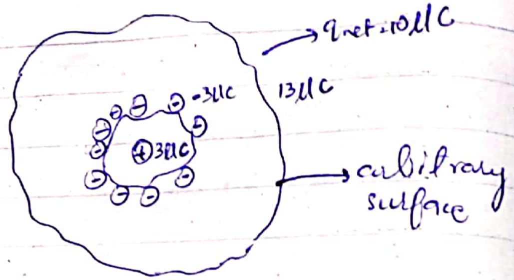
$\phi = ?$

Solution

$$\phi = \frac{q_{net}}{\varepsilon_0} = \frac{-2\times10^{-6}}{8.85\times10^{-12}}$$

$$\boxed{\phi = -2.25\times10^{5}\ Nm^2/C}$$

QUESTION 03:-



Data

$q_{net} = 10\mu C$

a) q on cavity wall = ?

b) q on outer surface = ?

Solution

a) Since the net charge inside a conductor is zero the charge on cavity walls will be

$$\boxed{q_{\substack{cavity\\wall}} = -3\mu C}$$

Ans

b) Since the net charge on the body must be equal to $10\mu C$ therefore,

~~$q_{net} = q_{outer\,surface} + q_{cavity\,wall} + q_{charge\,in\,cavity}$~~

$$q_{outer\,surface} = q_{net} - q_{inner\,wall}$$

$$= 10\mu C - (-3\mu C)$$

$$\boxed{q_{outer\,surface} = 13\mu C}$$

Ans

Question 04:-

Data

$q = 1.8 \mu C$

$l = 55 cm = 0.55 m$

Solution

$$\phi = \frac{q_{net}}{\varepsilon_0} = \frac{1.8 \times 10^{-6}}{8.85 \times 10^{-12}}$$

$$\boxed{\phi = 2.03 \times 10^{5} Nm^2/C}$$

Ans

Question 05

Data:-

$d = 1.2 m$

$r = 0.6 m$

$\sigma = 8.1 \mu C/m^2$

$q_{net} = ?$

$\phi = ?$

Solution:-

→ For q_{net}:-

we know that,

$$\sigma = \frac{q_{net}}{A} \quad (\because A = \pi r^3 \times 4/3)$$

$$q_{net} = \sigma \times \frac{4}{3} \pi r^3$$

$$q_{net} = 8.1 \times 10^{-6} \times \frac{4}{3} \times 3.142 \times (0.6)^3$$

$$q_{net} = 7.33 \times 10^{-6} C$$

$$\boxed{q_{net} = 7.33 \mu C}$$

Ans

→ FOR ϕ:-

Since $\phi = q_{net}/\varepsilon_0 = \frac{7.33 \times 10^{-6}}{8.85 \times 10^{-12}}$

$$\boxed{\phi = 8.28 \times 10^{5} Nm^2/C}$$

Ans

Question 06:

Data

$E = 4.52 \times 10^4$ N/C

$r = 1.96$ m

$\lambda = ?$

Solution:-

We know for a cylindrical charge distribution

$$E = \frac{2K\lambda}{r}$$

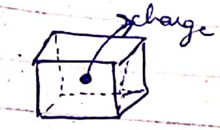
$$\lambda = \frac{E \times r}{2K}$$

$$= \frac{4.52 \times 10^4 \times 1.96}{2 \times 9 \times 10^9}$$

$$\lambda = 4.92 \times 10^{-6}\ C/m$$

$$\boxed{\lambda = 4.92\ \mu C/m}$$

Question 07:



Data

$q = 60\ \mu C$

$l = 10cm = 0.1m$

a) total flux = ?

b) flux through the face = ?

c) will flux change with position of charge = ?

Solution:

a) For total flux

$$\phi = \frac{q_{net}}{\varepsilon_0} = \frac{60 \times 10^{-6}}{8.85 \times 10^{-12}}$$

$$\boxed{\phi = 6.77 \times 10^6\ Nm^2/C}$$

b) For flux through each face:

Since the cube has 6 faces:

ϕ through each face $= \phi_{total}/6$

$= 6.77 \times 10^{6}/6$

$\boxed{\phi \text{ through each face} = 1.12 \times 10^{6}\ Nm^2/C}$

Ans

c)

No, the flux in parts a) & b) will not change since flux depends upon "charge enclosed" not on the position of the charge.

Ans

Question 08:

Data:

$r = 8cm = 0.08\,m$

$\sigma = 0.1\,nC/m^2$

a) Electric Field at surface $= ?$

b) Electric Field at 10cm from center $= ?$

Solution:

For q:

$q = \sigma \times A = 0.1 \times 10^{-9} \times 4 \times 3.142 \times (0.08)^2$

$\boxed{q = 8.04 \times 10^{-12}\, C}$

a) For E at surface:

$E = \frac{Kq}{r^2} = \frac{9 \times 10^{9} \times 8.04 \times 10^{-12}}{(0.08)^2}$

$\boxed{E = 11.306\ N/C}$

Ans

b) E at 10 cm from center

$$E = \frac{Kq}{r^2} = \frac{9\times10^{9} \times 8.04\times10^{-12}}{(0.1)^2}$$

$$\boxed{E = 7.236\,N/C}$$

Ans