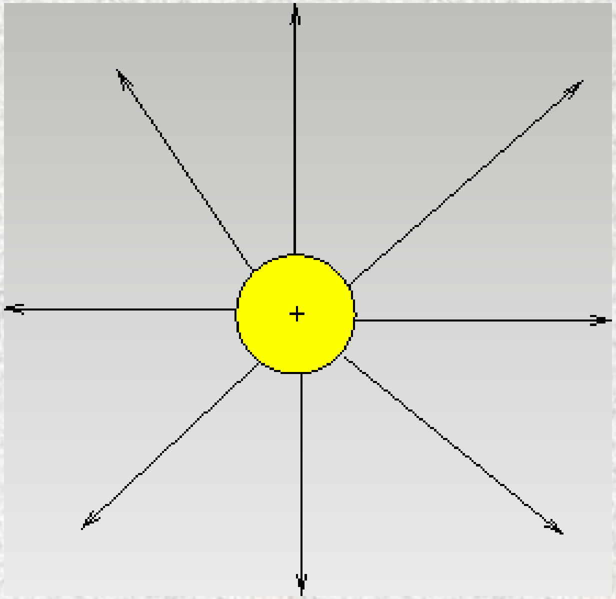
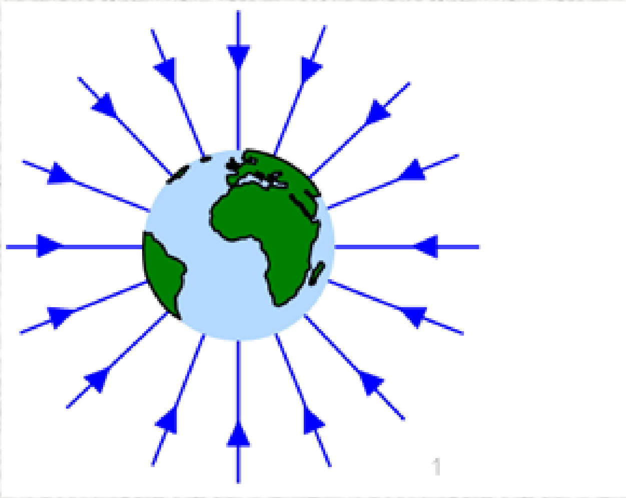
Analogy



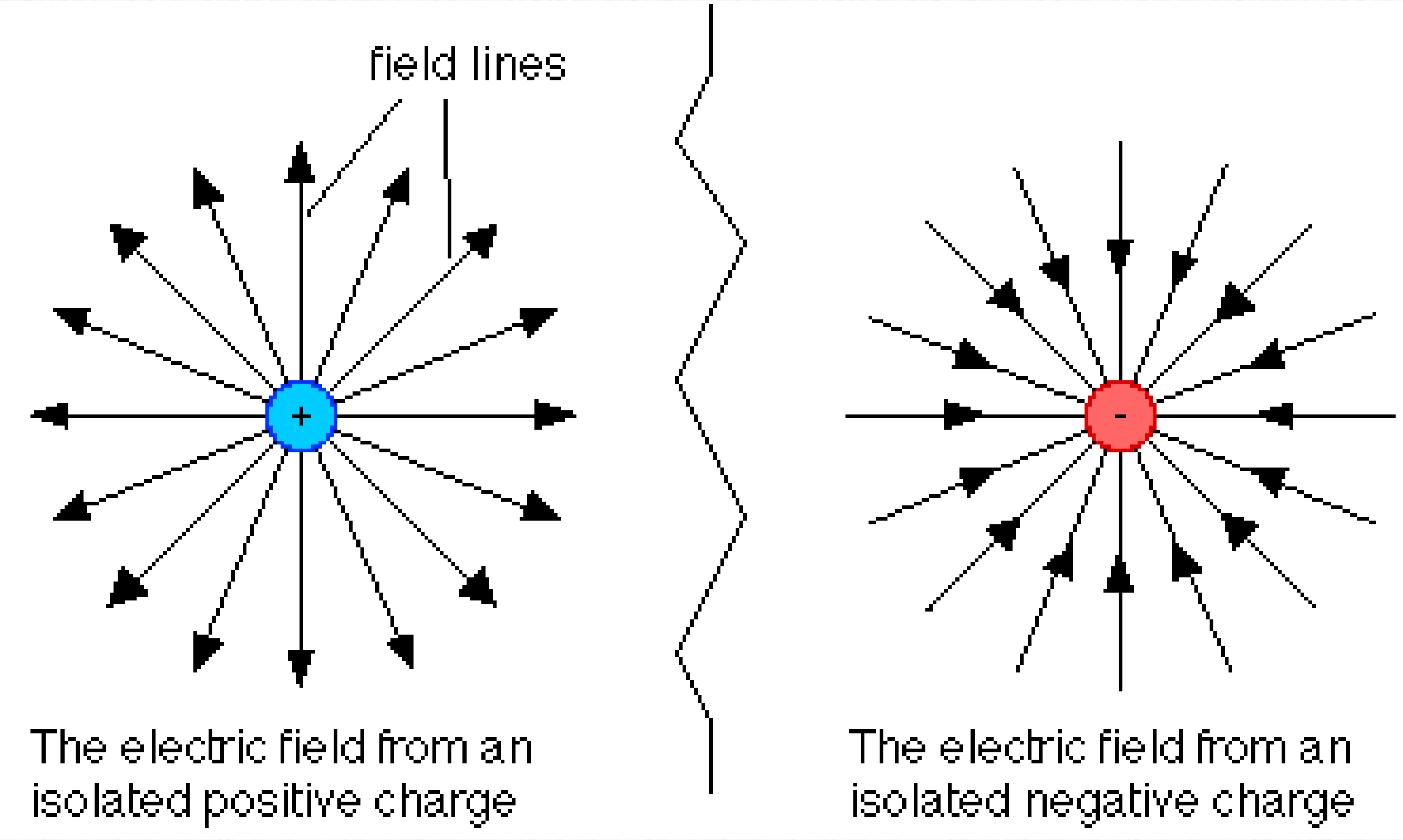
The electric field is the space around an **electrical charge**

just like

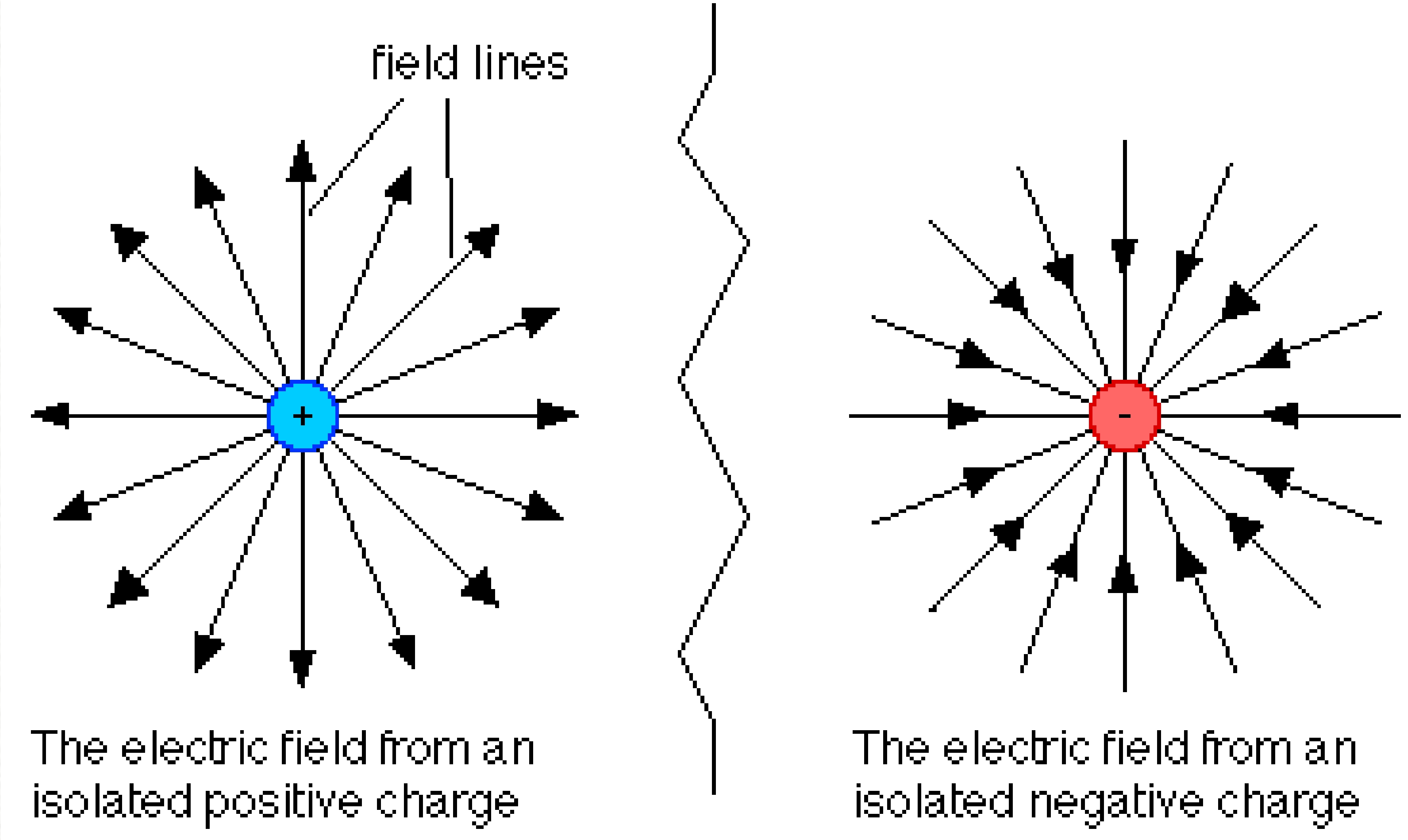
a gravitational field is the space around a **mass.**



Electric Field

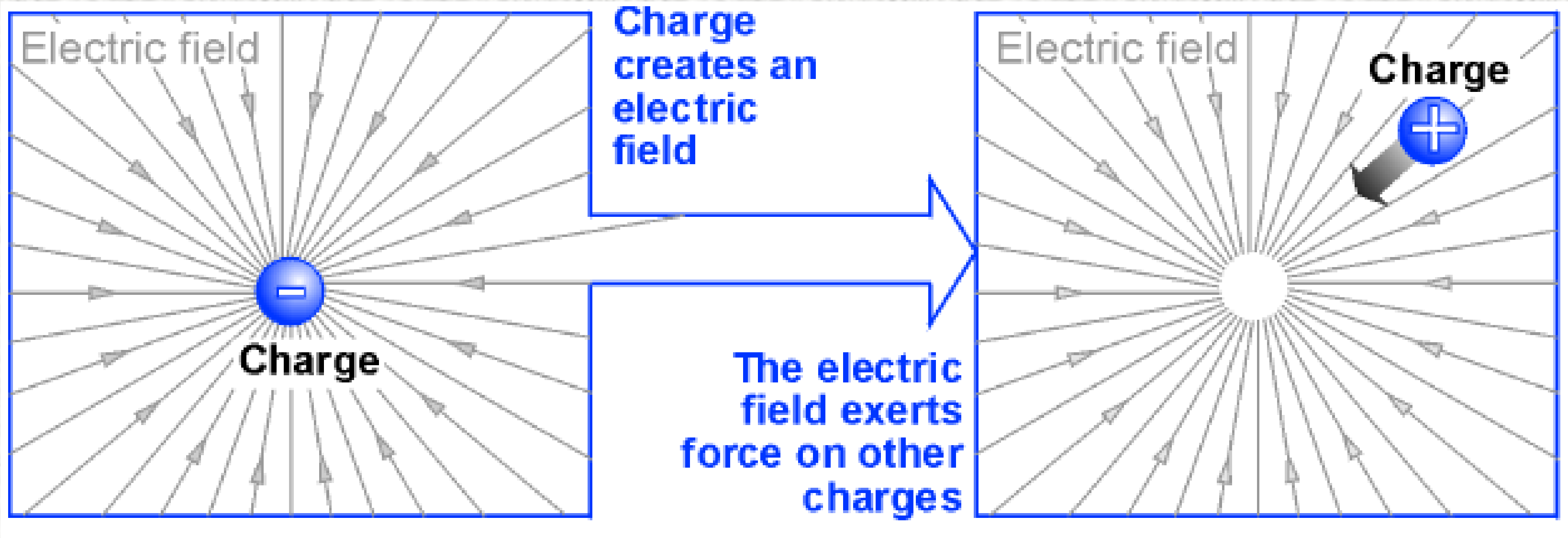


What is the difference?

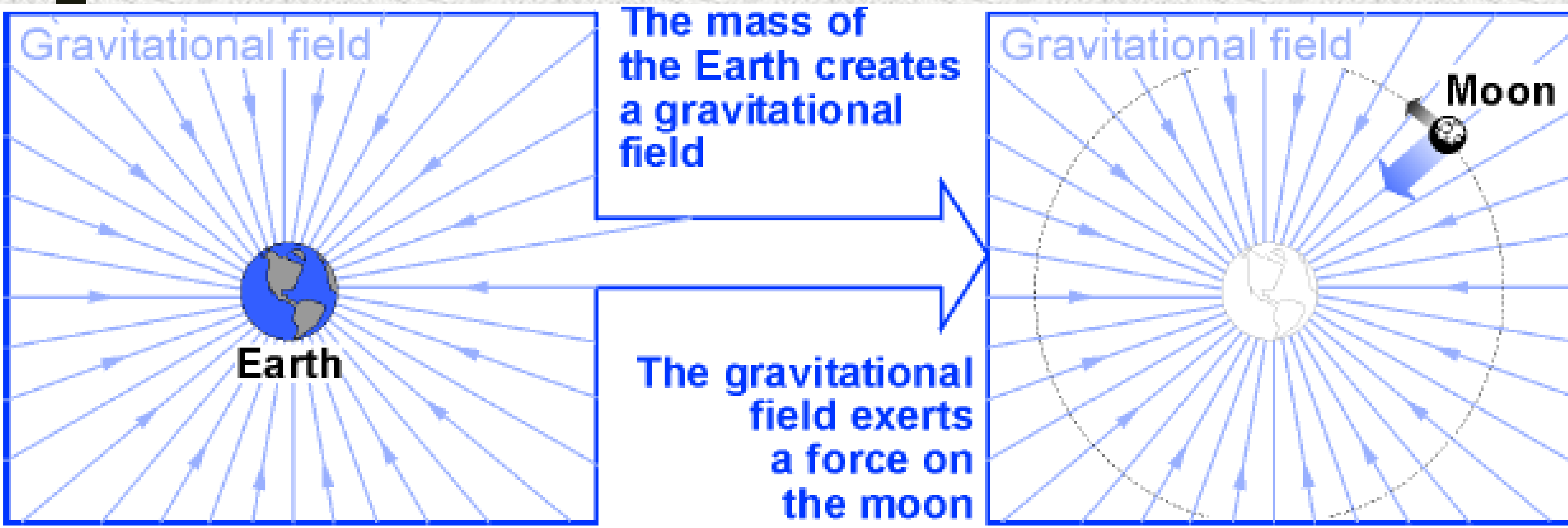


Fields and forces

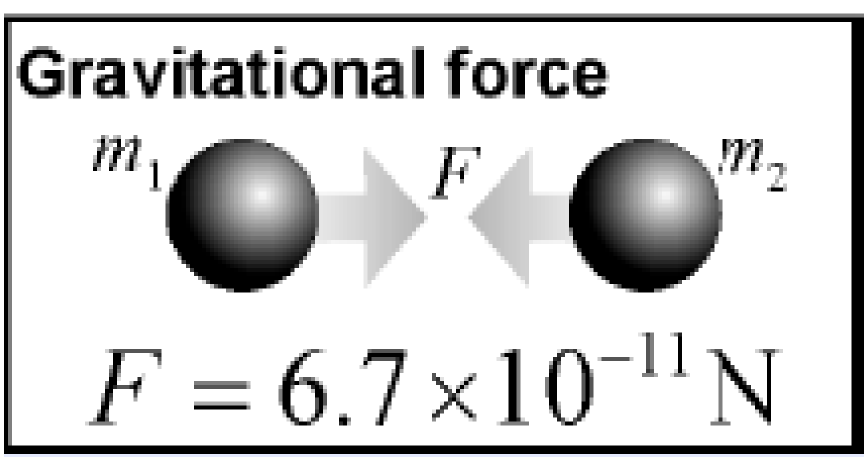
- The concept of a field is used to describe any quantity that has a value for all points in space.
- You can think of the field as the *way* forces are transmitted between objects.
- Charge creates an electric field that creates forces on other charges.

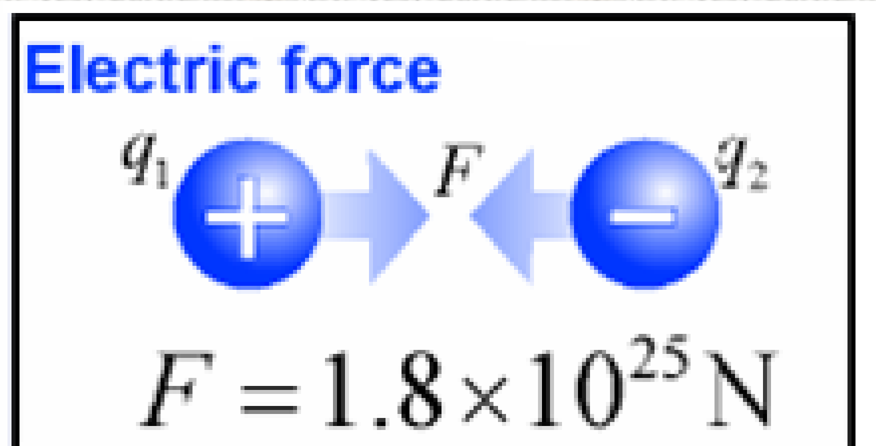


- Mass creates a gravitational field that exerts forces on other masses.



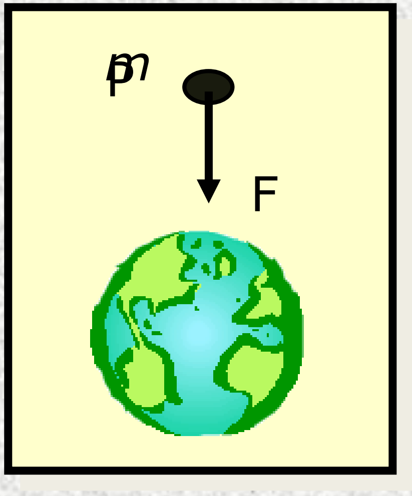
Gravitational forces are far weaker than electric f





The Concept of a Field

A field is defined as a property of space in which a material object experiences a force.



Above earth, we say there is a gravitational field at P.

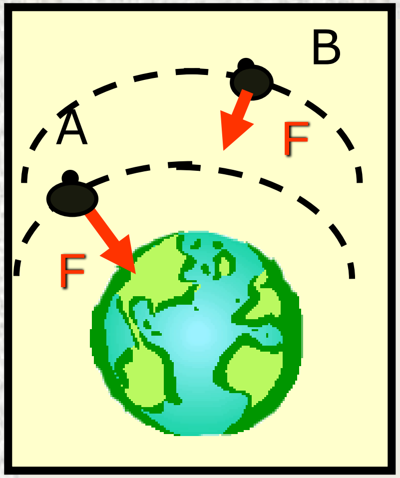
Because a mass *m* experiences a downward force at that point.

No force, no field; No field, no force!

The direction of the field is determined by the force.

The Gravitational Field

Consider points A and B above the surface of the earth—just points in space.



Note that the force F is real, but the field is just a convenient way of describing space.

The field at points A or B might be found from:

$$g = \frac{F}{m}$$

If g is known at every point above the earth then the force F on a given mass can be found.

The magnitude and direction of the field g is depends on the weight, which is the force F.

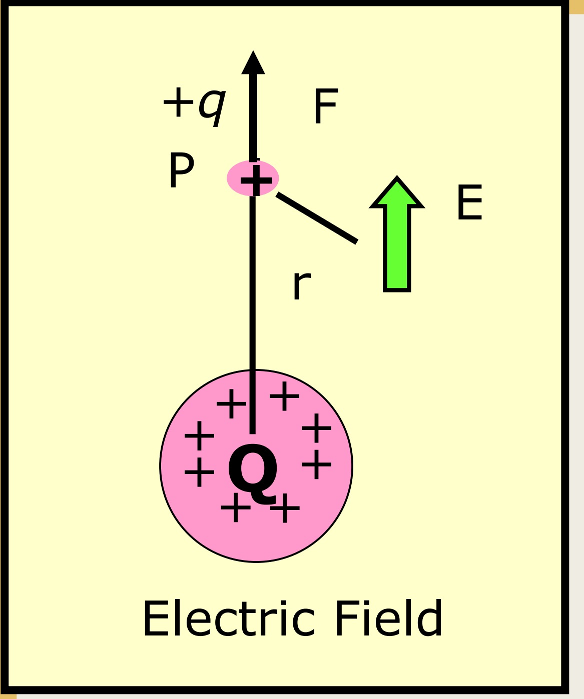
The Electric Field

1. Now, consider point P a distance *r* from +Q.

2. An electric field E exists at P if a test charge +q has a force F at that point.

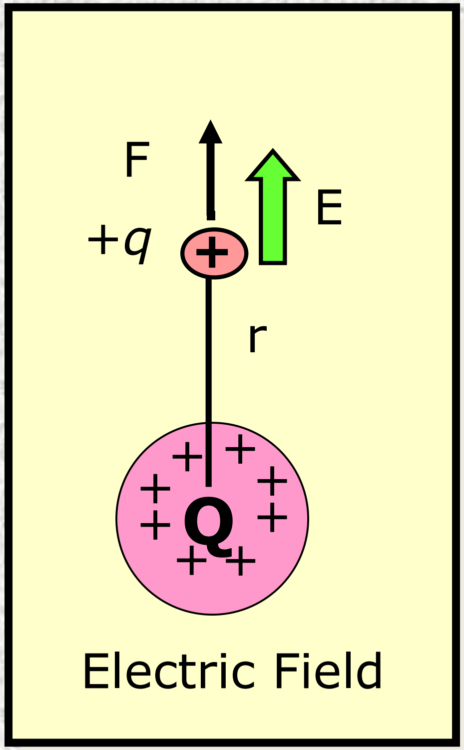
3. The direction of the E is the same as the direction of a force on + (pos) charge.

4. The magnitude of E is given by the formula:



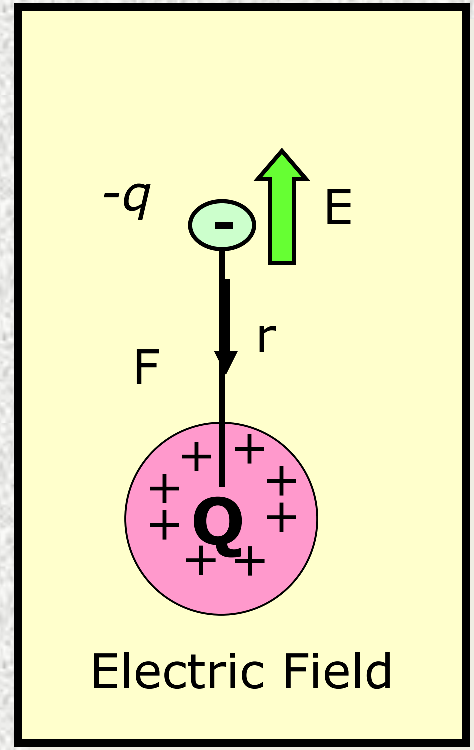
$$E = \frac{F}{q}; \text{ Units } \frac{\text{N}}{\text{C}}$$

Field is Property of Space



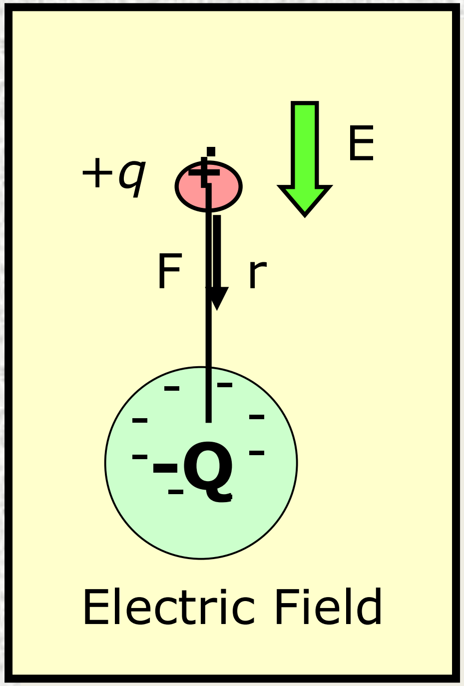
Force on +q is with field direction.

Force on -q is against field direction.



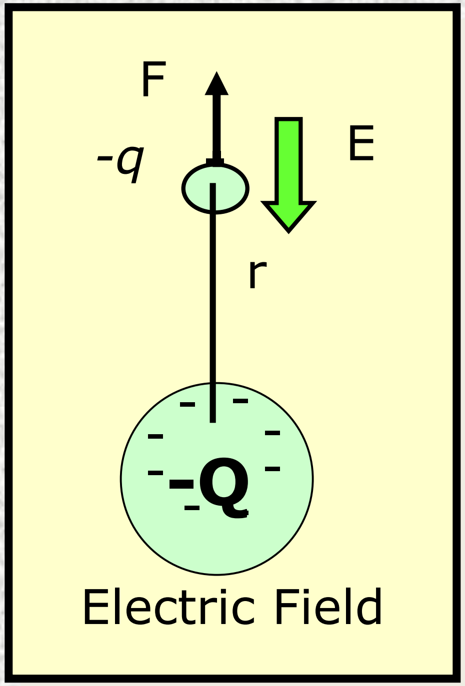
The field E at a point exists whether there is a charge at that point or not. The direction of the field is away from the +Q charge.

Field Near a Negative Charge



Force on +q is with field direction.

Force on -q is against field direction.



Note that the field E in the vicinity of a negative charge –Q is toward the charge—the direction that a +q test charge would move.

The Magnitude of E-Field

The magnitude of the electric field intensity at a point in space is defined as the force per unit charge (N/C) that would be experienced by any test charge placed at that point.

Electric Field Intensity E

$$E = \frac{F}{q};\ \text{Units}\left(\frac{\text{N}}{\text{C}}\right)$$

The direction of E at a point is the same as the direction that a positive charge would move IF placed at that point.

Relationship Between F and E

$$\vec{\mathbf{F}}_e = q\vec{\mathbf{E}}$$

- If q is placed in electric field , then we have
  - *This is valid for a point charge only*
  - *For larger objects, the field may vary over the size of the object*
- If *q* is positive, the force and the field are in the same direction
- If *q* is negative, the force and the field are in opposite directions

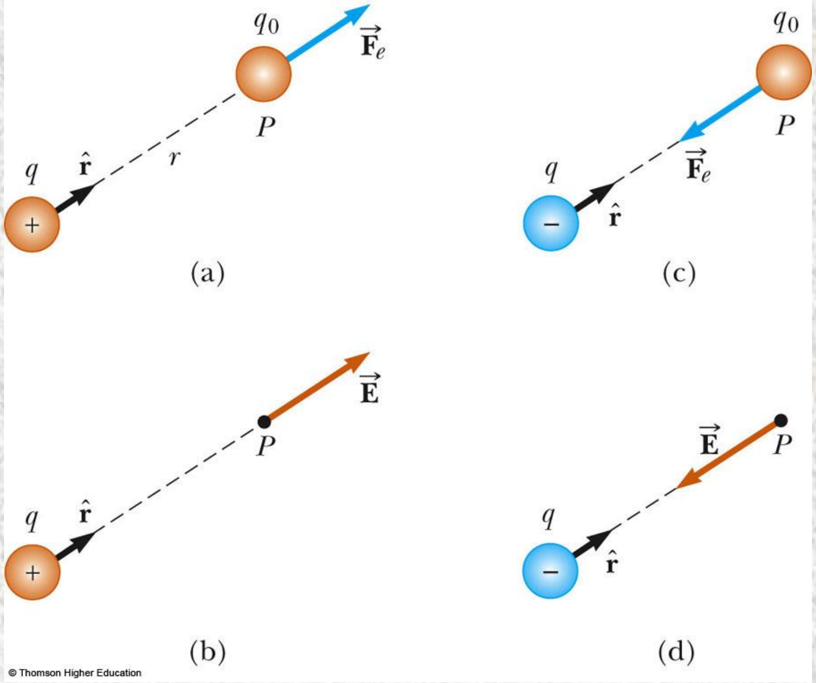
Electric Field, Vector Form

- From Coulomb's law, force between the source and test charges, can be expressed as

$$\vec{\mathbf{F}}_e = k_e \frac{q q_o}{r^2} \hat{\mathbf{r}}$$

- Then, the electric field will be

$$\vec{\mathbf{E}} = \frac{\vec{\mathbf{F}}_e}{q_o} = k_e \frac{q}{r^2} \hat{\mathbf{r}}$$



Superposition with Electric Fields

- At any point *P*, the total electric field due to a group of source charges equals the vector sum of the electric fields of all the charges

$$\vec{\mathbf{E}} = k_e \sum_i \frac{q_i}{r_i^{\,2}} \hat{\mathbf{r}}_i$$

Definition of electric field

the electric field E at a point in space is defined as the electric force $\mathbf{F}_e$ acting on a positive test charge q_0 placed at that point divided by the magnitude of the test charge:

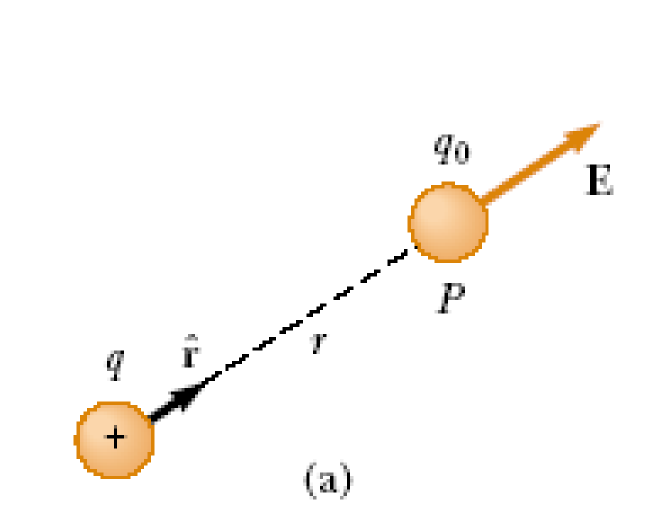
$$\mathbf{E} \equiv \frac{\mathbf{F}_e}{q_0}$$

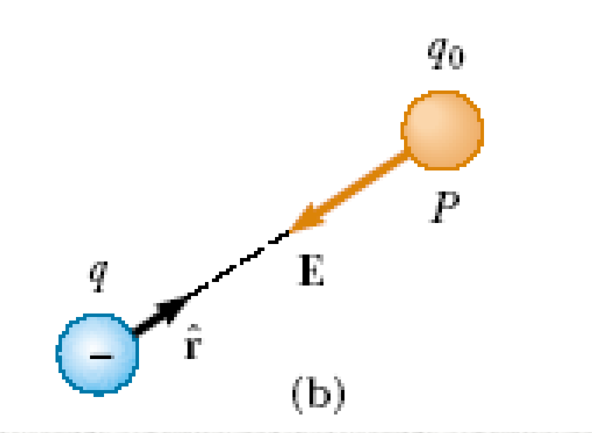
To determine the direction of an electric field, consider a point charge *q located* a distance *r from a test charge q0 located at a point P,* According to Coulomb's law, the force exerted by *q on the test charge is*

$$\mathbf{F}_e = k_e \frac{q q_0}{r^2}\,\hat{\mathbf{r}}$$

The electric field created by *q is*

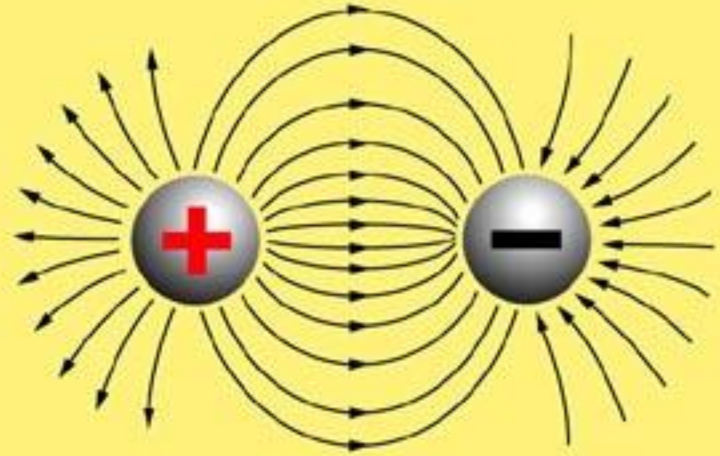
$$\mathbf{E} = k_e \frac{q}{r^2}\,\hat{\mathbf{r}}$$

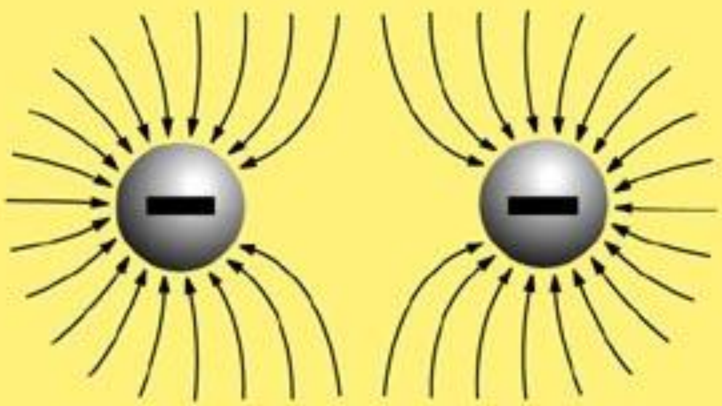


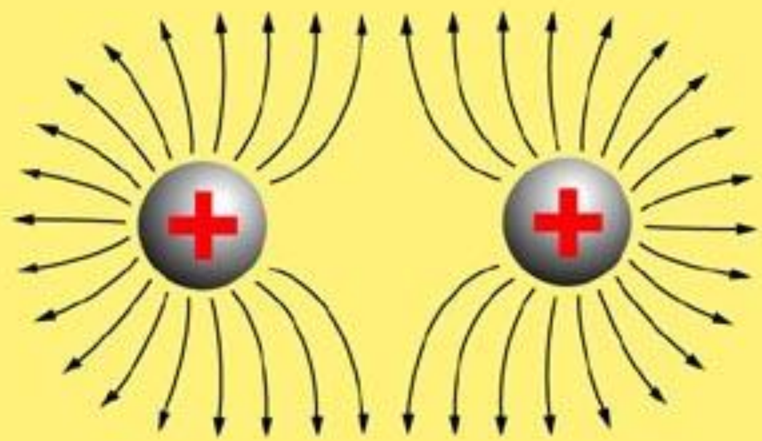


Drawing the Electric Field

Field lines point toward negative charges and away from positive charges.







Electric Field Due to Two Charges

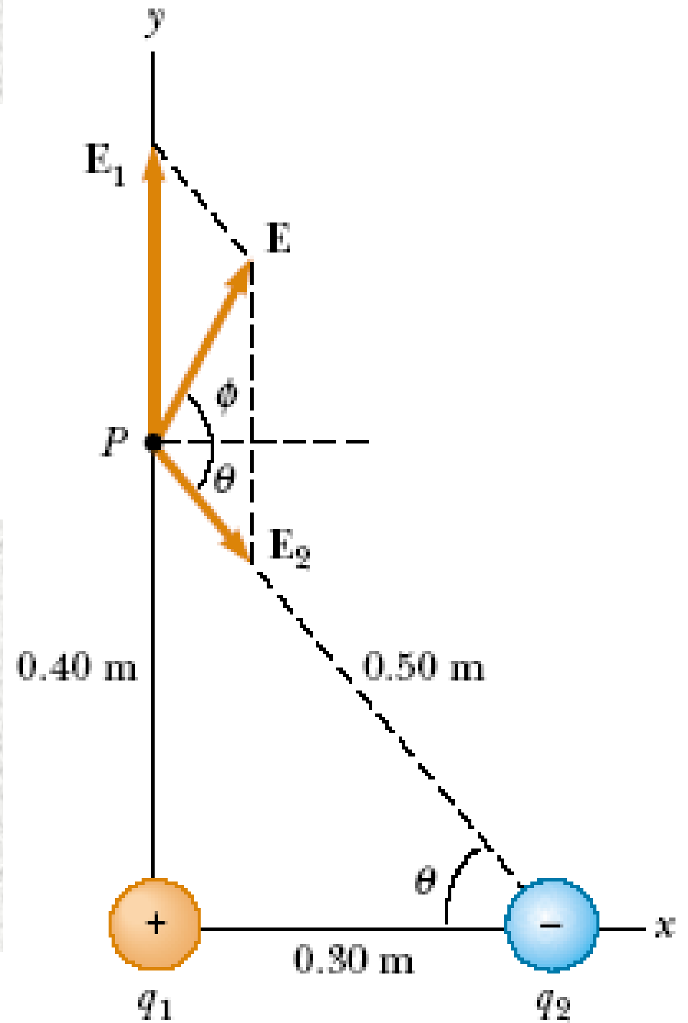
$$E_1 = k_e \frac{|q_1|}{r_1{}^2} = \left(8.99 \times 10^9 \ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right) \frac{(7.0 \times 10^{-6}\ \text{C})}{(0.40\ \text{m})^2}$$
$$= 3.9 \times 10^5\ \text{N/C}$$

$$E_2 = k_e \frac{|q_2|}{r_2{}^2} = \left(8.99 \times 10^9 \ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right) \frac{(5.0 \times 10^{-6}\ \text{C})}{(0.50\ \text{m})^2}$$
$$= 1.8 \times 10^5\ \text{N/C}$$

$$\mathbf{E}_1 = 3.9 \times 10^5 \mathbf{j}\ \text{N/C}$$

$$\mathbf{E}_2 = (1.1 \times 10^5 \mathbf{i} - 1.4 \times 10^5 \mathbf{j})\ \text{N/C}$$

$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 = (1.1 \times 10^5 \mathbf{i} + 2.5 \times 10^5 \mathbf{j})\ \text{N/C}$$



Electric Field of a Dipole

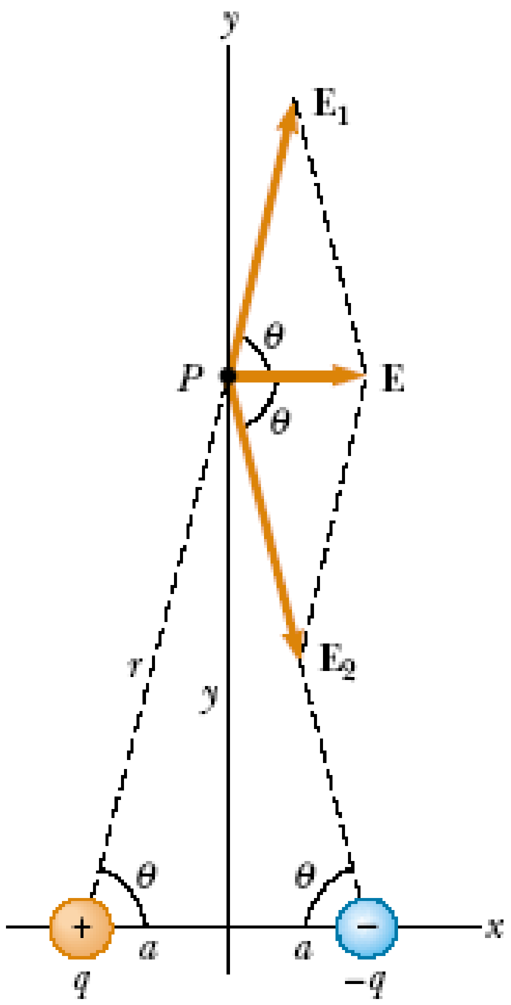
$$E_1 = E_2 = k_e \frac{q}{r^2} = k_e \frac{q}{y^2 + a^2}$$

$$E = 2E_1 \cos\theta = 2k_e \frac{q}{(y^2 + a^2)} \frac{a}{(y^2 + a^2)^{1/2}}$$

$$= k_e \frac{2qa}{(y^2 + a^2)^{3/2}}$$

Because $y \gg a$, we can neglect a^2 and write

$$E \approx k_e \frac{2qa}{y^3}$$



$\cos\theta = a/r = a/(y^2 + a^2)^{1/2}$.

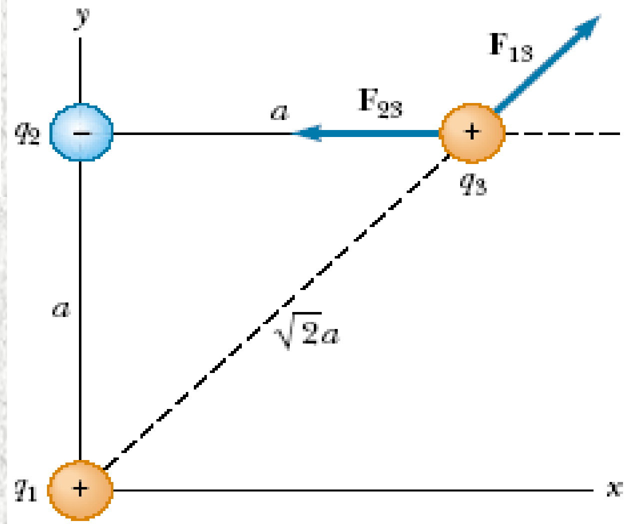
Calculate the Force

Consider three point charges located at the corners of a right triangle as shown in Figure. Find the resultant force exerted on *q3* .

$q_1 = q_3 = 5.0\ \mu C$, $q_2 = -2.0\ \mu C$, and $a = 0.10$ m.

$$F_{3x} = F_{13x} + F_{23} = 7.9\text{ N} - 9.0\text{ N} = -1.1\text{ N}$$

$$F_{3y} = F_{13y} = 7.9\text{ N}$$



$$\mathbf{F}_3 = \boxed{(-1.1\mathbf{i} + 7.9\mathbf{j})\text{ N}}$$

Find the magnitude and direction of the resultant force $\mathbf{F}_3$.

8.0 N at an angle of 98° with the *x* axis.

••7 SSM ILW WWW In Fig. 22-30, the four particles form a square of edge length $a = 5.00$ cm and have charges $q_1 = +10.0$ nC, $q_2 = -20.0$ nC, $q_3 = +20.0$ nC, and $q_4 = -10.0$ nC. In unit-vector notation, what net electric field do the particles produce at the square's center?

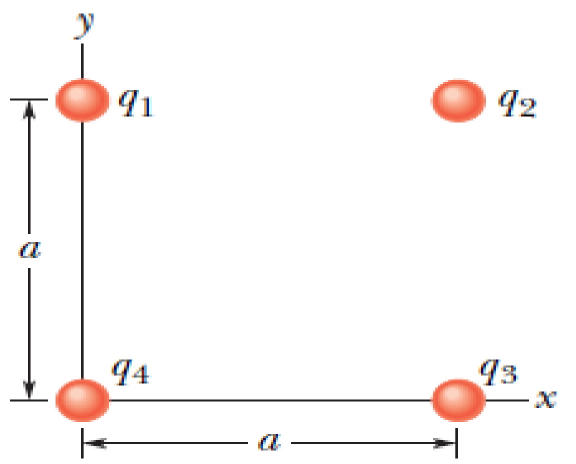


Fig. 22-30 Problem 7.

••8 GO In Fig. 22-31, the four particles are fixed in place and have charges $q_1 = q_2 = +5e$, $q_3 = +3e$, and $q_4 = -12e$. Distance $d = 5.0\ \mu\text{m}$. What is the magnitude of the net electric field at point P due to the particles?

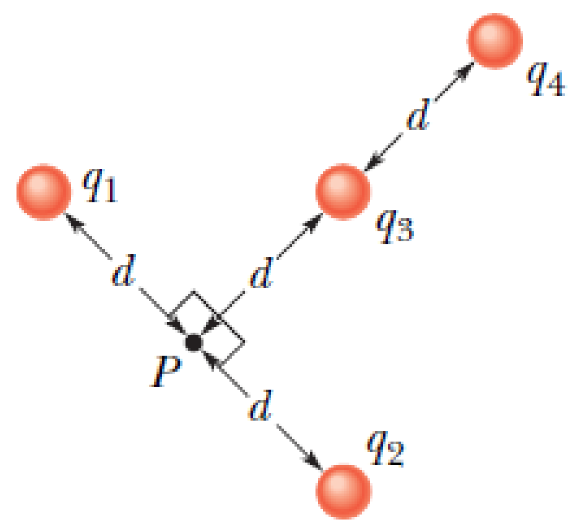
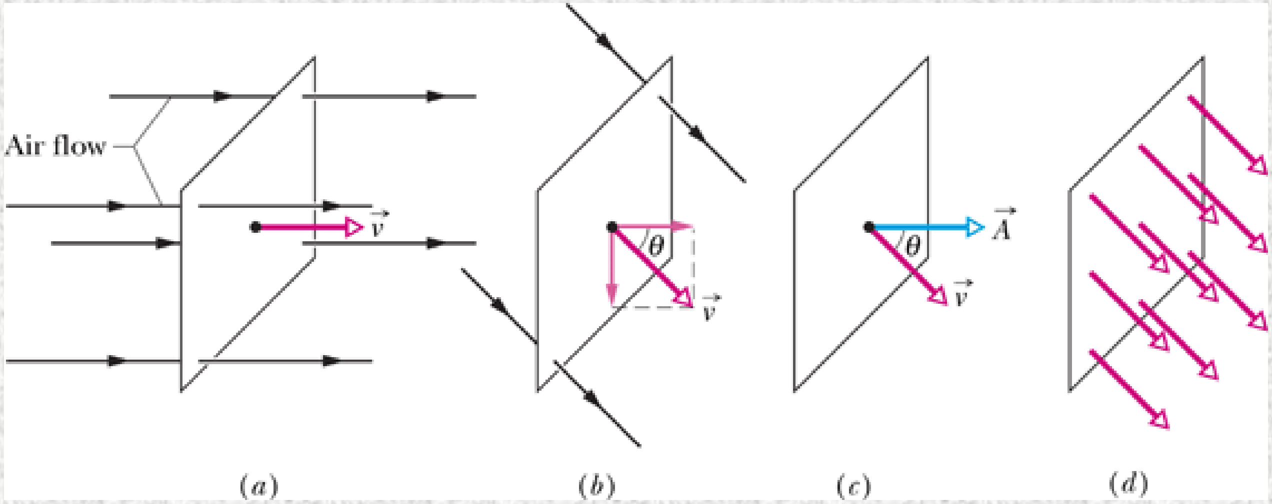


Fig. 22-31 Problem 8.

Flux



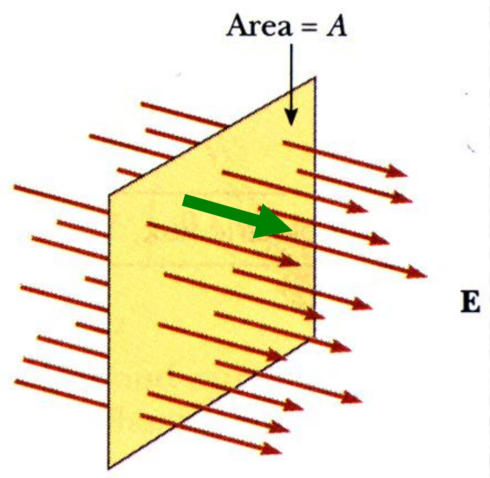
The rate of volume flow through the loop is :

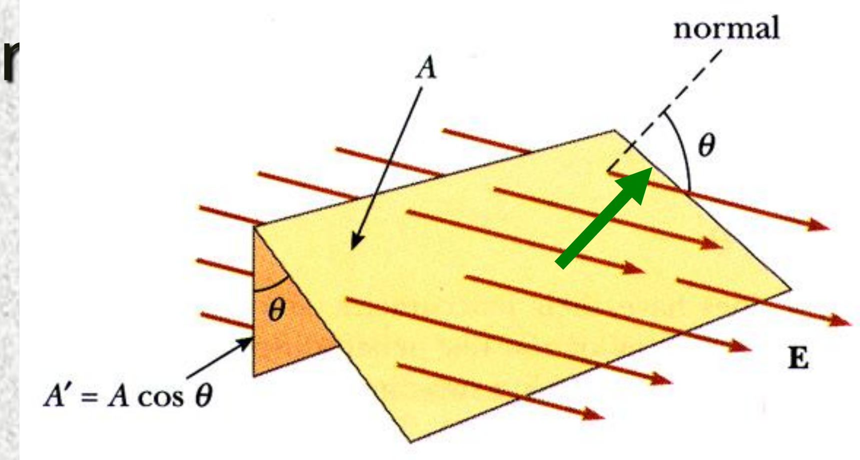
$$\Phi = (v\cos\theta)A .$$

$$\Phi = vA\cos\theta = \overrightarrow{v} \cdot \overrightarrow{A} ,$$

Definition of Electric Flux

- The amount of field, material or other physical entity passing through a surface.
- Surface area can be represented as vector defined normal to the surface it is describing
- Defined by the equatior





$$\Phi = \int_{\text{surface}} \vec{E} \cdot d\vec{A}$$

Flux of Electric Field

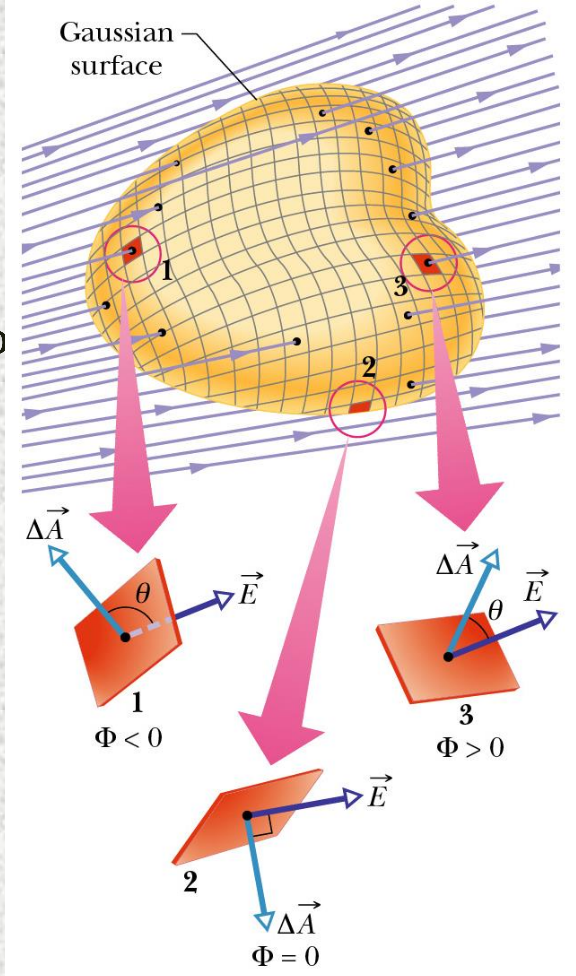
- Like the flow of water, or light energy, we can think of the electric field as flowing through a surface (although in this case nothing is actually moving).
- We represent the flux of electric field as Φ (greek letter phi), so the flux of the electric field through an element of area ΔA is

$$\Delta\Phi = \vec{E} \cdot \Delta\vec{A} = E\, \Delta A \cos\theta$$

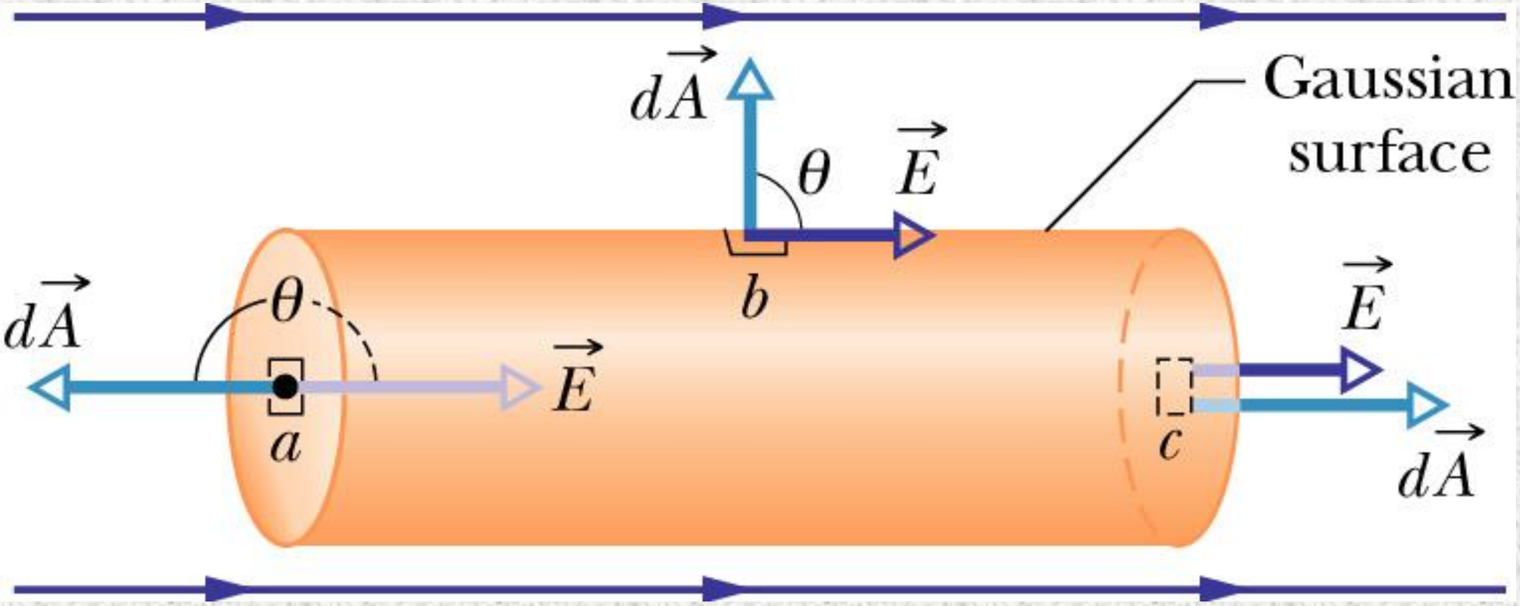
- When $\theta < 90^\circ$, the flux is positive (out of the surface), and when $\theta > 90^\circ$, the flux is negative.

$$d\Phi = \vec{E} \cdot d\vec{A} = E\, dA \cos\theta$$

- When we have a complicated surface, we can divide it up into tiny elemental areas:



Find the electric flux through a cylindrical surface in a uniform electric field **E**



$$\Phi = \oint \vec{E} \bullet d\vec{A} \qquad = \oint E\cos\theta dA$$

$$d\vec{A} = \hat{n} dA$$

a. $\Phi = \int E\cos 180 dA = -\int E dA = -E\pi R^2$

b. $\Phi = \int E\cos 90 dA = 0$

c. $\Phi = \int E\cos 180 dA = \int E dA = E\pi R^2$

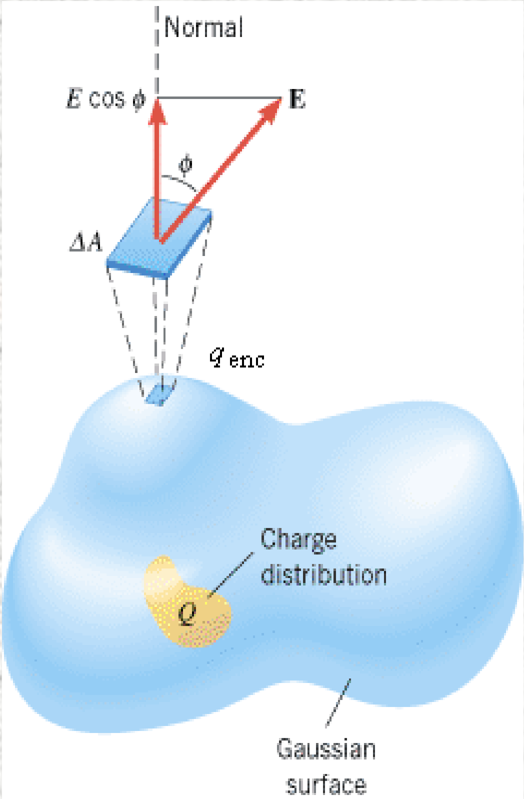
What is the flux if the cylinder were vertical ?

Suppose it were any shape?

Flux from a. + b. + c. = 0

Gauss' Law

For charge distribution Q:



The electric flux through a Gaussian surface times by ε_0 (the permittivity of free space) is equal to the net charge *Q* enclosed :

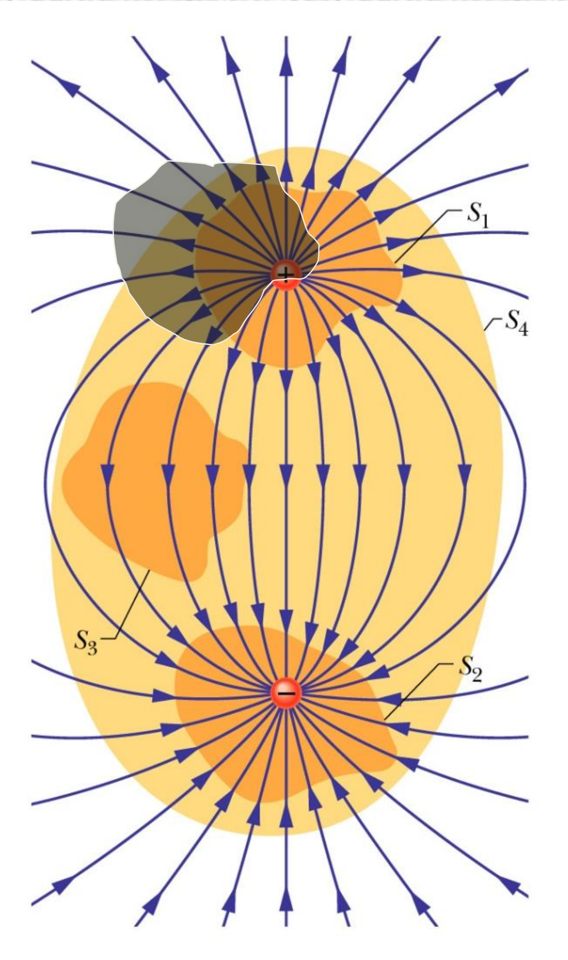
$$\varepsilon_0 \Phi = q_{enc} \qquad \text{(Gauss' law)},$$

$$\varepsilon_0 \oint \vec{E} \cdot d\vec{A} = q_{enc} \qquad \text{(Gauss ' law)}.$$

- The net charge q_{enc} is the algebraic sum of all the *enclosed* charges.
- Charge outside the surface, no matter how large or how close it may be, is not included in the term q_{enc}.

Example of Gauss’ Law

- Consider a dipole with equal positive and negative charges.
- Imagine four surfaces S_1, S_2, S_3, S_4, as shown.
- S_1 encloses the positive charge. Note that the field is everywhere outward, so the flux is positive.
- S_2 encloses the negative charge. Note that the field is everywhere inward, so the flux through the surface is negative.
- S_3 encloses no charge. The flux through the surface is negative at the upper part, and positive at the lower part, but these cancel, and there is no net flux through the surface.
- S_4 encloses both charges. Again there is no net charge enclosed, so there is equal flux going out and coming in—no net flux through the surface.



Electric lines of flux and Derivation of Gauss' Law using Coulombs law

- Consider a sphere drawn around a positive point charge. Evaluate the net flux through the closed surface.

Net Flux = $\Phi = \oint \vec{E} \bullet d\vec{A} = \oint E \cos\theta dA = \oint EdA$

E || n
Cos 0 = 1

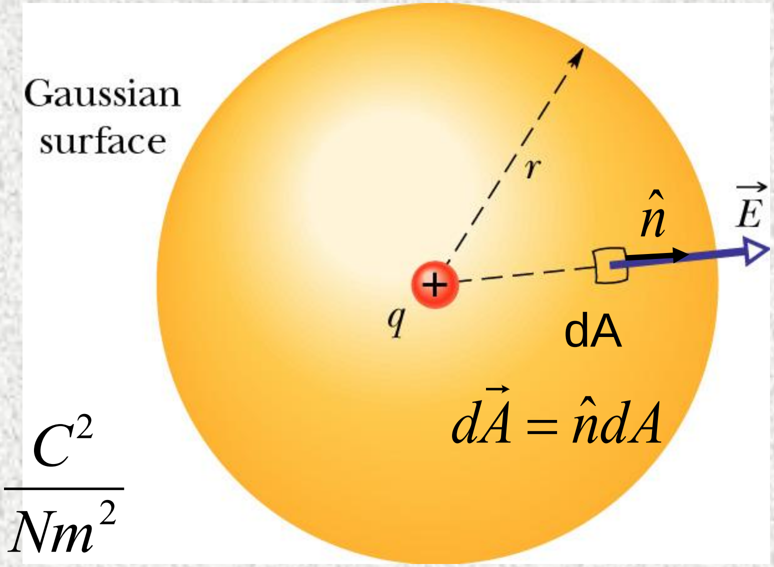
For a Point charge **E=kq/r²**

$$\Phi = \oint EdA = \oint kq/r^2 dA$$

$$\Phi = kq/r^2 \oint dA = kq/r^2(4\pi r^2)$$

$$\Phi = 4\pi kq$$

$$4\pi k = 1/\varepsilon_0 \text{ where } \varepsilon_0 = 8.85\text{x}10^{-12}\frac{C^2}{Nm^2}$$

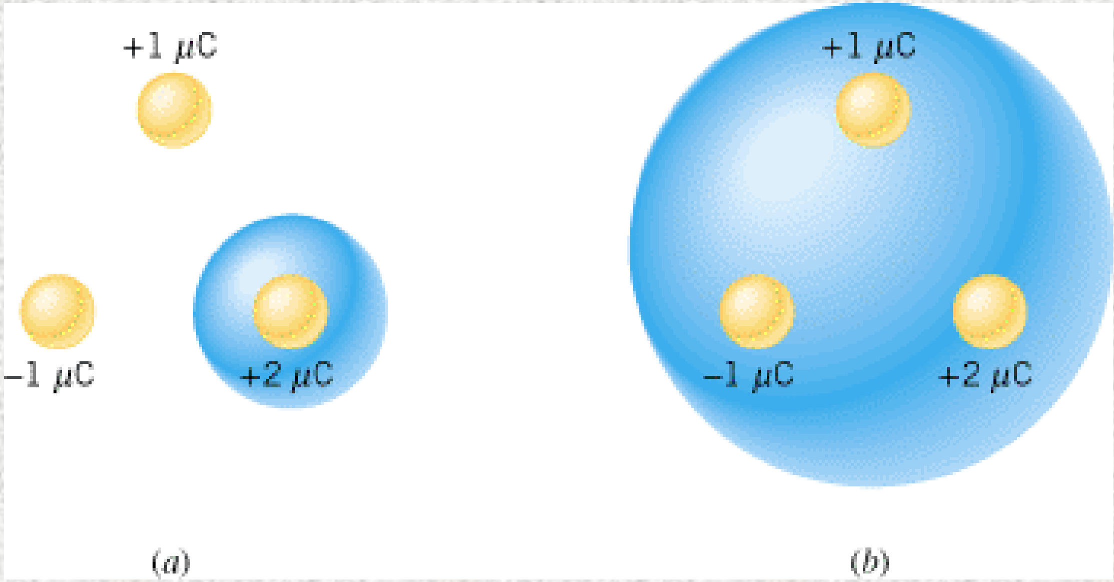


$$\boxed{\Phi_{net} = \frac{q_{enc}}{\varepsilon_0}}$$

Gauss' Law

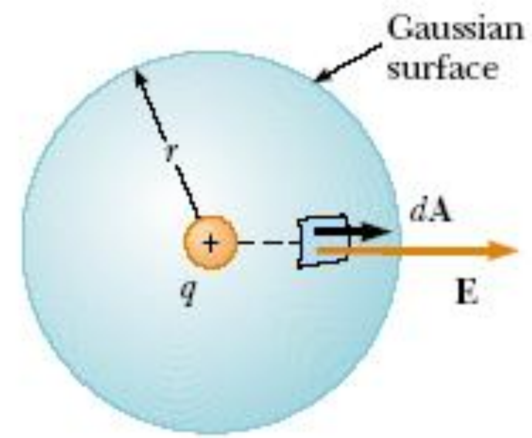
Check Your Understanding

The drawing shows an arrangement of three charges. In parts (*a*) and (*b*) different Gaussian surfaces are shown. Through which surface, if either, does the greater electric flux pass?



The Electric Field Due to a Point Charge

Starting with Gauss's law, calculate the electric field due to an isolated point charge q.



$$\Phi_E = \oint \mathbf{E} \cdot d\mathbf{A} = \oint E\, dA = \frac{q}{\epsilon_0}$$

$$\oint E\, dA = E \oint dA = E(4\pi r^2) = \frac{q}{\epsilon_0}$$

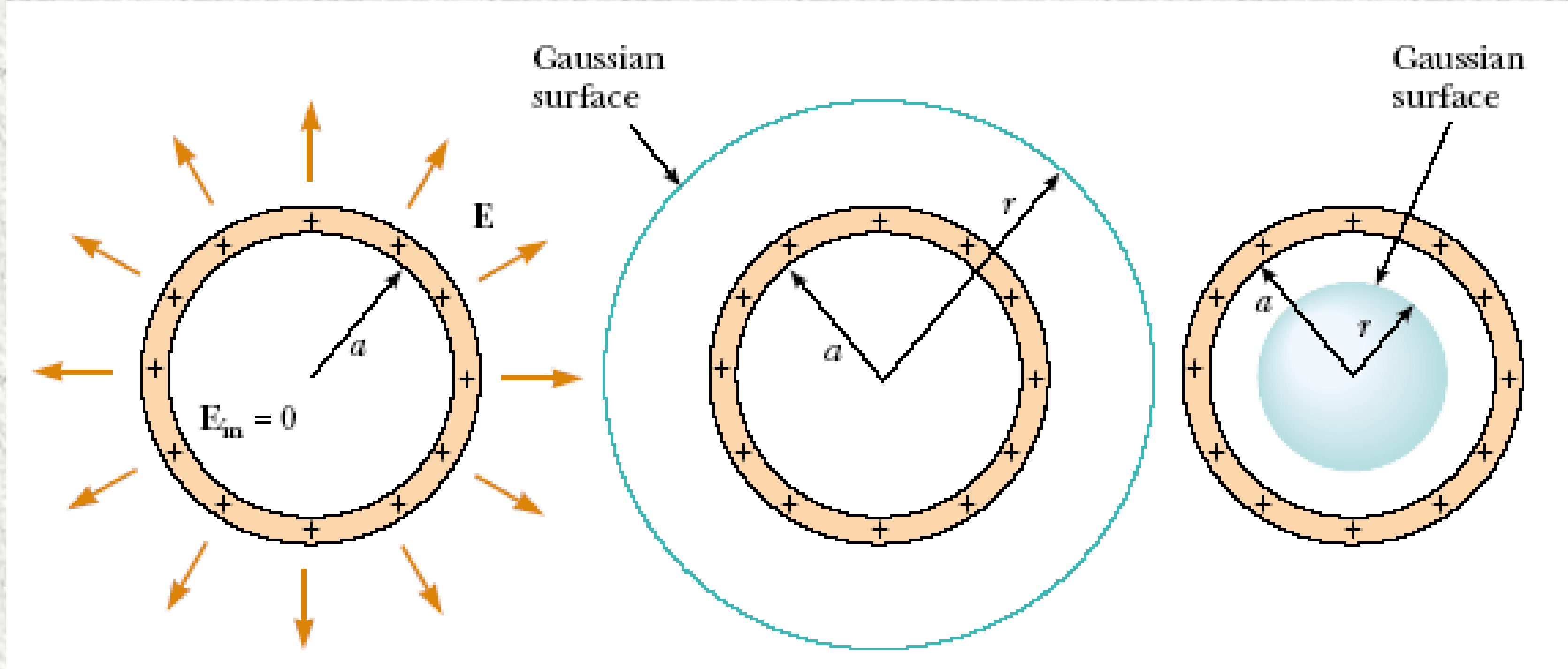
$$E = \frac{q}{4\pi\epsilon_0 r^2} = k_e \frac{q}{r^2}$$

The Electric Field Due to a Thin Spherical Shell

- A shell of uniform charge attracts or repels a charged particle that is outside the shell as if all the shell's charge were concentrated at the center of the shell.
- If a charged particle is located inside a shell of uniform charge, there is no electrostatic force on the particle from the shell.

$$E = k_e \frac{Q}{r^2} \quad (\text{for } r > a)$$

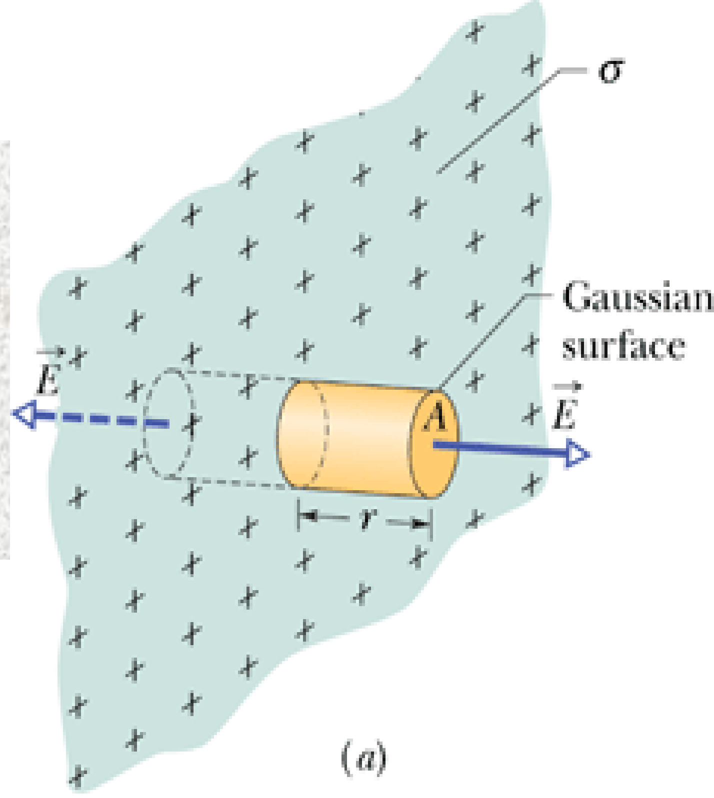
$E = 0$ in the region $r < a$.



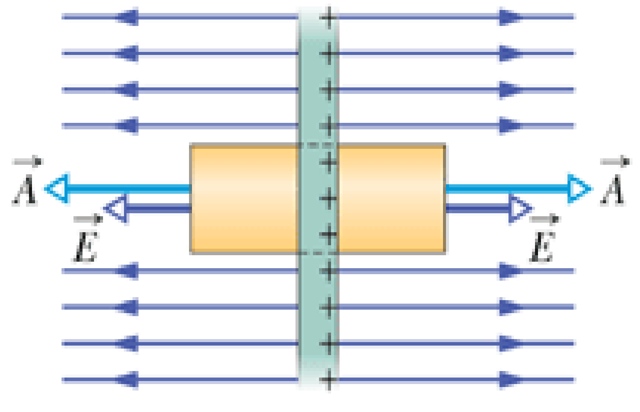
A Nonconducting Plane of Charge

$$\Phi_E = 2EA = \frac{q_{\text{in}}}{\epsilon_0} = \frac{\sigma A}{\epsilon_0}$$

$$E = \boxed{\frac{\sigma}{2\epsilon_0}}$$

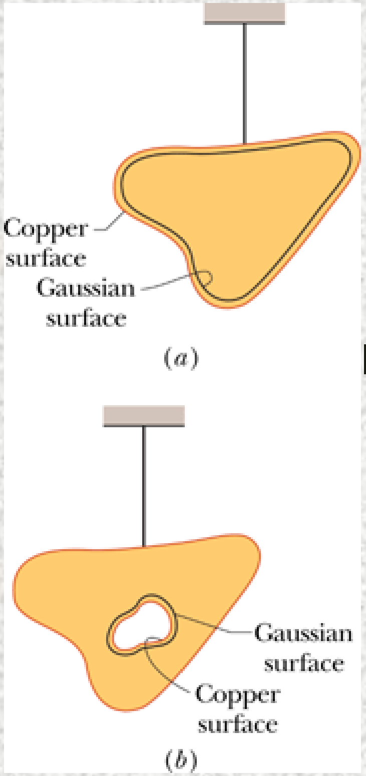


(a)

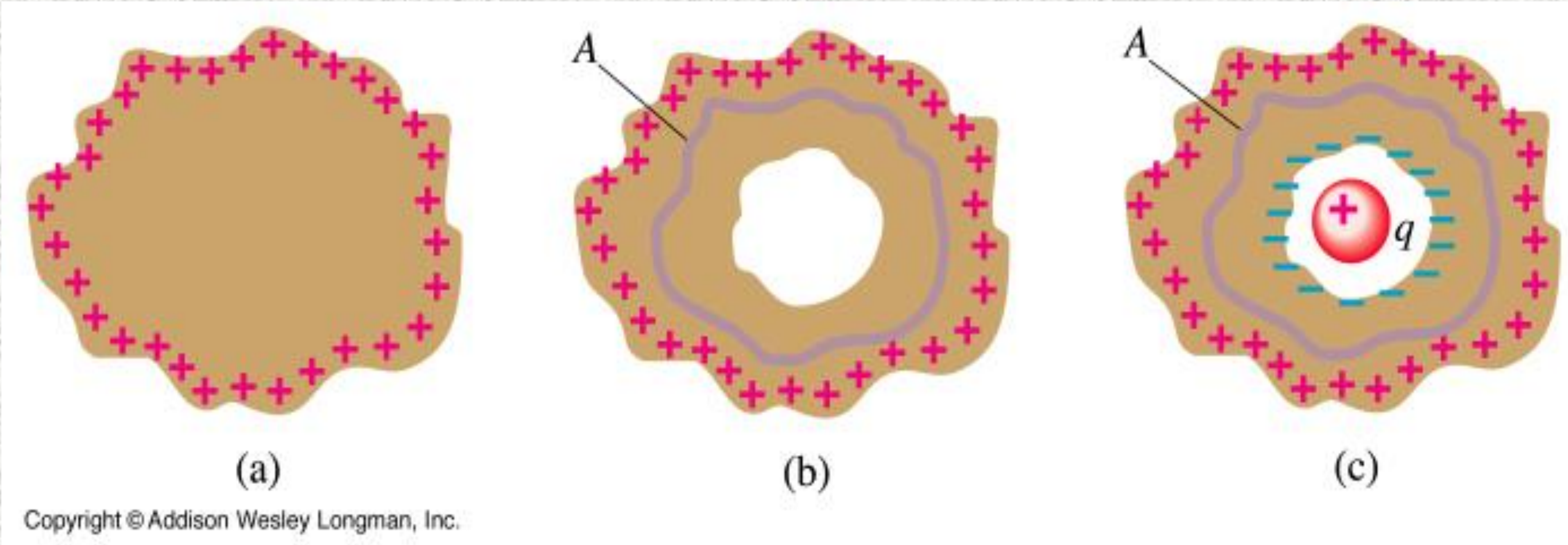


(b)

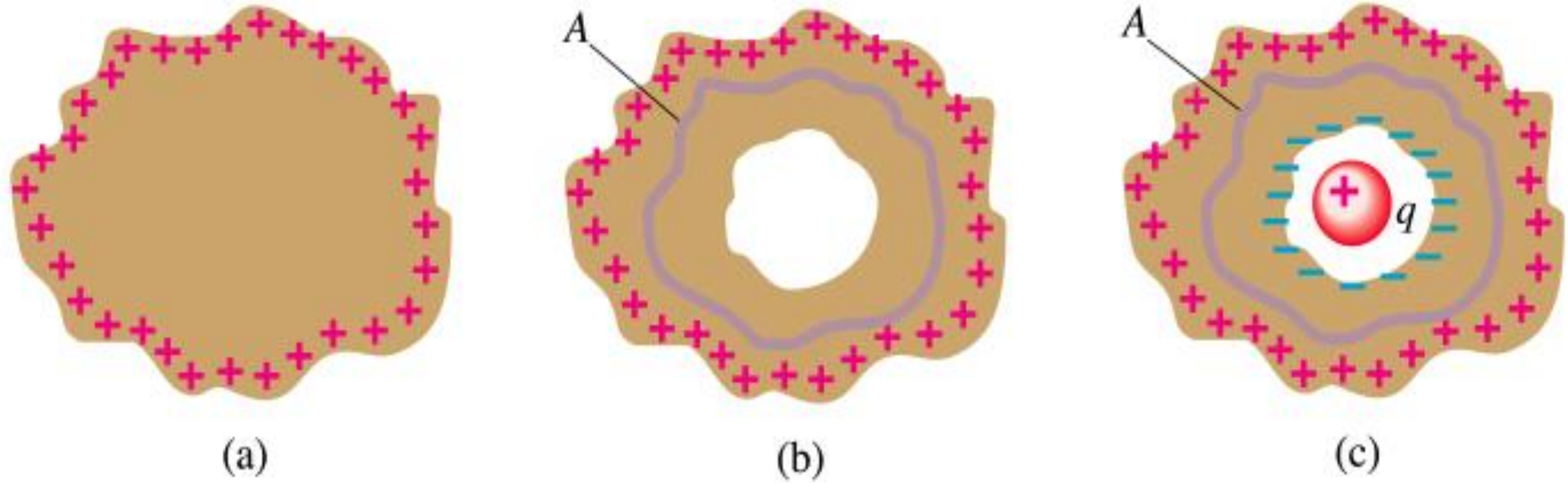
A Charged Isolated Conductor



- If an excess charge is placed on an isolated conductor, that amount of charge will move entirely to the surface of the conductor. None of the excess charge will be found within the body of the conductor.
- For an Isolated Conductor with a Cavity, There is no net charge on the cavity walls; all the excess charge remains on the outer surface of the conductor



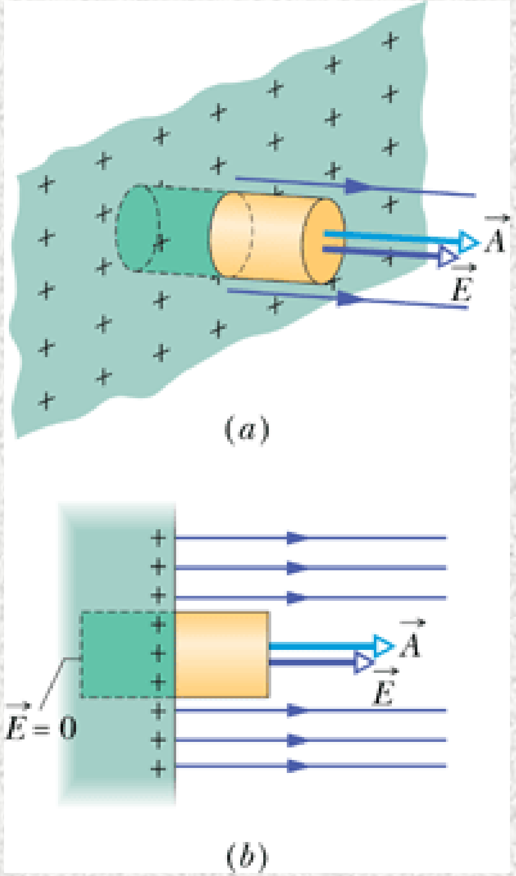
Find electric charge q on surface of hole in the charged conductor.



The solution of this problem lies in the fact that the electric field inside a conductor is zero and if we place our Gaussian surface inside the conductor (where the field is zero), the charge enclosed must be zero (+ q – q) = 0.

Find electric charge q on surface of hole in the charged conductor.

The External Electric Field of a Conductor



If σ is the charge per unit area,

according to Gauss' law,

$$\varepsilon_0 E A = \sigma A,$$

$$E = \frac{\sigma}{\varepsilon_0} \qquad \text{(conducting surface)}.$$

An isolated conductor of arbitrary shape carries a net charge +10 μC. Inside the conductor is a hollow cavity within which is a point charge q = +3 μC. What is the charge (a) on the cavity wall and (b) on the outer surface of the conductor?