VECTORS:

QUESTION 01:

a) For x & y component for vector $\vec{r}$

$$\tan\theta = \frac{r_y}{r_x}$$

$$\tan 30 = \frac{r_y}{r_x}$$

$$\boxed{r_x\, 0.577 = r_y}$$

$$\because |\vec{r}| = 7.3$$

$$\because |\vec{r}| = \sqrt{r_x^2 + r_y^2}$$

$$7.3 = \sqrt{r_x^2 + r_y^2} \quad (\because r_y = 0.577 r_x)$$

$$7.3^2 = \left(\sqrt{r_x^2 + r_y^2}\right)^2 \quad \left(\because \frac{r_y}{r_x} = 0.577\right)$$

$$53.29 = (r_x)^2 + (0.577 r_x)^2$$

$$53.29 = r_x^2 + 0.333 r_x^2$$

$$53.29 = 1.333 r_x^2$$

$$r_x^2 = \frac{53.29}{1.333}$$

$$r_x^2 = 39.98$$

Taking $\sqrt{\ }$ on b.s

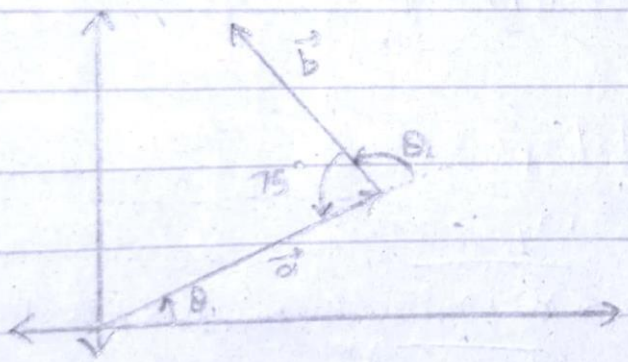
$\sqrt{r_x^2} = \sqrt{39.98}$

$\boxed{r_x = 6.32\,m}$ (x - component of displacement vector)

$\because r_y = 0.577\,(6.32)$

$\boxed{r_y = 3.647\,m}$ (y - component of displacement vector)

QUESTION 02:



$\theta = 180 - 105$

$\boxed{\theta = 75°}$

For x & y components of a:

$a_x = a\cos\theta = 10\cos 30°$

$\boxed{a_x = 8.66\,m\hat{i}}$

$a_y = a\sin\theta = 10\sin 30°$

$\boxed{a_y = 5\,m\hat{j}}$

For x and y component of b:

$\theta = \theta_1 + \theta_2$

$\boxed{\theta = 135^\circ}$

$b_x = b\cos\theta = 10\cos 135$

$\boxed{b_x = -7.07\,m\hat{i}}$

$b_y = b\sin\theta = 10\sin 135$

$\boxed{b_y = 7.07\,m\hat{j}}$

a) x & y components of vector sum $\vec{r}$.

i) x - component

$\vec{r}_x = a_x\hat{i} + b_x\hat{i}$

$= 8.66\,m\hat{i} + (-7.07)\,m\hat{i}$

$\boxed{\vec{r}_x = 1.59\,m\hat{i}}$ Ans

ii) y-component:

$\vec{r}_y = a_y\hat{j} + b_y\hat{j}$

$= 5\,m\hat{j} + 7.07\,m\hat{j}$

$\boxed{\vec{r}_y = 12.07\,m\hat{j}}$ Ans.

b) magnitude of $\vec{r}$

$$|\vec{r}| = \sqrt{r_x^2 + r_y^2}$$
$$= \sqrt{1.59^2 + 12.07^2}$$
$$\boxed{|\vec{r}| = 12.17m}$$

c) the angle r makes with the +ve x-axis

$$r_x = r\cos\theta$$
$$1.59 = 12.17\cos\theta$$
$$\theta = \cos^{-1}\left(\frac{1.59}{12.17}\right)$$
$$\boxed{\theta = 82.5^\circ}$$

QUESTION NO.03:

a) $a \times b$;

For magnitude;

$$|\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}| \sin\theta$$

$$\therefore a = 4;\ b = 3$$

$$= (4)(3)\sin(90)$$

$$\boxed{|\vec{a} \times \vec{b}| = 12}$$

For direction;

By right hand rule, the direction of the resultant vector will be into the page

QUICKBITS:

~~→ The full form of .ipynb is "Interactive Python NoteBook"~~

b) b×c;

i) For magnitude;

$\theta = \sin^{-1}\left(\frac{4}{5}\right)$

$\boxed{\theta = 53.13^\circ}$

$|\vec{b} \times \vec{c}| = bc \sin\theta$

$= (3)(5) \sin(53.13)$

$\boxed{|\vec{b} \times \vec{c}| = 11.99}$

ii) For direction.

By right hand rule, the direction of the resultant vector will be into the page

QUESTION 04:

a) For $\vec{a}.\vec{b}$:

$$\theta = 180 - \theta_2 = 180^\circ - 105^\circ$$

$$\boxed{\theta = 75^\circ}$$

$$\vec{a}.\vec{b} = |\vec{a}||\vec{b}|\cos\theta$$

$$= (10)(10)\cos(75^\circ)$$

$$\boxed{\vec{a}.\vec{b} = 25.88}$$

b) For $\vec{a} \times \vec{b}$:

$$\theta = 180 - \theta_2 = 180^\circ - 105^\circ$$

$$\boxed{\theta = 75^\circ}$$

$$\vec{a} \times \vec{b} = |\vec{a}||\vec{b}|\sin\theta$$

$$= (10)(10)(\sin 75^\circ)$$

$$\boxed{\vec{a} \times \vec{b} = 96.59}$$

c) angle between a & b

$\cancel{\theta = \theta_2}$ -

$\theta = 180 - \theta_2$

$= 180° - 105°$

$\boxed{\theta = 75°}$

QUESTION 05:

$A_x = 25m$

$A_y = 40m$

a) What is the magnitude of A

$|\vec{A}| = \sqrt{A_x^2 + A_y^2} = \sqrt{25^2 + 40^2}$

$\boxed{|\vec{A}| = 47.17}$

b) What is the direction b/w the direction of A & the positive direction of x

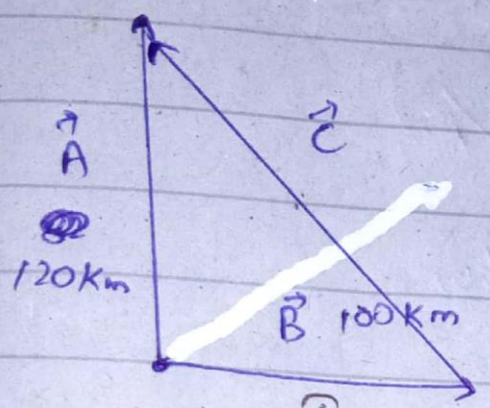
For direction,

$$\theta = \tan^{-1}(A_y/A_x)$$

$$= \tan^{-1}(40/25)$$

$$\theta = 57.99$$

QUESTION 06:



For magnitude:

By pythagoras theorem

$$H^2 = B^2 + P^2$$

$$H = \sqrt{100^2 + 120^2}$$

$$\boxed{H = 156.20\,Km}$$

For direction:

$$\theta = \tan^{-1}(y/x)$$

$$= \tan^{-1}\left(\frac{120}{100}\right) = \tan^{-1}(1.2)$$

$$\boxed{\theta = 50.19^\circ}$$

QUESTION 07:

$|\vec{a}| = 50m$

$|\vec{b}| = 50m$

$|\vec{c}| = 50m$

For a_x:

$\vec{a}_x = a\cos\theta = 50\cos 30°$

$\boxed{\vec{a}_x = 43.3m\hat{i}}$

For a_y:

$a_y = a\sin\theta = 50\sin 30°$

$\boxed{\vec{a}_y = 25m\hat{j}}$

For b_x:

$\vec{b}_x = b\cos\theta = 50\cos 195$

$\boxed{\vec{b}_x = -48.29m\hat{i}}$

For b_y:

$\vec{b}_y = b\sin\theta = 50\sin 195$

$\boxed{\vec{b}_y = -12.94m\hat{j}}$

For c_x:

$$\vec{c}_x = c\cos\theta = 50\cos 315^\circ$$

$$\boxed{\vec{c}_x = 35.55\,m\hat{i}}$$

For c_y:

$$c_y = c\sin\theta = 50\sin 315^\circ$$

$$\boxed{\vec{c}_y = -35.35\,m\hat{j}}$$

i) The magnitude and the angle of vector $\vec{a}+\vec{b}+\vec{c}$

$$\vec{a}+\vec{b}+\vec{c} = (a_x+b_x+c_x)\hat{i} + (a_y+b_y+c_y)\hat{j}$$

$$= (43.3+(-48.29)+35.35)\hat{i}$$

$$+(25+(-12.94)-35.35)\hat{j}$$

$$\boxed{|\vec{a}+\vec{b}+\vec{c}| = 30.36\,\hat{i}\,m - 23.29\,\hat{j}\,m}$$

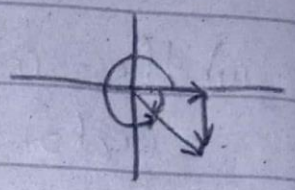
For magnitude:

$$|\vec{a}+\vec{b}+\vec{c}| = \sqrt{x^2+y^2}$$

$$= \sqrt{(30.36)^2+(23.29)^2}$$

$$\boxed{|\vec{a}+\vec{b}+\vec{c}| = 38.26\,m}$$

For direction:



$$\theta = \tan^{-1}\left(\frac{y}{x}\right)$$

$$= \tan^{-1}\left(\frac{-23.29}{30.36}\right)$$

$$\boxed{\theta = -37.49^\circ} \quad \text{or} \quad \boxed{\theta = 322.51^\circ}$$

ii) The magnitude and angle of a-b+c?

<u>For magnitude,</u>

$$\vec{a}-\vec{b}+\vec{c} = (a_x - b_x + c_x)\hat{i} + (a_y - b_y + c_y)\hat{j}$$

$$\boxed{\vec{a}-\vec{b}+\vec{c} = 126.94\,m\,\hat{i} - 2.59\,m\,\hat{j}}$$

$$\therefore |\vec{a}-\vec{b}+\vec{c}| = \sqrt{x^2+y^2} = \sqrt{(126.94)^2 + (2.59)^2}$$

$$\boxed{|\vec{a}-\vec{b}+\vec{c}| = 126.97\,m}$$

For direction;

$$\theta = \tan^{-1}(y/x) = \tan^{-1}\left(\frac{2.59}{126.97}\right)$$

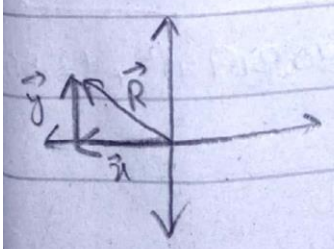
$$\boxed{\theta = 1.165^\circ}$$

iii) the magnitude & angle of a fourth vector d such that $(a+b)-(c+d)=0$.

$$\vec{a}+\vec{b}-(\vec{c}+\vec{d})=0$$
$$\vec{a}+\vec{b}-\vec{c}-\vec{d}=0$$
$$\boxed{\vec{d}=\vec{a}+\vec{b}-c}$$

$$\vec{d}=(a_x+b_x-c_x)\hat{i}+(a_y+b_y-c_y)\hat{j}$$
$$=(43.3+(-48.29)-35.35)\hat{i}$$
$$+(25-12.94+35.35)\hat{j}$$
$$\boxed{\vec{d}=-40.34\hat{i}+47.41\hat{j}}$$

For magnitude;



$$|\vec{d}|=\sqrt{d_x^2+d_y^2}=\sqrt{(-40.34)^2+(47.41)^2}$$
$$\boxed{|\vec{d}|=62.23m}$$

For direction;

$$\theta=\tan^{-1}(y/x)=\tan^{-1}(47.41/-40.34)$$

Either,

$$\boxed{\theta=-49.60^\circ}$$ OR $\theta=180-49.60$

$$\boxed{\theta=130.4^\circ}$$

Since the resultant vector is in the second quadrant

$\theta = 130.4°$

Answer.

QUESTION: 08

$\vec{A} = 2\hat{i} - 3\hat{j} + 5\hat{k}$

angle with x-axis = ?

angle with y-axis = ?

angle with z-axis = ?

i) For the angle with +ve x-axis

Consider $\vec{B} = \hat{i}$ as the unit vector along the x-axis

By taking dot product

$$\vec{A} \cdot \vec{B} = |\vec{A}|\,|\vec{B}| \cos\theta$$

$$(2\hat{i} - 3\hat{j} + 5\hat{k})(\hat{i} + 0\hat{j} + 0\hat{j}) = \left(\sqrt{2^2 + 3^2 + 5^2}\right)\left(\sqrt{1^2}\right)\cos\theta$$

$$2 = \sqrt{38}\,\cos\theta$$

$$\cos\theta = \frac{2}{\sqrt{38}}$$

$$\boxed{\theta = \cos^{-1}\left(\frac{2}{\sqrt{38}}\right)}$$

OR

$$\boxed{\theta = 71.06^\circ}$$

ii) For angle with +ve y-axis.

let $\vec{B} = \hat{j}$

By taking dot product

$$\vec{A}\cdot\vec{B} = |\vec{A}|\,|\vec{B}|\cos\theta$$

$$(2\hat{i} - 3\hat{j} + 5\hat{k})(0\hat{i} + \hat{j} + 0\hat{k}) = (\sqrt{4+9+25})\cos\theta$$

$$-3 = \sqrt{38}\cos\theta$$

$$\boxed{\theta = \cos^{-1}\left(\frac{-3}{\sqrt{38}}\right)}$$

OR

$$\boxed{\theta = 119.12^\circ}$$

iii) For angle with the +ve z-axis

let $\vec{C} = \hat{k}$

By taking dot product

$$\vec{A}\cdot\vec{B} = |\vec{A}|\,|\vec{B}|\cos\theta$$

$$(2\hat{i} - 3\hat{j} + 5\hat{k})\cdot(0\hat{i} + 0\hat{j} + \hat{k}) = (\sqrt{38})\cos\theta$$

$$\frac{5}{\sqrt{38}} = \cos\theta$$

$$\cos\theta = \frac{5}{\sqrt{38}}$$

$$\boxed{\theta = \cos^{-1}\left(\frac{5}{\sqrt{38}}\right)}$$

OR

$$\boxed{\theta = 35.79^\circ}$$

QUESTION 09:

For $\vec{r}$:-

$$\vec{r} = \vec{a} + \vec{b} + \vec{c}$$

$$= (5 + (-2) + 4)\hat{i} + (4 + 2 + 3)\hat{j} + (-6 + 3 + 2)\hat{k}$$

$$\boxed{\vec{r} = 7\hat{i} + 9\hat{j} - \hat{k}}$$

→ For the angle that makes with the +ve z-axis

let $\vec{q} = \hat{k}$

By taking the dot product

$$\vec{r}\cdot\vec{q} = |\vec{r}||\vec{q}|\cos\theta$$

$$(7\hat{i} + 9\hat{j} - \hat{k})(0\hat{i} + 0\hat{j} + \hat{k}) = (\sqrt{7^2 + 9^2 + 1^2})\cos\theta$$

$$1 = (\sqrt{131})\cos\theta$$

$$\boxed{\theta = \cos^{-1}\left(\frac{1}{\sqrt{131}}\right)}$$ OR

$$\boxed{\theta = 84.98^\circ}$$

c) angle b/w $\vec{a}$ and $\vec{b}$

Taking dot product of $\vec{a}$ & $\vec{b}$

$$\vec{a}.\vec{b} = |\vec{a}||\vec{b}|\cos\theta$$

$$(5\hat{i}+4\hat{j}-6\hat{k}).(2\hat{i}-3\hat{j}+5\hat{k}) = (\sqrt{5^2+4^2+6^2}) \times (\sqrt{2^2+3^2+5^2})\cos\theta$$

$$(10-12-30) = (\sqrt{77})(\sqrt{38})\cos\theta$$

$$\frac{-32}{\sqrt{2926}} = \cos\theta$$

$$\boxed{\theta = \cos^{-1}\left(\frac{-32}{\sqrt{2926}}\right)}$$

OR

$$\boxed{\theta = 126.26^\circ}$$

Ans

QUESTION 10:

$|\vec{A}| = 6$ units

$|\vec{B}| = 7$ units

$\vec{A}.\vec{B} = 14$ units

$\theta = ?$

By dot product

$\vec{A}.\vec{B} = |\vec{A}||\vec{B}| \cos\theta$

$$\boxed{\theta = \cos^{-1}\left(\frac{14}{42}\right)}$$

OR

$$\boxed{\theta = 70.53^\circ}$$ Ans

MOTION IN 1D

QUESTION 01:

$r = (2t^3 - 5t)\hat{i} + (6 - 7t^4)\hat{j}$; $t = 2s$

a) $r = ?$; b) $v = ?$ c) $a = ?$

a) To Find r;

$\because \vec{r} = (2t^3 - 5t)\hat{i} + (6 - 7t^4)\hat{j}$

$= \{2(2)^3 - 5(2)\}\hat{i} + (6 - 7(2)^4)\hat{j}$

$= 6\hat{i} + (-106\hat{j})$

$\boxed{\vec{r} = 6\hat{i} - 106\hat{j} \text{ m}}$

b) To find v;

$\vec{r} = (2t^3 - 5t)\hat{i} + (6 - 7t^4)\hat{j}$

diff w.r.t t

$\frac{d}{dt}\vec{r} = \frac{d}{dt}(2t^3 - 5t)\hat{i} + \frac{d}{dt}(6 - 7t^4)\hat{j}$

$\vec{v} = (6t^2 - 5)\hat{i} + (-28t^3)\hat{j}$ $(\because t = 2)$

$\boxed{\vec{v} = 19\hat{i} - 224\hat{j} \text{ m/s}}$

c) To find $\vec{a}$:

$\because \vec{v} = (6t^2 - 5)\hat{i} + (-28t^3)\hat{j}$

diff w.r.t t

$$\frac{d}{dt}\vec{v} = \frac{d}{dt}(6t^2-5)\hat{i} + \frac{d}{dt}(-28t^3)\hat{j}$$

$$\vec{a} = 12t\hat{i} - 84t^2\hat{j} \quad (\because t=2)$$

$$\boxed{\vec{a} = 24\hat{i} - 336\hat{j}\ m/s^2}$$

QUESTION 02:

$V_1 = 18m/s$; $V_2 = -30m/s$; $t = 2.4s$

$a = ?$

To find a

$$\vec{a} = \frac{V_2 - V_1}{t} = \frac{-30-18}{2.4} = -\frac{48}{2.4}$$

$$\boxed{a = -20m/s^2}$$

QUESTION 03:

$a = 9.8m/s^2$

i) $V_i = 0$; $V_f = 1/10\ C$; $t = ?$

ii) $S = ?$

To find t:

By 1st equation of motion

$$V_f = V_i + at$$

$$1/10\ C = 9.8t$$

$$t = \frac{c}{9.8} = \frac{3\times10^8}{98}$$

$$\boxed{t = 3.06\times10^6\ s}$$

For s:

$$s = v_i t + \frac{1}{2}at^2$$

$$s = 0 + \frac{1}{2}(9.8)(3.06\times10^6)^2$$

$$\boxed{s = 4.59\times10^{13}\ m}$$

QUESTION NO. 04

$V_f = 24\ m/s$; $V_i = 0\ m/s$; $a = g = 9.8\ m/s^2$

a) $h = ?$ b) $t = ?$

To find h:

$$2gh = V_f^2 - V_i^2$$

$$h = \frac{V_f^2 - V_i^2}{2g} = \frac{(24)^2}{2(9.8)}$$

$$\boxed{h = 29.38\ m}$$

To find t:

$$V_f = V_i + gt$$

$$24 = 0 + 9.8t$$

$$t = \frac{24}{9.8}$$

$$\boxed{t = 2.45\,s}$$

QUESTION 05:

$V_f = 0\,m/s$; $t = 2.25\,s$; $h = 36.8\,m$; $a = g = 9.8\,m/s^2$

a) $V_i = ?$ b) V at h c) height gained $(h_f - h)$

i) For V_i (Initial speed):

$$h = V_i t - \frac{1}{2} g t^2$$

$$\frac{h + \frac{1}{2} g t^2}{t} = V_i$$

$$V_i = \frac{36.8 + \frac{1}{2}(9.8)(2.25)^2}{2.25}$$

$$\boxed{V_i = 27.38\,m/s}$$ Ans.

ii) FOR speed at height (h) V_h:

$\because V_f = V_i - gt$

$V_h = V_i - gt = 27.38 - (9.8)(2.25)$

$\boxed{V_h = 5.33 \, m/s}$ Ans

iii) For height after h ($h_f - h$):

$h_f - h = ?$ —(1)

For h_f:

$2gh = V_f^2 - V_i^2$

$h_f = \frac{0 - (27.38)^2}{2(-9.8)}$

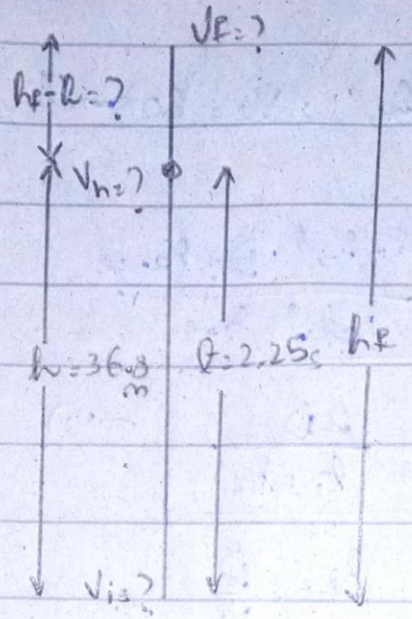
$\boxed{h_f = 38.24 m}$

Now,

height gained after h $= h_f - h = 38.24 - 36.8$

$\boxed{\text{height gained after } h = h_f - h = 1.44 m}$ Ans.

Diagram for Q5:



QUESTION 06:-

$V_1 = 40 km/h$; $V_2 = 60 km/h$; $D_1 = D_2 = D$

$$V_{avg} = \frac{\text{Total Distance}}{\text{total time}}$$

$$\boxed{V_{avg} = \frac{2D}{t_1 + t_2}} \quad -(1)$$

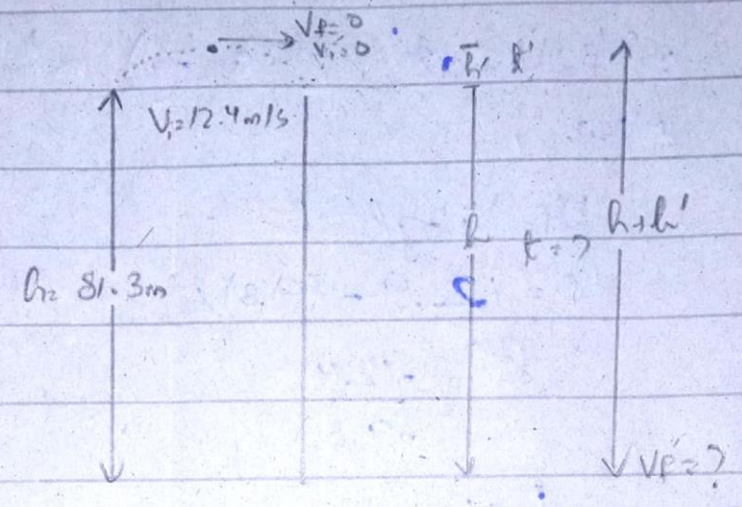
$$t_1 = \frac{D}{V_1} \; ; \quad t_2 = \frac{D}{V_2}$$

$$(1) \Rightarrow V_{avg} = \frac{2D}{\frac{D}{V_1} + \frac{D}{V_2}} = \frac{2D}{\frac{D}{40} + \frac{D}{60}} = \frac{2\cancel{D}}{\frac{\cancel{D}}{24}}$$

$$V_{avg} = \frac{2}{1/24}$$

$$\boxed{V_{avg} = 48 m/s} \text{ Ans.}$$

QUESTION 07:



$h = 81.3m$; $V = 12.4 m/s$

i) $V_f = ?$; ii) total $t = t' + t = ?$

$h' = ?$; $t' = ?$; $t = ?$

FOR h':

$$2gh' = V_f^2 - V_i^2$$

$$h' = \frac{0 - 12.4^2}{2(-9.8)}$$

$$\boxed{h' = 7.84m}$$

FOR t':

$\because V_f = V_i + at'$ $(\because a = -g)$

Now,

$V_f = V_i - gt'$

$0 = 12.4 - (9.8)t'$

$t' = \frac{-12.4}{-9.8}$

$\boxed{t' = 1.26s}$

For total height ($height_{Total}$):

$height_{Total} = h + h'$

$= 81.3 + 7.84$

$\boxed{height_{Total} = 89.14m}$

FOR V_f:

$2gh_{Total} = V_f^2 - V_i^2$

$2(+9.8)(89.14) = V_f^2 - 0$

$V_f^2 = \sqrt{1747.144}$

$\boxed{V_f = 41.8m/s}$ Ans.

FOR t (time to reach the ground):

$$t = \frac{V_f - V_i}{+g}$$

$$t = \frac{41.8 - 0}{9.8}$$

$$\boxed{t = 4.26s}$$

But

$$T_{total} = t + t'$$

$$= 4.26 + 1.26$$

$$\boxed{T_{total} = 5.52s}$$ Ans

QUESTION 08:

$V_f = 360km/h$; $V_i = 0$; $S = 1.8km$; $a = ?$

$= 100m/s$ $= 1800m$

$$a = \frac{V_f^2 - V_i^2}{2S} = \frac{(100)^2}{2(1800)}$$

$$= \frac{10000}{3600}$$

$$\boxed{a = 2.78m/s^2}$$ Ans.

QUESTION 09:

$a = +4.92 m/s^2$; $V_i = 24.6 m/s$; $V_f = 0 m/s$

$t = ?$; $S = ?$

FOR t:-

$\therefore V_f = V_i + at$

$\frac{V_f - V_i}{a} = t$

$t = \frac{24.6}{4.92}$

$t = 5s$

FOR S:

$S = \frac{V_f^2 - V_i^2}{2a} = \frac{(24.6)^2}{2(4.92)} = \frac{605.16}{9.84}$

$S = 61.5m$

QUESTION 10:

$x = 5t + 10t^2$

a) V_{avg} of particle during 1st 3s of its motion:

For V at 0s:-

$x = 50t + 10t^2$

diff w.r.t. t

$\frac{dx}{dt} = \frac{d}{dt}(50t) + 10\frac{d}{dt}t^2$

$V = 50 + 20(t)$ $(\because t = 0)$

$\boxed{V_0 = 50 \text{ m/s}}$

For V at 3s:

$x = 50(t) + 10(t)^2$

diff w.r.t t

$\frac{dx}{dt} = \frac{d}{dt}50t + 10\frac{d}{dt}t^2$

$V_{3s} = 50 + 20(t)$

$V_{3s} = 50 + 60$

$V_{3s} = 110 \text{ m/s}$

For V_{avg};

$$V_{avg} = \frac{V_0 + V_3}{2}$$

$$= \frac{50+110}{2}$$

$$\boxed{V_{avg} = 80 m/s}$$

b) V_{inst} at $t = 3s$

$x = 10t^2 + 50t$

diff w.r.t. t

$$\frac{dx}{dt} = \frac{d}{dt} 10t^2 + \frac{d}{dt} 50t$$

$V = 20t + 50$ ($\because t = 3s$)

$V_{3s} = 60 + 50$

$$\boxed{V_{3s} = 110 m/s}$$

c) Instantaneous acc of particle at $t = 3s$

$x = 10t^2 + 5t$

diff w.r.t. t

$$\frac{dx}{dt} = \frac{d}{dt} 10t^2 + \frac{d}{dt} 5t$$

$V_{ins} = 20t + 50$

diff. w.r.t t

$$\frac{d}{dt} V_{ins} = \frac{d}{dt} 20t + \frac{d}{dt} 50$$

$$\boxed{a_{ins} = 20 m/s^2}$$

—— The END ——