FORCE AND NEWTON'S LAW

Question 01:

Data:-

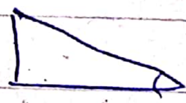
$m_1 = 5kg$

$m_2 = 13kg$

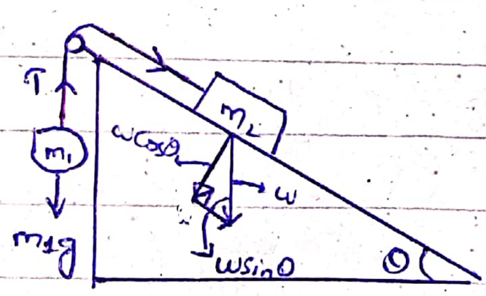
$\theta = 48°$

$T = ?$

$a = ?$



F.B.D:-



01) $m_1 a = T - m_1 g$ —(1)

02) $m_2 a = m_2 g \sin\theta - T$ —(2)

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For a:-

Adding eq.(1) & eq.(2)

$$m_1 a + m_2 a = \cancel{T} - m_1 g + m_2 g \sin\theta - \cancel{T}$$

$$a(m_1 + m_2) = \frac{m_2 g \sin\theta - m_1 g}{1}$$

$$a = \frac{g(m_2 \sin\theta - m_1)}{m_1 + m_2}$$

$$a = \frac{9.8(13 \sin 46 - 5)}{13 + 5}$$

$$\boxed{a = 2.53\,m/s^2}$$

Ans

For T:-

eq.(1) $\Rightarrow$ $T = m_1 a + m_1 g$

$T = 5(2.53) + 5(9.8)$

$$\boxed{T = 61.65\,N}$$

Ans

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QUESTION 02

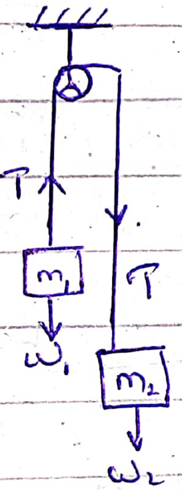
Data:-

$a = ?$

$T = ?$

$m_1 = 4kg$

$m_2 = 9.5kg$



01) $m_1 a = T - W_1$ —(1)

02) $m_2 a = W_2 - T$ —(2)

For a:- Adding eq (1) & (2)

$$m_1 a + m_2 a = \cancel{T} - m_1 g + m_2 g - \cancel{T}$$

$$a(m_1 + m_2) = g(m_2 - m_1)$$

$$a = \frac{9.8(9.5-4)}{9.5+4}$$

$$\boxed{a = 3.99 m/s^2}$$ Ans

FOR T:- (1) => $T = m_1 a + m_1 g = 4(3.99) + 4(9.8)$

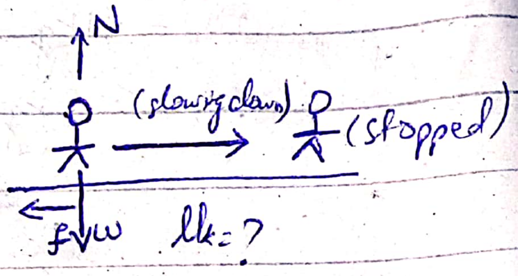
$$\boxed{T = 55.16 N}$$ Ans

QUESTION 03

$m = 79kg$

$f_k = 470N$

$\mu_k = ?$



we know that

$$f_k = \mu_k (mg)$$

$$\cancel{\mu_k = \frac{79(9.8)}{470}}$$

$$\cancel{\boxed{\mu_k = 1.647}}$$

$$\mu_k = \frac{470}{79(9.8)}$$

$$\boxed{\mu_k = 0.61}$$

QUESTION 04

$\mu_s = 0.62$

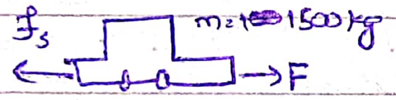
$m = 1500kg$

a) $f_{s_{max}}$ (on level ground) = ?

b) $f_{s_{max}}$ (on slope) = ?

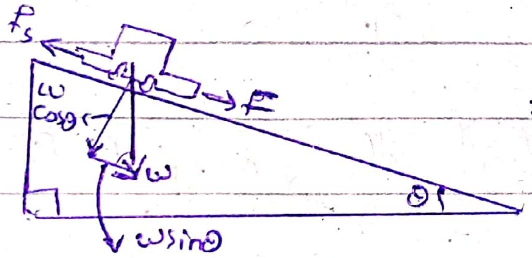
$\theta = 8.6°$

a)



1) $F = f_s$

b)



2) $F = f_s$

$f_s = w \sin\theta$

a) for $f_{s_{max}}$ (on level ground)

$F = f_{s_{max}} = \mu mg$

$F = f_{s_{max}} = (0.62)(1500)9.8$

$\boxed{f_{s_{max}} = 9114 N.F}$

b) for $f_{s_{max}}$ (on slope)

$f_{s_{max}} = \mu mg \cos\theta = \mu mg \cos\theta = (1500)(9.8) \cos 8.6 \times 0.62$

$\boxed{f_{s_{max}} = 9011.5 N.F}$

QUESTION 05

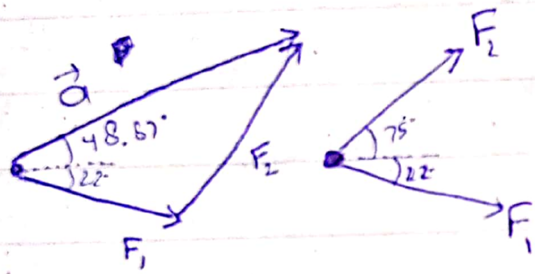
Data:-

$m = 0.5 kg$

$F_1 = 3.5 N$; $\theta_1 = -22^\circ$

$F_2 = 7.5 N$; $\theta_2 = 75^\circ$

F.B.D:-



$F_x = F_{1x} + F_{2x}$

$F_y = F_{1y} + F_{2y}$

To find the magnitude of F;

$F_{1x} = F_1 \cos\theta = 3.5 \cos(-22)$

$\boxed{F_{1x} = 3.24 N}$

$F_{1y} = F_1 \sin\theta = 3.5 (\cos(-22))$

$\boxed{F_{1y} = -1.311 N}$

$F_{2x} = F_2 \cos\theta = 7.5 \cos(75)$

$\boxed{F_{2x} = 1.94 N}$

$F_{2y} = F_2 \sin\theta = 7.5 \sin 75$

$\boxed{F_{2y} = 7.244N}$

For F_x:-

$F_x = F_{1x} + F_{2x} = 3.24 + 1.94$

$\boxed{F_x = 5.18N}$

For F_y:-

$F_y = F_{1y} + F_{2y} = 5.933$

$\boxed{F_y = 5.933N}$

For mag. of F:-

$F = \sqrt{F_x^2 + F_y^2} = \sqrt{5.18^2 + 5.933^2}$

$\boxed{F = 7.876 N}$

For mag of acceleration:-

$a = F/m = 7.876/0.5$

$\boxed{a = \cancel{17}\ 15.75 m/s^2}$

Ans

For direction of acceleration:-

The direction of acceleration is the same as the direction of force therefore

$$a = \tan^{-1}\left(\frac{F_y}{F_x}\right)$$

$$= \tan^{-1}\left(\frac{5.933}{5.18}\right)$$

$$\boxed{a = 48.876^\circ}$$

Ans.

QUESTIION 06

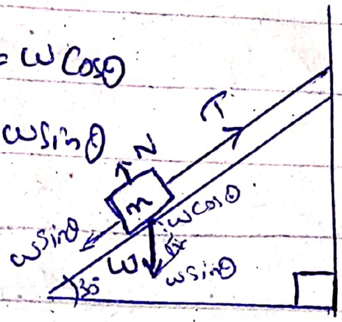
Data:-

$N = W\cos\theta$

$T = W\sin\theta$

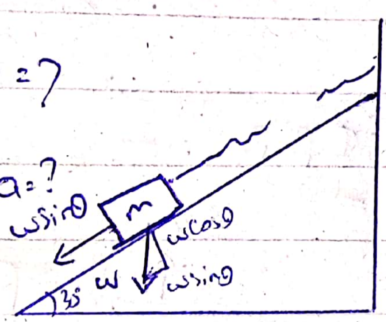
$m = 8.5\,kg$

$\theta = 30^\circ$



a) $T = ?$; b) $N = ?$

c) a (if the cord is cut) $= ?$



$a = \frac{mg\sin\theta}{m} = g\sin\theta$

a) For T:

$T = mg\sin\theta = 8.5(9.8)\sin 30^\circ$

$\boxed{T = 41.65N}$

b) FOR N:-

$N = mg\cos\theta = 8.5(9.8)\cos 30^\circ$

$\boxed{N = 72.14N}$

c) $a = g\sin\theta = 9.8\sin 30^\circ$

$\boxed{a = 4.9\,m/s^2}$

QUESTION 07

Data:- $m = 3\,kg$

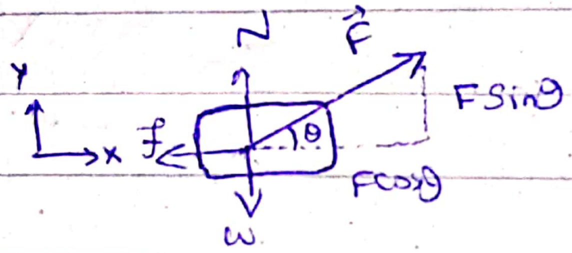
$F = 12\,N$

$\theta =$ may be 0 to $90°$

$\mu_k = 0.4$

$a = ?$

F.B.D.



The acceleration (along x-axis) is given by

$$ma = F\cos\theta - f$$

$$a = \frac{F\cos\theta}{m} - \frac{f}{m} \quad (\because f = \mu_k N)$$

$$\boxed{a = \frac{F\cos\theta}{m} - \frac{\mu_k N}{m}} \text{ —(1)}$$

FOR N:

consider the forces along y-axis.

$$N + F\sin\theta - mg = 0$$

$$\boxed{N = mg - F\sin\theta}$$

Put in eq(1)

$$(1) \Rightarrow \quad a = \frac{F\cos\theta}{m} - \frac{\mu_k(mg - F\sin\theta)}{m}$$

~~a Cos θ~~ diff w.r.t θ

$$\frac{da}{d\theta} = \frac{F}{m}(-\sin\theta) - \mu_k\left(\frac{d}{d\theta}g - \frac{F}{m}\frac{d}{d\theta}\sin\theta\right)$$

$$\frac{da}{d\theta} = -\frac{F}{m}\sin\theta - \mu_k\left(-\frac{F}{m}\cos\theta\right)$$

at max value of a $\frac{da}{d\theta}$ will be 0

$$0 = -\frac{F}{m}\sin\theta + \mu_k\frac{F}{m}\cos\theta$$

$$\mu_k \frac{F}{m}\cos\theta = \frac{F}{m}\sin\theta \; ; \; \mu_k = \tan\theta$$

$$\theta = \tan^{-1}\mu_k \; ; \; \theta = \tan^{-1}(0.4)$$

$$\boxed{\theta = 21.80^\circ}$$ Ans

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OSCILLATION

QUESTION 01:

<u>Data.</u> $T/2 = 0.25s$

~~$T = 0.5s$~~

Total distance from 1pt to another = $d = 36cm$

$d = 0.36m$

i) $T = ?$

ii) $f = ?$

iii) amplitude = ?

<u>Solution.</u>

i) For Time period (T)

The time taken to get from 1 extreme to another is

$\therefore T/2 = 0.25$

$$\boxed{T = 0.5s}$$

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ii) For frequency (f)

we know that

$f = 1/T = 1/0.5$

$\boxed{f = 2Hz}$

iii) For amplitude (x_m)

The amplitude is half of the distance between two extremes;

$x_m = d/2$

$\boxed{x_m = 0.18m}$

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QUESTION 02

Data:

$m = 0.12 kg$

$x_m = 8.5 cm = 0.085 m$

$T = 0.2 s$

i) $F_{max} = ?$

ii) $K = ?$

Solution:-

i) For F_{max}:

The max force is given by;

$$\cancel{F = \frac{1}{2} K x_m^2}$$

$$F = m a_{max} \quad (\because a_{max} = -\omega^2 x_m)$$

$$F = m(-\omega^2 x_m)$$

$$\because \omega = \frac{2\pi}{T}$$

$$F = -m x_m \left(\frac{4\pi^2}{T^2}\right)$$

$$= -(0.12)(0.085)\left(\frac{4\times(3.142)^2}{0.2^2}\right)$$

$$\boxed{F = -10.066 N}$$

Ans

ii) FOR K:-

we know that

$$\omega = \sqrt{\frac{K}{m}}$$

$$K = \omega^2 m$$

$$K = \left(\frac{2\pi}{T}\right)^2 m$$

$$K = \frac{4\pi^2}{(0.2)^2}(0.12)$$

$$\boxed{K = 118.45 \ N/m}$$

Ans

QUESTION 03

Data:-

$m = 1kg$

$T = 1s$

$V_{max} = 103 m/s$

a) $\omega = ?$

b) $x_m = ?$

Solution,

i) For ω,

we know that

$$\omega = \frac{2\pi}{T} = \frac{2 \times 3.142}{1}$$

$$\boxed{\omega = 6.284 \text{ rev/s}}$$

Ans

ii) For x_m:

we know that,

$V = r\omega$

but in S.H.M $r = x_m$, Thus

$x_m = V/\omega = 103/6.284$

$\boxed{x_m = 16.39m}$ Ans

QUESTION 04

Data:-

$x_m = 6cm = 0.06m$

$\phi = ?$

Solution:-

For ϕ:-

we are given that

$x = x_m \cos(\omega t + \phi)$

from graph at $t = 0$; $x = -0.02m$

so,

$-0.02 = 0.06\cos(\omega(0) + \phi)$

$\frac{-0.02}{0.06} = \cos\phi$

$\phi = \cos^{-1}(1/3)$

$\boxed{\phi = 109.47°}$ or $\boxed{\phi = 1.91 rad}$

Ans Ans

QUESTION 05

Data: $K_{max} = 6J$

$K = ?$

Solution

i) For K:-

we know that

$$K.E_{max} = \frac{1}{2} K x_m^2$$

from graph $x_m = 12cm = 0.12$

$$\cancel{6 = \frac{1}{2} K (144)}$$

$$\cancel{\frac{12}{144} = K}$$

$$\cancel{K = 0.0833 \, N/m}$$

Ans.

$$6 = \frac{1}{2} K (0.12)^2$$

$$K = \frac{12}{0.0144}$$

$$\boxed{K = 8.33 \times 10^2 \, N/m}$$

QUESTION NO. 06

Data:

$m = 1.5 kg$

$K = 8 N/m$

$b = 230 g/s = 0.23 kg/s$

$x_m = 12 cm = 0.12 m$

a) Amplitude $= \frac{1}{3} x_m$

b) No. of oscillations?

Solution:

~~a) Data~~

a) Amplitude $= \frac{1}{3} x_m$

$$\cancel{x_m} e^{-\frac{bt}{2m}} = \frac{\cancel{x_m}}{3}$$

taking ln on b.s

$$-\frac{bt}{2m} = \ln(1/3)$$

$$t = \frac{\ln(1/3) \times 2(1.5)}{-(0.23)}$$

$$\boxed{t = 14.32 s}$$

b) for number of oscillations

$$n = \frac{t}{T}$$

$$n = \frac{14.32}{2(3.142)\sqrt{\frac{1.5}{.8}}}$$

$$\boxed{n = 2.722}$$

Since $2.722 \approx 3$

$\boxed{\text{no. of oscillations} = 3}$

QUESTION 07

Data:-

$x_m = 6m$

$\omega = 3\pi$ rad/s

$\phi = \pi/3$ rad

$t = 2s$

a) $x = ?$

b) $V = ?$

c) $acc = ?$

d) phase = ?

e) frequency = ?

f) $T = ?$

Solution

a) ~~x~~ For x at $t = 2$

$$x = x_m \cos(\omega t + \phi)$$

$$= 6 \cos(3\pi(2) + \pi/3)$$

$$\boxed{x = 3m}$$

b) For V at $t = 2$

$$x = x_m \cos(\omega t + \phi)$$

$$\frac{dx}{dt} = x_m - \sin(\omega t + \phi).\omega$$

$V = -x_m \omega \sin(\omega t + \phi)$

$= -6(3\pi) \sin(6\pi + \pi/3)$

$\boxed{V = -48.97 m/s}$ Ans

c) For a at t = 2

$V = -x_m \omega \sin(\omega t + \phi)$

$\frac{dv}{dt} = -x_m \omega^2 \cos(\omega t + \phi)$

$a = -6(3\pi)^2 \cos(6\pi + \pi/3)$

$\boxed{a = -266.47 m/s^2}$ Ans

d) phase at t = 2 s

phase = $\omega t + \phi$

$= 3\pi(2) + \pi/3$

$\boxed{\text{phase} = \frac{19}{3}\pi \text{ rad}}$

e) frequency:-

$\omega = 2\pi f$

$\cancel{f = \frac{\omega}{2\pi} = \frac{6\pi}{2\pi}}$

$f = \frac{\omega}{2\pi} = \frac{3\pi}{2\pi}$

$\boxed{f = 1.5\,Hz}$ Ans

f) Time period:-

$T = \frac{2\pi}{\omega} = \frac{2\pi}{3\pi}$

$\boxed{T = 0.66s}$ Ans

QUESTION 08

Data:

$m = 3kg$

$x_m = 5m$

$\omega = \pi/3 \ rad/s$

$\phi = \pi/4 \ rad$

i) x at which P.E = 1/2 T.E.

2) time taken to reach x = ?

Sol.

a) x at which P.E = 1/2 T.E

we know that

$U = \frac{1}{2}Kx^2$

$T.E. = \frac{1}{2}Kx_m^2$

A/c to the condition

$U = \frac{1}{2}T.E.$

$\cancel{\frac{1}{2}}\cancel{K}x^2 = \frac{1}{2\cancel{4}}\cancel{K}x_m^2$

$x = \sqrt{\frac{x_m^2}{2}}$

$\boxed{x = 3.555m}$

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b) Time taken to reach x from the eq position

→ For t_x

$$x = x_m \cos(\omega t_x + \phi)$$

$$3.55 = 5 \cos(\pi/3\, t_x - \pi/4)$$

$$\frac{3}{\pi}\left(\cos^{-1}\left(\frac{3.55}{5}\right) + \frac{\pi}{4}\right) = t_x$$

$$\boxed{t_x = 1.496\,s}$$

→ For t_{eqr} (at eq $x = 0$)

$$0 = 5 \cos(\pi/3\, t - \pi/4)$$

$$\frac{3}{\pi}\left(\cos^{-1}(0) + \pi/4\right) = t$$

$$\boxed{t_{eq} = 2.25\,s}$$

$$T = t_{eq} - t_x = 2.25 - 1.496$$

$$\boxed{T = 0.754\,s}$$

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$2\pi\sqrt{\frac{l}{g}}$

QUESTION 09

Data:-

$m = 5\ kg$

$K = 1000\ N/m$

$x_m = 50cm = 0.5m$

$V = 10\ m/s$

a) freq = ?

b) U (at x_m) = ?

c) K (at x_m) = ?

d) amplitude = ?

Sol

a) for frequency

$\omega = 2\pi f$

$f = \frac{\omega}{2\pi}$ $\quad \left(\because \omega = \sqrt{\frac{K}{m}}\right)$

$f = \frac{1}{2\pi}\sqrt{\frac{m}{K}} = \frac{1}{2(3.142)}\sqrt{\frac{1000}{5}}$

$\boxed{f = 2.25\ Hz}$

b) U at x_m:-

The potential energy at max value of x is given by.

$$U = \frac{1}{2} K x_m^2$$

$$U = \frac{1}{2}(1000)(0.5)^2$$

$$\boxed{U = 125 J}$$

c) Initial K of the system

The K.E. depends upon the velocity of the body, therefore,

$$K.E = \frac{1}{2} m V^2 = \frac{1}{\cancel{2}} (\cancel{5}^{2.5})(10)^2$$

$$\boxed{K.E. = 250 N}$$

d) <u>amplitude.</u>

The total energy is given by

$$E = \frac{1}{2} K x_m^2$$

$$U + K.E = E$$

So,

$$U + K.E = \frac{1}{2} K x_m^2$$

$$375 = \frac{1}{2}(1000) x_m^2$$

$$\sqrt{\frac{2 \times 375}{1000}} = x_m$$

$$\boxed{x_m = 0.866 m}$$

QUESTION 10:

Data:-

$m = 80\ kg$

$K = 85\ N/m$

$b = 4\ kg/s$

a) $T = ?$

b) time for $x\, e^{\frac{bt}{2m}} = \frac{1}{2} x_m$

c) time for $M.E' = \frac{1}{2} M.E$

Sol

a) For time period.

$$T = \frac{2\pi}{\sqrt{\frac{K}{m} - \frac{b^2}{4m^2}}} = \frac{2(3.142)}{\sqrt{\frac{85}{80} - \frac{16}{4(80)^2}}}$$

$$\boxed{T = 6.098\,s}$$

b) For time to reach;

$$x_m e^{\frac{-bt}{2m}} = \frac{1}{2} x_m$$

taking ln b.s

$$\frac{-bt}{2m} = \ln\left(\frac{1}{2}\right)$$

$$t = \frac{\ln(1/2) \times 2m}{-b}$$

$$t = \frac{(-0.693) \times 2 \times 80}{-4}$$

$$\boxed{t = 27.72s}$$

c) time for M.E' $\frac{1}{2}$ M.E

The M.E of the system is given by, a/c to the condition given

$$\cancel{E} e^{-\frac{bt}{m}} = \frac{1}{2} \cancel{E}$$

taking ln b.s

$$\frac{-bt}{m} = \ln(1/2)$$

$$t = \frac{\ln(1/2)(80)}{-4}$$

$$\boxed{t = 13.86s}$$