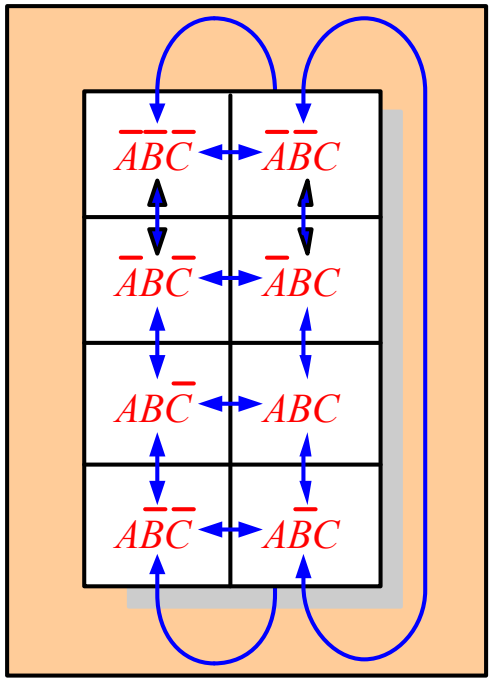
Summary

Karnaugh maps

The Karnaugh map (K-map) is a tool for simplifying combinational logic with 3 or 4 variables. For 3 variables, 8 cells are required (2^3).

The map shown is for three variables labeled *A, B,* and *C*. Each cell represents one possible product term.

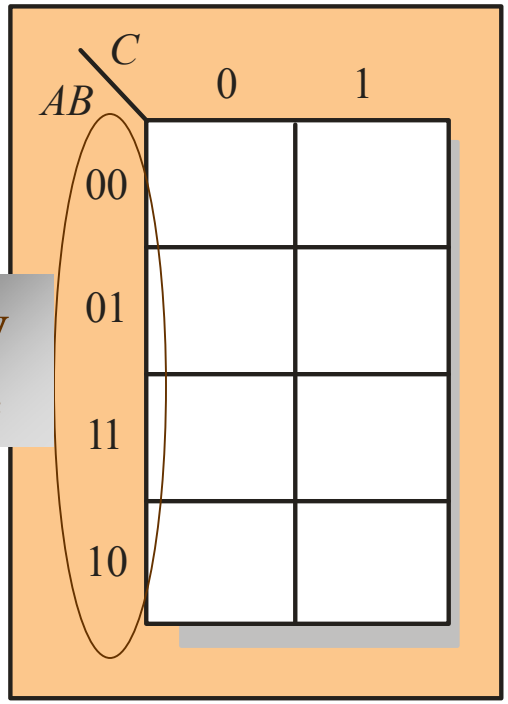
Each cell differs from an adjacent cell by only one variable.



Summary

Karnaugh maps

Cells are usually labeled using 0's and 1's to represent the variable and its complement.



Gray code

The numbers are entered in gray code, to force adjacent cells to be different by only one variable.

Ones are read as the true variable and zeros are read as the complemented variable.

Summary

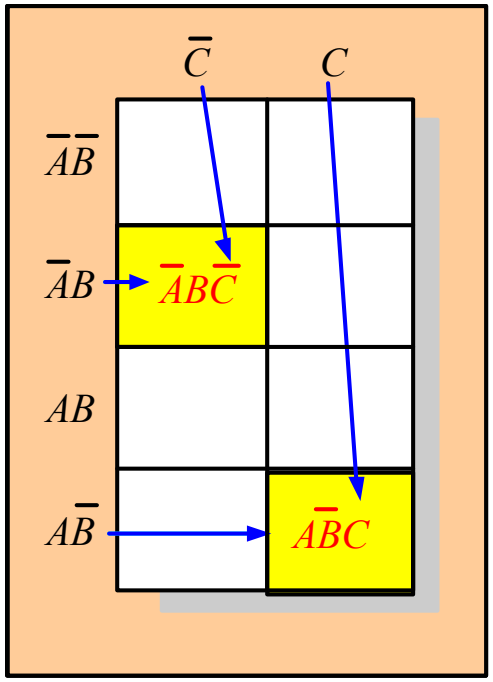
Karnaugh maps

Alternatively, cells can be labeled with the variable letters. This makes it simple to read, but it takes more time preparing the map.

Example Read the terms for the yellow cells.

Solution

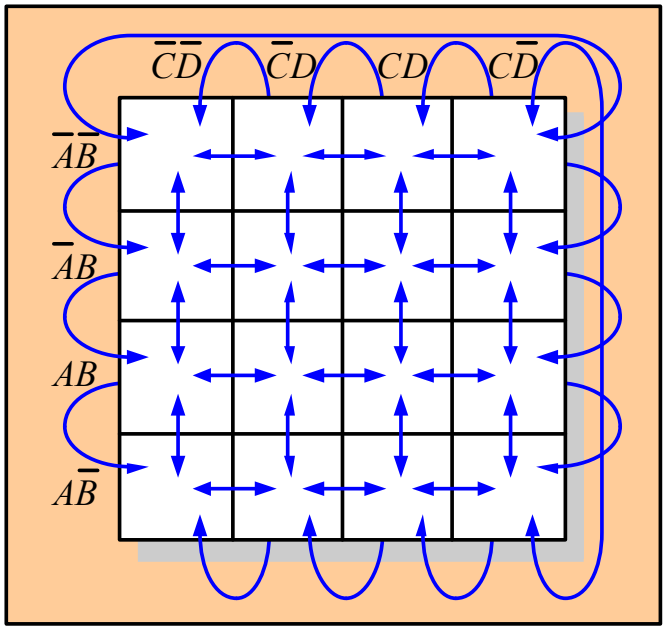
The cells are $\bar{A}B\bar{C}$ and $A\bar{B}C$.



Summary

Karnaugh maps

A 4-variable map has an adjacent cell on each of its four boundaries as shown.



Each cell is different only by one variable from an adjacent cell.

Grouping follows the rules given in the text.

The following slide shows an example of reading a four variable map using binary numbers for the variables…

Summary

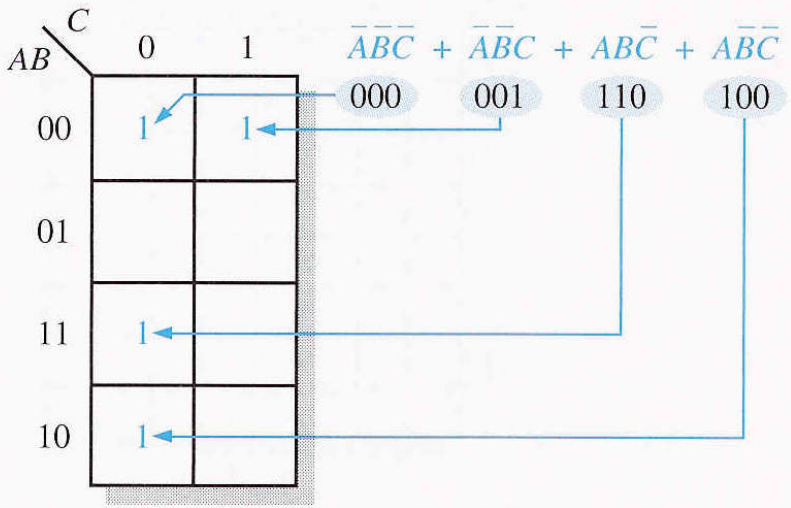
Mapping an SOP expression in K-Map

Step 1. Determine the binary value of each product term in the standard SOP expression. After some practice, you can usually do the evaluation of terms mentally.

Step 2. As each product term is evaluated, place a 1 on the Karnaugh map in the cell having the same value as the product term.

FIGURE 4–24

Example of mapping a standard SOP expression.



Summary

Map the following standard SOP expression on a Karnaugh map:

$$\overline{A}\,\overline{B}CD + \overline{A}B\overline{C}\,\overline{D} + AB\overline{C}D + ABCD + AB\overline{C}\,\overline{D} + \overline{A}\,\overline{B}\,\overline{C}D + A\overline{B}C\overline{D}$$

Summary

Map the following standard SOP expression on a Karnaugh map:

$$\overline{A}\,\overline{B}CD + \overline{A}B\overline{C}\,\overline{D} + AB\overline{C}D + ABCD + AB\overline{C}\,\overline{D} + \overline{A}\,\overline{B}\,\overline{C}D + A\overline{B}C\overline{D}$$

Evaluate the expression as shown below. Place a 1 on the 4-variable Karnaugh map in Figure 4–26 for each standard product term in the expression.

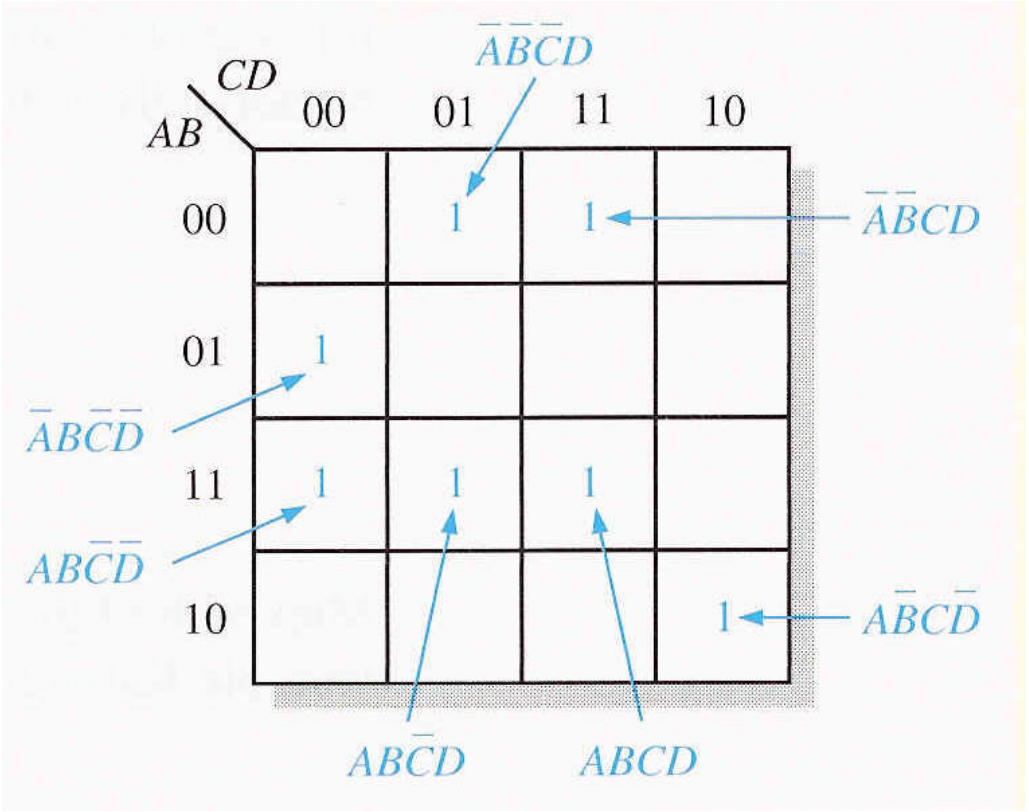
$$\overline{A}\,\overline{B}CD + \overline{A}B\overline{C}\,\overline{D} + AB\overline{C}D + ABCD + AB\overline{C}\,\overline{D} + \overline{A}\,\overline{B}\,\overline{C}D + A\overline{B}C\overline{D}$$

$$0011 \quad 0100 \quad 1101 \quad 1111 \quad 1100 \quad 0001 \quad 1010$$

Evaluate the expression as shown below. Place a 1 on the 4-variable Karnaugh map in Figure 4–26 for each standard product term in the expression.

$$\overline{A}\,\overline{B}CD + \overline{A}B\overline{C}\,\overline{D} + AB\overline{C}D + ABCD + AB\overline{C}\,\overline{D} + \overline{A}\,\overline{B}\,\overline{C}D + A\overline{B}C\overline{D}$$

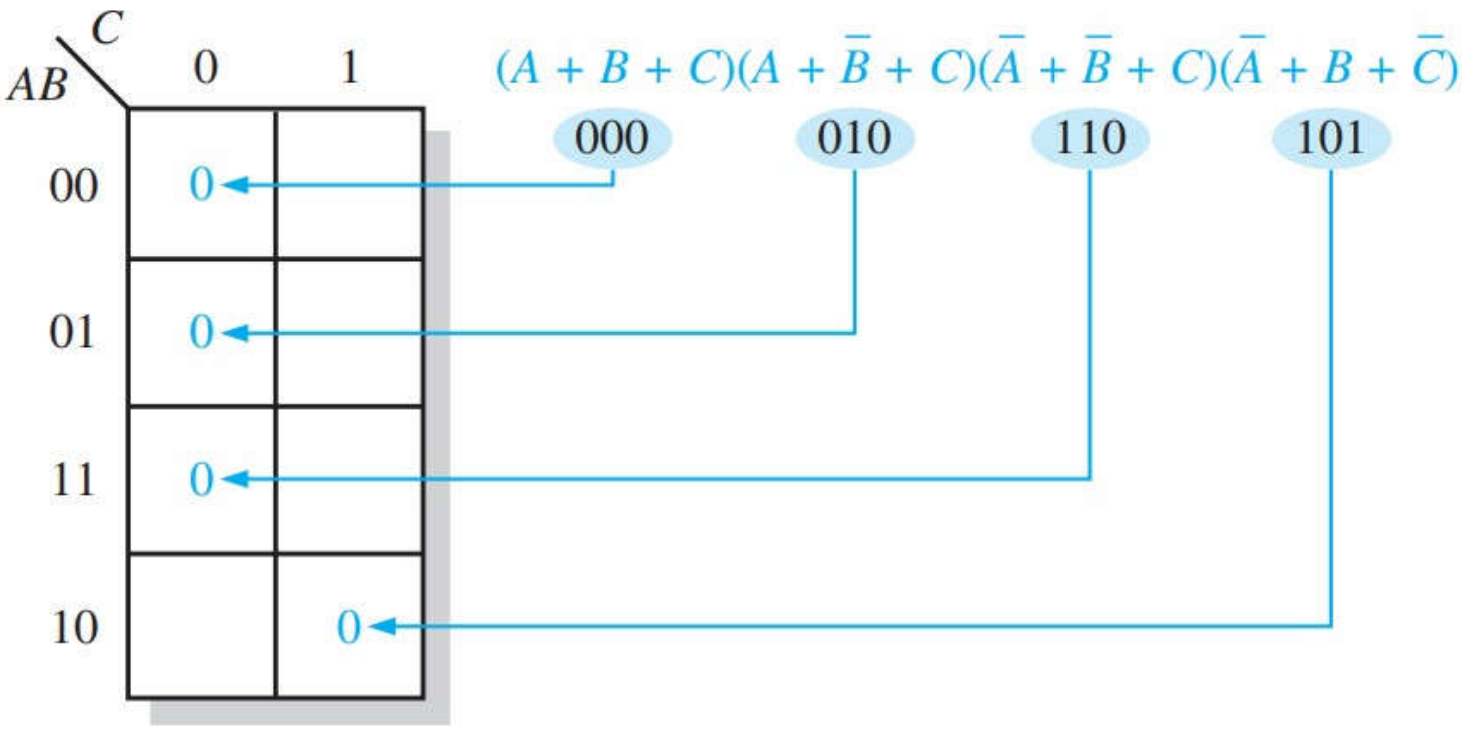
$$0011 \quad 0100 \quad 1101 \quad 1111 \quad 1100 \quad 0001 \quad 1010$$



Karnaugh Map for POS expressions

Step 1: Determine the binary value of each sum term in the standard POS expression. This is the binary value that makes the term equal to 0.

Step 2: As each sum term is evaluated, place a 0 on the Karnaugh map in the corresponding cell.



EXAMPLE 4–33

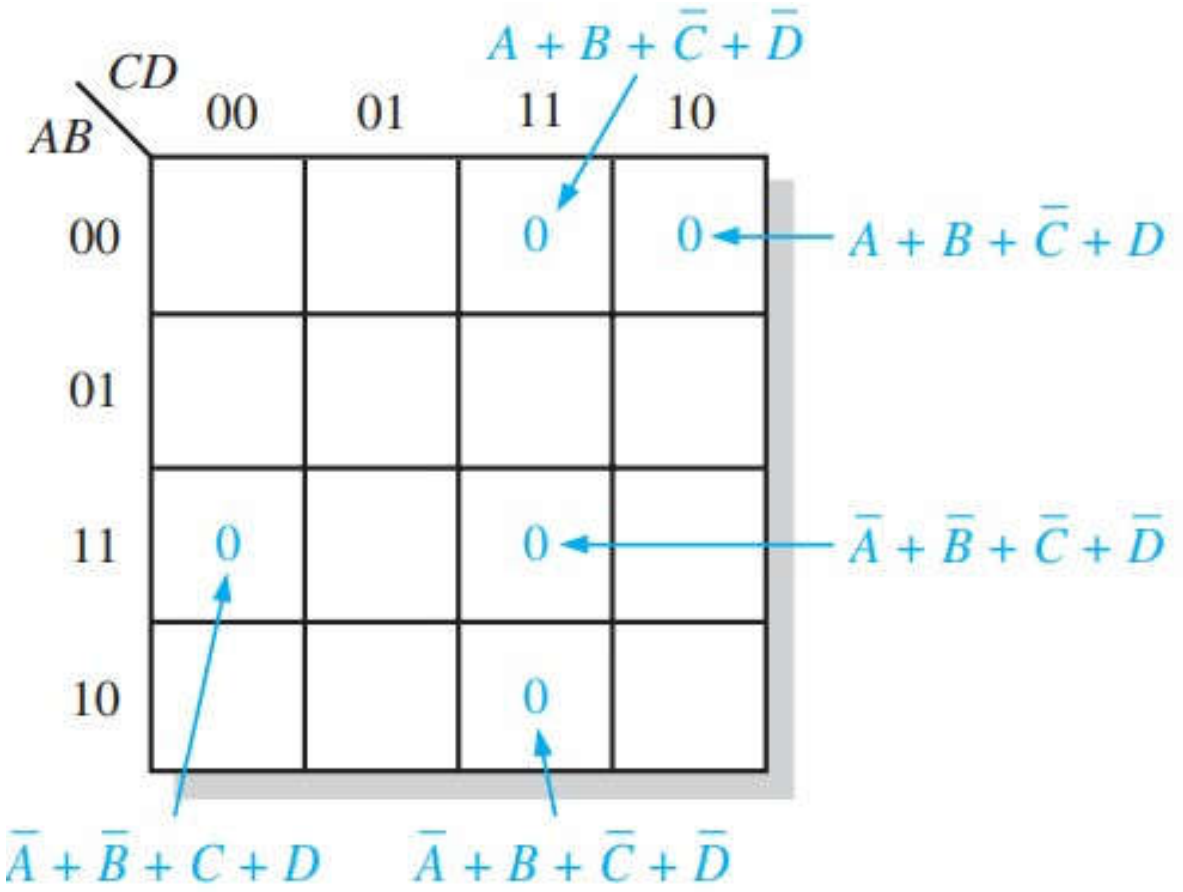
Map the following standard POS expression on a Karnaugh map:

$$(\overline{A} + \overline{B} + C + D)(\overline{A} + B + \overline{C} + \overline{D})(A + B + \overline{C} + D)(\overline{A} + \overline{B} + \overline{C} + \overline{D})(A + B + \overline{C} + \overline{D})$$

EXAMPLE 4–33

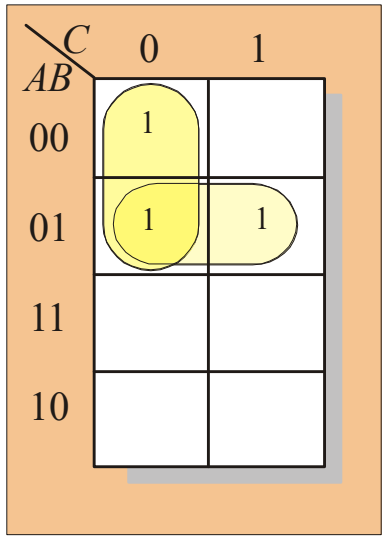
Map the following standard POS expression on a Karnaugh map:

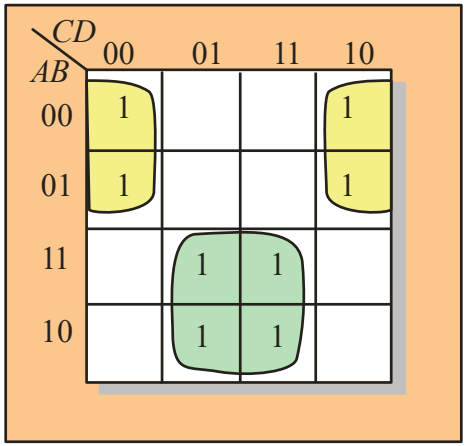
$$(\overline{A} + \overline{B} + C + D)(\overline{A} + B + \overline{C} + \overline{D})(A + B + \overline{C} + D)(\overline{A} + \overline{B} + \overline{C} + \overline{D})(A + B + \overline{C} + \overline{D})$$



Summary

Minimizing an SOP expression in K-Map





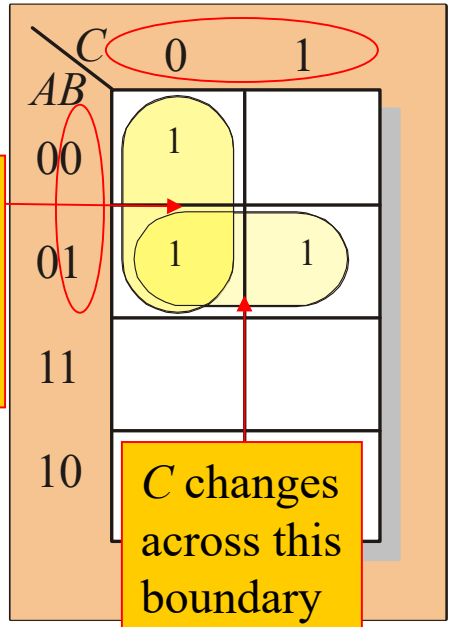
1. A group must contain either 1, 2, 4, 8, or 16 cells, which are all powers of two. In the case of a 3-variable map, $2^3 = 8$ cells is the maximum group.
2. Each cell in a group must be adjacent to one or more cells in that same group, but all cells in the group do not have to be adjacent to each other.
3. Always include the largest possible number of 1s in a group in accordance with rule 1.
4. Each 1 on the map must be included in at least one group. The 1s already in a group can be included in another group as long as the overlapping groups include noncommon 1s.

Summary

Karnaugh maps

K-maps can simplify combinational logic by grouping cells and eliminating variables that change.

Example Group the 1's on the map and read the minimum logic.



Solution

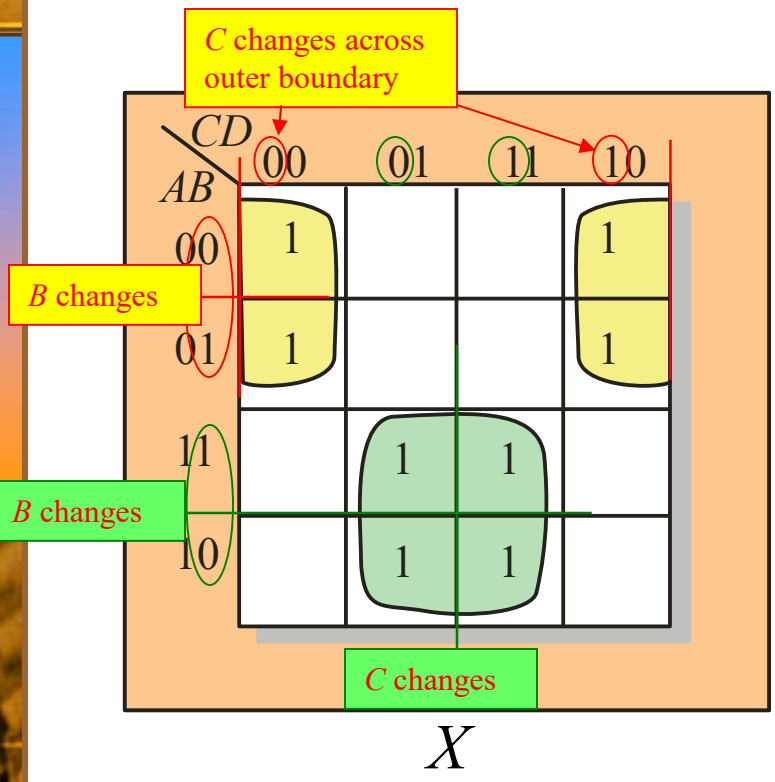
1. Group the 1's into two overlapping groups as indicated.
2. Read each group by eliminating any variable that changes across a boundary.
3. The vertical group is read $\overline{A}\,\overline{C}$.
4. The horizontal group is read $\overline{A}B$.

$$X = \overline{A}\,\overline{C} + \overline{A}B$$

Summary

Karnaugh maps

Example Group the 1's on the map and read the minimum logic.

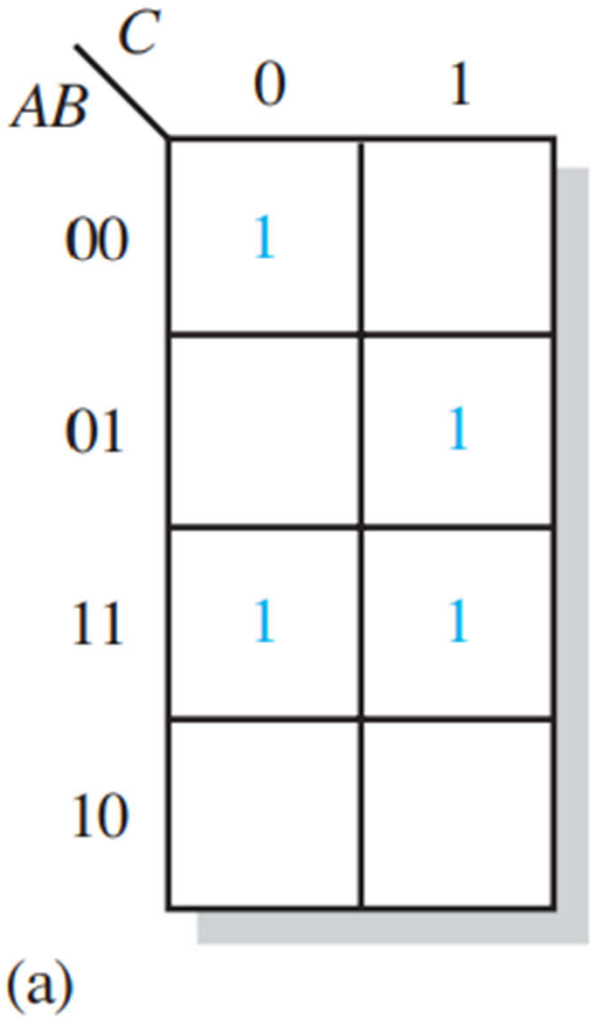


Solution

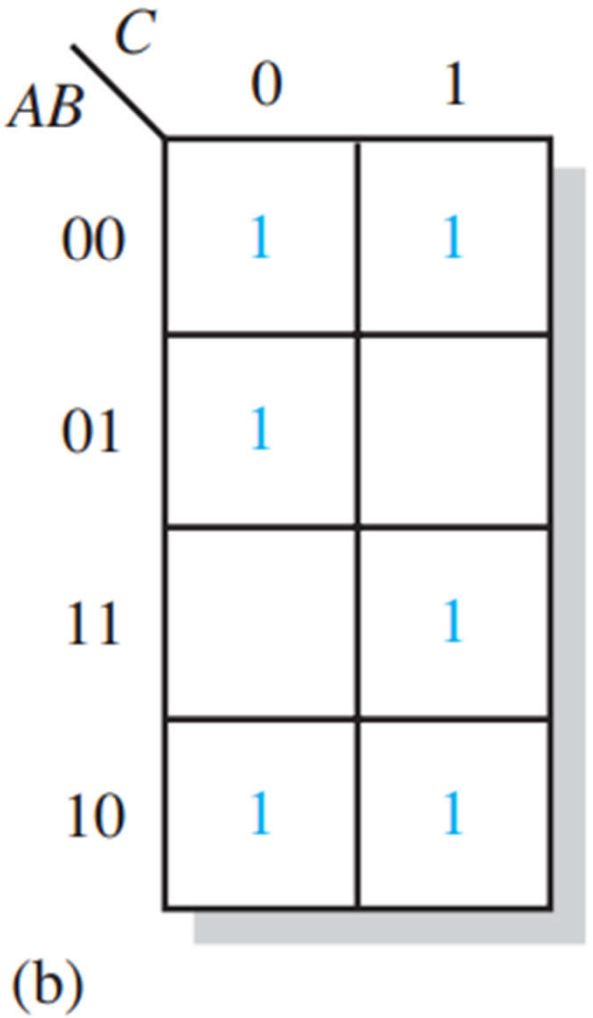
1. Group the 1's into two separate groups as indicated.
2. Read each group by eliminating any variable that changes across a boundary.
3. The upper (yellow) group is read as $\overline{A}\overline{D}$.
4. The lower (green) group is read as AD.

$$X = \overline{A}\overline{D} + AD$$

Group the 1s in each of the Karnaugh maps

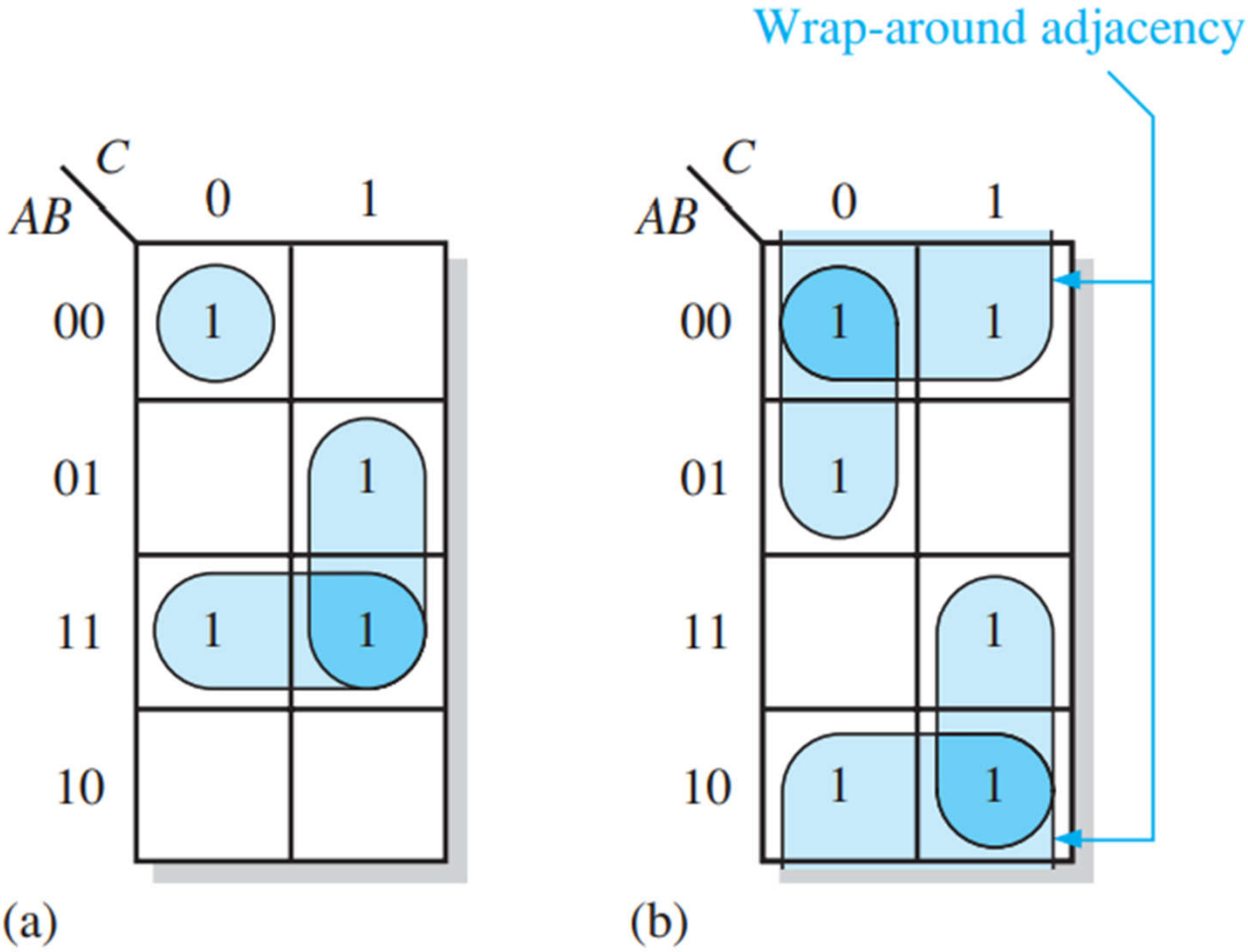


(a)

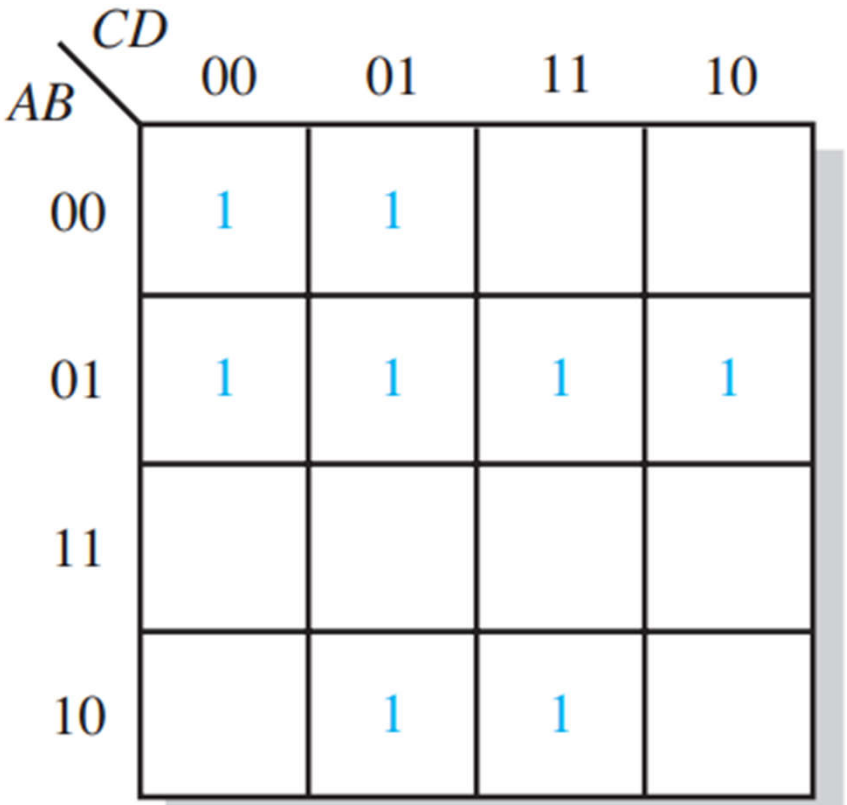


(b)

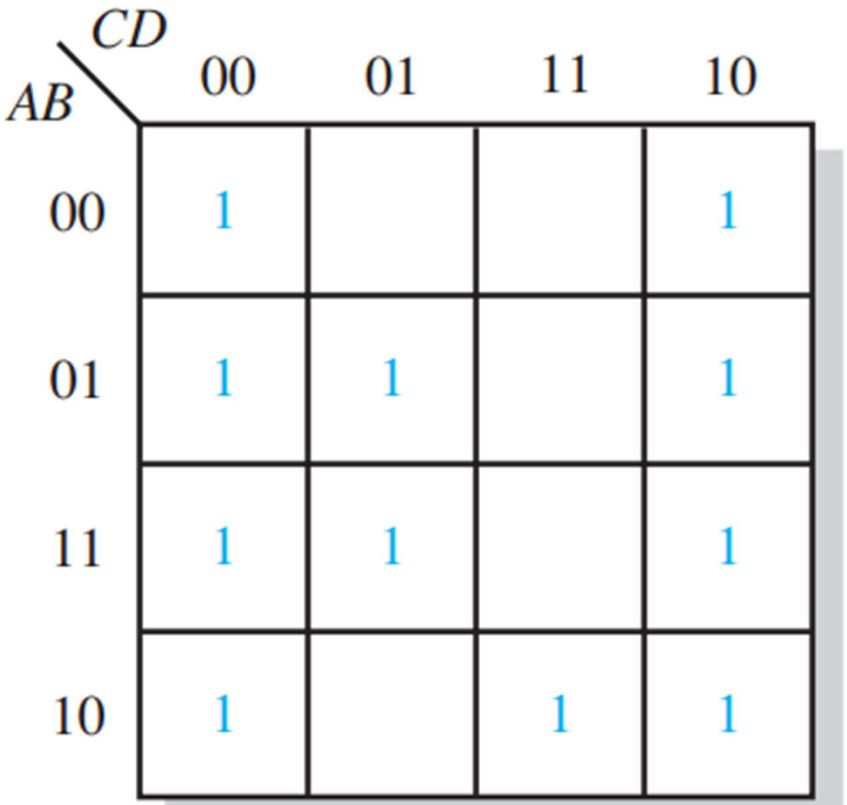
Group the 1s in each of the Karnaugh maps



Group the 1s in each of the Karnaugh maps

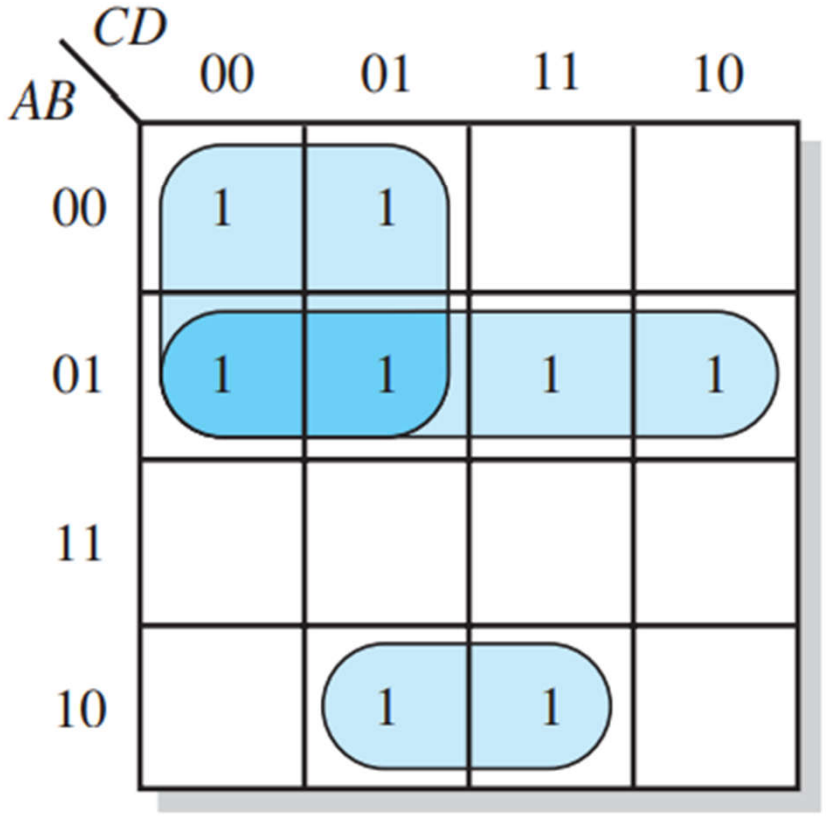


(c)

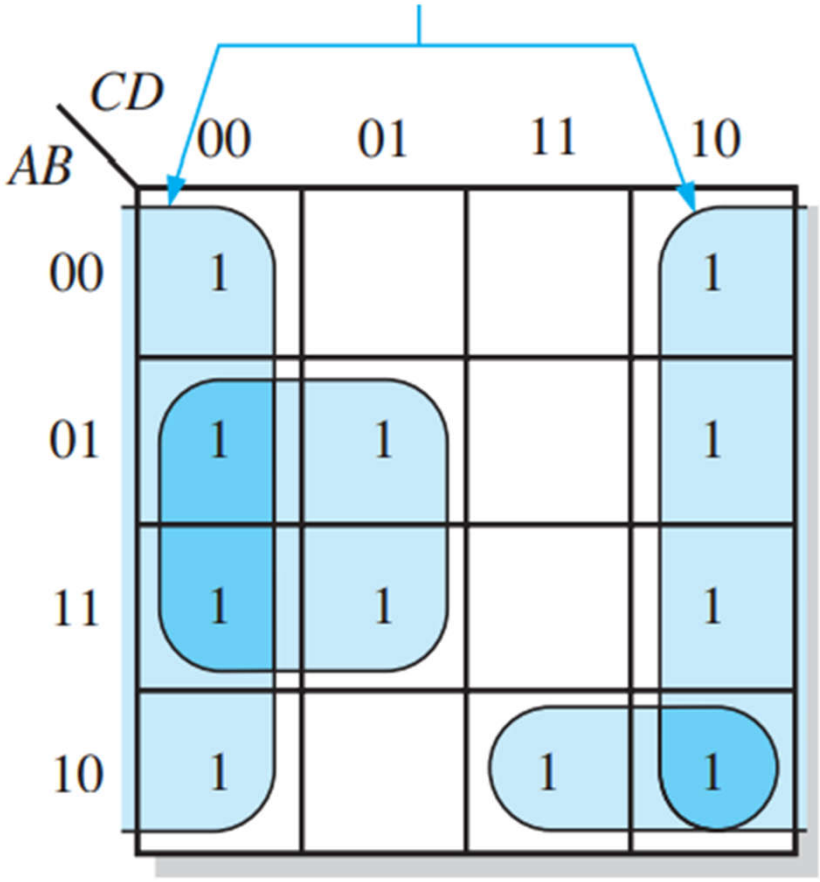


(d)

Group the 1s in each of the Karnaugh maps

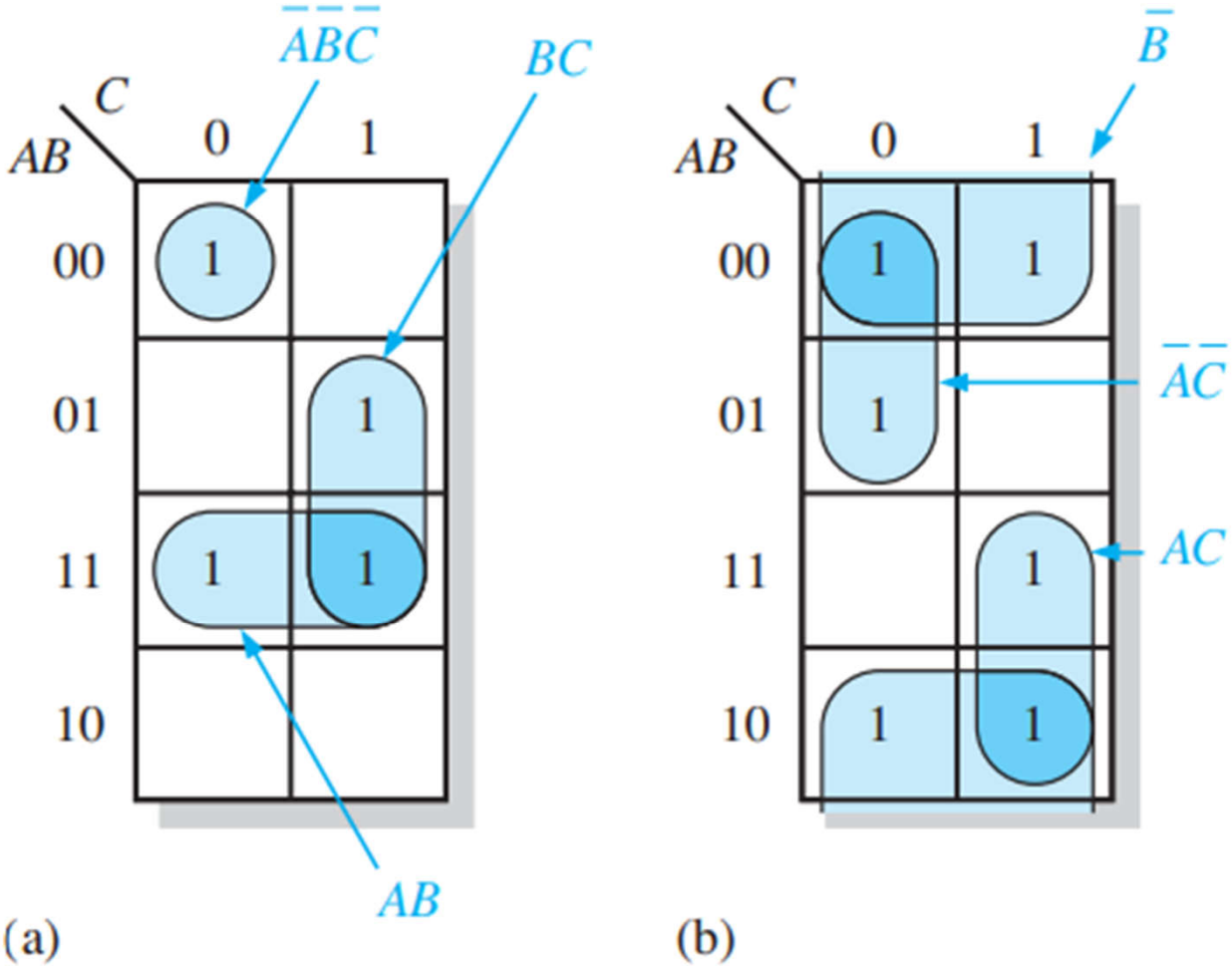


(c)

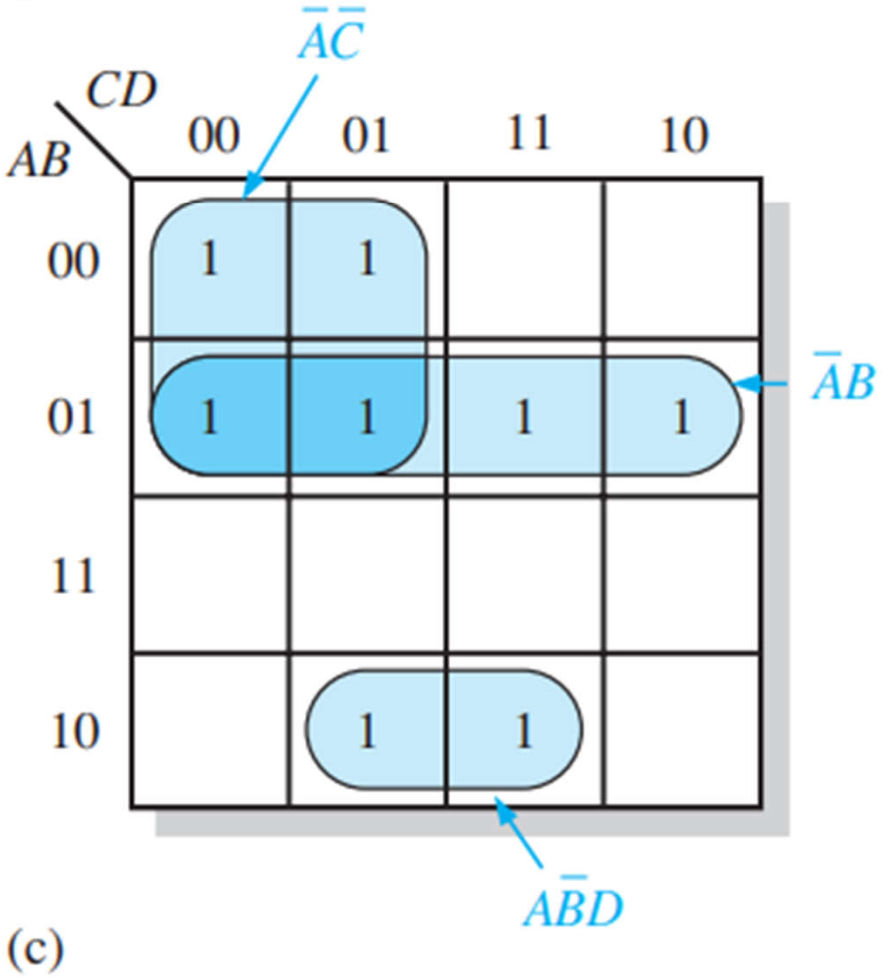


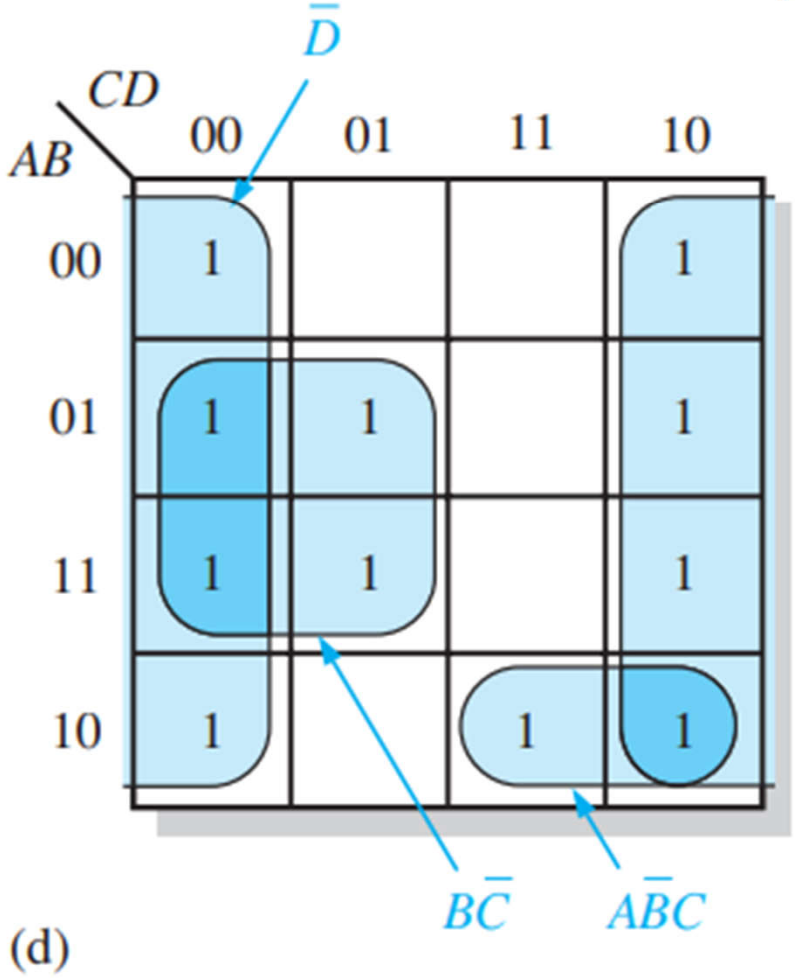
(d)

Group the 1s in each of the Karnaugh maps and write the resulting minimum SOP expression.



Group the 1s in each of the Karnaugh maps and write the resulting minimum SOP expression.





Group the 1s in each of the Karnaugh maps and write the resulting minimum SOP expression.

(a) $AB + BC + \overline{A}\,\overline{B}\,\overline{C}$

(b) $\overline{B} + \overline{A}\,\overline{C} + AC$

(c) $\overline{A}B + \overline{A}\,\overline{C} + A\overline{B}D$

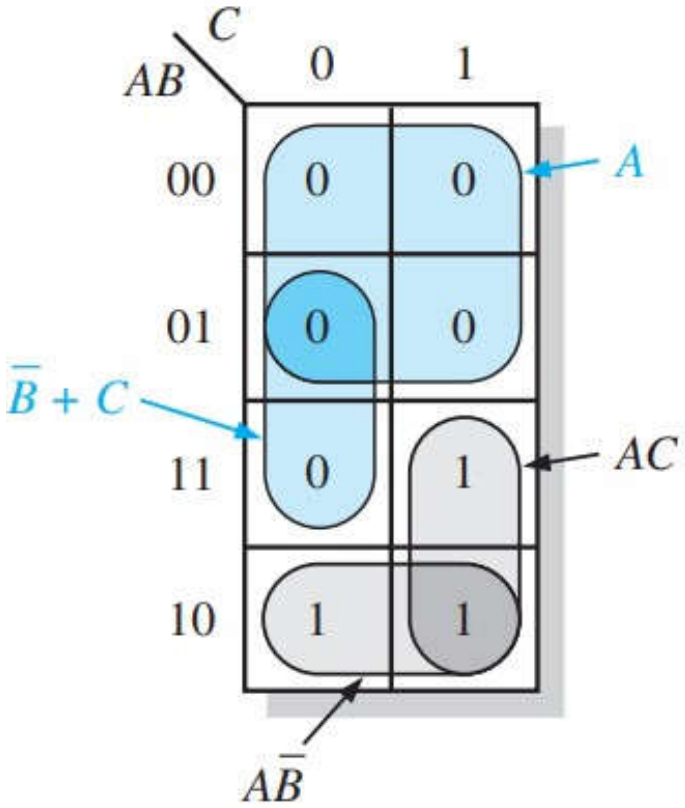
(d) $\overline{D} + A\overline{B}C + B\overline{C}$

Use a Karnaugh map to minimize the following SOP expression:

$$\overline{B}\,\overline{C}\,\overline{D} + \overline{A}B\overline{C}\,\overline{D} + AB\overline{C}\,\overline{D} + \overline{A}\,\overline{B}CD + A\overline{B}CD + \overline{A}\,\overline{B}C\overline{D} + \overline{A}BC\overline{D} + ABC\overline{D} + A\overline{B}C\overline{D}$$

$\overline{D} + \overline{B}C$

$$(A + B + C)(A + B + \overline{C})(A + \overline{B} + C)(A + \overline{B} + \overline{C})(\overline{A} + \overline{B} + C)$$



Minimized SOP form

$$AC + A\overline{B} = A(\overline{B} + C)$$

Minimized POS form

Use a Karnaugh map to simplify the following standard POS expression:

Homework $(X + \overline{Y} + Z)(X + \overline{Y} + \overline{Z})(\overline{X} + \overline{Y} + Z)(\overline{X} + Y + Z)$

EXAMPLE 4–36

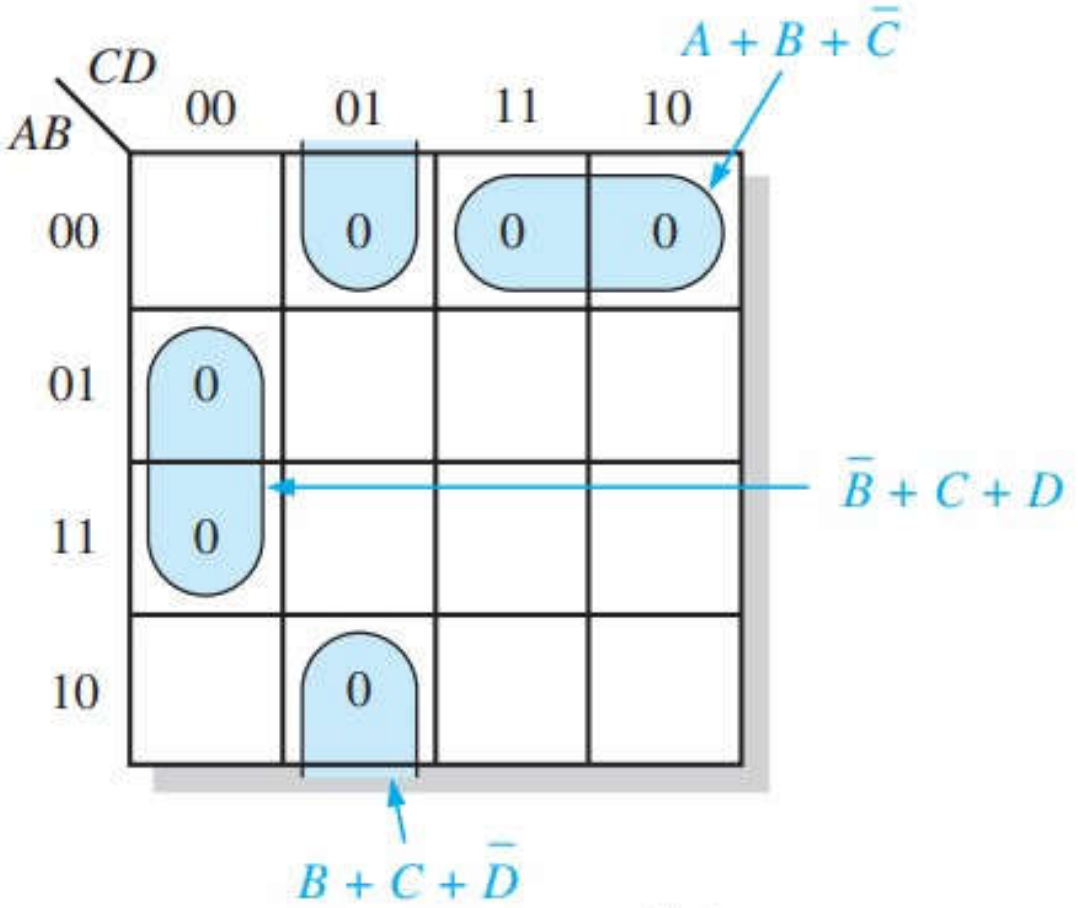
Using a Karnaugh map, convert the following standard POS expression into a minimum POS expression, a standard SOP expression, and a minimum SOP expression.

$$(\overline{A} + \overline{B} + C + D)(A + \overline{B} + C + D)(A + B + C + \overline{D})(A + B + \overline{C} + \overline{D})(\overline{A} + B + C + \overline{D})(A + B + \overline{C} + D)$$

EXAMPLE 4–36

Using a Karnaugh map, convert the following standard POS expression into a minimum POS expression, a standard SOP expression, and a minimum SOP expression.

$$(\overline{A} + \overline{B} + C + D)(A + \overline{B} + C + D)(A + B + C + \overline{D})(A + B + \overline{C} + \overline{D})(\overline{A} + B + C + \overline{D})(A + B + \overline{C} + D)$$

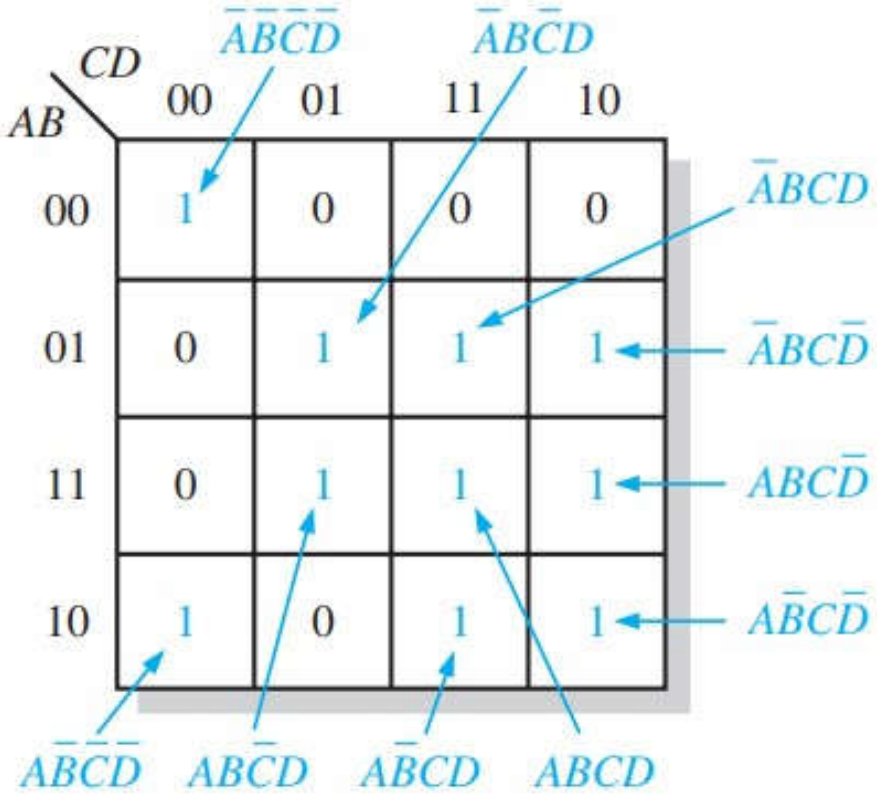


(a) Minimum POS: $(A + B + C)(\overline{B} + \overline{C} + D)(B + C + \overline{D})$

EXAMPLE 4–36

Using a Karnaugh map, convert the following standard POS expression into a minimum POS expression, a standard SOP expression, and a minimum SOP expression.

$$(\overline{A} + \overline{B} + C + D)(A + \overline{B} + C + D)(A + B + C + \overline{D})(A + B + \overline{C} + \overline{D})(\overline{A} + B + C + \overline{D})(A + B + \overline{C} + D)$$



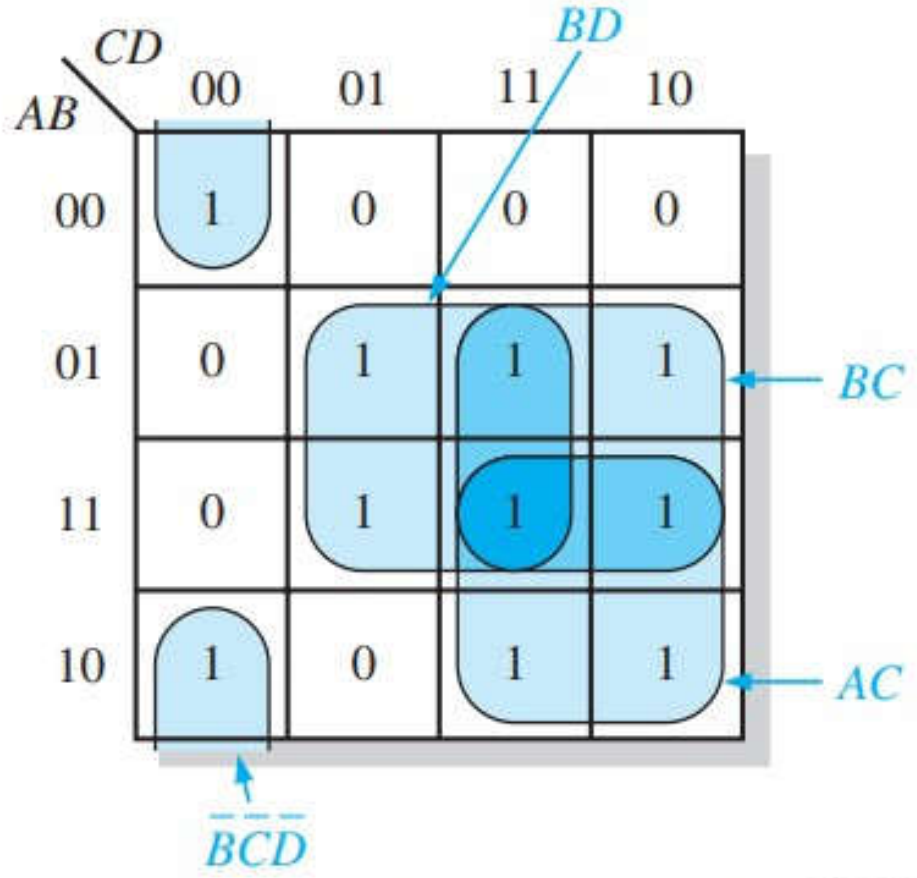
(b) Standard SOP:

$$\overline{A}\overline{B}\overline{C}\overline{D} + \overline{A}B\overline{C}D + \overline{A}BCD + \overline{A}BC\overline{D} + AB\overline{C}\overline{D} + A\overline{B}CD +$$
$$A\overline{B}\overline{C}\overline{D} + AB\overline{C}D + A\overline{B}CD + ABCD$$

EXAMPLE 4–36

Using a Karnaugh map, convert the following standard POS expression into a minimum POS expression, a standard SOP expression, and a minimum SOP expression.

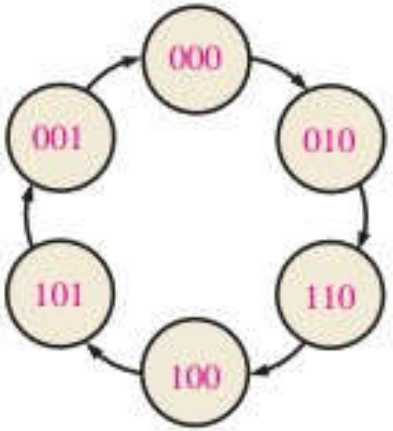
$$(\overline{A} + \overline{B} + C + D)(A + \overline{B} + C + D)(A + B + C + \overline{D})(A + B + \overline{C} + \overline{D})(\overline{A} + B + C + \overline{D})(A + B + \overline{C} + D)$$



(c) Minimum SOP: $AC + BC + BD + \overline{B}\overline{C}\overline{D}$

Highest level: The truth table or state diagram

A	B	C	D	X
0	0	0	0	0
0	0	0	1	0
⋮	⋮	⋮	⋮	⋮
1	1	1	1	1

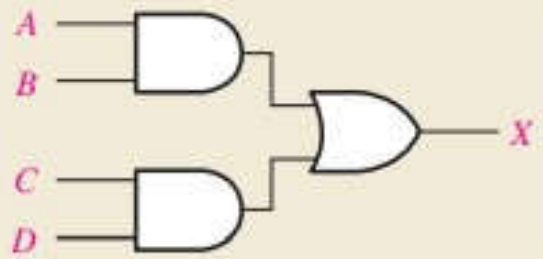


Middle level: The Boolean expression, which can be derived from a truth table or schematic

$X = AB + CD$

Logic function

Lowest level: The logic diagram (schematic)



Selected Key Terms

Variable A symbol used to represent a logical quantity that can have a value of 1 or 0, usually designated by an italic letter.

Complement The inverse or opposite of a number. In Boolean algebra, the inverse function, expressed with a bar over the variable.

Sum term The Boolean sum of two or more literals equivalent to an OR operation.

Product term The Boolean product of two or more literals equivalent to an AND operation.

Selected Key Terms

Sum-of-products (SOP)	A form of Boolean expression that is basically the ORing of ANDed terms.
Product of sums (POS)	A form of Boolean expression that is basically the ANDing of ORed terms.
Karnaugh map	An arrangement of cells representing combinations of literals in a Boolean expression and used for systematic simplification of the expression.
VHDL	A standard hardware description language. IEEE Std. 1076-1993.