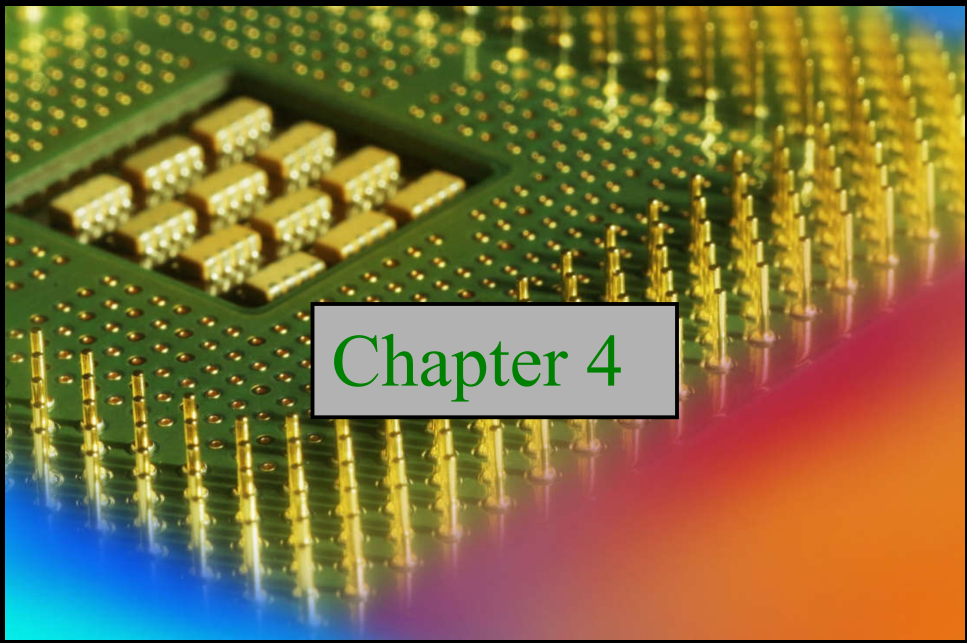
Digital Fundamentals

Tenth Edition

Floyd



Summary

Boolean Algebra 1854, George Boole

In Boolean algebra, a **variable** is a symbol used to represent an action, a condition, or data. A single variable can only have a value of 1 or 0.

The **complement** represents the inverse of a variable and is indicated with an overbar. Thus, the complement of A is $\overline{A}$.

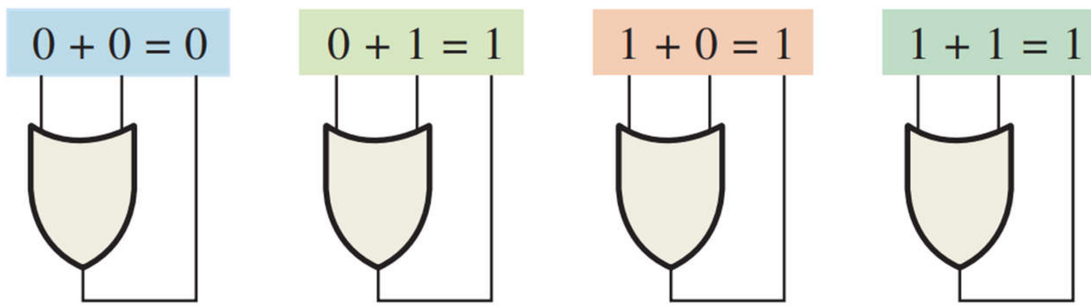
A **literal** is a variable or its complement.

Literal
- Variable
  - Action (0,1)
  - Condition (0,1)
  - Data (0,1)
- Complement → $\overline{variable}$

Summary

Boolean Addition

Addition is equivalent to the **OR operation**. The sum term is 1 if one or more of the literals are 1. The sum term is zero only if each literal is 0.



Example

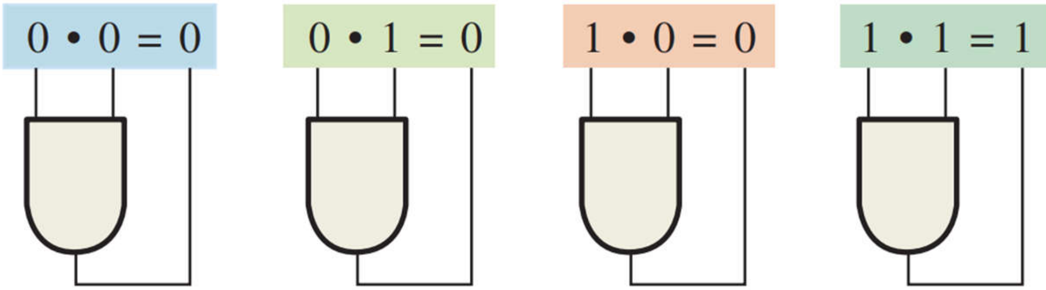
Determine the values of A, B, and C that make the sum term of the expression $\overline{A} + B + \overline{C} = 0$?

Solution

Summary

Boolean Multiplication

In Boolean algebra, **multiplication** is equivalent to the **AND** operation. The product of literals forms a product term. The product term will be 1 only if all of the literals are 1.



Example What are the values of the A, B and C if the product term of $A \cdot \overline{B} \cdot \overline{C} = 1$?

Solution

Summary

Commutative Laws

The **commutative laws** are applied to addition and multiplication. For addition, the commutative law states

In terms of the result, the order in which variables are ORed makes no difference.

$$\boxed{A + B = B + A}$$

For multiplication, the commutative law states

In terms of the result, the order in which variables are ANDed makes no difference.

$$\boxed{AB = BA}$$

Summary

Associative Laws

The **associative laws** are also applied to addition and multiplication. For addition, the associative law states

When ORing more than two variables, the result is the same regardless of the grouping of the variables.

$$A + (B + C) = (A + B) + C$$

For multiplication, the associative law states

When ANDing more than two variables, the result is the same regardless of the grouping of the variables.

$$A(BC) = (AB)C$$

Distributive Law

The **distributive law** is the factoring law. A common variable can be factored from an expression just as in ordinary algebra. That is

$$AB + AC = A(B + C)$$

The distributive law can be illustrated with equivalent circuits:

$$A(B + C) = AB + AC$$

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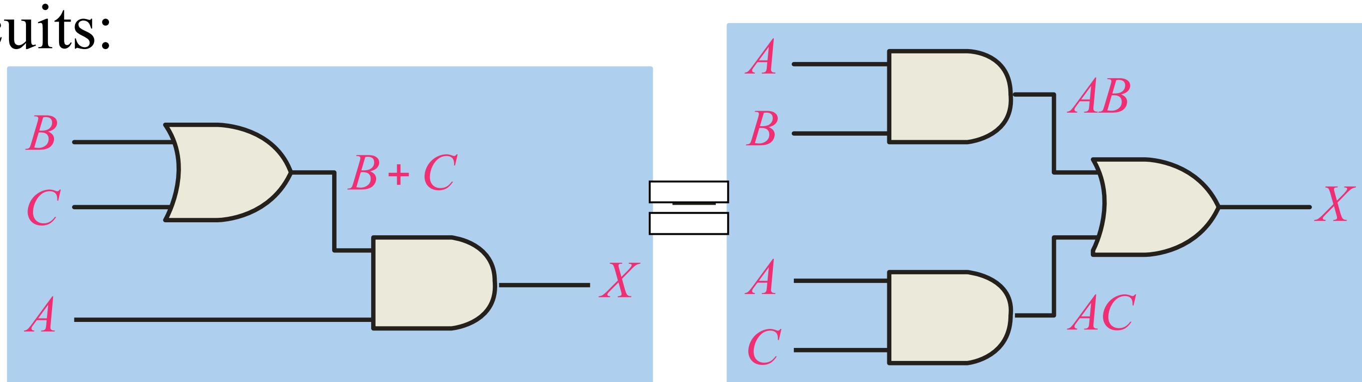


$A(B + C)$ $\quad$ $AB + AC$

Summary

Rules of Boolean Algebra

1. $A + 0 = A$
2. $A + 1 = 1$
3. $A \cdot 0 = 0$
4. $A \cdot 1 = A$
5. $A + A = A$
6. $A + \overline{A} = 1$
7. $A \cdot A = A$
8. $A \cdot \overline{A} = 0$
9. $\overline{\overline{A}} = A$
10. $A + AB = A$
11. $A + \overline{A}B = A + B$
12. $(A + B)(A + C) = A + BC$

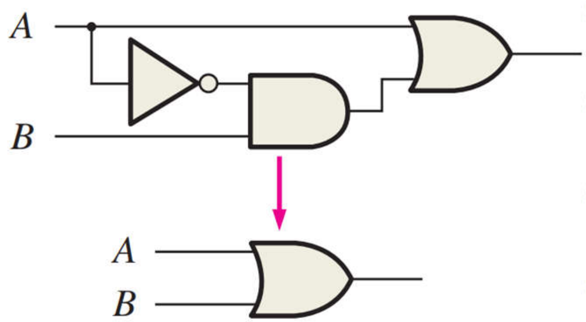
Rule 11: $A + \bar{A}B = A + B$ This rule can be proved as follows:

$A + \bar{A}B = (A + AB) + \bar{A}B$	Rule 10: $A = A + AB$
$= (AA + AB) + \bar{A}B$	Rule 7: $A = AA$
$= AA + AB + A\bar{A} + \bar{A}B$	Rule 8: adding $A\bar{A} = 0$
$= (A + \bar{A})(A + B)$	Factoring
$= 1 \cdot (A + B)$	Rule 6: $A + \bar{A} = 1$
$= A + B$	Rule 4: drop the 1

A	B	$\bar{A}B$	$A+\bar{A}B$	$A+B$
0	0			
0	1			
1	0			
1	1			

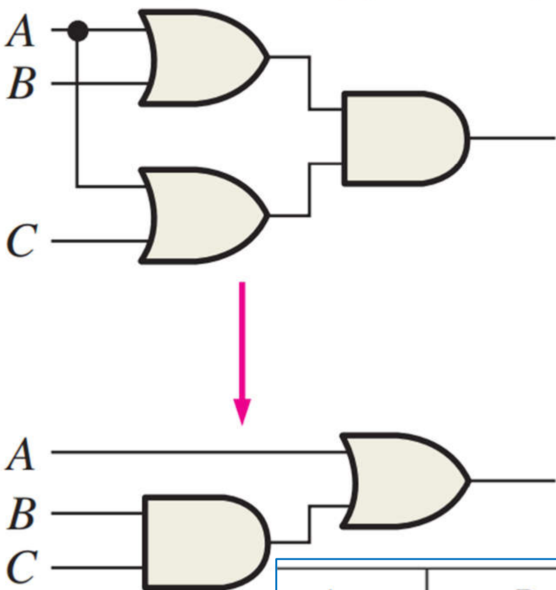
Rule 11: $A + \bar{A}B = A + B$ This rule can be proved as follows:

$$
\begin{aligned}
A + \bar{A}B &= (A + AB) + \bar{A}B && \text{Rule 10: } A = A + AB \\
&= (AA + AB) + \bar{A}B && \text{Rule 7: } A = AA \\
&= AA + AB + A\bar{A} + \bar{A}B && \text{Rule 8: adding } A\bar{A} = 0 \\
&= (A + \bar{A})(A + B) && \text{Factoring} \\
&= 1 \cdot (A + B) && \text{Rule 6: } A + \bar{A} = 1 \\
&= A + B && \text{Rule 4: drop the 1}
\end{aligned}
$$



A	B	$\bar{A}B$	$A + \bar{A}B$	$A + B$
0	0			
0	1			
1	0			
1	1			

Rule 12: $(A + B)(A + C) = A + BC$ This rule can be proved as follows:



$$
\begin{aligned}
(A + B)(A + C) &= AA + AC + AB + BC && \text{Distributive law} \\
&= A + AC + AB + BC && \text{Rule 7: } AA = A \\
&= A(1 + C) + AB + BC && \text{Factoring (distributive law)} \\
&= A \cdot 1 + AB + BC && \text{Rule 2: } 1 + C = 1 \\
&= A(1 + B) + BC && \text{Factoring (distributive law)} \\
&= A \cdot 1 + BC && \text{Rule 2: } 1 + B = 1 \\
&= A + BC && \text{Rule 4: } A \cdot 1 = A
\end{aligned}
$$

A	B	C	$A + B$	$A + C$	$(A + B)(A + C)$	BC	$A + BC$
0	0	0					
0	0	1					
0	1	0					
0	1	1					
1	0	0					
1	0	1					
1	1	0					
1	1	1					