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CLASS: BS(CS) "B"

ROLL NUMBER: 20K-0183

QUESTION 01

Part i)

$$\frac{dy}{dx} = \frac{x}{y}$$

$$y\,dy = x\,dx$$

Integrating b.s.

$$\int y\,dy = \int x.dx$$

$$\frac{y^2}{2} = \frac{x^2}{2} + C$$

$$y^2 = x^2 + 2C \qquad (2C = C_1)$$

$$\boxed{y^2 = x^2 + C_1}$$

Ans

Part ii)

$$x\frac{dy}{dx}+y=x^2y^2$$

$$x\left(\frac{dy}{dx}+\frac{y}{x}\right)=x^2y^2$$

$$\frac{dy}{dx}+\frac{y}{x}=xy^2$$

$$\frac{dy}{dx}+\left(\frac{1}{x}\right)y=xy^2$$

Std. form: $y'+P(x)y=Q(x)y^n$

Here $n=2$

let $V=y^{1-n}=y^{1-2}=y^{-1}$

$$\boxed{V=y^{-1}}$$

diff w.r.t x

$$\frac{dV}{dx}=-1(y^{-2})\frac{dy}{dx}$$

$$\frac{dV}{dx}=\frac{-1}{y^2}\frac{dy}{dx}$$

$$\boxed{\frac{dy}{dx}=\frac{dV}{dx}(-y^2)}$$

$$\frac{dV}{dx}(-y^2)+\frac{y}{x}=xy^2$$

Multiplying B.S. by $\frac{1}{y^2}$

$$\frac{1}{y^2}\left(\frac{dV}{dx}(-y^2)+\frac{y}{x}\right)=\cancel{xy^2}\times\frac{1}{y^2}$$

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$$\frac{-dV}{dx}+\frac{V}{yx}=x$$

$$\because V=\frac{1}{y}$$

$$\frac{-dV}{dx}+\frac{V}{x}=x$$

$$I.F.=e^{\int\frac{1}{x}dx}$$

$$I.F.=e^{\ln x}$$

$$\boxed{I.F.=x}$$

Multiplying B.S by x

$$x\left(-\frac{dV}{dx}+\frac{V}{x}\right)=x^2$$

$$-x\frac{dV}{dx}+V=x^2$$

~~$V=x^2+\frac{dV}{dx}(x$~~ $\quad$ ~~$\frac{-xdV}{dx}=x^2-V$~~

$$-xV'+V=x^2$$

$$\frac{-V'}{x}+\frac{V}{x^2}=1$$

$$\frac{-xV'+V}{x^2}=1$$

$$\frac{V-xV'}{x^2}=1$$

$$-\frac{(xV'-V)}{x^2}=1$$

$$\frac{xV'-V}{x^2}=-1$$

$$\because \frac{V'U-VU'}{V^2}=\frac{d}{dx}\left(\frac{U}{V}\right)$$

$$\frac{d}{dx}\left(\frac{V}{x}\right) = -1$$

Integrating B.S

$$\int \frac{d}{dx}\left(\frac{V}{x}\right) = -\int dx$$

$$\frac{V}{x} = -x + c$$

$$V = -x^2 + xc$$

$$\because V = \frac{1}{y}$$

$$\frac{1}{y} = -x^2 + xc$$

$$\boxed{y = \frac{1}{-x^2 + xc}}$$ Ans

Part iii)

$$(x^2+y^2)dx + xy\,dy = 0$$

$$\cancel{(x^2+y^2)dy}$$

$$(x^2+y^2)\,dx = -xy\,dy$$

$$\frac{-(x^2+y^2)}{xy} = \frac{dy}{dx}$$

$$\because y = Vx$$

$$\frac{dy}{dx} = V + x\frac{dV}{dx}$$

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$$\frac{-(x^2 + V^2x^2)}{x(Vx)} = V + x\frac{dV}{dx}$$

$$\frac{-\cancel{x^2}(1+V^2)}{V\cancel{x^2}} = V + x\frac{dV}{dx}$$

$$\frac{-(1+V^2)}{V} = V + x\frac{dV}{dx}$$

$$\frac{-1-V^2}{V} - V = x\frac{dV}{dx}$$

$$\frac{-1-V^2-V^2}{V} = x\frac{dV}{dx}$$

$$\frac{-1-2V^2}{V} = x\frac{dV}{dx}$$

$$\left(\frac{1}{x}\right)dx = -\left(\frac{V}{1+2V^2}\right)dV$$

$$\frac{1}{x}dx = \frac{-1}{4}\left(\frac{4V}{1+2V^2}\right)dV$$

Integrating B.S

$$\int \frac{1}{x}dx = \frac{-1}{4}\int\left(\frac{4V}{1+2V^2}\right)dV$$

$$\ln x = \frac{-1}{4}\ln(1+2V^2) + c$$

$$4\ln x = -\ln(1+2V^2) + c$$

$$\ln x^4 + \ln(1+2V^2) = c$$

$$\ln\{x^4 \cdot (1+2V^2)\} = c$$

Taking e on B.S

$$e^{\ln\{x^4\cdot(1+2V^2)\}} = e^c$$

$$x^4(1+2V^2) = e^c$$

$\because V = y/x$

$$x^4\left(1+\frac{2y^2}{x^2}\right) = c$$

$$x^{\cancel{4}\,2}\left(\frac{x^2+2y^2}{\cancel{x^4}}\right) = c$$

$$x^4 + 2y^2x^2 = c$$

$$2y^2x^2 = c - x^4$$

$$2y^2 = \frac{c}{x^2} - x^2$$

$$y^2 = \frac{c}{2x^2} - \frac{x^2}{2}$$

$$\because \frac{c}{2} = c_1$$

$$\boxed{y^2 = \frac{c}{x^2} - \frac{x^2}{2}}$$

Part iv)

$$(x^2-y^2)dx + 2xy\,dy = 0$$

$$(x^2-y^2)dx = -2xy\,dy$$

$$\frac{y^2-x^2}{2xy} = \frac{dy}{dx}$$

let $y = Vx$

$$\frac{dy}{dx} = V + x\frac{dV}{dx}$$

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$$\frac{V^2x^2-x^2}{2Vx^2} = V + x\frac{dV}{dx}$$

$$\frac{\cancel{x^2}(V^2-1)}{2V\cancel{x^2}} = V + x\frac{dV}{dx}$$

$$\frac{V^2-1}{2V} = V + x\frac{dV}{dx}$$

$$\frac{V^2-1}{2V} - V = x\frac{dV}{dx}$$

$$\frac{V^2-1-2V^2}{2V} = x\frac{dV}{dx}$$

$$\frac{-V^2-1}{2V} = x\frac{dV}{dx}$$

$$\frac{-(V^2+1)}{2V} = x\frac{dV}{dx}$$

$$\frac{1}{x}dx = \frac{-2V}{V^2+1}dV$$

Integrating b.s

$$\int \frac{1}{x}dx = -\int \frac{2V}{V^2+1}dV$$

$$c + \ln x = -\ln(V^2+1)$$

$$c = -\ln(V^2+1) - \ln x$$

$$c = -\{\ln(V^2+1) + \ln x\}$$

$$c = -\ln\{(V^2+1).x\}$$

$$-c = \ln\{(V^2+1).x\}$$

taking e on b.s

$$e^{-c} = e^{\ln((V^2+1).x)}$$

$$c = (V^2+1).x \quad (\because V = \frac{y}{x})$$

$$c = \left(\frac{y^2}{x^2}+1\right)x$$

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$$C = \left(\frac{y^2+x^2}{x^2}\right) \cdot x$$

$$Cx = y^2 + x^2$$

$$y^2 = Cx - x^2$$

$$\boxed{y^2 = -x^2 + Cx}$$

Part v)

$$e^y\left(\frac{dy}{dx} - 1\right) = e^x$$

$$\cancel{e^y}\frac{dy}{dx} - \cancel{e^y} = \cancel{e^x}$$

$$\cancel{\frac{dy}{dx} - 1 = \frac{e^x}{e^y}}$$

$$\cancel{\frac{dy}{dx} - \frac{e^x}{e^y} = 1}$$

~~Std form~~

$$\cancel{\frac{dy}{dx} - P(x)y = Q(x)}$$

$$\cancel{P(x) = e^x}$$

$$\frac{e^y}{e^x}\left(\frac{dy}{dx} - 1\right) = 1$$

$$\frac{e^y \cdot y'}{e^x} - \frac{e^y}{e^x} = 1$$

$$\frac{e^y \cdot y' - e^y}{e^x} = 1$$

multiplying & dividing by e^x

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$$\frac{e^x \cdot e^y \cdot y' - e^x e^y}{e^{x^2}} = 1$$

$$\because \frac{VU' - U \cdot V'}{V^2} = \frac{d}{dx}\left(\frac{U}{V}\right)$$

$$\frac{d}{dx}\left(\frac{e^y}{e^x}\right) = 1$$

Integrating both sides

$$\int \frac{d}{dx}\left(\frac{e^y}{e^x}\right) = \int 1$$

$$\frac{e^y}{e^x} = \int dx$$

$$\frac{e^y}{e^x} = x + c$$

taking ln B.S

$$\ln e^{y-x} = \ln(x+c)$$

$$y - x = \ln(x+c)$$

$$\boxed{y = \ln(x+c) + x}$$

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vi) $\sin y \frac{dy}{dx} = \cos x(2\cos y - \sin x)$

$\sin y \frac{dy}{dx} = 2\cos y\cos x - \cos x\sin x$

$\sin y\, dy = (2\cos y\cos x - \cos x\sin x)\, dx$

$(2\cos y\cos x - \cos x\sin x)dx - \sin y\, dy = 0 \quad \text{—(1)}$

Exact equation Std. form

$M dx + N dy = 0$

$M = 2\cos y\cos x - \cos x\sin x$

partial diff w.r.t y.

$\frac{\partial M}{\partial y} = \frac{\partial}{\partial y}(2\cos y\cos x - \cos x\sin x)$

$= 2\cos x \frac{\partial}{\partial y}\cos y - \frac{\partial}{\partial y}(\cos x\sin x)$

$\frac{\partial M}{\partial y} = 2\cos x(-\sin y)$.

$\boxed{\frac{\partial M}{\partial y} = -2\cos x\sin y}$

$N = -\sin y$

partial diff w.r.t. x

$\boxed{\frac{\partial N}{\partial x} = 0}$

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$\because \boxed{\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}}$

$U(x) = \frac{M_y - N_x}{N} = \frac{-2\cos x \sin y - 0}{-\sin y}$

$\boxed{U(x) = 2\cos x}$

$I.F. = e^{\int U(x)} = e^{\int 2\cos x} = e^{2\int \cos x}$

$\boxed{I.F. = e^{2\sin x}}$

multiplying b.s of eq(1) by I.F.

$e^{2\sin x}(2\cos x\cos y - \cos x\sin x)dx - e^{2\sin x}\sin y\, dy = 0$

$(2\cos x\cos y e^{2\sin x} - e^{2\sin x}\cos x\sin x)dx - e^{2\sin x}\sin y\, dy = 0$

$M = 2\cos x\cos y e^{2\sin x} - e^{2\sin x}\cos x\sin x$

$\frac{\partial M}{\partial y} = 2\cos x e^{2\sin x}\frac{\partial}{\partial y}\cos y - 0$

$\frac{\partial M}{\partial y} = 2e^{2\sin x}\cos x(-\sin y)$

$\boxed{\frac{\partial M}{\partial y} = -2e^{2\sin x}\cos x\sin y}$

$N = -e^{2\sin x}\sin y$

$\frac{\partial N}{\partial x} = -\sin y \frac{\partial}{\partial x}e^{2\sin x}$

$= -\sin y \cdot e^{2\sin x} \cdot 2\cos x$

$\boxed{\frac{\partial N}{\partial x} = -2e^{2\sin x}\cos x\sin y}$

$$\boxed{\because \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}}$$

$$Mdx = (2\cos x \cos y\, e^{2\sin x} - e^{2\sin x} \cos x \sin x)dx$$

Integrating B.S

$$\int Mdx = \int 2\cos x \cos y\, e^{2\sin x} dx - \int e^{2\sin x} \cos x \sin x\, dx$$

let $\sin x = t$

$$\frac{dt}{dx} = \cos x$$

$$\boxed{dx = \frac{dt}{\cos x}}$$

$$M = \int 2\cancel{\cos x} \cos y\, e^{2t} \frac{dt}{\cancel{\cos x}} - \int e^{2t} \sin x \cos x\, dx$$

$$M = 2\int \cos y\, e^{2t} dt - \int e^{2t}\, t\, \cancel{\cos x} \left(\frac{dt}{\cancel{\cos x}}\right)$$

$$M = 2\cos y \int e^{2t} dt - \int e^{2t}.t\, dt$$

$$I = \int e^{2t}.t\, dt$$

let $u = e^{2t}$, $v = t$

$$\therefore \int u.v = v\int u - \int \int u.v'$$

$$= t\int e^{2t} dt - \int \int e^{2t}.dt.\frac{d}{dt} t$$

$$= t.\frac{e^{2t}}{2} - \frac{1}{2}\int e^{2t}$$

$$= \frac{t.e^{2t}}{2} - \frac{1}{2}\frac{e^{2t}}{2}$$

$$\boxed{I = \frac{t.e^{2t}}{2} - \frac{e^{2t}}{4}}$$

Now,

$$M = \cancel{2}\cos y \frac{e^{2t}}{\cancel{2}} - \left(\frac{t.e^{2t}}{2} - \frac{e^{2t}}{4}\right)$$

$$M = \cos y\, e^{2t} - \frac{t.e^{2t}}{2} + \frac{e^{2t}}{4}$$

$$\because t = \sin x$$

$$\boxed{M = \cos y\, e^{2\sin x} - \frac{\sin x\, e^{2\sin x}}{2} + \frac{e^{2\sin x}}{4}}$$

$$Ndy = -e^{2\sin x} \sin y\, dy$$

Integrating B.S

$$\int Ndy = \int -e^{2\sin x} \sin y\, dy$$

$$N = -e^{2\sin x} \int \sin y\, dy$$

$$N = -e^{2\sin x}(-\cos y)$$

$$\boxed{N = \cos y.e^{2\sin x}}$$

Answer, $M \cup N$

$$\cos y.e^{2\sin x} - \frac{\sin x\, e^{2\sin x}}{2} + \frac{e^{2\sin x}}{4} = C$$

$$\cos y.e^{2\sin x} = C - \frac{e^{2\sin x}}{4} + \frac{\sin x\, e^{2\sin x}}{2}$$

$$\cos y\, e^{2\sin x} = \frac{4C - e^{2\sin x} + 2\sin x\, e^{2\sin x}}{4}$$

let $4C = C_1$

$$\cos y \, e^{2\sin x} = -\left(\frac{e^{2\sin x} - 2e^{2\sin x}\sin x + c}{4}\right)$$

$$-\cos y = \frac{e^{2\sin x} - 2e^{2\sin x}\sin x + c}{4e^{2\sin x}}$$

$$\boxed{y = -\cos^{-1}\left(\frac{e^{2\sin x} - 2e^{2\sin x}\sin x + c}{4e^{2\sin x}}\right)}$$

Ans.

vii) $x(3x + 2y^2)\,dx + 2y(1+x^2)\,dy = 0$

Std. Form:

$Mdx + Ndy = 0$

$M = 3x^2 + 2y^2 \cdot x$

part. diff w.r.t y

$$\frac{\partial M}{\partial y} = \frac{\partial}{\partial y}(3x^2) + 2x\frac{\partial}{\partial y}(y^2)$$

$$\boxed{\frac{\partial M}{\partial y} = 4xy}$$

$N = 2y + 2x^2 y$

$$\frac{\partial N}{\partial x} = \frac{\partial}{\partial x}(2y) + 2y\frac{\partial}{\partial x}(x^2)$$

$$\frac{\partial N}{\partial x} = 0 + 2y(2x)$$

$$\boxed{\frac{\partial N}{\partial x} = 4xy}$$

$$\boxed{\therefore \frac{\partial N}{\partial x} = \frac{\partial M}{\partial y}}$$

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$Mdx = (3x^2 + 2y^2 \cdot x)\,dx$

Integrating b.s

$\int M dx = 3\int x^2 dx + 2y^2 \int x\,dx$

$$M = \frac{\cancel{3}x^3}{\cancel{3}} + \frac{\cancel{2}y^2 \cdot x^2}{\cancel{2}}$$

$$\boxed{M = x^3 + y^2 \cdot x^2}$$

$Ndy = (2y + 2x^2 y)\,dy$

Integrating b.s.

$\int N dy = 2\int y\,dy + 2x^2 \int y\,dy$

$$N = \frac{\cancel{2}y^2}{\cancel{2}} + \frac{\cancel{2}x^2 \cdot y^2}{\cancel{2}}$$

$$\boxed{N = y^2 + x^2 y^2}$$

Answer = MUN

$x^3 + y^2 + x^2 y^2 = c$

$y^2(1+x^2) = c - x^3$

$$\boxed{y^2 = \frac{c - x^3}{1+x^2}}$$

viii) $e^{-y}\sec^2 y\,dy = dx + x\,dy$

$e^{-y}\sec^2 y\,dy - x\,dy = dx$

$dy(e^{-y}\sec^2 y - x) = dx$

$dx - (e^{-y}\sec^2 y - x)dy = 0 \quad -(1)$

$\because M\,dx + N\,dy = 0$

$M = 1$

part. diff w.r.t. y

$\frac{\partial M}{\partial y} = \frac{\partial}{\partial y}(1)$

$\boxed{\frac{\partial M}{\partial y} = 0}$

$N = (x - e^{-y}\sec^2 y)$

part diff w.r.t. x

$\frac{\partial N}{\partial x} = \frac{\partial}{\partial x}(x - e^{-y}\sec^2 y)$

$\frac{\partial N}{\partial x} = 1 - 0$

$\boxed{\frac{\partial N}{\partial x} = 1}$

$\boxed{\frac{\partial N}{\partial x} \neq \frac{\partial M}{\partial y}}$

$U(x) = \frac{0-1}{x - e^{-y}\sec^2 y}$

$\boxed{U(x) = \frac{1}{x - e^{-y}\sec^2 y}}$

$U(y) = \frac{1-0}{1}$

$\boxed{U(y) = 1}$

$I.F. = e^{\int U(y)} = e^{\int 1 dy} = e^y$

$\boxed{I.F. = e^y}$

Multiplying b.s by e^y

$e^y dx - e^y(e^{-y}\sec^2 y - x)dy = 0$

$e^y dx - (\sec^2 y - e^y x)dy = 0$

$M = e^y$

$\boxed{\frac{\partial M}{\partial y} = e^y}$

$N = e^y x - \sec^2 y$

$\boxed{\frac{\partial N}{\partial x} = e^y}$

$\boxed{\frac{\partial N}{\partial x} = \frac{\partial M}{\partial y}}$

$M\,dx = \int e^y\,dx$

Integrating b.s.

$\int M\,dx = e^y \int dx$

$\boxed{M = e^y x}$

$N dy = e^y \cdot x - \sec^2 y \, dy$

Integrating b.s

$\int N dy = x \int e^y \cdot dy - \int \sec^2 y \, dy$

$$\boxed{N = xe^y - \tan y}$$

Answer MUN

$xe^y - \tan y = c$

$-\tan y = c - xe^y$

$$\boxed{y = \tan(xe^y - c)}$$

ix) $(x^2+y^2)dx + (x^2 - xy)dy = 0$

$(x^2+y^2)dx = -(x^2 - xy)dy$

$(xy - x^2)dy = (x^2+y^2)dx$

$\frac{dy}{dx} = \frac{x^2+y^2}{xy - x^2}$

let $y = Vx$

$\frac{dy}{dx} = V + x\frac{dV}{dx}$

$V + x\frac{dV}{dx} = \frac{x^2 + V^2x^2}{x(Vx) - x^2}$

$V + x\frac{dV}{dx} = \frac{x^2(1+V^2)}{Vx^2 - x^2}$

$V + x\frac{dV}{dx} = \frac{\cancel{x^2}(1+V^2)}{\cancel{x^2}(V-1)}$

$V + x\frac{dV}{dx} = \frac{1+V^2}{V-1}$

$x\frac{dV}{dx} = \frac{1+V^2}{V-1} - V$

$x\frac{dV}{dx} = \frac{1+V^2 - V(V-1)}{V-1}$

$x\frac{dV}{dx} = \frac{1+\cancel{V^2} - \cancel{V^2} + V}{V-1}$

$x\frac{dV}{dx} = \frac{1+V}{V-1}$

$\left(\frac{V-1}{1+V}\right)dV = \frac{1}{x}dx$

$\left(\frac{V}{1+V}\right)dV - \frac{dV}{1+V} = \frac{1}{x}dx$

$\left(1 - \frac{1}{1+V}\right)dV - \frac{dV}{1+V} = \frac{1}{x}dx$

$dV - \frac{dV}{1+V} - \frac{dV}{1+V} = \frac{1}{x}dx$

$$\begin{array}{r|l} & 1 \\ 1+V & V \\ & -1+V \\ & -1 \end{array}$$

Integrating b.s

$V - \ln(1+V) - \ln(1+V) = \ln x$

$V - 2\ln(1+V) = \ln x + c$

$V - \ln(1+V)^2 = \ln x + c$

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$$V - \ln(1+V)^2 = \ln x + c$$

$$V = \ln x + \ln(1+V)^2 + c$$

$$\because \ln a + \ln b = \ln(ab)$$

$$V = \ln(x \cdot (1+V)^2) + c$$

$$\because V = \frac{y}{x}$$

$$\frac{y}{x} = \ln\left(x\left(1+\frac{y}{x}\right)^2\right) + c$$

$$\frac{y}{x} = \ln\left(x\left(\frac{x+y}{x}\right)^2\right) + c$$

$$\frac{y}{x} = \ln\left(\cancel{x}\frac{(x+y)^2}{x^{\cancel{2}}}\right) + c$$

$$\frac{y}{x} = \ln\left(\frac{(x+y)^2}{x}\right) + c$$

$$\frac{y}{x} = \ln\left(\frac{x^2+2xy+y^2}{x}\right) + c$$

$$\frac{y}{x} = \ln\left(x + 2y + \frac{y^2}{x}\right) + c$$

$$-\ln\left(x + 2y + \frac{y^2}{x}\right) + \frac{y}{x} = c$$

Subtracting 1 from b.s

$$-\ln\left(x+2y+\frac{y^2}{x}\right) + \frac{y}{x} - 1 = c - 1$$

let $c - 1 = c_1$

$$\boxed{-\ln\left(x+2y+\frac{y^2}{x}\right) + \frac{y}{x} - 1 = c_1}$$

Ans

x)

$$y - x\frac{dy}{dx} = a\left(y^2 + \frac{dy}{dx}\right)$$

$$y - x\frac{dy}{dx} = ay^2 + a\frac{dy}{dx}$$

$$y - ay^2 = a\frac{dy}{dx} + x\frac{dy}{dx}$$

$$y - ay^2 = \frac{dy}{dx}(a+x)$$

$$dx\,(y - ay^2) = dy\,(a+x)$$

$$\frac{dx}{a+x} = \frac{dy}{y(1-ay)} \quad \text{--(1)}$$

$$\frac{1}{y(1-ay)} = \frac{A}{y} + \frac{B}{1-ay}$$

$$\frac{1}{\cancel{y(1-ay)}} = \frac{A(1-ay) + By}{\cancel{y(1-ay)}}$$

$$1 = A - Aay + By$$

$$1 = A + y(B - Aa)$$

$$\boxed{A = 1}$$

$$B - Aa = 0 \quad (\because A = 1)$$

$$B - a = 0$$

$$\boxed{B = a}$$

$$\boxed{\frac{1}{y(1-ay)} = \frac{1}{y} + \frac{a}{1-ay}}$$

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$$\frac{dx}{x+a} = \left(\frac{1}{y} + \frac{a}{1-ay}\right) dy$$

$$\frac{dx}{x+a} = \frac{dy}{y} + \frac{a\,dy}{1-ay}$$

Taking Integration b.s

$$\cancel{\int \frac{dx}{x+a} = \int \frac{dy}{y} + a(-)\int \frac{-dy}{1-ay}}$$

$$\cancel{c + \ln(x+a) = \ln y - a\ln(1-ay)}$$

$$\int \frac{dx}{(x+a)} = \int \frac{dy}{y} - \int \frac{-a\,dy}{1-ay}$$

$$\ln(x+a) + c = \ln y - \ln(1-ay)$$

$$\ln(x+a) + c = \ln\left(\frac{y}{1-ay}\right)$$

$$c = \ln\left(\frac{y}{1-ay}\right) - \ln(x+a)$$

$$c = \ln\left(\frac{y}{(1-ay)(x+a)}\right)$$

Taking e on b.s

$$e^c = \frac{y}{(1-ay)(x+a)}$$

$$C = \frac{y}{(1-ay)(x+a)}$$

$$\frac{1}{C} = \frac{(1-ay)(x+a)}{y}$$

$$C = \frac{(1-ay)(x+a)}{y}$$

$$\frac{C}{x+a} = \frac{1-ay}{y}$$

$$\frac{C}{x+a} = \frac{1}{y} - a$$

$$\frac{C}{x+a} + a = \frac{1}{y}$$

$$\frac{C + a(x+a)}{x+a} = \frac{1}{y}$$

$$\frac{c + ax + a^2}{x+a} = \frac{1}{y}$$

$$\boxed{y = \frac{x+a}{a(x+a)+c}}$$

xi) $$(x+1)\frac{dy}{dx} + 1 = 2e^{-y}$$

$$(x+1)\frac{dy}{dx} = 2e^{-y} - 1$$

$$(x+1)\frac{dy}{dx} = \frac{2}{e^y} - 1$$

$$(x+1)\frac{dy}{dx} = \frac{2-e^y}{e^y}$$

$$\left(\frac{e^y}{2-e^y}\right) dy = \frac{dx}{(x+1)}$$

Integrating b.s.

$$-\int \frac{-e^y}{2-e^y}\,dy = \int \frac{dx}{x+1}$$

$$-\ln(2-e^y) = \ln(x+1) + c$$

$$c = \ln(x+1) + \ln(2-e^y)$$

$$\boxed{c = \ln(x+1)(2-e^y)}$$

xii) $x^2\frac{dy}{dx} + yx + y^2 = 0$

$x^2\left(\frac{dy}{dx} + \frac{y}{x}\right) = -y^2$

$\frac{dy}{dx} + \frac{y}{x} = \frac{-y^2}{x^2}$ ---(1)

Std. Bernaulli Eq^n

$y' + P(x)y = Q(x)y^n$

Here $n = 2$

Let $V = y^{1-n} = y^{1-2} = y^{-1}$

$\boxed{V = \frac{1}{y}}$

$\frac{dV}{dx} = \frac{d}{dx}(y^{-1})$

$\frac{dV}{dx} = -1y^{-2}\frac{dy}{dx}$

$\frac{dV}{dx} = \frac{-1}{y^2}\frac{dy}{dx}$

$\boxed{\frac{dy}{dx} = -y^2\frac{dV}{dx}}$

(1) $\Rightarrow$ $\frac{dy}{dx} + \frac{y}{x} = \frac{-y^2}{x^2}$

$-y^2\frac{dV}{dx} + \frac{y}{x} = \frac{-y^2}{x^2}$

Multiplying B.S by $\frac{1}{y^2}$

$\frac{1}{y^2}\left(-y^2\frac{dV}{dx} + \frac{y}{x}\right) = \left(\frac{-y^2}{x^2}\right)\frac{1}{y^2}$

$-\left(\frac{dV}{dx} - \frac{1}{yx}\right) = -\frac{1}{x^2}$ ---(1)

$\because \frac{1}{y} = V$

$\frac{dV}{dx} - \frac{V}{x} = \frac{1}{x^2}$

I.F. $= e^{\int \frac{1}{x}dx} = e^{\ln x} = x$

$\boxed{I.F = x}$

Multiplying B.S by x

$x\frac{dV}{dx} - V = \frac{1}{x}$

$xV' - V = \frac{1}{x}$

Multiplying B.S by $x^{\prime}$

$\frac{xV' - V}{x^2} = \frac{1}{x^3}$

$\because \frac{V.U' - UV'}{V^2} = \frac{d}{dx}\left(\frac{U}{V}\right)$

$\frac{d}{dx}\left(\frac{V}{x}\right) = \frac{1}{x^3}$

Integrating B.S

$\int\frac{d}{dx}\left(\frac{V}{x}\right) = \int\frac{1}{x^3}$

$\frac{V}{x} = \int\frac{dx}{x^3}$

$\frac{V}{x} = \frac{-x^{-2}}{2} + c$

$\because V = \frac{1}{y}$

Date

$$\frac{1}{xy} = \frac{-x^{-2}}{2} + c$$

$$\frac{1}{xy} = \frac{-1}{2x^2} + c$$

$$\frac{1}{xy} = \frac{-1+2x^2c}{2x^2}$$

$$xy = \frac{2x^2}{2x^2c-1}$$

$$\boxed{y = \frac{2x}{2x^2c-1}}$$

xiii) $(\sec x \tan x \tan y - e^x)\,dx + \sec x \sec^2 y\, dy = 0$

Exact Std. form

$M\,dx + N\,dy = 0$

Here,

$M = \sec x \tan x \tan y - e^x$

$N = \sec x \sec^2 y$

$$\frac{\partial M}{\partial y} = \frac{\partial}{\partial y}(\sec x \tan x \tan y - e^x)$$

$$= \sec x \tan x \frac{\partial}{\partial y}(\tan y) - 0$$

$$\boxed{\frac{\partial M}{\partial y} = \sec x \tan x \sec^2 y}$$

Date

$N = \sec x \sec^2 y$

$$\frac{\partial N}{\partial x} = \sec^2 y \frac{\partial}{\partial x} \sec x$$

$$\boxed{\frac{\partial N}{\partial x} = \sec^2 y \sec x \tan x}$$

$$\boxed{\because \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}}$$

$M\,dx = (\sec x \tan x \tan y - e^x)\,dx$

Integrating b.s

$\int M\,dx = \tan y \int \sec x \tan x\, dx - \int e^x dx$

$$\boxed{M = \tan y \sec x - e^x}$$

$N\,dy = (\sec x \sec^2 y)\,dy$

Integrating b.s

$\int N\,dy = \sec x \int \sec^2 y\, dy$

$$\boxed{N = \sec x \tan y}$$

Answer: $M \cup N = c$

$\tan y \sec x - e^x = c$

$$\tan y = \frac{c+e^x}{\sec x}$$

$$\boxed{y = \tan^{-1}\left(\frac{c+e^x}{\sec x}\right)}$$

Ans.

xiv) $x\cos x\dfrac{dy}{dx}+y(x\sin x+\cos x)=1$

dividing b.s by $x\cos x$

$$\frac{\cancel{x\cos x}}{\cancel{x\cos x}}\frac{dy}{dx}+y\left(\frac{\cancel{x}\sin x}{\cancel{x}\cos x}+\frac{\cancel{\cos x}}{x\cancel{\cos x}}\right)=\frac{1}{x\cos x}$$

$$\frac{dy}{dx}+y\left(\tan x+\frac{1}{x}\right)=\frac{1}{x\cos x}\quad-(1)$$

Std. linear form

$$y'+P(x)y=Q(x)$$

$$I.F.=e^{\int P(x)dx}$$

$$=e^{\int(\tan x+\frac{1}{x})dx}$$

$$=e^{\int\tan x\,dx+\int x^{-1}dx}$$

$$=e^{-\ln\cos x+\ln x}$$

$$=e^{\ln\left(\frac{x}{\cos x}\right)}$$

$$\boxed{I.F.=\frac{x}{\cos x}}$$

Multiplying b.s by I.F.

$$\frac{x}{\cos x}\left(\frac{dy}{dx}+y\left(\tan x+\frac{1}{x}\right)\right)=\frac{1}{\cancel{x}\cos x}\times\frac{\cancel{x}}{\cos x}$$

$$x\sec x\frac{dy}{dx}+y(\tan x\sec x.x+\sec x)=\frac{1}{\cos^2 x}$$

$$\frac{d}{dx}(x\sec x.y)=\sec^2 x$$

Integrating b.s

$$\int\frac{d}{dx}(x\sec x.y)=\int\sec^2 x$$

$$x\sec x.y=\int\sec^2 x\,dx$$

$$x\sec x\,y=\tan x+c$$

$$\boxed{y=\frac{\tan x+c}{x\sec x}}$$

xv) $x\ln x\dfrac{dy}{dx}+y=2\ln x$

dividing b.s by $x\ln x$

$$\frac{\cancel{x\ln x}}{\cancel{x\ln x}}\frac{dy}{dx}+\frac{y}{x\ln x}=\frac{2\cancel{\ln x}}{x\cancel{\ln x}}$$

$$\frac{dy}{dx}+\frac{y}{(x\ln x)}=\frac{2}{x}\quad-(1)$$

Std. linear form.

$$\frac{dy}{dx}+P(x)y=Q(x)$$

$$I.F.=e^{\int\frac{1}{x\ln x}dx}=e^{\ln(\ln x)}$$

$$\boxed{I.F.=\ln x}$$

multiplying b.s of eq (1) by $\ln x$

$$\ln x\frac{dy}{dx}+\frac{y}{x}=\frac{2\ln x}{x}$$

$$\frac{d}{dx}(\ln x.y)=\frac{2\ln x}{x}$$

Integrating B.S

$$\int \frac{d}{dx}(\ln x . y) = 2\int \frac{\ln x}{x}$$

$$\ln x . y = 2\int \frac{\ln x}{x} . dx$$

let $\ln x = t$

$$\frac{dt}{dx} = \frac{1}{x}$$

$$\boxed{dt . x = dx}$$

$$\ln x . y = 2\int \frac{t}{x} . dt . x$$

$$\ln x . y = \frac{\cancel{2} t^2}{\cancel{2}} + c$$

$\because t = \ln x$

$$\ln x . y = (\ln x)^2 + c$$

$$\boxed{y = \frac{(\ln x)^2 + c}{\ln x}}$$

xvi) $\frac{dy}{dx} + \frac{4}{x}y = x^3 y^2$ —(1)

Std Bernoulli eqn

$$y' + P(x)y = Q(x)y^n$$

Here $n = 2$

let

$$V = y^{1-n} = y^{1-2} = y^{-1}$$

$$\boxed{V = \frac{1}{y}}$$

$$\frac{dv}{dx} = \frac{-1}{y^2} . \frac{dy}{dx}$$

$$\boxed{\frac{dy}{dx} = -\frac{dv}{dx} y^2}$$

$$\frac{dy}{dx} + \frac{4}{x}y = x^3 y^2$$

$$-y^2 \frac{dV}{dx} + \frac{4}{x}y = x^3 y^2$$

multiplying B.S by $-\frac{1}{y^2}$

$$-\frac{1}{y^2}\left(-y^2\frac{dV}{dx} + \frac{4}{x}y\right) = \cancel{x^3 y^2} \times \frac{-1}{y^2}$$

$$\frac{dv}{dx} - \frac{4}{xy} = -x^3$$

$$I.F = e^{-4\int \frac{1}{x}dx} = e^{-4\ln x} = e^{-\ln x^4} = \frac{1}{x^4}$$

$$\boxed{I.F = \frac{1}{x^4}}$$

Multiplying B.S by $\frac{1}{x^4}$

$$\cancel{x^4\frac{dV}{dx} - \frac{4x^3}{y}}$$

$$\frac{1}{x^4}\frac{dV}{dx} - \frac{4}{x^5 y} = \frac{-1}{x}$$

$$\because \frac{1}{y} = V$$

$$\frac{1}{x^4}\frac{dV}{dx} - \frac{4V}{x^5} = \frac{-1}{x}$$

$$x^{-4}V' - 4x^{-5}V = \frac{-1}{x}$$

$$\frac{d}{dx}\left(\frac{V}{x^4}\right) = \frac{-1}{x}$$

Integrating B.S

$$\int \frac{d}{dx}\left(\frac{V}{x^4}\right) = \int \frac{-1}{x}$$

$$\frac{V}{x^4} = -\int \frac{dx}{x}$$

$$\frac{V}{x^4} = -\ln x + c$$

$$\because V = \frac{1}{y}$$

$$\frac{1}{x^4 y} = -\ln x + c$$

$$y x^4 = \frac{1}{c - \ln x}$$

$$\boxed{y = \frac{1}{x^4(c - \ln x)}}$$

Ans

Q2(i)

$$\frac{dP}{dt} \propto P$$

$$\boxed{\frac{dP}{dt} = kP} \quad -(1)$$

$$\frac{dP}{P} = k\,dt$$

Integrating B.S

$$\ln P = kt + c$$

taking e on B.S

$$P = e^{kt+c}$$

$$P = e^{kt}\cdot c \qquad (\text{let } c = P_0)$$

$$\boxed{P = e^{kt}\cdot P_0} \quad -(A)$$

$$\because P(5) = 2P_0$$

$$2\cancel{P_0} = e^{k(5)}\cdot\cancel{P_0}$$

taking ln B.S

$$\frac{\ln(2)}{5} = k$$

$$\boxed{k = 0.13863}$$

$$\boxed{P = e^{0.13863t}\cdot P_0}$$

i) $t = ?$ when $P = 3P_0$

$$P = e^{0.13863t} \cdot P_0$$

$$3\cancel{P_0} = e^{0.13863t} \cdot \cancel{P_0}$$

taking ln b.s

$$\frac{\ln(3)}{0.13863} = t \quad -(2)$$

$$\boxed{t = 7.92 \text{ yrs}}$$

ii) $t = ?$ when $P = 4P_0$

$$(2) \Rightarrow \frac{\ln(4)}{0.13863} = t$$

$$\boxed{t = 9.999 \text{ yrs} \approx 10 \text{ yrs}}$$

Q2 (ii)

$$\frac{dA}{dt} \propto A$$

$$\frac{dA}{dt} = KA$$

$$\frac{dA}{A} = Kt$$

Integrating b.s

$$\ln A = Kt + c$$

taking e b.s

(iii) $A = e^{Kt+c}$

$$A = e^{Kt} \cdot c \quad (\text{let } c = A_0)$$

$$\boxed{A = e^{Kt} \cdot A_0} \quad -(1)$$

$\frac{1}{2}$ life = 3.3 hrs then $A = \frac{1}{2}A_0$

$$(1) \Rightarrow \quad \frac{1}{2}\cancel{A_0} = e^{3.3K} \cdot \cancel{A_0}$$

taking ln b.s

$$\frac{\ln(\frac{1}{2})}{3.3} = K$$

$$\boxed{K = -0.21004}$$

Now $t = ?$ for $A = 0.1A_0$

$$(1) \Rightarrow \quad A = e^{-0.21004t} \cdot A_0$$

$$0.1\cancel{A_0} = e^{-0.21004t} \cdot \cancel{A_0}$$

taking ln b.s

$$\frac{\ln(0.1)}{-0.21004} = t$$

$$\boxed{t = 10.96 \text{ hrs} \approx 11 \text{ hrs}}$$

QUESTION 02 (iii)

By Newton's law of ~~B~~ cooling

$$\frac{dT}{dt} = K(T - T_m)$$

$$\frac{dT}{T - T_m} = K\,dt$$

Integrating b.s

$$\ln(T - T_m) = Kt + c$$

taking e on b.s

$$T - T_m = e^{Kt + c}$$

$$T - T_m = e^{Kt} \cdot e^{c}$$

let $e^{c} = C_1$

$$T - T_m = e^{Kt} \cdot C_1 \quad -(1)$$

Now,

$$C_1 = 70 - T_m$$

$$\boxed{T - T_m = e^{Kt}(70 - T_m)} \quad -(A)$$

Now $T(1/2) = 110$

$$110 - T_m = e^{\frac{1}{2}K}(70 - T_m) \quad - i$$

Now, $T(1) = 145$

$$145 - T_m = e^{K}(70 - T_m) \quad -(ii)$$

dividing eq (ii) by (i)

$$\frac{145 - T_m}{110 - T_m} = \frac{e^{K}\cancel{(70 - T_m)}}{e^{1/2K}\cancel{(70 - T_m)}}$$

$$\frac{145 - T_m}{110 - T_m} = e^{1/2K}$$

$$\because e^{1/2K} = \frac{110 - T_m}{70 - T_m}$$

$$\frac{145 - T_m}{110 - T_m} = \frac{110 - T_m}{70 - T_m}$$

$$(145 - T_m)(70 - T_m) = (110 - T_m)^2$$

$$10150 - 145T_m - 70T_m + \cancel{T_m^2} = 12100 - 220T_m + \cancel{T_m^2}$$

$$10150 - 215T_m = 12100 - 220T_m$$

$$220T_m - 215T_m = 12100 - 10150$$

$$5T_m = 1950$$

$$\boxed{T_m = 390\,°F}$$

Date

Q2(iv)

L-R Circuit equation is given by

$$L\frac{di}{dt} + Ri = E$$

Here $L = 0.1$; $R = 50$; $E = 30$

By putting values

$$0.1\frac{di}{dt} + 50i = 30$$

$$0.1\left(\frac{di}{dt} + 500i\right) = 30$$

$$\frac{di}{dt} + 500i = 300 \quad -(1)$$

$$\cancel{\frac{di}{dt} = 300 - 500i}$$

Std linear eqn

$$\frac{dy}{dx} + P(x)y = Q(x)$$

Here $P(x) = 500$

$$I.F. = \cancel{500}\ e^{\int 500\,dt}$$

$$\boxed{I.F. = e^{500t}}$$

Multiplying bs of eq(1) by I.F

$$e^{500t}\frac{di}{dt} + 500e^{500t}\cdot i = 300e^{500t}$$

$$\frac{d}{dt}\left(e^{500t}\cdot i\right) = e^{500t}\cdot 300$$

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Integrating b.s.

$$\int \frac{d}{dt}\left(e^{500}\cdot i\right) = 300\int e^{500t}$$

$$e^{500}i = \cancel{300}\,\frac{e^{500t}}{\cancel{500}} + c$$

$$\boxed{e^{500}\cdot i - \frac{3}{5}e^{500t} = c} \quad -(A)$$

$\because i(0) = 0$

$$(A) \Rightarrow \quad e^{500}(0) - \frac{3}{5}e^{0} = c$$

$$\boxed{C = -3/5}$$

$$(A) \Rightarrow \quad i.e^{500} - \frac{3}{5}e^{500t} = \frac{-3}{5}$$

$$ie^{500t} = \frac{3}{5}e^{500t} - \frac{3}{5}$$

$$\boxed{i(t) = \frac{3}{5} - \frac{3}{5e^{500t}}}$$

Ans

FOR $i(t)$ as $t \to \infty$

$$\lim_{t\to\infty} i(t) = \frac{3}{5} - \frac{3}{5e^{\infty}}$$

$$\boxed{\lim_{t\to\infty} i(t) = \frac{3}{5}}$$

Ans