Q4 $(x+1)\frac{dy}{dx}+1=2e^{-y}$

$$(x+1)\frac{dy}{dx}=2e^{-y}-1$$

$$(x+1)\frac{dy}{dx}=\frac{2}{e^{y}}-1$$

$$(x+1)\frac{dy}{dx}=\frac{2-e^{y}}{e^{y}}$$

$$\left(\frac{e^{y}}{2-e^{y}}\right)dy=\frac{dx}{(x+1)}$$

Integrating B.S

$$-1\int\frac{-e^{y}}{2-e^{y}}\,dy=\int\frac{dx}{(x+1)}$$

$$-1\ \ln(2-e^{y})=\ln(x+1)+c$$

$$-\ln(2-e^{y})=\ln(x+1)+c$$

$$C=\ln(x+1)+\ln(2-e^{y})$$

$$\boxed{C=\ln\{(x+1)(2-e^{y})}$$

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$\frac{d}{dx} 2 - e^y$

$0 - e^y$

Q_{12} $x^2 \frac{dy}{dx} + yx + y^2 = 0$

$x^2 \frac{dy}{dx} + yx = -y^2$

$x^2 \left(\frac{dy}{dx} + \frac{yx}{x^2} \right) = -y^2$

$\frac{dy}{dx} + \frac{1}{x} y = \left(\frac{1}{x^2} \right) -y^2 \quad —(1)$

Bernaulli Std form

$y' + P(x) y = Q(x) y^n$

Here $n = 2$

$V = y^{1-n} = y^{-1} = \frac{1}{y}$

$\boxed{V = \frac{1}{y}}$

~~$dy = y^2 . dV$~~ $\frac{dV}{dy} = 0 - \frac{d}{dy} (y^{-1})$

$\frac{dV}{dy} = (-1) y^{-2}$

$\frac{dV}{dy} = \frac{1}{y^2}$

$$V = -\frac{1}{y}$$

diff w.r.t x

$$\frac{dv}{dx} = 0 - \frac{d}{dx}(y^{-1})$$

$$\frac{dv}{dx} = -(-1)\, y^{-2} \frac{dy}{dx}$$

$$\frac{dv}{dx} = y^{-2} \frac{dy}{dx}$$

$$\boxed{\frac{dy}{dx} = \frac{1}{y^{-2}} \frac{dv}{dx}}$$

$$(1) \Rightarrow \frac{dy}{dx} + \frac{y}{x} = \frac{-y^2}{x^2}$$

$$\frac{1}{y^{-2}}\left(\frac{dv}{dx}\right) + \frac{y}{x} = \frac{-y^2}{x^2}$$

$$y^2 \frac{dv}{dx} + \frac{y}{x} = \frac{-y^2}{x^2}$$

Multiplyng B.s by $\frac{1}{y^2}$

$$\frac{1}{y^2}\left(y^2\frac{dv}{dx}+\frac{y}{x}\right)=\frac{-\cancel{y^2}}{x^2}\times\frac{1}{\cancel{y^2}}$$

$$\frac{dv}{dx}+\frac{1}{yx}=\frac{1}{x^2}$$

$$P(x)=\frac{1}{x}$$

$$I.F=e^{\int\frac{1}{x}dx}$$

$$IF=e^{\ln x}$$

$$\boxed{I.F=x}$$

multiplying B.S by x

$$x\frac{dv}{dx}+\frac{1}{yx}\cdot x=\frac{1}{x}$$

$$x\frac{dv}{dx}+\frac{1}{y}=\frac{1}{x}$$

$$\because \frac{1}{y}=V$$

$$x\frac{dv}{dx}+V=\frac{1}{x}$$

$$\frac{d}{dx}(x.V)=\frac{1}{x}$$

$$\frac{d}{dx}(x.V) = \frac{1}{x}$$

Integrating B.S

$$\int \frac{d}{dx}(x.V) = \int \frac{1}{x}$$

$$x.V = \int \frac{dx}{x}$$

$$xV = \ln x + c$$

$$\because V = \frac{1}{y}$$

$$\frac{x}{y} = \ln x + c$$

$$\frac{y}{x} = \frac{1}{\ln x + c}$$

$$\boxed{y = \frac{x}{\ln x + c}}$$

Ans

Q13 $(\sec x \tan x \tan y - e^x)dx + \sec x \sec^2 y\, dy = 0$

Exact std. form

$$M dx + N dy = 0$$

Here

$$M = \sec x \tan x \tan y - e^x$$
$$N = \sec x \sec^2 y$$

$$\frac{\partial M}{\partial y} = \frac{\partial}{\partial y}(\sec x \tan x \tan y - e^x)$$

$$\partial M = \sec x \tan x \frac{\partial}{\partial y} \tan y - 0$$

$$\boxed{\frac{\partial M}{\partial y} = \sec x \tan x \sec^2 y}$$

$$\frac{\partial N}{\partial x} = \frac{\partial}{\partial x}(\sec x \sec^2 y) = \sec^2 y \frac{\partial}{\partial x} \sec x$$

$$\boxed{\frac{\partial N}{\partial x} = \sec x \tan x \sec^2 y}$$

$$\boxed{\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}}$$

$$\partial M = (\sec x \tan x \tan y - e^x)\partial x$$

integrating ~~~~~~ b.s.

$$\int \partial M = \int \sec x \tan x \tan y \, \partial x - \int e^x \partial x$$

$$M = \tan y \int \sec x \tan x \, \partial x - e^x$$

$$\boxed{M = \tan y \sec x - e^x}$$

$$\int \partial N = \int \sec x \sec^2 y \, \partial y$$

$$N = \sec x \int \sec^2 y \, \partial y$$

$$\boxed{N = \sec x \tan y}$$

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$\therefore MUN = C$

$$\boxed{\sec x \tan y - e^x = c}$$

OR

$$\sec x \tan y = c_1 - e^x$$

$$\tan y = \frac{c_1 - e^x}{\sec x}$$

$$\boxed{y = \tan^{-1}\left(\frac{c_1 - e^x}{\sec x}\right)}$$