Summary

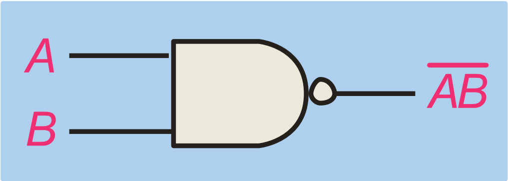
DeMorgan's Theorem

DeMorgan's 1st Theorem

The complement of a product of variables is equal to the sum of the complemented variables.

$$\overline{AB} = \overline{A} + \overline{B}$$

Applying DeMorgan's first theorem to gates:



NAND

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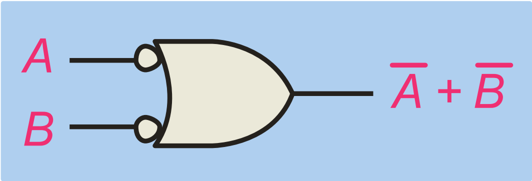


Negative-OR

Inputs		Output	
A	B	$\overline{AB}$	$\overline{A} + \overline{B}$
0	0	1	1
0	1	1	1
1	0	1	1
1	1	0	0

Summary

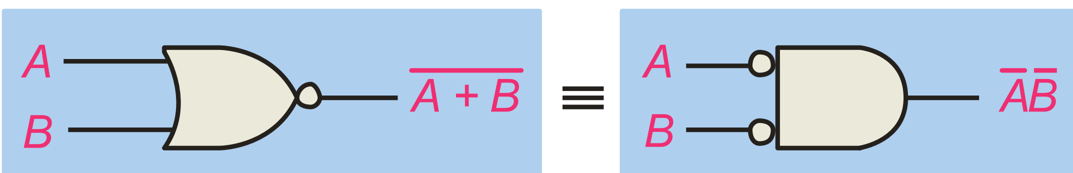
DeMorgan's Theorem

DeMorgan's 2nd Theorem

The complement of a sum of variables is equal to the product of the complemented variables.

$$\overline{A+B} = \overline{A} \cdot \overline{B}$$

Applying DeMorgan's second theorem to gates:



NOR Negative-AND

Inputs		Output	
A	B	$\overline{A+B}$	$\overline{A}\overline{B}$
0	0	1	1
0	1	0	0
1	0	0	0
1	1	0	0

Summary

DeMorgan's Theorem

Example Apply DeMorgan's theorem to remove the overbar covering both terms from the expression $X = \overline{\overline{C} + D}$.

Solution To apply DeMorgan's theorem to the expression, you can break the overbar covering both terms and change the sign between the terms. This results in $X = \overline{\overline{C}} \cdot \overline{D}$. Deleting the double bar gives $X = C \cdot \overline{D}$.

SECTION 4–3 CHECKUP **Homework**

1. Apply DeMorgan's theorems to the following expressions:

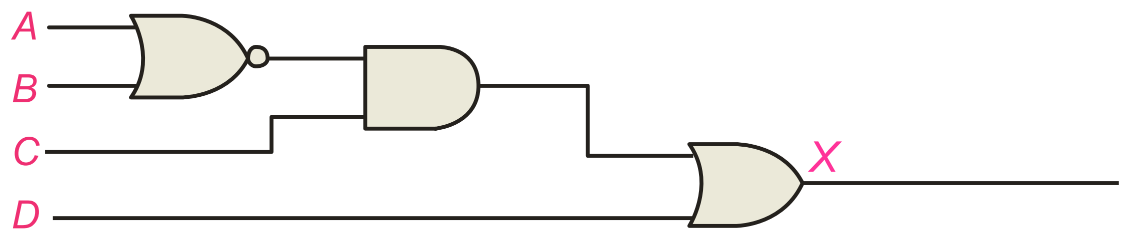
(a) $\overline{ABC} + \overline{(\overline{D} + E)}$ (b) $\overline{(A + B)C}$ (c) $\overline{A + B + C + \overline{DE}}$

Boolean Analysis of Logic Circuits

Combinational logic circuits can be analyzed by writing the expression for each gate and combining the expressions according to the rules for Boolean algebra.

Example Apply Boolean algebra to derive the expression for *X*.

Solution Write the expression for each gate:



Applying DeMorgan's theorem and the distribution law:

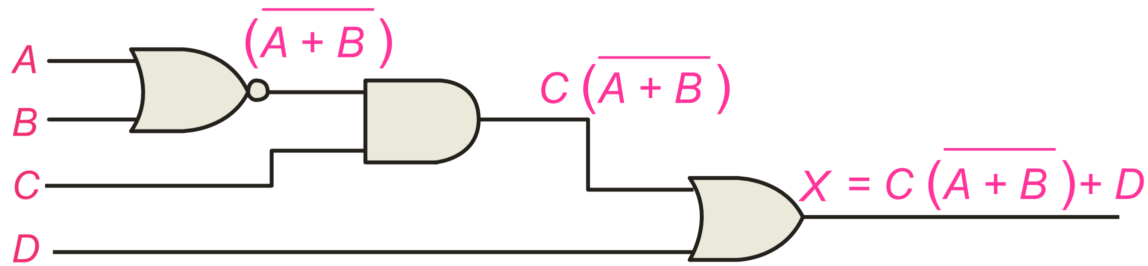
Summary

Boolean Analysis of Logic Circuits

Combinational logic circuits can be analyzed by writing the expression for each gate and combining the expressions according to the rules for Boolean algebra.

Example Apply Boolean algebra to derive the expression for X.

Solution Write the expression for each gate:



Applying DeMorgan's theorem and the distribution law:

$$X = C\,(\overline{A}\ \overline{B}) + D = \overline{A}\,\overline{B}\,C + D$$

EXAMPLE 4–9

Using Boolean algebra techniques, simplify this expression:

$$AB + A(B + C) + B(B + C)$$

EXAMPLE 4–9

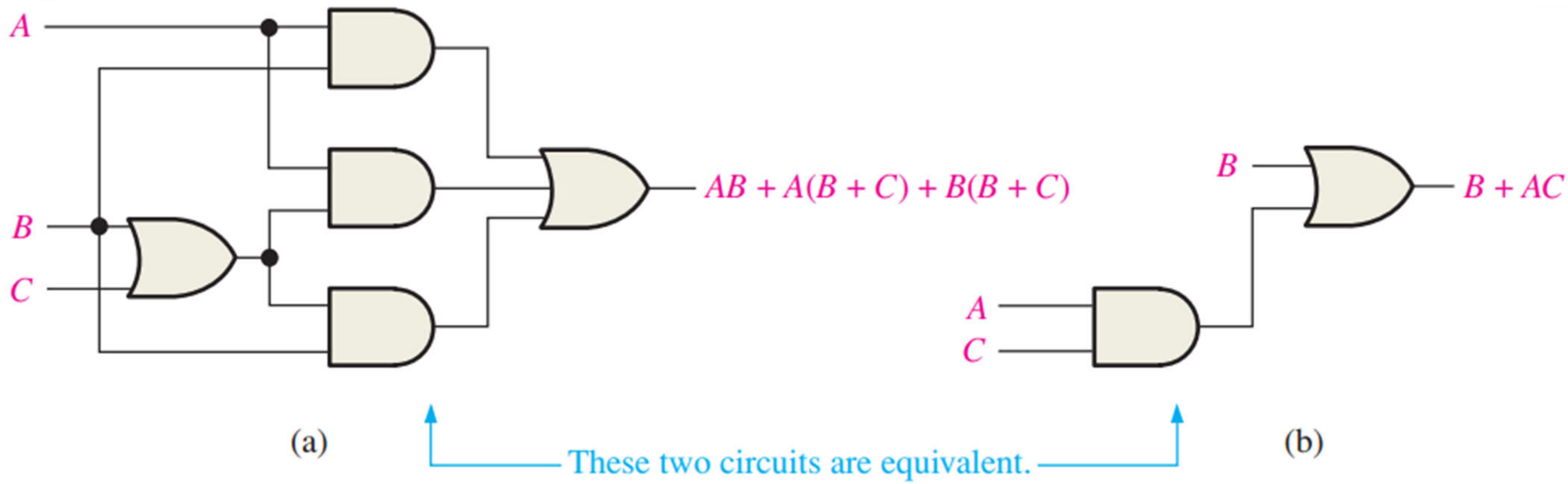
Using Boolean algebra techniques, simplify this expression:

$$AB + A(B + C) + B(B + C)$$

$$AB + AB + AC + BB + BC$$

$$AB + AB + AC + B + BC$$

$$AB + AC + B + BC = AB + AC + B = B + AC$$



(a) (b)

Summary

Practice Problems in DeMorgan's Theorem

Apply DeMorgan's theorems to the expressions $\overline{XYZ}$ and $\overline{X + Y + Z}$.

Solution

$$\overline{XYZ} = \overline{X} + \overline{Y} + \overline{Z}$$

$$\overline{X + Y + Z} = \overline{X}\,\overline{Y}\,\overline{Z}$$

Related Problem

Apply DeMorgan's theorem to the expression $\overline{\overline{X} + \overline{Y} + \overline{Z}}$.

Practice Problems in DeMorgan’s Theorem

The following procedure illustrates the application of DeMorgan’s theorems and Boolean algebra to the specific expression

$$\overline{\overline{A + B\overline{C}} + D(\overline{E + \overline{F}})}$$

The following procedure illustrates the application of DeMorgan's theorems and Boolean algebra to the specific expression

$$\overline{\overline{A + B\overline{C}} + D(\overline{E + \overline{F}})}$$

Step 1: Identify the terms to which you can apply DeMorgan's theorems, and think of each term as a single variable. Let $\overline{A + B\overline{C}} = X$ and $D(\overline{E + \overline{F}}) = Y$.

Step 2: Since $\overline{X + Y} = \overline{X}\,\overline{Y}$,

$$\overline{(\overline{A + B\overline{C}}) + (D(\overline{E + \overline{F}}))} = (\overline{\overline{A + B\overline{C}}})(\overline{D(\overline{E + \overline{F}})})$$

Step 3: Use rule 9 ($\overline{\overline{A}} = A$) to cancel the double bars over the left term (this is not part of DeMorgan's theorem).

$$(\overline{\overline{A + B\overline{C}}})(\overline{D(\overline{E + \overline{F}})}) = (A + B\overline{C})(\overline{D(\overline{E + \overline{F}})})$$

Step 4: Apply DeMorgan's theorem to the second term.

$$(A + B\overline{C})(\overline{D(\overline{E + \overline{F}})}) = (A + B\overline{C})(\overline{D} + (\overline{\overline{E + \overline{F}}}))$$

Step 5: Use rule 9 ($\overline{\overline{A}} = A$) to cancel the double bars over the $E + \overline{F}$ part of the term.

$$(A + B\overline{C})(\overline{D} + \overline{\overline{E + \overline{F}}}) = (A + B\overline{C})(\overline{D} + E + \overline{F})$$

Summary

Practice Problems in DeMorgan's Theorem

Try these expressions;

$$\overline{A\overline{B} + \overline{C}D + EF}$$

$$\overline{(A + B)\overline{C}\,\overline{D} + E + \overline{F}}$$

Summary

Practice Problems in DeMorgan's Theorem

EXAMPLE 4–7

The Boolean expression for an exclusive-OR gate is $A\overline{B} + \overline{A}B$. With this as a starting point, use DeMorgan's theorems and any other rules or laws that are applicable to develop an expression for the exclusive-NOR gate.

Practice Problems in DeMorgan's Theorem

Simplify the following Boolean expression:

$$[A\overline{B}(C + BD) + \overline{A}\,\overline{B}]C$$

Solve example problems in book for practice.

Summary

SOP and POS forms

Boolean expressions can be written in the **sum-of-products** form (**SOP**) or in the **product-of-sums** form (**POS**). These forms can simplify the implementation of combinational logic, particularly with PLDs. In both forms, **an overbar cannot extend over more than one variable.**

An expression is in SOP form when two or more product terms are summed as in the following examples:

$\overline{A}\,\overline{B}\,\overline{C} + A\,B$ $\qquad A\,B\,\overline{C} + \overline{C}\,\overline{D}$ $\qquad C\,D + \overline{E}$

An expression is in POS form when two or more sum terms are multiplied as in the following examples:

$(A + B)(\overline{A} + C)$ $\qquad (A + B + \overline{C})(B + D)$ $\qquad (\overline{A} + B)C$

Summary

SOP Standard form

In **SOP standard form**, every variable in the domain must appear in each term. This form is useful for constructing truth tables or for implementing logic in PLDs.

You can expand a **nonstandard** term to **standard form** by multiplying the term by a term consisting of the sum of the missing variable and its complement.

Example

Convert $X = \overline{A}\,\overline{B} + A\,B\,C$ to standard form.

Solution

The first term does not include the variable C. Therefore, multiply it by the $(C + \overline{C})$, which = 1:

$$X = \overline{A}\,\overline{B}\,(C + \overline{C}) + A\,B\,C$$

$$= \overline{A}\,\overline{B}\,C + \overline{A}\,\overline{B}\,\overline{C} + A\,B\,C$$

Summary

POS Standard form

In **POS standard form**, every variable in the domain must appear in each sum term of the expression.

You can expand a nonstandard POS expression to standard form by adding the product of the missing variable and its complement and applying rule 12, which states that $(A + B)(A + C) = A + BC$.

Example Convert $X = (\overline{A} + \overline{B})(A + B + C)$ to standard form.

Solution The first sum term does not include the variable C. Therefore, add $C\overline{C}$ and expand the result by rule 12.

$$X = (\overline{A} + \overline{B} + C\overline{C})(A + B + C)$$
$$= (\overline{A} + \overline{B} + C)(\overline{A} + \overline{B} + \overline{C})(A + B + C)$$

Summary

POS Standard form

Convert the following Boolean expression into standard POS form:

$$(A + \overline{B} + C)(\overline{B} + C + \overline{D})(A + \overline{B} + \overline{C} + D)$$

Convert the following Boolean expression into standard SOP form:

$$A\overline{B}C + \overline{A}\,\overline{B} + AB\overline{C}D$$

Summary

POS Standard form

Convert the following Boolean expression into standard POS form:

$$(A + \overline{B} + C)(\overline{B} + C + \overline{D})(A + \overline{B} + \overline{C} + D)$$

$$(A + \overline{B} + C + D)(A + \overline{B} + C + \overline{D})(A + \overline{B} + C + \overline{D})(\overline{A} + \overline{B} + C + \overline{D})(A + \overline{B} + \overline{C} + D)$$

Convert the following Boolean expression into standard SOP form:

$$A\overline{B}C + \overline{A}\,\overline{B} + AB\overline{C}D$$

$$= A\overline{B}CD + A\overline{B}C\overline{D} + \overline{A}\,\overline{B}CD + \overline{A}\,\overline{B}C\overline{D} + \overline{A}\,\overline{B}\,\overline{C}D + \overline{A}\,\overline{B}\,\overline{C}\,\overline{D} + AB\overline{C}D$$

Summary

Binary Representation

An SOP expression is equal to 1 only if one or more of the product terms in the expression is equal to 1.

$$\overline{X}YZ + X\overline{Y}Z + XY\overline{Z} + \overline{X}Y\overline{Z} + XYZ$$

A POS expression is equal to 0 only if one or more of the sum terms in the expression is equal to 0

$$(X + \overline{Y} + Z)(\overline{X} + Y + Z)(X + Y + \overline{Z})(\overline{X} + \overline{Y} + \overline{Z})(X + \overline{Y} + \overline{Z})$$

Also construct Truth Table using these combinations.

Converting Standard SOP to Standard POS

EXAMPLE 4–19

Convert the following SOP expression to an equivalent POS expression:

$$\bar{A}\bar{B}\bar{C} + \bar{A}B\bar{C} + \bar{A}BC + A\bar{B}C + ABC$$

Solution

The evaluation is as follows:

$$000 + 010 + 011 + 101 + 111$$

Since there are three variables in the domain of this expression, there are a total of eight (2^3) possible combinations. The SOP expression contains five of these combinations, so the POS must contain the other three which are 001, 100, and 110. Remember, these are the binary values that make the sum term 0. The equivalent POS expression is

$$(A + B + \bar{C})(\bar{A} + B + C)(\bar{A} + \bar{B} + C)$$

Summary

Karnaugh maps

The Karnaugh map (K-map) is a tool for simplifying combinational logic with 3 or 4 variables. For 3 variables, 8 cells are required (2^3).

The map shown is for three variables labeled *A, B,* and *C*. Each cell represents one possible product term.

Each cell differs from an adjacent cell by only one variable.