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DE ASSIGNMENT NO. 03

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Section: BCS-2B

Laplace by definition

Q1. $f(t) = \begin{cases} e^t & t \leq 2 \\ 3 & t > 2 \end{cases}$

$$\mathcal{L}\{f(t)\} = \int_0^2 e^t . e^{-st} + \int_2^\infty 3e^{-st}$$

$$\int e^{-t(s-1)} + 3\int_2^\infty e^{-st}$$

$$\left[\frac{e^{-2(s-1)} - 1}{-(s-1)}\right] + 3\left[\frac{0 - e^{-2s}}{-s}\right]$$

$$\frac{1-e^{-2(s-1)}}{s-1} + \frac{3}{s}.e^{-2s}$$

Q2. $f(t) = 3 + 2t^2$

$$\mathcal{L}\{f(t)\} = \int_0^\infty (3+2t^2) . e^{-st}$$

$$\mathcal{L}\{f(t)\} = \int_0^\infty 3e^{-st} + 2t^2 e^{-st}$$

$$\int_0^\infty 3e^{-st} + 2\int_0^\infty t^2 e^{-st} \rightarrow (A)$$

Solving A:

$v = t^2 \qquad u' = e^{-st}$

$v' = 2t \qquad u = \frac{e^{-st}}{-s}$

$$\frac{-t^2 e^{-st}}{s} + \frac{2}{s}\int te^{-st}$$

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$$-\frac{te^{-st}}{s} + \frac{1}{s}\int e^{-st}$$

$$-\frac{te^{-st}}{s} - \frac{e^{-st}}{s^2}$$

$$-\frac{t^2e^{-st}}{s} + \frac{2}{s}\left[-\frac{te^{-st}}{s} - \frac{e^{-st}}{s^2}\right]$$

$$-\frac{t^2e^{-st}}{s} - \frac{2te^{-st}}{s^2} - \frac{2e^{-st}}{s^3}$$

$$\left[\frac{3e^{-st}}{-s}\right]_0^\infty + 2\left[-\frac{t^2e^{-st}}{s} - \frac{2te^{-st}}{s^2} - \frac{2e^{-st}}{s^3}\right]_0^\infty$$

$$= -\frac{1}{s}\left[3e^{-s(\infty)} - 3e^0\right] + \frac{2}{s}\left[-\frac{t^2}{e^{st}} - \frac{2t}{se^{st}} - \frac{2e^{-st}}{s^2}\right]_0^\infty$$

$$-\frac{1}{s}[-3] - \frac{2}{s}\left[0 - \frac{2}{s^2}\right]$$

$$\frac{3}{s} - \frac{4}{s^3}$$

Q3. $f(t) = 5\sin 3t - 17e^{-2t}$

$$\mathcal{L}\{f(t)\} = \int_0^\infty (5\sin 3t - 17e^{-2t}).e^{-st}$$

$$\mathcal{L}\{f(t)\} = \int_0^\infty 5\sin 3t.e^{-st} - \int_0^\infty 17e^{-2t-st}$$

$$= 5\int_0^\infty \sin 3t.e^{-st} - 17\int_0^\infty e^{\cancel{-2t-st}\ -t(2+s)}$$

$V = \sin 3t$ $\quad U' = e^{-st}$

$V' = 3\cos 3t$ $\quad U = -e^{-st}$

Applying By parts

$$\int_0^\infty \sin 3t.e^{-st} = \frac{-e^{-st}\sin 3t}{s} + \frac{3}{s}\int \cos 3t.e^{-st}$$

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$v = \cos 3t \qquad u' = e^{-st}$

$v' = -3\sin 3t \qquad u = -\frac{e^{-st}}{s}$

$$\frac{-\cos 3t . e^{-st}}{s} - \frac{3}{s} \int \sin 3t . e^{-st}$$

let $I = \int_0^\infty \sin 3t . e^{-st}$

$$I = \frac{-e^{-st} \sin 3t}{s} + \frac{3}{s} \left[\frac{-\cos 3t e^{-st}}{s} - \frac{3}{s} I \right]$$

$$I = \frac{-e^{-st} \sin 3t}{s} - \frac{3\cos 3t . e^{-st}}{s^2} - \frac{9I}{s^2}$$

$$I + \frac{9I}{s^2} = \frac{-e^{-st} \sin 3t}{s} - \frac{3\cos 3t . e^{-st}}{s^2}$$

$$I\left(1 + \frac{9}{s^2}\right) = \left(\frac{-e^{-st} \sin 3t}{s} - \frac{3\cos 3t . e^{-st}}{s^2}\right) . \left(\frac{s^2}{s^2+9}\right)$$

$$\int_0^\infty \sin 3t . e^{-st} = \left[-e^{-st} \sin 3t - \frac{3\cos 3t . e^{-st}}{s} \right]_0^\infty . \left(\frac{s}{s^2+9}\right)$$

$$\int_0^\infty \sin 3t . e^{-st} = 0 + \frac{3}{\cancel{s}} \left(\frac{\cancel{s}}{s^2+9}\right)$$

$$5\left(\frac{3}{s^2+9}\right) - 17 \int_0^\infty e^{-t(2+s)}$$

$$\frac{15}{s^2+9} - 17 \left[\frac{0-1}{-(2+s)} \right]$$

$$\frac{15}{s^2+9} - 17\left(\frac{1}{2+s}\right)$$

$$\frac{15}{s^2+9} - \frac{17}{2+s}$$

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Q4. $f(t) = te^{4t}$

$$\mathcal{L}\{f(t)\} = \int_0^\infty te^{4t} \cdot e^{-st}$$

$$te^{4t-st}$$

$$te^{-t(s-4)}$$

$v = t \qquad u' = e^{-t(s-4)}$

$v' = 1 \qquad u = \dfrac{e^{-t(s-4)}}{-(s-4)}$

$$\cancel{\frac{-te^{-t(s-4)}}{s-4} + \frac{1}{s-4}\int \frac{e^{-t(s-4)}}{-(4-s)}}$$

$$\frac{-te^{-t(s-4)}}{s-4} + \int \frac{e^{-t(s-4)}}{s-4}$$

$$\frac{-te^{-t(s-4)}}{s-4} + \frac{1}{s-4}\left[\frac{e^{-t(s-4)}}{-(s-4)}\right]$$

$$\frac{-te^{-t(s-4)}}{s-4} - \frac{1}{(s-4)^2}e^{-t(s-4)}$$

$$\left[\frac{-te^{-t(s-4)}}{s-4} - \frac{e^{-t(s-4)}}{(s-4)^2}\right]_0^\infty$$

$$\frac{-1}{s-4}\left[te^{-t(s-4)} - \frac{e^{-t(s-4)}}{s-4}\right]_0^\infty$$

$$\mathcal{L}\{f(t)\} = \frac{-1}{s-4}\left[\frac{t}{e^{-t(s-4)}} - \frac{e^{-t(s-4)}}{s-4}\right]_0^\infty$$

$$-\frac{1}{s-4}\left[-\frac{1}{s-4}\right]$$

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Q4. (Continued)

$$\mathcal{L}\{f(t)\} = \frac{1}{(s-4)^2}$$

Find Laplace inverse of the following:

Q1. $\mathcal{L}^{-1}\left\{\frac{1}{s(s^2+2s+5)}\right\}$

$$1 = \frac{A}{s} + \frac{Bs+C}{s^2+2s+5}$$

$$1 = A(s^2+2s+5) + Bs^2+Cs$$

$A = \frac{1}{5}$ $\quad$ $A+B=0$ $\quad$ $2A+C=0$

$B = -\frac{1}{5}$ $\quad$ $2A=-C$

$C = -\frac{2}{5}$

$$\frac{1}{5}\mathcal{L}^{-1}\left\{\frac{1}{s}\right\} - \frac{1}{5}\mathcal{L}^{-1}\left\{\frac{s+2}{s^2+2s+5}\right\}$$

$$\frac{1}{5}(1) - \frac{1}{5}\left\{L^{-1}\left\{\frac{s+2}{(s+1)^2+4}\right\}\right.$$

$$\frac{1}{5} - \frac{1}{5}\mathcal{L}^{-1}\left\{\frac{s+1+1}{(s+1)^2+4}\right\}$$

$$\frac{1}{5} - \frac{1}{5}\left[\mathcal{L}^{-1}\left\{\frac{s+1}{(s+1)^2+2^2}\right\} + \mathcal{L}^{-1}\left\{\frac{1}{(s+1)^2+2^2}\right\}\right]$$

$$\frac{1}{5} - \frac{1}{5}\left[\cos 2t . e^{-t} + \frac{1}{2}\sin 2t . e^{-t}\right]$$

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$\frac{1}{5} - \frac{1}{5}\cos 2t . e^{-t} - \frac{1}{10}\sin 2t . e^{-t}$

Q2. $\mathcal{L}^{-1}\left\{\frac{7s-1}{(s+1)(s+2)(s-3)}\right\}$

$7s-1 = \frac{A}{s+1} + \frac{B}{s+2} + \frac{C}{s-3}$

$7s-1 = A(s+2)(s-3) + B(s+1)(s-3) + C(s+1)(s+2)$

$s = -1$	$s = -2$	$s = 3$
$-8 = -4A$	$-15 = 5B$	$20 = 20C$
$A = 2$	$B = -3$	$C = 1$

$2\mathcal{L}^{-1}\left\{\frac{1}{s+1}\right\} - 3\mathcal{L}^{-1}\left\{\frac{1}{s+2}\right\} + \mathcal{L}^{-1}\left\{\frac{1}{s-3}\right\}$

$2e^{-t} - 3e^{-2t} + e^{3t}$

Q3. $\mathcal{L}^{-1}\left\{\frac{s^2+9s+2}{(s-1)^2(s+3)}\right\}$

$s^2+9s+2 = \frac{A}{s-1} + \frac{B}{(s-1)^2} + \frac{C}{s+3}$

$s^2+9s+2 = A(s-1)(s+3) + B(s+3) + C(s-1)^2$

let $s = 1$	$s = -3$	$s = 0$
$12 = 4B$	$16C = -16$	$2 = -3A + 9 - 1$
$B = 3$	$C = -1$	$-6 = -3A$
		$A = 2$

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$$2\mathcal{L}^{-1}\left\{\frac{1}{s-1}\right\} + 3\mathcal{L}^{-1}\left\{\frac{1}{(s-1)^2}\right\} - \mathcal{L}^{-1}\left\{\frac{1}{s+3}\right\}$$

$$2e^{+t} + 3te^{t} - e^{-3t}$$

Q4. $\mathcal{L}^{-1}\left\{\frac{2s^2+10s}{(s^2-2s+5)(s+1)}\right\}$

$$\frac{2s^2+10s}{} = \frac{As+B}{s^2-2s+5} + \frac{C}{s+1}$$

$$2s(s+5) = (As+B)(s+1) + C(s^2-2s+5)$$

let $s=-1$ let $s=0$ let $s=-5$

$-8=8C$ $0=B-5$ $20A-4B+40C=0$

$C=-1$ $B=5$ $20A-20-40=$

$A=3$

$$\mathcal{L}^{-1}\left\{\frac{3s+5}{(s^2-2s+5)} - \frac{1}{(s+1)}\right\}$$

$$\mathcal{L}^{-1}\left\{\frac{3s-3+8}{s^2-2s+5} - \frac{1}{s+1}\right\}$$

$$\frac{3s-3}{s^2-2s+5} + \frac{8}{s^2-2s+5} - \frac{1}{s+1}$$

$$3\mathcal{L}^{-1}\left\{\frac{s-1}{(s-1)^2+2^2}\right\} + 4\mathcal{L}^{-1}\left(\frac{2}{(s-1)^2+2^2}\right) - e^{-t}$$

$$3e^{t}\cos 2t + 4e^{t}\sin 2t - e^{-t}$$

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Solve the following ~~by~~ DE by Laplace and then confirm the general solution by analytical method.

Q1 $y' - 5y = e^{5x}$ $y(0) = 0$

$sY(s) - y(0) - 5\mathcal{L}(y) = \mathcal{L}(e)^{5x}$

$Y(s) = \mathcal{L}\{y(x)\}$

$sY(s) - 0 - 5Y(s) = \frac{1}{s-5}$

$Y(s)\{s-5\} = \frac{1}{s-5}$

$\mathcal{L}\{y(x)\} = \frac{1}{(s-5)^2}$

$y(x) = \mathcal{L}^{-1}\left\{\frac{1}{(s-5)^2}\right\}$

$y(x) = \mathcal{L}^{-1}\left\{\frac{1}{s^2}\right\}_{s \to s-5}$

$y(x) = xe^{5x}$

Proving with analytical method:-

$\frac{dy}{dx} - 5y = e^{5x}$ $y(0) = 0$

$If = e^{\int -5\,dx}$

$If = e^{-5x}$

$e^{-5x}\frac{dy}{dx} - e^{-5x}.5y = 1$

$e^{-5x}.y = x + c$

$y = xe^{5x} + ce^{5x}$

$y(0) = 0$

Q1. (continued) $c = 0$

$$y = xe^{5x}$$

Q2. $y' + y = \sin x \quad , \quad y(0) = 1$

$$\mathcal{L}\{y'\} + \mathcal{L}\{y\} = \mathcal{L}\{\sin x\}$$

$$sY(s) - y(0) + Y(s) = \frac{1}{s^2+1}$$

$$Y(s)[1+s] - 1 = \frac{1}{s^2+1}$$

$$Y(s) = \frac{1}{(s^2+1)(s+1)} + \frac{1}{(s+1)}$$

$$Y(s) = \frac{As+B}{s^2+1} + \frac{C}{s+1}$$

$$1 = (As+B)(s+1) + C(s^2+1)$$

$s = -1$: $1 = 2C$, $C = \frac{1}{2}$

$s = 0$: $B + C = 1$, $B = +\frac{1}{2}$

$s = 1$: $1 = 2A + 1 + 1$, $-\frac{1}{2} = A$

$$Y(s) = -\frac{1}{2} \cdot \frac{s}{s^2+1} + \frac{1}{2} \cdot \frac{1}{s^2+1} + \frac{1}{2} \cdot \frac{1}{s+1} + 1 \cdot \frac{1}{s+1}$$

$$y(x) = -\frac{1}{2} \mathcal{L}^{-1}\left(\frac{s}{s^2+1}\right) + \frac{1}{2} \mathcal{L}^{-1}\left(\frac{1}{s^2+1}\right) + \frac{3}{2} \mathcal{L}^{-1}\left(\frac{1}{s+1}\right)$$

$$y(x) = -\frac{1}{2} \cos x + \frac{1}{2} \sin x + \frac{3}{2} e^{-x}$$

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$$\frac{dy}{dx} + y = \sin x$$

$$I.f = e^x$$

$$e^x \cdot \frac{dy}{dx} + e^x \cdot y = e^x \sin x$$

$$e^x \cdot y = \int e^x \sin x$$

$\cancel{u = e^x}$ $\quad V = \sin x \quad u' = e^x$

$V' = -\cos x \quad u = e^x$

$$e^x \sin x - \int e^x \cos x$$

$$\int e^x \cos x = e^x \cos x + \int e^x \sin x$$

$$\int e^x \sin x = e^x \sin x - e^x \cos x - \int e^x \sin x$$

$$2 \int e^x \sin x = e^x \sin x - e^x \cos x$$

$$e^x \sin x = \frac{e^x \sin x - e^x \cos x}{2}$$

$$e^x \cdot y = \frac{e^x \sin x - e^x \cos x}{2} + c$$

$$y(0) = 1$$

$$1 = -\frac{1}{2} + c$$

$$c = \frac{3}{2}$$

$$y = \frac{\sin x}{2} - \frac{\cos x}{2} + \frac{3}{2} e^{-x} \text{ [proven]}$$

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Q3. $y'' - y' = 2x \qquad y(0) = 1,\ y'(0) = -2$

$s^2 Y(s) - s y(0) - y'(0) - [s Y(s) - y(0)] = 2\mathcal{L}\{x\}$

$s^2 Y(s) - s + 2 - s Y(s) + 1 = \frac{2}{s^2}$

$Y(s)[s^2 - s] = \frac{2}{s^2} + s + 3$

$Y(s) = \frac{2 + s^3 + 3s^2}{s^3(s-1)}$

$2 + s^3 + 3s^2 = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s^3} + \frac{D}{s-1}$

$2 + s^3 - 3s^2 = A(s^2)(s-1) + B(s)(s-1) + C(s-1) + Ds^3$

$s = 0 \qquad 2 = -C$

$C = -2$

$s = 1 \qquad D = 0$

$s = -1 \qquad -2 = -2A + 2B - 2C$

$-1 = -A + B - C$

$-1 = -A + B + 2$

$B = A - 3$

$\cancel{A} = B = 1 - 3$

$B = -2$

$s = 2$

$-2 = 4A + 2B + C$

$-2 = 4A + 2A - 6 - 2$

$6A = 6$

$A = 1$

$y(x) = \mathcal{L}^{-1}\left\{\frac{1}{s}\right\} - 2\mathcal{L}^{-1}\left\{\frac{1}{s^2}\right\} - \mathcal{L}^{-1}\left\{\frac{2}{s^3}\right\}$

$y(x) = 1 - 2x - x^2$

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Proving by Analytical Method

$y'' - y' = 2x$

$D^2 - D = 0$

$D = 0 \quad D = 1$

$y_c = c_1 + c_2 e^x$

y_p :-

$y_p = Ax + B$ $\quad y_p = Ax^2 + Bx$

$y' = A$ $\quad y' = 2Ax + B$

$y'' = 0$ $\quad y'' = 2A$

$2A - 2Ax \mp B = 2x$

$-2A\cancel{x} = 2\cancel{x}$ $\quad 2A - B = 0$

$A = -1$ $\quad -2 - B = 0$

$B = -2$

$y_p = -x^2 - 2x$

$y = y_c + y_p$

$y = c_1 + c_2 e^x - x^2 - 2x$ $\quad y'(0) = -2$

$y' = 0 + c_2 e^x - 2x - 2$

$-2 = c_2 - 2$

$c_2 = 0$ $\quad c_1 = 1 \{ y(0) = +1 \}$

$y = 1 - x^2 - 2x$

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Q4. $y'' - 2y' + 5y = -8^{-(x-7)}$ $y(7) = 2$ $y'(7) = 12$

let $t = x - 7$

$s^2 Y(s) - sy(0) - y'(0) - 2[sY(0) - y(0)] + 5Y(s) = -8e^{-t}$

$s^2 Y(s) - 2s - 12 - 2sY(s) + 4 + 5Y(s) = \frac{-8}{s+1}$

$Y(s)[s^2 - 2s + 5] - 2s - 8 = \frac{-8}{s+1}$

$Y(s)[s^2 - 2s + 5] = \frac{-8}{s+1} + (2s+8)(s+1)$

$Y(s)[s^2 - 2s + 5] = \frac{-8 + 2s^2 + 2s + 8s + 8}{(s+1)}$

$Y(s) = \frac{2s^2 + 10s}{(s^2 - 2s + 5)(s+1)}$

$2s^2 + 10s = \frac{As + B}{s^2 - 2s + 5} + \frac{C}{s+1}$

$2s^2 + 10s = (As + B)(s+1) + C(s^2 - 2s + 5)$

$2s^2 + 10s = As^2 + As + Bs + B + Cs^2 - 2Cs + 5C$

$2 = A + C$ $10 = A + B - 2C$ $B = -5C$

$A = 2 - C$ $10 = 2 + B - 2C$ $B = 5$

$A = 3$ $10 = 2 - 8C$

$C = -1$

$\frac{3s + 5}{s^2 - 2s + 5} - \frac{1}{s+1}$

$\frac{3s + 5 + 3 - 3}{s^2 - 2s + 5} - \frac{1}{s+1}$

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$$\frac{3s-3}{(s-1)^2+2^2} + \frac{8}{(s-1)^2+2^2} - \frac{1}{s+1}$$

$$3\mathcal{L}^{-1}\left\{\frac{s-1}{(s-1)^2+2^2}\right\} + 4\mathcal{L}^{-1}\left\{\frac{2}{(s-1)^2+2^2}\right\} - e^{-t}$$

$$\cancel{y(t) = 3\cos 2t . e}$$

$$y(t) = 3\cos 2t . e^{t} + 4\sin 2t . e^{t} - e^{-t}$$

since $t = x-7$

$$y(t) = 3\cos 2(x-7).e^{x-7} + 4\sin 2(x-7)\, e^{x-7} - e^{-(x-7)}$$

with analytical method

$$y'' - 2y' + 5y = -8e^{7-x}$$

$$\cancel{D=0}, \cancel{D=1} \quad D = 1 \pm 2i$$

$$y_c = e^{x}\{c_1\cos 2x + c_2 \sin 2x\}$$

$$y_1 = e^{x}\cos 2x \qquad y_2 = e^{x}\sin 2x$$

for y_p:- $g(x) = -8e^{7-x}$

$$w_1 = -y_2\, g(x)$$

$$w_1 = -e^{x}\sin 2x . (-8e^{7-x})$$

$$w_1 = e^{x+7-x} . \sin 2x$$

$$w_1 = 8e^{7}\sin 2x$$

$$w_2 = e^{x}\cos 2x(-8e^{7-x})$$

$$w_2 = -8e^{7}\cos 2x$$

$$W = \begin{vmatrix} e^{x}\cos 2x & e^{x}\sin 2x \\ e^{x}\cos 2x - 2\sin 2x e^{x} & e^{x}\sin 2x + 2e^{x}\cos 2x \end{vmatrix}$$

ad - bc =

$$e^{x}\cos 2x(e^{x}\sin 2x + 2e^{x}\cos 2x) - e^{x}\sin 2x(e^{x}\cos 2x - 2\sin 2x\, e^{x})$$

$$W = e^{2x}[\sin 2x\cos 2x + 2\cos^2 2x] - e^{2x}(\sin 2x\cos 2x)$$

$$W = e^{2x}(\sin 2x\cos 2x + 2\cos^2 2x - e^{2}\sin 2x\cos 2x)$$

$$W = e^{2x}\,2(\cos^2 2x + \sin^2 2x)$$

$$W = 2e^{2x}$$

$$u_1' = \frac{W_1}{W} = \frac{8e^7\sin 2x}{2e^{2x}}$$

$$u_1' = \frac{W_1}{W} = \cancel{8e^7\sin}\ \frac{8e^7}{2} = e^{-2x}\sin 2x$$

$$u_1 = \frac{8e^7}{2}\ \cancel{e^{2x}}\int e^{-2x}\sin 2x\,dx$$

let $I = \int e^{-2x}\sin 2x\,dx$

$$I = \sin 2x\, e^{-2x} - \int 2\cos 2x\, \cancel{8} e^{-2x}\,dx$$

$$I = \frac{-\sin 2x e^{-2x}}{2} + \frac{\cos 2x e^{-2x}}{-2} - \int \frac{\cancel{+}\cancel{2}\sin 2x e^{-2x}}{\cancel{+2}}\,dx$$

$$I = \frac{-\sin 2x e^{-2x}}{2} + \frac{\cos 2x e^{-2x}}{-2} - \underbrace{\int \sin 2x e^{-2x}\,dx}_{I}$$

$$I + I = \frac{-\sin 2x e^{-2x}}{4} - \frac{\cos 2x e^{-2x}}{4}$$

$$u_1 = \frac{8e^7}{2}\left(-\frac{\sin 2x e^{-2x}}{4} - \frac{\cos 2x e^{-2x}}{4}\right)$$

$$u_2' = \frac{W_2}{W} = \frac{-8e^7\cos 2x}{2e^{2x}}$$

$$u_2' = \frac{\cancel{-8}}{\cancel{2}} - 4e^7 e^{-2x}\cos 2x$$

$$u_2 = -4e^7\underbrace{\int e^{-2x}\cos 2x\,dx}_{I}$$

$$I = \int e^{-2x}\cos 2x\,dx$$

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$$I = \frac{\cos 2x\, e^{-2x}}{-2} - \int \frac{-2\sin 2x\, e^{-2x}}{-2}\, dx$$

$$I = \frac{\cos 2x\, e^{-2x}}{-2} - \int \sin 2x\, e^{-2x}\, dx$$

$$I = \frac{\cos 2x\, e^{-2x}}{-2} - \frac{\sin 2x\, e^{-2x}}{-2} + \int \frac{\cancel{2}\cos 2x\, e^{-2x}}{\cancel{-2}}\, dx$$

$$I = \frac{\cos 2x\, e^{-2x}}{-2} - \frac{\sin 2x\, e^{-2x}}{-2} - \int \cos 2x\, e^{-2x}\, dx$$

$$I + I = -\frac{\cos 2x\, e^{-2x}}{\cancel{2}4} + \frac{\sin 2x\, e^{-2x}}{4}$$

$$u_2 = -4e^{7}\left(-\frac{\cos 2x\, e^{-2x}}{4} + \frac{\sin 2x\, e^{-2x}}{4}\right)$$

$$u_2 = \frac{-\cancel{4}}{\cancel{4}} e^{7}\left(-e^{-2x}\right)\left(\cos 2x - \sin 2x\right)$$

$$u_2 = e^{7-2x}\left(\cos 2x - \sin 2x\right)$$

$$u_1 = \frac{\cancel{8}e^{7}}{\cancel{2}}\left(\frac{-\sin 2x\, e^{-2x} - \cos 2x\, e^{-2x}}{\cancel{4}}\right)$$

$$u_1 = -e^{7-2x}\left(\sin 2x + \cos 2x\right)$$

$$y_p = y_1 u_1 + y_2 u_2$$

$$y_p = e^{x}\cos 2x\left(-e^{7-2x}\left(\sin 2x + \cos 2x\right)\right) + e^{x}\sin 2x\left(-\sin 2x + \cos 2x\right)e^{7-2x}$$

$$y_p = -e^{7-x}\left(\cos 2x \sin 2x + \cos^{2} 2x\right) + e^{7-x}\left(-\sin^{2} 2x + \sin^{2} x \cos 2x\right)$$

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$$y_p = e^{7-x}\left(-\sin 2x\cos 2x - \cos^2 2x - \sin^2 2x + \sin 2x\cos 2x\right)$$

$$y_p = e^{7-x}\left(-\left(\sin^2 2x + \cos^2 2x\right)\right)$$

$$y_p = -e^{7-x}$$

$$y = y_c + y_p$$

$$y = e^{x}(c_1\cos 2x + c_2\sin 2x) - e^{7-x}$$

$$y(7) = 2 \qquad y'(7) = 12$$

$$2 = e^7(c_1\cos 14 + c_2\sin 14) - e^{7-7}$$

$$2 = e^7(c_1\cos 14 + c_2\sin 14) - e^0$$

$$2 + 1 = e^7(c_1\cos 14 + c_2\sin 14)$$

$$y = e^x(c_1\cos 2x + c_2\sin 2x) - e^{7-x}$$

$$y' = e^x(c_1\cos 2x + c_2\sin 2x) + e^x(-2c_1\sin 2x + 2c_2\cos 2x) - e^{7-x}(-1)$$

$$y'(7) = 12$$

$$12 = e^7(c_1\cos 14 + c_2\sin 14) + e^7(-2c_1\sin 14 + 2c_2\cos 14) + e^0$$

$$12 - 1 = e^7(c_1\cos 14 - 2c_1\sin 14 + c_2\sin 14 + 2c_2\cos 14)$$

$$\frac{11}{e^7} = \cos 14(c_1 + 2c_2) + \sin 14(c_2 - 2c_1)$$

$$\frac{3}{e^7} = c_1\cos 14 + c_2\sin 14$$

$$\frac{11}{e^7} = c_1\cos 14 + 2c_2\cos 14 + c_2\sin 14 - 2c_1\sin 14$$

$$\frac{11}{e^7} = \underbrace{c_1\cos 14 + c_2\sin 14}_{3/e^7} + 2c_2\cos 14 - 2c_1\sin 14$$

$$\frac{11}{e^7} - \frac{3}{e^7} = 2(c_2\cos 14 - c_1\sin 14)$$

$$\frac{8}{e^7} = 2(c_2\cos 14 - c_1\sin 14) \quad \text{eq II}$$

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from eq 1:

$$\frac{3}{e^7} - c_1 \cos 14 = c_2 \sin 14$$

$$\frac{3}{e^7 \sin 14} - c_1 \frac{\cos 14}{\sin 14} = c_2$$

put c_2 in eq II.

$$\frac{8}{e^7} = 2\left(\left(\frac{3}{e^7 \sin 14} - \frac{c_1 \cos 14}{\sin 14}\right) \cos 14 - c_1 \sin 14\right)$$

$$\frac{8}{e^7} = 2\left(\frac{3\cos 14}{e^7 \sin 14} - \frac{c_1 \cos^2 14}{\sin 14} - c_1 \sin 14\right)$$

$$\frac{8}{2e^7} = \frac{3\cos 14}{e^7 \sin 14} - c_1\left(\frac{\cos^2 14 + \sin^2 14}{\sin 14}\right)$$

$$\frac{4\sin 14}{e^7} = \frac{3\cos 14}{e^7} - c_1$$

$$c_1 = \frac{3\cos 14 - 4\sin 14}{e^7}$$

$$c_2 = \frac{3}{e^7 \sin 14} - c_1 \frac{\cos 14}{\sin 14}$$

$$c_2 = \frac{3}{e^7 \sin 14} - \left(\frac{3\cos 14 - 4\sin 14}{e^7}\right) \frac{\cos 14}{\sin 14}$$

$$c_2 = \frac{3 - 3\cos^2 14 + 4\sin 14 \cos 14}{e^7 \sin 14}$$

$$c_2 = \frac{3\sin^{\cancel{2}} 14 + 4\cancel{\sin 14} \cos 14}{e^7 \cancel{\sin 14}}$$

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$$c_2 = \frac{3\sin 14 + 4\cos 14}{e^7}$$

$$y = e^{x}(c_1\cos 2x + c_2\sin 2x) - e^{7-x}$$

$$y = e^{x}\left(\left(\frac{3\cos 14 - 4\sin 14}{e^7}\right)\cos 2x + \left(\frac{3\sin 14 + 4\cos 14}{e^7}\right)\sin 2x\right) - e^{7-x}$$

$$y = e^{x-7}(3\cos 14\cos 2x - 4\sin 14\cos 2x + 3\sin 14\sin 2x + 4\cos 14\sin 2x) - e^{7-x}$$

$$y = e^{x-7}(3(\cos 14\sin 2x + \sin 14\sin 2x) + 4(\cos 14\sin 2x - \sin 14\cos 2x)) - e^{7-x}$$

$$y = e^{x-7}(3\cos(2x-14) + 4\sin(2x-14)) - e^{7-x}$$

$$y = -e^{-(x-7)} + 3e^{x-7}\cos(2(x-7)) + 4e^{x-7}\sin(2(x-7))$$