Assignment 3 DE

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Section: BCS-2B

Laplace By Derivation

Ans 1) $f(t) = \begin{cases} e^t & t \leq 2 \\ 3 & t > 2 \end{cases}$

$$\mathcal{L}\{f(t)\} = \int_0^2 e^t . e^{-st} + \int_2^{\infty} 3 . e^{-st}$$

$$\mathcal{L}\{f(t)\} = \int_0^2 e^{-t(s-1)} + 3\int_2^{\infty} e^{-st}$$

$$\mathcal{L}\{f(t)\} = \left[\frac{e^{-2(s-1)} - 1}{-(s-1)}\right] + 3\left[\frac{0 - e^{-2s}}{-s}\right]$$

$$\boxed{\mathcal{L}\{f(t)\} = \frac{1 - e^{-2(s-1)}}{s-1} + \frac{3}{s} . e^{-2s}}$$

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Ans2) $f(t) = 3 + 2t^2$

$$\mathcal{L}\{f(t)\} = \int_0^\infty (3+2t^2).e^{-st}$$

$$\mathcal{L}\{f(t)\} = \int_0^\infty 3e^{-st} + 2t^2 e^{-st}$$

$$\mathcal{L}\{f(t)\} = \int_0^\infty 3e^{-st} + 2\int_0^\infty t^2 e^{-st} \quad \text{-- (A)}$$

Solving A

$V = t^2 \qquad U' = e^{-st}$

$V' = 2t \qquad U = \dfrac{e^{-st}}{-s}$

$$\frac{-t^2e^{-st}}{s} + \frac{2}{s}\int t e^{-st}$$

$V = t \qquad U' = e^{-st}$

$V' = 1 \qquad U = \dfrac{e^{-st}}{-s}$

$$\frac{-te^{-st}}{s} + \frac{1}{s}\int \frac{e^{-st}}{}$$

$$\frac{-te^{-st}}{s} - \frac{e^{-st}}{s^2}$$

Subs :

$$\cancel{\frac{-t^2e^{-st}}{s}} + \frac{2}{s}\left[\cancel{\frac{-te^{-st}}{s}} - \cancel{\frac{e^{-st}}{s^2}}\right]$$

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$$-\frac{t^2e^{-st}}{s} + \frac{2}{s}\left[\frac{-te^{-st}}{s} \quad \frac{-e^{-st}}{s^2}\right]$$

$$\frac{-t^2e^{-st}}{s} - \frac{2te^{-st}}{s^2} - \frac{2e^{-st}}{s^3}$$

$$\left[\frac{3e^{-st}}{-s}\right]_0^\infty + 2\left[\frac{-t^2e^{-st}}{s} \frac{-2te^{-st}}{s^2} - \frac{2e^{-st}}{s^3}\right]_0^\infty$$

$$-\frac{1}{s}\left[3e^{-s(\infty)} - 3e^0\right] + \frac{2}{s}\left[\frac{-t^2}{e^{st}} \quad \frac{-2t}{se^{st}} \quad -\frac{2e^{-st}}{s^2}\right]_0^\infty$$

$$-\frac{1}{s}[-3] - \frac{2}{s}\left[0 - \frac{2}{s^2}\right]$$

$$\boxed{\frac{3}{s} + \frac{4}{s^3}}$$

Ans 3) $f(t) = 5\sin 3t - 17e^{-2t}$

$$\mathcal{L}\{f(t)\} = \int_0^\infty (5\sin 3t - 17e^{-2t}) \cdot e^{-st}$$

$$\mathcal{L}\{f(t)\} = \int_0^\infty 5\sin 3t . e^{-st} - \int_0^\infty 17 e^{-2t-st}$$

$$\mathcal{L}\{f(t)\} = 5\int_0^\infty \sin 3t . e^{-st} - 17\int_0^\infty e^{-t(2+s)}$$

$V = \sin 3t \qquad U' = e^{-st}$

$V' = 3\cos 3t \qquad U = \frac{-e^{-st}}{s}$

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$$\int_0^\infty \sin 3t . e^{-st} = \frac{-e^{-st}\sin 3t}{s} + \frac{3}{s}\int \cos 3t . e^{-st}$$

$$V = \int \cos 3t \qquad U' = e^{-st}$$

$$V' = -3\sin 3t \qquad U = \frac{-e^{-st}}{s}$$

$$\frac{-\cos 3t e^{-st}}{s} - \frac{3}{s}\int \sin 3t . e^{-st}$$

$$\text{let } I = \int_0^\infty \sin 3t . e^{-st}$$

$$I = \frac{-e^{-st}\sin 3t}{s} + \frac{3}{s}\left[\frac{-\cos 3t . e^{-st}}{s} - \frac{3}{s}I\right]$$

$$I = \frac{-e^{-st}\sin 3t}{s} - \frac{3\cos 3t . e^{-st}}{s^2} - \frac{9I}{s^2}$$

$$I + \frac{9I}{s^2} = \frac{-e^{-st}\sin 3t}{s} - \frac{3\cos 3t . e^{-st}}{s^2}$$

$$I\left(1 + \frac{9}{s^2}\right) = \left(\frac{-e^{-st}\sin 3t}{s} - \frac{3\cos 3t . e^{-st}}{s^2}\right) . \left(\frac{s^2}{s^2+9}\right)$$

$$\cancel{I = -e^{-st}}$$

$$\int_0^\infty \sin 3t . e^{-st} = \left[-e^{-st}\sin 3t - \frac{3\cos 3t . e^{-st}}{s}\right]_0^\infty . \left(\frac{s}{s^2+9}\right)$$

$$\int_0^\infty \sin 3t . e^{-st} = 0 + \frac{3}{\cancel{s}}\left(\frac{\cancel{s}}{s^2+9}\right)$$

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(1)

(5)

$$5\left(\frac{3}{s^2+9}\right) - 17\int_0^\infty e^{-t(2+s)}$$

$$\frac{15}{s^2+9} - 17\left[\frac{0-1}{-(2+s)}\right]$$

$$\boxed{\frac{15}{s^2+9} - 17\left[\frac{1}{2+s}\right]}$$

Ans4) $f(t) = te^{4t}$

$$\mathcal{L}\{f(t)\} = \int_0^\infty te^{4t} \cdot e^{-st}$$

$$\mathcal{L}\{f(t)\} = \int_0^\infty t e^{4t-st}$$

$V = t \qquad U' = e^{-t(s-4)}$

$V' = 1 \qquad U = \dfrac{e^{-t(s-4)}}{-(s-4)}$

$$\frac{-te^{-t(s-4)}}{s-4} + \int \frac{e^{-t(s-4)}}{(s-4)}$$

$$\frac{-te^{-t(s-4)}}{s-4} + \frac{1}{s-4}\left[\frac{e^{-t(s-4)}}{4-s}\right]$$

$$\frac{-te^{-t(s-4)}}{s-4} - \frac{e^{-t(s-4)}}{(s-4)^2}$$

$$\mathcal{L}\{f(t)\} = \left[\frac{-te^{-t(s-4)}}{s-4} - \frac{e^{-t(s-4)}}{(s-4)^2}\right]_0^\infty$$

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$$\mathcal{L}\{f(t)\} = \frac{-1}{s-4}\left[te^{-t(s-4)} - \frac{e^{-t(s-4)}}{s-4}\right]_0^\infty$$

$$\mathcal{L}\{f(t)\} = \frac{-1}{s-4}\left[\frac{t}{e^{-t(s-4)}} - \frac{e^{-t(s-4)}}{s-4}\right]_0^\infty$$

$$\mathcal{L}\{f(t)\} = \frac{-1}{s-4}\left[0 - \frac{1}{s-4}\right]$$

$$\boxed{\mathcal{L}\{f(t)\} = \frac{1}{(s-4)^2}}$$

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Laplace Inverse Following

Q1) $\mathcal{L}^{-1}\left\{\frac{1}{s(s^2+2s+5)}\right\}$

$$\frac{1}{} = \frac{A}{s} + \frac{Bs+C}{s^2+2s+5}$$

$$1 = A(s^2+2s+5) + Bs^2+Cs$$

$\boxed{A = 1/5}$ $\quad A+B=0$ $\quad 2A+C=0$

$\boxed{B = -1/5}$ $\quad 2A = -C$

$\boxed{C = -\frac{2}{5}}$

$$\frac{1}{5}\mathcal{L}^{-1}\left\{\frac{1}{s}\right\} - \frac{1}{5}\mathcal{L}^{-1}\left\{\frac{s+2}{s^2+2s+5}\right\}$$

$$\frac{1}{5}(1) - \frac{1}{5}\left[\mathcal{L}^{-1}\left\{\frac{s+2}{(s+1)^2+4}\right\}\right]$$

$$\frac{1}{5} - \frac{1}{5}\mathcal{L}^{-1}\left\{\frac{s}{(s+1)^2+4}\right\} - \frac{1}{5}\mathcal{L}^{-1}\left\{\frac{2}{(s+1)^2+2^2}\right\}$$

$$\frac{1}{5} - \frac{1}{5}\mathcal{L}^{-1}\left\{\frac{s}{s^2+2^2}\right\}_{s\to s+1} - \frac{1}{5}\mathcal{L}^{-1}\left\{\frac{2}{s^2+2^2}\right\}_{s\to s+1}$$

$$\frac{1}{5} - \frac{1}{5}\left\{\cos 2t . e^{-t}\right\} - \frac{1}{5}\left\{\sin 2t . e^{-t}\right\}$$

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$$\frac{1}{5} - \frac{1}{5}\mathcal{L}^{-1}\left\{\frac{s+1+1}{(s+1)^2+2^2}\right\}$$

$$\frac{1}{5} - \frac{1}{5}\left[\mathcal{L}^{-1}\left\{\frac{s+1}{(s+1)^2+2^2}\right\} + \mathcal{L}^{-1}\left\{\frac{1}{(s+1)^2+2^2}\right\}\right]$$

$$\frac{1}{5} - \frac{1}{5}\left[\mathcal{L}^{-1}\left\{\frac{s}{s^2+4}\right\}_{s\to s+1} + \mathcal{L}^{-1}\left\{\frac{1}{(s+1)^2+2^2}\right\}\right]$$

$$\frac{1}{5} - \frac{1}{5}\left[\cos 2t . e^{-t} + \frac{1}{2}\sin 2t . e^{-t}\right]$$

$$\boxed{\frac{1}{5} - \frac{1}{5}\cos 2t . e^{-t} - \frac{1}{10}\sin 2t . e^{-t}}$$

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Ans 2) $\mathcal{L}^{-1}\left\{\frac{7s-1}{(s+1)(s+2)(s-3)}\right\}$

$$\frac{7s-1}{} = \frac{A}{s+1} + \frac{B}{s+2} + \frac{C}{s-3}$$

$$7s-1 = A(s+2)(s-3) + B(s+1)(s-3) + C(s+1)(s+2)$$

let $s=-1$ let $s=-2$ let $s=3$

$-8 = -4A$ $-15 = +5B$ $20 = 20C$

$\boxed{A=2}$ $\boxed{B=-3}$ $\boxed{C=1}$

$$2\mathcal{L}^{-1}\left\{\frac{1}{s+1}\right\} - 3\mathcal{L}^{-1}\left\{\frac{1}{s+2}\right\} + \mathcal{L}^{-1}\left\{\frac{1}{s-3}\right\}$$

$$\boxed{2e^{-t} - 3e^{-2t} + e^{3t}}$$

Ans 3) $\mathcal{L}^{-1}\left\{\frac{s^2+9s+2}{(s-1)^2(s+3)}\right\}$

$$s^2+9s+2 = \frac{A}{s-1} + \frac{B}{(s-1)^2} + \frac{C}{s+3}$$

$\cancel{s^2+9s+2 = A(s-1)+B}$

$$s^2+9s+2 = A(s-1)(s+3) + B(s+3) + C(s-1)^2$$

let $s=1$ let $s=-3$ let $s=0$

$12 = 4B$ $-16 = 16C$ $2 = -3A+9+-4$

$\boxed{B=3}$ $\boxed{C=-1}$ $\cancel{-3 = -3A - 6} = -3A$

$\boxed{\cancel{A=1}}$ $A=2$

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$$2\mathcal{L}^{-1}\left\{\frac{1}{s-1}\right\} + 3\mathcal{L}^{-1}\left\{\frac{1}{(s-1)^2}\right\} - 1\mathcal{L}^{-1}\left\{\frac{1}{s+3}\right\}$$

$$\boxed{\cancel{e^{t} + 3te^{t} - 4e^{-3t}}}$$

$$\cancel{2e^{3t} + 3te^{t} - e^{-3t}}$$

$$\boxed{2e^{t} + 3te^{t} - e^{-3t}}$$

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Ans4) $\mathcal{L}^{-1}\left\{\dfrac{2s^2+10s}{(s^2-2s+5)(s+1)}\right\}$

$$2s^2+10s = \frac{As+B}{(s^2-2s+5)} + \frac{C}{s+1}$$

$$2s(s+5) = (As+B)(s+1) + C(s^2-2s+5)$$

let $s=-1$ let $s=0$ let $s=-5$

$-8 = 8C$ $\boxed{C=-1}$

$0 = B-5$ $\boxed{B=5}$

$0 = +20A-4B+40C$
$-20A = -20-40$
$-A = -3$
$\boxed{A=3}$

$$\mathcal{L}^{-1}\left\{\frac{3s+5}{(s^2-2s+5)} - \frac{1}{(s+1)}\right\}$$

$$\mathcal{L}^{-1}\left\{\frac{3s-3+8}{(s^2-2s+5)} - \frac{1}{(s+1)}\right\}$$

$$3\mathcal{L}^{-1}\left\{\frac{s-1}{(s^2-1)+2^2}\right\} + 4\mathcal{L}^{-1}\left\{\frac{2}{(s^2-1)+2^2}\right\} - \mathcal{L}^{-1}\left\{\frac{1}{s+1}\right\}$$

$$\boxed{3\cos 2t.e^t + 4\sin 2t.e^t - e^{-t}}$$

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Solving D.E Equations By laplace

Ans1) $y' - 5y = e^{5x}$ $\qquad y(0) = 0$

$$sY(s) - y(0) - 5\mathcal{L}(y) = \mathcal{L}(e)^{5x}$$

$$Y(s) = \mathcal{L}\{y(x)\}$$

$$sY(s) - 0 - 5Y(s) = \frac{1}{s-5}$$

$$Y(s)\{s-5\} = \frac{1}{s-5}$$

$$\mathcal{L}\{y(x)\} = \frac{1}{(s-5)^2}$$

$$y(x) = \mathcal{L}^{-1}\left\{\frac{1}{(s-5)^2}\right\}$$

$$y(x) = \mathcal{L}^{-1}\left\{\frac{1}{s^2}\right\}_{s \to s-5}$$

$$y(x) = x e^{5x}$$

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Ans1) $\frac{dy}{dx} - 5y = e^{5x}$ $\quad y(0)=0$

$I.F = \cancel{\int e^{-5}}\ e^{\int -5}$

$I.F = e^{-5x}$

$e^{-5x}.\frac{dy}{dx} - 5e^{-5x}.y = 1$

Applying $\int$ on both sides

$e^{-5x}.y = x + C$

$y = xe^{5x} + Ce^{5x}$

$\cancel{y = C}$ $C = 0$

$\boxed{y = xe^{5x}}$ hence same.

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Ans(2)) $y' + y = \sin x \quad , \quad y(0) = 1$

$$\mathcal{L}\{y'\} + \mathcal{L}\{y\} = \mathcal{L}\{\sin x\}$$

$$sY(s) - y(0) + Y(s) = \frac{1}{s^2+1}$$

$$Y(s)[1+s) - 1 = \frac{1}{s^2+1}$$

$$Y(s) = \frac{1}{(s^2+1)(1+s)} + \frac{1}{(1+s)}$$

$$\frac{1}{(s^2+1)(1+s)} = \frac{As+B}{s^2+1} + \frac{C}{s+1}$$

$$1 = (As+B)(s^2+1) + C(s^2+1)$$

let $s=-1$ | let $s=0$ | let $s=1$

$1 = 2C$ | $1 = B + C$ | $1 = 2A + 1 + 1$

$\boxed{C = 1/2}$ | $\boxed{B = 1/2}$ | $\boxed{\frac{-1}{2} = A}$

$$Y(s) = -\frac{1}{2} \cdot \frac{s}{s^2+1} + \frac{1}{2} \cdot \frac{1}{s^2+1} + \frac{1}{2} \cdot \frac{1}{s+1} + \frac{1}{1} \cdot \frac{1}{s+1}$$

$$y(x) = -\frac{1}{2}\mathcal{L}^{-1}\left\{\frac{s}{s^2+1}\right\} + \frac{1}{2}\mathcal{L}^{-1}\left\{\frac{1}{s^2+1}\right\} + \frac{3}{2}\mathcal{L}^{-1}\left\{\frac{1}{s+1}\right\}$$

$$\boxed{y(x) = -\frac{1}{2}\cos x + \frac{1}{2}\sin x + \frac{3}{2}e^{-x}}$$

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$$\frac{dy}{dx} + y = \sin x$$

$I.F = e^x$

$$e^x \cdot \frac{dy}{dx} + e^x . y = e^x \sin x$$

applying $\int$ on both sides

$$e^x . y = \int e^x \sin x$$

$V = \sin x \quad U' = e^x$

$V' = \cos x \quad U = e^x$

$$\int e^x \sin x = e^x \sin x - \int e^x \cos x$$

$$\int e^x \cos x = e^x \cos x + \int e^x \sin x$$

$$\int e^x \sin x = e^x \sin x - e^x \cos x - \int e^x \sin x$$

$$\int e^x \sin x = \frac{e^x \sin x - e^x \cos x}{2}$$

$$e^x . y = (e^x \sin x - e^x \cos x) / 2$$

$$1 = -\frac{1}{2} + c$$

$$\boxed{C = \frac{3}{2}} \qquad \boxed{y = \frac{\sin x}{2} - \frac{\cos x}{2} + \frac{3}{2} e^{-x}}$$

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Ans 3) $y'' - y' = 2x \qquad y(0) = 1 \;,\; y'(0) = -2$

$$[s^2 Y(s) - sy(0) - y'(0)] - [sY(s) - y(0)] = 2\mathcal{L}\{x\}$$

$$s^2 Y(s) - s + 2 - sY(s) + 1 = \frac{2}{s^2}$$

$$Y(s)[s^2 - s] = \frac{2}{s^2} + s + 3$$

$$Y(s) = \frac{2 + s^3 + 3s^2}{s^3(s-1)}$$

$$2 + s^3 + 3s^2 = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s^3} + \frac{D}{s-1}$$

$$2 + s^3 + 3s^2 = A(s^2)(s-1) + B(s)(s-1) + C(s-1) + Ds^3$$

let $s = 0$

$2 = -C$

$\boxed{C = -2}$

let $s = 1$

$\boxed{6 = D}$

let $s = -1$

$\cancel{4} = -2A + 2B - 2C$

$\cancel{2} = -A + B + 4$

$\boxed{B = 2 + A}$

let $s = 2$

$-2 = 4A + 2B - 2$

$\cancel{-1 = 2A + 2 + A - 1} \qquad \cancel{-2 = 4A + 4 + 2A - 2}$

$\cancel{A = \frac{1}{3}} \qquad B = \qquad -4 = 6A$

$\boxed{\cancel{A = -2/3}}$

$\cancel{A =} \; 0 = 2A + 2 + 2A$

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let $s=-1$

$-2=-2A+2B-2C$

$-1=-A+B-C$

$-1=-A+B+2$

$\boxed{B=A-3}$

let $s=2$

$-2=4A+2B+C$

$-2=4A+2A-6-2$

$6=6A$

$\boxed{A=1}$ $\boxed{B=-2}$

$$y(x)=\mathcal{L}^{-1}\left\{\frac{1}{s}\right\}-2\mathcal{L}^{-1}\left\{\frac{1}{s^2}\right\}-\mathcal{L}^{-1}\left\{\frac{2}{s^3}\right\}$$

$$\boxed{y(x)=1-2x-x^2}$$

Solving by analytical method :-

$y''-y'=2x$

For y_c:-

$D^2-D=0$

$D=0,1$

$y_c=C_1+C_2e^x$

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For yp :-

$yp = Ax + B$

$yp = Ax^2 + Bx$

$yp' = 2Ax + B$

$yp'' = 2A$

$2A - 2Ax - B = 2x$

$-2A = 2$

$\boxed{A = -1}$ $\quad -B + 2A = 0$

$-B = +2$

$\boxed{B = -2}$

$\boxed{yp = -x^2 - 2x}$

$y = C_1 + C_2 e^x - x^2 - 2x$

$1 = C_1 + C_2$

$\boxed{C_1 = 1 - C_2}$

$y' = C_2 e^x - 2x - 2$

$-2 = C_2 - 2$

$\boxed{C_2 = 0}$ $\quad \boxed{C_1 = 1}$

$\boxed{y = 1 - x^2 - 2x}$

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Q.4) $y'' - 2y' + 5y = -8^{-(x-7)}$

$\cancel{y(0)}$ let $t = x-7$

$y(0) = 2 \quad y'(0) = 12$

$s^2Y(s) - sy(0) - y'(0) - 2[sY(s) - y(0)] + 5Y(s) = -8e^{-t}$

$s^2Y(s) - 2s - 12 - 2sY(s) + 4 + 5Y(s) = -\frac{8}{s+1}$

$Y(s)[s^2 - 2s + 5] - 2s - 8 = \frac{-8}{s+1}$

$Y(s)[s^2 - 2s + 5] = \frac{-8}{s+1} + (2s+8)(s+1)$

$Y(s)[s^2 - 2s + 5] = \frac{-8 + 2s^2 + 2s + 8s + 8}{(s+1)}$

$Y(s) = \frac{2s^2 + 10s}{(s^2 - 2s + 5)(s+1)}$

$2s^2 + 10s = \frac{As + B}{(s^2 - 2s + 5)} + \frac{C}{s+1}$

$2s^2 + 10s = (As+B)(s+1) + C(s^2 - 2s + 5)$

$2s^2 + 10s = As^2 + As + Bs + B + Cs^2 - 2Cs + 5C$

$2 = A + C \qquad 10 = A + B - 2C \qquad B = -5C$

$\boxed{A = 2 - C} \qquad 10 = 2 + B - 3C \qquad \boxed{B = 5}$

$\boxed{A = 3} \qquad 10 = 2 - 8C \qquad \boxed{C = -1}$

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$$\frac{3s+5}{s^2-2s+5} - \frac{1}{s+1}$$

$$\frac{3\cancel{s}+\cancel{3}}{(\cancel{s^2-1})^2+4} \quad \frac{3s+5+3-3}{(s^2-2s+5)} - \frac{1}{s+1}$$

$$y(t) = \frac{3s-3}{(s^2-1)+2^2} + \frac{8}{(s^2-1)+2^2} - \frac{1}{s+1}$$

$$y(t) = 3\mathcal{L}^{-1}\left\{\frac{s-1}{(s-1)^2+2^2}\right\} + 4\mathcal{L}^{-1}\left\{\frac{2}{(s^2-1)+2^2}\right\} - e^{-t}$$

$$y\cancel{(t) = 3\sin 2t + 3\cos 2t + 4\sin 2t}$$

$$y(t) = 3\cos 2t \cdot e^{t} + 4\sin 2t \cdot e^{t} - e^{-t}$$

since ~~ke~~ $t = x-7$

$$\boxed{y(t) = 3\cos 2(x-7) \cdot e^{x-7} + 4\sin 2(x-7) \cdot e^{x-7} - e^{-(x-7)}}$$

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