Assignment # 1
Differential Equations (MT-224)
Date of Submission: 10th March, 2021
Total marks: 6 (CLO-2)

Q1: Evaluate the 1st order differential equations [marks: 16; weightage: 4]

i. $\frac{dy}{dx} = \frac{x}{y}$ **[Answer:** $\frac{y^2}{2} = \frac{x^2}{2} + C_1$**]**

ii. $x\frac{dy}{dx} + y = x^2y^2$ **[Answer:** $y = \frac{1}{x(-x+C_1)}$ **]**

iii. $(x^2+y^2)dx + xydy = 0$ **[Answer:** $y^2 = -\frac{x^2}{2} + \frac{C_1}{x^2}$**]**

iv. $(x - y^2)dx + 2xydy = 0$ **[Answer:** $y^2 = -x^2 + C_1x$**]**

v. $e^y\left(\frac{dy}{dx} - 1\right) = e^x$ **[Answer:** $y = \ln(x + C_1) + x$**]**

vi. $\sin y\frac{dy}{dx} = \cos x\,(2\cos y - \sin x)$ **[Answer:** $y = -\cos^{-1}\left(-\frac{C_1 - 2e^{2\sin x}\sin x + e^{2\sin x}}{4e^{2\sin x}}\right)$**]**

vii. $x(3x + 2y^2)dx + 2y(1 + x^2)dy = 0$ **[Answer:** $y^2 = -\frac{x^3}{1+x^2} + \frac{C_1}{1+x^2}$**]**

viii. $e^{-y}\,sec^2\,y\,dy = dx + xdy$ **[Answer:** $-xe^y + \tan y + C_1$**]**

ix. $(x^2 + y^2)dx + (x^2 - xy)dy = 0$ **[Answer:** $-ln\left(\frac{y^2}{x} + 2y + x\right) + \frac{y}{x} - 1 = C_1$**]**

x. $y - x\frac{dy}{dx} = a\left(y^2 + \frac{dy}{dx}\right)$ **[Answer:** $y = \frac{x}{a(x+a+e^{C_1}} + \frac{1}{x+a+e^{C_1}}$**]**

xi. $(x + 1)\frac{dy}{dx} + 1 = 2e^{-y}$ **[Answer:** $-y - \ln\big((2e^{-y} - 1)(x + 1)\big) = C_1$**]**

xii. $x^2\frac{dy}{dx} + y(x + y) = 0$ **[Answer:** $y = \frac{2x}{-1+2C_1x^2}$**]**

xiii. $(\sec x\tan x\tan y - e^x)dx + \sec x\ sec^2 ydy = 0$ **[Answer:** $y = \tan^{-1}\left(\frac{C_1+e^x}{\sec x}\right)$**]**

xiv. $x\cos x\frac{dy}{dx} + y(x\sin x + \cos x) = 1$ **[Answer:** $\frac{\tan x}{xsec(x)} + \frac{C_1}{xsec(x)}$**]**

xv. $x\ln x\frac{dy}{dx} + y = 2\ln x$ **[Answer:** $y = \ln(x) + \frac{C_1}{\ln(x)}$**]**

xvi. $y' + \frac{4}{x}y = x^3y^2$ **[Answer:** $y = \frac{1}{x^4(-\ln(x)+C_1)}$**]**

Q2: Mathematical Modelling of 1st order Differential Equations [marks: 4; weightage: 2]

i. The population of a community is known to increase at a rate proportional to the number of people present at the time t. If an initial population P_0 has doubled in 5 years, how long will it take to triple? To quadruple? **[Answer: 7.9 years; 10 years]**

ii. The radioactive isotope of lead, Pb-209, decays at a rate proportional to the amount present at the time t and has a half-life of 3.3 hours. If 1 gram of this isotope is present initially, how long will it take for 90% of the lead to decay? **[Answer: 11h]**

iii. A thermometer reading 70°F is placed in an oven preheated to a constant temperature. Through a glass window in the oven door, an observer records that the thermometer read 110°F after $\frac{1}{2}$ minute and 145°F after 1 minute. How hot is the oven? **[Answer: $\mathbf{390}$°F]**

iv. A 30-volt electromotive force is applied to an LR-series circuit in which the inductance is 0.1 henry and the resistance is 50 ohms. Find the current $i(t)$ if $i(0) = 0$. Determine the current as $t \to \infty$. **[Answer: $\boldsymbol{i(t) = \frac{3}{5} - \frac{3}{5}e^{-500t}}$, $\boldsymbol{i \to \frac{3}{5}}$ as $\boldsymbol{t \to \infty}$]**