Amin Farjad
2016-0344

Assignment

Date

Q.1

$$\frac{dy}{dx} = \frac{x}{y}$$

$$y\,dy = x\,dx$$

$$\frac{y^2}{2} = \frac{x^2}{2} + c \quad \text{or,} \quad y^2 = \frac{2x^2}{2} + 2c \qquad 2c = c_1$$

$$y^2 = x^2 + c_1$$

Q.2

$$x\frac{dy}{dx} + y = x^2y^2$$

divide by x

$$\frac{dy}{dx} + \frac{y}{x} = xy^2$$

$$n = 2$$

$$v = y^{1-n}$$

$$v = y^{1-2}$$

$$v = y^{-1}$$

$$\frac{dv}{dx} = -1\,y^{-2}\frac{dy}{dx}$$

$$\frac{dy}{dx} = -\frac{dv}{dx}y^2$$

$$-\frac{dv}{dx}y^2 + \frac{y}{x} = xy^2$$

divide by $-y^2$

$$\frac{dv}{dx} - \frac{y}{xy^2} = \frac{-x\cancel{y^2}}{\cancel{y^2}}$$

$$\frac{dv}{dx} - \frac{1}{x}\left(\frac{1}{y}\right) = -x \qquad \because v = \frac{1}{y}$$

$$\frac{dv}{dx} - \frac{1}{x}v = -x$$

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$$I.F = e^{\int -\frac{1}{x} dx}$$

$$I.F = e^{-\ln(x)}$$

$$I.F = \frac{1}{x}$$

Multiply equation by $\frac{1}{x}$

$$\frac{1}{x}\frac{dv}{dx} - \frac{1}{x^2} v = -1$$

$$\frac{d}{dx}\left(\frac{1}{x} v\right) = -1$$

$$\int d\left(\frac{1}{x} v\right) = -1\int dx$$

$$\frac{v}{x} = -x + c$$

$$v = x(-x+c) \qquad v = 1/y$$

$$\frac{1}{y} = x(-x+c)$$

OR

$$y = \frac{1}{x(-x+c)}$$

Q.3

$$(x^2+y^2)\,dx + xy\,dy = 0$$

$$M = x^2 + y^2 \qquad N = xy$$

$$\frac{\partial M}{\partial y} = 2y \qquad \frac{\partial N}{\partial x} = y$$

$$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$$

Non-exact equation

$$U(x) = \frac{M_y - N_x}{N}$$

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$$u(x) = \frac{2y - y}{xy}$$

$$I.F = e^{\int 1/x \, dx}$$

$$u(x) = \frac{y}{xy}$$

$$I.F = e^{\ln(x)}$$

$$I.F = x$$

$$u(x) = \frac{1}{x}$$

$$(x^2 + y^2)dx + xy\,dy = 0$$

multiply eq by x

$$(x^3 + xy^2)dx + x^2y\,dy = 0$$

$M = x^3 + xy^2$ $\qquad$ $N = x^2y$

$\frac{\partial M}{\partial y} = 2xy$ $\qquad$ $\frac{\partial N}{\partial x} = 2xy$

equation is exact.

$= \int M\,dx$ $\qquad$ $= \int N\,dy$

$= \int (x^3 + xy^2)\,dx$ $\qquad$ $= \int x^2y\,dy$

$= \frac{x^4}{4} + \frac{x^2y^2}{2}$ $\qquad$ $= \frac{x^2y^2}{2}$

Solve equation is $\frac{x^4}{4} + \frac{x^2y^2}{2} = c$

$$x^4 + 2x^2y^2 = 4c$$

$$2x^2y^2 = -x^4 + c_1 \qquad 4c = c_1$$

$$y^2 = \frac{-x^2}{2} + \frac{c_1}{2x^2}$$

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Q. iv

$$(x-y^2)\,dx + 2xy\,dy = 0$$

$M = x - y^2$ $\qquad N = 2xy$

$\frac{\partial M}{\partial y} = -2y$ $\qquad \frac{\partial N}{\partial x} = 2y$

$$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$$

Non-exact

$$U(x) = \frac{M_y - N_x}{N}$$

$U(x) = \frac{-2y - 2y}{2xy}$ $\qquad I.F = e^{\int U(x)\,dx}$

$U(x) = \frac{-\cancel{4}^{2}\cancel{y}}{\cancel{2}x\cancel{y}}$ $\qquad I.F = e^{\int -2/x\,dx}$

$\qquad I.F = e^{\ln(x^{-2})}$

$U(x) = \frac{-2}{x}$ $\qquad I.F = \frac{1}{x^2}$

Multiply whole eq by $\frac{1}{x^2}$

$$\left(\frac{1}{x} - \frac{y^2}{x^2}\right)dx + \frac{2y}{x}\,dy = 0$$

$M = \frac{1}{x} - \frac{y^2}{x^2}$ $\qquad N = \frac{2y}{x}$

$\frac{\partial M}{\partial y} = \frac{-2y}{x^2}$ $\qquad \frac{\partial N}{\partial x} = \frac{-2y}{x^2}$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

$= \int M\,dx$ $\qquad = \int N\,dy$

$\qquad = \int \frac{2y}{x}\,dy$

$= \int\left(\frac{1}{x} - \frac{y^2}{x^2}\right)dx$

$= \ln(x) + \frac{y^2}{x}$ $\qquad = \frac{y^2}{x}$

$$\ln(x) + \frac{y^2}{x} = C$$

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Q.v

$$e^y\left(\frac{dy}{dx}-1\right)=e^x$$

$$\frac{dy}{dx}-1=\frac{e^x}{e^y}$$

$$\cancel{\frac{dy}{dx}-\frac{e^x}{e^y}=1}$$

$$e^y\frac{dy}{dx}-e^y=e^x$$

$$\frac{e^y}{e^x}\frac{dy}{dx}-\frac{e^y}{e^x}=1$$

$$\underset{u}{\frac{1}{e^x}}\;\underset{v'}{e^y\frac{dy}{dx}}-\underset{u'}{\frac{1}{e^x}}\;\underset{v}{e^y}=1$$

$$\frac{d}{dx}\left(\frac{e^y}{e^x}\right)=1$$

$$\int d\left(\frac{e^y}{e^x}\right)=\int dx$$

$$\frac{e^y}{e^x}=x+c$$

$$e^y=e^x x+e^x c$$

$$e^y=e^x(x+c)$$

taking ln

$$y=\ln(e^x\cdot(x+c))$$

$$y=x+\ln(x+c)$$

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Q.vi

$$\sin y \frac{dy}{dx} = \cos x\,(2\cos y - \sin x)$$

$$\sin y\, dy = \cos x\,(2\cos y - \sin x)\, dx$$

$$(2\cos x\cos y - \cos x\sin x)dx - \sin y\, dy = 0$$

$M = 2\cos x\cos y - \cos x\sin x$ $\qquad N = -\sin y$

$\frac{\partial M}{\partial y} = -2\cos x\sin y$ $\qquad \frac{\partial N}{\partial x} = 0$

$$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x} \qquad I.F = e^{\int 2\cos x\, dx}$$

$$U(x) = \frac{M_y - N_x}{N} \qquad I.F = e^{2\sin x}$$

$$U(x) = \frac{-2\cos x \sin y}{-\sin y}$$

$$U(x) = 2\cos x$$

Multiply whole eq by $e^{2\sin x}$

$$(2\cos x\cos y - \cos x\sin x)dx - \sin y\, dy = 0$$

$$(e^{2\sin x}2\cos x\cos y - e^{2\sin x}\cos x\sin x)dx - e^{2\sin x}\sin y\, dy = 0$$

$M = e^{2\sin x}2\cos x\cos y - e^{2\sin x}\cos x\sin x$ $\qquad N = -e^{2\sin x}\sin y$

$\frac{\partial M}{\partial y} = -\sin y\, e^{2\sin x}2\cos x$ $\qquad \frac{\partial N}{\partial x} = -e^{2\sin x}2\cos x\sin y$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

$$= \int M\, dx$$

$$= \int (e^{2\sin x}2\cos x\cos y - e^{2\sin x}\cos x\sin x)dx$$

Let $u = \sin x$

$$\frac{du}{dx} = \cos x$$

$$du = \cos x\, dx$$

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$$= \int 2e^{2\sin x} \cos y \cos x \, dx - \int e^{2\sin x} \sin x \cos x \, dx$$

$$= 2\int e^{2u} \cos y \, du - \int e^{2u} u \, du$$

$$= 2\cos y \int e^{2u} du - \left[\frac{ue^{2u}}{2} - \frac{e^{2u}}{4} \right]$$

$$= \frac{\cancel{2}\cos y \, e^{2u}}{\cancel{2}} - \left[\frac{ue^{2u}}{2} - \frac{e^{2u}}{4} \right]$$

$u = \sin x$

$$= \cos y \, e^{2\sin x} - \frac{\sin x \, e^{2\sin x}}{2} + \frac{e^{2\sin x}}{4}$$

$$= \int N \, dy$$

$$= \int -e^{2\sin x} \sin y \, dy$$

$$= -e^{2\sin x} \int \sin y \, dy$$

$$= -e^{2\sin x} (-\cos y)$$

$$= e^{2\sin x} \cos y$$

$$\cos y \, e^{2\sin x} - \frac{\sin x \, e^{2\sin x}}{2} + \frac{e^{2\sin x}}{4} = c$$

$$\cos y \, e^{2\sin x} = c + \frac{\sin x \, e^{2\sin x}}{2} - \frac{e^{2\sin x}}{4}$$

$$\cos y \, e^{2\sin x} = \frac{4c + 2\sin x \, e^{2\sin x} - e^{2\sin x}}{4} \qquad \therefore 4c = c_1$$

$$y = \cos^{-1}\left(- \frac{c_1 - 2\sin x \, e^{2\sin x} + e^{2\sin x}}{4e^{2\sin x}} \right)$$

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Date

Q. vii

$$x(3x+2y^2)dx + 2y(1+x^2)dy = 0$$
$$(3x^2+2y^2x)dx + (2y+2yx^2)dy = 0$$

$M = 3x^2+2y^2x$ $\quad$ $N = 2y+2yx^2$

$\frac{\partial M}{\partial y} = 4yx$ $\quad$ $\frac{\partial N}{\partial x} = 4xy$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

eq is exact.

$= \int M\,dx$ $\quad$ $= \int N\,dy$

$= \int (3x^2+2y^2x)dx$ $\quad$ $= \int (2y+2yx^2)dy$

$= x^3+y^2x^2$ $\quad$ $= y^2+x^2y^2$

$$x^3+y^2+x^2y^2 = c$$
$$y^2(1+x^2) = -x^3+c$$
$$y^2 = \frac{-x^3}{1+x^2} + \frac{c}{1+x^2}$$

Q. viii

$$e^{-y}\sec^2 y\,dy = dx + x\,dy$$
$$dx + (x - e^{-y}\sec^2 y)dy = 0$$

$M = 1$ $\quad$ $N = x - e^{-y}\sec^2 y$

$\frac{\partial M}{\partial y} = 0$ $\quad$ $\frac{\partial N}{\partial x} = 1$

$$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$$

$U(y) = \frac{N_x - M_y}{M}$ $\quad$ $I.F = e^{\int U(y)dy}$

$U(y) = \frac{1-0}{1}$ $\quad$ $I.F = e^{y}$

$U(y) = 1$

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2012-0344

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Date

$$dx + (x - e^{-y}\sec^2 y)\,dy = 0$$

Multiply eq by e^y

$$e^y dx + (e^y x - \sec^2 y)\,dy = 0$$

$M = e^y$

$N = e^y x - \sec^2 y$

$\frac{\partial M}{\partial y} = e^y$

$\frac{\partial N}{\partial x} = e^y$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

$= \int M\,dx$

$= \int e^y\,dx$

$= xe^y$

$= \int N\,dy$

$= \int e^y x - \sec^2 y\,dy$

$= e^y x - \tan y$

$$xe^y - \tan y = C_1$$

$$xe^y - \tan y - C_1 = 0$$

OR,

$$-xe^y + \tan y + C_1 = 0$$

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Q.ix

$$(x^2+y^2)dx + (x^2-xy)dy = 0$$

$$(x^2-xy)dy = -(x^2+y^2)dx$$

$$\frac{dy}{dx} = \frac{\cancel{-}(x^2+y^2)}{\cancel{-}(xy-x^2)}$$

$$y = vx$$

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$v + x\frac{dv}{dx} = \frac{x^2+x^2v^2}{x(vx)-x^2}$$

$$x\frac{dv}{dx} = \frac{\cancel{x^2}(1+v^2)}{\cancel{x^2}(v-1)} - v$$

$$x\frac{dv}{dx} = \frac{1+v^2-v(v-1)}{v-1}$$

$$x\frac{dv}{dx} = \frac{1+\cancel{v^2}-\cancel{v^2}+v}{v-1}$$

$$\frac{v-1}{v+1}\,dv = \frac{dx}{x}$$

$$\frac{v+1-2}{v+1}\,dv = \frac{dx}{x}$$

$$\left(\frac{\cancel{v+1}}{\cancel{v+1}} - \frac{2}{v+1}\right)dv = \frac{dx}{x}$$

$$\int dv - 2\int\frac{1}{v+1}\,dv = \int\frac{dx}{x}$$

$\because v = \frac{y}{x}$

$$v - 2\ln(v+1) = \ln(x) + c$$

$$\frac{y}{x} = \ln x + \ln(v+1)^2 + c$$

$$\frac{y}{x} = \ln\left(x\cdot\left(\frac{y}{x}+1\right)^2\right) + c$$

$$\frac{y}{x} = \ln\left(\cancel{x}\cdot\frac{(y+x)^2}{x\cancel{x}}\right) + c$$

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$$\frac{y}{x} = \ln\left(\frac{(x+y)^2}{x}\right) + c$$

$$\frac{y}{x} = \ln\left(x + 2y + \frac{y^2}{x}\right) + c$$

Q. x

$$y - x\frac{dy}{dx} = a\left(y^2 + \frac{dy}{dx}\right)$$

$$y - x\frac{dy}{dx} = ay^2 + a\frac{dy}{dx}$$

$$-x\frac{dy}{dx} - a\frac{dy}{dx} = ay^2 - y$$

$$\frac{dy}{dx}(-x-a) = y(ay-1)$$

$$\frac{dy}{y(ay-1)} = \frac{dx}{(-x-a)}$$

$$1 = \frac{A}{y} + \frac{B}{ay-1}$$

$$1 = A(ay-1) + By$$

$y=0$

$$1 = A(-1)$$

$$A = -1$$

$$1 = A(ay-1) + By$$

$ay - 1 = 0$

$y = 1/a$

$$1 = B(1/a)$$

$$a = B$$

$$\left[\frac{-1}{y} + \frac{a}{ay-1}\right]dy = \frac{dx}{-(x+a)}$$

$$-\ln(y) + \ln(ay-1) = -\ln(x+a) + c$$

$$\ln\left(\frac{ay-1}{y}\right) + \ln(x+a) = c$$

$$\ln\left(\frac{(ay-1)(x+a)}{y}\right) = c$$

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$$\left(\frac{ay-1}{y}\right)(x+a) = e^{c}$$

$$\frac{ay}{y} - \frac{1}{y} = \frac{e^c}{x+a}$$

$$a - \frac{1}{y} = \frac{e^c}{x+a}$$

$$a - \frac{e^c}{x+a} = \frac{1}{y}$$

$$1/y = \frac{a(x+a) - e^c}{x+a}$$

$$y = \frac{x+a}{a(x+a) - e^c}$$

or

$$y = \frac{x}{a(x+a) - e^c} + \frac{a}{a(x+a) - e^c}$$

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Q.xi

$$(x+1)\frac{dy}{dx} + 1 = 2e^{-y}$$

$$\frac{(x+1)}{e^{-y}}\frac{dy}{dx} + \frac{1}{e^{-y}} = 2$$

~~divide~~

$$(x+1)\, e^{y} \frac{dy}{dx} + (1)e^{y} = 2$$

$$\frac{d}{dx}\left((x+1)e^{y}\right) = 2$$

$$\int d\,(x+1)e^{y} = \int 2\,dx$$

$$(x+1)\,e^{y} = 2x$$

$$e^{y} = \frac{2x}{x+1} + \frac{c}{x+1}$$

$$y = \ln\left(\frac{2x}{1+x} + \frac{c}{x+1}\right)$$

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Date

OR//

$$(x+1)\frac{dy}{dx} + 1 = 2e^{-y}$$

$$(x+1)\frac{dy}{dx} = 2e^{-y} - 1$$

$$(x+1)\frac{dy}{dx} = \frac{2}{e^y} - 1$$

$$(x+1)\frac{dy}{dx} = \frac{2 - e^y}{e^y}$$

$$\frac{e^y}{2-e^y}\,dy = \frac{dx}{x+1}$$

$$-\ln(2-e^y) = \ln(x+1) + c$$

$$-\ln(2-e^y) - \ln(x+1) = c$$

$$-\ln((2-e^y)(x+1)) = c$$

Q xii

$$x^2\frac{dy}{dx} + y(x+y) = 0$$

$$x^2\frac{dy}{dx} + xy + y^2 = 0$$

$$x^2\frac{dy}{dx} + xy = -y^2$$

divide eq by x^2

$$\frac{dy}{dx} + \frac{1}{x}y = \frac{-y^2}{x^2}$$

Bernaulli $n = 2$

$$v = y^{1-n}$$

$$v = y^{-1}$$

$$\frac{dv}{dx} = \frac{-1}{y^2}\frac{dy}{dx}$$

$$\frac{dy}{dx} = -\frac{dv}{dx}y^2$$

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Date

$$-\frac{dv}{dx}y^2 + \frac{y}{x} = -\frac{y^2}{x^2}$$

divide eq by $-y^2$

$$\frac{dv}{dx} - \frac{1}{x}\left(\frac{1}{y}\right) = \frac{1}{x^2}$$

$$\frac{dv}{dx} - \frac{1}{x}v = \frac{1}{x^2}$$

$$I.F = e^{\int -1/x\,dx}$$

$$= e^{-\ln(x)}$$

$$= \frac{1}{x}$$

Multiply eq by $\frac{1}{x}$

$$\frac{1}{x}\frac{dv}{dx} - \frac{1}{x^2}v = \frac{1}{x^3}$$

$$\frac{d}{dx}\left(\frac{v}{x}\right) = \frac{1}{x^3}$$

$$\int d\left(\frac{v}{x}\right) = \int \frac{dx}{x^3}$$

$$\frac{v}{x} = \frac{x^{-2}}{-2} + C$$

$$v = \frac{x}{-2x^2} + Cx$$

$$\frac{1}{y} = \frac{1}{-2x} + Cx$$

$$\frac{1}{y} = \frac{-1 + 2Cx^2}{2x}$$

$$y = \frac{2x}{-1 + 2Cx^2}$$

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Q. xiii

$(\sec x \tan x \tan y - e^x)dx + \sec x \sec^2 y\, dy = 0$

$M = \sec x \tan x \tan y - e^y$ $\qquad N = \sec x \sec^2 y$

$\frac{\partial M}{\partial y} = \sec x \tan x \sec^2 y$ $\qquad \frac{\partial N}{\partial x} = \sec x \tan x \sec^2 y$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

exact equation.

$= \int M\, dx$ $\qquad = \int N\, dy$

$= \int (\sec x \tan x \tan y - e^x) dx$ $\qquad = \int \sec x \sec^2 y\, dy$

$= \sec x \tan y - e^x$ $\qquad = \sec x \tan y$

$\sec x \tan y - e^x = c$

$\sec x \tan y = c + e^x$

$$y = \tan^{-1}\left(\frac{c+e^x}{\sec x}\right)$$

Q. iv

$x \cos x \frac{dy}{dx} + y(x \sin x + \cos x) = 1$

Divide by $x \cos x$

$\frac{dy}{dx} + y\left(\frac{x \sin x}{x \cos x} + \frac{\cos x}{x \cos x}\right) = \frac{1}{x \cos x}$

$\frac{dy}{dx} + \left(\tan x + \frac{1}{x}\right) y = \frac{1}{x \cos x}$

$I.F = e^{\int \tan x + 1/x\, dx}$

$I.F = e^{-\ln \cos x + \ln(x)}$

$= e^{\ln\left(\frac{x}{\cos x}\right)}$

$= \frac{x}{\cos x}$

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Date

Multiply by $\frac{x}{\cos x}$

$$\frac{x}{\cos x}\frac{dy}{dx} + y\left(\frac{\tan x}{\cos x} + \frac{1}{x}\frac{x}{\cos x}\right) = \frac{1}{\cancel{x}\cos x} \times \frac{\cancel{x}}{\cos x}$$

$$x \sec x \frac{dy}{dx} + y(x \tan x \sec x + \sec x) = \frac{1}{\cos^2 x}$$

$$\frac{d}{dx}(x \sec x\, y) = \sec^2 x$$

$$\int d(x \sec x\, y) = \int \sec^2 x\, dx$$

$$x \sec x\, y = \tan x + C$$

$$y = \frac{\tan x}{x \sec x} + \frac{C}{x \sec x}$$

Q. XV

$$x \ln x \frac{dy}{dx} + y = 2 \ln x$$

Divide by $x \ln x$

$$\frac{dy}{dx} + \frac{y}{x \ln x} = \frac{2}{x}$$

$$I.F = e^{\int \frac{1}{x \ln x} dx}$$

$$\ln x = u$$

$$\frac{du}{dx} = \frac{1}{x}$$

$$du = \frac{dx}{x}$$

$$I.F = e^{\int \frac{1}{u} du}$$

$$I.F = e^{\ln(u)}$$

$$I.F = u$$

$$I.F = \ln x$$

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(2012-0344)

Date

Multiply eq by $\ln x$

$$\ln x \frac{dy}{dx} + \frac{y}{x \cancel{\ln x}} \cancel{\ln x} = \frac{2}{x} \ln x$$

$$\frac{d}{dx}(\ln x \cdot y) = \frac{2}{x} \ln x.$$

$$\int d(\ln x \cdot y) = \int \frac{2}{x} \ln x \, dx$$

$$\ln x \cdot y = 2 \int \frac{\ln x}{x} dx$$

$$\ln x = u$$

$$\frac{du}{dx} = \frac{1}{x}$$

$$du = \frac{dx}{x}$$

$$\ln x \, y = 2 \int u \, du$$

$$\ln x \, y = \frac{2u^2}{2} + c$$

$$\ln x \, y = (\ln x)^2 + c$$

$$y = \frac{(\ln x)^2}{\ln(x)} + \frac{c}{\ln(x)}$$

$$y = \ln(x) + \frac{c}{\ln(x)}$$

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Q. xvi

$$y' + \frac{4}{x} y = x^3 y^2$$

$$n = 2$$

$$v = y^{1-n}$$

$$v = y^{1-2}$$

$$v = y^{-1}$$

$$\frac{dv}{dx} = \frac{-1}{y^2} \frac{dy}{dx}$$

$$\frac{dy}{dx} = -y^2 \frac{dv}{dx}$$

$$-y^2 \frac{dv}{dx} + \frac{4}{x} y = x^3 y^2$$

divide eq by $-y^2$

$$\frac{dv}{dx} \mp \frac{4}{x} \frac{1}{y} = -x^3$$

$$\frac{dv}{dx} - \frac{4}{x} v = -x^3$$

$$I.F = e^{\int 4/x \, dx}$$

$$I.F = e^{-4 \ln(x)}$$

$$I.F = \frac{1}{x^4}$$

Multiply eq by I.F

$$\frac{1}{x^4} \frac{dv}{dx} - \frac{4}{x^5} \cancel{y} = \frac{-1}{x}$$

$$\frac{d}{dx} y\left(\frac{v}{x^4}\right) = \frac{-1}{x}$$

$$\int d \frac{v}{x^4} = -\int \frac{1}{x} dx$$

$$\frac{v}{x^4} = -\ln(x) + C$$

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$$\frac{x^4}{v} = \frac{1}{-\ln(x)+C}$$

$$y = \frac{1}{x^4(-\ln(x)+C)}$$

Q.2 (i)

$$\frac{dp}{dt} \propto P(t)$$

$p(0)=P_0$

$p(5) = 2P_0$

$P(?) = 3P_0$

$P(?) = 4P_0$

$$\frac{dp}{dt} = kP$$

$$\frac{dp}{P} = k\,dt$$

integral both sides

$\ln(P) = kt + C$ — antiln both sides

$p = e^{kt+C}$

$p = e^{kt} \cdot e^{c}$ — $e^c = C_1$

$p = e^{kt} \cdot C_1$ — eqi

$p(0) = P_0$

$P_0 = e^0 C_1$

$C_1 = P_0$

$P = e^{kt} C_1$

$C_1 = P_0$ $\quad P(5) = 2P_0$

$2\cancel{P_0} = e^{5k}\cancel{P_0}$

ln both sides

$\ln(2) = 5k$

$$k = \frac{\ln(2)}{5}$$

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for population to triple

$P = e^{kt} \cdot C_1$

$C_1 = P_0$ $\quad P(t) = 3P_0$ $\quad k = \frac{\ln(2)}{5}$

$3\cancel{P_0} = e^{kt} \cancel{P_0}$

taking ln both sides

$\ln(3) = kt$

$\ln(3) = \frac{\ln(2)}{5} t$

$\frac{5 \ln(3)}{\ln(2)} = t$

$t = 7.92$ years for population to triple.

$P = e^{kt} \cdot C_1$

$C_1 = P_0$ $\quad P(t) = 4P_0$ $\quad k = \frac{\ln(2)}{5}$

$4\cancel{P_0} = e^{kt} \cancel{P_0}$

taking ln both sides

$\ln(4) = kt$

$\ln(4) = \frac{\ln(2)}{5} t$

$\frac{5 \ln(4)}{\ln(2)} = t$

$t = 10$ years for population to quadraple.

Amin Farjad
(20K-0344)

Date

Q. 2(ii)

let A be the amount of lead

$$\frac{dA}{dt} \propto A(t)$$

$A(0) = A_0$

$$\frac{dA}{dt} = kA$$

$A(3.3) = 0.5 A_0$

or

$$\frac{dA}{A} = k\,dt$$

$A(0) = 1$

$A(3.3) = 0.5$

$A(?) = 0.1$

$$\ln(A) = kt + c$$

$$A = e^{kt+c}$$

$$A = e^{kt} \cdot e^{c} \qquad \therefore e^{c} = c_1$$

$$A = e^{kt} \cdot c_1$$

$$A(0) = 1$$

$$1 = e^{0} c_1$$

$$\therefore c = 1$$

$$A = e^{kt} \cdot c_1$$

$c_1 = 1 \qquad A(3.3) = 0.5$

$$0.5 = e^{3.3k} \cdot (1)$$

ln both sides

$$\ln(0.5) = 3.3k$$

$$k = \frac{\ln(0.5)}{3.3}$$

$$A = e^{kt} \cdot c_1$$

$c_1 = 1 \qquad A(t) = 0.1 \qquad k = \frac{\ln(0.5)}{3.3}$

$$0.1 = e^{kt}(1)$$

ln bothsides

$$\ln(0.1) = kt$$

$$\ln(0.1) = \frac{\ln(0.5)}{3.3} t$$

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(20k-0344)

$$\frac{3.3 \ln(0.1)}{\ln(0.5)} = t$$

$t = 10.96$

or

$t \approx 11$ hours to 90% lead to decay.

Q.2 (iii)

Using Newton's law of cooling

$$\frac{dT}{dt} = k(T - T_m)$$

$T(0) = 70°$
$T(1/2) = 110°$
$T(1) = 145°$
$T_m = ?$

$$\frac{dT}{T - T_m} = k\,dt$$

integral both sides

$\ln(T - T_m) = kt + c$

$T - T_m = e^{kt} \cdot C_1 \qquad = e^c = C_1$

put $T(0) = 70°$

$70 - T_m = C_1 \quad \text{--- (i)}$

$T - T_m = e^{kt} \cdot C_1$

$70 - T_m = C_1 \qquad T(1/2) = 110°$

$110 - T_m = e^{\frac{1}{2}k} \cdot (70 - T_m)$

$$e^{\frac{1}{2}k} = \frac{110 - T_m}{70 - T_m}$$

So O.B.S

$$e^k = \left(\frac{110 - T_m}{70 - T_m}\right)^2$$

Amin Farjad
(20k-0344)

Date

$T - T_m = e^{kt} \cdot C_1$

$C_1 = 70 - T_m$ $\quad\quad T(1) = 145^\circ$

$145 - T_m = e^{kt}(70 - T_m)$

$\frac{145 - T_m}{70 - T_m} = e^{k}$

$\therefore e^{k} = \frac{(110 - T_m)^2}{(70 - T_m)^2}$

$\frac{145 - T_m}{\cancel{70 - T_m}} = \frac{(110 - T_m)^2}{(70 - T_m)^{\cancel{2}}}$

$(145 - T_m)(70 - T_m) = 110^2 - 220T_m + T_m^2$

$10150 - 145T_m - 70T_m + \cancel{T_m^2} = 12100 - 220T_m + \cancel{T_m^2}$

$-215T_m + 220T_m = 12100 - 10150$

$5T_m = 1950$

$T_m = \frac{1950}{5}$

$T_m = 390^\circ F$

(Temperature of oven).

Q.2(iv)

using LR circuit

$L\frac{di}{dt} + Ri = E(t)$

$L = 0.1 \quad R = 50 \quad E(t) = 30$

$0.1\frac{di}{dt} + 50i = 30$

divide eq by 0.1

$\frac{di}{dt} + 500i = 300$

Amin Farjad

20k-0344

Date

$$I.F = e^{\int 500 dt}$$

$$I.F = e^{500t}$$

$$e^{500t}\frac{di}{dt} + e^{500t} 500 i = 300 e^{500t}$$

$$\frac{d}{dt}\left(e^{500t} i\right) = 300 e^{500t}$$

$$\int d\left(e^{500t} i\right) = \int 300 e^{500t} dt$$

$$e^{500t} i = \frac{300}{500} e^{500t} + C$$

$$i = \frac{3}{5} + \frac{C}{e^{500t}}$$

$$i(0) = 0$$

$$0 = \frac{3}{5} + \frac{C}{e^{0}}$$

$$C = -3/5$$

at $t = t$

$$i(t) = \frac{3}{5} - \frac{3}{5e^{500t}}$$

as $t \to \infty$

$$i(t) = \frac{3}{5} - \frac{3}{5e^{\infty}}$$

$$i(t) = \frac{3}{5} - \frac{3}{\infty}$$

$$i(t) = \frac{3}{5}$$

as $t \to \infty$ $\quad i \to \frac{3}{5}$

Amin Farjad
20k-0344