DE Assignment (2016-0344)

Q.1

$$f(t) = \begin{cases} e^t & t \le 2 \\ 3 & t > 2 \end{cases}$$

$$\mathcal{L}\{f(t)\} = \int_0^2 e^{-st} e^t \, dt + \int_2^\infty e^{-st}\, 3 \, dt$$

$$= \int_0^2 e^{-t(s-1)} \, dt + 3\int_2^\infty e^{-st} \, dt$$

$$= \left[\frac{e^{-t(s-1)}}{-(s-1)} \right]_0^2 + 3\left[\frac{e^{-st}}{-s} \right]_2^\infty$$

$$= \left[\frac{e^{-2(s-1)}}{-(s-1)} - \frac{e^0}{-(s-1)} \right] + 3\left[\frac{e^{-\infty}}{-s} - \frac{e^{-2s}}{-s} \right]$$

$$= \frac{e^{2-2s}}{1-s} + \frac{e^0}{s-1} + 3\left(\frac{1}{\infty} + \frac{e^{-2s}}{s} \right)$$

$$= \frac{e^{-2(s-1)}}{-(s-1)} + \frac{1}{(s-1)} + \frac{3}{s} e^{-2s}$$

$$= \frac{1 - e^{-2(s-1)}}{(s-1)} + \frac{3}{s} e^{-2s} \quad \text{Ans.}$$

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Q.2

$f(t) = \cancel{3+t^2}\ 3+2t^2$

$$\mathcal{L}\{f(t)\} = \int_0^\infty e^{-st}(3+2t^2)\,dt$$

$$= 3\int_0^\infty e^{-st}\,dt + 2\int_0^\infty e^{-st}t^2\,dt$$

$$= 3\left[\frac{e^{-st}}{-s}\right]_0^\infty + 2\left[\frac{t^2e^{-st}}{-s} - \int_0^\infty \frac{2te^{-st}}{-s}\,dt\right]$$

$$= 3\left[\frac{e^{-st}}{-s}\right]_0^\infty + 2\left[-\frac{t^2e^{-st}}{s} + \frac{2}{s}\int_0^\infty te^{-st}\,dt\right]$$

$$= 3\left[\frac{e^{-st}}{-s}\right]_0^\infty + 2\left[\frac{-t^2e^{-st}}{s} + \frac{2}{s}\left(\frac{te^{-st}}{-s} - \frac{e^{-st}}{s^2}\right)\right]_0^\infty$$

$$= 3\left[\frac{e^{-st}}{-s}\right]_0^\infty + 2\lim_{a\to\infty}\left[\frac{-t^2e^{-st}}{s} - \frac{2te^{-st}}{s^2} - \frac{2e^{-st}}{s^3}\right]_0^a$$

$$= 3\left[\frac{e^{-st}}{-s}\right]_0^\infty + 2\lim_{a\to\infty}\left(-\frac{a^2e^{-as}}{s} - \frac{2ae^{-sa}}{s^2} - \frac{2e^{-sa}}{s^3} - \left(\frac{-0e^0}{s} - \frac{20e^0}{s^2} - \frac{2e^0}{s^3}\right)\right)$$

$$= 3\left[\frac{e^{-\infty}}{-s} - \frac{e^{-s0}}{-s}\right] + 2\left[\lim_{a\to\infty}\frac{-a^2e^{-as}}{s} - 2\lim_{a\to\infty}\frac{ae^{-as}}{s^2} - 2\lim_{a\to\infty}\frac{e^{-sa}}{s^3} + 0 + 0 + \frac{2}{s^3}\right]$$

hopital rule

$$= 3\left[\frac{1}{\infty} - \frac{1}{-s}\right] + 2\left[\lim_{a\to\infty} -\frac{2}{s^3e^{as}} - 2\lim_{a\to\infty}\frac{1}{s^3e^{as}} - 2\lim_{a\to\infty}\frac{e^{-sa}}{s^3} + 2/s^3\right]$$

Applying limit

$$= 3(1/s) + 2\left(-\frac{1}{\infty} - \frac{1}{\infty} - \frac{2}{\infty} + 2/s^3\right)$$

$$= \frac{3}{s} + \frac{4}{s^3} \quad \text{Ans.}$$

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Q.3

$f(t) = 5\sin 3t - 17e^{-2t}$

$$\mathcal{L}(f(t)) = \int_0^\infty e^{-st}(5\sin 3t - 17e^{-2t})\,dt$$

$$\mathcal{L}(f(t)) = 5\int_0^\infty \underbrace{e^{-st}\sin 3t\,dt}_{I} - 17\int_0^\infty e^{-2t}\cdot e^{-st}\,dt$$

Let

$$I = \int e^{-st}\sin 3t\,dt$$

$$I = \frac{\sin 3t\, e^{-st}}{-s} - \int \frac{3\cos 3t\, e^{-st}}{-s}\,dt$$

$$I = \frac{\sin 3t\, e^{-st}}{-s} + \frac{3}{s}\left[\frac{\cos 3t\, e^{-st}}{-s} - \int \frac{-3\sin 3t\, e^{-st}}{-s}\,dt\right]$$

$$I = \frac{\sin 3t\, e^{-st}}{-s} + \frac{3}{s}\left[\frac{\cos 3t\, e^{-st}}{-s} - \frac{3}{s}\int \sin 3t\, e^{-st}\,dt\right]$$

$$I + \frac{9}{s^2}\int \sin 3t\, e^{-st}\,dt = -\frac{\sin 3t\, e^{-st}}{+s} - \frac{3\cos 3t\, e^{-st}}{s^2}$$

$$\frac{(s^2+9)}{s^2} I = -\frac{\sin 3t\, e^{-st}}{s} - \frac{3\cos 3t\, e^{-st}}{s^2}$$

$$I = \frac{-s\sin 3t\, e^{-st}}{(s^2+9)} - \frac{3\cos 3t\, e^{-st}}{(s^2+9)}$$

$$\mathcal{L}(f(t)) = 5\left[\frac{-s\sin 3t\, e^{-st}}{(s^2+9)} - \frac{3\cos 3t\, e^{-st}}{s^2+9}\right]_0^\infty - 17\int_0^\infty e^{-t(s+2)}\,dt$$

$$\mathcal{L}(f(t)) = 5\left[-\frac{s\sin 3t\, e^{-st}}{s^2+9} - \frac{3\cos 3t\, e^{-st}}{s^2+9}\right]_0^\infty - 17\left[\frac{e^{-t(s+2)}}{-(s+2)}\right]_0^\infty$$

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$$\mathcal{L}(f(t)) = 5\left(\frac{-s\sin\infty\, e^{-\infty}}{s^2+9} - 3\frac{\cos 3\infty\, e^{-\infty}}{s^2+9} - \left(\frac{-s\sin 0\, e^{0}}{s^2+9} - \frac{3\cos 0\, e^{0}}{s^2+9} \right) \right)$$

$$-17\left(\frac{e^{-\infty}}{-(s+2)} + \frac{e^{0}}{s+2} \right)$$

$$\cancel{= 5\left(\frac{3}{s^2+9} \right) \left(-17\left(0 + \frac{1}{s+2} \right) \right)}$$

$$= 5\left(-0 - 0 + 0 + \frac{3}{s^2+9} \right) - 17\left(0 + \frac{1}{s+2} \right)$$

$$= \frac{15}{s^2+9} - \frac{17}{s+2}$$

Q.4

$$f(t) = te^{4t}$$

$$\mathcal{L}\{f(t)\} = \int_0^\infty e^{-st}\, te^{4t}\, dt$$

$$= \int_0^\infty t\, e^{-t(s-4)}\, dt$$

$$= \frac{te^{-t(s-4)}}{-(s-4)} - \int_0^\infty \frac{e^{-t(s-4)}}{-(s-4)}\, dt$$

$$= \left[\frac{te^{-t(s-4)}}{-(s-4)} - \frac{e^{-t(s-4)}}{(s-4)^2} \right]_0^\infty$$

$$= \lim_{a\to\infty} \left[\frac{te^{-t(s-4)}}{-(s-4)} - \frac{e^{-t(s-4)}}{(s-4)^2} \right]_0^\infty$$

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$$= \left[\lim_{a\to\infty} \left(\frac{4ae^{-a(s-4)}}{-(s-4)} - \frac{e^{-a(s-4)}}{(s-4)^2} \right) - \left(\frac{0e^{-0}}{-(s-4)} - \frac{e^{-0}}{(s-4)^2} \right) \right]$$

hopital rule

$$= \left[\lim_{a\to\infty} \left(\frac{1}{e^{-a(s-4)}(s-4)^2} - \frac{e^{-a(s-4)}}{(s-4)^2} \right) - \left(-\frac{1}{(s-4)^2} \right) \right]$$

Applying limit

$$= \left[\frac{1}{\infty} - \frac{1}{\infty} + \frac{1}{(s-4)^2} \right]$$

$$= \frac{1}{(s-4)^2}$$ Ans.

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Q.5

$$\mathcal{L}^{-1}\left\{\frac{1}{s(s^2+2s+5)}\right\}$$

$$=\mathcal{L}^{-1}\left\{\frac{1}{s(s^2+2s+5)}\right\}$$

$$\cancel{=\mathcal{L}^{-1}}\ \frac{1}{s(s^2+2s+5)} = \frac{A}{s}+\frac{Bs+C}{s^2+2s+5}$$

$$1 = A(s^2+2s+5)+(Bs+c)s$$

let $s=0$

$$1 = A(5)+0$$

$$A=1/5$$

$$1 = As^2+2sA+5A+Bs^2+Cs$$

$$1 = (A+B)s^2+(2A+c)s+5A$$

$A+B=0$ $\qquad$ $2A+C=0$

$B=-1/5$ $\qquad$ $2(\cancel{8}1/5)+C=0$

$c=-2/5$

$$=\mathcal{L}^{-1}\left\{\frac{1}{5s}+\frac{-1/5\,s+(-2/5)}{s^2+2s+5}\right\}$$

$$=\frac{1}{5}\mathcal{L}^{-1}\left(\frac{1}{s}\right)-\frac{1}{5}\mathcal{L}^{-1}\left(\frac{s+2}{s^2+2s+5}\right)$$

$$=\frac{1}{5}(1)-\frac{1}{5}\mathcal{L}^{-1}\left(\frac{s+2}{s^2+2s+1+4}\right)$$

$$=\frac{1}{5}-\frac{1}{5}\mathcal{L}^{-1}\left(\frac{s+1+1}{(s+1)^2+4}\right)$$

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$$= \frac{1}{5} - \frac{1}{5}\left[\mathcal{L}^{-1}\left(\frac{s+1}{(s+1)^2+4}\right) + \mathcal{L}^{-1}\left(\frac{1}{(s+1)^2+4}\right)\right]$$

$$\therefore \mathcal{L}^{-1}(f(s-a)) = e^{at} f(t)$$

$$= \frac{1}{5} - \frac{1}{5}\left[\mathcal{L}^{-1}\left(\frac{(s+1)}{(s+1)^2+4}\right) + \frac{1}{2}\mathcal{L}^{-1}\left(\frac{2}{(s+1)^2+4}\right)\right]$$

$$= \frac{1}{5} - \frac{1}{5}\left(e^{-t}\cos 2t + \frac{1}{2} e^{-t}\sin 2t \right)$$

$$= \frac{1}{5} - \frac{1}{5} e^{-t}\cos 2t - \frac{e^{-t}}{10}\sin 2t$$

$$= \frac{1}{5} - \frac{(2\cos 2t + \sin 2t)\,e^{-t}}{10}$$

Q.6

$$\mathcal{L}^{-1}\left\{ \frac{7s-1}{(s+1)(s+2)(s-3)} \right\}$$

$$= \mathcal{L}^{-1}\left\{ \frac{7s-1}{(s+1)(s+2)(s-3)} \right\}$$

$$\frac{7s-1}{(s+1)(s+2)(s-3)} = \frac{A}{(s+1)} + \frac{B}{(s+2)} + \frac{C}{s-3}$$

$$7s-1 = A(s+2)(s-3) + B(s+1)(s-3) + C(s+1)(s+2)$$

Put $s+2=0$

$s=-2$

$$7(-2)-1 = 0 + B(-2+1)(-2-3) + 0$$

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$-14-1 = B(-1)(-5)$

$-15 = 5B$

$B = \frac{-15}{5} = -3$

put $s-3=0$

$s=3$

$7(3)-1 = 0+0+c(3+1)(3+2)$

$21-1 = c(4)(5)$

$\cancel{20} = \cancel{20}c$

$c=1$

put $s+1=0 \Rightarrow s=-1$

$7(-1)-1 = A(-1+2)(-1-3)$

$-7-1 = A(1)(-4)$

$-8 = -4A$

$A = 2$

$= L^{-1}\left\{\frac{2}{s+1} + \frac{-3}{s+2} + \frac{1}{s-3}\right\}$

$= 2L^{-1}\left\{\frac{1}{s+1}\right\} - 3L^{-1}\left\{\frac{1}{s+2}\right\} + L^{-1}\left\{\frac{1}{s-3}\right\}$

$= 2L^{-1}\left\{\frac{1}{s-(-1)}\right\} - 3L^{-1}\left\{\frac{1}{s-(-2)}\right\} + L^{-1}\left\{\frac{1}{s-3}\right\}$

$= 2e^{-(1)t} - 3e^{-(2)t} + e^{+(3)t}$

$= 2e^{-t} - 3e^{-2t} + e^{3t}$

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Q.7

$$\mathcal{L}^{-1}\left\{\frac{s^2+9s+2}{(s-1)^2(s+3)}\right\}$$

* $$\frac{s^2+9s+2}{(s-1)^2(s+3)} = \frac{A}{s-1} + \frac{B}{(s-1)^2} + \frac{C}{s+3}$$

$$s^2+9s+2 = A(s-1)(s+3) + B(s+3) + C(s-1)$$

put $s-1=0$

$s=1$

$$1+9+2 = 0 + B(1+3) + 0$$

$$12 = B(4)$$

$$B = 12/4$$

$$B = 3$$

put $s+3=0$

$s=-3$

$$(-3)^2+9(-3)+2 = C(-3-1)^2$$

$$9-27+2 = C(16)$$

$$-\cancel{16} = \cancel{16}C$$

$$C=-1$$

$$s^2+9s+2 = A(s^2+2s-3) + B(s+3) + C(s^2-$$

$$s^2+9s+2 = As^2+2As-3A + Bs+3B + Cs^2-2Cs$$

$$s^2+9s+2 = (A+C)s^2 + (2A+B-2C)s$$

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$$= \mathcal{L}^{-1}\left\{ \frac{2}{s-1} + \frac{3}{(s-1)^2} + \frac{(-1)}{(s+3)} \right\}$$

$$= 2\,\mathcal{L}^{-1}\left(\frac{1}{s-1}\right) + 3\,\mathcal{L}^{-1}\left(\frac{1}{(s-1)^2}\right) - 1\,\mathcal{L}^{-1}\left(\frac{1}{s+3}\right)$$

$$e^{at} f(t) = \mathcal{L}^{-1}\{F(s-a)\}$$

$$= 2e^{t} + 3\,t\,e^{t} - e^{-3t}$$

$$= 2e^{t} + 3te^{t} - e^{-3t}$$

Q. 8

$$\mathcal{L}^{-1}\left\{ \frac{2s^2+10s}{(s^2-2s+5)(s+1)} \right\}$$

$$\frac{2s^2+10s}{(s^2-2s+5)(s+1)} = \frac{A}{s+1} + \frac{Bs+C}{s^2-2s+5}$$

$$2s^2+10s = A(s^2-2s+5) + (Bs+C)(s+1)$$

$$2s^2+10s = As^2 - 2As + 5A + Bs^2 + Bs + Cs + C$$

$$2s^2+10s = (A+B)s^2 + (-2A+B+C)s + 5A+C$$

$A+B=2$ $\quad -2A+B+C=10$ $\quad 5A+C=0$

$$\begin{array}{r} A+B=2 \\ +\;5A+C=0 \\ \hline 6A+B+C=2 \end{array} \qquad \begin{array}{r} -2A+\cancel{B}+\cancel{C}=10 \\ -6A\underline{+}\cancel{B}\underline{+}\cancel{C}=\underline{+}2 \\ \hline -8A\quad=8 \\ A=-1 \end{array}$$

$A+B=2$

$-1+B=2$

$B=3$

$5A+C=0$

$5(-1)+C=0$

$C=5$

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$$= \mathcal{L}^{-1}\left\{ \frac{-1}{(s+1)} + \frac{3s+5}{s^2-2s+5} \right\}$$

$$= \mathcal{L}^{-1}\left\{ \frac{-1}{(s+1)} + \frac{3s-3+8}{s^2-2s+5} \right\}$$

$$= -\mathcal{L}^{-1}\left(\frac{1}{s-(-1)} \right) + \mathcal{L}^{-1}\left\{ \frac{3s-3}{s^2-2s+5} \right\} + 8\mathcal{L}^{-1}\left\{ \frac{1}{s^2-2s+5} \right\}$$

$$= -\mathcal{L}^{-1}\left(\frac{1}{s-(-1)} \right) + 3\mathcal{L}^{-1}\left(\frac{(s-1)}{(s-1)^2+4} \right) + 4\mathcal{L}^{-1}\left(\frac{2}{(s-1)^2+4} \right)$$

$$= -\mathcal{L}^{-1}\left(\frac{1}{s-(-1)} \right) + 3\mathcal{L}^{-1}\left(\frac{s-1}{(s-1)^2+4} \right) + 4\mathcal{L}^{-1}\left(\frac{2}{(s-1)^2+2^2} \right)$$

$$= -e^{-t} + 3e^t \cos 2t + 4e^t \sin 2t$$

$$= -e^{-t} + 3e^t \cos 2t + 4e^t \sin 2t$$ Ans.

Q.9

$$y' - 5y = e^{5x} \qquad y(0) = 0$$

$$y' - 5y = e^{5x}$$

Applying laplace

$$\mathcal{L}(y') - 5\mathcal{L}(y) = \mathcal{L}(e^{5x})$$

$$sF(s) - f(0) - 5F(s) = \frac{1}{s-5}$$

$$(s-5)F(s) - 0 = \frac{1}{s-5}$$

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$$F(s) = \frac{1}{(s-5)^2}$$

taking laplace inverse

$$y = \mathcal{L}^{-1}\left(\frac{1}{(s-5)^2}\right)$$

$$y = x e^{5x} \quad \because e^{at} f(t) = \mathcal{L}^{-1}(F(s-a))$$

Analytical method

$$y' - 5y = e^{5x}$$

$$I.F = e^{-5x}$$

multiply eq by e^{-5x}

$$e^{-5x} y' - 5e^{-5x} y = \cancel{e^{5x}} \cdot \cancel{e^{-5x}}$$

$$\frac{d}{dx}(e^{-5x} y) = 1$$

$$\int d(e^{-5x} y) = \int dx$$

$$e^{-5x} y = x + c$$

$$y = \frac{x}{e^{-5x}} + \frac{c}{e^{-5x}}$$

$$y = xe^{5x} + ce^{5x}$$

$$y(0) = 0$$

$$0 = 0 + ce^{0}$$

$$c = 0$$

$$y = xe^{5x}$$

hence Solution is same.

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Q.10

$y' + y = \sin n \qquad y(0) = 1$

taking laplace

$L(y') + L(y) = L(\sin n)$

$sF(s) - f(0) + F(s) = \dfrac{1}{s^2+1}$

$(s+1)F(s) - 1 = \dfrac{1}{s^2+1}$

$(s+1)F(s) = 1 + \dfrac{1}{s^2+1}$

$F(s) = \dfrac{s^2+1+1}{(s^2+1)(s+1)}$

$F(s) = \dfrac{s^2+2}{(s^2+1)(s+1)}$

$\Rightarrow \dfrac{s^2+2}{(s^2+1)(s+1)} = \dfrac{As+B}{(s^2+1)} + \dfrac{C}{(s+1)}$

$s^2+2 = (As+B)(s+1) + C(s^2+1)$

$s^2+2 = As^2 + As + Bs + B + Cs^2 + C$

$s^2+2 = (A+C)s^2 + (A+B)s + (B+C)$

$\cancel{A}+C = 1 \qquad A+B = 0 \qquad \cancel{B}+C = 2$

$-\cancel{A} \pm B = -0 \qquad\qquad -\cancel{B}+C = 1$

$C - B = 1 \qquad\qquad 2C = 3$

$C - B = 1 \qquad\qquad C = 3/2$

$3/2 - 1 = B$

$B = 1/2 \qquad A = -1/2$

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$$F(s) = \frac{-\frac{1}{2}s + \frac{1}{2}}{(s^2+1)} + \frac{\frac{3}{2}}{(s+1)}$$

Laplace inverse

$$y = \mathcal{L}^{-1}\left\{ \frac{-\frac{1}{2}s + \frac{1}{2}}{(s^2+1)} + \frac{\frac{3}{2}}{(s+1)} \right\}$$

$$y = \frac{1}{2}\mathcal{L}^{-1}\left\{ \frac{1-s}{s^2+1} \right\} + \frac{3}{2}\mathcal{L}^{-1}\left\{ \frac{1}{s+1} \right\}$$

$$\cancel{y = \frac{1}{2}\mathcal{L}^{-1}\left\{ \frac{1-s}{s^2} \right.}$$

$$y = \frac{1}{2}\mathcal{L}^{-1}\left\{ \frac{1}{s^2+1} \right\} - \frac{1}{2}\mathcal{L}^{-1}\left\{ \frac{s}{s^2+1} \right\} + \frac{3}{2}\mathcal{L}^{-1}\left\{ \frac{1}{s+1} \right\}$$

$$y = \frac{1}{2}\sin x - \frac{1}{2}\cos x + \frac{3}{2}e^{-x}$$

Analytical method

$$y' + y = \sin x \qquad y(0) = 1$$

$$I.F = e^x$$

$$e^x y' + e^x y = e^x \sin x$$

$$\frac{d}{dx}(e^x y) = e^x \sin x$$

$$\int d(e^x y) = \int e^x \sin x \, dx$$

$$e^x y = \int e^x \sin x \, dx$$

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let $I = \int e^x \sin x \, dx$

$I = \sin x \, e^x - \int \cos x \, e^x \, dx$

$I = \sin x \, e^x - \cos x \, e^x + \int -\sin x \, e^x \, dx$

$I = \sin x \, e^x - \cos x \, e^x - \underbrace{\int \sin x \, e^x \, dx}_{I}$

$I + I = \sin x \, e^x - \cos x \, e^x$

$I = \frac{1}{2} \sin x \, e^x - \frac{1}{2} \cos x \, e^x$

$e^x y = \frac{1}{2} \sin x \, e^x - \frac{1}{2} \cos x \, e^x + C$

$y = \frac{1}{2} \frac{\sin x \, e^x}{e^x} - \frac{1}{2} \frac{\cos x \, e^x}{e^x} + \frac{C}{e^x}$

$y = \frac{1}{2} \sin x - \frac{1}{2} \cos x + Ce^{-x}$

$y(0) = 1$

$1 = 0 - \frac{1}{2} + Ce^0$

$1 + \frac{1}{2} = C$

$C = \frac{3}{2}$

$y = \frac{1}{2} \sin x - \frac{1}{2} \cos x + \frac{3}{2} e^{-x}$

hence the ~~solu~~ ans is same.

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Q.11

$y'' - y' = 2x \qquad y(0)=1 \quad y'(0)=-2$

Laplace

$\mathcal{L}(y'') - \mathcal{L}(y') = 2\mathcal{L}(x)$

$s^2F(s) - sf(0) - f'(0) - (sF(s) - f(0)) = \frac{2}{s^2}$

$s^2F(s) - s(1) - (-2) - (sF(s) - 1) = \frac{2}{s^2}$

$s^2F(s) - sF(s) - s + 2 + 1 = \frac{2}{s^2}$

$(s^2 - s)F(s) = -3 + s + \frac{2}{s^2}$

$F(s) = \frac{-3s^2 + s^3 + 2}{(s^2 - s)s^2}$

$F(s) = \frac{\cancel{s^3}}{\cancel{s^2}s(s-1)} + \frac{2-3s^2}{s^3(s-1)}$

$F(s) = \frac{1}{s-1} + \frac{2-3s^2}{s^3(s-1)}$

Partial fraction.

$\frac{2-3s^2}{s^3(s-1)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s^3} + \frac{D}{s-1}$

$2 - 3s^2 = As^2(s-1) + Bs(s-1) + C(s-1) + Ds^3$

$s=0$

$2 - 3(0) = 0 + 0 + C(0-1) + 0$

$2 = C(-1)$

$C = -2$

put $s=1$

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$2-3(1) = 0+0+0+D(1)$

$D=-1$

$2-3s^2 = As^3 - As^2 + Bs^2 - Bs + Cs - C + Ds^3$

$2-3s^2 = (A+D)s^3 + (-A+B)s^2 + (C-B)s - C$

$A+D=0 \qquad -A+B=-3 \qquad C-B=0$

$A-1=0 \qquad -1+B=-3$

$A=1 \qquad B=-3+1$

$B=-2$

$F(s) = \cancel{\frac{1}{s-1}} + \frac{1}{s} - \frac{2}{s^2} - \frac{2}{s^3} - \cancel{\frac{1}{(s-1)}}$

$F(s) = \frac{1}{s} - \frac{2}{s^2} - \frac{2}{s^3}$

Laplace inverse

$y = \mathcal{L}^{-1}\left(\frac{1}{s}\right) - 2\mathcal{L}^{-1}\left\{\frac{1}{s^2}\right\} - \mathcal{L}^{-1}\left(\frac{2}{s^3}\right)$

$y = 1 - 2x - x^2$

Analytical method

$y'' - y' = 2x$

for y_c

$y'' - y' = 0$

$D^2y - Dy = 0$

$y(D^2 - D) = 0$

$y \neq 0 \qquad \Rightarrow D^2 - D = 0$

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$$D(D-1)=0$$

$$\Rightarrow D=0 \qquad \Rightarrow D=1$$

$$y_c = c_1 + c_2 e^x$$

For y_p

$$y_p = Ax+B$$

$$y_p = Ax^2+Bx$$

$$y'_p = 2Ax+B$$

$$y''_p = 2A$$

$$y''_p - y'_p = 2x$$

$$2A-(2Ax+B) = 2x$$

$$2A-2Ax-B = 2x$$

$$(2A-B)+(-2A)x = 2x$$

$$-2A=2 \qquad 2A-B=0$$

$$A=-1 \qquad 2(-1)-B=0$$

$$-2=B$$

$$y_p = -x^2-2x$$

$$y = c_1+c_2 e^x - x^2-2x$$

$$y(0)=1 \qquad y'(0)=-2$$

$$1 = c_1 + c_2 e^0 - 0 - 0$$

$$1 = c_1+c_2$$

$$y' = c_2 e^x - 2x - 2$$

$$y'(0)=-2$$

$$-2 = c_2 e^0 - 2(0) - 2$$

Scanned with CamScanner

(20K-0344)

$-2+2 = c_2 e^0$

$c_2 = 0$

$1 = c_1 + c_2$

$c_1 = 1$

$\Rightarrow y = c_1 + c_2 e^x - x^2 - 2x$

$y = 1 - x^2 - 2x$

hence both ans are same.

Q.12

$y'' - 2y' + 5y = -8e^{7-x}$

$y(7) = 2 \quad y'(7) = 12$

$x = 7 \quad \Rightarrow x - 7 = 0 \quad$ let $m = x$

$y'' - 2y' + 5y = -8e^{7-x}$

Applying laplace

$\mathcal{L}(y'') - 2\mathcal{L}(y') + 5\mathcal{L}(y) = -8\mathcal{L}(e^{7-x})$

~~Ly''to~~

$\mathcal{L}(y'') - 2\mathcal{L}(y') + 5\mathcal{L}(y) = -8\mathcal{L}(e^{-m})$

$y(0) = 2 \quad y'(0) = 12$ where $m = x - 7$

$\Rightarrow m = 0$

$s^2 F(s) - sf(0) - f'(0) - 2(sF(s) - f(0))$
$+ 5F(s) = \dfrac{-8}{s+1}$

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$$s^2F(s) - s(2) - 12 - 2sF(s) + 2(2) + 5F(s) = \frac{-8}{s+1}$$

$$(s^2 - 2s + 5)F(s) - 2s - 12 + 4 = \frac{-8}{s+1}$$

$$(s^2 - 2s + 5)F(s) = 8 + 2s - \frac{8}{s+1}$$

$$(s^2 - 2s + 5)F(s) = \frac{8(s+1) + 2s(s+1) - 8}{s+1}$$

$$(s^2 - 2s + 5)F(s) = \frac{8s + \cancel{8} + 2s^2 + 2s - \cancel{8}}{s+1}$$

$$F(s) = \frac{2s^2 + 10s}{(s+1)(s^2 - 2s + 5)}$$

Partial Fractions were obtained in Q.8

$$F(s) = \left\{ \frac{-1}{s+1} + \frac{3s+5}{s^2 - 2s + 5} \right\}$$

taking inverse laplace

$$y = \mathcal{L}^{-1}\left\{ \frac{-1}{s+1} + \frac{3s - 3 + 8}{s^2 - 2s + 5} \right\}$$

$$y = -\mathcal{L}^{-1}\left\{ \frac{1}{s+1} \right\} + 3\mathcal{L}^{-1}\left\{ \frac{s-1}{s^2 - 2s + 5} \right\} + 8\mathcal{L}^{-1}\left\{ \frac{1}{s^2 - 2s + 5} \right\}$$

$$y = -\mathcal{L}^{-1}\left\{ \frac{1}{s+1} \right\} + 3\mathcal{L}^{-1}\left\{ \frac{s-1}{(s-1)^2 + 4} \right\} + 4\mathcal{L}^{-1}\left\{ \frac{2}{(s-1)^2 + 2^2} \right\}$$

$$y = -1e^{-m} + 3\cos 2m\, e^{m} + 4\sin 2m\, e^{m}$$

but $m = x - 7$

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$$y = -e^{-(x-7)} + 3e^{x-7}\cos(2(x-7)) + 4e^{x-7}\sin(2(x-7))$$