Assignment # 3
Differential Equations (MT-224)
Date of Submission: 25th May, 2021
Total marks: 16, Total weightage: 8
(CLO-1)

Find Laplace by the definition $\mathcal{L}\{f(t)\}$ for the following: **[marks: 4, weightage: 2]**

1. $f(t) = \begin{cases} e^t & t \leq 2 \\ 3 & t > 2 \end{cases}$ Answer: $\frac{1-e^{-2(s-1)}}{s-1} + \frac{3}{s}e^{-2s}$
2. $f(t) = 3 + 2t^2$ Answer: $\frac{3}{s} + \frac{4}{s^3}$
3. $f(t) = 5\sin 3t - 17e^{-2t}$ Answer: $\frac{15}{s^2+9} - \frac{17}{s+2}$
4. $f(t) = te^{4t}$ Answer: $\frac{1}{(s-4)^2}$

Find Laplace Inverse of the following: **[marks: 4, weightage: 2]**

1. $\mathcal{L}^{-1}\left\{\frac{1}{s(s^2+2s+5)}\right\}$ Answer: $\frac{1}{2}e^{-t}\sin 2t$
2. $\mathcal{L}^{-1}\left\{\frac{7s-1}{(s+1)(s+2)(s-3)}\right\}$ Answer: $2e^{-t} - 3e^{-2t} + e^{3t}$
3. $\mathcal{L}^{-1}\left\{\frac{s^2+9s+2}{(s-1)^2(s+3)}\right\}$ Answer: $2e^{-t} + 3te^{t} - e^{-3t}$
4. $\mathcal{L}^{-1}\left\{\frac{2s^2+10s}{(s^2-2s+5)}(s+1)\right\}$ Answer: $3e^{t}\cos 2t + 4e^{t}\sin 2t - e^{-t}$

Solve the following differential equations by Laplace and then confirm the general solution by analytical method.

[marks: 8, weightage: 4]

1. $y' - 5y = e^{5x}$ $y(0) = 0$
2. $y' + y = \sin x$ $y(0) = 1$
3. $y'' = y' = 2x$ $y(0) = 1,$ $y'(0) = -2$
4. $y'' - 2y' + 5y = -8e^{7-x}$ $y(7) = 2,$ $y'(7) = 12$

Answers:

1. xe^{x}
2. $\frac{3}{2}e^{-x} - \frac{1}{2}\cos x + \frac{1}{2}\sin x$
3. $1 - x^2 - 2x$
4. $3e^{x-7}\cos[2(x-7] + 4e^{x-7}\sin[2(x-7)] - e^{-(x-7)}$