Q12 $x^2 \frac{dy}{dx} + yx + y^2 = 0$

$$x^2\left(\frac{dy}{dx} + \frac{yx}{x^2}\right) = -y^2$$

$$\frac{dy}{dx} + \frac{y}{x} = \frac{-y^2}{x^2} \quad -(1)$$

Std. Bernoulli Eq^n

$$y' + P(x)\,y = Q(x)\,y^n$$

Here $n=2$

let $V = y^{1-n} = y^{1-2}$

$$\boxed{V = \frac{1}{y}}$$

$$\frac{dV}{dx} = \frac{d}{dx} y^{-1}$$

$$\frac{dV}{dx} = -1\, y^{-2} \cdot \frac{dy}{dx}$$

$$\frac{dV}{dx} = -y^{-2} \frac{dy}{dx}$$

$$\boxed{\frac{dy}{dx} = -y^2\frac{dv}{dx}}$$

(1) $\Rightarrow$ $$\frac{dy}{dx} + \frac{y}{x} = \frac{-y^2}{x^2}$$

$$-y^2\frac{dv}{dx} + \frac{y}{x} = \frac{-y^2}{x^2}$$

Multiplyg B.S by $\frac{1}{y^2}$

$$\frac{1}{y^2}\left(-y^2\frac{dv}{dx} + \frac{y}{x}\right) = \frac{1}{\cancel{y^2}}\left(\frac{-\cancel{y^2}}{x^2}\right)$$

$$-\frac{dv}{dx} + \frac{\cancel{y}}{xy^{2-1}} = -\frac{1}{x^2}$$

$$\cancel{-}\left(\frac{dv}{dx} - \frac{1}{xy}\right) = \cancel{-}\frac{1}{x^2}$$

$$\frac{dv}{dx} - \frac{1}{xy} = \frac{1}{x^2} \quad -(2)$$

$$P(x) = \frac{1}{x}$$

$$\boxed{I.F. = e^{\int 1/x \, dx} = e^{\ln x} = x}$$

Multiplyg B.S By x

$$\frac{vu' - uv'}{v^2}$$

$$x\left(\frac{dv}{dx} - \frac{1}{xy}\right) = \frac{1}{\cancel{x^2}} \times \cancel{x}$$

$$x\frac{dv}{dx} - \frac{\cancel{x}1}{\cancel{x}y} = \frac{1}{x}$$

$$\because \frac{1}{y} = v$$

$$x\frac{dv}{dx} - v = \frac{1}{x}$$

$$xv' - v = \frac{1}{x}$$

multiplying & dividing By x^2

$$x^2\left(\frac{xv' - v}{x^2}\right) = \frac{1}{x}$$

$$\frac{xv' - v}{x^2} = \frac{1}{x^3}$$

$$\because \frac{vu' - uv'}{v^2} = \frac{d}{dx}\left(\frac{u}{v}\right)$$

$$\frac{d}{dx}\left(\frac{v}{x}\right) = \frac{1}{x^3}$$

Scanned with CamScanner

Integrating B.S

$$\int \frac{d}{dx}\left(\frac{v}{x}\right) = \int \frac{1}{x^3}$$

$$\frac{v}{x} = \int \frac{1}{x^3}\,dx$$

$$\frac{v}{x} = \int x^{-3}\,dx$$

$$\frac{v}{x} = \frac{x^{-2}}{-2} + C$$

$$\because v = \frac{1}{y}$$

$$\frac{1}{xy} = \frac{-x^{-2}}{2} + C$$

$$\frac{1}{xy} = \frac{-1}{2x^2} + C$$

$$\frac{1}{xy} = \frac{-1+2x^2C}{2x^2}$$

$$\cancel{x}y = \frac{2x^{\cancel{2}\,1}}{-1+2x^2C}$$

$$\boxed{y = \frac{2x}{2x^2C-1}}$$