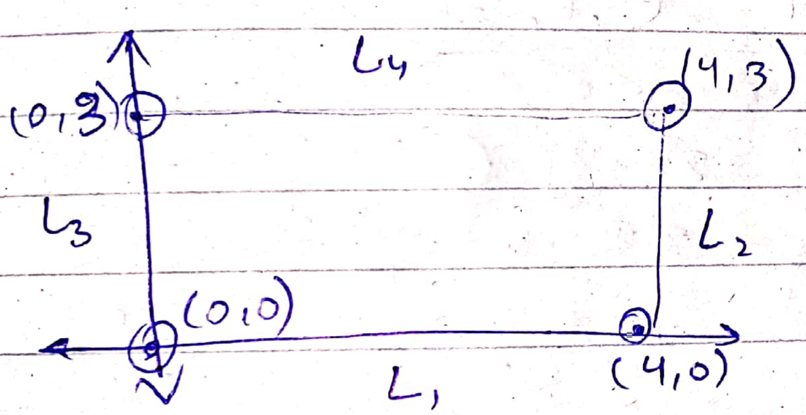
# FOR Questions 31-36:

Q. Find the absolute maximum & minimum of values of the function $f(x,y) = x^2 - 2xy + 4y$ on the rectangle

$D = \{(x,y) \mid 0 < x < 4,\ 0 < y < 3\}$




| $x$ | $y$ | $f$ |
|---|---|---|
| 0 | 0 | 0 |
| 4 | 0 | 16 |
| 4 | 3 | 12 |
| 6 | 3 | 24 |
| 2 | 2 | 4 |
| 3 | 3 | 3 |

write all the critical points in the table as well as the boundary values

## 1st partial derivatives:

$f_x = 2x - 2y$

$f_y = -2x + 4$

FOR C.P. :-

$2x - 2y = 0$

$-2x = -4$

$\boxed{x = 2}$

$4 = 2y$

$\boxed{y = 2}$  $(2,2)$ C.P.

NOW

**Consider $L_1$ $(y=0)$**

$f(x,0) = x^2 - 2xy + 4y$

$f(x,0) = x^2$

considering the end point of $L_1$

$f(0,0) = 0$

$f(4,0) = 16$

**Consider $L_2$ $(x=4)$**

$f(4,y) = 16 - 8y + 4y$

$f(4,y) = 16 - 4y$

Consider the end points of $L_2$

$f(4,0) = 16$

$f(4,3) = 4$

**Consider $L_3$ $(x=0)$**

$f(0,y) = 0 - 0 + 4y$

$f(0,y) = 4y$

Consider the boundary values of $L_3$

$f(0,0) = 0$

$f(0,3) = 12$

**Consider $L_4$ $(y=3)$**

~~$f(x,3) = x^2 - 6y +$~~

$\boxed{f(x,3) = x^2 - 6x + 12}$

Consider the endpoints of $L_4$

$f(0,3) = 12$

$f(4,3) = 4$

∴ Now since the equation is quadratic there may by a critical point b/w somewhere to find that we equate the 1st d~~~~ to 0

$f(x,3) = x^2 - 6x + 12$

$f(x,3) = 2x - 6$

$2x - 6 = 0$

$\boxed{x = 3}$

$f(3,3) = 3^2 - 6(3) + 12$

$= 9 - 18 + 12$

$\boxed{f(3,3) = 3}$

This point must be in the rectangular region D otherwise ignore it
In this case it is

Finally Putting in the C.P.

$f(2,2) = 2^2 - 2(2)(2) + 4(2)$

$\boxed{f(2,2) = 4}$

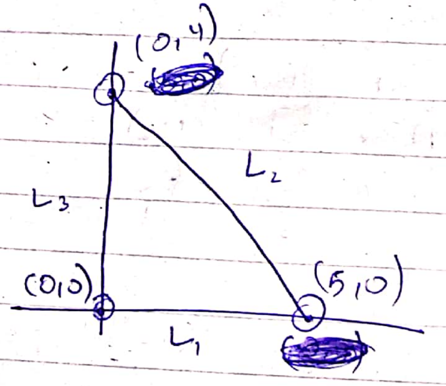
On the Basis of the table,

Absolute Min at (0,0)
Absolute Max at (4,0)

Ans.

Q31 – $f(x,y) = xy - x - 3y$

R is the triangular region within $(0,0)$ $(0,4)$ $(5,0)$.




| x | y | f |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 4 | -12 |
| 5 | 0 | -5 |
| 1 | 3 | -7 |
| $\frac{27}{8}$ | $\frac{13}{10}$ | -2.89 |

FOR C.P.s

$f_x = x - 1$

$f_y = y - 3$

equating equals to 0

$\boxed{x=1}$ $\boxed{y=3}$

C.P. $(1,3)$

~~Consider L1 (x=0)~~

~~$f(0,y) = xy - x - 3y = -3y$~~

~~$f(0,y) = -3y$~~

~~Putting the end points of L1 (y)~~

~~$f(0,0) = 0$~~

~~$f(0,5) = -15$~~

Consider $L_1$ $(y=0)$

$f(x,y) = xy - x - 3y$

$f(x,0) = -x$ (Putting the x-end values of $L_1$)

$\boxed{f(0,0) = 0}$

$\boxed{f(5,0) = -5}$

Consider $L_2$ (0,4) to (5,0)

To find the equation of line

$y = mx + c$

$m = \dfrac{4-0}{0-5}$

$\boxed{m = -\dfrac{4}{5}}$ $\boxed{c = y \text{ intercept} = 4}$

$y = \dfrac{-4}{5}x + 4$

Now

$f(x, -\frac{4}{5}x+4) = xy - x - 3y$

$= x(-\frac{4}{5}x+4) - x - 3(-\frac{4}{5}x+4)$

$= -\frac{4}{5}x^2 + 4x - x + \frac{12}{5}x - 12$

$$\boxed{f(x, -\tfrac{4}{5}x+4) = -\frac{4}{5}x^2 + \frac{27}{5}x - 12}$$

Since its a quadratic equation

$-\frac{8}{5}x + \frac{27}{5} = 0$

$+\frac{8}{\cancel{5}}x = +\frac{27}{\cancel{5}}$ $\boxed{x = \frac{27}{8}}$

$y = \frac{-4}{5}\left(\frac{27}{8}\right) + 4$

$\boxed{y = \frac{13}{10}}$ $\left(\frac{27}{8}, \frac{13}{10}\right)$

$f\left(\frac{27}{8}, +\frac{13}{10}\right) = \left(\frac{27}{8}\right)\left(\frac{13}{10}\right) - \frac{27}{8} - 3\left(\frac{13}{10}\right)$

$f\left(\frac{27}{8}, \frac{13}{10}\right) = -2.89$

Consider $L_3$ $(x=0)$

$f(0,y) = -3y$

$\boxed{f(0,0) = 0}$

$\boxed{f(0,4) = -12}$

Finally putting in C.P.

$f(1,3) = (1)(3) - 1 - (3)(3)$

$= 3 - 1 - 9$

$\boxed{f(1,3) = -7}$

Absolute max at (0,0)

Absolute min at (0,4)