EXERCISE 13.8

Critical Point:-

C(a,b) is found when
$f_x(x_0, y_0) = 0$ & $f_y(x_0, y_0) = 0$

1st partial derivative w.r.t x

first partial derivative w.r.t. y.

Finding local extrema, saddle points & critical points:

↳ $D = f_{xx} \cdot f_{yy} - f_{xy}^2$ → mixed partial derivative.

f_{xx}: 2nd partial derivative w.r.t x

f_{yy}: 2nd partial derivative w.r.t. y

↳ $D > 0$, $f_{xx} > 0$, $f(a,b)$ → local min

↳ $D > 0$, $f_{xx} < 0$, $f(a,b)$ → local max

↳ $D < 0$, $f(a,b)$ → Neither max nor min But a saddle point

↳ $D = 0$, no conclusion

Questions done using this method will be

Q1–4 & Q 9–20

Q1.

a) $f(x,y) = (x-2)^2 + (y+1)^2$

$\boxed{f(x,y) = x^2 - 4x + 4 + y^2 + 2y + 1}$

1st derivatives:-

$f_x = 2x - 4$ —— (i)

$f_y = 2y + 2$ —— (ii)

Finding critical points:-

(i) = 0 $\quad 2x - 4 = 0$

$\boxed{x = 2}$

$2y + 2 = 0$

$\boxed{y = -1}$

critical point $(2,-1)$

2nd Derivatives:-

$\boxed{f_{xx} = 2}$

$\boxed{f_{yy} = 2}$

$f_{xy} = \frac{d}{dy}(f_x) = \frac{d}{dy}(2x-4)$

$\boxed{f_{xy} = 0}$

$D = f_{xx} \cdot f_{yy} - f_{xy}^2 = (2\times2) - 0^2$

$\boxed{D = 4} > 0$

$f_{xx} > 0$

Since $f_{xx} > 0$ & $D > 0$
$(2,-1)$ is a relative minima while no maximas exist.

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b) $f = 1 - x^2 - y^2$

1st ders:-

$f_x = 0 - 2x - 0 = -2x$

$f_y = 0 - 0 - 2y$

Critical point:-

$-2x = 0$

$\boxed{x = 0}$

$-2y = 0$

$\boxed{y = 0}$

C.P. $(0,0)$

2nd ders:-

$f_{xx} = -2$

$f_{yy} = -2$

$f_{xy} = \frac{d}{dy}(f_x) = \frac{d}{dy}(-2x) = 0$

$\boxed{f_{xy} = 0}$

Now,

$D = f_{xx} f_{yy} - f_{xy}^2$

$= (-2)(-2) - 0$

$\boxed{D = 4 > 0}$

& since $f_{xx} < 0$

C.P. $(0,0)$ is a local maxima & there is no minima.

c) $f = x + 2y - 5$

1st ders:-

$f_x = 1$

$f_y = 2$

C.P.

$1 \neq 0$

$2 \neq 0$

C.P. donot exist.

Hence no max or min exist

Q2.

a) $f(x,y) = 1 - (x+1)^2 - (y-5)^2$

$f(x,y) = 1 - (x^2+2x+1) - (y^2-10y+25)$

$f(x,y) = \cancel{1} - x^2 - 2x - \cancel{1} - y^2 + 10y - 25$

$$\boxed{f(x,y) = -x^2 - 2x - y^2 + 10y - 25}$$

FOR 1st devs:-

$f_x = -2x - 2$

$f_y = -2y + 10$

FOR C.P.

$-2x - 2 = 0$

$\boxed{x = -1}$

$-2y + 10 = 0$

$\boxed{y = 5}$ C.P. $(-1,5)$

FOR 2nd devs

$f_{xx} = -2$

$f_{yy} = -2$

$f_{xy} = 0$

Now,

$D = f_{xx} f_{yy} - f_{xy}^2$

$= (-2)(-2) - 0$

$\boxed{D = 4 > 0}$

$f_{xx} < 0$

Thus C.P. $(-1,5)$ is a relative max & there are no minimas.

b) $f = e^{xy}$

<u>1st Derivr</u>

$f_{xx} = e^{xy}(x(0)+y(1))$

$\boxed{f_{x} = y e^{xy}}$

$\boxed{f_{y} = x e^{xy}}$

<u>FOR C.P.:</u>

$y \times e^{xy} = 0 \quad -(i)$

$x \times e^{xy} = 0 \quad -(ii)$

$y = \frac{0}{e^{xy}}$ | $x = \frac{0}{e^{xy}}$

$\boxed{y=0}$ | $\boxed{x=0}$

$\boxed{C.P. = (0,0)}$

<u>FOR 2nd devr</u>

i) $f_{xx} = y^2 e^{xy}$ Put $(0,0)$ $\boxed{f_{xx} = 0}$

ii) $f_{yy} = x^2 e^{xy}$ Put $(0,0)$ $\boxed{f_{yy} = 0}$

$f_{xy} = \frac{d}{dy}(ye^{xy})$

$= e^{xy}\frac{d}{dy}(y) + y\frac{d}{dy}e^{xy}$

$= e^{xy} + y[e^{xy}(x)]$

$\boxed{f_{xy} = e^{xy} + xye^{xy}}$

Put $x=0$, $y=0$

$f_{xy} = e^0 + 0$

$\boxed{f_{xy} = 1}$

Now,

$D = f_{xx}f_{yy} - f_{xy}^2$

$= 0 - 1^2$

$\boxed{D = -1 < 0}$

(0,0) is a saddle point & there is no max or min

c) $f = x^2 - y^2$

1st derivative:-

$f_x = 2x$

$f_y = -2y$

FOR C.P.:-

$2x = 0$; $\boxed{x=0}$

$-2y = 0$ $\boxed{y=0}$

$\boxed{C.P. (0,0)}$

2nd derivative:-

$f_{xx} = 2$

$f_{yy} = -2$

$f_{xy} = 0$

$D = f_{xx}f_{yy} - f_{xy}^2 = 2(-2) - 0^2$

$\boxed{D = -4 < 0}$

Since $D<0$

(0,0) is a saddle point

Q3.

$f = 13 - 6x + x^2 + 4y + y^2$

FOR 1st dev:-

$f_x = 0 - 6 + 2x$

$\boxed{f_x = 2x - 6}$

$f_y = 0 - 0 + 0 + 4 + 2y$

$\boxed{f_y = 2y + 4}$

FOR C.P.:-

$2x - 6 = 0$

$\boxed{x = 3}$

$2y + 4 = 0$

$\boxed{y = -2}$

$\boxed{C.P. (3, -2)}$

FOR 2nd dev:

$f_{xx} = 2$

$f_{yy} = 2$

$f_{xy} = 0$

$D = f_{xx} f_{yy} - f_{xy}^2$

$= 2(2) - 0^2$

$\boxed{D = 4 > 0}$ $f_{xx} > 0$

Since $D > 0$ & $f_{xx} > 0$ thus C.P. $(3, -2)$ is a local min & no local max exist

Q4.

$f = 1 - 2x - x^2 + 4y - 2y^2$

FOR 1st dev:

$f_x = 0 - 2 - 2x$

$\boxed{f_x = -2 - 2x}$

$f_y = 0 + 0 + 0 + 4 - 4y$

$\boxed{f_y = 4 - 4y}$

FOR C.P.:-

$-2 - 2x = 0$

$-2 = 2x$

$\boxed{x = -1}$

$4 - 4y = 0$

$\boxed{y = 1}$

C.P. $(-1, 1)$

FOR 2nd derivatives

$f_{xx} = -2$

$f_{yy} = -4$

$f_{xy} = 0$

Now

$D = f_{xx} f_{yy} - f_{xy}^2$

$= -2(-4) - 0^2$

$\boxed{D = 8 > 0}$

$f_{xx} < 0$

Thus $(-1,1)$ is a local max

Questions from Q9-20 will be done in the same way