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x ———— x ——— x

Part 01

Q1. $f(t) = \begin{cases} e^t & t \leq 2 \\ 3 & t > 2 \end{cases}$

Applying laplace transform

$$\mathcal{L}(f(t)) = \int_0^2 e^t e^{-st} + \int_2^\infty 3e^{-t}$$

$$= \int_0^2 e^{-st+t} + \int_2^\infty 3e^{-t}$$

$$= \int_0^2 e^{t(1-s)} + \int_2^\infty 3e^{-t}$$

$$= \frac{e^{2(1-s)} - 1}{1-s} + \frac{3}{s}e^{-2s}$$

$$= \frac{-(1-e^{2(1-s)})}{1-s} + \frac{3e^{-2s}}{s}$$

$$\boxed{\mathcal{L}(F(t)) = \frac{1-e^{2(1-s)}}{s-1} + \frac{3e^{-2s}}{s}}$$

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Q2. $f(t) = 3+2t^2$

Applying Laplace transform

$$\mathcal{L}(f(t)) = \int_0^\infty (3+2t^2)\, e^{-st}$$

$$= \int_0^\infty 3e^{-st} + 2\int_0^\infty t^2 e^{-st} \quad -(A)$$

FOR $\int_0^\infty t^2 e^{-st}$:- $\quad \int u.v = u\int v - \int\int u.v'$

$$\int_0^\infty t^2 e^{-st} = \frac{-t^2 e^{-st}}{s} + 2\int \frac{e^{-st}}{s}\, t$$

$$= \frac{-t^2 e^{-st}}{s} + \frac{2}{s}\int e^{-st}\, t$$

$$= \frac{-t^2 e^{-st}}{s} + \frac{2}{s}\left(-\frac{t e^{-st}}{s} - \frac{1}{s^2} e^{-st}\right)$$

$$\boxed{\int_0^\infty t^2 e^{-st} = \frac{-t^2 e^{-st}}{s} - \frac{2t e^{-st}}{s^2} - \frac{2 e^{-st}}{s^3}}$$

Putting values in eq (A)

$$= 3\left[\frac{e^{-st}}{s}\right]_0^\infty + 2\left(-\frac{t^2 e^{-st}}{s} - \frac{2te^{-st}}{s^2} - \frac{2e^{-st}}{s^3}\right)_0^\infty$$

$$= -3\left[\frac{e^{-st}}{s}\right]_0^\infty + \frac{2}{s}\left[-t^2e^{-st} - \frac{2te^{-st}}{s} - \frac{2e^{-st}}{s^2}\right]_0^\infty$$

$$\cancel{= \frac{3}{s}(e^{-\infty} - 1) + \frac{2\lim}{s}(}$$

$$= \frac{-3}{s}(e^{-\infty} - 1) + 2\lim_{a\to\infty}\left(-\frac{t^2 e^{-st}}{s} - \frac{2te^{-st}}{s^2} - \frac{2e^{-st}}{s^3}\right)_0^a$$

$$= \frac{-3}{s}(-1) + 2\lim_{a\to\infty}\left[-\frac{a^2e^{-as}}{s} - \frac{2ae^{-as}}{s^2} - \frac{2e^{-as}}{s^3} - \left(-\frac{0e^0}{s} - \frac{2\cdot 0e^0}{s^2} - \frac{2e^0}{s^3}\right)\right]$$

$$= \frac{3}{s} + 2\left(\lim_{a\to\infty} -\frac{a^2e^{-as}}{s} - 2\lim_{a\to\infty}\frac{ae^{-as}}{s^2} - 2\lim_{a\to\infty}\frac{e^{-sa}}{s^3} + \frac{2}{s^3}\right)$$

$$= \frac{3}{s} + 2\left(\lim_{a\to\infty} \frac{-a^2}{se^{+as}} - 2\lim_{a\to\infty}\frac{a}{s^2e^{+as}} - 2\lim_{a\to\infty}\frac{1}{s^3e^{as}} + \frac{2}{s^3}\right)$$

$$= \frac{3}{s} + 2\left(-\frac{1}{\infty} - \frac{1}{\infty} - \frac{2}{\infty} + \frac{2}{s^3}\right)$$

$$\boxed{\mathcal{L}(3+2t^2) = \frac{3}{s} + \frac{4}{s^3}}$$ Ans.

Q3. $f(t) = 5\sin 3t - 17e^{-2t}$

Applying Laplace transform

$$\mathcal{L}(f(t)) = \int_0^\infty (5\sin 3t - 17e^{-2t})e^{-st}$$

$$= 5\int_0^\infty \sin 3t\, e^{-st} - 17\int_0^\infty e^{-2t-st}$$

$$= 5\int_0^\infty \sin 3t\, e^{-st} - 17\int_0^\infty e^{-t(2+s)} \quad \text{—(A)}$$

$$\therefore \int U.V = V\int U - \int\int U.V'$$

FOR $\int \sin 3t\, e^{-st}$:-

$$\int \sin 3t\, e^{-st} = -\frac{\sin 3t\, e^{-st}}{s} + \frac{3}{s}\int \cos 3t\, e^{-st}$$

$$I = -\frac{\sin 3t\, e^{-st}}{s} + \frac{3}{s}\left(-\frac{\cos 3t\, e^{-st}}{s} - \frac{3}{s}\int \sin 3t\, e^{-st}\right)$$

$$\therefore \int \sin 3t\, e^{-st} = I$$

$$I = -\frac{\sin 3t\, e^{-st}}{s} - \frac{3\cos 3t\, e^{-st}}{s} - \frac{9}{s^2} I$$

$$I + \frac{9}{s^2} I = -\frac{\sin 3t\, e^{-st}}{s} - \frac{3\cos 3t\, e^{-st}}{s}$$

$$I\left(1+\frac{9}{s^2}\right) = -\frac{\sin 3t\, e^{-st}}{s} - \frac{3\cos 3t\, e^{-st}}{s^2}$$

$$I\left(\frac{s^2+9}{s^2}\right) = -\frac{\sin 3t\, e^{-st}}{s} - \frac{3\cos 3t\, e^{-st}}{s^2}$$

$$\int I = \left(-\frac{\sin 3t\, e^{-st}}{s} - \frac{3\cos 3t\, e^{-st}}{s^2}\right)\left(\frac{s^2}{s^2+9}\right)$$

$$\int_0^\infty I = \frac{s^2}{s^2+9}\left[-\frac{\sin 3t\, e^{-st}}{s} - \frac{3\cos 3t\, e^{-st}}{s^2}\right]_0^\infty$$

$$= \frac{s^2}{s^2+9}\left(-\frac{\sin 3\infty\, e^{-\infty}}{s} - \frac{3\cos 3t\, e^{-\infty s}}{s^2} - \left(-\frac{\sin 3(0)\, e^0}{s} - \frac{3\cos 0\, e^0}{s^2}\right)\right)$$

$$= \frac{\cancel{s^2}}{s^2+9}\left(0+0+0+\frac{3}{\cancel{s^2}}\right)$$

$$\boxed{\int_0^\infty \sin 3t \cdot e^{-st} = \frac{3}{s^2+9}}$$

Putting value in (A)

$$= 5\left(\frac{3}{s^2+9}\right) - 17\left(\frac{e^{-t(2+s)}}{-(2+s)}\right)_0^\infty$$

$$= \frac{15}{s^2+9} + \frac{17}{2+s}\left(e^{-\infty(2+s)} - 1\right)$$

$$\boxed{\mathcal{L}(5\sin 3t - 17e^{-2t}) = \frac{15}{s^2+9} - \frac{17}{2+s}}$$

Ans

Qu. $f(t) = te^{4t}$

Apply laplace transform

$$\mathcal{L}(f(t)) = \int_0^\infty te^{4t} \cdot e^{-st}$$

$$= \int_0^\infty te^{4t-st} = \int_0^\infty te^{t(4-s)}$$

$$\because \int U.V = V\int U - \int\int U.V'$$

$$= \frac{te^{t(4-s)}}{4-s} - \int \frac{e^{t(4-s)}}{4-s}$$

$$= \frac{te^{t(4-s)}}{4-s} - \frac{1}{4-s}\left(\frac{e^{t(4-s)}}{(4-s)}\right)$$

$$= \frac{t e^{t(4-s)}}{4-s} - \frac{e^{t(4-s)}}{(4-s)^2}$$

$$= \frac{1}{4-s}\left(t e^{t(4-s)} - \frac{e^{t(4-s)}}{4-s}\right]_0^{\infty}$$

$$= \frac{1}{4-s}\left(t e^{-t(s-4)} - \frac{e^{-t(s-4)}}{4-s}\right)_0^{\infty}$$

$$= \frac{1}{4-s}\left(\infty e^{-\infty} - \frac{e^{-\infty}}{4-s} - \left(0 - \frac{e^0}{4-s}\right)\right)$$

$$= \frac{1}{4-s}\left(\frac{1}{4-s}\right)$$

$$= \frac{-1}{s-4} \times \frac{-1}{s-4}$$

$$\boxed{\mathcal{L}(t e^{4t}) = \frac{1}{(s-4)^2}}$$

Ans.

Part 02 (laplace Inverse)

Q_5 $\mathcal{L}^{-1}\left(\frac{1}{s(s^2+2s+5)}\right)$

$$\frac{1}{s(s^2+2s+5)} = \frac{A}{s} + \frac{Bs+C}{s^2+2s+5}$$

$$1 = A(s^2+2s+5) + Bs^2 + Cs$$

$$= As^2 + 2As + A5 + Bs^2 + Cs$$

$$1 = s^2(A+B) + s(2A+C) + 5A$$

$\boxed{A = \frac{1}{5}}$ $\quad A+B=0 \quad$ $2A+C=0$

$\boxed{B = -\frac{1}{5}}$ $\quad$ $\boxed{C = -\frac{2}{5}}$

$$= \mathcal{L}^{-1}\left(\frac{1}{5s} + \frac{-\frac{1}{5}s + -\frac{2}{5}}{s^2+2s+5}\right)$$

$$= \mathcal{L}^{-1}\left(\frac{1}{5s}\right) - \frac{1}{5}\mathcal{L}^{-1}\left(\frac{s+2}{s^2+2s+5}\right)$$

$$= \frac{1}{5}\mathcal{L}^{-1}\left(\frac{1}{s}\right) - \frac{1}{5}\mathcal{L}^{-1}\left(\frac{s+2}{s^2+2s+5}\right)$$

$$= \frac{1}{5} - \frac{1}{5} \mathcal{L}^{-1}\left(\frac{s+1+1}{s^2+2s+1+4}\right)$$

$$= \frac{1}{5} - \frac{1}{5} \mathcal{L}^{-1}\left(\frac{s+1+1}{(s+1)^2+4}\right)$$

$$= \frac{1}{5} - \frac{1}{5}\left[\mathcal{L}^{-1}\left(\frac{s+1}{(s+1)^2+4}\right) + \frac{1}{2}\mathcal{L}^{-1}\left(\frac{2}{(s+1)^2+4}\right)\right]$$

$$\because \mathcal{L}^{-1}(f(s-a)) = e^{at} f(t)$$

$$= \frac{1}{5} - \frac{1}{5}\left[e^{-t}\cos 2t + \frac{1}{2} e^{-t} \sin 2t\right]$$

$$\boxed{\mathcal{L}^{-1}\left(\frac{1}{s(s^2+2s+5)}\right) = \frac{1}{5} - \frac{1}{5} e^{-t}\cos 2t - \frac{1}{10} e^{-t}\sin 2t}$$

Ans.

OR

$$\boxed{= \frac{1}{5} - \frac{e^{-t}(2\cos 2t + \sin 2t)}{10}}$$

Ans.

Q6. $\mathcal{L}^{-1}\left(\frac{7s-1}{(s+1)(s+2)(s-3)}\right)$

Sol.

$$\frac{7s-1}{(s+1)(s+2)(s-3)} = \frac{A}{s+1} + \frac{B}{s+2} + \frac{C}{s-3}$$

$$7s-1 = A(s+2)(s-3) + B(s+1)(s-3) + C(s+1)(s+2)$$

Put $s=-2$

$$-14-1 = 0 + B(-2+1)(-2-3) + 0$$

$$-15 = 5B$$

$$\boxed{B=-3}$$

let $\boxed{s=3}$

$$7(3)-1 = 0+0+C(4)(5)$$

$$20 = 20C$$

$$\boxed{C=1}$$

let $\boxed{s=-1}$

$$-7-1 = A(1)(-4)$$

$$-8 = -4A$$

$$\boxed{A=2}$$

$$= \mathcal{L}^{-1}\left[\frac{2}{s+1} + \frac{-3}{s+2} + \frac{1}{s-3}\right]$$

$$= 2\mathcal{L}^{-1}\left(\frac{1}{s+1}\right) - 3\mathcal{L}^{-1}\left(\frac{1}{s+2}\right) + \mathcal{L}^{-1}\left(\frac{1}{s-3}\right)$$

$$\boxed{= 2e^{-t} - 3e^{-2t} + e^{3t}}$$

Ans.

Q7. $\mathcal{L}^{-1}\left\{\frac{s^2+9s+2}{(s+1)^2(s+3)}\right\}$

$$\frac{s^2+9s+2}{(s+1)^2(s+3)} = \frac{A}{s-1} + \frac{B}{(s-1)^2} + \frac{C}{s+3}$$

$$s^2+9s+2 = A(s-1)(s+3) + B(s+3) + C(s-1)^2$$

put $s=1$

$$1+9+2 = 0 + 4B + 0$$

$$\boxed{B=3}$$

$\boxed{\text{put } s=-3}$

$$9-27+2 = C(-4)^2$$

$$-16 = 16C$$

$$\boxed{C=-1}$$

$$s^2+9s+2 = A(s^2+2s-3) + B(s+3) + C(s^2-2s+1)$$

$$= As^2 - 2As - 3A + Bs + B3 + Cs^2 - 2sC + C$$

$$s^2+9s+2 = s^2(A+C) + s(2A+B-2C) - 3A + 3B + C$$

~~$A+C=0$~~

~~$A=-C$~~

~~$2A+B-2C=0$~~

~~$2A+B+2A=0$~~

~~$B=-4A$~~

$A+C=1$

$A=1-C$

$\boxed{A=2}$

$2A+B-2C=9$

$2(1-C)+B-2C=9$

$2-2C+B-2C=9$

$2-4C+B=9$

$B=7+4C$

$\boxed{B=3}$

$-3A+3B+C=2$

$-3(1-C)+3(7+4C)+C=2$

$-3+3C+21+12C+C=2$

$16C+18=2$

$\boxed{C=-1}$

$$= 2\mathcal{L}^{-1}\left(\frac{1}{s-1}\right) + 3\mathcal{L}^{-1}\left(\frac{1}{(s-1)^2}\right) - 1\mathcal{L}\left(\frac{1}{s+3}\right)$$

$$\boxed{= 2e^{t} + 3te^{t} - e^{-3t}}$$

Ans.

Q5. $\mathcal{L}^{-1}\left(\frac{2s^2+10s}{(s^2-2s+5)(s+1)}\right)$

$$\frac{2s^2+10s}{(s^2-2s+5)(s+1)} = \frac{A}{s+1} + \frac{Bs+C}{s^2-2s+5}$$

$$2s^2+10s = A(s^2-2s+5) + (Bs+C)(s+1)$$

$$2s^2+10s = As^2-2As+5A+Bs^2+Cs+Bs+C$$

$$2s^2+10s = s^2(A+B)+s(-2A+B+C)+5A+C$$

$A+B=2$

$\boxed{A=2-B}$

$\boxed{A=-1}$

~~$A+B=10$~~

$-2A+B+C=10$

$-2(2-B)+B+C=10$

$-4+2B+B+C=10$

$3B+C=14$

$\boxed{C=14-3B}$

$\boxed{C=5}$

$5A+C=0$

$5(2-B)+14-3B=0$

$10-5B+14-3B=0$

$24-8B=0$

$\boxed{B=3}$

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$$= \mathcal{L}^{-1}\left(\frac{-1}{s+1} + \frac{3s+5}{s^2-2s+5}\right)$$

$$= -\mathcal{L}^{-1}\left(\frac{1}{s+1}\right) + \mathcal{L}^{-1}\left(\frac{3s-3+8}{s^2-2s+5}\right)$$

$$= -\mathcal{L}^{-1}\left(\frac{1}{s-(-1)}\right) + \mathcal{L}^{-1}\left(\frac{3s-3}{s^2-2s+5}\right)$$

$$+ 8\mathcal{L}^{-1}\left(\frac{1}{s^2-2s+1+4}\right)$$

$$= -e^{-t} + 3\mathcal{L}^{-1}\left(\frac{s-1}{s^2-2s+5}\right) + 4\mathcal{L}^{-1}\left(\frac{2}{(s-1)^2+2^2}\right)$$

$$\boxed{= -e^{-t} + 3e^{t}\cos 2t + 4e^{t}\sin 2t}$$

Ans

Part 03:

Q9 $y' - 5y = e^{5x}$ $\quad y(0) = 0$

$$y' - 5y = e^{5x}$$

Applying laplace transform.

$$\mathcal{L}(y') - 5\mathcal{L}(y) = \mathcal{L}(e^{5x})$$

$$sF(s) - f(0) - 5F(s) = \frac{1}{s-5}$$

$$F(s)(s-5) = \frac{1}{s-5}$$

$$F(s) = \frac{1}{(s-5)^2}$$

taking laplace inverse

$$\mathcal{L}^{-1}(F(s)) = \mathcal{L}^{-1}\left(\frac{1}{(s-5)^2}\right)$$

$$y = \mathcal{L}^{-1}\left(\frac{1}{s^2}\right)_{s \to s-5}$$

$$y = xe^{5x}$$

Confirmation by Analytical method

$$y' - 5y = e^{5x}$$

$$IF = e^{-5x}$$

$$e^{-5x} \cdot y' - 5e^{-5x} y = \cancel{e^{5x}} \cdot \cancel{e^{-5x}}$$

$$\frac{d}{dx}(e^{-5x} \cdot y) = 1$$

$$\int \frac{d}{dx}(e^{-5x} y) = \int 1$$

$$e^{-5x} \cdot y = x + c$$

$$y = xe^{5x} + ce^{5x}$$

$$y(0) = 0$$

$$0 = 0 + ce^{0}$$

$$C = 0$$

$$\boxed{y = xe^{5x}}$$

Ans.

Q1a $y'+y = \sin x \quad y(0)=1$

$y'+y = \sin x$

~~Tak~~ Applying laplace transform.

$\mathcal{L}(y') + \mathcal{L}(y) = \mathcal{L}(\sin x)$

$sF(s) - f(0) + F(s) = \frac{1}{s^2+1}$

$(s+1)F(s) - 1 = \frac{1}{s^2+1}$

$(s+1)F(s) = \frac{1}{s^2+1} + 1$

$(s+1)F(s) = \frac{1+s^2+1}{s^2+1}$

$\boxed{F(s) = \frac{s^2+2}{(s^2+1)(s+1)}}$

$\frac{s^2+2}{(s^2+1)(s+1)} = \frac{As+B}{s^2+1} + \frac{C}{s+1}$

$s^2+2 = (As+B)(s+1) + C(s^2+1)$

$s^2+2 = As^2+As+Bs+B+Cs^2+C$

$s^2+2 = (A+C)s^2 + (A+B)s + (B+C)$

$A+C=1 \quad A+B=0 \quad B+C=2$

$\boxed{A=1-C} \quad 1-C+B=0 \quad B+C=2$

$\boxed{B=C-1} \quad C-1+C=2$

$\boxed{A=\frac{-1}{2}} \quad \boxed{B=\frac{1}{2}} \quad 2C=3$

$\boxed{C=3/2}$

$F(s) = \frac{-\frac{1}{2}s+1/2}{s^2+1} + \frac{\frac{3}{2}}{s+1}$

$F(s) = \frac{1}{2}\left(\frac{1-s}{s^2+1}\right) + \frac{3}{2}\left(\frac{1}{s+1}\right)$

Applying laplace inverse

$\mathcal{L}^{-1}(F(s)) = \frac{1}{2}\mathcal{L}^{-1}\left(\frac{1-s}{s^2+1}\right) + \frac{3}{2}\mathcal{L}^{-1}\left(\frac{1}{s+1}\right)$

$$= \frac{1}{2}\mathcal{L}^{-1}\left(\frac{1}{s^2+1}\right) - \frac{1}{2}\mathcal{L}^{-1}\left(\frac{s}{s^2+1}\right) + \frac{3}{2}\mathcal{L}^{-1}\left(\frac{1}{s+1}\right)$$

$$\boxed{y = \frac{1}{2}\sin x - \frac{1}{2}\cos x + \frac{3}{2}e^{-x}}$$

Confirmation by analytical method

$$y' + y = \sin x \qquad y(0) = 0 1$$

$$I.F. = e^{x}$$

$$e^{x}.y' + e^{x}.y = e^{x}\sin x$$

$$\frac{d}{dx}(e^{x}y) = \sin x . e^{x}$$

$$e^{x}.y = \int \sin x . e^{x} \quad \text{—(A)}$$

let $I = \int \sin x . e^{x}$

$I = \int \sin x . e^{x}$ $\quad (\because \int U.V = V\int U - \int\int U.V')$

$= \sin x . e^{x} - \int \cos x . e^{x}$

$= \sin x\, e^{x} - \cos x\, e^{x} + \int -\sin x\, e^{x}\,dx$

$I = \sin x\, e^{x} - \cos x\, e^{x} - I$

$2I = \sin x\, e^{x} - \cos x\, e^{x}$

$I = \frac{1}{2}\sin x\, e^{x} - \frac{1}{2}\cos x\, e^{x} + c$

(A) $\Rightarrow$ $e^{x}y = \frac{1}{2}\sin x\, e^{x} - \frac{1}{2}\cos x\, e^{x} + c$

$y = \frac{1}{2}\sin x - \frac{1}{2}\cos x + ce^{-x}$

$y(0) = 1$

$1 = 0 - \frac{1}{2} + ce^{0}$

$\boxed{c = 3/2}$

$$\boxed{y = \frac{1}{2}\sin x - \frac{1}{2}\cos x + \frac{3}{2}e^{-x}}$$

Q11. $y'' - y' = 2x \qquad y(0) = 1 \; ; \; y'(0) = 2$

Applying laplace transform

$$\mathcal{L}(y'') - \mathcal{L}(y') = \mathcal{L}(2x)$$

$$s^2F(s) - sF(0) - f'(0) - (sF(s) - f(0)) = \frac{2}{s^2}$$

$$s^2F(s) - s(1) - (-2) - (sF(s) - 1) = \frac{2}{s^2}$$

$$s^2 F(s) - sF(s) = s + 3 = \frac{2}{s^2}$$

$$s^2 F(s) - sF(s) = \frac{2}{s^2} + -3 + s$$

$$F(s) \times (s^2 - s) = \frac{2 - 3s^2 + s^3}{s^2}$$

$$F(s) = \frac{s^3 - 3s^2 + 2}{s^2(s^2 - s)}$$

$$F(s) = \frac{\cancel{s^3}}{\cancel{s^2}(s^2 - s)} + \frac{2 - 3s^2}{s^2(s^2 - s)}$$

$$F(s) = \frac{1}{s-1} + \frac{2 - 3s^2}{s^2(s^2 - s)}$$

$\therefore F(s) = \frac{1}{s-1} + \frac{2-3s^2}{s^3(s-1)}$

Now,

$$\frac{2-3s^2}{s^3(s-1)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s^3} + \frac{D}{s-1}$$

$$2-3s^2 = As^2(s-1) + Bs(s-1) + C(s-1) + Ds^3$$

Let $s=0$,

$$2 = 0 + 0 + C(0-1) + 0$$

$$\boxed{C = -2}$$

Put $s=1$,

$$2-3 = 0 + 0 + 0 + D$$

$$\boxed{D = -1}$$

$$2-3s^2 = As^3 - As^2 + Bs^2 - Bs + Cs - C + Ds^3$$

$$2-3s^2 = (A+D)s^3 + (B-A)s^2 + (C-B)s - C$$

$A - D = 0$ $\quad$ $B - A = 3$

$A - 1 = 0$ $\quad$ $-1 + B = 3$

$\boxed{A = 1}$ $\quad$ $\boxed{B = 4}$

$$F(s) = \cancel{\frac{1}{s-1}} + \frac{1}{s} - \frac{2}{s^2} - \frac{2}{s^3} - \cancel{\frac{1}{s-1}}$$

$$F(s) = \frac{1}{s} - \frac{2}{s^2} - \frac{2}{s^3}$$

Applying laplace inverse

$$y = \mathcal{L}^{-1}\left(\frac{1}{s}\right) - 2\mathcal{L}^{-1}\left(\frac{1}{s^2}\right) - \mathcal{L}^{-1}\left(\frac{2}{s^3}\right)$$

$$\boxed{y = 1 - 2x - x^2}$$

Ans

Proving Analytically,

$$y'' - y' = 2x$$

For y_c:-

$$y'' - y' = 0$$

$$D^2y - Dy = 0$$

$$D^2 - D = 0$$

$\boxed{D = 0}$ or $\boxed{D = 1}$

$$\boxed{y_c = C_1 + C_2 e^x}$$

FOR y_p:

$y_p = Ax + B$

$y_p = Ax^2 + Bx$

$y_p' = 2Ax + B$

$y_p'' = 2A$

$y_p'' - y_p' = 2x$

$2A - (2Ax + B) = 2x$

$2A - 2Ax - B = 2x$

$2A - B - 2Ax = 2x$

$-2A = 2$ $\qquad$ $2A - B = 0$

$\boxed{A = -1}$ $\qquad$ $2(-1) - B = 0$

$\boxed{B = -2}$

$\boxed{y_p = -x^2 - 2x}$

$y = y_c + y_p$

$y = C_1 + C_2 e^x - x^2 - 2x$

$y(0) = 1$

$1 = C_1 + C_2 e^0 - 0 - 0$

$1 = C_1 + C_2$

$y' = C_2 e^x - 2x - 2$

$y(0) = -2$

$-2 = C_2 e^0 - 2(0) - 2$

$C_2 = 0$

$C_1 = 1$

NOW,

$\boxed{y = 1 - x^2 - 2x}$

Proved!

Q12. $y'' - 2y' + 5y = -8e^{7-x}$

$y(7) = 2 \; ; \; y'(7) = 12$

Applying laplace transform

$\mathcal{L}(y'') - 2\mathcal{L}(y') + 5\mathcal{L}(y) = -8\mathcal{L}(e^{7-x})$

let $m = x - 7$

$s^2 F(s) - sf(0) - f'(0) - 2(sF(s) - f(0))$
$+ 5F(s) = -8\mathcal{L}(e^{-m})$

$s^2 F(s) - s(2) - 12 - 2sF(s) + 4 + 5F(s) = \frac{-8}{s+1}$

$(s^2 - 2s + 5)F(s) - 2s - 8 = \frac{-8}{s+1}$

$(s^2 - 2s + 5)F(s) = \frac{-8}{s+1} + 8 + 2s$

$(s^2 - 2s + 5)F(s) = \frac{8(s+1) + 2s(s+1) - 8}{s+1}$

$(s^2 - 2s + 5)F(s) = \frac{8s + 8 + 2s^2 + 2s - 8}{s+1}$

$F(s) = \frac{8s + 2s^2 + 2s}{(s+1)(s^2 - 2s + 5)}$

$F(s) = \frac{2s^2 + 10s}{(s+1)(s^2 - 2s + 5)}$

Copying the result from Question 05

$F(s) = \left\{ \frac{-1}{s+1} + \frac{3s+5}{s^2 - 2s + 5} \right\}$

taking inverse laplace

$y = \mathcal{L}^{-1}\left(\frac{-1}{s+1}\right) + \mathcal{L}^{-1}\left(\frac{3s - 3 + 8}{s^2 - 2s + 5}\right)$

$= -e^{-m} + 3\mathcal{L}^{-1}\left(\frac{s-1}{s^2 - 2s + 1 + 4}\right) + 8\mathcal{L}^{-1}\left(\frac{1}{s^2 - 2s + 5}\right)$

$= -e^{-m} + 3\mathcal{L}^{-1}\left(\frac{s-1}{(s-1)^2 + 4}\right) + 4\mathcal{L}^{-1}\left(\frac{2}{(s-1)^2 + 2^2}\right)$

$y = -e^{-m} + 3\cos 2m\, e^m + 4\sin 2m \cdot e^m$

$\because m = x - 7$

$\boxed{y = -e^{-(x-7)} + 3\cos(2x - 14)e^{x-7} + 4\sin(2x - 14)\, e^{x-7}}$

Now, proving with analytical method.

$$y'' - 2y' + 5y = -8e^{7-x}$$

FOR y_c:

$D = 1 \pm 2i$

$y_c = e^x(C_1 \cos 2x + C_2 \sin 2x)$

$y_1 = e^x \cos 2x \;;\; y_2 = e^x \sin 2x$

FOR y_p:

$g(x) = -8e^{7-x}$

$W_1 = -y_2\, g(x)$

$= -e^x \sin 2x\,(-8e^{7-x})$

$$\boxed{W_1 = 8e^7 \sin 2x}$$

$W_2 = y_1\, g(x)$

$= e^x \cos 2x\,(-8e^{7-x})$

$$\boxed{W_2 = 8e^7 \cos 2x}$$

$$W = \begin{vmatrix} e^x \cos 2x & e^x \sin 2x \\ e^x \cos 2x - 2\sin 2x\, e^x & e^x \sin 2x + 2e^x \cos 2x \end{vmatrix}$$

$= e^x \cos 2x\,(e^x \sin 2x + 2e^x \cos 2x)$
$\quad - e^x \sin 2x\,(e^x \cos 2x - 2\sin 2x\, e^x)$

$= e^{2x}(\sin 2x \cos 2x + \cos^2 2x)$
$\quad - e^{2x}(\sin 2x \cos 2x + 2\sin^2 2x)$

$= e^{2x}(\sin 2x \cancel{\cos 2x} + 2\cos^2 2x - \sin 2x \cancel{\cos 2x} + 2\sin^2 2x)$

$W = 2e^x(1)$

$$\boxed{W = 2e^x}$$

$U_1' = \dfrac{W_1}{W} = \dfrac{8e^7 \sin 2x}{2e^{2x}}$

$U_1' = \dfrac{8e^7}{2} \times \sin 2x\, e^{-2x}$

$U_1 = \dfrac{8e^7}{2} \int \sin 2x\, e^{-2x}$

let $I = \int \sin 2x\, e^{-2x}$

$\cancel{I = \int \sin 2x\, e^{-2x}}$

$\cancel{= \dfrac{-\sin 2x\, e^{-2x}}{2} - \int \dfrac{2\cos 2x\, e^{-2x}}{-2} = \int \dfrac{\cos 2x}{e^{x}}}$

$$I = \int \sin 2x \cdot e^{-2x}$$

$$= -\frac{\sin 2x \cdot e^{-2x}}{2} - \int \frac{e^{-2x}}{-2} \cdot \cos 2x \cdot 2$$

$$= -\frac{\sin 2x \cdot e^{-2x}}{2} + \int e^{-2x} \cos 2x$$

$$= -\frac{\sin 2x e^{-2x}}{2} + \frac{\cos 2x \cdot e^{-2x}}{(-2)} - \int \sin 2x \, e^{-2x}$$

$$2I = \frac{-\sin 2x e^{-2x}}{2} - \frac{\cos 2x e^{-2x}}{2}$$

$$\boxed{I = \frac{-\sin 2x e^{-2x}}{4} - \frac{\cos 2x e^{-2x}}{4}}$$

$$\boxed{u_1 = \frac{\cancel{8} e^{7x}}{4\cancel{2}} \left(\frac{-\sin 2x e^{-2x}}{4} - \frac{\cos 2x e^{-2x}}{4} \right)}$$

$$u_1 = \frac{\cancel{4} e^{7x}}{\cancel{4}} \left(-\sin 2x e^{-2x} - \cos 2x e^{-2x}\right)$$

$$\boxed{u_1 = e^{7x} \left(-\sin 2x e^{-2x} - \cos 2x \, e^{-2x}\right)}$$

$$u_2' = \frac{W_2}{W} = \frac{-8e^{?}\cos 2x}{2e^{2x}}$$

$$= -4e^{?}\cos 2x \cdot e^{-2x}$$

$$u_2 = -4e^{?}\int \cos 2x e^{-2x}$$

Let $I = \int \cos 2x e^{-2x}$

$$I = \int \cos 2x e^{-2x}$$

$$= \frac{\cos 2x\, e^{-2x}}{-2} - \int \frac{e^{-2x}}{-x} \cdot -\sin 2x(2)$$

$$= \frac{-\cos 2x e^{-2x}}{2} - \int e^{-2x}\sin 2x$$

$$= \frac{-\cos 2x e^{-2x}}{2} - \frac{\sin 2x\, e^{-2x}}{-2} - \int \cos 2x e^{-2x}$$

$$I + I = -\frac{\cos 2x e^{-2x}}{2} + \frac{\sin 2x e^{-2x}}{2}$$

$$\boxed{I = \frac{-\cos 2x e^{-2x}}{4} + \frac{\sin 2x e^{-2x}}{4}}$$

$$u_2 = -4e^{?}\left(-\frac{\cos 2x e^{-2x}}{4} + \frac{\sin 2x e^{-2x}}{4}\right)$$

$$\boxed{u_2 = e^{?-2x}(\cos 2x - \sin 2x)}$$

$$y_p = y_1 u_1 + y_2 u_2$$

$$= e^x\cos 2x\left(-e^{?-2x}(\sin 2x + \cos 2x)\right) + e^x \sin 2x\,(-\sin 2x + \cos 2x)e^{?-2x}$$

$$= -e^{?-x}(\cos 2x \sin 2x + \cos^2 2x) + e^{?x}(-\sin^2 2x + \sin 2x\cos 2x)$$

$$= e^{?-x}(-\cancel{\sin 2x\cos x} - \cos^2 2x - \sin^2 x + \cancel{\sin 2x\cos 2x})$$

$$= -e^{2x}(\sin^2 2x + \cos^2 2x)$$

$$\boxed{y_p = -e^{?-x}}$$

$$y = y_c + y_p$$

$$y = e^{x}(C_1\cos 2x + C_2 \sin 2x) - e^{?-x}$$

$$\boxed{y = e^{x}(C_1\cos 2x + C_2\sin 2x) - e^{?-x}}$$

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Now $y(7)=2$

$2 = e^7(C_1\cos 14 + C_2\sin 14) - e^{7-7}$

$1+2 = e^7(C_1\cos 14 + C_2\sin 14)$

$\boxed{3 = e^7(C_1\cos 14 + C_2\sin 14)}$ —(1)

$y' = e^x(C_1\cos 2x + C_2\sin 2x)$
$+e^x(-2C_1\sin 2x + 2C_2\cos 2x)$
$-e^{7-x}(-1)$

$y'(7) = 12$

$12 = e^7(C_1\cos 14 + C_2\sin 14)$
$+e^7(-2C_1\sin 14 + 2C_2\cos 14)$

$11 = e^7(C_1\cos 14 + C_2\sin 14$
$-2C_1\sin 14 + 2C_2\cos 14)$

$\frac{11}{e^7} = \cos 14(C_1+2C_2) + \sin 14(C_2 -$

$\Rightarrow \frac{3}{e^7} = C_1\cos 14 + C_2\sin 14$

$\frac{11}{e^7} - \frac{3}{e^7} = 2(C_2\cos 14 - C_1\sin 14)$

$\frac{8}{e^7} = 2(C_2\cos 14 - C_1\sin 14)$ —(2)

(1) $\Rightarrow \frac{3}{e^7} - C_1\cos 14 = C_2\sin 14$

$\frac{3}{e^7\sin 14} - \frac{C_1\cos 14}{\sin 14} = C_2$

Put in (2)

$\frac{8}{e^7} = 2\left(\left(\frac{3}{e^7\sin 14} - \frac{C_1\cos 14}{\sin 14}\right)\cos 14 - C_1\sin 14\right)$

$\frac{4}{e^7} = \frac{3\cos 14}{e^7\sin 14} - \frac{C_1\cos^2 14}{\sin 14} - C_1\sin 14$

$\frac{4}{e^7} = \frac{3\cos 14}{e^7\sin 14} - C_1\left(\frac{\cos^2 14 - \sin^2 14}{\sin 14}\right)$

$$\frac{4\sin 14}{e^7} = \frac{3\cos 14}{e^7} - C_1$$

$$\boxed{C_1 = \frac{3\cos 14 - 4\sin 14}{e^7}}$$

$$C_2 = \frac{3}{e^7\sin 14} - C_1\frac{\cos 14}{\sin 14}$$

$$C_2 = \frac{3}{e^7\sin 14} - \left(\frac{3\cos 14 - 4\sin 14}{e^7}\right)\frac{\cos 14}{\sin 14}$$

$$C_2 = \frac{3 - 3\cos^2 14 + 4\sin 14\cos 14}{e^7\sin 14}$$

$$C_2 = \frac{3\sin^2 14 + 4\sin 14\cos 14}{e^7\sin 14}$$

$$\boxed{C_2 = \frac{3\sin 14 + 4\cos 14}{e^7}}$$

$$y = e^x(C_1\cos 2x + C_2\sin 2x) - e^{7-x}$$

$$y = e^x\left(\left(\frac{3\cos 14 - 4\sin 14}{e^7}\right)\cos 2x + \left(\frac{3\sin 14 + 4\cos 14}{e^7}\right)\sin 2x\right) - e^{7-x}$$

$$y = e^{x-7}(3(\cos(2x-14)) + 4\sin(2x-14)) - e^{7-x}$$

$$\boxed{y = -e^{-x-7} + 3xe^{x-7}\cos(2(x-7)) + 4e^{x-7}\sin(2(x-7))}$$

⟵ THE END ⟶