Laplace Transform

Ex # 4.1

$k(s,t) \rightarrow f(x,y)$

$y = x^2 + 2x \ldots$ 'y' is replaced by 't' as the independent variable

$t = s^2 + 2s$ Same goes for 'x' to 's'

$$\boxed{\mathcal{L}\{f(t)\} = \int_0^\infty e^{-st} f(t) . dt}$$

// s will be used as a Constant

$e^{\infty} = \infty$ $e^{-\infty} = 0$

Ex # 01:-

$f(t) = 1$

$$\mathcal{L}\{1\} = \int_0^\infty e^{-st}(1) . dt = \int_0^\infty e^{-st} . dt$$

$$= \left[\frac{e^{-st}}{-s}\right]_0^\infty = \frac{-1}{s}\left[e^{-\infty} - e^{0}\right]$$

$$= \frac{-1}{s}[0 - 1] = 1/s$$

$$\boxed{\text{So } \mathcal{L}\{1\} = 1/s}$$

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$\mathcal{L}\{2\} = 2/s$

$\mathcal{L}\{3\} = 3/s$

Ex # 02 :-

$f(t) = t$

$\mathcal{L}\{f(t)\} = \int_0^\infty e^{-st}(t)\, dt$

$= \left[\frac{te^{-st}}{-s} \right]_0^\infty - \int_0^\infty \frac{(1)e^{-st}}{-s} . dt$

$= \left[\frac{\infty . e^{-\infty} - (0)(e^0)}{-s} \right] + \frac{1}{s} \int_0^\infty e^{-st} . dt$

$= 0 + \frac{1}{s} \left[\frac{e^{-st}}{-s} \right]_0^\infty = \frac{-1}{s^2} \left[e^{-st} \right]_0^\infty$

$= \frac{-1}{s^2} \left[e^{-\infty} - e^0 \right] = \frac{1}{s^2}$

$$\boxed{\mathcal{L}\{f(t)\} = \frac{1}{s^2}}$$

$$\mathcal{L}\{t^n\} = \frac{n!}{s^{n+1}}$$ General formula

Ex # 03 :-

$$f(t) = e^{at}$$

$$\mathcal{L}\{e^{at}\} = \int_0^\infty e^{-st}(e^{at})\,.dt$$

$$= \int_0^\infty e^{(-s+a)t}\,.dt$$

$$= \int_0^\infty e^{-(+s+a)t}\,.dt$$

$$= \int_0^\infty \underline{e^{-(s-a)t}}\,.dt$$

$$= \cancel{\frac{e^{-(s-a)t}}{-(s(}} \left[\frac{e^{-(s-a)t}}{-(s-a)}\right]_0^\infty$$

$$= \frac{e^{-\infty} - e^{0}}{-(s-a)} = \frac{-1}{-(s-a)}$$

$$\boxed{\mathcal{L}\{(e^{at})\} = \frac{1}{s-a}}$$

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Ex #04:-

$f(t) = \sin kt$

$\mathcal{L}\{\sin kt\} = \int_0^\infty e^{-st} (\sin kt) \, dt$

Solving using integration by parts

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$$= \int_0^\infty e^{-st} \left(\frac{e^{ikt} - e^{-ikt}}{2i} \right) . dl$$

$$= \frac{1}{2i} \int_0^\infty e^{-st+ikt} - e^{-st-ikt}$$

$$= \frac{1}{2i} \int_0^\infty e^{-(s-ik)t} - e^{-(s+ik)t} \, . dt$$

$$= \frac{1}{2i} \left[\frac{e^{-(s-ik)t}}{-(s-ik)} - \frac{e^{-(s+ik)t}}{-(s+ik)} \right]_0^\infty$$

$$= \frac{1}{2i} \left[\frac{e^{-\infty} - e^{0}}{-(s-ik} + \frac{e^{-\infty} - e^{0}}{(s+ik)} \right]$$

$$= \frac{1}{2i} \left[\frac{0-1}{-(s-ik)} + \frac{0-1}{s+ik} \right]$$

$$= \frac{1}{2i} \left[\frac{1}{s-ik} - \frac{1}{s+ik} \right]$$

$$\frac{1}{2i}\left[\frac{s+ik-s+ik}{s^2-i^2k^2}\right]=\frac{1}{2i}\left[\frac{2ik}{s^2+k^2}\right]$$

$$=\frac{k}{s^2+k^2}$$

$$\mathcal{L}\{\sin kt\}=\frac{k}{s^2+k^2}$$

$$e^{i\theta}=\cos\theta+i\sin\theta$$
$$e^{-i\theta}=\cos\theta-i\sin\theta$$
$$e^{i\theta}+e^{-i\theta}=2\cos\theta$$

$$\boxed{\cos\theta=\frac{e^{i\theta}+e^{-i\theta}}{2}}$$

$$e^{i\theta}-e^{-i\theta}=2i\sin\theta$$

$$\boxed{\sin\theta=\frac{e^{i\theta}-e^{-i\theta}}{2i}}$$

Since $\theta=kt$

So

$$\boxed{\sin(kt)=\frac{e^{ikt}-e^{-ikt}}{2i}}$$

$$\sinh(kt) = \frac{e^{kt} - e^{-kt}}{2}$$

$$\cosh(kt) = \frac{e^{kt} + e^{-kt}}{2}$$

$$\sin(kt) = \frac{e^{ikt} - e^{-ikt}}{2i}$$

$$\cos(kt) = \frac{e^{ikt} + e^{-ikt}}{2i}$$

in piece-wise function break the limits

Q) $f(t) \begin{cases} -1, & t < 1 \\ 1, & t \geq 1 \end{cases}$

$$= \int_0^1 e^{-st}(-1) + \int_1^\infty e^{-st}(1).dt$$