NAME: Bilal Ahmed Khan

SECTION: B

ROLL NO.: 20K-0183

DE-ASSIGNMENT NO. 02

1. $y''+4y'+3y=0$

$y''+4y'+3y=0$

$D^2y+4Dy+3y=0$

$D^2+4D+3=0$

$\boxed{D_1=-1}$ or $\boxed{D_2=-3}$

$$\boxed{y=C_1e^{-x}+C_2e^{-3x}}$$

Ans.

2. $y'''-y''+y'-y=0$

Sol

$y'''-y''+y'-y=0$

$D^3y-D^2y+Dy-y=0$

D^3-D^2+D-1

$\boxed{D_1=1}$ $\boxed{D_2=i}$ $\boxed{D_3=-i}$

$y=y_{c_1}+y_{c_2}$

$\boxed{y_{c_1}=C_1e^{x}}$

$y_{c_2}=e^{0}(C_2\cos x+C_3\sin x)$

$\boxed{y_{c_2}=C_2\cos x+C_3\sin x}$

$$\boxed{y_c=C_1e^{x}+C_2\cos x+C_3\sin x}$$

Ans.

3. $2x^2y'' + 3xy - 15y = 0$

$$2x^2y'' + 3xy' - 15y = 0$$

$$y = x^m$$

$$y' = m x^{m-1}$$

$$y'' = (m^2 - m)x^{m-2}$$

$$2\cancel{x^2}(m^2 - m)\frac{x^m}{\cancel{x^2}} + 3\cancel{x}m \cdot \frac{x^m}{\cancel{x}} - 15 x^m = 0$$

$$(2m^2 - 2m)x^m + 3m x^m - 15x^m = 0$$

$$x^m(2m^2 + m - 15) = 0$$

$x^m \neq 0$ $\qquad$ $2m^2 + m - 15 = 0$

$$\boxed{m_1 = 5/2} \quad \boxed{m_2 = -3}$$

$$\boxed{y = C_1 x^{5/2} + C_2 x^{-3}}$$

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4. $y'' - 3y' + 2y = x^2 e^x$ —(A)

FOR y_c:-

$y'' - 3y' + 2y = 0$

$D^2 y - 3Dy + 2y = 0$

$D^2 - 3D + 2 = 0$

$\boxed{D_1 = 1}$ $\boxed{D_2 = 2}$

$\boxed{y_c = c_1 e^x + c_2 e^{2x}}$

FOR y_p:-

$y_p = (Ax^2 + Bx + C)e^x$

$y_p = Ax^3 e^x + Bx^2 e^x + Cxe^x$

$y_p' = A(x^3 e^x + 3x^2 e^x) + B(x^2 e^x + 2xe^x)$
$+ C(xe^x + e^x)$

$y_p'' = A[x^3 e^x + 3x^2 e^x + 3(x^2 e^x + 2xe^x)]$
$+ B[x^2 e^x + 2xe^x + 2(xe^x + e^x)]$
$+ c(xe^x + 2e^x)$

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Putting y_p' & y_p'' & y_p in eq (A)

$$y'' - 3y' + 2y = x^2e^x$$

$$x^2e^x = Ax^3e^x + 3Ax^2e^x + 3Ax^2e^x + 6Axe^x + Bx^2e^x + 2Bxe^x + 2Bxe^x + 2Be^x + Cxe^x + 2Cxe^x - 3(Ax^3e^x + 3Ax^2e^x + Bx^2e^x + 2Bxe^x + Cxe^x + Ce^x) + 2(Ax^3e^x + Bx^2e^x + Cxe^x)$$

$$= \cancel{Ax^3e^x} + 6Ax^2e^x + 6Axe^x + \cancel{Bx^2e^x} + 4Bxe^x + 2Be^x + \cancel{Cxe^x} + \cancel{2Cxe^x} - 3Ax^3e^x - 9Ax^2e^x - \cancel{3Bx^2e^x} - 6Bxe^x - \cancel{3Cxe^x} - 3Ce^x + \cancel{2Ax^3e^x} + \cancel{2Bx^2e^x} + \cancel{2Cxe^x}$$

$$-x^2e^x$$

$$-3Ax^2e^x + 6Axe^x + 4Bxe^x - 6Bxe^x + 2Be^x - Ce^x = x^2e^x$$

$$(-3A)e^x x^2 + 6Axe^x + 4Bxe^x - 6Bxe^x + 4Be^x - Ce^x = x^2e^x$$

$$(-3A)x^2e^x + (6A - 2B)xe^x + (2B - C)e^x = x^2e^x$$

$$-3A = 1$$

$$\boxed{A = -1/3}$$

$$6A - 2B = 0$$

$$2\cancel{6}\left(-\tfrac{1}{\cancel{3}}\right) - 2B = 0$$

$$-\cancel{2} = \cancel{2}B$$

$$\boxed{B = -1}$$

$$2B - C = 0$$

$$2(-1) = C$$

$$\boxed{C = -2}$$

$$y_p = Ax^3e^x + Bx^2e^x + Cxe^x$$

$$\boxed{y_p = -\frac{1}{3}x^3e^x - x^2e^x + 2xe^x}$$

$$y = y_c + y_p$$

$$\boxed{y = C_1e^x + C_2e^{2x} - \frac{1}{3}x^3e^x - x^2e^x - 2xe^x}$$

Ans

5. $y'' + 4y = xe^x + x\sin 2x$

For y_c:-

$y'' + 4y = 0$

$D^2 y + 4y = 0$

$D^2 + 4 = 0$

$\boxed{D = \pm 2i}$

$y_c = c_1 \cos 2x + c_2 \sin 2x$

FOR y_p:-

$$y_p = (Ax+B)e^x + (Cx^2+Dx)\cos 2x + (Ex^2+Dx)\sin 2x$$

$$y_p' = (Ax+B)e^x + Ae^x + (Cx^2+Dx)(-2\sin 2x) + (2Cx+D)(\cos 2x) + (Ex^2+Dx)(2\cos 2x) + (2Ex+F)\sin 2x$$

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$y_p' = (Ax+B+A)e^x + (-2Cx^2-2Dx+2Ex+F)\sin 2x$
$+(2Cx+D+2Ex^2+2Fx)\cos 2x$

$$\boxed{\begin{aligned} y_p'' &= (Ax+A+B)e^x + Ae^x + (-2Cx^2-2Dx+2Ex+F)\,2\cos 2x \\ &+(-4Cx-2D+2E)\sin 2x - 2(2Cx+D+2Ex^2+2Fx)\sin 2x \\ &+(2C+4Ex+2F)\cos 2x \end{aligned}}$$

Putting the value of y_p, y_p' & y_p'' in Equation (A)

$xe^x + x\sin 2x = Axe^x + Ae^x + Be^x + Ae^x - 4Cx^2\cancel{\cos 2x}$
$-4Dx\cancel{\cos} 2x + 4Ex\cos 2x + 2F\cos 2x$
$-4Cx\sin 2x - 2D\sin 2x + 2E\sin 2x$
$-4Cx\sin 2x - 2D\sin 2x - 4Ex^2\cancel{\sin 2x}$
$-4Fx\cancel{\sin} 2x - 2C\cos 2x + 4Ex\cos 2x$
$+2F\cos 2x + 4Ae^x + 4Be^x +$
$+4Cx^2\cancel{\cos} 2x + 4D\cancel{x\cos} 2x + 4E\cancel{x^2\sin} 2x$
$+4Fx\cancel{\sin} 2x$

$5Ae^x + 5Be^x + 2Ae^x + 8Ex\cos 2x + 4F\cos 2x$
$-8Cx\sin 2x - 4D\sin 2x + E\sin 2x + 2C\cos 2x$
$= xe^x + x\sin 2x$

$xe^x + x\sin 2x = e^x(5B+2A) + xe^x(5A) + \cos 2x(4F+2C)$
$+\sin 2x(2E-4D) + (-8C)x\sin 2x + 8Ex\cos 2x$

$5A = 1$
$\boxed{A = 1/5}$

$5B+2A = 0$
$B = \frac{-2A}{5}$
$= -2\left(\frac{1}{5}\right)\left(\frac{1}{5}\right)$
$\boxed{B = \frac{-2}{25}}$

$-8C = 1$
$\boxed{C = \frac{-1}{8}}$

$8E = 0$
$\boxed{E = 0}$

$4F + 2C = 0$
$F = \frac{-2C}{4}$
$= -\cancel{2}\left(\frac{1}{8}\right)\frac{1}{\cancel{4}\,2}$
$\boxed{F = \frac{-1}{16}}$

$2E - 4D = 0$
$0 - 4D = 0$
$\boxed{D = 0}$

Putting the values of A, B, C, D, E & F in y_p

$$y_p = (Ax+B)e^x + (Cx^2+Dx)\cos 2x + (Ex^2+Fx)\sin 2x$$

$$y_p = \left(\frac{1}{5}x - \frac{2}{25}\right)e^x + \left(-\frac{1}{8}x^2+0\right)\cos 2x + \left(0+\frac{1}{16}x\right)\sin 2x$$

$$\boxed{y_p = \frac{1}{5}xe^x - \frac{2}{25}e^x - \frac{1}{8}x^2\cos 2x + \frac{1}{16}x\sin 2x}$$

Since

$$y = y_c + y_p$$

$$\boxed{y = C_1\cos 2x + C_2\sin 2x + \frac{1}{5}xe^x - \frac{2}{25}e^x - \frac{1}{8}x^2\cos 2x + \frac{1}{16}x\sin 2x}$$

6. $y'' - 2y' + y = xe^x \ln x$

FOR y_c:

$y'' - 2y' + y = 0$

$D^2 - 2D + 1 = 0$

$\boxed{D = 1}$

$$\boxed{y_c = (c_1 + x c_2) e^x}$$

FOR y_p:-

$$W = \begin{vmatrix} e^x & xe^x \\ e^x & xe^x + e^x \end{vmatrix}$$

$= (\cancel{xe^x} + e^x) e^x - \cancel{xe^{2x}}$

$\boxed{W = e^{2x}}$

$W_1 = -g(x) \cdot y_2$

$= -xe^x \ln x \cdot xe^x$

$\boxed{W_1 = -x^2 e^{2x} \ln x}$

$W_2 = g(x)\, y_1 = xe^x \ln x \cdot e^x$

$\boxed{W_2 = xe^{2x} \ln x}$

$u_1' = \frac{W_1}{W} = \frac{-x^2 \cancel{e^{2x}} \ln x}{\cancel{e^{2x}}}$

$u_1' = -x^2 \ln x$

$\int u \cdot v = v \int u - \int \int u \cdot v'$

$-u_1 = \ln x \cdot \left(\frac{x^3}{3}\right) - \int \frac{x^{\cancel{3}\,2}}{3} \cdot \frac{1}{\cancel{x}}$

$= \frac{\ln x \cdot x^3}{3} - \frac{1}{3} \int x^2$

$= \frac{x^3 \ln x}{3} - \frac{1}{3}\left(\frac{x^3}{3}\right)$

$-u_1 = \frac{x^3 \ln x}{3} - \frac{x^3}{9}$

$$\boxed{u_1 = -\frac{x^3 \ln x}{3} + \frac{x^3}{9}}$$

$$u_2' = \frac{W_2}{W} = \frac{x\cancel{e^{2x}}\ln x}{\cancel{e^{2x}}} = x\ln x$$

$$u_2 = \int x\ln x = \ln x\left(\frac{x^2}{2}\right) - \int \frac{x^2}{2}\left(\frac{1}{x}\right)$$

$$= \frac{x^2\ln x}{2} - \frac{1}{2}\int x$$

$$\boxed{u_2 = \frac{x^2\ln x}{2} - \frac{x^2}{4}}$$

$$y_p = y_1u_1 + y_2u_2$$

$$= e^x\left(-\frac{x^3}{3}\ln x + \frac{x^3}{9}\right) + xe^x\left(\frac{x^2\ln x}{2} - \frac{x^2}{4}\right)$$

$$= e^x\left(-\frac{x^3\ln x}{3} + \frac{x^3\ln x}{2} + \frac{x^3}{9} - \frac{x^3}{4}\right)$$

$$y_p = e^x\left(\frac{x^3\ln x}{6} - \frac{5}{36}x^3\right)$$

$$y = y_c + y_p$$

$$\boxed{y = (C_1 + xC_2)e^x + e^x\left(\frac{x^3\ln x}{6} - \frac{5}{36}x^3\right)}$$

7. $x^2y'' - xy' + y = x^3$ —(A)

FOR y_c:-

$$x^2y'' - xy' + y = 0$$

let,

$$y = x^m$$

$$y' = mx^{m-1}$$

$$y'' = (m^2 - m)x^{m-2}$$

$$\cancel{x^2}(m^2 - m)\frac{x^m}{\cancel{x^2}} - \cancel{x}.\frac{mx^m}{x} + x^m = 0$$

$$x^m(m^2 - m - m + 1) = 0$$

$$x^m(m^2 - 2m + 1) = 0$$

$x^m \neq 0$ $\quad m^2 - 2m + 1 = 0$

$$\boxed{m = 1}$$

$$\boxed{y_c = C_1x + C_2x\ln x}$$

FOR y_c:-

(A) $\Rightarrow$ $x^2y'' - xy' + y = x^3$

Converting into std. form

$$y'' - \frac{y'}{x} + \frac{y}{x^2} = x$$

$$y_1 = x \; ; \; y_2 = x\ln x \; ; \; g(x) = x$$

FOR W:-

$$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$$

$$= \begin{vmatrix} x & x\ln x \\ 1 & x(1/x) + \ln x \end{vmatrix}$$

$$= x + x\ln x - x\ln x$$

$$\boxed{W = x}$$

For W_1:

$$W_1 = -g(x)y_2$$

$$= -x(x\ln x)$$

$$\boxed{W_1 = -x^2\ln x}$$

FOR W_2:-

$$W_2 = g(x)y_1 = x(x)$$

$$\boxed{W_2 = x^2}$$

Now,

$$u_1' = \frac{W_1}{W} = \frac{-x^2\ln x}{x} = -x\ln x$$

$$\int u_1' = -\int x\ln x$$

$$\int U.V = V\int U - \int \int U.V'$$

$$-u_1 = \ln x . \frac{x^2}{2} - \int \frac{x^2}{2}\left(\frac{1}{x}\right)$$

$$= \frac{x^2\ln x}{2} - \frac{1}{2}\int x$$

$$-u_1 = \frac{1}{2}x^2\ln x - \frac{x^2}{4}$$

$$u_1 = -\left(\frac{1}{2}x^2\ln x - \frac{x^2}{4}\right)$$

$$\boxed{u_1 = \frac{x^2}{4} - \frac{1}{2}x^2\ln x}$$

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$u_2' = \frac{W_2}{W} = \frac{x^2}{x} = x$

$$\boxed{u_2 = \frac{x^2}{2}}$$

$y_p = y_1 u_1 + y_2 u_2$

$= -x\left(\frac{1}{2}x^2 \ln x - \frac{1}{4}x^2\right) + x\ln x \cdot \frac{x^2}{2}$

$= \cancel{-\frac{1}{2}x^3 \ln x} - \frac{1}{4}x^3 + \cancel{\ln x \frac{x^3}{2}}$

$$\boxed{y_p = -\frac{1}{4}x^3}$$

$y = y_c + y_p$

$$\boxed{y = c_1 x + c_2 x \ln x + \frac{1}{4}x^3}$$

8. $y'' - 4y' - 12y = 2t^3 - t + 3$

FOR y_c:

$y'' - 4y' - 12y = 0$

$D^2 y - 4Dy - 12y = 0$

$D^2 - 4D - 12 = 0$

$\boxed{D_1 = 6}$ $\boxed{D_2 = -2}$

FOR y_p:

$y_p = At^3 + Bt^2 + Ct + D$

$y_p' = 3At^2 + 2Bt + C$

$y_p'' = 6At + 2B$

Putting the values on y_p, y_p' & y_p''

$6At + 2B - 12At^2 - 8Bt - 4C - 12At^3$
$- 12Bt^2 - 12Ct - 12D = 2t^3 - t + 3$

$(-12A)t^3 + (-12A - 12B)t^2 + (-8B - 12C + 6A)t$
$+ 2B - 4C - 12D = 2t^3 - t + 3$

$-12A = 2$

$\boxed{A = -\frac{1}{6}}$

$-12A - 12B = 0$

$-\cancel{12}^{2}\left(-\frac{1}{\cancel{6}}\right) = 12B$

$\boxed{B = \frac{1}{6}}$

$-8B - 12C + 6A = -1$

$-\frac{4}{3} - 12C - 1 = -1$

$-12C = -\cancel{1} + \cancel{1} + \frac{4}{3}$

$-12C = \frac{4}{3}$

$C = \frac{-4}{\cancel{36}\,9}$

$\boxed{C = -\frac{1}{9}}$

$2B - 4C - 12D = 3$

$\frac{1}{3} + \frac{4}{9} - 12D = 3$

$-12D = 3 - \frac{1}{3} - \frac{4}{9}$

$-12D = \frac{20}{9}$

$D = \frac{-20}{108}$

$\boxed{D = -\frac{5}{27}}$

$y_p = Al^3 + Bl^2 + Cl + D$

$\boxed{y_p = -\frac{1}{6}l^3 + \frac{1}{6}l^2 - \frac{1}{9}l - \frac{5}{27}}$

Ans

9. $y'' + 5y' + 6y = 2x$ — (A)

FOR y_c:

$y'' + 5y' + 6y = 0$

$D^2y + 5Dy + 6y = 0$

$D^2 + 5D + 6 = 0$

$\boxed{D_1 = -2}$ $\boxed{D_2 = -3}$

$$\boxed{y_c = C_1 e^{-3x} + C_2 e^{-2x}}$$

FOR y_p:

$y_p = Ax + B$

$y_p' = A$

$y_p'' = 0$

Putting y_p, y_p', y_p'' in (A)

(A) ⇒ $y'' + 5y' + 6y = 2x$

$0 + 5A + 6(Ax + B) = 2x$

$5A + 6B + 6Ax = 2x$

Comparing co-efficients

$6A = 2$

$\boxed{A = \frac{1}{3}}$

$5A + 6B = 0$

$\frac{5}{3} + 6B = 0$

$6B = -\frac{5}{3}$

$\boxed{B = -\frac{5}{18}}$

$$\boxed{y_p = \frac{x}{3} - \frac{5}{18}}$$

$y = y_c + y_p$

$$\boxed{y = C_1 e^{-2x} + C_2 e^{-3x} + \frac{x}{3} - \frac{5}{18}}$$

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Q10. $y'' + 5y' - 9y = e^{-2x} + 2 - x \quad -(A)$

FOR y_c:

$$y'' + 5y' - 9y = 0$$

$$D^2y + 5Dy - 9y = 0$$

$$D^2 + 5D - 9 = 0$$

$$\boxed{D_1 = 1.40} \quad \boxed{D_2 = -6.40}$$

$$\boxed{y_c = C_1 e^{1.40x} + C_2 e^{-6.40x}}$$

FOR y_p:

$$y_p = Ae^{-2x} + Bx + C$$

$$y_p' = -2Ae^{-2x} + B$$

$$y_p'' = 4Ae^{-2x}$$

Putting y_p, y_p' & y_p'' in eq (A)

(A) => $y'' + 5y' - 9y = e^{-2x} + 2 - x$

$$4Ae^{-2x} + 5(-2Ae^{-2x} + B) - 9(Ae^{-2x} + Bx + C) = e^{-2x} + 2 - x$$

$$4Ae^{-2x} - 10Ae^{-2x} + 5B - 9Ae^{-2x} - 9Bx - 9C = e^{-2x} + 2 - x$$

$-15A = 1$

$$\boxed{A = \frac{-1}{15}}$$

$-9B = -1$

$$\boxed{B = \frac{1}{9}}$$

$5B - 9C = 2$

$\frac{5}{9} - 9C = 2$

$-9C = 2 - \frac{5}{9}$

$-9C = \frac{13}{9}$

$$\boxed{C = -\frac{13}{81}}$$

$$y_p = \cancel{\#} Ae^{-2x} + Bx + C$$

$$\boxed{y_p = \frac{-1}{15} e^{-2x} + \frac{1}{9} x - \frac{13}{81}}$$

$y = y_p + y_c$

$$\boxed{y = C_1 e^{1.40x} + C_2 e^{6.40x} - \frac{1}{15} e^{-2x} + \frac{1}{9} x - \frac{13}{81}}$$

Ans.

11. $y'' - 100y = 9t^2 e^{10t} + \cos t - t\sin t \rightarrow (A)$

FOR y_c:-

$y'' - 100y = 0$

$D^2 y - 100y = 0$

$D^2 - 100 = 0$

$\boxed{D_1 = 10}$ $\boxed{D_2 = -10}$

$$\boxed{y_c = C_1 e^{10t} + C_2 e^{-10t}}$$

FOR y_p:-

$y_p = (At^3 + Bt^2 + Ct)e^{10t} + (Dt+E)\cos t + (Ft+G)\sin t$

$y_p' = (3At^2 + 2Bt + C)e^{10t} + 10e^{10t}(At^3 + Bt^2 + Ct) + D\cos t - (Dt+E)\sin t + F\sin t + (Ft+G)\cos t$

$y_p'' = (6At + 2B)e^{10t} + 10e^{10t}(3At^2 + 2Bt + C) + 10e^{10t}(3At^2 + 2Bt + C) + 100e^{10t}(At^3 + Bt^2 + Ct) - D\sin t - D\sin t - (Dt+E)\cos t + F\cos t + F\cos t - (Ft+G)\sin t$

$y_p'' = (6At + 2B)e^{10t} + 20e^{t}(3At^2 - 2Bt + C) + 100e^{10t}(At^2 + Bt^2 + Ct) - 2D\sin t - (Dt+E)\cos t + 2F\cos t - (Ft+G)\sin t$

Putting the value of y_p, y_p' & y_p'' in (A)

$y'' - 100y = 9t^2 e^{10t} + \cos t - t\sin t$

$(6At + 2B)e^{10t} + 20e^{10t}(3At^2 + 2Bt + C) + 100e^{10t}(At^3 + Bt^2 + Ct) - 2D\sin t - (Dt+E)\cos t + 2F\cos t - (Ft+G)\sin x - 100[(At^3 + Bt^2 + Ct)e^{10t} + (Dt+E)\cos t + (Ft+G)\sin t]$

$= t^2 e^{10t}(60A + 100B - 100B)$
$+ te^{10t}(40B + 6A + 100C - 100C) + \cos t(-E + 2F + \ldots$
$+ t\cos t(-t - 100t) + \sin t(-G - 100G - 2D)$
$+ t\sin t(-F + 100F) + e^{10t}(2B + 20C)$

$= t^2 e^{10t}(60A) + te^{10t}(40B + 6A) + \cos t(2F - 101E)$
$t\cos t(-101D) + \sin t(-2D - 101G) + t\sin t(-101F)$
$+ e^{10t}(2B + 20C) = 9t^2 e^{10t} + \cos t - t\sin t$

$60A = 9$

$\boxed{A = 3/20}$

$2F - 101E = 1$

$\boxed{F = \frac{1 + 101E}{2}}$

$F = \frac{1 + 101E}{2}$

$\frac{1}{101} = \frac{1 + 101E}{2}$

$\frac{2}{101} - 1 = 101E$

$\frac{-99}{101} = 101E$

$\boxed{E = \frac{-99}{10201}}$

$-101F = -1$

$\boxed{F = \frac{1}{101}}$

$40B + 6A = 0$

$40B = \frac{-9}{10}$

$\boxed{B = \frac{-9}{400}}$

$2B + 20C = 0$

$\frac{-9}{200} + 20C = 0$

$20C = \frac{9}{200}$

$\boxed{C = \frac{9}{4000}}$

$-101D = 0$

$\boxed{D = 0}$

$-2D - 101G = 0$

$0 - 101G = 0$

$\boxed{G = 0}$

$y_p = At^3 e^{10t} + Bt^2 e^{10t} + Cte^{10t} + Dt\cos t + E\cos t$
$+ Ft\sin t + G\sin t$

$= \frac{3}{20} t^3 e^{10t} - \frac{9}{400} t^2 e^{10t} + \frac{9}{4000} te^{10t} + 0 - \frac{99}{10201}\cos t$
$+ \frac{1}{101} t\sin t + 0$

$$\boxed{y_p = \frac{3}{20} t^3 e^{10t} - \frac{9}{400} t^2 e^{10t} + \frac{9}{4000} te^{10t} - \frac{99}{10201}\cos t + \frac{t}{101}\sin t}$$

$y = y_c + y_p$

$$\boxed{y = C_1 e^{10t} + C_2 e^{-10t} + \frac{3}{20} t^3 e^{10t} - \frac{9}{400} t^2 e^{10t} + \frac{9}{4000} te^{10t} - \frac{99}{10201}\cos t + \frac{t}{101}\sin t}$$

AG.

12. $y'' - 2y' + 2y = e^x \tan x$

FOR y_c:-

$y'' - 2y' + 2y = 0$

$D^2y - 2Dy + 2y = 0$

$D^2 - 2D + 2 = 0$

$\boxed{D = 1 \pm i}$

$y_c = e^x \{ C_1 \cos x + C_2 \sin x \}$

$\boxed{y_c = C_1 e^x \cos x + C_2 e^x \sin x}$

FOR y_p:-

$y_1 = e^x \cos x$

$y_2 = e^x \sin x$

$g(x) = e^x \tan x$

$$W = \begin{vmatrix} w_1 & w_2 \\ w_1' & w_2' \end{vmatrix}$$

$$= \begin{vmatrix} e^x \cos x & e^x \sin x \\ e^x \cos x - e^x \sin x & e^x \sin x + e^x \cos x \end{vmatrix}$$

$= e^x \cos x (e^x \sin x + e^x \cos x)$
$- e^x \sin x (e^x \cos x - e^x \sin x)$

$= e^{2x} \cancel{\cos x \sin x} + e^{2x} \cos^2 x - e^{2x} \cancel{\cos x \sin x}$
$+ e^{2x} \sin^2 x$

$= e^{2x} \cos^2 x + e^{2x} \sin^2 x = e^{2x} (\cos^2 x + \sin^2 x)$

$\boxed{W = e^{2x}}$

$W_1 = -g(x) y_2 = -e^x \tan x . e^x \sin x$

$\boxed{W_1 = -e^{2x} \tan x \sin x}$

$W_2 = g(x) y_1 = e^x \tan x . e^x \cos x$

$\boxed{W_2 = e^{2x} \tan x \cos x}$

$u_1' = \frac{W_1}{W} = \frac{-\cancel{e^{2x}} \tan x \sin x}{\cancel{e^{2x}}} = \tan x \sin x$

$-u_1' = \frac{\sin x}{\cos x} . \sin x = \frac{\sin^2 x}{\cos x}$

$$-u_1' = \frac{1-\cos^2 x}{\cos x}$$

$$-u_1' = \frac{1}{\cos x} - \cos x$$

$$-\int u_1' = \int \frac{1}{\cos x} - \int \cos x$$

$$\boxed{u_1 = -\ln(\sec x + \tan x) + \sin x}$$

$$u_2' = \frac{W_2}{W} = \frac{\cancel{e^{2x}} \tan x \cos x}{\cancel{e^{2x}}}$$

$$u_2' = \frac{\sin x}{\cancel{\cos x}} \cdot \cancel{\cos x}$$

$$\int u_2' = \int \sin x$$

$$\boxed{u_2 = -\cos x}$$

$$y_p = y_1 u_1 + y_2 u_2$$

$$= e^x \cos x \left[-\ln(\sec x + \tan x) + \sin x\right]$$

$$+ e^x \sin x(-\cos x)$$

$$= -e^x \cos x\left[\ln(\sec x + \tan x)\right] + e^x \cancel{\cos x \sin x}$$

$$- e^x \cancel{\cos x \sin x}$$

$$\boxed{y_p = -e^x \cos x \ln(\sec x + \tan x)}$$

$$y = y_c + y_p$$

$$\boxed{y = c_1 e^x \cos x + c_2 e^x \sin x - e^x \cos x \ln(\sec x + \tan x)}$$

B. $x^2y'' - 4xy' + 6y = 2x^4 + x^2$

Std form:

$$y'' - \frac{4y'}{x} + \frac{6y}{x^2} = 2x^2 + 1$$

FOR y_c:-

$$x^2y'' - 4xy' + 6y = 0$$

Put $y = x^m$

$y' = mx^{m-1}$

$y'' = (m^2 - m)x^{m-2}$

$$\cancel{x^2}(m^2-m)\frac{x^m}{\cancel{x^2}} - 4\cancel{x}m\frac{x^m}{\cancel{x}} + 6x^m = 0$$

$x^m(m^2 - m - 4m + 6) = 0$

$x^m(m^2 - 5m + 6) = 0$

$x^m \neq 0$ $\boxed{m_1 = 3}$ $\boxed{m_2 = 2}$

$$\boxed{y_c = c_1x^2 + c_2x^3}$$

FOR y_p:-

$\boxed{y_1 = x^2}$; $\boxed{y_2 = x^3}$; $\boxed{g(x) = 2x^2 + 1}$

$$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = \begin{vmatrix} x^2 & x^3 \\ 2x & 3x^2 \end{vmatrix}$$

$= 3x^4 - 2x^4$

$\boxed{W = x^4}$

$W_1 = -g(x)y_2 = -(2x^2+1)x^3$

$\boxed{W_1 = -2x^5 - x^3}$

$W_2 = g(x)y_1 = (2x^2+1)x^2$

$\boxed{W_2 = 2x^4 + x^2}$

$$u_1' = \frac{W_1}{W} = -\frac{(2x^5 - x^3)}{x^4}$$

$-u_1' = 2x + \frac{1}{x}$

$-\int u_1' = 2\int x + \int \frac{1}{x}$

$-u_1 = \frac{2x^2}{2} + \ln x$

$\boxed{u_1 = -(x^2 + \ln x)}$

$$U_2' = \frac{W_2}{W} = \frac{2x^4 + x^2}{x^4}$$

$$U_2' = 2 + \frac{1}{x^2}$$

$$U_2 = 2x + (-1)x^{-1}$$

$$\cancel{U_2 = 2x +} \quad \boxed{U_2 = 2x - \frac{1}{x}}$$

$$y_p = U_1 y_1 + U_2 y_2$$

$$= (-x^2 - \ln x)x^2 + (2x - \frac{1}{x})x^3$$

$$= -x^4 - x^2 \ln x + 2x^4 - x^2$$

$$\boxed{y_p = x^4 - x^2(\ln x + 1)}$$

$$y = y_c + y_p$$

$$\boxed{y = C_1 x^2 + C_3 x^3 + x^4 - x^2(\ln x + 1)}$$

14. $x^2y'' + 10xy' + 8y = x^2$

FOR y_c:-

std form:- $y'' + \frac{10y'}{x} + \frac{8y}{x^2} =$

$x^2y'' + 10xy' + 8y = 0$

let $y = x^m$

$y' = m x^{m-1}$

$y'' = (m^2 - m) x^{m-2}$

$\cancel{x^2}(m^2-m)\frac{x^m}{\cancel{x^2}} + 10\cancel{x}.m.\frac{x^m}{\cancel{x}} + 8x^m = 0$

$x^m(m^2 - m + 10m + 8) = 0$

$x^m \neq 0$ $m^2 + 9m + 8 = 0$

$\boxed{m_1 = -1}$ $\boxed{m_2 = -8}$

$\boxed{y_c = C_1 x^{-1} + C_2 x^{-8}}$

FOR y_p:-

$y_1 = x^{-1}$; $y_2 = x^{-8}$; $g(x) = 1$

$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$

$= \begin{vmatrix} x^{-1} & x^{-8} \\ -x^{-2} & -8x^{-9} \end{vmatrix} = -8x^{-10} + x^{10}$

$\boxed{W = -7x^{-10}}$

$W_1 = -g(x)y_2 = -x^{-8}$

$W_2 = g(x)y_1 = x^{-1}$

$u_1' = \frac{W_1}{W} = \frac{-x^{-8}}{-7x^{-10}} = \frac{1}{7}x^2$

$\int u_1' = \frac{1}{7}\frac{x^3}{(3)}$

$\boxed{u_1 = \frac{x^3}{21}}$

$$U_2' = \frac{x^{-1}}{-7x^{-10}}$$

$$-U_2' = \frac{1}{7}x^9$$

$$-\int U_2' = \frac{1}{7}\frac{x^{10}}{10}$$

$$-U_2 = \frac{x^{10}}{70}$$

$$\boxed{U_2 = \frac{-x^{10}}{70}}$$

$$y_p = y_1 U_1 + y_2 U_2$$

$$= x^{-1}\cdot\frac{x^3}{21} + x^{-8}\cdot\left(\frac{-x^{10}}{70}\right)$$

$$= \frac{x^2}{21} - \frac{x^2}{70}$$

$$\boxed{y_p = \frac{x^2}{30}}$$

$$y = y_c + y_p$$

$$\boxed{y = C_1 x^{-1} + C_2 x^{-8} + \frac{x^2}{30}}$$

15. $x^2y'' - 3xy' + 13y = 4 + 3x$

Std form:

$$y'' - \frac{3y'}{x} + \frac{13y}{x^2} = \frac{4}{x^2} + \frac{3}{x}$$

For y_c:-

$$x^2y'' - 3xy' + 13y = 0$$

let $y = x^m$

$y' = mx^{m-1}$

$y'' = (m^2 - m)x^{m-2}$

$$\cancel{x^2}(m^2 - m)\frac{x^m}{\cancel{x^2}} - 3\cancel{x}\frac{mx^m}{\cancel{x}} + 13x^m = 0$$

$$x^m(m^2 - m - 3m + 13) = 0$$

$x^m \neq 0$ $\quad m^2 - 4m + 13 = 0$

$$\boxed{m = 2 \pm 3i}$$

$$\boxed{y = x^2\{C_1\cos 3\ln x + C_2\sin 3\ln x\}}$$

FOR y_p:

$$y_1 = x^2 \cos 3\ln x$$
$$y_2 = x^2 \sin 3\ln x$$
$$g(x) = \frac{4}{x^2} + \frac{3}{x}$$

$$W = \begin{vmatrix} x^2 \cos 3\ln x & x^2 \sin(3\ln x) \\ 2x\cos(3\ln x) - x^2 \sin 3\ln x \cdot \frac{3}{x} & 2x \sin(3\ln x) + x^2\cos(3\ln x)\cdot\frac{3}{x} \end{vmatrix}$$

$$= x^2 \cos(3\ln x)\left[2x \sin(3\ln x) + 3x\cos(3\ln x)\right]$$
$$- x^2 \sin(3\ln x)\left[2x \cos(3\ln x) + 3x\sin(3\ln x)\right]$$

$$= \cancel{2x^3 \sin(3\ln x)\cos(3\ln x)} + 3x^3 \cos^2(3\ln x)$$
$$- \cancel{2x^3 \sin(3\ln x)\cos(3\ln x)} + 3x^3 \sin^2(3\ln x)$$
$$= 3x^3\left(\cos^2(3\ln x) + \sin^2(3\ln x)\right)$$

$$\boxed{W = 3x^3}$$

$$W_1 = -g(x)\, y_2$$

$$= -\sin(3\ln x)\cdot x^2 \cdot \left(\frac{4}{x^2} + \frac{3}{x}\right)$$

$$= -\sin(3\ln x)\cdot \cancel{x^2}\left(\frac{4 + 3x}{\cancel{x^2}}\right)$$

$$\boxed{W_1 = -\left[\sin(3\ln x)(4+3x)\right]}$$

$$W_2 = g(x)\, y_1$$
$$= \cancel{x^2}\cos(3\ln x)\left(\frac{4+3x}{\cancel{x^2}}\right)$$
$$\boxed{W_2 = \cos(3\ln x)(4+3x)}$$

$$u_1' = \frac{W_1}{W} = -\frac{(\sin(3\ln x)\cdot(4+3x))}{3x^3}$$

$$u_1' = \frac{-4\sin(3\ln x)}{3x^3} - \frac{3\sin(3\ln x)}{x^2}$$

By integrating Both sides & solving

$$\boxed{u_1 = \frac{(39x + 80)\sin(3\ln x) + (117x + 120)\cos(3\ln}{390x^2}}$$

$U_2 = g(x) y_1$

$= \left(\frac{4+3x}{x^2}\right)$

$$U_2' = \frac{W_2}{W} = \frac{(4+3x)\cos(3\ln x)}{3x^3}$$

Integrating both sides we get

$$U_2 = \frac{(117x+120)\sin(3\ln x) + (-39x-80)\cos(3\ln x)}{390x^2}$$

$y_p = y_1 U_1 + y_2 U_2$

$$= \cancel{x^2}\cos(3\ln x)\left\{\frac{(39x+80)\sin(3\ln x) + (117x+120)\cos(3\ln x)}{390\cancel{x^2}}\right.$$

$$+ x^2\sin(3\ln x)\left\{\frac{(117x+120)\sin(3\ln x) + (-39x-80)\cos(3\ln x)}{390x^2}\right\}$$

* After simplification

$$\boxed{y_p = \frac{4}{13} + \frac{3}{10}x}$$

$y = y_c + y_p$

$$\boxed{y = x^2\left[C_1\cos(3\ln x) + C_2\sin(3\ln x)\right] + \frac{4}{13} + \frac{3}{10}x}$$

Ans.

Q16 $x^3y''' - 3x^2y'' + 6xy' - 6y = 3 + \ln x^3$

$$x^3y''' - 3x^2y'' + 6xy' - 6y = 3 + 3\ln x$$

Std form:

$$y''' - \frac{3y''}{x} - \frac{6y'}{x^2} - \frac{6y}{x^3} = \frac{3 + 3\ln x}{x^3}$$

FOR y_c:-

$$x^3y''' - 3x^2y'' + 6xy' - 6y = 0$$

$$\boxed{y = x^m}$$

$$\boxed{y' = mx^{m-1}}$$

$$\boxed{y'' = m(m-1)x^{m-2} = (m^2-m)x^{m-2}}$$

$$y''' = \cancel{m}(m^2-m)(m-2)x^{m-3}$$

$$y''' = (m^3 - 2m^2 - m^2 + 2m)x^{m-3}$$

$$\boxed{y''' = (m^3 - 3m^2 + 2m)x^{m-3}}$$

Now,

$$\cancel{x^3}(m^3 - 3m^2 + 2m)\frac{x^m}{\cancel{x^3}} - 3\cancel{x^2}(m^2-m)\frac{x^m}{\cancel{x^2}} + 6x\,m\frac{x^m}{x} - 6x^m = 0$$

$$(m^3 - 3m^2 + 2m - 3m^2 + 3m + 6m - 6)x^m = 0$$

$$(m^3 - 6m^2 + 11m - 6)x^m = 0$$

Either,

$x^m \neq 0$

$$m^3 - 6m^2 + 11m - 6 = 0$$

$\boxed{m_1 = 1}$ $\boxed{m_2 = 2}$ $\boxed{m_3 = 3}$

$$\boxed{y_c = c_1x + c_2x^2 + c_3x^3}$$

FOR y_p:

$\boxed{y_1 = x}$ $\boxed{y_2 = x^2}$ $\boxed{y_3 = x^3}$

$$\boxed{g(x) = \frac{3 + 3\ln x}{x^3}}$$

$$W = \begin{vmatrix} y_1 & y_2 & y_3 \\ y_1' & y_2' & y_3' \\ y_1'' & y_2'' & y_3'' \end{vmatrix}$$

$$W_1 = \begin{vmatrix} 0 & y_2 & y_3 \\ 0 & y_2' & y_3' \\ g(x) & y_2'' & y_3'' \end{vmatrix}$$

$$W_1 = g(x)\begin{vmatrix} y_2 & y_3 \\ y_2' & y_3' \end{vmatrix}$$

$$\boxed{W_1 = g(x)\{y_2y_3' - y_2'y_3\}}$$

$$W_2 = \begin{vmatrix} y_1 & 0 & y_3 \\ y_1' & 0 & y_3' \\ y_1'' & g(x) & y_3'' \end{vmatrix}$$

$$= -g(x) \begin{vmatrix} y_1 & y_3 \\ y_1' & y_3' \end{vmatrix}$$

$$\boxed{W_2 = -g(x) \{ y_1 y_3' - y_1' y_3 \}}$$

$$W_3 = \begin{vmatrix} y_1 & y_2 & 0 \\ y_1' & y_2' & 0 \\ y_1'' & y_2'' & g(x) \end{vmatrix}$$

$$= g(x) \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$$

$$\boxed{W_3 = g(x) (y_1 y_2' - y_1' y_2)}$$

Now putting the values.

$$W = \begin{vmatrix} x & x^2 & x^3 \\ 1 & 2x & 3x^2 \\ 0 & 2 & 6x \end{vmatrix}$$

$$= x \begin{vmatrix} 2x & 3x^2 \\ 2 & 6x \end{vmatrix} - 1 \begin{vmatrix} x^2 & x^3 \\ 2 & 6x \end{vmatrix}$$

$$= x(12x^2 - 6x^2) - (6x^3 - 2x^3)$$

$$= x(6x^2) - (4x^3)$$

$$= 6x^3 - 4x^3$$

$$\boxed{W = 2x^3}$$

$$W_1 = g(x) \{ y_2 y_3' - y_2' y_3 \}$$

$$= \frac{3+3\ln x}{x^3} \{ x^2(3x^2) - 2x(x^3) \}$$

$$= \frac{3+3\ln x}{x^3} (3x^4 - 2x^4) = \frac{3+3\ln x}{\cancel{x^3}} (x^{4})$$

$$\boxed{W_1 = x (3+3\ln x)}$$

$$W_2 = -g(x) \{ y_1 y_3' - y_1' y_3 \}$$

$$= -\left(\frac{3+3\ln x}{x^3}\right) \{ x(3x^2) - 1(x^3) \}$$

$$= -\left(\frac{3+3\ln x}{x^3}\right)(3x^3 - x^3) = -\left(\frac{3+3\ln x}{\cancel{x^3}}\right)(2\cancel{x^3})$$

$$\boxed{W_2 = -6 - 6\ln x}$$

$$W_3 = g(x) \{ y_1 y_2' - y_1' y_2 \}$$

$$= \left(\frac{3+3\ln x}{x^3}\right) \{ x(2x) - 1(x^2) \}$$

$$= \left(\frac{3+3\ln x}{x^3}\right)(2x^2 - x^2) = \left(\frac{3+3\ln x}{x^{\cancel{3}\,1}}\right) \times \cancel{x^2}$$

$$\boxed{W_3 = (3+3\ln x)x^{-1}}$$

$$U_1' = \frac{W_1}{W} = \frac{x(3+3\ln x)}{2x^{3}2}$$

$$= \frac{3}{2x^2} + \frac{3}{2}\frac{\ln x}{x^2}$$

$$\int U_1' = \frac{3}{2}\int \frac{1}{x^2} + \frac{3}{2}\int \ln x \cdot x^{-2}$$

$$= \frac{3}{2}\frac{(x)^{-1}}{(-1)} + \frac{3}{2}\left\{\ln x \int x^{-2} - \int \int x^{-2} \cdot \frac{d}{dx}(\ln x)\right\}$$

$$= \frac{-3}{2x} + \frac{3}{2}\left\{\ln x \frac{(x^{-1})}{-1} - \int \frac{x^{-1}}{(-1)} \times \frac{1}{x}\right\}$$

$$= \frac{-3}{2x} + \frac{3}{2}\left\{-\frac{\ln x}{x} + \int \frac{1}{x^2}\right\}$$

$$= \frac{-3}{2x} + \frac{3}{2}\left\{-\frac{\ln x}{x} + \frac{(x^{-1})}{-1}\right)\Big\}$$

$$= \frac{-3}{2x} + \frac{3}{2}\left(-\frac{\ln x}{x} - \frac{1}{x}\right)$$

$$= -\frac{3}{2x} - \frac{3\ln x}{2x} - \frac{3}{2x}$$

$$\boxed{U_1 = -\frac{3}{x} - \frac{3}{2}\frac{\ln x}{x}}$$

$$U_2' = \frac{W_2}{W} = \frac{-6-6\ln x}{2x^3} = \frac{2(-3-3\ln x)}{2x^3}$$

$$= \frac{-3}{x^3} - \frac{3\ln x}{x^3}$$

$$\int U_2' = -3\int \frac{1}{x^3} - 3\int \ln x \cdot x^{-3}$$

$$= -3\frac{(x)^{-2}}{-2} - 3\left\{\ln x \int x^{-3} - \int \int x^{-3} \cdot \frac{d}{dx}\ln x\right\}$$

$$= \frac{3}{2x^2} - 3\left\{\ln x \frac{(x^{-2})}{(-2)} - \int\left(\frac{x^{-2}}{(-2)} \cdot \frac{1}{x}\right)\right\}$$

$$= \frac{3}{2x^2} - 3\left\{-\frac{\ln x}{2x^2} + \frac{1}{2}\int \frac{1}{x^3}\right\}$$

$$= \frac{3}{2x^2} - 3\left\{-\frac{\ln x}{2x^2} + \frac{1}{2}\frac{(x^{-2})}{(-2)}\right\}$$

$$= \frac{3}{2x^2} - 3\left\{-\frac{\ln x}{2x^2} - \frac{1}{4x^2}\right\}$$

$$= \frac{3}{2x^2} + \frac{3}{2}\frac{\ln x}{x^2} + \frac{3}{4x^2}$$

$$\boxed{U_2 = \frac{9}{4x^2} + \frac{3}{2}\frac{\ln x}{x^2}}$$

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$$U_3' = \frac{(3+3\ln x)x^{-1}}{2x^3}$$

$$= \frac{3}{2x^4} + \frac{3}{2}\frac{\ln x}{x^4}$$

$$\int U_3' = \frac{3}{2}\int \frac{1}{x^4} + \frac{3}{2}\int \ln x \cdot x^{-4}$$

$$= \frac{\cancel{3}}{2}\frac{(x^{-3})}{(-\cancel{3})} + \frac{3}{2}\left\{\ln x \int x^{-4} - \int \left[\int x^{-4} \cdot \frac{d}{dx}(\ln x)\right]\right\}$$

$$= \frac{-1}{2x^3} + \frac{3}{2}\left[\ln x\left(\frac{x^{-3}}{-3}\right) - \int \frac{(x^{-3})}{-3}\cdot\frac{1}{x}\right]$$

$$= \frac{-1}{2x^3} + \frac{3}{2}\left(-\frac{\ln x}{3x^3} + \frac{1}{3}\int\frac{1}{x^4}\right)$$

$$= \frac{-1}{2x^3} + \frac{3}{2}\left(-\frac{\ln x}{3x^3} + \frac{1}{3}\frac{(x^{-3})}{-3}\right)$$

$$= \frac{-1}{2x^3} + \frac{3}{2}\left(-\frac{\ln x}{3x^3} - \frac{1}{9x^3}\right)$$

$$= \frac{-1}{2x^3} - \frac{\ln x}{2x^3} - \frac{1}{6x^3}$$

$$\boxed{U_3 = \frac{-2}{3x^3} - \frac{\ln x}{2x^3}}$$

$$y_p = y_1U_1 + y_2U_2 + y_3U_3$$

$$= x\left(-\frac{3}{x} - \frac{3}{2}\frac{\ln x}{x}\right) + x^2\left(\frac{15}{4x^2} + \frac{3}{2}\frac{\ln x}{x^2}\right) + x^3\left(-\frac{2}{3x^3} - \frac{\ln x}{2x^3}\right)$$

$$= -3 - \frac{3\cancel{\ln x}}{\cancel{2}} + \frac{9}{4} + \frac{3\cancel{\ln x}}{\cancel{2}} - \frac{2}{3} - \frac{\ln x}{2}$$

$$\boxed{y_p = \frac{-17}{12} - \frac{\ln x}{2}}$$

$$y = y_c + y_p$$

$$\boxed{y = C_1x + C_2x^2 + C_3x^3 - \frac{\ln x}{2} - \frac{17}{12}}$$

Q17. $y'' - 2y' + y = \frac{1}{x}e^x$

$y(1) = 0$; $y'(1) = 1$

FOR y_c:-

$y'' - 2y' + y = 0$

$D^2y - 2Dy + y = 0$

$D^2 - 2D + 1 = 0$

$\boxed{D = 1}$

$$\boxed{y_c = (c_1 + c_2 x)e^x}$$

FOR y_p:-

$g(x) = \frac{1}{x}e^x$; $y_1 = e^x$; $y_2 = xe^x$

$$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = \begin{vmatrix} e^x & xe^x \\ e^x & xe^x + e^x \end{vmatrix}$$

$= e^x(xe^x + e^x) - xe^x(e^x)$

$\cancel{W = x^2e^x + xe^x - xe^{2x}}$

$W = \cancel{xe^{2x}} + e^{2x} - \cancel{xe^{2x}}$

$\boxed{W = e^{2x}}$

$W_1 = -g(x)y_2$

$= \frac{1}{\cancel{x}}e^x \times \cancel{x}e^x$

$\boxed{W_1 = e^{2x}}$

$W_2 = g(x)y_1 = \frac{1}{x}e^x \times e^x$

$\boxed{W_2 = \frac{e^{2x}}{x}}$

$u_1' = \frac{W_1}{W} = \frac{\cancel{e^{2x}}}{\cancel{e^{2x}}}$

$u_1' = 1$

$\boxed{u_1 = x}$

$u_2' = \frac{W_2}{W} = \frac{\cancel{e^{2x}} \cdot x^{-1}}{\cancel{e^{2x}}} = \frac{1}{x}$

$\boxed{u_2 = \ln x}$

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$y_p = y_1 u_1 + y_2 u_2$

$y_p = e^x(x) + xe^x(\ln x)$

$\boxed{y_p = xe^x + xe^x \ln x}$

$y = y_c + y_p$

$$\boxed{y = e^x c_1 + c_2 e^x x + xe^x + xe^x \ln x}$$

$y' = c_1 e^x + c_2(xe^x + e^x) + xe^x + e^x$
$+ xe^x\left(\frac{1}{x}\right) + \ln x\{xe^x + e^x\}$

$$\boxed{y' = c_1 e^x + c_2 xe^x + c_2 e^x + xe^x + 2e^x + x\ln x e^x + \ln x e^x}$$

Now, $y(1) = 0$

$0 = ec_1 + c_2 e + e$

$0 = e(c_1 + c_2 + 1)$

$\boxed{c_2 = -1 - c_1}$

$y'(1) = 1$

$y' = c_1 e^x + c_2 xe^x + c_2 e^x + xe^x + 2e^x$
$+ x\ln x e^x + \ln(x) e^x$

$1 = c_1 e + c_2 e + c_2 e + e + 2e$

$1 = c_1 e + 2c_2 e + 3e$

$1 - 3e = c_1 e + 2c_2 e$

$1 - 3e = e(c_1 + 2c_2)$

$\frac{1-3e}{e} = c_1 + 2c_2$

$\frac{1-3e}{e} = c_1 + 2(-1 - c_1)$

$\frac{1-3e}{e} = c_1 - 2 - 2c_1$

$\frac{1-3e}{e} = -2 - c_1$

$\frac{3e-1}{e} = 2 + c_1$

$\frac{3e-1}{e} - 2 = c_1$

$\frac{3e-1-2e}{e} = c_1$

$$\boxed{\frac{e-1}{e} = c_1}$$

$$C_2 = -1 - \left(\frac{e-1}{e}\right)$$

$$= \frac{-e - e + 1}{e}$$

$$\boxed{C_2 = \frac{1 - 2e}{e}}$$

$$y = e^x \left(\frac{e-1}{e}\right) + \left(\frac{1-2e}{e}\right) e^x . x + x e^x + x e^x \ln x$$

$$= e^{x-1}(e-1) + e^{x-1}(1-2e)x + x e^x + x e^x \ln x$$

$$\cancel{= e^{x-1}\left(e-1+(1-2e)x\right)+}$$

$$= e^{x-1}(e-1) + e^{x-1}(1-2e)x + x e^x (\ln x + 1)$$

$$\boxed{y = e^{x-1}(e-1+x-2e.x) + x e^x \ln x}$$

~~16. $x^3y'' - 3x^2y$~~

18. $y'' + 4y = \sin^2 2x$

$y(1)=0 \; ; \; y'(0)=0$

FOR y_c:-

$y'' + 4y = 0$

$D^2y + 4Dy = 0$

$D^2 + 4 = 0$

$\boxed{D = \pm 2i}$

$\boxed{y_c = c_1 \cos 2x + c_2 \sin 2x}$

FOR y_p:-

$y_1 = \cos 2x \; ; \; y_2 = \sin 2x \; ; \; g(x) = \sin^2 2x$

$W = \begin{vmatrix} \cos 2x & \sin 2x \\ -2\sin 2x & 2\cos 2x \end{vmatrix}$

$= 2\cos^2 2x + 2\sin^2 2x$

$= 2(\cos^2 2x + \sin^2 2x)$

$\boxed{W = 2}$

$W_1 = -g(x)y_2$

$= -\sin^2(2x)\sin 2x$

$\boxed{W_1 = -\sin^3 2x}$

$W_2 = g(x)\,y_1$

$\boxed{W_2 = \sin^2 2x \cdot \cos 2x}$

NOW, $u_1' = \frac{W_1}{W} = \frac{-\sin^3 2x}{2}$

$\int u_1' = -\frac{1}{2}\int \sin^3 2x$

$= -\frac{1}{2}\left(-\frac{\cos(2x)}{2} + \frac{\cos^3(2x)}{6}\right)$

$\boxed{u_1 = \frac{1}{4}\left(\cos(2x) - \frac{\cos^3(2x)}{3}\right)}$

$$u_2' = \frac{w_2^x}{w} = \frac{+\sin^2 2x \cos 2x}{2}$$

$$\int u_2' = \frac{1}{2}\int (\sin 2x)^2 \cos 2x$$

let $\sin 2x = u$

$$\frac{du}{dx} = \cos 2x \cdot 2$$

$$\frac{du}{\cos 2x \cdot 2} = dx$$

$$u_2 = \frac{1}{4}\int u^2 \cdot \cancel{\cos 2x} \cdot \frac{du}{\cancel{\cos 2x}}$$

$$u_2 = \frac{1}{4}\int u^2 du$$

$$= \frac{1}{4}\left(\frac{u^3}{3}\right)$$

$$\boxed{u_2 = \frac{1}{12}\sin^3 2x}$$

$$y_p = y_1 u_1 + y_2 u_2$$

$$= \cos 2x\left(\frac{\cos 2x}{4} - \frac{\cos^3 2x}{12}\right) + \sin 2x\left(\frac{\sin^3 2x}{12}\right)$$

$$= \frac{3\cos^2 2x - \cos^4 2x}{12} + \frac{\sin^4 2x}{12}$$

$$\boxed{y_p = \frac{3\cos^2 2x - \cos^4 2x + \sin^4 2x}{12}}$$

$$y = y_c + y_p$$

$$y = c_1 \cos 2x + c_2 \sin 2x + \frac{3\cos^2 2x - \cos^4 2x + \sin^4 2x}{12}$$

$$\boxed{y = \frac{12c_1 \cos 2x + 12c_2 \sin 2x + 3\cos^2 2x - \cos^4 2x + \sin^4 2x}{12}}$$

$$\boxed{y' = \frac{-24c_1 \sin 2x + 24c_2 \cos 2x + 12\cos 2x \sin 2x - 8\cos^3 2x \sin x + 8\sin^3 2x \cos 2x}{12}}$$

Now $y(0)=0$;

$\cos(0)=1 \quad \sin(0)=0$

$$0 = \frac{12C_1+3-1}{12}$$

$$12C_1+2=0$$

$$\boxed{C_1 = \frac{-1}{16}}$$

Now $y'(0)=0$

$$0 = \frac{24C_2}{12}$$

$$\boxed{C_2=0}$$

Putting the values of C_1 & C_2

$$y = -\frac{1}{6}\cos 2x + \frac{3\cos^2 2x - \cos^4 2x + \sin^4 2x}{12}$$

$$= -\frac{1}{6}\cos 2x + \frac{3\cos^2 2x - \cos^4 2x + (\sin^2 2x)^2}{12}$$

$$= -\frac{1}{6}\cos 2x + \frac{3\cos^2 2x - \cos^4 2x + (-\cos^2 2x + 1)^2}{12}$$

$$= -\frac{1}{6}\cos 2x + \frac{3\cos^2 2x - \cancel{\cos^4 2x} + \cancel{\cos^4 2x} - 2\cos^2 2x + 1}{12}$$

$$= -\frac{1}{6}\cos 2x + \frac{\cos^2 2x + 1}{12}$$

$$= -\frac{1}{6}\cos 2x + \frac{\cos^2 2x + \cos^2 2x + \sin^2 2x}{12}$$

$$\boxed{y = -\frac{1}{6}\cos 2x + \frac{2\cos^2 2x + \sin^2 2x}{12}}$$

Ans

Q19. $y'' - 6y' - 7y = -9e^{-2x}$

$y(0) = -2;\ y'(0) = -13$

FOR y_c:-

$y'' - 6y' - 7y = 0$

$D^2y - 6Dy - 7y = 0$

$D^2 - 6D - 7 = 0$

$\boxed{D_1 = 7}$ $\boxed{D_2 = -1}$

$$\boxed{y_c = C_1 e^{7x} + C_2 e^{-x}}$$

FOR y_p:-

$y_1 = e^{7x};\ y_2 = e^{-x};\ g(x) = -9e^{-2x}$

$$W = \begin{vmatrix} e^{7x} & e^{-x} \\ 7e^{7x} & -e^{-x} \end{vmatrix}$$

$= -e^{6x} - 7e^{6x}$

$$\boxed{W = -8e^{6x}}$$

$W_1 = -g(x)\, y_2 = +9e^{-2x} \cdot e^{-x}$

$$\boxed{W_1 = +9e^{-3x}}$$

$W_2 = g(x)\, y_1 = -9e^{-2x} \cdot e^{7x}$

$$\boxed{W_2 = -9e^{5x}}$$

$u_1' = \frac{W_1}{W} = \frac{9e^{-3x}}{-8e^{6x}} = -\frac{9}{8} e^{-9x}$

$-\int u_1' = \frac{9}{8}\int e^{-9x} = \frac{9}{8} \frac{e^{-9}}{(-9)}$

$+u_1 = +\frac{e^{-9}}{8}$

$$\boxed{u_1 = \frac{e^{-9}}{8}}$$

$u_2' = \frac{W_2}{W} = \frac{-9e^{5x}}{-8e^{6x}} = \frac{9}{8} e^{-x}$

$\int u_2' = \frac{9}{8}\int e^{-x}$

$$\boxed{u_2 = -\frac{9}{8} e^{-x}}$$

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Q19. $y'' - 6y' - 7y = -9e^{-2x}$

$y(0) = -2$; $y'(0) = -13$

FOR y_c:-

$y'' - 6y' - 7y = 0$

$D^2y - 6Dy - 7y = 0$

$D^2 - 6D - 7 = 0$

$\boxed{D_1 = 7}$ $\boxed{D_2 = -1}$

$$\boxed{y_c = C_1 e^{7x} + C_2 e^{-x}}$$

FOR y_p:-

$y_1 = e^{7x}$; $y_2 = e^{-x}$; $g(x) = -9e^{-2x}$

$$W = \begin{vmatrix} e^{7x} & e^{-x} \\ 7e^{7x} & -e^{-x} \end{vmatrix}$$

$= -e^{6x} - 7e^{6x}$

$$\boxed{W = -8e^{6x}}$$

$W_1 = -g(x)\, y_2 = +9e^{-2x} \cdot e^{-x}$

$$\boxed{W_1 = +9e^{-3x}}$$

$W_2 = g(x) y_1 = -9e^{-2x} \cdot e^{7x}$

$$\boxed{W_2 = -9e^{5x}}$$

$u_1' = \frac{W_1}{W} = \frac{9e^{-3x}}{-8e^{6x}} = -\frac{9}{8} e^{-9x}$

$-\int u_1' = \frac{9}{8} \int e^{-9x} = \frac{9}{8} \frac{e^{-9}}{(-9)}$

$+u_1 = + \frac{e^{-9}}{8}$

$$\boxed{u_1 = \frac{e^{-9}}{8}}$$

$u_2' = \frac{W_2}{W} = \frac{-9e^{5x}}{-8e^{6x}} = \frac{9}{8} e^{-x}$

$\int u_2' = \frac{9}{8} \int e^{-x}$

$$\boxed{u_2 = -\frac{9}{8} e^{-x}}$$

$y_p = y_1 u_1 + y_2 u_2$

$y_p = e^{7x}\left(\frac{e^{-9x}}{8}\right) + e^{-x}\left(-\frac{9}{8}e^{-x}\right)$

$= \frac{e^{-2x}}{8} - \frac{9}{8}e^{-2x}$

$\boxed{y_p = -e^{-2x}}$

$y = y_c + y_p$

$\boxed{y = c_1 e^{7x} + c_2 e^{-x} - e^{-2x}}$

$\boxed{y' = 7c_1 e^{7x} - c_2 e^{-x} + 2e^{-2x}}$

Now $y(0) = -2$

$-2 = c_1 + c_2 - 1$

$\boxed{-1 - c_2 = c_1}$

Now $y'(0) = -13$

$-13 = 7c_1 - c_2 + 2$

$-13 = -7 - 7c_2 - c_2 + 2$

$-8 = +8c_2$

$\boxed{c_2 = 1}$

$\boxed{c_1 = -2}$

$\boxed{y = -2e^{+7x} + e^{-x} - e^{-2x}}$

A.

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20. $y''-4y'+4y=2e^{2x}-12\cos 3x-5\sin 3x$ — (A)

$y(0)=-2;\ y'(0)=4$

FOR y_c:-

$y''-4y'+4y=0$

$D^2y-4Dy+4y=0$

$D^2-4D+4=0$

$\boxed{D=2}$

$\boxed{y_c = C_1e^{2x}+C_2xe^{2x}}$

FOR y_p:-

$y_p = Ax^2e^{2x}+B\cos 3x+C\sin 3x$

$y_p' = A(2xe^{2x}+x^2 2e^{2x})-3B\sin 3x+3C\cos 3x$

$\boxed{y_p'' = A(2e^{2x}+4xe^{2x}+4xe^{2x}+4x^2e^{2x})-9B\cos 3x-9C\sin 3x}$

Putting the values of y_p, y_p' & y_p'' in (A)

(A) $\Rightarrow$ $8Axe^{2x}+2Ae^{2x}+4Ax^2e^{2x}-9B\cos 3x-9C\sin 3x$
$-8Axe^{2x}-8Ax^2e^{2x}+12B\sin 3x-12C\cos 3x$
$+4Ax^2e^{2x}+4B\cos 3x+4C\sin 3x=2e^{2x}-12\cos 3x$
$-5\sin 3x$

By comparing co-efficients

$2A=2$

$\boxed{A=1}$

$-9B-12C+4B=-12$

$5B+12C=12$

$\boxed{B=\frac{12-12C}{5}} \Rightarrow \boxed{B=0}$ $\because C=1$

$-9C+12B+4C=-5$

$-5C+12B=-5$

$-5C+\frac{144-144C}{5}=-5$

$-25C+144-144C=-25$

$-169C=-169$

$\boxed{C=1}$

$$\boxed{y = x^2 e^{2x} + \sin 3x}$$

$$\boxed{y = y_c + y_p}$$

$$\boxed{y = c_1 e^{2x} + x e^{2x} C_2 + x^2 e^{2x} + \sin 3x}$$

$$\boxed{y' = 2C_1 e^{2x} + e^{2x} C_2 + 2x e^{2x} C_2 + 2x e^{2x} + 2x^2 e^{2x} + 3\cos 3x}$$

Now $y(0) = -2$;

$-2 = C_1 + 0 + 0 + 0$

$$\boxed{C_1 = -2}$$

Now $y'(0) = 4$

$4 = 2C_1 + C_2 + 3$

$4 = -4 + C_2 + 3$

$$\boxed{C_2 = 5}$$

$$\boxed{y = -2e^{2x} + 5x e^{2x} + x^2 e^{2x} + \sin 3x}$$

Ans