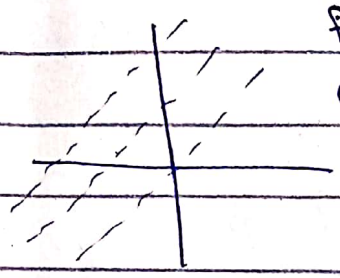
D.E Post Mid 2 Notes

Orthogonal Trajectories

- Perpendicular
- Function / Equation
  - $y = mx + C$
  - $y^2 + x^2 = r^2$

For $y = mx + C$

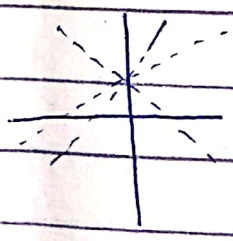
if $m = 1$



family of Curves/Lines

So the Standard Form is $y = mx + C$

- if C is constant and m changes



Q) Find the orthogonal trajectories of the family of straight lines $y=Cx$, where C is a parameter.

Step 1:-

First, we construct the differential equation for the family of straight lines $y=Cx$, By differentiating the last equation with respect to x, we get:

$$y' = C = \text{Const}$$

Since $C = y/x$

$$\frac{dy}{dx} = y/x$$

For O.T:

$$\frac{dy}{dx} = -\frac{dx}{dy}$$

$$\frac{dy}{dx} = \frac{y}{x}$$

$$\frac{-dx}{dy} = \frac{y}{x}$$

$$-x \cdot dx = y \cdot dy$$

Applying $\int$ on both sides

$$\frac{-x^2}{2} + C = \frac{y^2}{2}$$

$$C = \frac{y^2}{2} + \frac{x^2}{2}$$

$$\boxed{C_1 = x^2 + y^2}$$