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FA21 BEE 038 BILAL AHMED

ASSIGNMENT. 2

Question No 1

Part (a)

$$\lim_{(x,y)\to(2,1)} \frac{x^2-2xy}{x^2-4y^2}$$

$$= \frac{(2)^2-2(2)(1)}{(2)^2-4(1)^2} \Rightarrow \frac{4-4}{4-4} = 0/0$$

let $x^2-4y^2=0$

$x = 4y$

$y = x/4$

$$\lim_{x\to 0} \frac{x^2-2(x)(x)}{x^2-4(x/4)^2} \Rightarrow \lim_{x\to 0} \frac{x^2 - x^2/8}{x^2-4(x^2/16)}$$

$$\lim_{x\to 0} \frac{8x^2}{8} \Big/ x^2 - x^2/4$$

$$\lim_{x\to 0} \frac{8x^2-x^2}{8} \Big/ \frac{4x^2-x^2}{4}$$

$$\lim_{x\to 0} \frac{7x^2}{2(3)x^2} = 7/6 \text{ Ans}$$

Part (b)

$$\lim_{x,y\to(0,0)} \frac{x-4y}{6y+7x}$$

$6y = 7x$

$y = 7/6\, x$

$$\lim_{x\to 0} \frac{x-4(7x/6)}{6(7x)+7x} \Rightarrow \lim_{x\to 0} \frac{x-28/6\,x}{7x+7x}$$

$$\lim_{x\to 0} \frac{-22x}{\cancel{6}(14x)} \Rightarrow \lim_{x\to 0} \frac{-11x}{42x}$$

$$= -11/42 \text{ Ans}$$

Part c

$$\lim_{(x,y)\to(0,0)} \frac{x^2 - y^6}{x y^3}$$

let $y = mx$

$$\frac{x^2 - m^6x^6}{x(m^3x^3)} \Rightarrow \frac{x^2(1 - m^6x^3)}{x^2 m^3}$$

$$= \frac{1 - m^6x^3}{x^2 m^3}$$

Part d

$$\lim_{(x,y,z)\to(-1,0,4)} \frac{x^3 - 2e^{2y}}{6x + 2y - 3z}$$

$$= \frac{(-1)^3 - 4e^2(0)}{6(-1) + 0 - 3(4)} \Rightarrow \frac{-1}{-18} = 1/18$$

Question No 2

Determine $D_{\vec{u}} f$ for the given function in indicated direction

(a) $f(x,y) = \cos(x/y)$ is in direction of $v = (3,-4)$

$$\vec{V} = \hat{i}\frac{\partial f}{\partial x} + \hat{j}\frac{\partial f}{\partial y} + k\frac{\partial f}{\partial z}$$

$$\vec{V} = \hat{i}\left(\frac{\partial}{\partial x}\cos(x/y)\right) + \hat{j}\,\partial\left(\cos(x/y)\right)$$

$$\vec{V} = \hat{i}\left[\left(-\sin x/y\right) + \hat{j}\left(-\sin x/y\right)x - y^{-2}\right)\right]$$

$$\vec{V} = \left(-\frac{\sin x/y}{y}\right)\hat{i} + \hat{j}\left(\frac{x \sin x/y}{y}\right)$$

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$$\vec{V} = \left[-\frac{\sin(3/4)}{-4}\right] i + \hat{j}\left[\frac{3\sin(3/y)}{(-y)^2}\right]$$

$$= 1/_4 \left[-0.0131\, i + 3/_{16}(-0.0131)\right] j$$

$$\vec{V} = -\frac{0.0131}{4} i - \frac{0.0393}{16} j$$

Part b

$$f(x,y,z) = x^3y^3 - 4xz$$

$$\vec{V} = (-1, 2, 0)$$

$$\vec{V} = i\frac{\partial f}{\partial x} + j\left(\frac{\partial f}{\partial y}\right) + k\left(\frac{\partial f}{\partial z}\right)$$

$$\vec{V} = \hat{i}\,\partial/_{\partial x}(x^2y^3 - 4xz) + \hat{j}\,\partial/_{\partial y}(x^2y^3 - 4xz) +$$

$$\hat{k}\,\partial/_{\partial z}(x^2y^3 - 4xz)$$

$$= i(2xy^3 - 4z) + j(3x^2y^2) + k(-4x)$$

$$\vec{V} = i(2(-1)(2)^3 - 4(0) + \hat{j}(3(-1)^2(2)^2 + k(-4(-1)$$

$$\vec{V} = \hat{i}(-2\times 8) + (3\times 4)\hat{j} + 4\hat{k}$$

$$\vec{V} = -16i + 12j + 4k$$

Question No 3

Determine directional derivatives of $f(x,y,z) = 4xy^2e^{3x2z}$ at $(3-10)$ in the direction of $v(-1,4,2)$

$f(x y z) = 4xy^2 e^{3xz}$

$\vec{v} = -\hat{i} + 4\hat{j} + 2\hat{k}$

$\hat{v} = \dfrac{-i + 4j + 2k}{\sqrt{(-1)^2 + (4)^2 + (2)^2}}$

$= \dfrac{-i + 4j + 2k}{\sqrt{1+16+4}} \Rightarrow \dfrac{i + 4j + 2k}{\sqrt{21}}$

(a)

grad f at $(3,-1,0)$

grad f $\hat{i}\dfrac{\partial f}{\partial x} + \hat{j}\dfrac{\partial f}{\partial y} + \hat{k}\dfrac{\partial f}{\partial z}$

$= \hat{i}\dfrac{\partial}{\partial x} 4xy^2 e^{3xz} + \hat{j}\dfrac{\partial}{\partial x}(4xy^2 e^{3xz}) +$

$\hat{k}\left(\dfrac{\partial}{\partial z}(4xy^2 e^{3xz})\right)$

$\hat{i}(4y^2 e^{3xz} + 4xy^2 e^{3xz} \cdot 3z) + \hat{j}$
$(4x e^{3xz} \cdot 2y) + k(4xy^2 e^{3xz} \cdot 3x)$

$\hat{i}(4y^2 e^{3xz} + 12xy^2 z e^{3xz}) + \hat{j}(8x e^{3xz}) +$
$k(12x^2 y^2 e^{3xz}$

$e^{3(3)(0)}\Big[\hat{i}(4(-1)^2 + 12(3) + (-1)^2(0) +$
$\hat{j}(8(3)(-1) + k(12(3)^2(-1)^2$

$= e^0 [4\hat{i} - 24\hat{j} + 108k]$

$= 4\hat{i} - 24\hat{j} + 108\hat{k}$

directional derivative $\hat{a} \cdot$ grad f

$= \dfrac{1}{\sqrt{21}}\left[(-\hat{i} + 4\hat{j} + 2\hat{k}) \cdot (4\hat{i} - 24\hat{j} - 108k)\right]$

$= \dfrac{1}{\sqrt{21}}[-4 - 96 + 216]$

$= \dfrac{1}{\sqrt{21}}(116) = 116/\sqrt{21}$

Question No 4

find the maximum rate of change of the function at the indicated point and direction in which the rate of change occurs?

(a) $f(x,y) = \sqrt{x^2+y^2}$ at $(-2,3)$

$\nabla f(x,y) = |\text{grad} f|$

$\text{grad} f = i\frac{\partial f}{\partial x} + J\frac{\partial f}{\partial y}$

$= \hat{i}\frac{\partial}{\partial x}(x^2+y^2)^{1/2} + \hat{j}\left(\frac{\partial}{\partial y}(x^2+y^2)^{1/2}\right)$

$= \hat{i}\left[\frac{1}{2\sqrt{x^2+y^2}}\cdot 2x\right] + \hat{j}\left[\frac{1}{2\sqrt{x^2+y^2}}(2y)\right]$

$\hat{i}\left[\frac{x}{\sqrt{x^2+y^2}}\right] + J\left[\frac{y}{\sqrt{x^2+y^2}}\right]$

$\frac{-2i}{\sqrt{4+9}} + J\frac{3}{\sqrt{4+9}}$

$= -\frac{2}{\sqrt{13}}\hat{i} + \frac{3}{\sqrt{13}}\hat{j}$

$\text{grad} f = \sqrt{\left(\frac{-2}{\sqrt{13}}\right)^2 + \left(\frac{3}{\sqrt{13}}\right)^2}$

$= \sqrt{4/13 + 9/13} \Rightarrow \sqrt{13/13} = 1$

direction in which rate of change occurs

$\frac{\nabla \text{grad} f}{|\nabla \text{grad} f|} = \frac{\frac{-2i}{\sqrt{13}} + \frac{3J}{\sqrt{13}}}{1}$

$\Rightarrow \frac{-2\hat{i}}{\sqrt{13}} + \frac{3J}{\sqrt{13}}$ Ans

Part b.

$f(x,y,z) = e^x\cos(y-2z)$ at $(4, \ldots)$

$\nabla f(x,y,z) = |\text{grad } f|$

$$\text{grad } f = i\frac{\partial f}{\partial x} + j\frac{\partial f}{\partial y} + k\frac{\partial f}{\partial z}$$

$$= i\frac{\partial}{\partial x}\left[e^x\cos(y-2z)\right] + j\frac{\partial}{\partial y}(e^x\cos y-2z)$$

$$+ k\frac{\partial}{\partial z}(e^x\cos y - 2z)$$

$$i(e^x\cos(y-2z)] + j[e^x - \sin(y-2z) + k[e^x - \sin(y-2x) - -2)$$

$$i[e^x\cos(y-2z)] + j[e^x\sin(y-2x) + k[e^x 2\sin(y-2z))$$

$$e^4[i(\cos(-2(0) + j(\sin(-2)-2(0) + k(2\sin(-2-2)$$

$$5.46[(1i) + j(-0.035) + k(-0.07)]$$

$$5.46\hat{i} - 1.911j - 3.8k$$

$$\text{grad } f = \sqrt{5(.46)^2 + (1.911)^2 + (3.8)^2}$$

$$\text{grad } f (= 54.8)$$

The direction in which rate of change occur

$$\frac{\nabla\, \text{grad } f}{}$$

$$= \frac{54.6i - 1.91j - 3.8k}{54.8}$$

$$= \frac{54.6}{54.8}i - \frac{1.911}{54.8}j - \frac{3.8}{54.8}k$$

a)

Question No 7

a) $z = \frac{x^2 - w}{y^4}$, $x = t^3 + 7$

$y = \cos(2t)$, $w = 4t$

$\frac{dz}{dt} = ?$

$\frac{dz}{dt} = \frac{2x}{y^4}$, $\frac{\partial z}{\partial y} = (x^2 - w) - 4y^{-5}$

$\frac{dx}{dt} = 3t^2$, $\frac{dz}{dy} = (x^2 - w) - 4y^{-5}$

$\frac{dz}{dy} = -4\frac{(x^2 - w)}{y^5}$

$\frac{dy}{dt} = \frac{d}{dt}\cos 2t = -\sin 2t \cdot 2$

$= -2\sin 2t$

$\frac{\partial z}{\partial w} = -1/y^4$ $\frac{dw}{dt} = 4$

$\frac{dz}{dt} = \frac{dz}{dx} \cdot \frac{dx}{dt} + \frac{dz}{dy} \cdot \frac{dy}{dt} + \frac{\partial z}{\partial w} \frac{dw}{dt}$

$= \frac{2x}{y^4} \cdot 3t^2 + \left(-\frac{4(x^2 - w)}{y^5} \cdot -2\sin 2t\right)$

$+ 1/y^4 \cdot 4$

$= \frac{6x^2t^2}{y^4} + \frac{8(x^2 - w)\sin 2t}{y^5} - \frac{4w}{y^4}$

(b) $z = x^2y^4 - 2y$, $y = \sin(x^2)$

$\frac{dz}{dx} = ?$

$$\frac{dz}{dx} = \frac{dz}{dy} \cdot \frac{dy}{dx}$$

$$\frac{dz}{dy} = \frac{d}{dy}(x^2y^4 - 2y)$$

$$= 4y^3(x^2) - 2)$$

$$= 4x^2y^3 - 2$$

$$\frac{dy}{dx} = \frac{d}{dx} \sin x^2 = \cos x^2 \cdot 2x$$

$$= 2x \cos x^2$$

$$\frac{dz}{dx} = \frac{dz}{dy} \cdot \frac{dy}{dx}$$

$$= (4x^2y^3 - 2)(2x \cos x^2)$$

$$= 8x^3y^3 - 4x \cos x^2$$

Question No 5

Compute div $\vec{F}$ and cur $\vec{F}$

(a) $\vec{F} = x^2yi - (z^3 - 3x)\hat{j} + 4y^2\hat{k}$

$$\vec{F} = \frac{\partial}{\partial x} f + \frac{\partial f}{\partial y} + \frac{\partial}{\partial z} f$$

$$\nabla F = \frac{\partial}{\partial x} x^2y + \frac{\partial}{\partial y}(z^3 - 3x) + \frac{\partial}{\partial z} 4y^2$$

$$\nabla F = 2xy\,i - 0 + 0$$

Div f = $\Delta f \cdot f$

$$= (2xy\,i) \cdot [x^2yi - (z^3 - 3x)j + 4y^2k$$

$$= 2x^3y^2$$

ans: $\nabla f \times f$

$$= \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x z^2 y & -z^3-3x & 4y^2 \end{vmatrix}$$

$$i \begin{vmatrix} \partial/\partial y & \partial/\partial z \\ z^3-3x & 4y^3 \end{vmatrix} - j \begin{vmatrix} \partial/\partial x & \partial/\partial z \\ x^2 y & 4y^2 \end{vmatrix} + k \begin{vmatrix} \partial/\partial x & \partial/\partial y \\ x^2 y & -z^3-3x \end{vmatrix}$$

$i(8y+3z^2) - j(0) + k(-(-3) - x^2)$

$= (8y+3z^2)i + (3-x^2)k$

Part (b):

$F = (8x+2z^2)i + \frac{x^3y^2}{2} j - 2(7x)k$

div $\nabla f \cdot f$

$\nabla f = \frac{\partial f}{\partial x} i + \frac{\partial f}{\partial y} j + \frac{\partial f}{\partial z} k$

$i \frac{\partial}{\partial x}(8x+2z^2) + j\left(\frac{\partial}{\partial y} \frac{x^3y^2}{2}\right) + k \frac{\partial}{\partial z}(z-2x)$

$8i + \frac{x^3 \cdot 2y\, j}{z} + -14$

$8i + \frac{2x^3y}{z} j + k$

$8i + \frac{2x^3y}{z} - k \cdot (8x+2z)i + \left(\frac{x^3y^2}{z}\right)j - (z-7x)k$

$16x + 16z + 2x^6y^4/z + z - 7x$

$9x + 17z + \frac{2x^6y^4}{z}$

ans: $\nabla f \times f$

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$$\begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 3x+2z^2 & \frac{x^3y^2}{z} & \frac{-z}{7x} \end{vmatrix}$$

$$i\begin{vmatrix} \partial/\partial y & \partial/\partial z \\ \frac{x^3y^2}{z} & -(z-7x) \end{vmatrix} - j\begin{vmatrix} \partial/\partial x & \partial/\partial z \\ 3x+2z & -(z(-7x)) \end{vmatrix} + k\begin{vmatrix} \partial/\partial x & \partial/\partial y \\ 3x+2z & \frac{x^3y^2}{z} \end{vmatrix}$$

$$= i\left[\partial/\partial y -(z-7x) - \partial/\partial z \left(\frac{x^3y^2}{z}\right)\right] - j\left[\partial/\partial x (z-7x) - \partial\, 3x+2z^2\right)$$

$$+ k\left[\partial/\partial x \frac{x^3y^2}{z} - \partial/\partial y\ 3x+2z^2\right]$$

$$= i(0 - x^3y^2 - z^{-2}] - j[+7-4z] + k\left(\frac{3x^2}{z}y^2 - 0\right)$$

$$= i\frac{x^3y^2}{z^2} - j(7-4z) + \left(\frac{3x^2y^2}{z}\right) k$$

$$= 8\hat{i} + \frac{x^3 \cdot 2y}{z} i + i - 1k$$

$$= 8i + \frac{2x^3y}{z}\hat{j} + k$$

$$= 8i + \frac{2x^3}{z}y - k \cdot (3x+2z)i + \left(\frac{x^3y^2}{z}\right)j - z - 7x\, k$$

$$= 16x + 16z + 2x^2y^4/z + z - 7x$$

$$= 90x + 17z + \frac{2x^2y^4}{z}$$

Curve $\nabla f \times f$
$$\begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 3x+2z^2 & \frac{x^3y^3}{} & -(z \times z) \end{vmatrix}$$

$$i(0 - x^3y^2, -(z^{-2}) - \hat{j}(7-4z) + k^2\left(\frac{3x^2y^2}{z}\right) - 0$$

$$\hat{i}\,\frac{x^3y^2}{z^2} - j(7-4z) + \left(\frac{3x^2y^2}{z}\right)k$$

Dete

(a) f

$\hat{F}$

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Question No 6

Determine if the vector field is conservative

(a) $f = x^2 y z^i$

$$\hat{F} = \left(4x^2 + \frac{3x^2y}{z^2}\right)i + \left(8xy + \frac{x^3}{z^2}\right)j + \left(11 - \frac{2x^3y}{z^3}\right)k$$

The vector field is conservative if and only if

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}, \quad \frac{\partial N}{\partial z} = \frac{\partial P}{\partial y}, \quad \frac{\partial M}{\partial z} = \frac{\partial P}{\partial x}$$

$$\vec{F} = (4y^2 + 3x^2y)\hat{i} + 8\left(xy + \frac{x^3}{z^3}\right)j + \left(11 - \frac{2x^2y}{z^3}\right)k$$

$$\frac{\partial M}{\partial y} = 8y + \frac{3x^2}{z}, \quad \frac{\partial N}{\partial x} = 8y - \frac{3x^2}{z^2}$$

$$\frac{\partial N}{\partial z} = x^3 z^{-2} = x^3(-2)z^{-3} = \frac{-2x^3}{z^3}$$

$$\frac{\partial P}{\partial y} = -\frac{2x^3}{z^3}$$

$$\frac{\partial M}{\partial z} = 4x^2 + \frac{3x^2y}{z^2} = 3x^2y(-2)z^{-3}$$

$$= -\frac{6x^3y}{z^3}$$

$$\frac{\partial P}{\partial x} = \partial\left(1 - \frac{2x^2y}{z^3}\right) = -\frac{6x^2y}{z^3}$$

Hence

$$\frac{dM}{dy} = \frac{\partial N}{\partial x}, \quad \frac{dN}{dz} = \frac{\partial P}{\partial y} = \frac{\partial M}{\partial z} = \frac{dP}{dx}$$

since the vector field is conservative

b $\vec{F} = 6xi\,(2xy^2)\,j + (6z - x^3)\,\hat{k}$

The vector field is conservative if and only if

$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$, $\frac{dN}{\partial z} = \frac{dP}{dy}$, $\frac{\partial M}{\partial z} = \frac{\partial P}{\partial x}$

$\therefore F = \underbrace{6xi}_{M} + \underbrace{(2x - y^2)J}_{N} + \underbrace{(6z - x^2)k}_{P}$

$\frac{\partial M}{\partial y} = \frac{\partial}{\partial y}(6x) = 0$ $\qquad \frac{\partial N}{\partial x} = \frac{d}{dx}(2x - y^2)$

$\frac{\partial N}{\partial z} = \frac{\partial}{\partial z}(2x - y^2) = 0$ $\qquad = 2$

$\frac{\partial P}{\partial y} = \frac{\partial}{\partial x}(6z - x^3) = -3x^2$

$\frac{\partial M}{\partial z} = \frac{\partial\, 6x}{\partial z} = 0$

So $\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$, $\frac{dN}{\partial z} = \frac{\partial P}{\partial y}$, $\frac{\partial M}{\partial z} \neq \frac{\partial P}{\partial x}$

Since the vector is not conservative