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FA20-BEE-038

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Multi

Question 1

A (4, -4, 1)
B (-4, 3, -4)
(4, -1, 2)

$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$

$$\begin{bmatrix} 4 \\ 3 \\ -4 \end{bmatrix} - \begin{bmatrix} -4 \\ -4 \\ 1 \end{bmatrix} = -8i + 7j - 5k$$

$\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA}$

$$\begin{bmatrix} 4 \\ -1 \\ -2 \end{bmatrix} - \begin{bmatrix} 4 \\ -3 \\ 1 \end{bmatrix} = 3j - 3k$$

$\overrightarrow{AB} \times \overrightarrow{AC}$

$$\begin{vmatrix} i & j & k \\ -8 & 7 & -5 \\ 0 & 3 & -3 \end{vmatrix}$$

$+6i - 24j - 24k$

$n = i + 4j + 4k$

$d = a \cdot n$

$= 4 - 4j + 4k \cdot (i + 4j + 4k)$

$4 - 16 + 4 = -8$

Eq of plane

$r \cdot n = d$

$r(i + 4j + 4k) = -8$

$= x + 4y + 4z = -8$

$= x + 4y + 4z + 8 = 0$

(b)

perp $= \frac{d}{n} = \frac{8}{\sqrt{1+4^2+4^2}} = \frac{8}{33} = 1.39$

(c) line ; $r = a + \lambda b$

$$r = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 3 \\ -3 \end{pmatrix}$$

$$\begin{pmatrix} 2\lambda \\ 3\lambda \\ -3\lambda \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 4 \\ 4 \end{pmatrix} = -8$$

$2\lambda + 3(2\lambda) - 12\lambda = -8$

$\boxed{\lambda = -4}$

$$r = \begin{pmatrix} 2(-4) \\ 3(-4) \\ -3(-4) \end{pmatrix} = -8, -12, 12$$

Question no 2

(a)

$r_a = OA - OD$

$AB = OB - OA$

$$\begin{bmatrix} 11 \\ 3 \\ 0 \end{bmatrix} - \begin{bmatrix} 7 \\ 4 \\ -1 \end{bmatrix} = \begin{bmatrix} 4 \\ -1 \\ 1 \end{bmatrix}$$

$$r_1 = \begin{bmatrix} 7 \\ 4 \\ -1 \end{bmatrix} + \lambda \begin{bmatrix} 4 \\ -1 \\ 1 \end{bmatrix}$$

$r_2 = OC - CD$ $\quad CD = OD - OC$

$$\begin{bmatrix} 2 \\ 7 \\ \lambda \end{bmatrix} - \begin{bmatrix} 2 \\ 6 \\ 3 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ \lambda - 3 \end{bmatrix}$$

$$b_1 \times b_2 = \begin{vmatrix} i & j & k \\ 4 & -1 & 1 \\ 0 & 1 & \lambda-3 \end{vmatrix}$$

$$= i\begin{vmatrix} -1 & 1 \\ 1 & \lambda-3 \end{vmatrix} - j\begin{vmatrix} 4 & 1 \\ 0 & \lambda-3 \end{vmatrix} + k\begin{vmatrix} 4 & -1 \\ 0 & 1 \end{vmatrix}$$

$$= i\{-(\lambda-3)-1\} - j[4(\lambda-3)] + k(4)$$

$$a_2 - a_1 = \begin{bmatrix} 2 \\ 6 \\ 3 \end{bmatrix} - \begin{bmatrix} 7 \\ 4 \\ -1 \end{bmatrix} = \begin{bmatrix} -5 \\ 2 \\ 4 \end{bmatrix}$$

$$|b_1 \times b_2| = \sqrt{(2-\lambda)^2 + (4\lambda-12)^2 + (4)^2}$$

$$= \sqrt{4 + \lambda^2 - 4\lambda + 16\lambda^2 + 144 - 96\lambda + 16}$$

$$= \sqrt{17\lambda^2 - 100\lambda + 164}$$

$$(b_1 \times b_2) \cdot a_1 - a_2 \left[(2-\lambda)i - j(4\lambda-12) + 4k\right] \cdot (-5i + 2j + 4k)$$

$$= -5(2-\lambda) - 2(4\lambda-12) + 16$$

$$= 30 - 3\lambda$$

$$d = \frac{(b_1 \times b_2) \cdot (a_1 - a_2)}{|b_1 \times b_2|}$$

$$3 = \frac{30 - 3\lambda}{\sqrt{17\lambda^2 + 100\lambda + 164}}$$

Square

$$9 = \frac{900 + 9\lambda^2 - 180\lambda}{17\lambda^2 - 100\lambda + 164}$$

$$9(17\lambda^2 - 100\lambda + 164) = 900 + 9\lambda^2 - 180\lambda$$

$$144\lambda^2 - 720\lambda + 576 = 0$$

$$16(9\lambda^2 - 45\lambda + 36) = 0$$

$$9\lambda^2 - 45\lambda + 36$$

$$\lambda^2 - 5\lambda + 4 = 0$$

$$\lambda(\lambda - 4) - 1(\lambda - 4) = 0$$

$\lambda = 4$ $\lambda = 1$

$\lambda = 4$ in eq (1)

$$(4)^2 - 5(4) + 4 = 0$$

$0 = 0$

$\lambda = 1$ in eq ①

$1 - 5 + 4 = 0$

$0 = 0$

parth

Let π_1 ~~and~~ be plane where $\lambda = 1$

π_2 where $\lambda = 4$

① $\pi_1 = r_1 = OA + sAB + tAD$

$\lambda = 1$

$AD = OD - OA$

$$\begin{bmatrix} 2 \\ 7 \\ 1 \end{bmatrix} - \begin{bmatrix} 7 \\ 4 \\ 1 \end{bmatrix} = \begin{bmatrix} -5 \\ 3 \\ 2 \end{bmatrix}$$

$\pi_1 = 7i + 4j - k + s(4i - j + k) + t(-5i + 3j + 2k)$

② $\pi_2 = r_2 = OA + \lambda AB + \mu AD$

$\lambda = 4$

$AD = OD - OA$

no. 02

$I_2 = r_2 = OA + \lambda AB + \mu AD$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7 \\ 4 \\ -1 \end{bmatrix} + \lambda \begin{bmatrix} 4 \\ -1 \\ 1 \end{bmatrix} + \mu \begin{bmatrix} -5 \\ 3 \\ 5 \end{bmatrix}$$

$x = 7 + 4\lambda - 5\mu$ — (i)

$y = 4 - \lambda + 3\mu$ — (ii)

$z = -1 + \lambda + 5\mu$ — (iii)

eq (ii) to (iii)

$y = 4 - \lambda + 3\mu$

$z = -1 + \lambda + 5\mu$

$y + z = 3 + 8\mu$ — (IV)

eq (ii) by 2

$y = 4 - \lambda + 3\mu$

$\times 4$

$\mu y = 16 - 4\lambda + 12\mu$ — (V)

$40 + 325 + 49 = 414$

$|n_1| = \sqrt{5^2 + 13^2 + (-7)^2} = 9\sqrt{3}$

$|n_2| = \sqrt{8^2 + 25^2 + (-7)^2} = 3\sqrt{82}$

$\theta = \cos^{-1}\left(\frac{414}{27\sqrt{246}}\right)$

$\theta = 12.15^\circ$

(c)

$\bar{\lambda} = r_1 = \overrightarrow{OA} + \lambda \overrightarrow{AB} + \mu \overrightarrow{AD}$

$x = 7 + 4\lambda - 5\mu$ (1)

$y = 4 - \lambda + 3\mu$ (2)

$z = -1 + \lambda + 2\mu$ (3)

eq (1) to eq (3)

$y = 4 - \lambda + 3\mu$

$z = -1 + \lambda + 2\mu$

$y + z = 3 + 5\mu$ — (4)

$$x \quad 7 + 4\lambda - 5\mu$$
$$4y \quad 16 - 4\lambda + 12\mu$$

$$x + 4y = 23 + 7\mu$$
$$x = \frac{x + 4y - 23}{4}$$

values in eq (iii)

$$y + z = 3 + 8\left(\frac{x + 4y - 23}{7}\right)$$

$$y + z = 3 + \frac{8x + 32y - 184}{7}$$

$$7y + 7z = 21 + 8x + 8y - 184$$

$$-8x - 25y + 7z + 163 = 0$$
$$-(8x + 25y - 7z - 163) = 0$$
$$8x + 25y - 7z = 168$$

$$-(5x + 13y - 7z - 94)$$

$$\theta = \cos^{-1}\frac{\vec{n_1} \cdot \vec{n_2}}{|\vec{n_1}| \cdot |\vec{n_2}|}$$

$$\vec{n_1} \cdot \vec{n_2} \quad \begin{bmatrix} 5 \\ 13 \\ -7 \end{bmatrix} \begin{bmatrix} 8 \\ -25 \\ -7 \end{bmatrix}$$

eq (2) by 4

$4y = 16 - 4\lambda + 12$

add eq 1 and eq (1)

$x = 7 + 4\lambda - 5y$

$16 - 4\lambda + 12$

$x + 4y = 23 + 7y$

$x + 4y - 23 = y$

$y + z = 3 + \left(\frac{5x + 20y - 115}{7}\right)$

$7y + 7z - 21 = 5x + 20y - 115$

$-5x - 13y + 7z + 94 = 0$