

Date _____

- Question #2 :-

$$= x = \begin{bmatrix} 0.34 \\ 0.55 \end{bmatrix}$$

$$= w_1 = \begin{bmatrix} 0.42 & 0.72 \\ 0 & 0.30 \\ 0.15 & 0.09 \end{bmatrix}, w_2 = \begin{bmatrix} 0.44 & 0.03 & 0.55 \end{bmatrix}$$

$$= w_3 = \begin{bmatrix} 0.55 \\ 0.71 \\ 0.29 \end{bmatrix}, w_4 = \begin{bmatrix} 0.97 & 0.55 & 0.9 \\ 0.71 & 0.70 & 0.22 \end{bmatrix}$$

• Forward propagation :

• L₁ :

$$= z_1 = \begin{bmatrix} 0.42 & 0.72 \\ 0 & 0.30 \\ 0.15 & 0.09 \end{bmatrix} \begin{bmatrix} 0.34 \\ 0.55 \end{bmatrix}$$

$$= z_1 = \begin{bmatrix} 0.53 \\ 0.16 \\ 0.1005 \end{bmatrix} = a_1$$

• L₂

$$= z_2 = \begin{bmatrix} 0.44 & 0.03 & 0.55 \end{bmatrix} \begin{bmatrix} 0.53 \\ 0.16 \\ 0.1005 \end{bmatrix}$$

$$= z_2 = 0.297 = a_2$$

• L3 :

$$z_3 = \begin{bmatrix} 0.55 \\ 0.71 \\ 0.29 \end{bmatrix} \begin{bmatrix} 0.297 \end{bmatrix}$$

$$= z_3 = \begin{bmatrix} 0.163 \\ 0.211 \\ 0.086 \end{bmatrix} = a_3$$

• L4 :

$$z_4 = \begin{bmatrix} 0.97 & 0.55 & 0.97 \\ 0.71 & 0.70 & 0.22 \end{bmatrix} \begin{bmatrix} 0.163 \\ 0.211 \\ 0.086 \end{bmatrix}$$

$$= z_4 = \begin{bmatrix} 0.358 \\ 0.282 \end{bmatrix} = a_4 = \hat{x}$$

• Error Reconstruction :-

$$J = -\sum x_i \log(\hat{x}_i)$$

$$J = -(0.34 \log(0.358) + 0.55 \log(0.282))$$

$J = 1.043$ is the reconstructed error

• Backpropagation :-

$$d = \begin{bmatrix} 0.358 \\ 0.282 \end{bmatrix} - \begin{bmatrix} 0.34 \\ 0.55 \end{bmatrix}$$

$$= \begin{bmatrix} 0.018 \\ 0.267 \end{bmatrix}$$

• w_4 gradient:

$$\delta w_4 = \begin{bmatrix} 0.002 & 0.003 & 0.001 \\ -0.043 & -0.05 & -0.023 \end{bmatrix}$$

$$\delta z = (w_4)^T \delta u \odot g'(z_3)$$

$$\text{Here, } g'(z_3) = 1$$

$$(w_4)^T \delta u = \begin{bmatrix} 0.97 & 0.71 \\ 0.55 & 0.70 \\ 0.97 & 0.22 \end{bmatrix} \begin{bmatrix} 0.018 \\ -0.267 \end{bmatrix}$$

$$\delta q = \begin{bmatrix} -0.171 \\ -0.176 \\ -0.04 \end{bmatrix}$$

• w_3 gradient:

$$\delta w_3 = \delta_3 (a_2)^T$$

$$\delta w_3 = \begin{bmatrix} -0.051 \\ -0.052 \\ -0.012 \end{bmatrix}$$

$$\delta_2 = (w_3)^T \delta_3 \cdot g'(z_2)$$

$$\delta_2 = \begin{bmatrix} 0.55 & 0.71 & 0.29 \end{bmatrix} \begin{bmatrix} -0.171 \\ -0.176 \\ -0.040 \end{bmatrix}$$

$$\delta_2 = [-0.232]$$

• w_2 gradient:

$$\delta w_2 = \delta_2 (a_1)^T$$

Date _____

$$\Delta w_2 = [-0.128 \quad +0.038 \quad -0.023]$$

$$\delta_1 = (w_2)^T \delta_2 \odot g'(z_1)$$

$$\delta_1 = \begin{bmatrix} 0.44 \\ 0.03 \\ 0.55 \end{bmatrix} [-0.232]$$

$$\delta_1 = \begin{bmatrix} -0.102 \\ -0.006 \\ -0.127 \end{bmatrix}$$

• w_1 gradient:

$$\Delta w_1 = \delta_1 (x)^T$$

$$\Delta w_1 = \begin{bmatrix} -0.03 & -0.056 \\ -0.002 & -0.003 \\ -0.043 & -0.07 \end{bmatrix}$$

• Weight updates:

$$\Delta \alpha =$$

• w_4 :

$$w_4' = \begin{bmatrix} 0.968 & 0.548 & 0.969 \\ 0.731 & 0.728 & 0.231 \end{bmatrix}$$

• w_3 :

$$w_3' = \begin{bmatrix} 0.575 \\ 0.736 \\ 0.296 \end{bmatrix}$$

Date _____

• w_2 :

$$w_2' = [0.502 \quad 0.049 \quad 0.561]$$

• w_1 :

$$w_1' = \begin{bmatrix} 0.437 & 0.748 \\ 0.001 & 0.301 \\ 0.171 & 0.125 \end{bmatrix}$$