Linear Algebra

Assignment 2

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Q:- What is a matrix determinant?

The Determinant is a scalar value. It can be calculated using square matrix. It is function of the elements of a square matrix

1. Reflection Property:-

The determinant remains unchanged if it rows are ~~un~~changed into columns and vice versa i.e $\det A^T = \det A$

e.g $A = \begin{bmatrix} 2 & 5 \\ 4 & 3 \end{bmatrix}$, $A^T = \begin{bmatrix} 2 & 4 \\ 5 & 3 \end{bmatrix}$

$|A| = -14$ $\qquad |A^T| = -14$

2. Multiplicative Property:-

This property states that $\det(AB) = \det(A) \cdot \det(B)$

$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 4 \\ 2 & 2 \end{bmatrix}$

$AB = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 3 & 4 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} 3+4 & 4+4 \\ 9+8 & 12+8 \end{bmatrix}$

$AB = \begin{bmatrix} 7 & 8 \\ 17 & 20 \end{bmatrix}$, $\det(AB) = 4$

$|A| = -2$ $\qquad$ L.H.S = R.H.S => Proved

$|B| = -2$

3. **Determinant of Inverse:-**

Let A be $n \times n$ matrix, then A is inversible iff $\det(A) \neq 0$

e.g $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \Rightarrow |A| = -2 \neq 0$, A is inversible

4. **Identity matrix:-**

Determinant of the Identity matrix is always 1

e.g ~~I =~~ $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 1 \times 1 - 0 \times 0 = 1$

Same for all orders.

5. **Triangle Property:-**

If all the elements of a determinant above or below the main diagonal consists of zero, then the determinant is equal to the product of diagonal elements.

e.g $\begin{vmatrix} a_1 & a_2 & a_3 \\ 0 & b_2 & b_3 \\ 0 & 0 & c_3 \end{vmatrix} = \begin{vmatrix} a_1 & 0 & 0 \\ a_2 & b_2 & 0 \\ a_3 & b_3 & c_3 \end{vmatrix} = a_1 b_2 c_3$

6. **Repitition Property:-**

If the elements of a row (or column) are identical to the elements of some other row (or column), then the determinant is zero.

e.g $A = \begin{bmatrix} 4 & 2 & 1 \\ 3 & 1 & 0 \\ 4 & 2 & 1 \end{bmatrix}$

$|A| = 4[(1)(1) - 2(0)] - 2[3(1) - 4(0)] + 1[2(3) - 4(1)]$

$|A| = 4 - 6 + 2 = 0$

7. Property of Invariance

The determinant remains un-changed under on operation of the form:-

$C_i \to C_i + \alpha C_j + \beta C_k$, where $j, k \neq i$ or $R_i \to R_i + \alpha R_j + \beta R_k$, $j, k \neq i$

8. Factor Property:-

If a determinant is a polynomial in x, then $(x-\alpha)$ is a factor of the determinant if its value is zero when we put $x=\alpha$.

9. Sum property:-

The sum of the product of the elements of any row (or column) with the cofactors of the corresponding elements is zero.

$$\begin{vmatrix} a_1+b_1 & c_1 & d_1 \\ a_2+b_2 & c_2 & d_2 \\ a_3+b_3 & c_3 & d_3 \end{vmatrix} = \begin{vmatrix} a_1 & c_1 & d_1 \\ a_2 & c_2 & d_2 \\ a_3 & c_3 & d_3 \end{vmatrix} + \begin{vmatrix} b_1 & c_1 & d_1 \\ b_2 & c_2 & d_2 \\ b_3 & c_3 & d_3 \end{vmatrix}$$