Todo list

These might need to be replaced with variants for Reals/Integers	2
Do we want to have the length of our vectors as a type argument?	2
define criteria for what a well-formed expression language should provide	2
Quantities discussion – remaining untyped	2
modulo, remainder, etc	4
discuss vectors in general	5
oddity: topology appears as a constructor arg and signature arg but can	
desync – can we just remove the constructor arg?	5
addressed in "misc" section	6

1 Syntax

1.1 Current

Type	au	::=	Integer		Integer numbers	
			Real		Real numbers	
		ĺ	String		Text	
		ĺ	Bool		Truth values (true/false)	
		İ	$\texttt{Vector}[\tau]$		Vectors	
Literal	l	::=	Integer[n]	n	Integer number	
			$\mathtt{Real}[r]$	r	Real number	
		j	String[s]	"s"	Text	
		j	$Bool[\bar{b}]$	b	Boolean value	
		İ	$\texttt{Vector}(l_1l_n)$	$\langle l_1,,l_n \rangle$	Vectors	
Expr	e	::=	$\mathtt{Literal}(l)$	l	Literal values	
•			${\tt UnaryOp}(f,e)$	f(e)	Unary operations	
		j	$\mathtt{BinaryOp}(\oplus, e_1, e_2)$	$e_1 \oplus e_2$	Binary operations	
		j	AssocOp(\otimes , e_1e_n)	$e_1 \otimes \otimes e_n$	Associative binary operations	
		j	$Case(\stackrel{-}{e_{1c}}e_{1e}e_{nc}e_{ne})$	if e_{1c} then e_{2e} elif e_{2c}	If-then-else-if-then-else (Switch-like	

2 Typing Rules

2.1 Literal

1. Integers:

$$\frac{i: {\tt Integer}}{Integer[i]: {\tt Literal \ Integer}} \tag{1}$$

2. Strings (Text):

$$\frac{s: \mathtt{String}}{Str[s]: \mathtt{Literal String}} \tag{2}$$

3. Real numbers:

$$\frac{d: \mathtt{Double}}{Dbl[d]: \mathtt{Literal Real}} \tag{3}$$

4. Whole numbered reals $(\mathbb{Z} \subset \mathbb{R})$:

$$\frac{d: \texttt{Integer}}{ExactDbl[d]: \texttt{Literal Real}} \tag{4}$$

5. Percentages:

$$\frac{n: \texttt{Integer} \quad d: \texttt{Integer}}{Perc[n, d]: \texttt{Literal Real}} \tag{5}$$

2.2 Miscellaneous

Vectors

As of right now, Drasil/GOOL only supports lists and arrays as "code types", which would be the representations used for representing "vectors" in Drasil.

For now, the below type rules define vectors with Haskell lists. We can choose to create our own type with the length of the vector as a parameter – likely going "too far into Haskell".

Functions

Presently, functions are defined through "QDefinitions", where a list of UIDs used in an expression are marked as the parameters of the function. Function "calls"/applications are captured in "Expr" (the expression language) by providing a list of input expressions and a list of named inputs (expressions) – f(x, y, z, a = b)").

A few solutions:

- 1. Leave expressions in general untyped in Haskell, and rely on calculating the "space" of an expression dynamically to ensure that expressions are well-formed. If runtime (drasil's compiling-knowledge-time) type analysis is ever needed, this will prove much easier to use in general.
- 2. Push the typing rules into Haskell via Generalized Algebraic Data Types (GADTs). Here, a larger question appears regarding functions how should we handle function creation, application, and typing?
 - (a) Currying and applying arguments (allowing partial function applications): This would work well if we only generated functional languages, but it might prove problematic for GOOL if expressions are left with partial function applications.

Type Rules

These might need to be replaced with variants for Reals/Integers

Do we want to have the length of our vectors as a type argument?

define criteria for what a wellformed expression language should provide

Quantities discussion – remaining untyped 1. Completeness:

$$\overline{Complete[]: Completeness}$$
 (6)

$$\overline{Incomplete[]: \texttt{Completeness}} \tag{7}$$

2. AssocOp:

(a) Numerics:

$$\frac{x: \texttt{Numerics}(\mathbf{T})}{Add[]: \texttt{AssocOp x}} \tag{8}$$

$$\frac{x: \texttt{Numerics}(\mathbf{T})}{Mul[]: \texttt{AssocOp x}} \tag{9}$$

(b) Bool:

$$\overline{And}[]: AssocOp Bool$$
 (10)

$$\overline{Or}[]$$
: AssocOp Bool (11)

3. UnaryOp:

(a) Numerics:

$$\frac{x: \texttt{NumericsWithNegation(T)}}{Neg[]: \texttt{UnaryOp} \ \texttt{x} \ \texttt{x}} \tag{12}$$

$$\frac{x: \texttt{NumericsWithNegation(T)}}{Abs[]: \texttt{UnaryOp x x}} \tag{13}$$

$$\frac{x: \texttt{Numerics(T)}}{Exp[]: \texttt{UnaryOp x Real}} \tag{14}$$

For Log, Ln, Sin, Cos, Tan, Sec, Csc, Cot, Arcsin, Arccos, Arctan, and Sqrt, please use the following template, replacing "TRG" with the desired operator:

$$\overline{\$TRG[]}: \mathtt{UnaryOp} \ \mathtt{Real} \ \mathtt{Real}$$

$$RtoI[]$$
: UnaryOp Real Integer (16)

$$\overline{ItoR[]}$$
: UnaryOp Integer Real (17)

$$Floor[]$$
: UnaryOp Real Integer (18)

		$\overline{Ceil[]}: exttt{UnaryOp Real Integer}$	(19)
		$\overline{Round[]}: {\tt UnaryOp\ Real\ Integer}$	(20)
(b)		$\overline{Trunc[]}: exttt{UnaryOp Real Integer}$	(21)
	Vectors:	$rac{x: exttt{NumericsWithNegation(T)}}{NegV[]: exttt{UnaryOp [x] [x]}}$	(22)
		$\frac{x: \texttt{Numerics}(\mathbf{T})}{Norm[]: \texttt{UnaryOp [x] Real}}$	(23)
		$\frac{\mathtt{x} : \tau}{Dim[] : \mathtt{UnaryOp} \ [\mathtt{x}] \ \mathtt{Integer}}$	(24)
(c)	Booleans:	$\overline{Not[]: exttt{UnaryOp Bool Bool}}$	(25)
4. Bina	aryOp:		
(a)	Arithmetic:		
	\overline{Fr}	cacI[]: BinaryOp Integer Integer Integer	(26)
		$\overline{FracR[]: exttt{BinaryOp Real Real Real}}$	(27) modulo,
(b)	Bool:		remainder,
		$\overline{Impl[]}: exttt{BinaryOp Bool Bool Bool}$	(28) etc.
		$\overline{Iff[]}: exttt{BinaryOp Bool Bool Bool}$	(29)
(c)	Equality:	$\mathtt{x}:\tau$	
		$\overline{Eq[]: exttt{BinaryOp x x Bool}}$	(30)
		$\mathtt{x}: au$	
		$NEq[]: exttt{BinaryOp} exttt{ x Bool}$	(31)

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(d) Ordering:
                                        x: \mathtt{Numerics}(\mathbf{T})
                                   Lt[]: BinaryOp x x Bool
                                                                                       (32)
                                        x: \mathtt{Numerics}(\mathbf{T})
                                   Gt[]: BinaryOp x x Bool
                                                                                       (33)
                                        x: \texttt{Numerics}(T)
                                 LEq[]: BinaryOp x x Bool
                                                                                       (34)
                                        x: \mathtt{Numerics}(\mathbf{T})
                                 GEq[]: BinaryOp x x Bool
                                                                                       (35)
     (e) Indexing:
                                               \mathtt{x}:\tau
                             Index[]: BinaryOp [x] Integer x
                                                                                       (36)
     (f) Vectors:
                                                                                                discuss
                                        x: Numerics(T)
                                                                                                vectors in
                              Cross[]: BinaryOp [x] [x] [x]
                                                                                       (37)
                                                                                                general
                                        x: Numerics(T)
                                 Dot[]: BinaryOp [x] [x] x
                                                                                       (38)
                                        x: \texttt{Numerics}(T)
                                Scale[]: BinaryOp [x] x [x]
                                                                                       (39)
5. RTopology:
                                 \overline{Discrete[]}: \mathtt{RTopology}
                                                                                       (40)
                                \overline{Continuous[]: RTopology}
                                                                                       (41)
6. DomainDesc:
                                                                                                oddity:
                                                                                                topology
                top: 	au_1 \quad bot: 	au_2 \quad s: 	exttt{Symbol} \quad rtop: 	exttt{RTopology}
                                                                                                appears
         \overline{BoundedDD[s,rtop,top,bot]}: 	exttt{DomainDesc Discrete } 	au_1 \ 	au_2
                                                                                                as a con-
                                                                                       (42)
                                                                                                structor
                                                                                                arg and
                 \mathtt{topT}: \tau \quad \mathtt{botT}: \tau \quad s: \mathtt{Symbol} \quad rtop: \mathtt{RTopology}
                                                                                                signature
               AllDD[s,rtop]: DomainDesc Continuous topT botT
                                                                                       (43)
                                                                                                arg but
                                                                                                can desync
7. Inclusive:
                                                                                                 - can we
                                     Inc[]: \mathtt{Inclusive}
                                                                                       (44)
                                                                                                just re-
                                                                                                move the
                                                                                                construc-
                                    \overline{Exc[]}: \mathtt{Inclusive}
                                                                                       (45)
                                                                                                tor arg?
```

8. RealInterval:

$$\frac{\mathtt{a}:\tau\quad \mathtt{b}:\tau\quad top: (\mathtt{Inclusive, a})\quad bot: (\mathtt{Inclusive, b})}{Bounded[top,bot]: \mathtt{RealInterval a b}} \tag{46}$$

$${\tt a: \tau \quad b: \tau \quad top: (Inclusive, a)} \over UpTo[top]: {\tt RealInterval \ a \ b}$$

$$\frac{\mathtt{a}:\tau\quad\mathtt{b}:\tau\quad bot: (\mathtt{Inclusive, b})}{UpFrom[bot]:\mathtt{RealInterval a b}} \tag{48}$$

2.3 Expr

1. Literals:

$$\frac{\mathbf{x}:\tau\quad l: \mathtt{Literal}\ \mathbf{x}}{Lit[l]: \mathtt{Expr}\ \mathbf{x}} \tag{49}$$

2. Associative Operations:

$$\frac{\mathbf{x} : \tau \quad op : \mathsf{AssocOp} \ \mathbf{x} \quad args : [\mathsf{Expr} \ \mathbf{x}]}{\mathsf{Assoc}[op, args] : \mathsf{Expr} \ \mathbf{x}} \tag{50}$$

3. Symbols:

$$\begin{array}{ccc} \underline{\mathbf{x}} : \tau & u : \mathtt{UID} \\ C[u] : \mathtt{Expr} & \mathbf{x} \end{array} \tag{51}$$

4. Function Call:

addressed in "misc" section

5. Case:

$$\frac{\mathbf{x}:\tau\quad c: \texttt{Completeness}\quad ces: \texttt{[(Expr Bool, Expr x)]}}{Case[c,ces]: \texttt{Expr x}} \tag{52}$$

6. Matrices:

$$\frac{\mathbf{x}:\tau\quad es: \texttt{[[Expr x]]}}{Matrix[es]: \texttt{Expr x}} \tag{53}$$

7. Unary Operations:

$$\frac{\mathbf{x}:\tau\quad\mathbf{y}:\tau\quad op: \mathtt{Unary0p}\ \mathbf{x}\ \mathbf{y}\quad e: \mathtt{Expr}\ \mathbf{x}}{Unary[op,e]: \mathtt{Expr}\ \mathbf{y}} \tag{54}$$

8. Binary Operations:

$$\frac{\mathbf{x}:\tau\quad\mathbf{y}:\tau\quad\mathbf{z}:\tau\quad op: \texttt{BinaryOp}\;\mathbf{x}\;\mathbf{y}\;\mathbf{z}\quad l: \texttt{Expr}\;\mathbf{x}\quad r: \texttt{Expr}\;\mathbf{y}}{Binary[op,l,r]: \texttt{Expr}\;\mathbf{z}} \tag{55}$$

9. "Big" Operations:

$$\frac{\texttt{x}:\tau \quad op: \texttt{AssocOp x} \quad dom: \texttt{DomainDesc Discrete (Expr x) (Expr x)}}{BigOp[op, dom]: \texttt{Expr x}} \tag{56}$$

10. "Is in interval" operator:

$$\frac{\mathtt{x}:\tau\quad u:\mathtt{UID}\quad itvl:\mathtt{RealInterval}\ (\mathtt{Expr}\ \mathtt{x})\ (\mathtt{Expr}\ \mathtt{x})}{RealI[u,itvl]:\mathtt{Expr}\ \mathtt{x}} \tag{57}$$

2.4 ModelExpr

1. $\underline{B} \underbrace{C}_{A}$

2.5 CodeExpr

1. $\underline{B} \underbrace{C}_{A}$