

## Todo list

Discuss Numerics-(T) and Numerics-With-Negation-(T) . . . . .	2
discuss vectors in general . . . . .	3
discuss functions in general . . . . .	5

## 1 Typing Rules

### 1.1 Literal

1. Integers:

$$\frac{i : \text{Integer}}{\text{Integer}[i] : \text{Literal Integer}} \quad (1)$$

2. Strings (Text):

$$\frac{s : \text{String}}{\text{Str}[s] : \text{Literal String}} \quad (2)$$

3. Real numbers:

$$\frac{d : \text{Double}}{\text{DbI}[d] : \text{Literal Real}} \quad (3)$$

4. Whole numbered reals ( $\mathbb{Z} \subset \mathbb{R}$ ):

$$\frac{d : \text{Integer}}{\text{ExactDbI}[d] : \text{Literal Real}} \quad (4)$$

5. Percentages:

$$\frac{n : \text{Integer} \quad d : \text{Integer}}{\text{Perc}[n, d] : \text{Literal Real}} \quad (5)$$

### 1.2 Miscellaneous

1. Completeness:

$$\overline{\text{Complete[]} : \text{Completeness}} \quad (6)$$

$$\overline{\text{Incomplete[]} : \text{Completeness}} \quad (7)$$

2. AssocOp:

- (a) Numerics:

$$\frac{x : \text{Numerics(T)}}{\text{Add[]} : \text{AssocOp } x} \quad (8)$$

$$\frac{x : \text{Numerics(T)}}{\text{Mul[]} : \text{AssocOp } x} \quad (9)$$

(b) Bool:

$$\overline{And[] : AssocOp \text{ Bool}} \quad (10)$$

$$\overline{Or[] : AssocOp \text{ Bool}} \quad (11)$$

3. UnaryOp:

(a) Numerics:

$$\frac{x : \text{NumericsWithNegation}(T)}{Neg[] : \text{UnaryOp } x \ x} \quad (12)$$

$$\frac{x : \text{NumericsWithNegation}(T)}{Abs[] : \text{UnaryOp } x \ x} \quad (13)$$

$$\frac{x : \text{Numerics}(T)}{Exp[] : \text{UnaryOp } x \ \text{Real}} \quad (14)$$

For Log, Ln, Sin, Cos, Tan, Sec, Csc, Cot, Arcsin, Arccos, Arctan, and Sqrt, please use the following template, replacing “\$TRG” with the desired operator:

$$\overline{\$TRG[] : \text{UnaryOp } \text{Real } \text{Real}} \quad (15)$$

$$\overline{RtoI[] : \text{UnaryOp } \text{Real } \text{Integer}} \quad (16)$$

$$\overline{ItoR[] : \text{UnaryOp } \text{Integer } \text{Real}} \quad (17)$$

$$\overline{Floor[] : \text{UnaryOp } \text{Real } \text{Integer}} \quad (18)$$

$$\overline{Ceil[] : \text{UnaryOp } \text{Real } \text{Integer}} \quad (19)$$

$$\overline{Round[] : \text{UnaryOp } \text{Real } \text{Integer}} \quad (20)$$

$$\overline{Trunc[] : \text{UnaryOp } \text{Real } \text{Integer}} \quad (21)$$

(b) Vectors:

$$\frac{x : \text{NumericsWithNegation}(T)}{NegV[] : \text{UnaryOp } [x] \ [x]} \quad (22)$$

$$\frac{x : \text{Numerics}(T)}{Norm[] : \text{UnaryOp } [x] \ \text{Real}} \quad (23)$$

$$\frac{x : \tau}{Dim[] : \text{UnaryOp } [x] \ \text{Integer}} \quad (24)$$

Discuss  
Numerics-  
(T) and  
Numerics-  
With-  
Negation-  
(T)

(c) Booleans:

$$\overline{Not[] : \text{UnaryOp Bool Bool}} \quad (25)$$

4. BinaryOp:

(a) Arithmetic:

$$\overline{FracR[] : \text{BinaryOp Real Real Real}} \quad (26)$$

(b) Bool:

$$\overline{Impl[] : \text{BinaryOp Bool Bool Bool}} \quad (27)$$

$$\overline{Iff[] : \text{BinaryOp Bool Bool Bool}} \quad (28)$$

(c) Equality:

$$\overline{x : \tau} \quad \overline{Eq[] : \text{BinaryOp } x \ x \ \text{Bool}} \quad (29)$$

$$\overline{x : \tau} \quad \overline{NEq[] : \text{BinaryOp } x \ x \ \text{Bool}} \quad (30)$$

(d) Ordering:

$$\overline{x : \text{Numerics}(T)} \quad \overline{Lt[] : \text{BinaryOp } x \ x \ \text{Bool}} \quad (31)$$

$$\overline{x : \text{Numerics}(T)} \quad \overline{Gt[] : \text{BinaryOp } x \ x \ \text{Bool}} \quad (32)$$

$$\overline{x : \text{Numerics}(T)} \quad \overline{LEq[] : \text{BinaryOp } x \ x \ \text{Bool}} \quad (33)$$

$$\overline{x : \text{Numerics}(T)} \quad \overline{GEq[] : \text{BinaryOp } x \ x \ \text{Bool}} \quad (34)$$

(e) Indexing:

$$\overline{x : \tau} \quad \overline{Index[] : \text{BinaryOp } [x] \ \text{Integer } x} \quad (35)$$

(f) Vectors:

$$\overline{x : \text{Numerics}(T)} \quad \overline{Cross[] : \text{BinaryOp } [x] \ [x] \ [x]} \quad (36)$$

$$\overline{x : \text{Numerics}(T)} \quad \overline{Dot[] : \text{BinaryOp } [x] \ [x] \ x} \quad (37)$$

$$\overline{x : \text{Numerics}(T)} \quad \overline{Scale[] : \text{BinaryOp } [x] \ x \ [x]} \quad (38)$$

discuss  
vectors in  
general

5. RTopology:

$$\frac{}{\text{Discrete[]} : \text{RTopology}} \quad (39)$$

$$\frac{}{\text{Continuous[]} : \text{RTopology}} \quad (40)$$

6. DomainDesc:

$$\frac{\text{top} : \tau_1 \quad \text{bot} : \tau_2 \quad s : \text{Symbol} \quad \text{rtop} : \text{RTopology}}{\text{BoundedDD}[s, \text{rtop}, \text{top}, \text{bot}] : \text{DomainDesc} \quad \text{Discrete} \quad \tau_1 \quad \tau_2} \quad (41)$$

$$\frac{\text{topT} : \tau \quad \text{botT} : \tau \quad s : \text{Symbol} \quad \text{rtop} : \text{RTopology}}{\text{AllDD}[s, \text{rtop}] : \text{DomainDesc} \quad \text{Continuous} \quad \text{topT} \quad \text{botT}} \quad (42)$$

7. Inclusive:

$$\frac{}{\text{Inc[]} : \text{Inclusive}} \quad (43)$$

$$\frac{}{\text{Exc[]} : \text{Inclusive}} \quad (44)$$

8. RealInterval:

$$\frac{\text{a} : \tau \quad \text{b} : \tau \quad \text{top} : (\text{Inclusive}, \text{a}) \quad \text{bot} : (\text{Inclusive}, \text{b})}{\text{Bounded}[\text{top}, \text{bot}] : \text{RealInterval} \quad \text{a} \quad \text{b}} \quad (45)$$

$$\frac{\text{a} : \tau \quad \text{b} : \tau \quad \text{top} : (\text{Inclusive}, \text{a})}{\text{UpTo}[\text{top}] : \text{RealInterval} \quad \text{a} \quad \text{b}} \quad (46)$$

$$\frac{\text{a} : \tau \quad \text{b} : \tau \quad \text{bot} : (\text{Inclusive}, \text{b})}{\text{UpFrom}[\text{bot}] : \text{RealInterval} \quad \text{a} \quad \text{b}} \quad (47)$$

### 1.3 Expr

1. Literals:

$$\frac{\text{x} : \tau \quad l : \text{Literal} \quad \text{x}}{\text{Lit}[l] : \text{Expr} \quad \text{x}} \quad (48)$$

2. Associative Operations:

$$\frac{\text{x} : \tau \quad \text{op} : \text{AssocOp} \quad \text{x} \quad \text{args} : [\text{Expr} \quad \text{x}]}{\text{Assoc}[\text{op}, \text{args}] : \text{Expr} \quad \text{x}} \quad (49)$$

3. Symbols:

$$\frac{\text{x} : \tau \quad u : \text{UID}}{C[u] : \text{Expr} \quad \text{x}} \quad (50)$$

4. Function Call:

discuss  
functions  
in general

5. Case:

$$\frac{x : \tau \quad c : \text{Completeness} \quad ces : [(\text{Expr Bool}, \text{Expr } x)]}{\text{Case}[c, ces] : \text{Expr } x} \quad (51)$$

6. Matrices:

$$\frac{x : \tau \quad es : [[\text{Expr } x]]}{\text{Matrix}[es] : \text{Expr } x} \quad (52)$$

7. Unary Operations:

$$\frac{x : \tau \quad y : \tau \quad op : \text{UnaryOp } x \ y \quad e : \text{Expr } x}{\text{Unary}[op, e] : \text{Expr } y} \quad (53)$$

8. Binary Operations:

$$\frac{x : \tau \quad y : \tau \quad z : \tau \quad op : \text{BinaryOp } x \ y \ z \quad l : \text{Expr } x \quad r : \text{Expr } y}{\text{Binary}[op, l, r] : \text{Expr } z} \quad (54)$$

9. “Big” Operations:

$$\frac{x : \tau \quad op : \text{AssocOp } x \quad dom : \text{DomainDesc Discrete (Expr } x) \ (\text{Expr } x)}{\text{BigOp}[op, dom] : \text{Expr } x} \quad (55)$$

10. “Is in interval” operator:

$$\frac{x : \tau \quad u : \text{UID} \quad itvl : \text{RealInterval (Expr } x) \ (\text{Expr } x)}{\text{RealI}[u, itvl] : \text{Expr } x} \quad (56)$$

## 1.4 ModelExpr

1.

$$\frac{B \ C}{A}$$

## 1.5 CodeExpr

1.

$$\frac{B \ C}{A}$$