Todo list

These might need to be replaced with variants for Reals/Integers	3
Do we want to have the length of our vectors as a type argument?	3
define criteria for what a well-formed expression language should provide	3
Quantities discussion – remaining untyped	3
modulo, remainder, etc	5
discuss vectors in general	6
oddity: topology appears as a constructor arg and signature arg but can	
desync – can we just remove the constructor arg?	6
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1 Syntax

1.1 Current

An idealized version of the current syntax.

```
Real
                                                                            \mathbb{R}
                                                                                               Real numbers
                              String
                                                                         String
                                                                                                Text
                                                                            \mathbb{B}
                                                                                               Truth values (true/false)
                              Vector(\tau, n)
                                                                           [\tau]_n
                                                                                               Vectors (single element t
                              Tuple(\tau_1...\tau_n)
                                                                   \tau_1 \times \tau_2 \times ... \times \tau_n
                                                                                                Alternative vectors/tuple
      Literal l
                              Integer[n]
                                                                                               Integer number
                                                                            n
                                                                                               Real number
                              Real[r]
                              String[s]
                                                                                               Text
                              Bool[b]
                                                                            b
                                                                                               Boolean value
                              Vector(l_1...l_n)
                                                                     < l_1, ..., l_n >
                                                                                                Vectors*
                              Tuple (l_1...l_n)
                                                                       (l_1, ..., l_n)
                                                                                               Tuples*
   UnaryOp
                                                                                               Logical negation
                              Not
                                                                                               Numeric negation
                              Neg
                              . . .
                                                                                               omitted for brevity
  BinaryOp
                                                                                               Subtraction
                              Sub
                              Pow
                                                                                               Powers
                                                                                               omitted for brevity
AssocBinOp
                              Add
                                                                                               Addition
                              Mul
                                                                                               Multiplication
                                                                                               omitted for brevity
                                                                      UID ''s''
                                                                                               UIDs
         UID
                       ::=
                              UID(s)
        Expr
                              Literal(l)
                                                                                               Literal values
                  e
                              Vector(e_1...e_n)
                                                                                               Vectors
                                                                     < e_1, ..., e_n >
                              Var(u)
                                                                                                Variable (QuantityDict C
                                                                            u
                                                                                                "Complete" function app
                              FuncCall(f, e_1...e_n)
                                                                       f(e_1...e_n)
                              UnaryOp(\ominus, e)
                                                                                                Unary operations
                                                                          \ominus e
                              BinaryOp(\oplus, e_1, e_2)
                                                                                               Binary operations
                                                                        e_1 \oplus e_2
                                                                     e_1 \otimes ... \otimes e_n
                                                                                               Associative binary opera-
                              AssocOp(\otimes, e_1...e_n)
                              \mathtt{Case}(e_{1c}e_{1e}...e_{nc}e_{ne})
                                                             if e_{1c} then e_{2e} elif e_{2c} ... \bigotimes_{i=e_1}^{e_2} i
                                                                                               If-then-else-if-then-else (S
                              \mathtt{BigAsBinOp}(\otimes,e_1,e_2)
                                                                                               Apply a "big" op to a dis
                              IsInRlItrvl(u, e_1, e_2)
                                                                      u \in [e_1, e_2]
                                                                                                Variable in range
```

 \mathbb{Z}

Integer numbers

 $\ast\colon$ does not currently appear in the code at the moment, but would be needed/desired

Type τ

Integer

2 Typing Rules

2.1 Literal

1. Integers:

$$\frac{i: \mathtt{Integer}}{Integer[i]: \mathtt{Literal\ Integer}} \tag{1}$$

2. Strings (Text):

$$\frac{s: \mathtt{String}}{Str[s]: \mathtt{Literal String}} \tag{2}$$

3. Real numbers:

$$\frac{d: \mathtt{Double}}{Dbl[d]: \mathtt{Literal Real}} \tag{3}$$

4. Whole numbered reals $(\mathbb{Z} \subset \mathbb{R})$:

$$\frac{d: \texttt{Integer}}{ExactDbl[d]: \texttt{Literal Real}} \tag{4}$$

5. Percentages:

$$\frac{n: \texttt{Integer} \quad d: \texttt{Integer}}{Perc[n,d]: \texttt{Literal Real}} \tag{5}$$

2.2 Miscellaneous

Vectors

As of right now, Drasil/GOOL only supports lists and arrays as "code types", which would be the representations used for representing "vectors" in Drasil.

For now, the below type rules define vectors with Haskell lists. We can choose to create our own type with the length of the vector as a parameter – likely going "too far into Haskell".

Functions

Presently, functions are defined through "QDefinitions", where a list of UIDs used in an expression are marked as the parameters of the function. Function "calls"/applications are captured in "Expr" (the expression language) by providing a list of input expressions and a list of named inputs (expressions) – f(x, y, z, a = b)").

A few solutions:

1. Leave expressions in general untyped in Haskell, and rely on calculating the "space" of an expression dynamically to ensure that expressions are well-formed. If runtime (drasil's compiling-knowledge-time) type analysis is ever needed, this will prove much easier to use in general.

These might need to be replaced with variants for Reals/Integers

Do we want to have the length of our vectors as a type argument?

define criteria for what a wellformed expression language should provide

Quantities discussion – remain-

- 2. Push the typing rules into Haskell via Generalized Algebraic Data Types (GADTs). Here, a larger question appears regarding functions how should we handle function creation, application, and typing?
 - (a) Currying and applying arguments (allowing partial function applications): This would work well if we only generated functional languages, but it might prove problematic for GOOL if expressions are left with partial function applications.

Type Rules

1. Completeness:

$$\overline{Complete[]: Completeness}$$
 (6)

$$\overline{Incomplete[]: \texttt{Completeness}} \tag{7}$$

- 2. AssocOp:
 - (a) Numerics:

$$\frac{x: \texttt{Numerics}(T)}{Add[]: \texttt{AssocOp x}}$$
 (8)

$$\frac{x: \texttt{Numerics}(T)}{Mul||: \texttt{AssocOp x}} \tag{9}$$

(b) Bool:

$$\overline{And[]}: \texttt{AssocOp Bool}$$
 (10)

$$\overline{Or[]: \mathtt{AssocOp\ Bool}}$$
 (11)

- 3. UnaryOp:
 - (a) Numerics:

$$\frac{x: \texttt{NumericsWithNegation(T)}}{Neg[]: \texttt{UnaryOp x x}} \tag{12}$$

$$\frac{x: \texttt{NumericsWithNegation(T)}}{Abs[]: \texttt{UnaryOp x x}} \tag{13}$$

$$\frac{x: \texttt{Numerics(T)}}{Exp[]: \texttt{UnaryOp x Real}} \tag{14}$$

For Log, Ln, Sin, Cos, Tan, Sec, Csc, Cot, Arcsin, Arccos, Arctan, and Sqrt, please use the following template, replacing "TRG" with the desired operator:

$$\overline{\$TRG[]}: \mathtt{UnaryOp} \ \mathtt{Real} \ \mathtt{Real}$$

	$\overline{RtoI[]}: exttt{UnaryOp Real Integer}$	(16)
	$\overline{ItoR[]}: exttt{UnaryOp Integer Real}$	(17)
	$\overline{Floor[]}: {\tt UnaryOp\ Real\ Integer}$	(18)
	$\overline{Ceil[]}: exttt{UnaryOp Real Integer}$	(19)
	$\overline{Round[]}: {\tt UnaryOp\ Real\ Integer}$	(20)
(1) 77	$\overline{Trunc[]}: {\tt UnaryOp\ Real\ Integer}$	(21)
(b) Vectors:	N . H. IN (TD)	
	$rac{x: exttt{NumericsWithNegation(T)}}{NegV[]: exttt{UnaryOp [x] [x]}}$	(22)
	$\frac{x: \texttt{Numerics}(\mathbf{T})}{Norm[]: \texttt{UnaryOp} \texttt{ [x] Real}}$	(23)
	$\frac{\mathtt{x} : \tau}{Dim[] : \mathtt{UnaryOp} \ [\mathtt{x}] \ \mathtt{Integer}}$	(24)
(c) Booleans:		
(c) Booleans.	$\overline{Not[]}: { t UnaryOp Bool Bool}$	(25)
4. BinaryOp:		
(a) Arithmetic:		
\overline{Fr}	acI[]: BinaryOp Integer Integer Integer	(26)
	$\overline{FracR[]}: exttt{BinaryOp Real Real Real}$	(27) modulo,
(b) Bool:		remainder,
. ,	$\overline{Impl[]}: exttt{BinaryOp Bool Bool Bool}$	(28) etc.
	$\overline{Iff[]: exttt{BinaryOp Bool Bool Bool}}$	(29)

(c) Equality: $\mathtt{x}:\tau$ $\overline{Eq[]}: BinaryOp x x Bool$ (30) $\overline{NEq[]}: \mathtt{BinaryOp} \ \mathtt{x} \ \mathtt{x} \ \mathtt{Bool}$ (31)(d) Ordering: x: Numerics(T)Lt[]: BinaryOp x x Bool (32)x: Numerics(T)Gt[]: BinaryOp x x Bool (33) $x: \mathtt{Numerics}(\mathbf{T})$ LEq[]: BinaryOp x x Bool (34) $x: \mathtt{Numerics}(\mathbf{T})$ $\overline{GEq[]}: \texttt{BinaryOp} \ \texttt{x} \ \texttt{x} \ \texttt{Bool}$ (35)(e) Indexing: Index[]: BinaryOp [x] Integer x (36)(f) Vectors: discuss x: Numerics(T)vectors in Cross[]: BinaryOp [x] [x] [x] (37)general $x: \mathtt{Numerics}(\mathbf{T})$ $\overline{Dot}[]: \texttt{BinaryOp} \ [\texttt{x}] \ \texttt{x}$ (38) $x: \mathtt{Numerics}(\mathbf{T})$ $\overline{Scale[]}$: BinaryOp [x] x [x] (39)5. RTopology: Discrete[]: RTopology(40) $\overline{Continuous}[]: RTopology$ (41)6. DomainDesc: ___ oddity: topology $top: au_1 \quad bot: au_2 \quad s: exttt{Symbol} \quad rtop: exttt{RTopology}$ appears $\overline{BoundedDD[s,rtop,top,bot]}: exttt{DomainDesc D} exttt{Discrete} \ au_1 \ au_2$ as a con-(42)structor arg and $\mathsf{topT}: \tau \quad \mathsf{botT}: \tau \quad s: \mathsf{Symbol} \quad rtop: \mathsf{RTopology}$ signature AllDD[s, rtop]: DomainDesc Continuous topT botT (43)arg but can desync - can we 6 just re-

move the constructor arg?

7. Inclusive:

$$\overline{Inc[]}: \mathtt{Inclusive}$$
 (44)

$$\overline{Exc[]}:$$
 Inclusive (45)

8. RealInterval:

$$\frac{\mathtt{a}:\tau\quad\mathtt{b}:\tau\quad top: (\texttt{Inclusive, a})\quad bot: (\texttt{Inclusive, b})}{Bounded[top,bot]: \texttt{RealInterval a b}} \tag{46}$$

$$\frac{\mathtt{a}:\tau\quad\mathtt{b}:\tau\quad top: (\texttt{Inclusive, a})}{UpTo[top]: \texttt{RealInterval a b}} \tag{47}$$

$$\frac{\mathtt{a}:\tau\quad\mathtt{b}:\tau\quad bot: (\mathtt{Inclusive, b})}{UpFrom[bot]:\mathtt{RealInterval a b}} \tag{48}$$

2.3 Expr

1. Literals:

$$\frac{\mathbf{x}:\tau\quad l: \mathtt{Literal}\ \mathbf{x}}{Lit[l]: \mathtt{Expr}\ \mathbf{x}} \tag{49}$$

2. Associative Operations:

$$\frac{\mathbf{x}:\tau \quad op: \texttt{AssocOp} \ \mathbf{x} \quad args: [\texttt{Expr} \ \mathbf{x}]}{Assoc[op, args]: \texttt{Expr} \ \mathbf{x}} \tag{50}$$

3. Symbols:

$$\frac{\mathbf{x}:\tau\quad u:\mathtt{UID}}{C[u]:\mathtt{Expr}\ \mathbf{x}}\tag{51}$$

4. Function Call:

addressed in "misc" section

5. Case:

$$\frac{\mathbf{x}:\tau\quad c: \texttt{Completeness}\quad ces: \texttt{[(Expr Bool, Expr x)]}}{Case[c,ces]: \texttt{Expr x}} \tag{52}$$

6. Matrices:

$$\frac{\mathbf{x}:\tau \quad es: \texttt{[[Expr x]]}}{Matrix[es]: \texttt{Expr x}} \tag{53}$$

7. Unary Operations:

$$\frac{ \texttt{x} : \tau \quad \texttt{y} : \tau \quad op : \texttt{UnaryOp} \ \texttt{x} \ \texttt{y} \quad e : \texttt{Expr} \ \texttt{x} }{Unary[op, e] : \texttt{Expr} \ \texttt{y} } \tag{54}$$

8. Binary Operations:

$$\frac{\mathbf{x}:\tau\quad\mathbf{y}:\tau\quad\mathbf{z}:\tau\quad op: \texttt{BinaryOp}\;\mathbf{x}\;\mathbf{y}\;\mathbf{z}\quad l: \texttt{Expr}\;\mathbf{x}\quad r: \texttt{Expr}\;\mathbf{y}}{Binary[op,l,r]: \texttt{Expr}\;\mathbf{z}} \tag{55}$$

9. "Big" Operations:

$$\frac{\texttt{x}:\tau \quad op: \texttt{AssocOp x} \quad dom: \texttt{DomainDesc Discrete (Expr x) (Expr x)}}{BigOp[op, dom]: \texttt{Expr x}} \tag{56}$$

10. "Is in interval" operator:

$$\frac{\mathtt{x}:\tau\quad u:\mathtt{UID}\quad itvl:\mathtt{RealInterval}\ (\mathtt{Expr}\ \mathtt{x})\ (\mathtt{Expr}\ \mathtt{x})}{RealI[u,itvl]:\mathtt{Expr}\ \mathtt{x}} \tag{57}$$

2.4 ModelExpr

1.

$$\frac{B C}{A}$$

2.5 CodeExpr

1.

$$\frac{B C}{A}$$