

## Todo list

These might need to be replaced with variants for Reals/Integers . . . . .	1
Do we want to have the length of our vectors as a type argument? . . . . .	1
define criteria for what a well-formed expression language should provide	2
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## 1 Typing Rules

### 1.1 Literal

1. Integers:

$$\frac{i : \text{Integer}}{\text{Integer}[i] : \text{Literal Integer}} \quad (1)$$

2. Strings (Text):

$$\frac{s : \text{String}}{\text{Str}[s] : \text{Literal String}} \quad (2)$$

3. Real numbers:

$$\frac{d : \text{Double}}{\text{Dbl}[d] : \text{Literal Real}} \quad (3)$$

4. Whole numbered reals ( $\mathbb{Z} \subset \mathbb{R}$ ):

$$\frac{d : \text{Integer}}{\text{ExactDbl}[d] : \text{Literal Real}} \quad (4)$$

5. Percentages:

$$\frac{n : \text{Integer} \quad d : \text{Integer}}{\text{Perc}[n, d] : \text{Literal Real}} \quad (5)$$

### 1.2 Miscellaneous

**Sorts Legend**    **Numerics(T)** : any numeric type  
                          **NumericsWithNegation(T)** : any signed numeric type

#### Vectors

As of right now, Drasil/GOOL only supports lists and arrays as “code types”, which would be the representations used for representing “vectors” in Drasil.

For now, the below type rules define vectors with Haskell lists. We can choose to create our own type with the length of the vector as a parameter – likely going “too far into Haskell”.

These might need to be replaced with variants for Reals/Integers

Do we want to have the length of our vectors as a type argument?

## Functions

Presently, functions are defined through “QDefinitions”, where a list of UIDs used in an expression are marked as the parameters of the function. Function “calls”/applications are captured in “Expr” (the expression language) by providing a list of input expressions and a list of named inputs (expressions) –  $f(x, y, z, a = "b")$ .

A few solutions:

1. Leave expressions in general untyped in Haskell, and rely on calculating the “space” of an expression dynamically to ensure that expressions are well-formed. If runtime (drasil’s compiling-knowledge-time) type analysis is ever needed, this will prove much easier to use in general.
2. Push the typing rules into Haskell via Generalized Algebraic Data Types (GADTs). Here, a larger question appears regarding functions – how should we handle function creation, application, and typing?
  - (a) Currying and applying arguments (allowing partial function applications): This would work well if we only generated functional languages, but it might prove problematic for GOOL if expressions are left with partial function applications.

define  
criteria  
for what  
a well-  
formed  
expression  
language  
should  
provide

Quantities  
discussion  
– remain-  
ing un-  
typed

## Type Rules

1. Completeness:

$$\overline{Complete[] : Completeness} \quad (6)$$

$$\overline{Incomplete[] : Completeness} \quad (7)$$

2. AssocOp:

- (a) Numerics:

$$\frac{x : \text{Numerics}(T)}{Add[] : \text{AssocOp } x} \quad (8)$$

$$\frac{x : \text{Numerics}(T)}{Mul[] : \text{AssocOp } x} \quad (9)$$

- (b) Bool:

$$\overline{And[] : \text{AssocOp } Bool} \quad (10)$$

$$\overline{Or[] : \text{AssocOp } Bool} \quad (11)$$

3. UnaryOp:

(a) Numerics:

$$\frac{x : \text{NumericsWithNegation}(T)}{\text{Neg}[] : \text{UnaryOp } x \ x} \quad (12)$$

$$\frac{x : \text{NumericsWithNegation}(T)}{\text{Abs}[] : \text{UnaryOp } x \ x} \quad (13)$$

$$\frac{x : \text{Numerics}(T)}{\text{Exp}[] : \text{UnaryOp } x \ \text{Real}} \quad (14)$$

For Log, Ln, Sin, Cos, Tan, Sec, Csc, Cot, Arcsin, Arccos, Arctan, and Sqrt, please use the following template, replacing “\$TRG” with the desired operator:

$$\overline{\text{\$TRG}[] : \text{UnaryOp } \text{Real } \text{Real}} \quad (15)$$

$$\overline{\text{RtoI}[] : \text{UnaryOp } \text{Real } \text{Integer}} \quad (16)$$

$$\overline{\text{ItoR}[] : \text{UnaryOp } \text{Integer } \text{Real}} \quad (17)$$

$$\overline{\text{Floor}[] : \text{UnaryOp } \text{Real } \text{Integer}} \quad (18)$$

$$\overline{\text{Ceil}[] : \text{UnaryOp } \text{Real } \text{Integer}} \quad (19)$$

$$\overline{\text{Round}[] : \text{UnaryOp } \text{Real } \text{Integer}} \quad (20)$$

$$\overline{\text{Trunc}[] : \text{UnaryOp } \text{Real } \text{Integer}} \quad (21)$$

(b) Vectors:

$$\frac{x : \text{NumericsWithNegation}(T)}{\text{NegV}[] : \text{UnaryOp } [x] \ [x]} \quad (22)$$

$$\frac{x : \text{Numerics}(T)}{\text{Norm}[] : \text{UnaryOp } [x] \ \text{Real}} \quad (23)$$

$$\frac{x : \tau}{\text{Dim}[] : \text{UnaryOp } [x] \ \text{Integer}} \quad (24)$$

(c) Booleans:

$$\overline{\text{Not}[] : \text{UnaryOp } \text{Bool } \text{Bool}} \quad (25)$$

4. BinaryOp:

(a) Arithmetic:

$$\overline{FracI[] : \text{BinaryOp Integer Integer Integer}} \quad (26)$$

$$\overline{FracR[] : \text{BinaryOp Real Real Real}} \quad (27)$$

(b) Bool:

$$\overline{Impl[] : \text{BinaryOp Bool Bool Bool}} \quad (28)$$

$$\overline{If[] : \text{BinaryOp Bool Bool Bool}} \quad (29)$$

(c) Equality:

$$\overline{x : \tau} \quad \overline{Eq[] : \text{BinaryOp } x \ x \ \text{Bool}} \quad (30)$$

$$\overline{x : \tau} \quad \overline{NEq[] : \text{BinaryOp } x \ x \ \text{Bool}} \quad (31)$$

(d) Ordering:

$$\overline{x : \text{Numerics}(T)} \quad \overline{Lt[] : \text{BinaryOp } x \ x \ \text{Bool}} \quad (32)$$

$$\overline{x : \text{Numerics}(T)} \quad \overline{Gt[] : \text{BinaryOp } x \ x \ \text{Bool}} \quad (33)$$

$$\overline{x : \text{Numerics}(T)} \quad \overline{LEq[] : \text{BinaryOp } x \ x \ \text{Bool}} \quad (34)$$

$$\overline{x : \text{Numerics}(T)} \quad \overline{GEq[] : \text{BinaryOp } x \ x \ \text{Bool}} \quad (35)$$

(e) Indexing:

$$\overline{x : \tau} \quad \overline{Index[] : \text{BinaryOp } [x] \ \text{Integer } x} \quad (36)$$

(f) Vectors:

$$\overline{x : \text{Numerics}(T)} \quad \overline{Cross[] : \text{BinaryOp } [x] \ [x] \ [x]} \quad (37)$$

$$\overline{x : \text{Numerics}(T)} \quad \overline{Dot[] : \text{BinaryOp } [x] \ [x] \ x} \quad (38)$$

$$\overline{x : \text{Numerics}(T)} \quad \overline{Scale[] : \text{BinaryOp } [x] \ x \ [x]} \quad (39)$$

modulo,  
remainder,  
etc.

discuss  
vectors in  
general

5. RTopology:

$$\overline{Discrete[] : \text{RTopology}} \quad (40)$$

$$\overline{Continuous[] : \text{RTopology}} \quad (41)$$

6. DomainDesc:

$$\frac{top : \tau_1 \quad bot : \tau_2 \quad s : \text{Symbol} \quad rtop : \text{RTopology}}{BoundedDD[s, rtop, top, bot] : \text{DomainDesc Discrete } \tau_1 \tau_2} \quad (42)$$

$$\frac{topT : \tau \quad botT : \tau \quad s : \text{Symbol} \quad rtop : \text{RTopology}}{AllDD[s, rtop] : \text{DomainDesc Continuous topT botT}} \quad (43)$$

7. Inclusive:

$$\overline{Inc[] : \text{Inclusive}} \quad (44)$$

$$\overline{Exc[] : \text{Inclusive}} \quad (45)$$

8. RealInterval:

$$\frac{a : \tau \quad b : \tau \quad top : (\text{Inclusive}, a) \quad bot : (\text{Inclusive}, b)}{Bounded[top, bot] : \text{RealInterval } a \ b} \quad (46)$$

$$\frac{a : \tau \quad b : \tau \quad top : (\text{Inclusive}, a)}{UpTo[top] : \text{RealInterval } a \ b} \quad (47)$$

$$\frac{a : \tau \quad b : \tau \quad bot : (\text{Inclusive}, b)}{UpFrom[bot] : \text{RealInterval } a \ b} \quad (48)$$

oddity:  
topology  
appears  
as a con-  
structor  
arg and  
signature  
arg but  
can desync  
– can we  
just re-  
move the  
construc-  
tor arg?

### 1.3 Expr

1. Literals:

$$\frac{x : \tau \quad l : \text{Literal } x}{Lit[l] : \text{Expr } x} \quad (49)$$

2. Associative Operations:

$$\frac{x : \tau \quad op : \text{AssocOp } x \quad args : [\text{Expr } x]}{Assoc[op, args] : \text{Expr } x} \quad (50)$$

3. Symbols:

$$\frac{x : \tau \quad u : \text{UID}}{C[u] : \text{Expr } x} \quad (51)$$

4. Function Call:

addressed  
in “misc”  
section

5. Case:

$$\frac{x : \tau \quad c : \text{Completeness} \quad ces : [(\text{Expr Bool}, \text{Expr } x)]}{\text{Case}[c, ces] : \text{Expr } x} \quad (52)$$

6. Matrices:

$$\frac{x : \tau \quad es : [[\text{Expr } x]]}{\text{Matrix}[es] : \text{Expr } x} \quad (53)$$

7. Unary Operations:

$$\frac{x : \tau \quad y : \tau \quad op : \text{UnaryOp } x \ y \quad e : \text{Expr } x}{\text{Unary}[op, e] : \text{Expr } y} \quad (54)$$

8. Binary Operations:

$$\frac{x : \tau \quad y : \tau \quad z : \tau \quad op : \text{BinaryOp } x \ y \ z \quad l : \text{Expr } x \quad r : \text{Expr } y}{\text{Binary}[op, l, r] : \text{Expr } z} \quad (55)$$

9. “Big” Operations:

$$\frac{x : \tau \quad op : \text{AssocOp } x \quad dom : \text{DomainDesc Discrete (Expr } x) \ (\text{Expr } x)}{\text{BigOp}[op, dom] : \text{Expr } x} \quad (56)$$

10. “Is in interval” operator:

$$\frac{x : \tau \quad u : \text{UID} \quad itvl : \text{RealInterval (Expr } x) \ (\text{Expr } x)}{\text{RealI}[u, itvl] : \text{Expr } x} \quad (57)$$

## 1.4 ModelExpr

1.

$$\frac{B \ C}{A}$$

## 1.5 CodeExpr

1.

$$\frac{B \ C}{A}$$