1 Typing Rules

1.1 Literal

1. Integers:

$$\frac{i : \texttt{Integer}}{Integer[i] : \texttt{Literal Integer}}$$

2. Strings (Text):

$$\frac{s : \texttt{String}}{Str[s] : \texttt{Literal String}}$$

3. Real numbers:

$$\frac{d : \texttt{Double}}{Dbl[d] : \texttt{Literal Real}}$$

4. Whole numbered reals ($\mathbb{Z} \subset \mathbb{R}$):

$$\frac{d : \texttt{Integer}}{ExactDbl[d] : \texttt{Literal Real}}$$

5. Percentages:

$$\frac{n : \texttt{Integer} \quad d : \texttt{Integer}}{Perc[n, d] : \texttt{Literal Real}}$$

1.2 Miscellaneous

1. Completeness:

$$\frac{}{Complete[] : \texttt{Completeness}}$$

$$\frac{}{Incomplete[] : \texttt{Completeness}}$$

2. AssocOp:

   (a) Numerics:

$$\frac{x : \texttt{Numerics}(\tau)}{Add[] : \texttt{AssocOp x}}$$

$$\frac{x : \texttt{Numerics}(\tau)}{Mul[] : \texttt{AssocOp x}}$$

   (b) Bool:

$$\frac{}{And[] : \texttt{AssocOp Bool}}$$

$$\frac{}{Or[] : \texttt{AssocOp Bool}}$$

3. UnaryOp:

   (a) Numerics:

$$\frac{x : \texttt{NumericsWithNegation(x)}}{Neg[] : \texttt{UnaryOp x x}}$$

$$\frac{x : \texttt{NumericsWithNegation(x)}}{Abs[] : \texttt{UnaryOp x x}}$$

   For Log, Ln, Sin, Cos, Tan, Sec, Csc, Cot, Arcsin, Arccos, Arctan, and Sqrt, please use the following template, replacing "$\$TRG$" with the desired operator:

$$\frac{}{\$TRG[] : \texttt{UnaryOp Real Real}}$$

   (b) Vectors:

$$\frac{x : \texttt{NumericsWithNegation(x)}}{NegV[] : \texttt{UnaryOp [x] [x]}}$$

$$\frac{x : \texttt{Numerics(x)}}{Norm[] : \texttt{UnaryOp [x] Real}}$$

$$\frac{\texttt{x} : \tau}{Dim[] : \texttt{UnaryOp [x] Integer}}$$

   (c) Booleans:

$$\frac{}{Not[] : \texttt{UnaryOp Bool Bool}}$$

4. BinaryOp:

   (a) Arithmetic:

   (b) Bool:

   (c) Equality:

   (d) Ordering:

   (e) Indexing:

   (f) Vectors:

5. RTopology:

$$\frac{}{Discrete[] : \texttt{RTopology}}$$

$$\frac{}{Continuous[] : \texttt{RTopology}}$$

6. DomainDesc:

$$\frac{top : \tau_1 \quad bot : \tau_2 \quad s : \texttt{Symbol} \quad rtop : \texttt{RTopology}}{BoundedDD[s, rtop, top, bot] : \texttt{DomainDesc Discrete } \tau_1 \ \tau_2}$$

$$\frac{\texttt{topT} : \tau \quad \texttt{botT} : \tau \quad s : \texttt{Symbol} \quad rtop : \texttt{RTopology}}{AllDD[s, rtop] : \texttt{DomainDesc Continuous topT botT}}$$

7. Inclusive:

$$\frac{}{Inc[] : \texttt{Inclusive}}$$

$$\frac{}{Exc[] : \texttt{Inclusive}}$$

8. RealInterval:

$$\frac{\texttt{a} : \tau \quad \texttt{b} : \tau \quad top : \texttt{(Inclusive, a)} \quad bot : \texttt{(Inclusive, b)}}{Bounded[top, bot] : \texttt{RealInterval a b}}$$

$$\frac{\texttt{a} : \tau \quad \texttt{b} : \tau \quad top : \texttt{(Inclusive, a)}}{UpTo[top] : \texttt{RealInterval a b}}$$

$$\frac{\texttt{a} : \tau \quad \texttt{b} : \tau \quad bot : \texttt{(Inclusive, b)}}{UpFrom[bot] : \texttt{RealInterval a b}}$$

1.3 Expr

1. Literals:

$$\frac{\texttt{x} : \tau \quad l : \texttt{Literal x}}{Lit[l] : \texttt{Expr x}}$$

2. Associative Operations:

$$\frac{\texttt{x} : \tau \quad op : \texttt{AssocOp x} \quad args : \texttt{[Expr x]}}{Assoc[op, args] : \texttt{Expr x}}$$

3. Symbols:

$$\frac{\texttt{x} : \tau \quad u : \texttt{UID}}{C[u] : \texttt{Expr x}}$$

4. Function Call:

5. Case:

$$\frac{\texttt{x} : \tau \quad c : \texttt{Completeness} \quad ces : \texttt{[(Expr Bool, Expr x)]}}{Case[c, ces] : \texttt{Expr x}}$$

6. Matrices:

$$\frac{\texttt{x} : \tau \quad es : \texttt{[[Expr x]]}}{Matrix[es] : \texttt{Expr x}}$$

7. Unary Operations:

$$\frac{\texttt{x} : \tau \quad \texttt{y} : \tau \quad op : \texttt{UnaryOp x y} \quad e : \texttt{Expr x}}{Unary[op, e] : \texttt{Expr y}}$$

8. Binary Operations:

$$\frac{\texttt{x} : \tau \quad \texttt{y} : \tau \quad \texttt{z} : \tau \quad op : \texttt{BinaryOp x y z} \quad l : \texttt{Expr x} \quad r : \texttt{Expr y}}{Binary[op, l, r] : \texttt{Expr z}}$$

9. "Big" Operations:

$$\frac{\texttt{x} : \tau \quad op : \texttt{AssocOp x} \quad dom : \texttt{DomainDesc Discrete (Expr x) (Expr x)}}{BigOp[op, dom] : \texttt{Expr x}}$$

10. "Is in interval" operator:

$$\frac{\texttt{x} : \tau \quad u : \texttt{UID} \quad itvl : \texttt{RealInterval (Expr x) (Expr x)}}{RealI[u, itvl] : \texttt{Expr x}}$$

1.4 ModelExpr

1.

$$\frac{B \quad C}{A}$$

1.5 CodeExpr

1.

$$\frac{B \quad C}{A}$$