Chapter 5

5.2 $\mu = \lim_{t\to 0} \frac{e^t(1-e^{kt}-ke^{kt}+ke^{t-kt})}{(e^t-1)^2 k} = \frac{k+1}{2}$

5.3 $f(0) = 1-\theta,\quad f(1) = \theta$

(a) $\sum_{x=0}^{1} x^r f(x) = 0^r(1-\theta) + 1^r \cdot \theta = \theta$

(b) $M_x(t) = \sum_{x=0}^{1} e^{tx} f(x) = (1-\theta) + e^t \cdot \theta = 1 + \theta(e^t - 1)$

$$= 1 + \theta\left(t + \frac{t^2}{2!} + \frac{t^3}{3!} + \frac{t^4}{4!} + \ldots\right)$$

$\mu_r' = \theta$

5.4 $\mu = \theta \qquad \mu_2' = \theta \qquad \sigma^2 = \theta - \theta^2 = \theta(1-\theta)$

(a) $\mu_3' = \theta \qquad \mu_3 = \theta - 3\theta\cdot\theta + 28^3 = \theta(1-3\theta+2\theta^2) = \theta(1-2\theta)(1-\theta)$

$$\alpha_3 = \frac{\theta(1-\theta)(1-2\theta)}{\theta(1-\theta)\sqrt{\theta(1-\theta)}} = \frac{1-2\theta}{\sqrt{\theta(1-\theta)}}$$

$$\mu_4 = \theta - 4\theta^2 + 6\theta^3 - 3\theta^4 = \theta(1-4\theta+6\theta^2-3\theta^2)$$
$$= \theta(1-\theta)[1-3\theta(1-\theta)]$$

(b) $\alpha_4 = \frac{\theta(1-\theta)[1-3\theta(1-\theta)]}{\theta^2(1-\theta)^2} = \frac{1-3\theta(1-\theta)}{\theta(1-\theta)}$

5.5 **(a)** $b(n-x;\ n,\ 1-\theta) = \binom{n}{n-x}(1-\theta)^{n-x}\theta^{n-(n-x)}$

$$= \binom{n}{x}\theta^x(1-\theta)^{n-x} = b(x;\ n,\ \theta)$$

(b) $B(x;\ n,\ \theta) - B(x-1;\ n,\ \theta) = \sum_{i=1}^{x} - \sum_{i=1}^{x-1} b = b(x;\ n, \theta)$

(c) $B(n-x;\ n,\ 1-\theta) = B(n-x-1;\ n,\ 1-\theta)$

$$= b(n-x;\ n,\ 1\text{-}\theta) == \binom{n}{n-x}(1-\theta)^{n-x}\theta^{n-(n-x)}$$

$$= \binom{n}{x}\theta^x(1-\theta)^{n-x} = b(x;\ n,\ \theta)$$

(d) $1-B(n-x-1;\ n,\ 1-\theta)=1-\sum_{k=0}^{n-x-1} b(k;\ n,\ 1-\theta)$

$$=\sum_{k=n=x}^{n} b(k;\ n,\ 1-\theta)$$

$$=\sum_{r=x}^{0} b(n-r;\ n,\ 1-\theta)=\sum_{\text{r}=0}^{\text{x}} b(r;\ n,\ \theta)=B(x;\ n,\ \theta) \qquad \text{QED}$$

5.6 **(a)** $B(x;\ n,\ \theta)-B(x-1;\ n,\ \theta)=\sum_{i=1}^{x}-\sum_{i=1}^{x-1}=b(x;n,\theta)$

(b) $B(n-x;n,\ 1-\theta)-B(n-x-1;\ n,\ 1-\theta)$

$$=b(n-x;\ n,\ 1-\theta)=\binom{n}{n-x}(1-\theta)^{n-x}\theta^{n-(n-x)}$$

$$=\binom{n}{x}\theta^{x}(1-\theta)^{n-x}=b(x;\ n,\ \theta)$$

(c) $1-B(n-x-1;\ n,\ 1-\theta)=1-\sum_{k=0}^{n-x-1} b(k;\ n,\ 1-\theta)$

$$=\sum_{k=n-x}^{n} b(k;\ n,\ 1-\theta)$$

$$=\sum_{r=x}^{0} b(n-r;\ n,\ 1-\theta)=\sum_{r=0}^{x} b(r;\ n,\ \theta)=B(x;\ n,\ \theta) \qquad \text{QED}$$

5.7 $E(Y)=E\left(\frac{X}{n}\right)=\frac{1}{n}E(X)=\frac{n\theta}{n}=\theta$

$$\mu_2'=E\left(\frac{X^2}{n^2}\right)=\frac{1}{n^2}[n\theta(1-\theta)+n^2\theta^2$$

$$\sigma_Y^2=\frac{1}{n^2}[n\theta-n\theta^2+n^2\theta^2-n^2\theta^2]=\frac{\theta(1-\theta)}{n}$$

5.8 $b(x+1;\ n,\theta)=\binom{n}{x+1}\theta^{x+1}(1-\theta)^{n-x-1}$

$$=\frac{n!}{(x+1)!(n-x-1)!}\theta^{x+1}(1-\theta)^{n-x-1}$$

$$=\frac{\theta}{1-\theta}\cdot\frac{n-x}{(x+1)}\cdot\binom{n}{x}\theta^{x}(1-\theta)^{n-x}=\frac{\theta(n-x)}{(x+1)(1-\theta)}b(x;\ n,\ \theta)$$

5.9 $\dfrac{b(x)}{b(x-1)} \ge 1 \qquad \dfrac{b(x+1)}{b(x)} \le 1 \qquad \dfrac{\theta(n-x-1)}{x(1-\theta)} \ge 1 \qquad \dfrac{\theta(n-x)}{(x+1)(1-\theta)} \le 1$

$\theta n - \theta x - \theta \ge x - \theta x \qquad\qquad \theta n - \theta x \le x + 1 - \theta x - \theta$

$x \le \theta(n-1) \qquad\qquad \theta n \le x + 1 - \theta$

$x \le \dfrac{n-1}{2} \qquad\qquad \theta(n+1) - 1 \le x$

(b) odd maximum at $\dfrac{n-1}{2} \qquad\qquad \dfrac{1}{2}n + \dfrac{1}{2} \le x \qquad x \ge \dfrac{n+1}{2}$

(a) even maximum at $\dfrac{n-1}{2}$ and $\dfrac{n+1}{2}$

5.10 $b(x;\, n,\, \theta) = \dbinom{n}{x}\theta^x (1-\theta)^{n-x}$

$\ln b = \ln\dbinom{n}{x} + x\ln\theta + (n-x)\ln(1-\theta)$

$\dfrac{\partial b}{\partial\theta} = \dfrac{x}{\theta} - \dfrac{n-x}{1-\theta} = 0 \qquad x - \theta x = n\theta - \theta x \qquad x = n\theta$ and $\theta = \dfrac{x}{n}$

5.11 $\mu_2' = E(x^2) = E(x^2 - x + x) = \mu'_{(2)} + \mu'_{(1)}$ Since $x^2 = x(1-x) + x$

let $x^3 = x(x-1)(x-2) + ax(x-1) + bx$

$x = 1 \qquad 1 = b$

$b = 1 \qquad a = 3 \qquad\qquad \mu_3' = \mu'_{(3)} + 3\mu'_{(2)} + \mu'_{(1)}$

$x = 2 \qquad 8 - 2a + 2$

$x^4 = x(x-1)(x-2)(x-3) + ax(x-1)(x-2) + bx(x-1) + cx$

$x = 1 \qquad 1 = c \qquad\qquad \mu_4' = \mu'_{(4)} + 6\mu'_{(3)} + 7\mu'_{(2)} + \mu'_{(2)}$

$x = 2 \qquad 16 = 2b + 2 \qquad b = 7$

$x = 3 \qquad 81 = 6a + 6b + 3c = 6a + 42 + 3$

$36 = 6a \qquad a = 6$

5.12 $F'(x) = \sum x t^{x-1} f(x) \qquad\qquad F'(1) = \sum x f(x) = \mu'_{(1)}$

$F''(x) = \sum x(x-1) t^{x-2} f(x) \qquad\qquad F''(1) = \sum x(x-1) f(x) = \mu'_{(2)}$

$F'''(x) = \sum x(x-1)(x-2) t^{x-3} f(x) \qquad\qquad F'''(1) = \sum x(x-1)(x-2) f(x) = \mu'_{(3)}$

etc.

5.13 **(a)** $F_x(t) = t^\theta \cdot (1-\theta) + t\theta = 1 - \theta + \theta t \qquad F' = \theta \qquad F'' = 0$ etc.

$\mu'_{(1)} = \theta \qquad \mu'_{(r)} = 0$ for $r > 1$

(b) $F_x(t) = \sum_x t^x \binom{n}{x} \theta^X (1-\theta)^{n-x} = \sum_x \binom{n}{x} (\theta t)^x (1-\theta)^{n-x}$

$$= [\theta t + 1 - \theta]^n$$

$$= [1 + \theta(t-1)]^n$$

$F' = n[1+\theta(t-1)]^{n-1}\theta \qquad F'(1) = n\theta$

$F'' = n(n-1)[1+\theta(t-1)]^{n-2}\theta^2 \qquad F''(1) = n(n-1)\theta^2$

$\mu = \mu'_{(1)} = n\theta \qquad \mu'_2 = \mu'_{(2)} + \mu'_{(1)} = n(n-1)\omega^2 + n\theta$

$\sigma^2 = n(n-1)\theta^2 + n\theta - n^2\theta^2 = n\theta - n\theta^2 = n\theta(1-\theta)$

5.14 $M'_Y = e^{-\mu t} M'_X(t) + M_X(t)(-\mu)e^{-\mu t} = e^{-\mu t}[M'_X(t) - \mu M_X(t)]$

(a) Expand series.

(b) $M_{X-\mu}(t) = e^{-n\theta t}[1+\theta(e^t-1)]^n$

$$M'_{x-\mu}(t) = e^{-n\theta t} \cdot n[1+\theta(e^t-1)^{n-1} \cdot \theta e^t - n\theta e^{-n\theta t}[1+\theta(e^t-1)]^n$$

$$= n\theta e^{-n\theta t}[1+\theta(e^t-1)^{n-1}\left\{1-[1+\theta(e^t-1)\right\}$$

$$= n\theta e^{-n\theta t}[1+\theta(e^t-1)^{n-1}\left\{e^t(1-\theta)-(1-\theta)\right\} \qquad M'_{x-\mu}(0) = 0$$

$$= -n\theta^2 e^{-n\theta t}(e^t-1)[1+\theta(e^t-1)]^{n-1}$$

$$M''_{x-\mu}(t) = -n\theta^2 e^{-n\theta t}(e^t-1)(n-1)[1+\theta(e^t-1)]^{n-2}(e^t-1)$$

$$- n\theta^2[1+\theta(e^t-1)]^{n-1}\left\{e^{-n\theta t} \cdot e^t + (e^t-1)(-n\theta e^{-n\theta t})\right\}$$

$$= e^{-n\theta t}[1+\theta(e^t-1)]^n$$

5.15 **(a)** $\theta = \frac{1}{2},\ \alpha_3 = 0;$ **(b)** $\alpha_3 \to 0$ as $n \to \infty$

5.16 $f(y) = \binom{y+k+1}{k-1}\theta^k(1-\theta)^y \qquad \begin{array}{l} y = x-k \\ y = 0,\ 1,\ 2,\ \ldots \end{array}$

5.17 $E(Y) = E(X) - k = \frac{k}{\theta} - k = k\left(\frac{1}{\theta} - 1\right)$

$\sigma_Y^2 = \sigma_X^2 = \frac{k}{\theta}\left(\frac{1}{\theta} - 1\right)$

5.18 $b^*(x;k,\theta) = \binom{x-1}{k-1}\theta^k(1-\theta)^{x-k} = \frac{k}{x}\binom{x}{k}\theta^k(1-\theta)^{x-k} = \frac{k}{x}b(k;x,\theta)$ QED

5.19 $E(x) = \sum_{x=k}^{\infty} x \binom{x-1}{k-1} \theta^k (1-\theta)^{x-k} = \frac{k}{\theta} \sum_{x=k}^{\infty} \binom{x}{k} \theta^{x+1} (1-\theta)^{x-k}$ $\qquad y = x+1$, $m = k+1$

$$= \frac{k}{\theta} \sum_{y=m}^{\infty} \binom{y-1}{m-1} \theta^y (1-\theta)^{y-m} = \frac{k}{\theta}$$

$$E[x(x+1)] = \sum_{x=k}^{\infty} x(x+1) \binom{x-1}{k-1} \theta^k (1-\theta)^{x-k}$$

$$= \frac{k(k+1)}{\theta^2} \sum_{x=k}^{\infty} \binom{x+1}{k+1} \theta^{x+2} (1-\theta)^{x-k} \qquad y = x+2, \quad m = k+2$$

$$= \frac{k(k+1)}{\theta^2} \sum_{y=m}^{\infty} \binom{y-1}{m-1} \theta^{x+2} (1-\theta)^{y-k} = \frac{k(k+1)}{\theta^2}$$

$$\sigma^2 = \frac{k(k+1)}{\theta^2} - \frac{k}{\theta} - \frac{k^2}{\theta^2} = \frac{k^2 + k - k\theta - k^2}{\theta^2} = \frac{k(1-\theta)}{\theta^2} = \frac{k}{\theta}\left(\frac{1}{\theta} - 1\right)$$

5.20 $g(x) = \theta(1-\theta)^{x-1} \qquad x = 1, 2, 3, \ldots$

$$M = \sum_{x=1}^{\infty} e^{tx} \theta(1-\theta)^{x-1} = \sum_{x=1}^{\infty} \theta \frac{[e^t(1-\theta)]^x}{1-\theta} = \frac{\theta}{1-\theta} \sum_{x=1}^{\infty} [e^t(1-\theta)]^x$$

$$= \frac{\theta}{1-\theta} \cdot \frac{e^t(1-\theta)}{1-e^t(1-\theta)} = \frac{\theta e^t}{1-e^t(1-\theta)} \qquad \text{QED}$$

5.21 $M' = \dfrac{[1-e^t(1-\theta)\theta e^t + \theta e^t(1-\theta)e^t}{[1-e^t(1-\theta)]^2} = \dfrac{\theta e^t - \theta e^{2t}(1-\theta) + \theta e^{2t} - \theta^2 e^{2t}}{[1-e^t(1-\theta)]^2}$

$$= \frac{\theta e^t}{[1-e^t(1-\theta)]^2}$$

$$M'(0) = \frac{\theta}{\theta^2} = \frac{1}{\theta}$$

$$M'' = \frac{[1-e^t(1-\theta)]^2 \theta e^t - \theta e^t \cdot 2[1-e^t(1-\theta)][-e^t(1-\theta)]}{[1-e^t(1-\theta)]^4}$$

$$M''(0) = \frac{\theta^2 - 2\theta \cdot \theta(1-\theta)}{\theta^4} - \frac{2-\theta}{\theta^2} \qquad \sigma^2 = \frac{2-\theta}{\theta^2} - \frac{1}{\theta^2} = \frac{1-\theta}{\theta^2}$$

5.22 $\sum_{x=1}^{\infty}\theta(1-\theta)^{x-1}=1$

$\theta+\sum_{x=2}^{\infty}\theta(1-\theta)^{x-1}=1 \qquad y=x-1$

$\sum_{y=1}^{\infty}\theta(1-\theta)^{y}=1-\theta$

$\sum_{y=1}^{\infty}[(1-\theta)^{y}+\theta y(1-\theta)^{y-1}(-\theta)]-1$

$\sum_{y=1}^{\infty}(1-\theta)^{y}-\sum_{y=1}^{\infty}\theta(1-\theta)^{y-1}=-1$

$\frac{1-\theta}{\theta}-\mu=-1$

$-\mu=-\frac{1}{\theta}$ and $\mu=\frac{1}{\theta}$

$\theta+\theta(1-\theta)+\sum_{x=3}^{\infty}\theta(1-\theta)^{x-1}=1 \quad y=x-2$

$\theta+\theta(1-\theta)+\sum_{y=1}^{\infty}\theta(1-\theta)^{y+1}=1$

$\sum_{y=1}^{\infty}\theta(1-\theta)^{y+1}=1-\theta-\theta(1-\theta)=(1-\theta)^2$

then differentiate *twice* with respect to θ.

5.23 $P(X=x+n|x>n)=\frac{P(X=x+n)}{P(X>n)}=\frac{\theta(1-\theta)^{x+n}}{(1-\theta)^{n}}=\theta(1-\theta)^{x}$ QED

$P(X>n)=\frac{\theta(1-\theta)^{n}}{1-(1-\theta)}=(1-\theta)^{n}$

5.24 $f(x)=\theta(1-\theta)^{x-1}$ $F(x)=\sum_{t=1}^{x}\theta(1-\theta)^{tx-1}=\theta\cdot\frac{1-(1-\theta)^{x}}{1-(1-\theta)}=1-(1-\theta)^{x}$

$z(x)=\frac{\theta(1-\theta)^{x-1}}{(1-\theta)^{x-1}}=\theta$

5.25 $X=X_1+X_2=\ldots X_n$

(a) $E(X)=\sum E(X_i)=\sum\theta_i=n\frac{\sum\theta_i}{n}=n\theta$

(b) $\sigma_X^2=\sum\sigma_i^2=n\sum\theta_i(1-\theta_i)=n\sum\theta_i-\sum\theta_i^2$

$=n\theta-n\sigma_\theta^2+n\theta^2=n\theta(1-\theta)-n\sigma_\theta^2$

5.26 $h(x+1)=\dfrac{\dbinom{M}{x+1}\dbinom{n-M}{n-x-1}}{\dbinom{N}{n}}$

$$=\frac{\dfrac{M!}{(x+1)!(M-x-1)!}\cdot\dfrac{(N-M)!}{(n-x-1)!(N-M-n+x+1)!}}{\dbinom{N}{n}}$$

$$=\frac{\dfrac{M-x}{x+1}\cdot\dfrac{M!}{x!(M-x)!}\cdot\dfrac{n-x}{N-M-n+x-1}\cdot\dfrac{(N-M)!}{(n-x)!(N-M-n+x)!}}{\dbinom{N}{n}}$$

$$=\frac{\dfrac{(M-x)(n-x)}{(x+1)(N-M-n+x+1)}\dbinom{M}{x}\dbinom{n-M}{n-x}}{\dbinom{N}{n}}$$

$$=\frac{(M-x)(n-x)}{(x+1)(N-M-n+x+1)}\cdot h(x)$$

$n=4,\ N=9,\ M=5$

$$h(0)=\frac{\dbinom{5}{0}\dbinom{4}{4}}{\dbinom{9}{4}}=\frac{1}{126},\quad h(1)=\frac{5\cdot 4}{1\cdot 1}\cdot\frac{1}{126}=\frac{20}{126}$$

$$h(2)=\frac{4\cdot 3}{2\cdot 2}\cdot\frac{20}{126}=\frac{60}{126},\ h(3)=\frac{3\cdot 2}{3\cdot 3}\cdot\frac{60}{126}=\frac{40}{126}$$

$$h(4)=\frac{2\cdot 1}{4\cdot 4}\cdot\frac{40}{126}=\frac{5}{126}$$

5.27 $E[X(X-1)] = \sum_{x=0}^{n} x(x-1)\frac{\binom{M}{x}\binom{N-M}{n-x}}{\binom{N}{n}}$

$$= \sum_{x=2}^{n} M(M-1)\frac{\binom{M-2}{x-2}\binom{N-M}{n-x}}{\binom{N}{n}} \qquad y = x-2, \quad m = n-2$$

$$= M(M-1)\sum_{y=0}^{m}\frac{\binom{M-2}{y}\binom{N-M}{m-y}}{\binom{N}{n}}$$

$$= \frac{M(M-1)n(n-1)}{N(N-1)}\sum_{y=0}^{m}\frac{\binom{M-2}{y}\binom{N-M}{m-y}}{\binom{N-2}{m}}$$

$$= \frac{M(M-1)n(n-1)}{N(N-1)} \qquad \text{QED}$$

5.28 $\theta = \frac{M}{N} \qquad \mu = n\frac{M}{N} = n\theta$

$$\sigma^2 = n\cdot\frac{M}{N}\cdot\left(1-\frac{M}{N}\right)\cdot\frac{N-n}{N-1} = n\theta(1-\theta)\cdot\frac{N-n}{N-1}$$

5.29 $P(x+1;\ \lambda) = \frac{\lambda^{x+1}e^{-\lambda}}{(x+1)!} = \frac{\lambda}{x+1}\cdot\frac{\lambda^{x}e^{-\lambda}}{x!} = \frac{\lambda}{x+1}\cdot p(x;\ \lambda)$

5.30 $p(3;\ 10) = \frac{10^3 e^{-10}}{6} = \frac{1000(0.000045)}{6} = \frac{0.045}{6} = 0.0075$

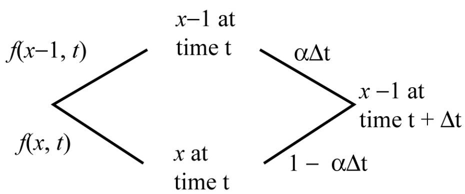
Table II yields 0.0076

(a) $\binom{100}{3}(0.1)^3(0.90)^{97} = \frac{100!}{3!97!}(0.1)^2(0.9)^{97}$

$$\log p = 157.97000 - 0.77815 - 151.98314 + 3(-1) + 97(0.95424) - 1$$
$$= 5.20871 - 3 + 92.56128 - 97$$
$$= 0.77699 - 3,\ p = 0.0060$$

(b) $p = 0.00598$

5.31



(a) $f(x,\ t+\Delta t) = f(x,\ t)(1-\alpha\Delta t) + f(x-1,\ t)\alpha\Delta t$

$$f(x,\ t+\Delta t) - f(x,\ t) = -\alpha\Delta t\, f(x,\ t) + \alpha\Delta t\, f(x-1,\ t)$$

$$\lim_{\Delta t\to 0}\frac{f(x,\ t+\Delta t)-f(x,\ t)}{\Delta t} = \alpha[f(x-1,\ t) - f(x,\ t)]$$

(b) $f(x,\ \alpha t) = \dfrac{(\alpha t)^x e^{-\alpha t}}{x!}\ \dfrac{\partial f}{\partial t} = \dfrac{\alpha^x x\ t^{x-1}\ e^{-at} + \alpha^x t^x(-\alpha\ e^{-\alpha t})}{x!}$

$$= \frac{\alpha^x x\ t^{x-1}\ e^{-at} - \alpha^{x+1} t^x\ e^{-\alpha t}}{x!}$$

$$\alpha[f(x-1,\ t) - f(x,\ t)] = \frac{\alpha\cdot(\alpha t)^{x-1}e^{-\alpha t}}{(x-1)!} - \frac{\alpha(\alpha t)^x e^{-\alpha t}}{x!}$$

$$= \frac{\alpha^x\cdot x\ t^{x-1}e^{-\alpha t} - \alpha^{x+1}t^x e^{-\alpha t}}{x!} \qquad \text{QED}$$

5.32 $u = t^x dv = e^{-t}dt \qquad v = -e^{-t}du = x\ t^{x-1}dt$

$$\frac{1}{x!}\int_\lambda^\infty t^x e^{-t}dt = \frac{\lambda^x e^{-\lambda}}{x!} + \frac{1}{(x-1)!}\int_\lambda^\infty t^{x-1}e^{-t}dt$$

$$= \frac{\lambda^x e^{-\lambda}}{x!} + \frac{\lambda^{x-1}e^{-\lambda}}{(x-1)!} + \frac{1}{(x-2)!}\int_\lambda^\infty t^{x-2}e^{-t}dt$$

$$= \frac{\lambda^x e^{-\lambda}}{x!} + \ldots + \frac{\lambda^0 e^{-\lambda}}{0!} = \sum_{y=0}^{x}\frac{\lambda^x e^{-\lambda}}{y!} \qquad \text{QED}$$

5.33 $E(X) = \displaystyle\sum_{x=0}^{\infty} x\cdot\frac{\lambda^x e^{-\lambda}}{x!} = \sum_{x=1}^{\infty}\lambda\cdot\frac{\lambda^{x-1}e^{-\lambda}}{(x-1)!} = \sum_{y=0}^{\infty}\lambda\cdot\frac{\lambda^y e^{-y}}{y!} = \lambda\cdot 1 = \lambda$

$$E[X(X-1)] = \sum_{x=2}^{\infty} x(x-1)\frac{\lambda^x e^{-\lambda}}{x!}$$

$$= \sum_{x=2}^{\infty}\lambda^2\frac{\lambda^{x-2}e^{-\lambda}}{(x-2)!} = \sum_{y=0}^{\infty}\lambda^2\frac{\lambda^y e^{-y}}{y!} = \lambda^2$$

$$\mu = \lambda,\ \sigma^2 = \lambda^2 + \lambda - \lambda^2 = \lambda$$

5.34 $n \to \infty,\ \theta \to 0,\ n\theta = \lambda$

$$M_x = [1 + \lambda(e^t - 1)]^n$$

$$= \left[1 + \frac{n\lambda(e^t - 1)}{n}\right]^n = \left[1 + \frac{\lambda(e^t - 1)}{n}\right]^n$$

$$\lim_{n\to\infty} = e^{\lambda(e^{t-1})} \text{ QED}$$

5.35 $M = e^{\lambda(e^t - 1)}$

$$M' = \lambda\, e^t e^{\lambda(e^t-1)} \qquad M'(0) = \lambda$$

$$M'' = (\lambda e^t)^2 e^{\lambda(e^t-1)} + \lambda e^t e^{\lambda(e^t-1)} \quad M''(0) = \lambda^2 + \lambda$$

$$M''' = (\lambda e^t)^3 e^{\lambda(e^t-1)} + 2(\lambda e^t)^2 e^{\lambda(e^t-1)} \qquad M'''(0) = \lambda^3 + 3\lambda^2 + \lambda$$

$$+(\lambda e^t)^2 e^{\lambda(e^t-1)} + \lambda e^t e^{\lambda(e^t-1)}$$

$$\mu = \lambda,\ \sigma^2 = \lambda^2 + \lambda - \lambda^2 = \lambda,\ \mu^3 = \lambda^3 + 3\lambda^2 + \lambda - 3\lambda(\lambda^2 + \lambda) + 2\lambda^2 = \lambda$$

$$\alpha_3 = \frac{1}{\left(\sqrt{\lambda}\right)^3} = \frac{1}{\sqrt{\lambda}}$$

5.36 $$\frac{d\mu_r}{d\lambda} = \sum_{x=0}^{\infty} r(x-\lambda)^{r-1} \cdot \frac{\lambda^x e^{-x}}{x!} + \frac{(x-\lambda)^r}{x!}\left\{x\lambda^{x-1}e^{-\lambda} - \lambda^x e^{-\lambda}\right\}$$

$$= -r\mu_{r-1} + \sum_{x=0}^{\infty} \frac{(x-\lambda)^r}{x!}\lambda^{x-1}e^{-\lambda}(x-\lambda)$$

$$= -r\mu_{r-1} + \sum_{x=0}^{\infty} (x-\lambda)^{r+1}\frac{\lambda^{x-1}e^{-x}}{x!}$$

$$= -r\mu_{r-1} + {}_{\lambda}\,\mu_{r+1} \qquad \mu_{r+1} = \lambda\left[r\mu_{r-1} + \frac{d\mu_r}{d\lambda}\right]$$

$$\mu_0 = 1,\ \mu_1 = 0,\ r = 1,\ \mu_2 = \lambda\left[1\cdot\mu_0 + \frac{d\mu_1}{d\lambda}\right] = \lambda$$

$$r = 2,\ \mu_3 = \lambda\left[2\cdot\mu_1 + 1\right] = \lambda$$

$$r = 3,\ \mu_4 = \lambda\left[3\cdot\lambda + 1\right] = \lambda + 3\lambda^2$$

5.57 $M_x = E(e^{xt}) = e^{\lambda(e^t-1)}$

$$M_Y = E[e^{(x-\lambda)t}] = e^{-\lambda t}E(e^{xt}) = e^{-\lambda t}e^{\lambda(e^t-1)} = e^{\lambda(e^t-t-1)}$$

$$M'_Y = \lambda(e^t - 1)e^{\lambda(e^t-t-1)}$$

$$M''_Y = \lambda^2(e^t-1)^2 e^{\lambda(e^t-t-1)} + \lambda e^t e^{\lambda(e^t-t-1)}$$

$$M'_Y(0) = \lambda$$

5.38 Marginal distribution of x_i is binomial distribution with parameter n and θ_i; therefore

$$\mu_1 = n\theta_i$$

5.39 $E(x_i x_j) = \sum\sum x_i x_j \frac{n!}{x_i!x_j!(n-x_i-x_j)!}\theta_i^{x_i}\ \theta_j^{x_j}(1-\theta_i-\theta_j)^{n-x_i-x_j}$

$= n(n-1)\theta_i\theta_j \sum\sum \frac{(n-2)!}{(x_i-1)!(x_j-1)!(n-x_i-x_j)!}\ \theta_i^{x_i-1}\theta_j^{x_j-1}(1-\theta_i-\theta_j)^{n-x_i-x_j}$

$= n(n-1)(\theta_i)(\theta_j)$

$\text{cov}(x_i,\ x_j) = n(n-1)\theta_i\theta_j - (n\theta_i)(n\theta_j)$

$= -n\theta_i\theta_j$

5.40 $\binom{8}{4}\left(\frac{1}{3}\right)^4\left(\frac{2}{3}\right)^4 = \frac{70\cdot 16}{6561} = 0.1707$

5.41 $\binom{5}{3}(0.1)^3(0.9)^2 + \binom{5}{4}(0.1)^4(0.9) + \binom{5}{5}(0.1)^5$

$= 10(0.001)(0.81) + 5(0.0001)(0.9) + (0.00001)$

$= 0.0081 + 0.00045 + 0.00001 = 0.0086$

5.42 **(a)** $\binom{6}{5}(0.7)^5(0.3) = 0.3025$

(b) 0.3025

5.43 **(a)** $\binom{15}{6}(0.4)^6(0.6)^6 = 5005(0.004096)(0.01008) = 0.2066$

(b) 0.2066

5.44 **(a)** 0.1669

(b) $0.1669 + 0.1214 + 0.0708 + 0.0327 + 0.0117 + 0.0031 + 0.0006 + 0.0001 = 0.4073$

(c) $0.0000 + 0.0001 + \ldots + 0.1669 = 0.4073$

5.45 **(a)** $0.1529 + 0.0578 + 0.0098 = 0.2205$

(b) $1 - 0.7794 = 0.2206$

5.46 **(a)** $0.0285 + 0.0849 + 0.1734 = 0.2868$

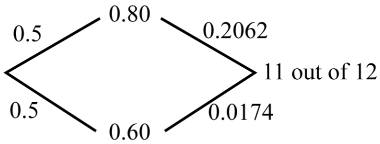
(b) $0.2939 - 0.0071 = 0.2868$

5.47 $p = 0.42,\ n = 15,\ x = 6,\ 0.2041$

5.48 $p = 0.51 \qquad n = 18$

(a) $x = 10$ 0.1731, **(b)** $1 - 0.5591 = 0.4409$, **(c)** 0.3742

5.49



$$\frac{2062}{2236} = 0.9222$$
$$1 - 0.9222 = 0.0778$$

5.50 **(a)** $\sigma_{\text{orig}} = \sqrt{np(1-p)}$. If $\sigma_{\text{new}} = \frac{1}{2}\sigma_{\text{orig}} = \frac{1}{2}\sqrt{np(1-p)} = \sqrt{\frac{n}{4}\,p(1-p)}$, then $n_{\text{new}} = \frac{1}{4} n_{\text{orig}}$

(b) $\sigma_{\text{orig}} = \sqrt{np(1-p)}$; $\sigma_{\text{new}} = \sqrt{nkp(1-p)} = \sqrt{k}\cdot\sqrt{np(1-p)} = \sqrt{k}\cdot\sigma_{\text{orig}}$

5.51 $P(x \ge 3) = 1 - b(0;20,0.05) - (b(1;20,0.05) - b(2;20,0.05)$
$= 1 - 0.3585 - 0.3774 - 0.1887 = 0.0754$

Thus, there is only a 0.0754 chance of obtaining 3 or more failures if the manufacturer's claim is correct.

5.52 Using MINITAB software we first enter 13 and 18 in C1 and then give the commands:

```
MTB> CDF C1;
SUBC> BINOMIAL 100 .16667.
```

obtaining

```
K  P(X LESS THAN OR = K)
13   .2000
10    .6964
```

(a) $P(x \le 18) = 0.6984$; $P(x \le 13) = 0.2000$
thus, $P(14 \le x \le 18) = 0.6954 - 0.2000 = 0.4964$

(b) No. The probability of obtaining more than 18 "sevens" is $1 - 0.6964 = 0.3036$

5.53 Using MINITAB with the number 12 entered into C1 and the commands:

```
MTB> CDF C1;
SUBC> BINOMIAL 80 .10.
```

we get

```
K  P(X LESS THAN OR = K)
12   .9462
```

(a) $P(x \le 12) = 0.9462$; thus $P(x > 12) = 1 - 0.9462 = 0.0538$

(b) With a probability of only 0.0538 the assumption is borderline questionable.

5.54 $k = 6$

(a) $\mu = 450$; $\sigma = 15$ $\quad \frac{450 \pm 90}{900}$ or 0.40 to 0.60

(b) $\mu = 5{,}000$; $\sigma = 50$ $\quad \frac{5{,}000 \pm 300}{10{,}000}$ or 0.47 to 0.53

(c) $\mu = 500{,}000$; $\sigma = 500$ $\quad \frac{500{,}000 \pm 3{,}000}{100{,}000}$ or 0.497 to 0.503

5.57 **(a)** $\theta = 0.5,\ x = 4,\ k = 1$

$$b^* = \binom{3}{0}(0.5)^1(0.25)^3 = 1\cdot(0.5)(0.125) = 0.0625$$

(b) $\theta = 0.5,\ x = 7,\ k = 2$

$$b^* = \binom{6}{1}(0.5)^1(0.5)^5 = 6(0.25)(0.003125) = 0.0469$$

(c) $\theta = 0.5,\ x = 10,\ k = 4$ and 5

$$b^* = \binom{9}{3}(0.5)^4(0.5)^6 = \binom{9}{4}(0.5)^5(0.5)^5$$

$$= (84+126)(0.5)^{10} = 210(0.0009765) = 0.2051$$

5.58 **(a)** $\theta = 0.75,\ x = 8,\ k = 5$

$$b^* = \binom{7}{4}(0.75)^5(0.25)^3 = 35(0.2373)(0.015625) = 0.1298$$

(b) $\theta = 0.75,\ x = 15,\ k = 10$

$$b^* = \binom{14}{9}(0.75)^{10}(0.25)^5 = 2002(0.05631)(0.0009765) = 0.1101$$

5.59 $b^* = \binom{6-1}{1-1}(0.3)^1(0.7)^5 = 1-(0.3)(0.16807) = 0.0504$

5.60 $\theta = 0.05,\ x = 15,\ k = 2$

(a) $b^* = \binom{14}{1}(0.5)^2(0.95)^{13} = 14(0.0025)(0.51334) = 0.0180$

(b) $b^* = \frac{2}{15}\cdot(2;\ 15,\ 0.05) = \frac{2}{15}(0.1348) = 0.0180$

5.61 $g(x;\ 1,\ \theta) = \frac{1}{x}b(x;\ 1,\ \theta)$

(a) $x = 4,\ \theta = 0.75$

$$g = \frac{1}{4}b(1;\ 4,\ 0.75)$$

$$= \frac{1}{4}\binom{4}{1}(0.75)^1(0.25)^3 = 0.0117$$

(b) $x = 6,\ \theta = 0.30$

$$g = \frac{1}{6}b(1;\ 6,\ 0.30)$$

$$= \frac{1}{6}\binom{6}{1}(0.3)(0.70)^5 = 0.0504$$

5.62 $g = (0.999)^{800}$

$$\begin{aligned}\log g &= 800(\log 0.999)\\ &= 800(0.99957-1)\\ &= 799.656-800 = 0.656-1\\ g &= 0.4529 \quad \text{(depends on rounding)}\end{aligned}$$

5.63 **(a)** $$\frac{\binom{14}{2}\binom{4}{0}}{\binom{18}{2}} = \frac{91}{153} = 0.5948$$

(b) $$\frac{\binom{10}{2}\binom{8}{0}}{\binom{18}{2}} = \frac{45}{153} = 0.2941$$

(c) $$\frac{\binom{6}{2}\binom{12}{0}}{\binom{18}{2}} = \frac{15}{153} = 0.980$$

5.64 **(a)** $$\frac{\binom{10}{0}\binom{6}{3}}{\binom{16}{3}} = \frac{1\cdot 20}{560} = \frac{2}{56} = \frac{1}{28}$$

(b) $$\frac{\binom{10}{1}\binom{6}{2}}{\binom{16}{3}} = \frac{10\cdot 15}{560} = \frac{15}{56}$$

(c) $$\frac{\binom{10}{2}\binom{6}{1}}{\binom{16}{3}} = \frac{45\cdot 6}{560} = \frac{27}{56}$$

(d) $$\frac{\binom{10}{3}\binom{6}{0}}{\binom{16}{3}} = \frac{120}{560} = \frac{3}{14}$$

5.65 **(a)** $\mu = 0\cdot\frac{2}{56} + 1\cdot\frac{15}{56} + 2\cdot\frac{27}{56} + 3\cdot\frac{12}{56} = \frac{105}{56} = \frac{15}{8}$

$\mu_2' = 0^2\cdot\frac{2}{56} + 1^2\cdot\frac{15}{56} + 2^2\cdot\frac{27}{56} + 3^2\cdot\frac{12}{56} = \frac{231}{56}$

$\sigma^2 = \frac{231}{50} - \left(\frac{15}{8}\right)^2 = \frac{1848-1575}{448} = \frac{273}{448} = \frac{39}{64}$

(b) $\mu = \frac{3\cdot 10}{16} = \frac{15}{8}$

$\sigma^2 = \frac{3\cdot 10\cdot 6\cdot 13}{16\cdot 16\cdot 15} = \frac{39}{64}$

5.66 $\frac{\binom{9}{2}\binom{6}{3}}{\binom{15}{5}} = \frac{36\cdot 20}{3003} = 0.2398$

5.67 **(a)** $12 > 0.05(200) = 10$; condition *not* satisfied

(b) $20 < 0.05(500) = 25$; condition satisfied

(c) $30 < 0.05(640) = 32$; condition satisfied

5.68 **(a)** $\frac{\binom{4}{1}\binom{76}{2}}{\binom{80}{3}} = \frac{4\cdot 76\cdot 75}{2} = \frac{6}{80\cdot 79\cdot 78} = \frac{285}{2054} = 0.1388$

(b) $\binom{3}{1}(0.05)(0.95)^2 = 0.1354$

5.69 $n = 300,\ M = 240,\ n = 6,\ x = 4$

(a) $\frac{\binom{240}{4}\binom{60}{2}}{\binom{300}{6}} = \frac{240\cdot 239\cdot 238\cdot 237\cdot 60\cdot 59\cdot 720}{24\cdot 2\cdot 300\cdot 299\cdot 298\cdot 297\cdot 296\cdot 295} = 0.2478$

(b) $\binom{6}{4}(0.80)^4(0.2)^2 = 15(0.4096)(0.04) = 0.2458$

5.70 $\dfrac{\binom{30}{1}\binom{270}{11}}{\binom{300}{12}} \div \dfrac{\binom{30}{0}\binom{270}{12}}{\binom{300}{12}} = \dfrac{360}{259} = 1.39$, and hence, less than 3 to 2

5.71 Good $n \geq 20$ and $\theta \leq 0.05$ excellent $n \geq 100$ and $n\theta < 10$

(a) $125 \geq 20$ and $0.10 > 0.05$, also $n\theta = 12.5 > 10$; neither rule is satisfied

(b) $25 > 20,\ 0.04 \leq 0.05$; good approximation

(c) $120 > 100,\ n\theta = 6 < 10$; excellent approximation

(d) $0.06 > 0.05,\ 40 < 100$; neither rule is satisfied

5.72 $\lambda = 150(0.014) = 2.1$ from Table II

$p(2;\ 2.1) = 0.2700$

5.73

5 $\dfrac{0.1904 - 0.1088}{0.1088} \cdot 100 = 0.55\%$

6 $\dfrac{0.1367 - 0.1384}{0.1384} \cdot 100 = -1.23\%$

7 $\dfrac{0.1465 - 0.1499}{0.1499} \cdot 100 = -2.27\%$

8 $\dfrac{0.1373 - 0.1410}{0.1410} \cdot 100 = -2.62\%$

9 $\dfrac{0.1144 - 0.1171}{0.1171} \cdot 100 = -2.31\%$

10 $\dfrac{0.0858 - 0.0869}{0.0869} \cdot 100 = -1.27\%$

11 $\dfrac{0.0585 - 0.0582}{0.0582} \cdot 100 = 0.52\%$

12 $\dfrac{0.0366 - 0.0355}{0.0355} \cdot 100 = 3.10\%$

13 $\dfrac{0.0211 - 0.0198}{0.0198} \cdot 100 = 6.57\%$

14 $\dfrac{0.0113 - 0.0102}{0.0102} \cdot 100 = 10.78\%$

15 $\dfrac{0.0057 - 0.0049}{0.0049} \cdot 100 = 16.33\%$

$x = 15$

5.74 $\lambda = 150(0.04) = 6$ from Table II

(a) 0.1606

(b) $0.0025 + 0.0149 + 0.0446 + 0.892 = 0.1512$

5.75 $\lambda = 1000(0.0012) = 1.2$ from Table II

$p(0) + p(1) + p(2) = 0.3012 + 0.3614 + 0.2169 = 0.8795$

5.76 **(a)** $0.1373 + 0.1144 + 0.0858 + 0.0585 + 0.0366 = 0.4326$

(b) $0.9573 - 0.5246 = 0.4327$

5.77 $f(2;\ 3.3) = \dfrac{3.3^2 e^{-3.3}}{2!} = (5.445)(0.037) = 0.201$

5.78 **(a)** $f(0;\ 1.8) = \dfrac{(1.8)^0 e^{-1.8}}{0!} = 0.165$

(b) $f(1;\ 1.8) = \dfrac{1.8\, e^{-1.8}}{1} = 0.297$

5.79 **(a)** 0.1653; **(b)** 0.2975

5.80 **(a)** $\lambda = 0.5 \qquad 0.6065 + 0.3033 = 0.9098$

(b) $\dfrac{(0.5)^0 e^{-0.5}}{0!} + \dfrac{(0.5)e^{-0.5}}{1!} = 1.5(0.607) = 0.9105$

5.81 **(a)** $f(3;\ 5.2) = 0.1293$

(b) $0.0220 + 0.0104 + 0.0045 + 0.0018 + 0.0007 + 0.0002 + 0.0001 = 0.0397$

(c) $0.1681 + 0.1748 + 0.1515 = 0.4944$

5.82 **(a)**

$$h(0;\ 100,\ 100,\ 6) = \frac{\binom{6}{0}\binom{994}{100}}{\binom{1000}{100}}$$

Calculation of such large binomial coefficients is not possible with MINITAB. However, other statistical (e.g., MICROSTAT) yield 3.3876×10^{139} for the large coefficient in the numerator and 6.3850×10^{139} for denominator. Thus, the required probability is given by

$$1 - h(0;\ 100,\ 1000,\ 6) = 1 - \frac{1 \cdot 3.3876}{6.3850} = 0.4695$$

(b) Using MINITAB software we enter 1 in C1 and give commands:

MTB> CDF C1;
SUBC? Binomial 100 .006.

obtaining K P(X LESS THAN OR = K)
1.5478

Thus, the approximate probability is $1 - 0.5478 = 0.4522$

(c) Using the Poisson distribution having the mean $100 \times 0.006 = 0.6$, we obtain the probability $1 - 0.5478 = 0.4522$ from Table II.

5.83 $\dfrac{10!}{3!\ 6!\ 1!}(0.40)^3(0.50)^6(0.10) = 840(0.064)(0.015625)(0.10) = 0.0840$

5.84 $\dfrac{12!}{5!\ 4!\ 2!\ 1!}(0.6)^5(0.2)^4(0.1)^2(0.1) = 83160(0.07776)(0.0016)(0.001) = 0.0103$

5.85 $\dfrac{9!}{4!\ 3!\ 2!\ 0!}\left(\dfrac{9}{16}\right)^4\left(\dfrac{3}{16}\right)^3\left(\dfrac{3}{16}\right)^2 = 1260(0.1001128)(0.0002317) = 0.0292$

5.86 **(a)** $\dfrac{\binom{15}{4}\binom{7}{1}\binom{3}{0}}{\binom{25}{5}} = \dfrac{1365 \cdot 7 \cdot 24 \cdot 5}{25 \cdot 24 \cdot 23 \cdot 22 \cdot 21} = 0.1798$

(b) $\dfrac{\binom{15}{3}\binom{7}{1}\binom{3}{1}}{\binom{25}{5}} = \dfrac{455 \cdot 7 \cdot 3 \cdot 120}{25 \cdot 24 \cdot 23 \cdot 22 \cdot 21} = 0.1798$

5.87 $\dfrac{\binom{10}{3}\binom{5}{1}\binom{3}{2}}{\binom{18}{6}} = \dfrac{120 \cdot 5 \cdot 3}{18564} = 0.0970$

5.88 $P(\text{rejection} \mid \% \text{ defective} = 0.01) = 0.10,$ thus the producer's risk is 0.10.
$P(\text{rejection} \mid \% \text{ defective} = 0.03) = 0.95,$ thus the consumer's risk is $1 - 0.95 = 0.05$.

5.89 **(a)** Since producer's risk = 0.05 with an AQL of 0.03, the probability is $1 - 0.95 = 0.05$.
(b) Since the consumer's risk is 0.10 with an LTPD of 0.07, the probability is 0.10.

5.90 If $c = 2$, we get the following from Table I.

p	0	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
$L(p)$	1	0.9245	0.6769	0.4049	0.2061	0.0913	0.0355	0.0121	0.0036	0.0009	0.0002

Sketching the OC curve and finding values of p for $L(p) = 1 - 0.05 = 0.95$ and 0.10, we obtain: AQL = 0.03 and LTPD = 0.26.

5.91 **(a)** Producer's risk = 1 value of $L(p)$ when $p = 0.10$, or 0.17.
(b) LTPD = value of p for which $L(p) = 0.05$

5.92 If $n = 10$ and $c = 1$, we get the following from Table I.

p	0	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
$L(p)$	1	0.9139	0.7361	0.5443	0.3758	0.2440	0.1493	0.0860	0.0464	0.0009	0.0002

5.93 If $n = 15$ and $c = 2$, we get the following from Table I.

p	0	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
$L(p)$	1	0.9638	0.8159	0.6042	0.3980	0.2361	0.1268	0.0617	0.0271	0.0107	0.0037

5.94 If $n = 8$ and $c = 0$, we get the following from Table I.

p	0	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45
$L(p)$	1	0.6634	0.4305	0.2725	0.1678	0.1001	0.0576	0.0319	0.0168	0.0084

5.95 The AQL is the value of p for which $L(p) = 1 - 0.10 = 0.90$, or 0.07.
The LTPD is the value of p for which $L(p) = 0.10$ or 0.33.

5.96 The producer's risk is $1-$ value of $L(p)$ for which p = 0.10, or $1 - 0.74 = 0.26$.
The consumer's risk is the value of $L(p)$ = 0.25, or 0.24.

5.97 **(a)** If n = 10 and c = 0, we get the following from Table I.

p	0	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45
$L(p)$	1	0.5987	0.3487	0.1969	0.1074	0.0563	0.0282	0.0135	0.0060	0.0010

(b) For plan 1 (n = 10, c = 1, see Exc. 5.93), the producer's risk = $1 - 0.9139 = 0.0861$ and the consumer's risk = 0.1493.

(c) For plan 2 (n = 10, c = 0, see preceding table), the producer's risk = $1 - 0.5987 = 0.4013$ and the consumer's risk = 0.0282.