IBA

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Quiz 4

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**Question 1** 1.6

A continuous Random variable has p.d.f:

$$f(x) = \begin{cases} \frac{3}{4}(2x - x^2) & for\ 0 \le x \le 2 \\ 0 & otherswise \end{cases}$$

Find first three moments about origin.

**Question 2** 1.7

A variable has p.d.f :

$$f(x) = \begin{cases} 0 & for\ x < 0 \\ \frac{3}{8}(x-2)^2 & for\ 0 \le x \le 2 \\ 0 & for\ x > 2 \end{cases}$$

Find the expected value of X and its Standard deviation.

**Question 3** 1.7

Suppose the joint p.d.f of (X, Y) is given by

$$f(x,y) = \begin{cases} \{3x^2y + 3xy^2) & for\ 0 \le x \le 1 and\ 0 \le y \le 1 \\ 0 & otherwise \end{cases}$$

Compute (ii) P $(\frac{1}{2} \le X \le \frac{3}{4} | \frac{1}{2} \le Y \le \frac{2}{3})$.

Solution: Q1 - we know that $\mu_r'$ represents the $r^{th}$ moment of X about origin. By def$^n$ $\mu_r' = E(X^r) = \sum_x x^r f(x)$ for X discrete

& $\mu_r' = E(X^r) = \int x^r f(x)dx$ for X continuous.

so in our case, $\mu_r' = \int_0^2 \frac{3}{4} x^r (2x - x^2)dx = \frac{3}{4}\int_0^2 (2x^{r+1} - x^{r+2})dx$

$= \frac{3}{4}\left[\frac{2}{r+2}x^{r+2} - \frac{1}{r+3}x^{r+3}\right]_0^2 = \frac{3}{4}\left[\frac{2^{r+3}}{r+2} - \frac{2^{r+3}}{r+3}\right]$

$= \frac{3}{4}\frac{2^{r+3}}{(r+2)(r+3)}$

So $\mu_1' = \frac{3}{4}\frac{16}{3\times4} = \boxed{1}$, $\mu_2' = \frac{3}{4}\frac{32}{4\times5} = \boxed{\frac{6}{5}}$, $\mu_3' = \frac{3}{4}\frac{64}{5\times6} = \boxed{\frac{8}{5}}$

$= \boxed{1.2}$ $= \boxed{1.6}$

Q2 — $E(X) = \frac{3}{8}\int_0^2 x(x-2)^2\,dx = \frac{3}{8}\int_0^2 x(x^2-4x+4)\,dx = \frac{3}{8}\int_0^2 (x^3-4x^2+4x)\,dx$

$= \frac{3}{8}\left(\frac{1}{4}x^4 - \frac{4}{3}x^3 + 2x^2\right)\Big|_0^2 = \frac{3}{8}\left(4 - \frac{32}{3} + 8\right) = \frac{12-32+24}{8}$

$= 4/8 = 1/2 = 0.5$

$E(X^2) = \frac{3}{8}\int_0^2 x^2(x-2)^2\,dx = \frac{3}{8}\int_0^2 (x^4-4x^3+4x^2)\,dx = \frac{3}{8}\left[\frac{x^5}{5} - x^4 + \frac{4}{3}x^3\right]_0^2$

$= \frac{3}{8}\left[\frac{32}{5} - 16 + \frac{32}{3}\right] = \frac{12}{5} - 6 + 4 = \frac{12}{5} - 2 = \frac{2}{5} = 0.4$

So $\sigma = \sqrt{E(X^2) - (E(x))^2} = \sqrt{0.4 - 0.25} = \sqrt{0.15} = 0.3873$

Q3 — $P(1/2 \le X \le 3/4 \mid 1/2 \le Y \le 2/3) = \dfrac{P(1/2 \le X \le 3/4,\ 1/2 \le Y \le 2/3)}{P(1/2 \le Y \le 2/3)}$

$= \int_{1/2}^{3/4}\int_{1/2}^{2/3} (3x^2y + 3xy^2)\,dy\,dx \Big/ \int_0^1\int_{1/2}^{2/3} (3x^2y + 3xy^2)\,dy\,dx$

Now $\int_{1/2}^{2/3} (3x^2y + 3xy^2)\,dy = \frac{3}{2}x^2y^2 + xy^3\Big|_{y=1/2}^{y=2/3}$

$= \left(\cancel{\frac{3}{2}} \times \cancel{\frac{2}{3}} \times \frac{2}{3} - \frac{3}{2} \times \frac{1}{2} \times \frac{1}{2}\right)x^2 + \left(\frac{8}{27} - \frac{1}{8}\right)x = \left(\frac{2}{3} - \frac{3}{8}\right)x^2 + \left(\frac{8}{27} - \frac{1}{8}\right)x$

$= \frac{7}{24}x^2 + \frac{37}{216}x$ Now $\int\left(\frac{7}{24}x^2 + \frac{37}{216}x\right)dx = \frac{7x^3}{72} + \frac{37x^2}{432}$

So $P(1/2 \le X \le 3/4 \mid 1/2 \le Y \le 2/3) = \left.\frac{7}{72}x^3 + \frac{37x^2}{432}\right|_{1/2}^{3/4} \Big/ \left.\frac{7}{72}x^3 + \frac{37x^2}{432}\right|_0^1$

$= \left\{\frac{7}{\cancel{72}_{8}}\left(\frac{\cancel{27}^{3}}{64}\right) + \left(\frac{37}{\cancel{432}_{48}}\right)\frac{\cancel{9}}{16} - \frac{7}{72}\times\frac{1}{8} - \frac{37}{432}\times\frac{1}{4}\right\} \Big/ \left(\frac{7}{72} + \frac{37}{432}\right)$

$= 0.0556279 / 0.182870\text{4}$

$= 0.304193$ Ans.