Chapter 2

2.1 **(a)** $P[A] = P[(A \cap B) \cup (A \cup B')] = P(A \cap B) + P(A \cap B') \geq P(A \cap B)$

(b) $A \cup B = (A \cap B) \cup (A \cap B) \cup (A' \cap B) = A \cup (A' \cap B)$

2.6 $P(A) - P(A \cap B) = (a+b) - a = b = P(A \cap B')$
$P(A \cup B) = P(A) + P(A' \cap B) \geq P(A)$

2.7 $1 - P(A) - P(B) + P(A \cap B) = (a+b+c+d) - (a+b) - (a+c) + a = d$
$= P(A' \cap B')$

2.8 $P[(A \cap B') \cup (A' \cap B)] = b + c = (a+b) + a + c) - 2a$
$= P(A) + P(B) - 2P(A \cap B)$ Refer to Figure 2.6

2.9 **(a)** $P(A) + P(B) - P(A \cap B) \geq 0 \rightarrow P(A \cap B) \leq P(A) + P(B)$

(b) $P(A) + P(B) - P(A \cap B) \leq 1 \quad P(A \cap B) \geq P(A) + P(B) - 1$

2.10 Refer to Figure 2.7 $P(A) = 1 \rightarrow e = c = f = 0$
$P(B) = 1 \rightarrow d = f = g = 0$
$P(C) = 1 \rightarrow b = e = g = 0$
Therefore $P(A) = a + b + d + g = a = 1$ QED

2.11 $P(A \cup B) = P(A) + P[A' \cap B)$
$= P(A) + P(A' \cap B) + P(A \cap B) - P(A \cap B)$
$= P(A) + P(B) - P(A \cap B)$ QED

2.12 $P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$
$= (a+b+d+g+(a+b+c+e)+(a+c+d+f)-(a+b)$
$-(a+d)-(a+c)+a = a+b+c+d+e+f$
$= P(A \cup B \cup C \cup D)$

2.13 $P(A)+P(B)+P(C)+P(D)-P(A\cap B)-P(A\cap C)-P(A\cap D)-P(B\cap C)$

$$-P(B\cap D)-P(C\cap D)+P(A\cap B\cap C)+P(A\cap B\cap D)$$
$$+P(A\cap C\cap D)+P(B\cap C\cap D)-P(A\cap B\cap C\cap D)$$
$$=(a+b+d+g+i+j+l+o)+(a+b+c+e+i+j+k+m)$$
$$+(a+c+d+f+i+k+l+n)+(a+b+c+d+e+f+g+h)$$
$$-(a+b+i+j)-(a+d+i+l)-(a+b+d+g)$$
$$-(a+c+i+k)-(a+b+c+e)-(a+c+d+f)$$
$$+(a+i)+(a+b)+(a+d)+(a+c)-a$$
$$=a+b+c+d+e+f+g+h+i+j+k+l+m+n+o$$
$$=P(A\cup B\cup C\cup D)$$

2.14 For $n=2,\ P(E_1\cup E_2)=P(E_1)+P(E_2)-P(E_1\cap E_2)\le P(E_1)+P(E_2)$

Assume that for some n: $P(E_1\cup E_2\cup\ldots\cup E_n)=\sum_{j=1}^{n}P(E_j)$, then

$$P\big((E_1\cup E_2\cup\ldots\cup E_n)\cup E_{n+1}\big)=P\big[(E_1\cup E_2\cup\ldots\cup E_n)\cup E_{n+1}\big]$$
$$\le P(E_1\cup E_2\cup\ldots\cup E_n)+P(E_{n+1})\le\sum_{j=1}^{n+1}P(E_j)$$

where the first inequality follows from the first step of the induction, and the second inequality comes from the second step of the induction.

2.15 $\dfrac{p}{1-p}=\dfrac{A}{B}$, $pb=A-Ap$, $PA+pB=A$, $p(A+B)=A$, $p=\dfrac{A}{A+B}$

2.16 **(a)** P:ostulate 1 $P(A)=\dfrac{a}{a+b}\ge 0$

(B) Postulate 2 $P(A)=\dfrac{a}{a+b}$, $P(A')=\dfrac{b}{a+b}$

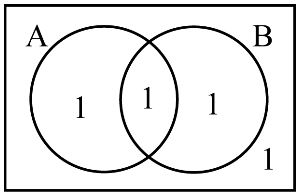
$$P(A)+P(A')=\frac{a}{a+b}+\frac{b}{a+b}=1=P(S)$$

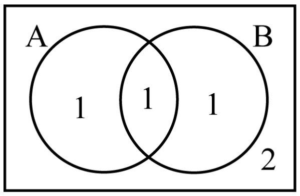
2.17 **(a)** $P(A\,|\,B)=\dfrac{P(A\cap B)}{P(B)}\ge 0$; **(b)** $P(B\,|\,B)=\dfrac{P(B\cap B)}{P(B)}=\dfrac{P(B)}{P(B)}=1$

(c) $P(A_1\cup A_2\cup\ldots|B)=\dfrac{P[A_1\cup A_2\cup\ldots)\cap B]}{P(B)}$

$$=\frac{P(A_1\cap B)}{P(B)}+\frac{P(A_2\cap B)}{P(B)}+\cdots$$
$$=P(A_1|B)+P(A_2|B)+\cdots$$

2.18





For example

(a) If $P(A \cap B) = P(A \cap B') = P(A' \cap B)$

$= P(A' \cap B') = \frac{1}{4}$ so that

$P(B|A) = \frac{1}{2}, \quad P(B|A') = \frac{1}{2}$, and

$P(B|A) + P(B|A') = 1$

(b) If $P(A \cap B) = P(A \cap B') = P(A' \cap B) = \frac{1}{5}$

and $P(A' \cap B') = \frac{2}{5}$

$P(B|A) = \frac{1}{2}, \quad P(B|A') = \frac{1}{3}$, and

$P(B|A) + P(B|A') = \frac{5}{6}$

2.19
$$\begin{aligned} P(A \cap B \cap C \cap D) &= P(A \cap B \cap C)P(D|A \cap B \cap C) \\ &= P(A \cap B)P(C|A \cap B)P(D|A \cap B \cap C) \\ &= P(A)P(B|A)P(C|A \cap B)P(D|A \cap B \cap C) \end{aligned}$$

2.20 $P(C|A \cap B) = P(C|B) \to \frac{P(A \cap B \cap C)}{P(A \cap B)} = \frac{P(B \cap C)}{P(B)} \to \frac{P(A \cap B \cap C)}{P(B \cap C)} = \frac{P(A \cap B)}{P(B)}$

$\to P(A|B \cap C) = P(A|B)$

2.21 $P(B|A) = \frac{P(A \cap B)}{P(A)} = P(B) \to \frac{P(A \cap B)}{P(B)} = P(A) \to P(A|B) = P(A)$

2.22 **(a)** $P(B) = P(A \cap B) + P(A' \cap B) = P(A)P(B) + P(A' \cap B)$

$P(A' \cap B) = P(B) - P(A)P(B) = P(B)[(1 - P(A)] = P(B)P(A')$ QED

(b) $P(B') = P(A \cap B') + P(A' \cap B') = P(A' \cap B') + P(B')P(A)$

$P(A' \cap B') = P(B') - P(B')P(A\,|B') = P(B')[(1 - P(A)] = P(B')P(A')$ QED

2.23 Assume that A and B' are independent and show that this leads to contradiction.

$P(A) = P(A \cap B) + P(A \cap B') = P(A \cap B) + P(A)P(B')$

$P(A \cap B) = P(A) - P(A)P(B') = P(A)[1 - P(B')] = P(A)P(B)$ and A and B are *independent*

2.24 $P(A) = 0.60, \quad P(B) = 0.80, \quad P(C) = 0.50, \quad P(A \cap B) = 0.48, \quad P(A \cap C) = 0.30$

$P(B \cap C) = 0.38, \quad P(A \cap B \cap C) = 0.24$

$P(A \cap B \cap C) = 0.24, \quad P(A)(B)(C) = (0.6)(0.8)(0.5) = 0.24$

$P(B \cap C) = 0.38, \quad P(B)P(C) = (0.8)(0.5) = 0.40$ B and C *not independent*

2.25 Refer to 2.21

$P(A \cap B) = 0.48,\ \ P(A)P(B) = (0.6)(0.8) = 0.48$ A and B *independent*

$P(A \cap C) = 0.30,\ \ P(A)P(C) = (0.6)(0.5) = 0.30$ A and C *independent*

$P(B \cap C) = 0.38,\ \ P(B)P(C) = (0.8)(0.5) = 0.40$ B and C *not independent*

2.26 (Refer to 2.24 and 2.25) Already showed that A and B *independent*,.
A and C *independent*
$P[(A \cap (B \cap C)] = 0.54,\ \ P(A) = 0.60,\ \ P(B \cup C) = 0.92,\ \ (0.6)(0.92) = 0.552 \neq 0.54$

2.27 **(a)** $P[(A \cap (B \cap C)] = P(A \cap B \cap C) = P(A)P(B)P(C) = P(A)P(B \cap C)$ QED

(b)
$$\begin{aligned} P[(A \cap (B \cup C)] &= P[(A \cap B) \cup (A \cap C)] \\ &= P(A \cap B) + P(A \cap C) - P(A \cap B \cap C) \\ &= P(A)P(B) + P(A)P(C) - P(A)P(B)P(C) \\ &= P(A)[P(B) + P(C) - P(B \cap C)] \\ &= P(A)P(B \cup C) \quad \text{QED} \end{aligned}$$

2.28 $P(A|B) \to \dfrac{P(A \cap B)}{P(A)} < P(B) \to P(B|A) < P(B)$

2.29 Proof by induction: If $n = 2$, then $P(A_1 \cup A_2) = P(A_1) + P(A_2) - P(A_1) \cdot P(A_2)$
and $1 - [1 - P(A_1)] \cdot [1 - P(A_2)] = 1 - 1 + P(A_1) + P(A_2) - P(A_1)P(A_2)$.
Assuming $P(A_1 \cup A_2 \cup \ldots \cup A_n) = 1 - [1 - P(A_1)] \cdot [1 - P(A_2)] \cdot \ldots \cdot [1 - P(A_n)]$.
we can write
$$\begin{aligned} P(A_1 \cup A_2 \cup \ldots \cup A_n \cup A_{n+1}) &= P(A_1 \cup A_2 \cup \ldots \cup A_n) + P(A_{n+1}) \\ &\quad - P(A_1 \cup A_2 \cup \ldots A_n) \cdot P(A_{n+1}) \\ &= P(A_1 \cup A_2 \cup \ldots \cup A_n) \cdot [1 - P(A_{n+1})] + P(A_{n+1}) \end{aligned}$$
$$= \{1 - [P(A_1)] \cdot [1 - P(A_2)] \cdot \ldots \cdot [1 - P(A_n)]\} \cdot [1 - P(A_{n+1})] + P(A_{n+1})$$
$$= 1 - [1 - P(A_1)] \cdot [1 - P(A_2)] \cdot \ldots \cdot [1 - P(A_n)] \cdot [1 - P(A_{n+1})]$$

2.30 Two at time $\binom{k}{2}$

Three at time $\binom{k}{3}$

k at time $\binom{k}{k}$

$$\binom{k}{2} + \binom{k}{3} + \ldots + \binom{k}{k} = 2^k - \binom{k}{0} - \binom{k}{1} = 2^k - 1 - k$$

2.31 $P(A \cap \varnothing) = P(A) \cdot P(\varnothing|A) = P(A) \cdot P(\varnothing)$, since $P(\varnothing|A) = P(\varnothing) = 0$.

2.32 Since $B_1 \cup B_2 \cup \ldots \cup B_k = S$, $A \cap (B_1 \cup B_2 \cup \ldots \cup B_k) = A$. Thus, by the distributive property, $(A \cap B_1) \cup (A \cap B_2) \cup \ldots \cup (A \cap B_k) = A$, and
$P(A) = P(B_1)P(A|B_1) + P(B_2)P(A|B_2) + \ldots + P(B_k)P(A|B_k)$ QED

2.33 The probability of no matches on any given trial is $\frac{n-1}{n}$; since the trials are independent, the probability of no match in n trials is $\left(\frac{n-1}{n}\right)^n = \left(1-\frac{1}{n}\right)^n$.

2.34 $P(A \cup B) = P(A) + P(B) - P(A \cap B) = [1 - P(A')] + [1 - P(B')] - P(A \cap B)$
$= 1 - P(A') - P(B') + [1 - P(A \cap B)]$.
Since $1 - P(A \cap B) \geq 0$, $P(A \cup B) \leq 1 - P(A') - P(B')$ QED

2.35 **(a)** $\{6, 8, 9\}$; **(b)** **{8};** **(c)** $\{1, 2, 3, 4, 5, 8\}$; **(d)** $\{1, 5\}$;
(e) $(2, 4, 8\}$; **(f)** $\varnothing$

2.36 **(a)** Los Angeles, Long Beach, Pasadena, Anaheim, Santa Maria, Westwood;
(b) San Diego, Long Beach, Pasadena, Anaheim, Santa Maria, Westwood;
(c) Santa Barbara; **(d)** $\varnothing$; **(e)** San Diego, Long Beach, Santa Barbara, Anaheim;
(f) San Diego, Santa Barbara, Long Beach; **(g)** Los Angeles, Santa Barbara, Anaheim;
(h) Los Angeles, Pasadena, Santa Maria, Westwood; **(i)** Los Angeles, Pasadena, Santa Maria, Westwood.

2.37 **(a)** $\{5, 6, 7, 8\}$; **(b)** $\{2, 4, 5, 7\}$; **(c)** $\{1, 8\}$ **(d)** $(3, 4, 7, 8\}$

2.38 **(a)** He chooses a car with air conditioning.
(b) He chooses a car with bucket seats or no power steering.
(c) He chooses a car with bucket seats that is 2 or 3 years old.
(d) He chooses a car with bucket seats that is 2 or 3 years old.

2.39 **(a)** House has fewer than three baths;
(b) does not have fire place;
(c) does not cost more than $200,000
(d) is not new;
(e) has three or more baths and fire place;
(f) has three more baths and costs more than $200,000
(g) costs more than $200,000 but has no fire place;
(h) is new or costs more than $200,000
(i) is new or costs $200,000 or less
(j) has 3 or more baths and/or fire place;
(k) has 3 or more baths and/or costs more than $200,000;
(l) is new and costs more than $200,000

2.41 **(a)** (H,1), (H,2), (H,3), (H,4), (H,5), (H,6)
(T,H,H), (T,H,T), (T,T,H), (T,T,T)
(b) (H,1), (H,2), (H,3), (H,4), (H,5), (H,6), (T,H,T), (T,T,H)
(c) (H,5), (H,6), (T,H,T), (T,T,H), (T,T,T)

2.42 **(a)** S = {(0,0,0)…(1,1,1)}
A = {(1,0,1),(0,1,1),(1,1,1)}
B = {(0,1,1)}
C = {(1,0,1)}
(b) A & B *not* mutually exclusive, A & C *not* mutually exclusive, B & C are mutually exclusive.

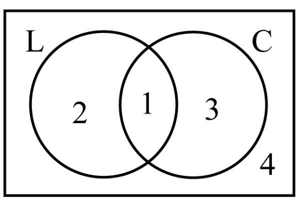
2.43 $3,\ x_1 3,\ x_1 x_2 3,\ x_1 x_2 x_3 3, \ldots$ where $x_1 = 1, 2, 4, 5, 6,$ for all i

(a) 5^{k-1}; **(b)** $1 + 5 + \ldots 5^k = \dfrac{5^k - 1}{4}$

2.44 $S = \{(x, y) \mid (x-2)^2 + (y+3)^2 \le 9\}$

2.45 **(a)** $(x \mid 3 < x < 10)$; **(b)** $(x \mid 5 < x \le 8)$; **(c)** $(x \mid 3 < x \le 5)$;
(d) $(x \mid 0 < x \le 3 \text{ or } 5 < x < 10)$

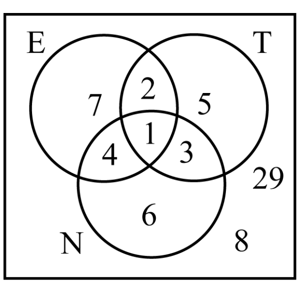
2.46



1 A driver has liability insurance and collision insurance.
2 A driver has liability insurance but not collision insurance.
3 A driver has collision insurance but not liability insurance.
4 A driver has neither liability insurance nor collision insurance.

2.47 **(a)** A driver has liability insurance.
(b) A driver does not have collision insurance.
(c) A driver has either liability or collision insurance, but not both.
(d) A driver does not have both kinds of insurance.

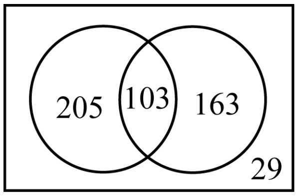
2.48



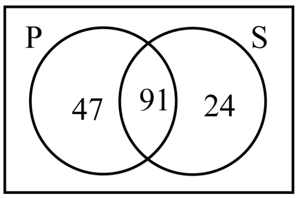
(a) A car brought to the garage needs engine overhaul, transmission repairs, and new tires.
(b) A car brought to the garage needs transmission repairs, new tires, but no engine overhaul.
(c) A car brought to the garage needs engine overhaul, but neither transmission repairs nor new tires.
(d) A car brought to the garage needs engine overhaul and new tires.
(e) A car brought to the garage needs transmission repairs, but no new tires.
(f) A car brought to the garage does not need engine overhaul.

2.49 **(a)** 5; **(b)** 1 and 2 together **(c)** 3, 5, and 6 together; **(d)** 1, 3, 4, and 6 together

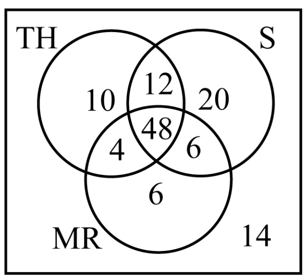
2.50 $500-(308+266)+103=29 \neq 59$ results are *in*consistent



2.51 $200-(1.38+115)+91=38$



2.52 **(a)** 12; **(b)** 6; **(c)** 20



2.53 **(a)** permissible;
(b) not permissible because the sum of the probabilities exceeds 1;
(c) permissible;
(d) not permissible because $P(E)$ is negative
(e) not permissible because the sum of the probabilities is less than 1.

2.54 **(a)** $1-0.37=0.63$; **(b)** $1-0.44=0.56$; **(c)** $0.37+0.44=0.81$;
(d) 0; **(e)** 0.37, $P(A \cap B')=P(A)$ for mutually exclusive events;
(f) $1-0.81=0.19$

2.55 **(a)** Probability cannot be negative.
(b) $0.77+0.08=0.85 \neq 0.95$
(c) $0.12+0.25+0.36+0.14+0.09+0.07=1.03>1$
(d) $0.08+0.21+0.29+0.40=0.98<1$

2.56 **(a)** $0.12+0.17=0.29$; **(b)** $0.17+0.34+0.29=0.80$
(c) $0.34+0.17+0.12=0.63$; **(d)** $0.34+0.29+0.08+0.71$

2.57 (0,0), (1,0), (2,0), (3,0), (4,0), (5,0), (0,1), (1,1), (2,1), (3,1), (4,1), (5,1), (0,2), (1,2), (2,2), (3,2), (4,2), (5,2), (0,3), (1,3), (2,3), (3,3), (4,3), (5,3), (0,4), (1,4), (2,4), (3,4), (4,4), (5,4)

(a) $\frac{10}{30}=\frac{1}{3}$; **(b)** $\frac{5}{30}=\frac{1}{6}$; **(c)** $\frac{15}{30}=\frac{1}{2}$; **(d)** $\frac{10}{30}=\frac{1}{3}$

2.58 **(a)** $\frac{20+10}{80}=\frac{3}{8}$; $\frac{4\cdot 5}{80}=\frac{1}{4}$; **(c)** $\frac{2\cdot 4}{80}=\frac{1}{10}$; **(d)** $\frac{4+2+1+1}{80}=\frac{1}{10}$;

(e) $\frac{8+14}{80}=\frac{22}{80}=\frac{11}{40}$

2.59 **(a)** $0.24+0.22=0.46$; **(b)** $0.15+0.03+0.22=0.40$
(c) $0.03+0.08=0.11$; **(d)** $0.15+0.03+0.28+0.22=0.68$

2.60 $\dfrac{\binom{16}{2}}{\binom{52}{2}}=\frac{120}{1326}=\frac{20}{221}$

2.61 Let $P(A)=4p$, $P(B)=2p, P(C)=2p$, and $P(D)=p$.

Then $9p=1$ and $p=\frac{1}{9}$;

(a) $\frac{2}{9}$; **(b)** $1-\frac{4}{9}=\frac{5}{9}$

2.62 **(a)** $\dfrac{\binom{13}{2}\binom{4}{2}\binom{4}{2}44}{\binom{52}{5}}=\frac{13\cdot 12\cdot 6\cdot 6\cdot 44\cdot 120}{2\cdot 52\cdot 51\cdot 50\cdot 49\cdot 48}=\frac{198}{4165}=0.0475$

(b) $\dfrac{13\cdot 48}{\binom{52}{5}}=\frac{13\cdot 48\cdot 120}{51\cdot 51\cdot 50\cdot 49\cdot 48}=\frac{1}{4165}$

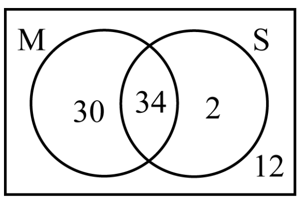
2.63 **(a)** $\dfrac{\binom{6}{2}\binom{5}{2}\binom{3}{2}\cdot 4}{6^5} = \dfrac{15\cdot 10\cdot 3\cdot 4}{6\cdot 6\cdot 6\cdot 6\cdot 6} = \dfrac{25}{108}$

(b) $\dfrac{6\binom{5}{3}\cdot 5\cdot 4}{6^5} = \dfrac{6\cdot 10\cdot 5\cdot 4}{6\cdot 6\cdot 6\cdot 6\cdot 6} = \dfrac{25\cdot 4}{648} = \dfrac{25}{162}$

(c) $\dfrac{6\cdot 5\binom{5}{3}\binom{2}{2}}{6^5} = \dfrac{6\cdot 5\cdot 10}{6\cdot 6\cdot 6\cdot 6\cdot 6} = \dfrac{25}{648}$

(d) $\dfrac{6\binom{5}{4}\cdot 5}{6^5} = \dfrac{6\cdot 5\cdot 5}{6\cdot 6\cdot 6\cdot 6\cdot 6} = \dfrac{25}{1296}$

2.65



$$\frac{78-[64+36-34]}{78} = \frac{12}{78} = \frac{2}{13}$$

2.64 **(a)** $P(A\cup B)$ is less than $P(A)$.
(b) $P(A\cap B)$ exceeds $P(A)$.
(c) $P(A\cup B) = 0.72+0.84-0.52 = 1.04$ exceeds 1

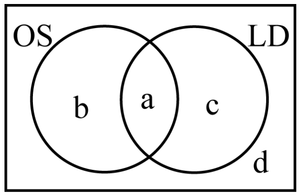
2.66 $\dfrac{2}{3}, 0$

2.67 The area of the triangle is $\dfrac{4\cdot 3}{2} = 6$; If the point is a distance x from the vertex on the longer leg, then is will be $\dfrac{3x}{4}$ units from the vertex on the other leg. The area of the required triangle is $x\cdot\dfrac{3x}{4\cdot 2} = \dfrac{3x^2}{8}$. For this to be greater than 3, or half the area of the triangle, $x^2 > 8$, or $x > 2\sqrt{2}$.
Thus, the probability of the line segment dividing the area in at least one-half is
$\dfrac{4-2\sqrt{2}}{4} = 1-\dfrac{\sqrt{2}}{2}$

2.68 $0.21+0.28-0.15 = 0.34$

2.69 **(a)** $0.59+0.30-0.21 = 0.68$; **(b)** $0.59-0.21 = 0.38$
(c) $1-0.21 = 0.79$; **(d)** $1-0.68 = 0.32$

2.70



$b+d=\frac{1}{3}$

$c+d=\frac{5}{9}$

$a+b+c=\frac{3}{4}$; hence $d=\frac{1}{4}$

$b=\frac{1}{3}-\frac{1}{4}=\frac{1}{12},\ c=\frac{5}{9}-\frac{1}{4}=\frac{11}{36},\ a=1-\frac{1}{12}-\frac{11}{36}-\frac{1}{4}=\frac{13}{36}$

$a = P$(out of state living on campus)
$b = P$(out of state not living on campus)
$c = P$(from Virginia living on campus)
$d = P$(from Virginia not living on campus)

2.71 **(a)** $(0.08)+0.05-0.02=0.11$; **(b)** $1-0.02=0.98$
(c) $0.08+0.05-2(0.02)=0.09$

2.72 $0.74+0.70+0.62-0.52-0.45-0.44+0.34=0.98$

2.73 $0.70+0.64+0.58+0.58-0.45-0.42-0.41-0.35-0.39-0.32$
$+0.23+0.26+0.21+0.20-0.12=0.94$

2.74 **(a)** The probability is $\frac{34}{34+21}=\frac{34}{55}$ that one of the eggs will be cracked.

(b) The probability is $\frac{11}{11+2}=\frac{11}{13}$ that they will not all be $1 bills.

(c) The probability is $\frac{5}{5+1}=\frac{5}{6}$ that we will not get a meaningful word and $1-\frac{5}{6}=\frac{1}{6}$ that we will get a meaningful word.

2.75 **(a)** The odds are $\frac{6}{10}$ to $\frac{4}{10}$ or 3 to 2;

(b) The odds are $\frac{11}{16}$ to $\frac{5}{16}$ or 11 to 5;

(c) The odds are $\frac{7}{9}$ to $\frac{2}{9}$ or 7 to 2 against it.

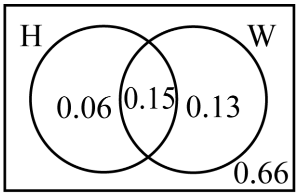
2.76 **(a)** $\frac{18+36}{90}=\frac{54}{90}=\frac{3}{5}$; **(b)** $\frac{36+27}{90}=\frac{63}{90}=\frac{7}{10}$; **(c)** $\frac{18}{90}=\frac{2}{10}=\frac{1}{5}$;

(d) $\frac{27}{90}=\frac{3}{10}$; **(e)** $\frac{18}{18+36}=\frac{18}{54}=\frac{1}{3}$; **(f)** $\frac{27}{27+36}=\frac{27}{63}=\frac{3}{7}$

2.77 **(a)** $\frac{1}{3} = \frac{1/5}{3/5}$ **(b)** $\frac{3}{7} = \frac{3/10}{7/10}$

2.78 $\frac{34}{34+2} = \frac{34}{36} = \frac{17}{18}$

2.79



$$\frac{0.15}{0.15+0.13} = \frac{0.15}{0.28} = \frac{15}{28}$$

2.80 $\frac{a}{a+b} = \frac{13/36}{13/36+1/12} = \frac{13/36}{13/36+3/36} = \frac{13}{16}$

2.81
$$P(R \cap W) = \frac{25 \cdot 40}{\binom{100}{2}}$$
$$= \frac{20}{99}$$

2.82 **(a)** $\frac{5}{12} + \frac{1}{12} = \frac{6}{12} = \frac{1}{2}$; consistent

(b) $\frac{1}{3} + \frac{1}{5} = \frac{8}{15} \neq \frac{7}{12}$; not consistent

2.83 **(a)**

Outcome	2	3	4	5	6	7	8	9	10	11	12
No.Combinations	1	2	3	4	5	6	5	4	3	2	1
Probability	1/36	1/18	1/12	1/9	5/36	1/6	5/36	1/9	1/12	1/18	1/36

(b) $(1+2+3+4+5+6+5+4+3+2+1)/36 = 1$

2.84 $\frac{1}{4} + \frac{3}{8} = \frac{5}{8}$; odds are 5 to 3 that either car will win.

2.85 Using MINITAB software, first we generate 1,000 uniformly distributed pseudo-random numbers, putting them in Column 1 (C1) as follows:

MTB> Random 1000 C1;
SUBC> Uniform 0.0 10.0.

Sorting these numbers facilitates counting the number that are less than 1. The sort is accomplished as follows:

MTB> Sort C1, C2;
SUBC> by C1.

When we did this, we obtained 111 numbers less than 1; thus, the required probability is estimated to be 111/1,000 = 0.111.

2.86 **(a)** Repeating the work of Exercise 2.59, we found the corresponding probability for the second set to be 99/1,000 = 0.099. Obtaining $P(A \cup B)$ is facilitated by using the LET command to add the two columns of random numbers and then sorting the resulting column. When we performed these operations, we noted that there were 22 cases in which the sum column contained a number less than 2. Thus, we estimated the required probability as 22/1,000 = 0.022.

(b) Using Theorem 2.7 with $P(A) = P(B) = 0.1$ we obtain $0.01 + 0.01 - 0.001 = 0.019$

2.87 $\dfrac{0.20}{0.20 + 0.30 + 0.10} = \dfrac{0.20}{0.60} = \dfrac{1}{3}$

2.88 **(a)** $\dfrac{0.52}{0.74} = \dfrac{25}{37}$; **(b)** $\dfrac{0.34}{0.52} = \dfrac{17}{26}$; **(c)** $\dfrac{0.18 + 0.16 - 0.10}{0.70 + 0.62 - 0.44} = \dfrac{0.24}{0.88} = \dfrac{3}{11}$

(d) $\dfrac{0.46 - 0.34}{0.30} = \dfrac{0.12}{0.30} = \dfrac{2}{5}$

2.89 $\dfrac{\binom{110}{3}}{\binom{120}{3}} = \dfrac{110 \cdot 109 \cdot 108}{120 \cdot 119 \cdot 118} = 0.7685$

2.90 $(0.55)(0.80) = 0.44$

2.91 **(a)** $(0.8)(0.2)(0.6) = 0.096$; **(b)** $(0.20)(0.40)(0.60) = 0.048$;
(c) $(0.8)(0.8)(0.2)(0.4) = 0.0512$; **(d)** $(0.8)(0.8) + (0.2)(0.6) = 0.76$

2.92 $\dfrac{15}{20} \cdot \dfrac{14}{19} \cdot \dfrac{13}{18} \cdot \dfrac{12}{17} = \dfrac{91}{323}$

2.93 **(a)** $\dfrac{3}{4} \cdot \dfrac{1}{4} \cdot \dfrac{1}{4} = \dfrac{3}{64}$; **(b)** $3 \cdot \left(\dfrac{3}{4}\right)^2 \cdot \dfrac{1}{4} = \dfrac{27}{64}$

2.94 A even first, B even second, C same number both

$P(A) = \dfrac{1}{2}$, $P(B) = \dfrac{1}{2}$, $P(C) = \dfrac{1}{6}$, $P(A \cap B) = \dfrac{1}{4}$, $P(A \cap C) = \dfrac{3}{36} = \dfrac{1}{12}$

11, 12, 13, 14, 15, 16, 21, 22, 23, 24, 25, 26, 31, 32, 33, 34, 35, 36,
41, 42, 43, 44, 45, 46, 51, 52, 53, 54, 55, 56, 61, 62, 63, 64, 65, 66

$P(B \cap C) = \dfrac{1}{12}$, $P(A \cap B \cap C) = \dfrac{3}{36} = \dfrac{1}{12} \neq \dfrac{1}{2} \cdot \dfrac{1}{2} \cdot \dfrac{1}{6} = \dfrac{1}{24}$

(a) Since $\dfrac{1}{2} \cdot \dfrac{1}{2} = \dfrac{1}{4}$, $\dfrac{1}{2} \cdot \dfrac{1}{6} = \dfrac{1}{12}$, and $\dfrac{1}{2} \cdot \dfrac{1}{6} = \dfrac{1}{12}$, events are pairwise independent.

(b) Since $\dfrac{1}{2} \cdot \dfrac{1}{2} \cdot \dfrac{1}{6} \neq \dfrac{1}{12}$ the events are *not* independent.

2.95 (a) The required probability is approximately $(0.99)^4 = 0.9606$ (assuming independence). The exact probability is

$$\frac{990}{1{,}000} \cdot \frac{989}{999} \cdot \frac{988}{998} \cdot \frac{987}{997} = 0.9605$$

(b) The required probability is approximately $(0.99)^3(0.01) = 0.0097$ (assuming independence). The exact probability is

$$\frac{990}{1{,}000} \cdot \frac{989}{999} \cdot \frac{988}{998} \cdot \frac{10}{997} = 0.0097$$

2.96 (a) $(0.52)^3 = 0.1406$; **(b)** $(0.48)^2(0.52) = 0.1198$

2.97 $\frac{5}{10} \cdot \frac{4}{9} \cdot \frac{3}{8} = \frac{1}{12}$

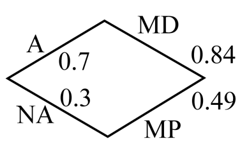
2.98 $1 - (0.9)^{12} = 1 - 0.2824 = 0.7176$

2.99 $\frac{6}{15} \cdot \frac{5}{14} \cdot \frac{4}{13} \cdot \frac{3}{12} = \frac{1}{91}$

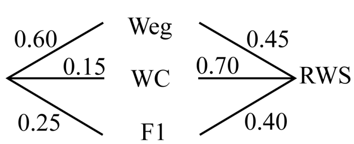
2.100 (a) $(0.9)(0.9)(0.9) = 0.729$

(b) $(0.6)(0.6)(0.4) = 0.144$

2.101 $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{2}$, $P(C) = \frac{1}{3}$, $P(D) = \frac{1}{3}$, $P(A \cap B) = \frac{1}{4}$, $P(A \cap C) = \frac{1}{6}$,
$P(A \cap D) = \frac{1}{6}$, $P(B \cap C) = \frac{1}{6}$, $P(B \cap D) = \frac{1}{6}$, $P(C \cap D) = \frac{1}{9}$,
$P(A \cap B \cap C) = \frac{1}{12}$, $P(A \cap B \cap D) = \frac{1}{12}$, $P(A \cap C \cap D) = \frac{1}{18}$,
$P(B \cap C \cap D) = \frac{1}{18}$, $P(A \cap B \cap C \cap D) = \frac{1}{36}$. Substitution shows that all conditions for independence are satisfied.

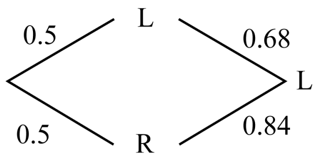
2.102 

$(0.7)(0.84) + (0.3)(0.49) = 0.735$

2.103 

$$(0.60)(0.45) + (0.15)(0.70) + (0.25)(0.40) = 0.27 + 0.105 + 0.1 = 0.475$$

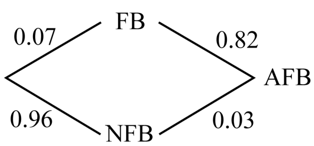
2.104



$(0.5)(0.68)+(0.5)(0.84)=0.76$

2.105 $\dfrac{0.27}{0.475}=0.5684$

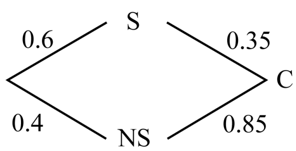
2.106



(a) $(0.04)(0.82)+(0.96)(0.03)$
$=0.0328+0.0288=0.0616$

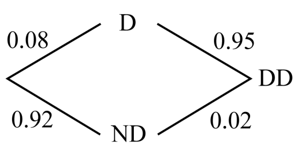
(b) $\dfrac{0.0328}{0.0616}=0.5325$

2.107



$$\frac{(0.6)(0.35)}{(0.6)(0.35)+(0.4)(0.85)}=\frac{0.21}{0.21+0.34}=\frac{0.21}{0.55}=0.3818$$

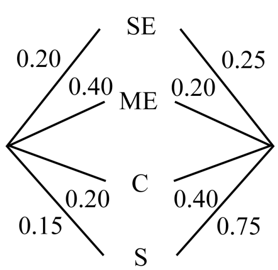
2.108



(a) $(0.08)(0.95)+(0.92)(0.02)$
$=0.076+0.0814=0.0944$

(b) $\dfrac{0.076}{0.0944}=0.8051$

2.109

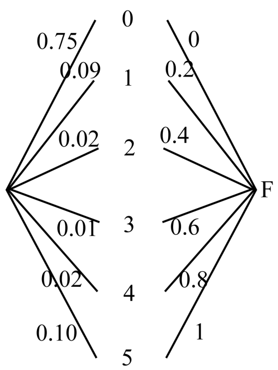


$0.05/0.3425=0.1460$
$0.08/0.3425=0.2336$
$0.10/0.3425=0.2920$
$0.1125/0.3425=0.3285$

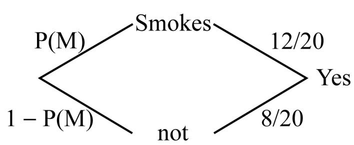
(a) Most likely cause is sabotage.
(b) Least likely cause is static electricity.

2.110 (a) 0.032; **(b)** 0.09375; **(c)** 0.625

2.111 $\dfrac{(0.10)1}{(0.09)(0.2)+(0.02)(0.4)+(0.01)(0.6)+(0.02)(0.8)+0.10}=\dfrac{0.10}{0.148}=0.6757$



2.112



$P(Y)=0.6\ P(M)+0.4[1-P(M)]$

(a) $P(Y)=0.4+0.2\ P(M)$

(b) $5P(Y)=2+P(M)$

$P(M)=5\cdot\dfrac{106}{250}-2=0.12$

2.113 $(0.95)^3(0.99)^3=0.832$

2.114 $(0.995)(0.990)(0.992)(0.995)(0.998)=0.970$

2.115 $R^6=0.95 \therefore R=(0.95)^{1/6}=0.991$

2.116 $R^{10}=0.90 \therefore R=(0.90)^{0.1}=0.990$

2.117 $1-(1-0.8)(1-0.7)(1-0.65)=0.979$

2.118 $1-(1-0.85)(1-0.80)(1-0.65)(1-0.60)(1-0.70)=0.999$

2.119 $(0.95)(0.90)\left[1-(1-0.60)^4\right]\left[1-(1-0.75)^2\right]=0.781$

2.120 $(0.98)(0.99)\left[1-(1-0.75)(1-0.60)(1-0.65)(1-0.70)(1-0.60)\right]=0.966$