Chapter 13

13.1 Test statistic $z = \dfrac{\bar{x} - \mu_0}{\sigma / \sqrt{n}}$

Then by Theorem 8.7 $\left(\dfrac{\bar{x} - \mu_0}{\sigma / \sqrt{n}}\right)^2$ is random variable having χ^2 distribution with $\nu = 1$. So rejection criterion becomes $\dfrac{n(\bar{x} - \mu_0)^2}{\sigma^2} \geq \chi^2_{\alpha,1}$

13.2 $K = \mu_0 + z_\alpha \dfrac{\sigma}{\sqrt{n}}$ and $K = \mu_1 - z_\beta \dfrac{\sigma}{\sqrt{n}}$

$$\mu_0 + z_\alpha \frac{\sigma}{\sqrt{n}} = \mu_1 - z_\beta \frac{\sigma}{\sqrt{n}}$$

$$\mu_1 - \mu_0 = (z_\alpha + z_\beta)\frac{\sigma}{\sqrt{n}} \rightarrow \sqrt{n} = \frac{\sigma(z_\alpha + z_\beta)}{\mu_1 - \mu_0}$$

and $n = \dfrac{\sigma^2 (z_\alpha + z_\beta)^2}{(\mu_1 - \mu_0)^2}$

13.3 $n = \dfrac{9^2(1.645 + 2.33)^2}{5^2} = \dfrac{81(3.975)^2}{25} = 51.19 \qquad n = 52$

13.4 $K = \delta + z_\alpha \sqrt{\dfrac{\sigma_1^2 + \sigma_2^2}{n}} \qquad K = \delta' - z_\beta \sqrt{\dfrac{\sigma_1^2 + \sigma_2^1}{n}}$

$$\delta + z_\alpha \sqrt{\frac{\sigma_1^2 + \sigma_2^2}{n}} = \delta' - z_\beta \sqrt{\frac{\sigma_1^2 + \sigma_2^2}{n}}$$

$$\delta - \delta' = (z_\alpha + z_\beta)\sqrt{\frac{\sigma_1^2 + \sigma_2^2}{n}}$$

$\sqrt{n} = \dfrac{(z_\alpha + z_\beta)\sqrt{\sigma_1^2 + \sigma_2^2}}{\delta - \delta'}$ and $n = \dfrac{(\sigma_1^2 + \sigma_2^2)(z_\alpha + z_\beta)^2}{(\delta - \delta')^2}$

13.5 $n = \dfrac{(81 + 169)(2.33 + 2.33)^2}{6^2} = \dfrac{(250(21.7156)}{36} = 150.80 = 151$

13.6 $\dfrac{(n-1)s^2}{\sigma_0^2}$ has chi square distribution with $(n-1)$ degrees of freedom, so that according to corollary 2 to Theorem 6.3

$\mu = n-1$ and $\sigma = \sqrt{2(n-1)}$

Using normal approximation, critical region is

$$\frac{(n-1)s^2}{\sigma_0^2} \ge n-1+z_\alpha\sqrt{2(n-1)}$$

or $s^2 \ge \sigma_0^2\left[1+z_\alpha\sqrt{\dfrac{2}{n-1}}\right]$

For $H_1:\sigma^2 < \sigma_0^2$ critical region is $s^2 \le \sigma_0^2\left[1-z_\alpha\sqrt{\dfrac{2}{n-1}}\right]$

For $H_1:\sigma^2 \ne \sigma_0^2$ critical region is $s^2 \le \sigma_0^2\left[1-z_{\alpha/2}\sqrt{\dfrac{2}{n-1}}\right]$ or $s^2 \ge \sigma_0^2\left[1+z_{\alpha/2}\sqrt{\dfrac{2}{n-1}}\right]$

13.7 If x has χ^2 distribution with $n-1$ degrees of freedom, then according to Example 8.42 $\sqrt{2x}-\sqrt{2(n-1)} \to$ standard normal distribution.

Since $\dfrac{(n-1)s^2}{\sigma_0^2}$ has chi square distribution with $n-1$ degrees of freedom.

$\sqrt{\dfrac{2(n-1)s^2}{\sigma_0^2}}-\sqrt{2(n-1)}$ has approximately standard normal distribution

$\dfrac{s}{\sigma_0}\sqrt{2(n-1)}-\sqrt{2(n-1)}$ has approximately standard normal distribution

$\left(\dfrac{s}{\sigma_0}-1\right)\sqrt{2(n-1)}$ has approximately standard normal distribution

13.8 $e_{i1}=n_1\hat{\theta},\ e_{i2}=n_i(1-\hat{\theta}),\ f_{i1}=x_i,\ f_{i2}=n_i-x_i$

$$\chi^2=\sum_{i=1}^{k}\sum_{j=1}^{2}\frac{(f_{ij}-e_{ij})^2}{e_{ij}}=\sum_{i=1}^{k}\frac{(x_i-n_i\hat{\theta})^2}{n_i\hat{\theta}}+\frac{[n_i-x_i-n_i(1-\hat{\theta})]^2}{n_i(1-\hat{\theta})}$$

$$=\sum_{i=1}^{k}\frac{(x_i-n_i\hat{\theta})^2+\hat{\theta}(x_i-n_i\hat{\theta})^2}{n_i\hat{\theta}(1-\hat{\theta})}$$

$$=\sum_{i=1}^{k}\frac{(x_i-n_i\hat{\theta})^2}{n_i\hat{\theta}(1-\hat{\theta})} \qquad \text{QED}$$

13.9 $H_1 : \lambda > \lambda_0$, Reject null hypothesis if $\sum_{i=1}^{n} x_i \geq k_\alpha$, where k_α is smallest integer for which

$\sum_{y=k_\alpha}^{\infty} p(y; n\lambda_0) \leq \alpha$.

$H_1 : \lambda < \lambda_0$, Reject null hypothesis if $\sum_{i=1}^{n} x_i \leq k'_\alpha$, where k'_α is smallest integer for which

$\sum_{y=0}^{k'_\alpha} p(y; n\lambda_0) \leq \alpha$.

$H_1 : \lambda \neq \lambda_0$, Reject null hypothesis if $\sum x \leq k'_{\alpha/2}$ or $\sum x \geq k_{\alpha/2}$

13.10 From Table II with $\lambda = 5(3.6) = 18$

$k_{0.025} = 25$ (Probability $X \geq 28 = 0.0173,\ x \geq 27 = 0.0282$)

$k'_{0.025} = 9$ (Probability $X \leq 9 = 0.0153,\ x \leq 10 = 0.0303$)

13.11 Substitute $e_{11} = \dfrac{n_1(x_1 + x_2)}{n_1 + n_2}, f_{11} = e_{21} = \dfrac{n_2(x_1 + x_2)}{n_1 + n_2}$

$f_{21} = x_2,\ e_{12} = \dfrac{n_1[(n_1 + n_2) - (x_1 + x_2)]}{n_1 + n_2},\ f_{12} = n_1 - x_1$

$e_{22} = \dfrac{n_2[(n_1 + n_2) - (x_1 + x_2)]}{n_1 + n_2},\ f_{22} = n_2 - x_2$ into

$\chi^2 = \sum_{i=1}^{k} \sum_{j=1}^{2} \dfrac{(f_{ij} - e_{ij})^2}{e_{ij}}$ and simplify algebraically

13.12 $E\left(\dfrac{x_1}{n_1} - \dfrac{x_2}{n_2}\right) = \theta_1 - \theta_2 = 0$

$$\operatorname{var}\left(\frac{x_1}{n_1} - \frac{x_2}{n_2}\right) = \operatorname{var}\left(\frac{x_1}{n_1}\right) + \operatorname{var}\left(\frac{x_2}{n_2}\right)$$

$$= \frac{\theta_2(1-\theta_2)}{n_1} + \frac{\theta_2(1-\theta_2)}{n_2}$$

$\theta_1 = \theta_2 = \theta$ estimated by $\hat{\theta} = \dfrac{x_1 + x_2}{n_1 + n_2}$

$$\hat{\theta}(1-\hat{\theta})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)$$

Thus, $z = \dfrac{\dfrac{x_1}{n_1} - \dfrac{x_2}{n_2} - 0}{\sqrt{\hat{\theta}(1-\hat{\theta})\left(\dfrac{1}{n_1} + \dfrac{1}{n_2}\right)}} = \dfrac{\dfrac{x_1}{n_1} - \dfrac{x_2}{n_2}}{\sqrt{\hat{\theta}(1-\hat{\theta})\left(\dfrac{1}{n_1} + \dfrac{1}{n_2}\right)}}$

has approximately standard normal distribution.

13.13 $\chi^2 = \dfrac{(x_1 - n_1\hat{\theta})^2}{n_1\hat{\theta}(1-\hat{\theta})} + \dfrac{(x_2 - n_2\hat{\theta})^2}{n_2\hat{\theta}(1-\hat{\theta})}$

$$= \frac{\left[x_1 - \dfrac{n_1(x_1+x_2)^2}{n_1+n_2}\right]^2}{n_1\hat{\theta}(1-\hat{\theta})} + \frac{\left[x_2 - \dfrac{n_2(x_1+x_2)^2}{n_1+n_2}\right]^2}{n_2\hat{\theta}(1-\hat{\theta})}$$

$$= \frac{\left[\dfrac{x_1 n_2}{n_1+n_2} - \dfrac{n_1 x_2}{n_1+n_2}\right]^2}{n_1\hat{\theta}(1-\hat{\theta})} + \frac{\left[\dfrac{x_2 n_1}{n_1+n_2} - \dfrac{n_2 x_1}{n_1+n_2}\right]^2}{n_2\hat{\theta}(1-\hat{\theta})}$$

$$= \frac{\dfrac{n_1^2 \cdot n_2}{n_1^2(n_1+n_2)^2}\left(\dfrac{x_1}{n_1} - \dfrac{x_2}{n_2}\right)^2 + \dfrac{n_2^2 \cdot n_1}{n_1^2(n_1+n_2)}\left(\dfrac{x_1}{n_1} - \dfrac{x_2}{n_2}\right)}{n_1 n_2\hat{\theta}(1-\hat{\theta})} = \frac{\left(\dfrac{x_1}{n_1} - \dfrac{x_2}{n_2}\right)^2}{\dfrac{(n_1+n_2)}{n_1 n_2}\hat{\theta}(1-\hat{\theta})}$$

$$= \frac{\left(\dfrac{x_1}{n_1} - \dfrac{x_2}{n_2}\right)^2}{\left(\dfrac{1}{n_1} + \dfrac{1}{n_2}\right)\hat{\theta}(1-\hat{\theta})} = Z^2 \qquad \text{QED}$$

13.14 $e_{ij} = \dfrac{\sum\limits_i f_{ij} \sum\limits_j f_{ij}}{n}$

$$\sum_i e_{ij} = \frac{\sum\limits_i f_{ij} \cdot \sum\limits_i \sum\limits_j f_{ij}}{n} = \frac{\sum\limits_i f_{ij} \cdot n}{n} = \sum_i f_{ij}$$

$$\sum_i e_{ij} = \frac{\sum\limits_j f_{ij} \cdot \sum\limits_i \sum\limits_j f_{ij}}{n} = \frac{\sum\limits_j f_{ij} \cdot n}{n} = \sum_j f_{ij}$$

13.15 Under $H_o : e_{1j} = \theta_{2j} = \ldots = \theta_{nj}$ for $j = 1, 2, \ldots.$

$$= \theta_j$$

$$\hat{\theta}_j = \frac{\sum\limits_i f_{ij}}{n} \qquad e_{ij} = \frac{\sum\limits_i f_{ij}}{n} \cdot \sum_j f_{ij} = \frac{\sum\limits_i f_{ij} \cdot \sum\limits_j f_{ij}}{n}$$

13.16 $\chi^2 = \sum\limits_i \sum\limits_j \dfrac{(f_{ij} - e_{ij})^2}{e_{ij}} = \sum\limits_i \sum\limits_j \dfrac{f_{ij}^2}{e_{ij}} - 2\sum\limits_i \sum\limits_j f_{ij} + \sum\limits_i \sum\limits_j e_{ij}$

$$= \sum_i \sum_j \frac{f_{ij}^2}{e_{ij}} - 2f + f \qquad \text{(see Ex 13.14)}$$

$$= \sum_i \sum_j \frac{f_{ij}^2}{e_{ij}} - f \qquad \text{QED}$$

13.17 $\chi^2 = \frac{232^2}{212} + \frac{260^2}{265} + \frac{197^2}{212} + \frac{168^2}{188} + \frac{240^2}{235} + \frac{203^2}{188} - 1300$

$= 253.887 + 255.094 + 183.061 + 150.128 + 245.106 + 219.197 - 1300$

$= 6.473$ (differs due to rounding)

13.18 (a)

$f/2$ $f/4$	0 $f/4$
0 $f/4$	$f/2$ $f/4$

$$\chi^2 = \frac{(f/4)^2}{f/4} + \frac{(f/4)^2}{f/4} + \frac{(f/4)^2}{f/4} + \frac{(f/4)^2}{f/4}$$

$$= f$$

$$C = \sqrt{\frac{f}{f+f}} = \sqrt{\frac{1}{2}} = \frac{\sqrt{2}}{2}$$

(b)

$f/3$ $f/9$	0 $f/9$	0 $f/9$
0 $f/9$	$f/3$ $f/9$	0 $f/9$
0 $f/9$	0 $f/9$	$f/3$ $f/9$

$$\chi^2 = 3 \cdot \frac{\left(\frac{2f}{9}\right)^2}{f/9} + 6 \cdot \frac{\left(\frac{f}{9}\right)^2}{f/9}$$

$$= \frac{4}{3}f + \frac{2}{3}f = 2f$$

$$C = \sqrt{\frac{2f}{2f+f}} = \sqrt{\frac{2}{3}} = \frac{1}{3}\sqrt{6}$$

13.19 (a) not necessarily; **(b)** yes

13.20 (a) No, since $0.0316 > 0.01$
(b) Yes, since $0.0316 < 0.05$
(c) Yes, since $0.0316 < 0.10$

13.21 Normal curve area corresponding to $z = 2.84$ is 0.4977
p-value is $2(0.5000 - 0.4977) = 0.0046$

13.22 Normal curve area corresponding to 1.40 is 0.4192
p-value is $0.5000 - 0.4192 = 0.0808$

13.23 p-value is $\frac{1 - 0.3502}{2} = 0.3249$. As is exceeds 0.05, null hypothesis *cannot* be rejected.

13.24 $H_0: \mu = 10$; $H_1: \mu < 10$

$$z = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}} = \frac{8.4 - 10}{3.2/\sqrt{16}} = -2.0$$

Since $z_{0.05} = 1.645$, we reject H_0 in favor of H_1.

13.25 1. $H_0: \mu = 84.3,\ \ H_1: \mu > 84.3,\ \ \alpha = 0.01$

2. Reject null hypothesis if $z \geq 2.33$

3. $z = \dfrac{87.5 - 84.3}{8.6/\sqrt{45}} = 2.73$

4. Since 2.73 exceeds 2.33, null hypothesis must be rejected.

13.26 2. $z = \dfrac{\overline{x} - \mu_0}{\sigma/\sqrt{n}}$

3. $z = \dfrac{87.5 - 84.3}{8.6/\sqrt{45}} = 2.73$, p-value = $0.5000 - 0.4968 = 0.0032$

4. Since $0.0032 < 0.01$, null hypothesis must be rejected.

13.27 1. $H_0: \mu = 30,\ \ H_1: \mu \neq 30,\ \ \alpha = 0.01$

2. Reject null hypothesis if $z \leq -2.575$ or $z \geq 2.575$

3. $z = \dfrac{30.8 - 30}{1.5/\sqrt{32}} = \dfrac{0.8\sqrt{32}}{1.5} = 3.02$

4. Since $3.02 > 2.575$, null hypothesis must be rejected.

13.28 2. $z = \dfrac{\overline{x} - \mu_0}{\sigma/\sqrt{n}}$

3. $z = \dfrac{30.8 - 30}{1.5/\sqrt{32}} = 3.02$, p-value = $2(0.5 - 0.4987) = 0.0026$

4. Since 0.0026 is less than 0.005, null hypothesis must be rejected.

13.29 1. $H_0: \mu = 35,\ \ H_1: \mu < 35,\ \ \alpha = 0.05$

2. Reject null hypothesis if $t \leq -t_{0.05,11} = -1.796$

3. $t = \dfrac{33.6 - 35}{2.3/\sqrt{12}} = \dfrac{-1.4}{2.3\sqrt{12}} = -2.11$

4. Since $-2.11 < -1.796$, the null hypothesis must be rejected.

13.30 $n = 5,\ \ \overline{x} = 14.4,\ \ s = 0.158$

1. $H_0: \mu = 14,\ \ H_1: \mu \neq 14,\ \ \alpha = 0.05$

2. Reject null hypothesis if $t \leq -2.776$ or $t \geq 2.776$

3. $t = \dfrac{14.4 - 14}{0.158/\sqrt{5}} = 5.66$

4. Since 5.66 exceeds 2.776, null hypothesis must be rejected.

13.31 $n = 5,\ \ \overline{x} = 14.7,\ \ s = 0.742$

3. $t = \dfrac{14.7 - 14}{0.742/\sqrt{5}} = 2.11$

4. Since $t = 2.11$ falls between -2.776 and 2.776, null hypothesis cannot be rejected.

$x - \mu_0$ has increased from 14.4 to 14.7 but s has increased from 0.158 to 0.742.

13.32 $t = 5.66$, d.f. $= 4$
p-value $= 1 - 0.9952 = 0.0048$
Since $0.0048 < 0.05$, null hypothesis must be rejected.

13.33 **(a)** $P(\text{reject } H_0 | H_0 \text{ is true}) = 0.05$ (by definition)

(b) $P(\text{reject } H_0$ on experiment 1 or experiment 2 (or both) $|H_0$ is true) =
$0.05 + 0.05 - .0025 = 0.0975$

(c) Reject H_0 on one or more of 30 experiments $|H_0$ is true) =
$1 - P($ do not reject H_0 on any experiment $|H_0$ is true)$=1-(0.95)^{30} = 0.79$.

13.34 **(a)** $P(\text{reject } H_0$ on exactly one factor $|H_0$ is true for all 48 factors) =
$$\binom{48}{1}(0.01)^1(0.99)^{47} = 0.30$$

(b) $P(\text{reject } H_0$ on more than one factor $|H_0$ is true for all 48 factors) = $1 - 0.30 = 0.70$.

13.35 $$\frac{(\bar{x}_1 - \bar{x}_2) - 0.20}{\sqrt{\frac{(0.12)^2}{50} + \frac{(0.14)^2}{40}}} \le -1.96 \text{ or } \ge 1.96$$
$$\frac{(\bar{x}_1 - \bar{x}_2) - 0.20}{0.0279} \le -1.96 \text{ or } \ge 1.96$$
$\bar{x}_1 - \bar{x}_2 \le 0.20 - 0.0547 = 0.145$
or $\bar{x}_1 - \bar{x}_2 \ge 0.20 + 0.0547 = 0.255$

(a) $$z = \frac{0.145 - 0.12}{0.0279} = 0.90 \text{ and } z = \frac{0.255 - 0.12}{0.0279} = 4.84$$
$\beta = 0.5 - 0.3159 = 0.1841 = 0.18$

(b) $$z = \frac{0.145 - 0.16}{0.0279} = -0.54 \text{ and } z = \frac{0.255 - 0.16}{0.0279} = 3.405$$
$\beta = 0.2054 + 0.5 = 0.7054 = 0.71$

(c) $$z = \frac{0.145 - 0.24}{0.0279} = -3.40 \text{ and } z = \frac{0.255 - 0.24}{0.0279} = 0.54$$
$\beta = 0.2054 + 0.5 = 0.7054 = 0.71$

(d) $$z = \frac{0.145 - 0.28}{0.0279} = -4.84 \text{ and } z = \frac{0.255 - 0.28}{0.0279} = -0.90$$
$\beta = 0.5 - 0.3159 = 0.1841 = 0.18$

13.36 1. $H_0: \mu_1 - \mu_2 = 0,\ H_1: \mu_1 - \mu_2 \neq 0,\ \alpha = 0.05$

2. Reject null hypothesis if $z \leq -1.96$ or $z \geq 1.96$

3. $$z = \frac{9.1 - 8}{\sqrt{\frac{1.9^2}{40} + \frac{2.1^2}{50}}} = \frac{1.1}{0.4224} = 2.60$$

4. Since $2.60 > 1.96$, null hypothesis must be rejected.

13.37 $z = 2.60,\ p-\text{value} = 2(0.5 - 0.4953) = 0.0094$

Since $0.0094 < 0.05$, null hypothesis must be rejected.

13.38 1. $H_0: \mu_1 - \mu_2 = -0.05,\ H_1: \mu_1 - \mu_2 < -0.05,\ \alpha = 0.05$

2. Reject null hypothesis if $z \leq -1.645$

3. $$z = \frac{(53.8 - 54.5) + 0.05}{\sqrt{\frac{2.4^2}{400} + \frac{2.5^2}{500}}} = \frac{-0.20}{0.164} = -1.22$$

4. Since $-1.22 > -1.645$, null hypothesis cannot be rejected.

13.39 $z = -1.22,\ p-\text{value} = 0.5 - 0.3888 = 0.1112$

Since $0.1112 > 0.05$, null hypothesis cannot be rejected.

13.40 1. $H_0: \mu_1 - \mu_2 = 0,\ H_1: \mu_1 - \mu_2 \neq 0,\ \alpha = 0.01$

2. Reject null hypothesis if $t \leq -t_{0.005} = -3.169$ or $t > t_{0.005} = 3.169$

3. $$s_p^2 = \frac{5(3.3)^2 + 5(2.1)^2}{10} = 7.65 \text{ and } s_p = 2.766$$

$$t = \frac{77.4 - 72.2}{2.766\sqrt{\frac{1}{6} + \frac{1}{6}}} = \frac{5.2}{(2.766)(0.577)} = 3.26$$

4. Since $3.26 > 3.169$, null hypothesis must be rejected.

13.41 $t = 2.67,\ \text{d.f.} = 6,\ \alpha = 0.05$

$$p\text{-value} = \frac{1}{2}(1 - 0.9630) = 0.0185$$

13.42 $\bar{x}_1 = 144,\ s_1 = 19.06,\ \bar{x}_2 = 149,\ s_2 = 14.21$

1. $H_0: \mu_1 = \mu_2,\ H_1: \mu_1 \neq \mu_2,\ \alpha = 0.01$

2. Reject null hypothesis if $t \leq -3.169$ or $t \geq 3.169$

3. $$s_p^2 = \frac{5(19.06)^2 + 5(14.21)^2}{10} = 282.604 \text{ and } s_p = 16.802$$

$$t = \frac{144 - 149}{16.802\sqrt{\frac{1}{6} + \frac{1}{6}}} = \frac{-5}{(16.802)(0.577)} = -0.52$$

4. Since -0.52 falls between -3.169 and 3.169, null hypothesis cannot be rejected.

13.43 $t = -0.52,\ \text{d.f.} = 10$
p-value $= 1 - 0.3856 = 0.61$
Since $0.61 > 0.01$, null hypothesis cannot be rejected.

13.44 13, 7, −1, 5, 3, 2, −1, 0, 6, 1, 4, 3, 2, 6, 12, 4
$\bar{x} = 4.125,\ s = 4.064,\ n = 16$

1. $H_0: \mu = 0,\ H_1: \mu > 0,\ \alpha = 0.05$
2. Reject null hypothesis if $t \geq t_{0.05,15} = 1.753$
3. $t = \dfrac{4.125 - 0}{4.064/\sqrt{16}} = 4.06$
4. Since $4.06 > 1.753$, null hypothesis must be rejected. Exercises are effective in reducing weight.

13.45 9, 13, 2, 5, −2, 6, 6, 5, 2, 6
$n = 10,\ \bar{x} = 5.2,\ s = 4.08$

1. $H_0: \mu = 0,\ H_1: \mu > 0,\ \alpha = 0.05$
2. Reject null hypothesis if $t > t_{0.05,9} = 1.833$
3. $t = \dfrac{5.2 - 0}{4.08/\sqrt{10}} = 4.03$
4. Since $4.03 > 1.833$, null hypothesis must be rejected. Safety program is effective.

13.46 $t = 4.03,\ \text{d.f.} = 9$
$p\text{-value} = \dfrac{1}{2}(1 - 0.997) = 0.0015$

13.47
1. $H_0: \sigma = 0.0100,\ H_1: \sigma < 0.0100,\ \alpha = 0.05$
2. Reject null hypothesis if $\chi^2 \leq \chi^2_{0.95,8} = 2.733$
3. $\chi^2 = \dfrac{8(0.0086)^2}{(0.0100)^2} = 5.92$
4. Since $5.92 > 2.733$, null hypothesis cannot be rejected.

13.48 $s = 238,\ n = 24$

1. $H_0: \sigma = 250,\ H_1: \sigma \neq 250,\ \alpha = 0.01$
2. Reject null hypothesis if $\chi^2 \leq \chi^2_{0.995,23} = 9.260$ or $\chi^2 \geq \chi^2_{0.005,23} = 44.181$
3. $\chi^2 = \dfrac{23(238)^2}{(250)^2} = 20.84$
4. Since $9.260 < 20.84 < 44.181$, null hypothesis cannot be rejected.

13.49 $s = 2.53,\ n = 30,\ \alpha = 0.05$

1. $H_0: \sigma = 2.85,\ H_1: \sigma < 2.85,\ \alpha = 0.05$
2. Reject null hypothesis if $\chi^2 \leq \chi^2_{0.95,29} = 17.708$
3. $\chi^2 = \dfrac{29(2.53)^2}{(2.85)^2} = 22.85$
4. Since $22.85 > 17.708$, null hypothesis cannot be rejected.

13.50 1. $H_0: \sigma = \sigma_0,\ H_1: \sigma < \sigma_0,\ \alpha = 0.05$

2. Reject null hypothesis if $z \le -z_{0.05} = -1.645$

3. $z = \left(\frac{2.53}{2.85} - 1\right)\sqrt{2 \cdot 29} = -0.1123(7.616) = -0.85$

4. Since $-0.85 > -1.645$, null hypothesis cannot be rejected.

13.51 $n = 50,\ s = 0.49$

1. $H_0: \sigma = 0.41,\ H_1: \sigma > 0.41,\ \alpha = 0.05$

2. Reject null hypothesis if $z \ge z_{0.05} = 1.645$

3. $z = \left(\frac{0.49}{0.41} - 1\right)\sqrt{2 \cdot 49} = (0.1951)(9.8995) = 1.93$

4. Since $1.93 > 1.645$, null hypothesis must be rejected.

13.52 p-value = $0.5 - 0.4732 = 0.0268$

Since $0.0268 < 0.05$, null hypothesis must be rejected.

13.53 $n_1 = 4,\ s_1 = 31,\ n_2 = 4,\ s_2 = 26,\ \alpha = 0.05$

1. $H_0: \sigma_1 - \sigma_2 = 0,\ H_1: \sigma_1 - \sigma_2 > 0,\ \alpha = 0.05$

2. Reject null hypothesis if $\frac{s_1^2}{s_2^2} \ge F_{0.05,3,3} = 9.28$

3. $\frac{s_1^2}{s_2^2} = 1.42$

4. Since 1.42 does not exceed 9.28, null hypothesis cannot be rejected.

13.54 1. $H_0: \sigma_1 - \sigma_2 = 0,\ H_1: \sigma_1 - \sigma_2 \ne 0,\ \alpha = 0.10$

2. Reject null hypothesis if $\frac{s_1^2}{s_2^2} \ge F_{0.05,5,5} = 5.05$

3. $\frac{s_1^2}{s_2^2} = \frac{3.3^2}{2.1^2} = 2.47$

4. Since 2.47 < 5.05, null hypothesis cannot be rejected. Assumption was reasonable.

13.55 $s_1 = 19.06,\ s_2 = 14.21,\ n_1 = n_2 = 6$

1. $H_0: \sigma_1 - \sigma_2 = 0,\ H_1: \sigma_1 - \sigma_2 \ne 0,\ \alpha = 0.02$

2. Reject null hypothesis if $\max\left(\frac{s_1^2}{s_2^2}, \frac{s_2^2}{s_1^2}\right) \ge F_{0.01,5,5} = 11.0$

3. $\frac{s_1^2}{s_2^2} = 1.80$

4. Since 1.80 < 11.0, null hypothesis cannot be rejected.

13.56 $n = 20,\ \theta = 0.5$ against $\theta \neq 0.50,\ \alpha = 0.05$

$p(x \le 5) = 0.0207$ Critical region is $x \le 5$ or $x \ge 15$

$p(x \le 6) = 0.0507$ $a = 0.0207 + 0.0207 = 0.0414$

$p(x \ge 15) = 0.0207$

$p(x \ge 14) = 0.0507$

13.57
1. $H_0 : \theta = 0.40,\ H_1 : \theta > 0.40,\ \alpha = 0.05$
2. Observed number of successes in n = 18 trials
3. $x = 10$ $P(X \ge 10) = 0.1348$ p-value 0.1348
4. Since 0.1348 > 0.05, null hypothesis cannot be rejected.

13.58 $p(X \ge 12) = 0.0203$ Critical region is $x \ge 12$

$p(X \ge 11) = 0.0577$ $a = 0.0203$

13.59
1. $H_0 : \theta = 0.30,\ H_1 : \theta < 0.30,\ \alpha = 0.05$
2. Observed number of successes in n = 19 trials
3. $x = 1$ p-value is 0.0011 + 0.0093 = 0.0104
4. Since 0.0104 < 0.05, null hypothesis must be rejected.

13.60 $p(x \le 2) = 0.0462$ Critical region is $x \le 2$

$p(x \le 3) = 0.1331$ $\alpha = 0.0462$

13.61
1. $H_0 : \theta = 0.40,\ H_1 : \theta \neq 0.40,\ \alpha = 0.01$
2. Observed number of successions in n = 14 trials
3. $p(x \ge 12) = 0.0006$, p-value = 0.0012
4. Since 0.0012 < 0.01, null hypothesis must be rejected.

13.62 $P(x \le 0) = 0.0008,\ P(x \ge 11) = 0.0039$, Critical region is $x = 0$, or $x \ge 11$

$P(x \le 1) = 0.0081,\ P(x \ge 10) = 0.0175,\ \alpha = 0.008 + 0.0039 = 0.0047$

13.63 $H_0 : \theta = 0.35;\ H_1 : \theta < 0.35.$ Using the normal approximation

$$z = \frac{x - n\theta_0}{\sqrt{n\theta_0(1-\theta_0)}} = \frac{290 - 350}{\sqrt{(350)(0.65)}} = -3.98$$

Since $z_{0.05} = 1.645$, we reject H_0 at the 0.05 level of significance and conclude that $\theta < 0.35$; thus, the statement can be refuted.

13.64
1. $H_0 : \theta = 0.20,\ H_1 : \theta > 0.20,\ \alpha = 0.01$
2. Number of successes in n = 12 trials
3. $x = 6,\ p(X \ge 6) = 0.0194 = p$-value
4. Since 0.0194 > 0.01, null hypothesis cannot be rejected.

13.65 1. $H_0: \theta = 0.60,\ H_1: \theta \neq 0.60,\ \alpha = 0.05$

2. Number of failures in $n = 18$ trials

3. $x = 7,\ n - x = 18 - 7 = 11$

$P(X \geq 11);\ \theta = 0.40) = 0.0577$

p-value is 2(0.0577) = 0.1154

4. Since 0.1154 > 0.05, null hypothesis cannot be rejected.

13.66 1. $H_0: \theta = 0.30,\ H_1: \theta \neq 0.30,\ \alpha = 0.05$

2. Reject if $z \leq -1.96$ or $z \geq 1.96$

3. $z = \dfrac{157 - 600(0.30)}{\sqrt{600(0.3)(0.7)}} = -2.05$

4. Since $-2.05 < -1.96$, null hypothesis must be rejected.

13.67 1. $H_0: \theta = 0.90,\ H_1: \theta < 0.90,\ \alpha = 0.05$

2. Reject if $z < -1.645$

3. $z = \dfrac{174 - 200(0.9)}{\sqrt{200(0.9)(0.1)}} = -\dfrac{6}{4.2426} = -1.41$

4. Since $-1.41 > -1.645$, null hypothesis cannot be rejected.

13.68 1. $H_0: \theta_1 = \theta_2,\ \ H_1: \theta_1 \neq \theta_2,\ \ \alpha = 0.01$

2. Reject null hypothesis if $\chi^2 \geq \chi^2_{0.01,1} = 6.635$

74 (83)	92 (83)	166
176 (167)	158 (167)	334
250	250	500

$e_{11} = \dfrac{166 \cdot 250}{500} = 83$, others by subtraction

$\chi^2 = \dfrac{9^2}{83} + \dfrac{9^2}{83} + \dfrac{9^2}{167} + \dfrac{9^2}{167} = 2.92$

4. Since $2.92 < 6.635$, null hypothesis cannot be rejected.

13.69 1. $H_0: \theta_1 = \theta_2,\ H_1: \theta_1 \neq \theta_2,\ \alpha = 0.01$

2. Reject null hypothesis if $z \leq -z_{0.005}$ or $z \geq z_{0.005}$

$\hat{\theta} = \dfrac{74 + 92}{500} = 0.332$

3. $z = \dfrac{\dfrac{74}{250} - \dfrac{92}{250}}{\sqrt{(0.332)(0.668)(0.008)}} = -\dfrac{0.072}{0.04212} = -1.71$

4. Since -1.71 falls between -2.575 and 2.575, null hypothesis cannot be rejected.

13.70 1. $H_0: \theta_1 = \theta_2,\ H_1: \theta_1 \neq \theta_2,\ \alpha = 0.05$

2. Reject null hypothesis if $\chi^2 \geq \chi^2_{0.05,1} = 3.841$

3.

	46 (42.7)	18 (21.3)	64
	354 (357.3)	182 (178.7)	536
	400	400	600

$e_{11} = \dfrac{64 \cdot 400}{600} = 42.7$, others by subtraction

$$\chi^2 = \frac{3.3^2}{42.7} + \frac{3.3^2}{21.3} + \frac{3.3^2}{357.3} + \frac{3.3^2}{178.7}$$
$$= 0.255 + 0.511 + 0.030 + 0.061$$
$$= 0.86$$

4. Since 0.86 < 3.841, null hypothesis cannot be rejected.

13.71 1. $H_0: \theta_1 = \theta_2,\ H_1: \theta_1 \neq \theta_2,\ \alpha = 0.05$

2. Reject null hypothesis if $z \leq -1.96$ or $z \geq 1.96$

$$\hat{\theta} = \frac{74+92}{500} = 0.332$$

3. $$z = \frac{\dfrac{46}{400} - \dfrac{18}{200}}{\sqrt{(0.107)(0.893)(0.0075)}} = \frac{0.025}{0.0268} = 0.93$$

$$z^2 = (0.93)^2 = 0.8649 = 0.86 = \chi^2$$

13.72 $H_0: \theta_1 = \theta_2,\ H_1: \theta_1 > \theta_2,\ \alpha = 0.05$

2. Reject null hypothesis if $z \geq 1.645$

3. $\hat{\theta} = \dfrac{169}{500} = 0.338 \qquad z = \dfrac{\dfrac{82}{200} - \dfrac{87}{300}}{\sqrt{(0.338)(0.662)(0.00833)}} = 2.78$

4. Since 2.78 > 1.645, null hypothesis must be rejected.

13.73 $H_0: \theta_1 = \theta_2 = \theta_3 = \theta_4,\ H_1:$ not all equal, $\alpha = 0.05$

2. Reject null hypothesis if $\chi^2 \geq \chi^2_{0.05,3} = 7.815$

3.

26 (24)	23 (24)	15 (24)	32 (24)	96
174 (176)	177 (176)	185 (176)	168 (176)	704
200	200	200	200	800

$e_{11} = \dfrac{96 \cdot 200}{800} = 24$ etc.

$$\chi^2 = \frac{4+1+81+64}{24} + \frac{4+1+81+64}{24} = 7.10$$

4. Since 7.10 < 7.818, null hypothesis cannot be rejected.

13.74 $H_0: \theta_1 = \theta_2 = \theta_3$, $H_1:$ not all equal, $\alpha = 0.05$

2. Reject null hypothesis if $\chi^2 \geq \chi^2_{0.05,2} = 5.991$

3.

155 150	118 120	87 90	360
95 100	82 80	63 60	240
250	200	150	600

$$e_{11} = \frac{360 \cdot 250}{600}$$

$$\chi^2 = \frac{25}{100} + \frac{4}{120} + \frac{9}{90} + \frac{25}{100} + \frac{4}{80} + \frac{9}{60} = 0.75$$

4. Since 0.75 < 5.991, null hypothesis cannot be rejected.

13.75 In the following contingency table, the expected frequency is given below the observed frequency in each cell.

				TOTALS
	45 45.0	58 49.8	49 57.3	152
	21 21.0	15 23.2	35 26.7	71
TOTALS	66	73	84	223

The expected frequencies were calculated as $\frac{152 \times 66}{223} = 45.0$, etc.

Thus, $\chi^2 = \frac{(45-45.0)^2}{45.0} + \frac{(58-49.8)^2}{49.8} + \ldots + \frac{(35-26.7)^2}{26.7}$

$= 0.00 + 1.35 + 1.20 + 0.00 + 2.90 + 2.58 = 8.03$

Since $\chi^2_{0.01} = 9.210$, we cannot reject H_0, and we have no reason to conclude that the three processes have different probabilities of passing the strength standard.

13.76

48 44.4	40 38.6	12 17.0	100
55 60.9	53 52.9	29 23.2	137
57 54.7	46 47.5	20 20.8	123
160	139	61	360

1. $H_0:$ independent, $H_1:$ not independent, $\alpha = 0.05$
2. Reject null hypothesis, if $\chi^2 \geq \chi^2_{0.05,4} = 9.488$
3. $\chi^2 = 0.292 + 0.051 + 1.471 + 0.572$
   $+ 0.000 + 1.450 + 0.097$
   $+ 0.047 + 0.031$
   $= 4.01 = 4.0$
4. Since 4.0 < 9.488, null hypothesis cannot be rejected.

13.77

7 / 15	12 / 22.1	31 / 12.9	50
35 / 33.6	59 / 49.5	18 / 28.9	112
15 / 8.4	13 / 12.4	0 / 7.2	28
57	8.4	49	190

1. H_0: independent, H_1: not independent, $\alpha = 0.01$
2. Reject null hypothesis, if $\chi^2 \geq \chi^2_{0.01,4} = 13.277$
3. $\chi^2 = 4.27 + 4.62 + 25.40 + 0.06 + 1.82$
   $+ 4.11 + 5.19 + 0.029 + 7.2$
   $= 52.7$
4. Since 52.7 > 13.277, null hypothesis must be rejected.

13.78

12 / 13.5	23 / 21.4	89 / 89.1	124
8 / 8.9	12 / 14.2	62 / 58.9	82
21 / 18.6	30 / 29.4	119 / 122.0	170
41	65	270	376

1. H_0: Venders ship equal quantities
   H_1: Venders do not ship equal quantities; $\alpha = 0.01$
2. Reject null hypothesis, if $\chi^2 \geq \chi^2_{0.01,4} = 13.277$
3. $\chi^2 = 0.17 + 0.12 + 0.00 + 0.09 + 0.34$
   $+ 0.16 + 0.31 + 0.01 + 0.07$
   $= 1.27 = 1.3$
4. Since 1.3 < 13.277, null hypothesis cannot be rejected.

13.79

174 / 159.4	93 / 99.1	133 / 141.5	400
196 / 199.2	124 / 123.8	180 / 177.0	500
148 / 159.4	105 / 99.1	147 / 141.5	400
518	322	460	1300

1. H_0: percentages same for three cities
   H_1: percentages *not* same for three cities $\alpha = 0.05$
2. Reject null hypothesis, if $\chi^2 \geq \chi^2_{0.05,4} = 9.488$
3. $\chi^2 = 1.34 + 0.38 + 0.51 + 0.05 + 0.00$
   $+ 0.05 + 0.82 + 0.35 + 0.21$
   $= 3.71$
4. Since 3.71 < 9.488, null hypothesis cannot be rejected.

13.80

	f	prob	e
0	19	1/16	10
1	54	4/16	40
2	58	10/16	60
3	23	4/16	40
4	6	1/16	10

1. H_0: coins are balanced
   H_1: coins are *not* balanced
   $\alpha = 0.05$
2. Reject null hypothesis if $\chi^2 \geq \chi^2_{0.05,4} = 9.488$
3. $\chi^2 = \frac{81}{10} + \frac{196}{40} + \frac{4}{60} + \frac{289}{40} + \frac{16}{10} = 8.1 + 4.9 + 0.1 + 7.2 + 1.6 = 21.9$
4. Since 21.9 > 9.488, null hypothesis must be rejected.

13.81

	f		prob	e	
0	19		0.0907	27.2	
1	48		0.2177	65.3	
2	66		0.2613	78.4	
3	74		0.2090	62.7	
4	44		0.1254	37.6	
5	35		0.0602	18.1	
6	10	14	0.0241	7.2	10.7
7 or more	4		0.0117	3.5	
	300				

1. H_0: Poisson distribution with $\lambda = 2.4$
   H_1: *not* Poisson distribution with $\lambda = 2.4$
   $\alpha = 0.05$
2. Reject null hypothesis if $\chi^2 \geq \chi^2_{0.05,6} = 12.592$
3. $\chi^2 = 2.47 + 4.58 + 1.96 + 2.04 + 1.09 + 15.78 + 1.02 = 28.9$
4. Since 28.9 > 12.592, null hypothesis must be rejected.

13.82 $\bar{x} = \dfrac{0 \cdot 1 + 1 \cdot 16 + 2 \cdot 55 + 3 \cdot 228}{300} = \dfrac{810}{300} = 2.7$ $\qquad \hat{\theta} = \dfrac{2.7}{3} = 0.9$

	f		prob	e	
0	1	17	0.001	0.3	8.4
1	16		0.027	8.1	
2	55		0.243	72.9	
3	228		0.729	218.7	

1. H_0: binomial distribution
   H_1: *not* binomial distribution
   $\alpha = 0.05$
2. Reject null hypothesis if $\chi^2 \geq \chi^2_{0.05,1} = 3.841$
3. $\chi^2 = 8.80 + 4.40 + 0.40 = 13.6$
4. Since 13.6 > 3.841, null hypothesis must be rejected.

13.83 (a) $\bar{x} = 20$ and $s = 5.025 = 5$

using $\bar{x} = \dfrac{\sum xf}{n}$ and $s = \sqrt{\dfrac{n\left(\sum x^2 f\right) - \left(\sum xf\right)^2}{n(n-1)}}$

where x's are the class marks (midpoints)

(b)

	z			e
9.5	−2.1	0.4821	0.0179	1.8
14.5	−1.1	0.3643	0.1178	11.8
19.5	−0.1	0.0398	0.3245	32.4
24.5	0.9	0.3159	0.3557	35.5
29.5	1.9	0.4713	0.1554	15.5
34.5	2.9	0.4981	0.0268	2.7
			0.0019	0.2

Probabilities are 0.0179, 0.1178, 0.3245, 0.3557, 0.1554, 0.0268, 0.0019.

(c) Expected frequencies are 1.8, 11.8, 32.4, 35.6, 15.5, 2.7, 0.2

1. $H_0:$ normally distributed random variables
   $H_A:$ *not* normally distributed random variables, $\alpha = 0.05$

f	e
11	13.6
37	32.4
36	35.6
16	18.4

2. Reject null hypothesis if $\chi^2 \geq \chi^2_{0.05,1} = 3.841$
3. $\chi^2 = 0.50 + 0.65 + 0.00 + 0.31 = 1.46$
4. Since $1.46 < 3.841$, null hypothesis cannot be rejected.

13.84 $H_0: \mu = 300$; $H_1: \mu < 300$. Using MINITAB:

```
MTB> Ttest 300 C1;
SUBC> Alternative -1.
```

we get

N	MEAN	ST DEV	SEMEAN	T	P VALUE
38	284.553	104.220	16.907	-0.91	0.18

With a *P*-value of 0.18, the mean failure time is not significantly less than 300 hours at the 0.01 level of significance.

13.85 $H_0: \mu_1 = \mu_2$; $H_1: \mu_1 \neq \mu_2$ Using MINITAB:

```
MTB> TwosampleT for C1 vs C2
```

we get

	N	MEAN	ST DEV	SEMEAN
C1	20	57.76	3.66	0.82
C2	20	52.75	5.01	1.1

TTEST MUC1=MUC2: T = 3.61 P = 0.0009 DF = 38

With a *P*-value of 0.0009, we conclude that the difference between the mean drying times is significant at the 0.05 level of significance.

13.86 Using MINITAB, we enter the three columns in this table into C1, C2, and C3, respectively.

```
MTB> Chisquare C1 C2 C3
```

Expected counts are printed below observed counts.

	C1	C2	C3	Total
1	36	22	18	76
	35.32	23.91	16.77	
2.	63	45	29	137
	63.68	43.09	30.23	
Total	99	67	47	213

Chisq = 0.013 + 0.152 + 0.090 + 0.007 + 0.084 + 0.050 = 0.397

From Table V with df = 2, $\chi^2_{0.05,2} = 5.991$, and we cannot reject the null hypothesis that the three materials have the same probability of leaking at the 0.05 level of significance.