Chapter 3

3.1 **(a)** No, because $f(4)$ is negative; **(b)** Yes; **(c)** No, because $f(1) + f(2) + f(3) + f(4) = \frac{18}{19}$ is less than 1.

3.2 **(a)** No, because $f(1)$ is negative; **(b)** Yes; **(c)** No, because $f(0) + f(1) + f(2) + f(3) + f(4) + f(5)$ is greater than 1.

3.3 $f(x) > 0$ for each value of x and

$$\sum_{x=1}^{k} f(x) = \frac{2}{k(k+1)}(1+2+\ldots+k) = \frac{2}{k(k+1)} \cdot \frac{k(k+1)}{2} = 1$$

3.4 **(a)** $c(1+2+3+\ldots 5) = 1$; thus $C = \frac{1}{15}$

(b) $c\left(5+\frac{5}{2}+\frac{5}{3}+\frac{5}{4}+1\right) = 1$**; thus,** $c = \frac{12}{137}$

(c) $$\sum_{x=1}^{k} f(x) = c\sum_{x=1}^{k} x^2 = cS(k,2)$$

From Theorem A.1 we obtain $S(k,2) = \frac{1}{6}k(k+1)(2k+1)$

Thus, for $f(x)$ to be a distribution function, $c = \frac{6}{k(k+1)(2k+1)}$, $k \neq 0$.

(d) $$\sum_{x=1}^{\infty} f(x) = c\sum_{x=1}^{\infty}\left(\frac{1}{4}\right)^x$$

The right-hand sum is a geometric progression with $a = 1$ and $r = 1/4$.
For $x = 1$ to n, this sum equals

$$S_n = \frac{1-\left(\frac{1}{4}\right)^n}{1-\frac{1}{4}} \to \frac{1/4}{3/4} = \frac{1}{3} \quad \text{as } n \to \infty.$$ Therefore, $c = 3$.

3.5 For $f(x) = (1-k)k^x$ to converge to 1, $0 < k < 1$.

3.6 For $c > 0$, $f(x)$ diverges. For $c = 0$, $f(x) = 0$ for all x, and it cannot be a density function

3.9 **(a)** No, because $F(4) > 1$; **(b)** No, because $F(2) < F(1)$; **(c)** Yes.

3.10 $f(0) = \dfrac{4}{20} = \dfrac{1}{5}$; $\quad f(1) = \dfrac{2 \cdot 6}{20} = \dfrac{12}{20} = \dfrac{3}{5}$, $\quad F(2) = \dfrac{4}{20} = \dfrac{1}{5}$

$$F(x) = \begin{cases} 0 & x < 0 \\ 1/5 & 0 \le x < 1 \\ 4/5 & 1 \le x < 2 \\ 1 & 2 \le x \end{cases}$$

3.11 **(a)** $\dfrac{5}{6} - \dfrac{1}{3} = \dfrac{1}{2}$; **(b)** $\dfrac{1}{2} - \dfrac{1}{3} = \dfrac{1}{6}$; **(c)** $f(1) = \dfrac{1}{3}$, $f(4) = \dfrac{1}{6}$, $f(6) = \dfrac{1}{3}$ and $f(10) = \dfrac{1}{6}$,
0 elsewhere.

3.12

$$F(x) = \begin{cases} 0 & x < 1 \\ 1/15 & 1 \le x < 2 \\ 3/15 & 2 \le x < 3 \\ 6/15 & 3 \le x < 4 \\ 10/15 & 4 \le x < 5 \\ 1 & 5 \le x \end{cases}$$

3.13 **(a)** $\dfrac{3}{4}$ **(b)** $\dfrac{3}{4} - \dfrac{1}{2} = \dfrac{1}{4}$ **(c)** $\dfrac{1}{2}$ **(d)** $1 - \dfrac{1}{4} = \dfrac{3}{4}$

(e) $\dfrac{3}{4} - \dfrac{1}{4} = \dfrac{1}{2}$ **(f)** $1 - \dfrac{3}{4} = \dfrac{1}{4}$

3.14 $f(1) = \dfrac{3}{25}$, $f(2) = \dfrac{4}{25}$, $f(3) = \dfrac{5}{25}$, $f(4) = \dfrac{6}{25}$, $f(5) = \dfrac{7}{25}$

$$F(x) = \begin{cases} 0 & x < 1 \\ 3/25 & 1 \le x < 2 \\ 7/25 & 2 \le x < 3 \\ 12/25 & 3 \le x < 4 \\ 18/25 & 4 \le x < 5 \\ 1 & 5 \le x \end{cases}$$

$F(1) = \dfrac{6}{50} = \dfrac{3}{25}$, $F(2) = \dfrac{14}{50} = \dfrac{7}{25}$, $F(3) = \dfrac{24}{50} = \dfrac{12}{25}$, $F(4) = \dfrac{36}{50} = \dfrac{18}{25}$, $F(5) = \dfrac{50}{50} = 1$, checks

3.15 **(a)** $P(x > x_1) = 1 - P(x \le x_1) = 1 - F(x_1)$ for $i = 1, 2, \ldots, n$

(b) $P(x > x_1) = 1 - P(x < x_i) = 1 - F(x_{i-1})$ for $i = 2, \ldots, n$ and

$P(x \ge x_1) = 1$

3.16 $F(x) = \begin{cases} 0 & x \le 2 \\ \frac{1}{5}(x-2) & 2 < x < 7 \\ 1 & 7 \le x \end{cases}$

3.17 **(a)** $\int_{-\infty}^{\infty} f(x)dx = \int_{2}^{7} \frac{1}{5} dx \frac{1}{5} \cdot x \Big|_{2}^{7} = \frac{1}{5}(7-2) = 1$

(b) $\int_{3}^{5} \frac{1}{5} dx = \frac{1}{5}(5-3) = \frac{2}{5}$

3.18 **(a)** $f(x) \ge 0, \quad 0 < x < \infty,$ and $\int_{0}^{\infty} f(x)dx = \int_{0}^{\infty} e^{-x} dx = e^{0} = 1$

(c) $P(x > 1) = \int_{1}^{\infty} e^{-x} dx = e^{-1}$

3.19 **(a)** $f(x) \ge 0, \quad 0 < x < 1$ and $\int_{0}^{1} f(x)dx = 1$

(c) $P(0.1 < x < 0.5) = \int_{0.1}^{0.5} 3x^2 dx = 0.124$

3.20 **(a)** $\int_{2}^{3.2} \frac{1}{8}(y+1)dy = \frac{1}{8}\left(\frac{y^z}{2} + y\right)\Big|_{2}^{3.2} = \frac{1}{8}(8.32-4) = 0.54$

(b) $\int_{2.9}^{3.2} \frac{1}{8}(y+1)dy = \frac{1}{8}\left(\frac{y^x}{2} + y\right)\Big|_{2.9}^{3.2} = \frac{1}{8}(8.32-7.105) = 0.1519$

3.21 $\int_{2}^{y} \frac{1}{8}(t+1)dt = \frac{1}{8}\left(\frac{t^2}{2} + y\right)\Big|_{2}^{y} = \frac{1}{8}\left(\frac{y^2}{2} + y\right) - \frac{1}{8} \cdot 4 = \frac{1}{8}\left(\frac{y^2}{2} + y - 4\right)$

$$F(y) = \begin{cases} 0 & y \le 2 \\ \frac{1}{8}\left(\frac{y^2}{2} + y - 4\right) & 2 < y < 4 \\ 1 & 4 \le y \end{cases}$$

(a) $F(3.2) = \frac{1}{8}\left(\frac{3.2^2}{2} + 3.2 - 4\right) = 0.54$

(b) $F(3.2) = F(2.9) = 0.54 - \frac{1}{8}\left(\frac{2.9^2}{2} + 2.9 - 4\right) = 0.54 - 0.3881 = 0.1519$

3.22 **(a)** $1 = \int_0^4 \frac{c}{\sqrt{x}} dx = c\int_0^4 x^{-1/2} dx = c\left.\frac{x^{1/2}}{1/2}\right|_0^4 = 4c \qquad c = \frac{1}{4}$

(b) $P\left(x < \frac{1}{4}\right) = \int_0^{1/4} \frac{1}{4\sqrt{x}} dx = \frac{1}{4}\int x^{-1/2} dx = \frac{1}{4}\left.\frac{\sqrt{x}}{1/2}\right|_0^{1/4} = \frac{1}{2}\cdot\frac{1}{2} = \frac{1}{4}$

$P(x > 1) = 1 - \int_0^1 \frac{1}{4\sqrt{x}} dx = 1 - \frac{1}{2}\left.\sqrt{x}\right|_0^1 = \frac{1}{2}$

3.23 $F(x) = \frac{1}{2}\sqrt{x}$

$$F(x) = \begin{cases} 0 & x \le 0 \\ \frac{1}{2}\sqrt{x} & 0 < x < 4 \\ 1 & 4 \le x \end{cases}$$

$F\left(\frac{1}{4}\right) = \frac{1}{2}\cdot\frac{1}{2} = \frac{1}{4}$ and $1 = F(1) = 1 - \frac{1}{2} = \frac{1}{2}$

3.24 $F(z) = k\int_0^z ze^{-z^2} dz = k\int_0^z \frac{1}{2}e^{-u} du = \frac{k}{2}[1 - e^{-u}] = \frac{k}{2}(1 - e^{-z^2}) \qquad k = 2$

3.25 $F(z) = \begin{cases} 0 & z \le 0 \\ 1 - e^{-z^2} & z > 0 \end{cases}$

3.26 $P\left(x < \frac{1}{4}\right) = (3x^2 - 2x^3)\Big|_0^{1/4} = \frac{3}{16} - \frac{1}{32} = \frac{5}{32}$

$P\left(x > \frac{1}{2}\right) = \int_{1/2}^1 6x(1-x)dx = (3x^2 - 2x^3)\Big|_{1/2}^1 = 1 - \left(\frac{3}{4} - \frac{1}{4}\right) = \frac{1}{2}$

3.27 $F(x) = \int_0^x 6x(1-x)dx = 3x^2 - 2x^3 \qquad F(x) = \begin{cases} 0 & x \le 0 \\ 3x^2 - 2x^3 & 0 < x < 1 \\ 1 & 1 \le x \end{cases}$

$P\left(x < \frac{1}{4}\right) = \frac{3}{16} - \frac{2}{64} = \frac{5}{32}$ and $P\left(x > \frac{1}{2}\right) = 1 - \left(\frac{3}{4} - \frac{2}{8}\right) = \frac{1}{2}$

3.28 $F(x) = \int_0^x x\,dx = \frac{x^2}{2} \quad 0 \text{ to } 1$

$$F(x) = \frac{1}{2} + \int_1^x (2-x)dx = \frac{1}{2} + \left(2x - \frac{x^2}{2}\right)\Bigg|_1^x = \frac{1}{2} + 2x - \frac{x^2}{2} - \frac{3}{2}$$

$$= 2x - \frac{x^2}{2} - 1$$

$$F(x) = \begin{cases} 0 & x \le 0 \\ \frac{x^2}{2} & 0 < x < 1 \\ 2x - \frac{x^2}{2} - 1 & 1 \le x < 2 \\ 1 & 2 \le x \end{cases}$$

3.29 $F(x) = \int_0^x \frac{1}{3}dx = \frac{1}{3}x \qquad 0 \text{ to } 1 \quad F(x) = \frac{1}{3} \qquad 1 \text{ to } 2$

$$F(x) = \frac{1}{3}(x-2) \quad 2 \text{ to } 4 \qquad F(x) = \begin{cases} 0 & x \le 0 \\ \frac{1}{3}x & 0 < x < 1 \\ \frac{1}{3} & 1 \le x \le 2 \\ \frac{1}{3}(x-1) & 2 < x < 4 \\ 1 & 4 \le x \end{cases}$$

$$= \frac{1}{3}(x-1$$

3.30 **(a)** $$\int_{0.8}^{1} x\,dx + \int_1^{1.2}(2-x)dx = \frac{x^2}{2}\Bigg|_{0.8}^{1} + \left(2x - \frac{x^2}{2}\right)\Bigg|_1^{1.2} = \left(\frac{1}{2} - 0.32)\right) + \left(2.4 - 0.72 - 2 + \frac{1}{2}\right) = 0.36$$

(b) $$F(1.2) - F(0.8) = 2(1.2) - \frac{(1.2)^2}{2} - 1 - \left(\frac{(0.8)^2}{2}\right)$$

$$= 2.4 - 0.72 - 1 - 0.32 = 0.36$$

3.31

$x \le 0$	$F(x) = 0$	
$0 < x \le 1$	$F(x) = \frac{x^2}{4}$	$F(1) = \frac{1}{4}$
$1 < x \le 2$	$F(x) = \frac{1}{2}x - \frac{1}{4}$	$F(2) = \frac{3}{4}$
$2 < x < 3$	$F(x) = \frac{3}{2}x - \frac{x^2}{4} - \frac{5}{4}$	$F(3) = 1$
$3 \le x$	$F(x) = 1$	

3.32 **(a)** $F\left(\frac{1}{2}\right)-F\left(-\frac{1}{2}\right)=\frac{3}{4}-\frac{1}{4}=\frac{1}{2}$; $F(3)-F(2)=1-1=0$

3.33 $\frac{\text{dF}}{\text{dx}}=\frac{1}{2}$, $f(x)=\frac{1}{2}$ for $-1<x<1$; 0 elsewhere

$P\left(-\frac{1}{2}<x<\frac{1}{2}\right)=\frac{1}{2}\cdot 1=\frac{1}{2}$; $P(2<x<3)=0$

3.34 **(a)** $F(5)=1-\frac{9}{25}=\frac{16}{25}$

(b) $1-F(8)=1-1+\frac{9}{64}=\frac{9}{64}$

3.35 $\frac{\text{dF}}{dy}=\frac{18}{y^2}$ for $y>0$; elsewhere

(a) $\int_3^5 \frac{18}{y^2}dy=-\frac{9}{y^2}\Big|_3^5=-\frac{9}{25}+1=\frac{16}{25}$; **(b)** $\int_8^\infty \frac{18}{y^2}dy=-\frac{9}{y^2}\Big|_8^\infty=0+\frac{9}{64}=\frac{9}{64}$

3.37 $P(x\le 2)=F(2)=1-3e^{-2}=1-3(0.1353)=1-0.4074=0.5926$

$P(1<x<3)=F(3)-F(1)=1-4e^{-2}-1+2e^{-1}-4e^{-2}$
$=2(0.3679)-4(0.0498)=0.7358-0.1992=0.5366$

$P(x>4)=1-F(4)=5e^{4}=5(0.0183)=0.0915$

3.38 $\frac{dF}{dx}=xe^{-x}$ for >0; 0 elsewhere

3.39 **(a)** for $x\le 0$ $F(x)=0$

(b) for $0<x<0.5$ $F(x)=\frac{1}{2}x$

(c) for $0.5\le x<1$ $F(x)=\frac{1}{2}\left(x-\frac{1}{2}\right)+\frac{3}{4}=\frac{1}{2}(x+1)$

(d) for $x\ge 1$ $f(x)=0$

3.40 **(a)** $f(x)=0$; **(b)** $f(x)=\frac{1}{2}$; **(c)** $f(x)=\frac{1}{2}$; **(d)** $f(x)=0$

3.41 $P(Z=-2)=\frac{-2+4}{8}=\frac{1}{4}$, $P(Z=2)=\frac{1}{4}$, $P(-2<Z<1)=\frac{5}{8}-\frac{1}{4}=\frac{3}{8}$

and $P(0\le z\le 2)=1-\frac{1}{2}=\frac{1}{2}$

3.42 **(a)** $\frac{1}{20}$; **(b)** $\frac{1}{4}+\frac{1}{8}=\frac{3}{8}$; **(c)** $\frac{1}{6}+\frac{1}{4}+\frac{1}{12}=\frac{1}{2}$; **(d)** $\frac{1}{6}+\frac{1}{24}+\frac{1}{40}=\frac{28}{120}=\frac{7}{30}$

3.43 **(a)** $\frac{1}{6}+\frac{1}{12}=\frac{1}{4}$; **(b)** 0; **(c)** $\frac{1}{12}+\frac{1}{6}+\frac{1}{24}=\frac{7}{24}$; **(d)** $1-\frac{1}{120}=\frac{119}{120}$

3.44 $c(2+5+10+1+4+9+2+5+10+10+13+18)=1$

$c=\frac{1}{89}$

3.45 **(a)** $\frac{1}{89}(10+9+10)=\frac{29}{89}$; **(b)** $\frac{1}{89}(1+4)=\frac{5}{89}$

(c) $\frac{1}{89}(9+5+10+13+18)=\frac{55}{89}$

3.46 **(a)** $k(0+2+8+0-1+2)=1$

$f(3, 1)$ differs in sign from all other terms

3.47

density

y \ x	0	1	2	3
0	0	$\frac{1}{30}$	$\frac{1}{15}$	$\frac{1}{10}$
1	$\frac{1}{30}$	$\frac{1}{15}$	$\frac{1}{10}$	$\frac{2}{15}$
2	$\frac{1}{15}$	$\frac{1}{10}$	$\frac{2}{15}$	$\frac{1}{6}$

joint distribution function

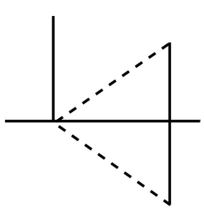
x \ y	0	1	2	3
0	0	$\frac{1}{30}$	$\frac{1}{10}$	$\frac{1}{5}$
1	$\frac{1}{30}$	$\frac{2}{15}$	$\frac{3}{10}$	$\frac{8}{15}$
2	$\frac{1}{10}$	$\frac{3}{10}$	$\frac{3}{5}$	1

3.48 **(a)** $P(x\le -\infty, y\le -\infty)=0$

(b) $P(x\le \infty, y\le \infty)=1$

(c) $F(b,c)=F(a,c)+$ three probabilities

$F(b,c)\ge F(a,c)$

3.49 

$$k\int_0^1\int_{-x}^{x} x(x-y)dy\ dx = k\int_0^1\left(x^2y-\frac{xy^2}{2}\right)\Big|_{-x}^{x} dx$$

$$k\int_0^1\left(x^3-\frac{x^3}{2}+x^3+\frac{x^3}{2}\right)dx = k\int_0^1 2x^3\ dx=\frac{k}{2}=1$$

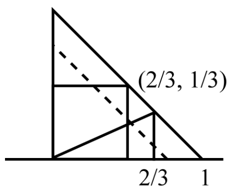
$k=2$

3.50 $24\int_0^{1/2}\int_0^{1/2-x} xy\,dy\,dx = 24\int_0^{1/2}\frac{xy^2}{2}\Bigg|_0^{1/2-x} dx = 12\int_0^{1/2} x\left(\frac{1}{2}-x\right)^2 dx$

$= 12\int_0^{1/2}\left(\frac{x}{4}-x^2+x^3\right)dx = 12\left[\frac{x^2}{8}-\frac{x^2}{3}+\frac{x^4}{4}\right]\Bigg|_0^{1/2} = 12\left[\frac{1}{32}-\frac{1}{24}+\frac{1}{64}\right]$

$= \frac{12}{64\cdot 3}(6-8+3) = \frac{12}{3\cdot 64} = \frac{1}{16}$

3.51



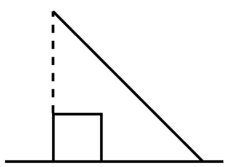
(a) $\frac{1}{2}$

(b) $1-2\cdot\frac{1}{2}\cdot\frac{2}{3}\cdot\frac{2}{3} = 1-\frac{4}{9} = \frac{5}{9}$

(c) $2\left(\frac{1}{2}\cdot\frac{2}{3}\cdot\frac{1}{3}+\frac{1}{2}\cdot\frac{1}{3}\cdot\frac{1}{3}\right) = \frac{3}{9} = \frac{1}{3}$

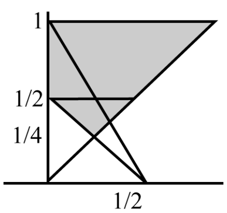
$F(x,y) = 2xy$ for $x>0,\ y>0,\ x+y<1$

3.52



(a) $2\cdot\frac{1}{2}\cdot\frac{1}{2} = \frac{1}{2}$

3.53



$\int_{1/4}^{1/2}\frac{1}{y}\int_{1/2-y}^{y} dx\,dy + \int_{1/2}^{1}\frac{1}{y}\int_0^y dx\,dy$

$= 1-\frac{1}{2}\ln\ 2$=1-0.3466=0.6534

3.54 $\frac{\partial F}{\partial y\,\partial x} = 2xe^{=x^2}\cdot 2ye^{-y^2} = 4xy^{-x^2}e^{-y^2} = 4xye^{-(x^2+y^2)}$ $\quad x>0,\ y>0$

and $f(x,y)=0$ elsewhere

3.55 $\int_1^2 2xe^{-x^2}dx\int_1^2 2ye^{-y^2}dy = \left[\int_1^4 e^{-u}du\right]^2 = \left(-e^{-u}\Big|_1^4\right)^2 = (e^{-1}-e^{-4})^2$

3.56 $\frac{\partial F}{\partial x} = e^{-x}-e^{-x-y}\ \frac{\partial^2 F}{\partial x\partial y} = e^{-x-y}$ $\quad x>0,\ y>0$

= 0 elsewhere

3.57 $\int_2^3 e^{-X}dx\int_2^3 e^{-y}dy = \left[-e^{-x}\Big|_2^3\right]^2 = (e^{-2}=e^{-3})^2$

3.58 $F(b,d)-F(a,d)-F(b,c)+F(a,c)$

3.59 $a = 1, b = 3, c = 1, d = 2$

$$F(3,2) - F(1,2) - F(3,1) + F(1,1)$$
$$= (1-e^{-3})(1-e^{-2}) - (1-e^{-1})(1-e^{-2}) - (1-e^{-3})(1-e^{-1}) + (1-e^{-1})(1-e^{-1})$$
$$= (1-e^{-2})\left[(1-e^{-3}) - (1-e^{-1})\right] - (1-e^{-1})\left[(1-e^{-2}) - (1-e^{-1})\right]$$
$$= \left[(1-e^{-2})(1-e^{-1})\right]\left[(1-e^{-3}) - (1-e^{-1})\right]$$
$$= (e^{-1} - e^{-2})(e^{-1} - e^{-3}) = 0.074$$

3.60 $F(2,2) - F(1,2) - F(2,1) + F(1,1)$

$$= (1-e^{-4})(1-e^{-4}) - (1-e^{-1})(1-e^{-4}) - (1-e^{-1})(1-e^{-4}) + (1-e^{-1})(1-e^{-1})$$
$$= (1-e^{-4})\left[(1-e^{-4}) - (1-e^{-1})\right] - (1-e^{-1})\left[(1-e^{-4}) - (1-e^{-1})\right]$$
$$= (1-e^{-4})(e^{-1} - e^{-4}) - (1-e^{-1})(e^{-1} - e^{-4})$$
$$= (e^{-1} - e^{-4})(e^{-1} - e^{-4}) = (e^{-1} - e^{-4})^2$$

3.61 $F(3,3) - F(2,3) - F(3,2) + F(2,2)$

$$= (1 - e^{-3} - e^{-3} + e^{-6})$$
$$-(1 - e^{-2} - e^{-3} + e^{-5}) - (1 - e^{-2} - e^{-2} + e^{-5} + (1 - e^{-2} - e^{-2} + e^{-4})$$
$$= e^{-4} - 2e^{-5} + e^{-6} = (e^{-2} - e^{-3})^2 \qquad \text{QED}$$

3.62 $x = 1, 2$
$y = 1, 2, 3$
$z = 1, 2$

$$(1+2+2+4+3+6+2+4+4+8+6+12)k = 1$$
$$k = \frac{1}{54}$$

3.63 **(a)** $\frac{1}{54}(1+2) = \frac{1}{18}$

(b) $\frac{1}{54}(8+6) = \frac{14}{54} = \frac{7}{27}$

3.64 **(a)** $\frac{1}{54}(1+2+2+4) = \frac{9}{54} = \frac{1}{6}$ **;** (b) 0; **(c)** 1

3.65 $$\int_0^1 \int_0^{1-z} \int_0^{1-y-z} xy(1-z)\,dx\,dy\,dz$$

$$\int_0^1 \int_0^{1-z} \frac{1}{2}(1-y-z)^2 y(1-z)\,dy\,dz$$

$$k\int_0^1 \int_0^{1-z} \int_0^{1-y-z} xy(1-z)\,dx\,dy\,dz = 1 \quad k = 144$$

3.66 $\int_0^{1/2}\int_0^{1/2-x}\int_0^{1-x-y} 144\, xy(1-z)\, dz\, dy\, dx = 0.15625$

3.68 **(a)** $\frac{1}{3}\int_0^{1/2}\int_0^{1/2}\int_0^{1/2}(2x+3y+z)\, dz\, dy\, dx$

$$= \frac{1}{3}\int_0^{1/2}\int_0^{1/2}\left[(2x+3y)z+\frac{z^2}{2}\right]_0^{1/2} dy\, dx$$

$$= \frac{1}{3}\int_0^{1/2}\int_0^{1/2}\left(x+\frac{3}{2}y+\frac{1}{8}\right)dy\, dx$$

$$= \frac{1}{3}\int_0^{1/2}\left(xy+\frac{3}{4}y^2+\frac{1}{8}y\right)\Bigg|_0^{1/2} dx = \frac{1}{3}\int_0^{1/2}\left(\frac{1}{2}x+\frac{3}{16}+\frac{1}{16}\right)dx$$

$$= \frac{1}{3}\left(\frac{1}{16}+\frac{3}{32}+\frac{1}{32}\right) = \frac{1}{3}\cdot\frac{6}{32} = \frac{1}{16}$$

3.69 **(a)** $g(-1) = \frac{1}{4},\ g(1) = \frac{3}{4}$

(b) $h(-1) = \frac{5}{8},\ h(0) = \frac{1}{4},\quad h(1) = \frac{1}{8}$

(c) $f(-1|-1) = \frac{1/8}{1/8+1/2} = \frac{1}{5};\quad f(1|-1) = \frac{1/2}{1/8+1/2} = \frac{4}{5}$

3.70 **(a)** $g(0) = \frac{1}{12}+\frac{1}{4}+\frac{1}{8} = \frac{1}{120} = \frac{7}{15};\quad g(1) = \frac{1}{6}+\frac{1}{4}+\frac{1}{20} = \frac{7}{15}$

$g(2) = \frac{1}{24}+\frac{1}{40} = \frac{1}{15}$

(b) $h(0) = \frac{1}{12}+\frac{1}{6}+\frac{1}{24} = \frac{7}{24};\quad h(1) = \frac{1}{4}+\frac{1}{4}+\frac{1}{40} = \frac{21}{40}$

$h(2) = \frac{1}{8}+\frac{1}{20} = \frac{7}{40};\quad h(3) = \frac{1}{120}$

(c) $f(0|1) = \frac{1/4}{21/40} = \frac{10}{21};\quad f(1|1) = \frac{10}{21};\quad f(2|1) = \frac{1/40}{21/20} = \frac{1}{21}$

(d) $w(0|0) = \frac{1/12}{56/120} = \frac{5}{28};\quad w(1|0) = \frac{1/4}{56/120} = \frac{15}{28};\quad w(2|0) = \frac{1/8}{56/120} = \frac{15}{56}$

$w(3|0) = \frac{1/120}{56/120} = \frac{1}{56}$

3.71 **(a)** $m(x,y)=\frac{xy}{108}(1+2)=\frac{xy}{36}$ for $x=1,2,3$; $y=1,2,3$

(b) $n(x,z)=\frac{xz}{108}(1+2+3)=\frac{xz}{18}$ for $x=1,2,3$; $z=1,2$

(c) $g(x)=\frac{x}{36}(1+2+3)=\frac{x}{6}$ for $x=1,2,3$

(d) $\phi(z|1,2)=\frac{z/64}{2/36}=\frac{z}{3}$ for $z=1,2$

(e) $\psi(y,z|3)=\frac{yz/36}{1/2}=\frac{yz}{18}$ for $y=1,2,3$; $z=1,2$

3.72 **(a)** $g(0)=\frac{5}{12}$, $g(1)=\frac{1}{2}$; $g(2)=\frac{1}{12}$

$$G(x)=\begin{cases}0 & x<0\\ 5/12 & 0\le x<1\\ 11/12 & 1\le x<2\\ 1 & 2\le x\end{cases}$$

(b) $f(0|1)=\frac{2/9}{7/18}=\frac{4}{7}$

$f(1|1)=\frac{1/6}{7/18}=\frac{3}{7}$

$$F(x|1)=\begin{cases}0 & x<0\\ 4/7 & 0\le x<1\\ 1 & 1\le x\end{cases}$$

3.73 **(a)** $f(x)=\frac{1}{2}$ for $x=-1,\ 1$; $g(y)=\frac{1}{2}$ for $y=-1,\ 1$; $\frac{1}{2}\cdot\frac{1}{2}=\frac{1}{4}$, independent

(b) $f(0)=\frac{2}{3}$, $f(1)=\frac{1}{3}$, $g(0)=\frac{1}{3}$, $g(1)=\frac{2}{3}$

$f(0,0)=\frac{1}{3}\neq\frac{2}{3}\cdot\frac{1}{3}=\frac{2}{9}$ *not* independent

3.74 **(a)** $\frac{1}{4}\int_0^2(2x+y)\,dy=\frac{1}{4}\left[2xy+\frac{y^2}{2}\right]\Big|_0^2=\frac{1}{4}(4x+2)=\frac{1}{2}(2x+1)$ for $0<x<1$

= 0 elsewhere

(b) $f\left(y\middle|\frac{1}{4}\right)=\frac{\frac{1}{4}\left(\frac{1}{2}+y\right)}{\frac{1}{2}\cdot\frac{3}{2}}=\frac{1}{6}(2y+1)$ for $0<y<2$

= 0 elsewhere

3.75 **(a)** $\frac{1}{4}\int_0^1 (2x+y)\,dx = \frac{1}{4}(x^2+xy)\Big|_0^1 = \frac{1}{4}(1+y)$ for $0<y<2$

$= 0$ elsewhere

(b) $f(x|1) = \dfrac{\frac{1}{4}(2x+1)}{\frac{1}{4}(2)} = \frac{1}{2}(2x+1)$ for $0<x<1$

$= 0$ elsewhere

3.76 **(a)** $f(x) = 24\int_0^{1-x}(y-xy-y^2)dy = 24\left[\frac{y}{2}-\frac{xy^2}{2}-\frac{y^3}{3}\right]\Big|_0^{1-x}$

$= 12(1-x)^2 - 12x(1-x)^2 - 8(1-x)^3$

$= 12(1-x)^3 - 8(1-x)^3 = 4(1-x)^3$

$$f(x) = \begin{cases} 4(1-x)^3 & 0<x<1 \\ 0 & \text{elsewhere} \end{cases}$$

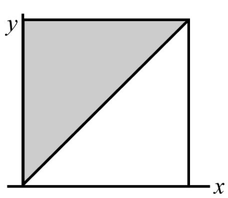
(b) $g(y) = 24\int_0^{1-y}(y-xy-y^2)dy = 24\left[y(1-y)-\frac{1}{2}y(1-y)^2-y^2(1-y)\right]$

$= 24(1-y)\left[1-\frac{1}{2}(1-y)-y\right] = 24y\left(\frac{1}{2}-\frac{y}{2}\right)(1-y)$

$$= \begin{cases} 12y(1-y)^2 & 0<y<1 \\ 0 & \text{elsewhere} \end{cases}$$

$f(x,y) \neq f(x)\cdot g(y)$ not independent

3.77



(a) $g(x) = \int_x^1 \frac{1}{y}\,dy = \ln y\Big|_x^1 = \ln 1 - \ln x = \begin{cases} -\ln x & 0<x<1 \\ 0 & \text{elsewhere} \end{cases}$

(b) $h(y) = \int_0^y \frac{1}{y}\,dx = \frac{1}{y}(y-0) = \begin{cases} 1 & 0<x<1 \\ 0 & \text{elsewhere} \end{cases}$

$\frac{1}{y} \neq 1\cdot(-\ln x)$ *not* independent

3.78 **(a)** $$f(x_2|x_1,x_3)=\frac{(x_1+x_2)e^{-x_3}}{\left(x_2+\frac{1}{2}\right)e^{x_2}}=\frac{x_1+x_2}{x_1+\frac{1}{2}}$$

$$f\left(x_2\middle|\frac{1}{3},2\right)=\frac{\frac{1}{3}+x_2}{\frac{1}{3}+\frac{1}{2}}=\begin{cases}\dfrac{2+6x^2}{5} & 0<x_2<1\\ 0 & \text{elsewhere}\end{cases}$$

(b) $$g(x_2,x_3|x_1)=\frac{(x_1+x_2)e^{-x_3}}{x_2+\quad 2}=\begin{cases}\left(\frac{1}{2}+x_2\right)e^{-x_3} & 0<x_2<1, x_3>0\\ 0 & \text{elsewhere}\end{cases}$$

3.79 $$g(x)=\int_{-\infty}^{\infty} f(x,y)\,dy \qquad G(x)=\int_{0}^{x}\int_{-\infty}^{\infty} f(x,y)\,dy=F(x,\infty)$$

$$G(x)=F(x,\infty)=\begin{cases}1-e^{-x^2} & x>0\\ 0 & \text{elsewhere}\end{cases}$$

3.80 $$M(x_1,x_3)=\int_{-\infty}^{\infty} f(x_1,x_2,x_3)\,dx_2=F(x_1,\infty,x_3)$$

$$G(x_1)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x_1,x_2,x_3)\,dx_2\;dx_3=F(x_1,\infty,\infty)$$

(a) $$M(x_2,x_3)=\begin{cases}0 & x_1\le 0 \text{ or } x_3\le 0\\ \frac{1}{2}x_1(x_1+1)(-1-e^{-x_3}) & 0<x_1<1, x_3>0\\ 1-e^{-x_3} & \mathrm{x}_1\ge 1, x_3>0\end{cases}$$

(b) $$G(x_1)=\begin{cases}0 & x_1\le 0\\ \frac{1}{2}x_1(x_1+1) & 0<x_1<1\\ 1 & 1\le x_1\end{cases}$$

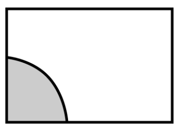
3.81 $g(x_1) = \begin{cases} x_1 + \quad 2 & 0 < x_1 < 1 \\ 0 & \text{elsewhere} \end{cases}$ $\qquad h(x_2) = \begin{cases} x_2 + \quad 2 & 0 < x_2 < 1 \\ 0 & \text{elsewhere} \end{cases}$

$\phi(x_3 \;= \begin{cases} e^{-x_3} & x_3 > 1 \\ 0 & \text{elsewhere} \end{cases}$

$f(x_{1,} x_2, x_3) \neq g(x_1) \cdot h(x_2) \cdot \phi(x_3)$ *not* independent

$m(x_1, x_3) = g(x_1)\phi(x_3)$ independent

$n(x_2, x_3) = h(x_2)\phi(x_3)$ independent

3.82 

(a) $g(x, y) = \begin{cases} \frac{1}{6} & 0 < x < 2,\ 0 < y < 3 \\ 0 & \text{elsewhere} \end{cases}$

(b) $1 - \frac{\pi / 4}{6} = 1 - \frac{\pi}{24}$

(a) $g(0) = \frac{5}{14}, \quad g(1) = \frac{5}{28}, \quad g(2) = \frac{3}{28}$

(b) $\phi(0|0) = \frac{3.28}{10/28} = \frac{3}{10}, \quad \phi(1|0) = \frac{6/28}{10/28} = \frac{6}{10}, \quad \phi(2|0) = \frac{1/28}{10/28} = \frac{1}{10}$

3.83

Heads	Tails	Probability	H-T
0	4	1/16	–4
1	3	4/16	–2
2	2	6/16	0
3	1	4/16	2
4	0	1/16	4

3.84

1	2	3
1	3	4
1	4	5
2	3	5
2	4	6
3	4	7

(a)

x	3	4	5	6	7
$f(x)$	1/6	1/6	2/6	1/6	1/6

$$F(x) = \begin{cases} 0 & x < 3 \\ 1/6 & 3 \le x < 4 \\ 2/6 & 4 \le x < 5 \\ 4/6 & 5 \le x < 6 \\ 5/6 & 6 \le x < 7 \\ 1 & 7 \le x \end{cases}$$

3.85 $P(H) = \dfrac{2}{3}$

(a) $P(0) = \dfrac{1}{27}$, $P(1) = \dfrac{6}{27}$, $P(2) = 3, \dfrac{1}{3}, \dfrac{2}{3}, \dfrac{2}{3} = \dfrac{12}{27}$, $P(3) = \dfrac{8}{27}$

(b) $\dfrac{1}{27} + \dfrac{6}{27} + \dfrac{12}{27} = \dfrac{19}{27}$

3.86 $F(x) = \begin{cases} 0 & x < 0 \\ 1/27 & 0 \le x < 1 \\ 7/27 & 1 \le x < 2 \\ 19/27 & 2 \le x < 3 \\ 1 & 3 \le x \end{cases}$

(a) $1 - \dfrac{7}{27} = \dfrac{20}{27}$

(b) $1 - \dfrac{19}{27} = \dfrac{8}{27}$

3.87 $F(V) = \begin{cases} 0 & V < 0 \\ 0.40 & 0 \le V < 1 \\ 0.70 & 1 \le V < 2 \\ 0.90 & 2 \le V < 3 \\ 1 & 3 \le V \end{cases}$

3.88 **(a)** $0.20 + 0.10 = 0.30$
(b) $1 - 0.70 = 0.30$

3.89 Yes; $f(x) \ge 0$ for $x = 2, 3, \ldots 12$ and $\sum_{x=2}^{12} f(x) = 1$

3.90

S	2	3	4	5	6	7	8	9	10	11	12
	1/36	3/36	6/36	10/36	15/36	21/36	26/36	30/36	33/36	35/36	1

3.91 **(a)** $\dfrac{1}{5}(228.65 - 227.5) = 0.23$; **(b)** $\dfrac{1}{5}(231.66 - 229.34) = 0.464$;

(c) $\dfrac{1}{5}(232.5 - 229.85) = 0.53$

3.92 $F(x) = \dfrac{1}{288}\displaystyle\int (36 - x^2)dx + c = \dfrac{1}{288}\left(36x - \dfrac{x^3}{3}\right) + \dfrac{1}{2}$ so that $F(-6) = 0$ and $F(6) = 1$.

(a) $F(-2) = \dfrac{1}{288}(-72 + \dfrac{8}{3}) + \dfrac{1}{2} = \dfrac{1}{288} \cdot \dfrac{-208}{3} + \dfrac{1}{2} = \dfrac{7}{27}$

(b) $F(6) - F(1) = 1 - \dfrac{1}{288}(36 - \dfrac{1}{3}) - \dfrac{1}{2} = 1 - \dfrac{1}{288} \cdot \dfrac{107}{3} - \dfrac{1}{2} = \dfrac{757}{864} - \dfrac{1}{2} = \dfrac{325}{854}$

(c) $F(3) - F(1) = \dfrac{1}{288}(108 - 9) - \dfrac{1}{288}\left(36 - \dfrac{1}{3}\right) = \dfrac{99}{288} - \dfrac{1}{288} \cdot \dfrac{107}{3} = \dfrac{190}{288 \cdot 3} = \dfrac{95}{432}$

(d) 0

3.93 $F(x) = \int \frac{1}{30} e^{-x/30} dx + c = \frac{1}{30} \frac{e^{-x/30}}{-1/30} + c = c - e^{-x/30} = 1 - e^{-x/30}$

(a) $F(18) = 1 - e^{-18/30} = 1 - e^{-0.6} = 1 - 0.5488 = 0.4512$

(b) $F(36) - F(27) = e^{-27/30} - e^{-36/30} = e^{-0.9} - e^{-1.2} = 0.4066 - 0.3012 = 0.1054$

(c) $1 - F(48) = e^{-48/30} = e^{-1.6} = 0.2019$

3.94 $F(x) = \int \frac{20,000}{(x+100)^3} dx + c = \frac{20,000}{-2(x+100)^2} + 1 = -\frac{10,000}{(x+100)^2} + 1$

(a) $1 - F(200) = \frac{10,000}{300^2} = \frac{1}{9}$

(b) $f(100) = 1 - \frac{10,000}{40,000} = \frac{3}{4}$

3.95 **(a)** $1 - F(10) = \frac{25}{10^2} = 0.25 = \frac{1}{4}$

(b) $F(8) = 1 - \frac{25}{8^2} = \frac{39}{64}$

(c) $F(15) - F(12) = \frac{25}{12^2} - \frac{25}{15^2} = \frac{25(25-16)}{15^2 - 16} = \frac{1}{16}$

3.96 $F(x) = \frac{1}{9} \int_0^x x\ e^{-x/3} dx + c = \frac{1}{9} \frac{e^{-x/3}}{1/9} \left(-\frac{1}{3} x - 1 \right) + c = c - e^{-x/3} \left(\frac{1}{3} x + 1 \right)$

$c = 1$

(a) $F(6) = 1 - 3e^{-2} = 1 - 3e^{-2} = 1 - 3(0.1353) = 0.5491$

(b) $1 - F(9) = 4e^{-3} = 4(0.0498) = 0.1992$

3.97 $(0,0,2) = \binom{3}{0}\binom{2}{0}\binom{3}{2} = 3$ $\quad f(0,0) = \frac{3}{28}, \quad f(0,1) = \frac{6}{28}, \quad f(0,2) = \frac{1}{28}$

$(1,0,1) = \binom{3}{1}\binom{2}{0}\binom{3}{1} = 9$ $\quad f(1,0) = \frac{9}{28}, \quad f(2,0) = \frac{3}{28}, \quad f(1,1) = \frac{6}{28}$

$(0,1,1) = \binom{3}{0}\binom{2}{1}\binom{3}{1} = 6$

$(2,0,0) = \binom{3}{2}\binom{2}{0}\binom{3}{0} = 3$

$(1,1,0) = \binom{3}{1}\binom{2}{1}\binom{3}{0} = 6$

$(0,2,0) = \binom{3}{0}\binom{2}{2}\binom{3}{0} = 1$

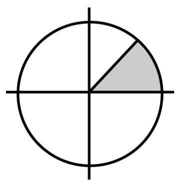
3.98 **(b)**

		$x=0$	$x=1$	$x=2$
y	0	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{36}$
	1	$\frac{1}{3}$	$\frac{1}{6}$	
	2	$\frac{1}{4}$		

3.99 $f(0,3)=\frac{1}{8},\quad f(1,2)=\frac{3}{8},\quad f(2,1)=\frac{3}{8},\quad f(3,0)=\frac{1}{8}$

$g(0,-3)=\frac{1}{8},\quad g(1,1)==\frac{3}{8},\quad g(2,1)=\frac{3}{8}, g(3,3)=\frac{1}{8}$

3.100 **(a)** Probability = 1/8



(b) $\frac{1}{\pi}\cdot\pi\cdot\frac{1}{4}=\frac{1}{4}$

3.101 **(a)**

$$\int_{0.2}^{0.3}\int_{2}^{\infty} 5pe^{-ps}ds\ dp = \int_{0.2}^{0.3} -5e^{-ps}\Big|_{2}^{\infty} dp$$

$$= \int_{0.2}^{0.3} 5e^{-2p}dp = \frac{5\cdot e^{-2p}}{-2}\Big|_{0.2}^{0.3} = \frac{5}{2}\left(e^{-0.4}-e^{-0.6}\right) = 0.3038$$

(b)

$$\int_{0.25}^{0.30}\int_{0}^{1} 5pe^{-ps}ds\ dp = \int_{0.25}^{0.30} -5e^{-ps}\Big|_{0}^{1} dp = \int_{0.25}^{0.30} 5(1-e^{-p})\ dp$$

$$= 5[p+e^{-p}]^{0.30} = 5(0.30+e^{-0.30}-0.25-e^{-0.25}] = 0.01202$$

3.102 **(a)**

$$\frac{2}{5}\int_{0}^{0.4}\int_{0}^{0.4}(2x+3y)\ dx\ dy = \frac{2}{5}\int_{0}^{0.4}(x^2+3xy)\Big|_{0}^{0.4} dy$$

$$= \frac{2}{5}\int_{0}^{0.4}\left((0.16+1.2y\right)dy$$

$$= \frac{2}{5}\left[(0.16)(0.4)+\frac{1.2(0.16)}{2}\right] = 0.064$$

(b) $$\frac{2}{5}\int_0^{0.5}\int_{0.8}^{1}(2x+3y)\,dx\,dy = \frac{2}{5}\int_0^{0.5}(x^2+3xy)\Big|_{0.8}^{1}dy$$
$$= \frac{2}{5}\int_0^{0.5}\left[(1+3y)-(0.64+2.4y)\right]dy = \frac{2}{5}\int_0^{0.5}(0.6y+0.36)\,dy$$
$$= \frac{2}{5}(0.3y^2+0.36y)\Big|_0^{0.5} = \frac{2}{5}(0.075+0.18) = 0.102$$

3.103 (a) $g(0)=\frac{5}{14}$, $g(1)=\frac{15}{28}$ and $g(2)=\frac{3}{28}$

(b) $\phi(0|0)=\frac{3}{10}$, $\phi(1|0)=\frac{6}{10}$, and $\phi(2|0)=\frac{1}{10}$

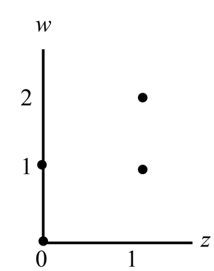
3.104 (a) $$\int_{0.3}^{1}\int_0^1\frac{2}{5}(x+4y)\,dy\,dx = \frac{2}{5}\int_{0.3}^{1}(xy+2y^2)\Big|_0^1 dx = \frac{2}{5}\int_{0.3}^{1}(x+2)\,dx$$
$$= \frac{2}{5}\left[\frac{x^2}{2}+2x\right]\Big|_{0.3}^{1}$$
$$= \frac{2}{5}\left(\frac{1}{2}+2-\frac{0.09}{2}-0.6\right) = \frac{2}{5}(1.855) = 0.742$$

(b) $$g(x)=\frac{2}{5}\int_0^1(x+4y)\,dy = \frac{2}{5}(x+2)$$
$$g(y|x)=\frac{(2/5)(x+4y)}{(2/5)(x+2)},\quad g(y|0.2)=\frac{4y+0.2}{2.2}$$
$$\frac{1}{2.2}\int_0^{0.5}(4y+0.2)\,dy = \frac{1}{2.2}(0.5+0.1) = \frac{0.6}{2.2} = 0.273$$

3.105 (a) $$f(0,0)=\frac{48}{52}\cdot\frac{47}{51}=\frac{188}{221},\quad f(0,1)=\frac{48}{52}\cdot\frac{4}{51}=\frac{16}{221}$$
$$f(1,0)=\frac{4}{52}\cdot\frac{48}{51}=\frac{16}{221},\quad f(1,1)=\frac{48}{52}\cdot\frac{4}{51}=\frac{16}{221},\quad f(1,2)=\frac{4}{52}\cdot\frac{3}{51}=\frac{1}{221}$$

(b) $g(0)=\frac{188+16}{221}=\frac{204}{221}$, $g(1)=\frac{16+1}{221}=\frac{17}{221}$

(c) $\phi(0|1)=\frac{16/221}{17/221}=\frac{16}{17}$, $\phi(1|1)=\frac{1/221}{17/221}=\frac{1}{17}$



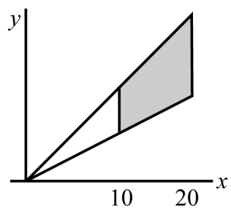
3.106 $f(p,s) = 5pe^{-ps} \quad 0.2 < p < 0.4 \quad s > 0$

(a) $$5p\int_0^{\infty} e^{-ps}\,ds = 5p\frac{e^{-ps}}{-p} = -5e^{-ps}\Big|_0^{\infty} = \begin{cases} 5 & 0.2 < p < 0.4 \\ 0 & \text{elsewhere} \end{cases}$$

(b) $$\frac{f(p,s)}{g(s)} = \frac{5pe^{-ps}}{5} = \begin{cases} pe^{-ps} & \text{for } s > 0 \\ 0 & \text{elsewhere} \end{cases}$$

(c) $$\int_0^3 \frac{1}{4}e^{-(1/4)s}ds = \left[e^{-s/4}\right]\Big|_0^3 = 1 - e^{-0.75}$$

3.107



(a) $$\frac{1}{25}\frac{20-x}{x}\int_{x/2}^{x} dy = \begin{cases} \dfrac{20-x}{50} & 10 < x < 20 \\ 0 & \text{elsewhere} \end{cases}$$

(b) $$\phi(y|x) = \frac{\frac{1}{25}\left(\frac{20-x}{x}\right)}{\frac{20-x}{50}} = \frac{2}{x}, \quad \phi(y|12) = \begin{cases} 1/6 & 6 < y < 12 \\ 0 & \text{elsewhere} \end{cases}$$

(c) $$\frac{1}{6}(12-8) = \frac{1}{6}\cdot 4 = \frac{2}{3}$$

3.108 $f(x,y) = \frac{2}{5}(2x+3y)$

$$g(x) = \frac{2}{5}\left[2xy + \frac{3y^2}{2}\right]\Bigg|_0^1 = \frac{2}{5}\left(2x + \frac{3}{2}\right)$$

$$= \begin{cases} \dfrac{4}{5}x + \dfrac{3}{5} & 0 < x < 1 \\ 0 & \text{elsewhere} \end{cases}$$

$$h(y) = \frac{2}{5}(x^2 + 3xy)\Big|_0^1$$

$$= \begin{cases} (2/5)(1+3y) & 0 < y < 1 \\ 0 & \text{elsewhere} \end{cases}$$

$f(x,y) \neq g(x)h(y)$

3.109 **(a)**

$$f(x_1, x_2, x_3) = \begin{cases} \dfrac{(20{,}000)^3}{(x_2+100)^3(x_2+100)^3(x_3+100)^3} & x_1 > 0,\ \ x_2 > 0,\ \ x_3 > 0 \\ 0 & \text{elsewhere} \end{cases}$$

(b)

$$\int_0^{100} \frac{20{,}000}{(x_1+100)^3}\,dx_1 \int_0^{100} \frac{20{,}000}{(x_2+100)^3}\,dx_2 \int_{200}^{\infty} \frac{20{,}000}{(x_3+100)^3}\,dx_3$$

$$= \frac{3}{4}\cdot\frac{3}{4}\cdot\frac{1}{9} = \frac{1}{16}$$

3.110 **(a)**

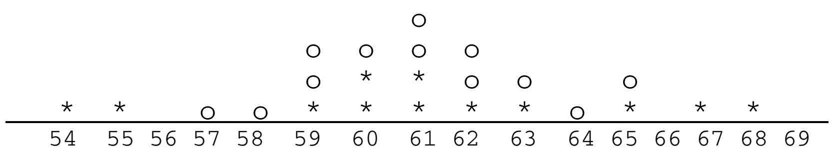
5 | 9 4 5 7 9 9 8
6 | 1 3 5 0 2 1 7 0 8 4 5 2 0 2 1 3 1

(b)

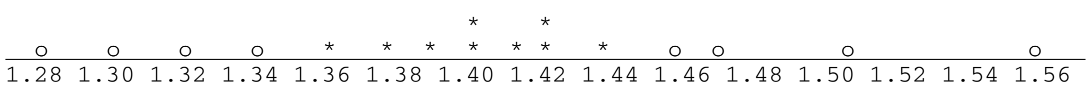
5f | 4
5s | 9 5 7 9 9 8
6f | 1 3 0 2 0 1 0 4 2 0 2 1 3 1
6s | 5 7 8 5

(c) The double-stem display is more informative.

3.111 *=Station 105 ○ = Station 107



3.112 *=Lathe A ○ = Lathe B



3.115

Class Limits	Frequency
40.0 – 44.9	5
45.0 – 49.9	7
50.0 – 54.9	15
55.0 – 59.9	23
60.0 – 64.9	29
65.0 – 69.9	12
70.0 – 74.9	8
75.0 – 79.9	1
	100

3.116

Class Limits	Frequency
3.0 – 4.9	15
5.0 – 6.9	25
7.0 – 8.9	17
9.0 – 10.9	11
11.0 – 12.9	8
13.0 – 14.9	4
	80

3.117 The class boundaries are: 39.95, 44.95, 49.95, 54.95, 59.95, 64.95, 69.95, 79.95;
the class interval is 5;
the class marks are: 42.45, 47.45, 52.45, 57.45, 62.45, 67.45, 72.45, 77.45.

3.118 The class boundaries are: 2.95, 4.95, 6.95, 8.95, 10.95, 12.95, 14.95;
the class interval is 2;
the class marks are: 3.95, 5.95, 7.95, 9.95, 9.95, 11.95, 13.95.

3.119

Class Limits	Frequency	Class Boundary	Class Mark
0 – 1	12	–0.5 – 1.5	0.5
2 – 3	7	1.5 – 3.5	2.5
4 – 5	4	3.5 – 5.5	4.5
6 – 7	5	5.5 – 7.5	6.5
8 – 9	1	7.5 – 9.5	8.5
10 – 11	0	9.5 – 11.5	10.5
12 – 13	1	11.5 – 13.5	12.5
	30		

3.120

Class Limits	Frequency	Percentage
3.0 – 4.9	15	18.75%
5.0 – 6.9	25	31.25
7.0 – 8.9	17	21.25
9.0 – 10.9	11	13.75
11.0 – 12.9	8	10.00
13.0 – 14.9	4	5.00
	80	100.00

3.121

Class Limits	Frequency	Percentage
40.0 – 44.9	5	5.0%
45.0 – 49.9	7	7.0
50.0 – 54.9	15	15.0
55.0 – 59.9	23	23.0
60.0 – 64.9	29	29.0
65.0 – 69.9	12	12.0
70.0 – 74.9	8	8.0
75.0 – 79.9	1	1.0
	100	100.0

3.122

Class Limits	Percentage: Shipping Department	Security Department
0 – 1	43.3%	45.0%
2 – 3	30.0	27.5
4 – 5	16.7	17.5
6 – 7	6.7	7.5
8 – 9	3.3	2.5
	100.0	100.0

The patterns seem comparable for the two departments.

3.123

Upper Class Boundary	Frequency	Cumulative Frequency
44.95	5	5
49.95	7	12
54.95	15	27
59.95	23	50
64.95	29	79
69.95	12	91
74.95	8	99
79.95	1	100
	100	

3.124

Upper Class Boundary	Frequency	Cumulative Frequency
4.95	15	15
6.95	25	40
8.95	17	57
10.95	11	68
12.95	8	76
14.95	4	80
	100	

3.125

Class Limits	Cumulative Percentage: Shipping Department	Security Department
1.5	43.3%	45.0%
3.5	73.3	72.5
5.5	90.0	90.0
7.5	96.7	97.5
9.5	100.0	100.0

3.126 **(a)**

Class Limits	Frequency
0 – 1	12
2 – 3	7
4 – 5	4
6 – 7	5
8 – 13	2
	30

(b) No. The class interval of the last class is greater than that of the others.

3.127 **(a)**

Class Limits	Frequency	Class Marks
0 – 99	4	49.5
100 – 199	3	149.5
200 – 299	4	249.5
300 – 324	7	312.0
325 – 349	14	337.0
350 – 399	6	374.5
	38	

(b) Yes, [see part (a)..

3.130 The class marks are found from the class boundaries by averaging them; thus, the first class mark is (2.95 + 4.95)/2 = 3.95, and so forth.

3.135 The MINITAB output is:

MIDDLE OF INTERVAL	NUMBER OF OBSERVATIONS	
6.0	2	**
6.5	5	*****
7.0	4	****
7.5	5	*****
8.0	5	*****
8.5	3	***
9.0	2	**
9.5	2	**
10.02	2	**

3.136 The MINITAB output is:

MIDDLE OF INTERVAL	NUMBER OF OBSERVATIONS	
40	1	*
45	7	*******
50	11	***********
55	21	*********************
60	21	*********************
65	23	***********************
70	10	**********
75	6	******