Sol. 5/5

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Quiz 3

Linear Algebra Nov 24, 2022

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Question 1

1/1.5 If the joint probability distribution of x and y is given by

$$f(x, y) = \frac{x+y}{30} \qquad \text{for } x = 0, 1, 2, 3 \text{ and } y = 0, 1, 2.$$

construct a table showing the values of the joint probability distribution of the two random variables at 12 points (0,0), (0,1), ..., (3,2).

Question 2

1.5/2

Let x and y have the joint probability distribution,

$$f(x, y) = \frac{xy^2}{30} \qquad \text{for } x = 1, 2, 3 \text{ and } y = 1, 2.$$

find the marginal probability functions of x and y and find out if the product of these two marginal probability functions is the same as $f(x, y)$.

Question 3

2.5

Suppose the joint probability distribution $f(x, y)$ is given by

$$f(x, y) = \frac{xy+3x^2}{3}, 0 \le x \le 1, 0 \le y \le 2$$

.9 a) Check that $\iint f(x,y)dxdy = 1$

.8+.8 b) Compute (i) $P(X > \frac{1}{2})$ (ii) $P(X < \frac{1}{2}, Y < \frac{1}{2})$

Solution: Q1) .8 .8

y \ x	0	1	2	3	
0	0	1/30	2/30	3/30	6/30
1	1/30	2/30	3/30	4/30	10/30
2	2/30	3/30	4/30	5/30	14/30
	3/30	6/30	9/30	12/30	1

Q2

$y \backslash x$	1	2	3	$h(y)$
1	1/30	2/30	3/30	6/30
2	4/30	8/30	12/30	24/30
$g(x)$	5/30	10/30	15/30	1

$$f(x,y) = \begin{cases} xy^2/30 & \text{for } x=1,2,3 \text{ and } y=1,2. \\ 0 & \text{o.w.} \end{cases}$$

$$\text{So } g(x) = \begin{cases} 5/30 & x=1 \\ 10/30 & x=2 \\ 15/30 & x=3 \\ 0 & \text{o.w.} \end{cases} = \begin{cases} x/6 & \text{for } x=1,2,3 \\ 0 & \text{o.w.} \end{cases}$$

Another way $g(x) = \sum_y f(x,y) = \frac{x}{30} + \frac{4x}{30} = \frac{5x}{30} = \frac{x}{6}$

$$h(y) = \begin{cases} 6/30 & y=1 \\ 24/30 & y=2 \\ 0 & \text{o.w.} \end{cases} = \begin{cases} y^2/5 & \text{for } y=1,2 \\ 0 & \text{o.w.} \end{cases}$$

Another way $h(y) = \sum_x f(x,y) = (1+2+3)y^2/30 = 6y^2/30 = y^2/5$

From above $g(x)h(y) = xy^2/30 = f(x,y)$. So, yes.

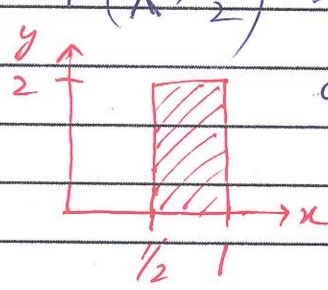
Q3 a) $\iint f(x,y)\,dx\,dy = \frac{1}{3}\int_{x=0}^{1}\int_{y=0}^{2}(xy+3x^2)\,dy\,dx$

$$= \frac{1}{3}\int_{x=0}^{1} \left. \frac{xy^2}{2} + 3x^2y \right|_{y=0}^{2} dx$$

$$= \frac{1}{3}\int_0^1 (2x+6x^2)\,dx$$

$$= \frac{1}{3}\left[x^2+2x^3\right]_0^1 = \frac{1}{3}(1+2) = \boxed{1} \checkmark$$

b)(i) $P(X > \frac{1}{2}) = \frac{1}{3}\int_0^2\int_{1/2}^{1}(xy+3x^2)\,dx\,dy = \int_0^2 \frac{1}{3}\left[\frac{x^2}{2}y + x^3\right]_{1/2}^{1} dy$

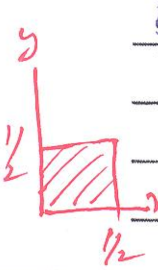


$$= \int_0^2 \frac{1}{3}\left[\frac{1}{2}y + 1 - \frac{1}{8}y - \frac{1}{8}\right] dy$$

$$= \frac{1}{3}\int_0^2\left(\frac{3}{8}y + \frac{7}{8}\right)dy = \frac{1}{3}\left[\frac{3y^2}{16} + \frac{7}{8}y\right]_0^2$$

$$= \frac{1}{3}\left[\frac{3}{4} + \frac{7}{4}\right] = \frac{10}{3\times 4} = \boxed{5/6}.$$

b) (ii) $P(X < 1/2, Y < 1/2) = \frac{1}{3}\int_0^{1/2}\int_0^{1/2}(xy+3x^2)\,dx\,dy$



$$= \frac{1}{3}\int_0^{1/2}\left[\frac{x^2 y}{2} + x^3\right]_0^{1/2} dy = \frac{1}{3}\int_0^{1/2}\left[\frac{y}{8} + \frac{1}{8}\right]dy = \frac{1}{3}\left[\frac{y^2}{16} + \frac{y}{8}\right]_0^{1/2}$$

$$= \frac{1}{3}\left[\frac{1}{64} + \frac{1}{16}\right] = \frac{1}{3}\left[\frac{1+4}{64}\right] = 5/192.$$