Chapter 12

12.1 **(a)** simple; **(b)** composite (β *not* specified); **(c)** composite (parameter *not* specified); **(d)** composite (parameter *not* specified).

12.2 **(a)** simple; **(b)** composite (parameter *not* specified); **(c)** composite (σ *not* specified); **(d)** composite (θ *not* specified).

12.3
$$\alpha = -\frac{\binom{2}{2}\binom{5}{0}}{\binom{7}{2}} = \frac{1\cdot 1}{21} = \frac{1}{21}$$

$$\beta = \frac{\binom{4}{0}\binom{3}{2}}{\binom{7}{2}} + \frac{\binom{4}{1}\binom{3}{1}}{\binom{7}{2}} = \frac{1\cdot 3}{21} + \frac{4.3}{21} = \frac{15}{21} = \frac{5}{7}$$

12.4
$$\alpha = p(x \le 16; \theta = 0.90) = p(x \ge 4; \theta = 0.10)$$
$$= 1 - (0.1216 + 0.2702 + 0.2852 + 0.1901)$$
$$= 1 - 0.8671 = 0.1329$$
$$\beta = p(x > 16; \theta = 0.60) = p(x < 4; \theta = 0.40)$$
$$= 0.000 + 0.0005 + 0.0031 + 0.0123 = 0.0159$$

12.5
$$\alpha = p(x \ge k; \theta_0) = \frac{a}{1-r} = \frac{\theta_0(1-\theta_0)^{k-1}}{1-(1-\theta_0)} = (1-\theta_0)^{k-1}$$

$$\beta = p(x < k; \theta_1) = a\frac{1-r^n}{1-r} = \theta_1 \cdot \frac{1-(1-\theta_1)^{k-1}}{1-(1-\theta_1)} = 1-(1-\theta_1)^{k-1}$$

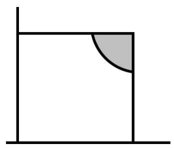
12.6
$$\alpha = p(x > 3; \theta = 2)$$
$$= \int_3^\infty \frac{1}{2}e^{-x/2}dx = -e^{-x/2}\Big|_3^\infty e^{-1.5} = 0.223$$
$$\beta = p(x \le 3; \theta = 5)$$
$$= \int_0^3 \frac{1}{5}e^{-x/5}dx = -e^{-x/5}\Big|_0^3 = 1 - e^{-0.6} = 1 - 0.549 = 0.451$$

12.7
$$\bar{x} > \mu_0 + z_\alpha \cdot \frac{\sigma}{\sqrt{n}}$$

$$z_\alpha \cdot \frac{1}{\sqrt{2}} = 1 \qquad z_\alpha = \sqrt{2} = 1.414$$
$$a = 0.5000 - 0.4207 = 0.8$$

12.8 $p(x > \beta_0 + 1; \beta_0) = 0$

$$p(x \le \beta_0 + 1; \beta_0 + 2) = (\beta_0 + 1) \cdot \frac{1}{\beta_0 + 2} = \frac{\beta_0 + 1}{\beta_0 + 2}$$

12.9 

$$1 - \beta = 4 \int_{3/4}^{1} x_2 \int_{3/4x_2}^{1} x_1 \; dx_1 \; dx_2$$

$$= 4 \int_{3/4}^{1} x_2 \left[\frac{1}{2} - \frac{9}{32x_2^2} \right] dx_2$$

$$1 - \beta = \int_{3/4}^{1} 2x_2 \; dx_2 - \frac{9}{8} \int_{3/4}^{1} \frac{dx_2}{x_2}$$

$$= 1 - \frac{9}{16} + \frac{9}{8} \ln 0.75$$

$$= \frac{7}{16} - \frac{9}{8}(0.28768) = 0.114$$

12.10 Proof same as in Example 12.4 except that the quantity $n(\mu_0 - \mu_1)$ is now *positive* and the inequalities are

$\bar{x} \le K$ inside c
$\bar{x} \ge K$ outside c

where $k = \mu_0 - z_\alpha \frac{1}{\sqrt{n}}$. So, critical region is

$$\bar{x} \le \mu_0 - z_\alpha \frac{1}{\sqrt{n}}$$

12.11 $L_0 = \frac{1}{\theta_0^n} e^{-(1/\theta)\sum x_i}$ $\qquad$ $L_1 = \frac{1}{\theta_1^n} e^{-(1/\theta_1)\sum x_i}$

$$\frac{L_0}{L_1} = \left(\frac{\theta_1}{\theta_0} \right)^n e^{-\sum x_i (1/\theta_0 - 1/\theta_1)} \le k$$

$$n \ln \frac{\theta_1}{\theta_0} - \sum x_i \left(\frac{1}{\theta_0} - \frac{1}{\theta_1} \right) \le \ln k$$

$$\sum x_i \ge \frac{n \ln \frac{\theta_1}{\theta_0} \ln k}{\frac{1}{\theta_0} - \frac{1}{\theta_1}} = K$$

Critical region is $\sum_{i=1}^{n} x_i \ge K$, where K can be determined by making use of fact that $\sum_{i=1}^{n} x_i$ has the gamma distribution with $\alpha = n$ and $\beta = \theta_0$.

12.12 $L_0 = \binom{n}{x}\theta_0^x(1-\theta_0)^{n-x}$ $\qquad$ $L_1 = \binom{n}{x}\theta_1^x(1-\theta_1)^{n-x}$

$$\frac{L_0}{L_1} = \left[\frac{\theta_0(1-\theta_1)}{\theta_1(1-\theta_0)}\right]^x \left(\frac{1-\theta_0}{1-\theta_1}\right)^n \le k$$

$$x \cdot \ln\frac{\theta_0(1-\theta_1)}{\theta_1(1-\theta_0)} + n \cdot \ln\frac{1-\theta_0}{1-\theta_1} \le \ln k$$

$$x \le \frac{\ln k - n\ln\frac{1-\theta_0}{1-\theta_1}}{\ln\frac{\theta_0(1-\theta_1)}{\theta_1(1-\theta_0)}} = K$$

Critical region is $x \le K$, where K can be determined from table of binomial probabilities.

12.13 $\frac{K-100(0.40)}{\sqrt{100(0.4)(0.6)}} = -1.645$, $K = 40\ -1.645(4.90) = 31.94$

Critical region $x \le 31$

$z = \frac{31.5-30}{\sqrt{100(0.3)(0.7)}} = \frac{1.5}{4.58} = 0.33$ $\qquad$ $\theta = 0.5 - 0.1293 = 0.37$

12.14 $f(x) = \theta(1-\theta)^{x-1}$ $\qquad$ $x = 1, 2, 3, \ldots$

$L_0 = \theta_0(1-\theta_0)^{x-1}$ $\qquad$ $L_1 = \theta_1(1-\theta_1)^{x-1}$

$$\frac{L_0}{L_1} = \left[\frac{\theta_0(1-\theta_1)}{\theta_1(1-\theta_0)}\right]\left[\frac{1-\theta_0}{1-\theta_1}\right]^x \le k$$

$$\ln\left[\frac{\theta_0(1-\theta_1)}{\theta_1(1-\theta_0)}\right] + x \cdot \ln\frac{1-\theta_0}{1-\theta_1} \le \ln k$$

$$x \le \frac{\ln k - \ln\frac{\theta_0(1-\theta_1)}{\theta_1(1-\theta_0)}}{\frac{1-\theta_0}{1-\theta_1}} = K$$

Critical region is $x \le K$, where K can be determined using formula for sum of terms of geometric progression.

12.15 $L_0 = \frac{1}{(\sqrt{2\pi})^n\sigma_0^n}e^{-(1/2\sigma_0^2)\sum x^2}$ $\qquad$ $L_1 = \frac{1}{\sqrt{2\pi}^n\sigma_1^n}e^{-(1/2\sigma_1^2)\sum x^2}$

$$\frac{L_0}{L_1} = \left(\frac{\sigma_1}{\sigma_0}\right)^n e^{-\left(\sum x^2/2\right)\left(1/\sigma_0^2 - 1/\sigma_1^2\right)} \le k$$

$$n\ln\frac{\sigma_1}{\sigma_2} - \frac{\sum x^2}{2}\left(\frac{1}{\sigma_0^2} - \frac{1}{\sigma_1^2}\right) \le \ln k$$

$$\sum x^2 \geq \frac{n\ln\frac{\sigma_1}{\sigma_0} - \ln k}{\left(\frac{1}{\sigma_0^2} - \frac{1}{\sigma_1^2}\right)} = K$$

Critical region is $\sum x^2 \geq K$, where K is determined using the fact that $\sum x^2 = (n-1)s^2$ and $\frac{(n-1)s^2}{\sigma_0^2}$ is random variable having χ^2 distribution with $n-1$ degrees of freedom. Therefore, critical region is $\sum x^2 \geq \sigma_0^2 \cdot \chi^2_{\alpha,n-1}$.

12.16 The probabilities of making wrong decisions are

	$\theta = 0.9$	$\theta = 0.6$
d_1	0.0114	0.1255
d_2	0.0433	0.0509
d_3	0.0025	0.2499

(a) (0.0114)(0.8) + (0.1255)(0.2)=0.034
(b) (0.0433)(0.8) + (0.0509)(0.2) = 0.045
(c) (0.0025)(0.8) + (0.2499)(0.2) = 0.052

12.17 (a) $\frac{\binom{0}{2}\binom{7}{0}}{\binom{7}{2}} = 0 \qquad \frac{\binom{1}{2}\binom{6}{0}}{\binom{7}{2}} = 0 \qquad \frac{\binom{2}{2}\binom{5}{0}}{\binom{7}{2}} = \frac{1}{21}$

(b) $1 - \frac{\binom{4}{2}\binom{3}{0}}{\binom{7}{2}} = \frac{5}{7} \qquad 1 - \frac{\binom{5}{2}\binom{2}{0}}{\binom{7}{2}} = \frac{11}{21} \qquad 1 - \frac{\binom{6}{2}\binom{1}{0}}{\binom{7}{2}} = \frac{2}{7}$

$1 - \frac{\binom{7}{2}\binom{0}{0}}{\binom{7}{2}} = 0$

12.18
$\theta = 0.95 \qquad \alpha = 0.0022 + 0.0003 = 0.0025$
$\theta = 0.90 \qquad \alpha = 0.0319 + 0.0089 + 0.0020 + 0.0004 + 0.0001 = 0.0433$
$\theta = 0.85 \qquad 1 - \beta = 1 - (0.0388 + 0.1368 + 0.2293 + 0.2428 + 0.1821) = 0.1702$
$\theta = 0.80 \qquad 1 - \beta = 1 - (0.0115 + 0.0576 + 0.1369 + 0.2054 + 0.2182) = 0.3704$
$\theta = 0.75 \qquad 1 - \beta = 1 - (0.0032 + 0.0211 + 0.0669 + 0.1339 + 0.1897) = 0.5852$
$\theta = 0.70 \qquad 1 - \beta = 1 - (0.0008 + 0.0068 + 0.0278 + 0.0716 + 0.1304) = 0.7626$
$\theta = 0.65 \qquad 1 - \beta = 1 - (0.0002 + 0.0020 + 0.0100 + 0.0323 + 0.0738) = 0.8817$
$\theta = 0.60 \qquad 1 - \beta = 1 - (0.0005 + 0.0031 + 0.0123 + 0.0350) = 0.9491$
$\theta = 0.55 \qquad 1 - \beta = 1 - (0.0001 + 0.0008 + 0.0040 + 0.0139) = 0.9812$
$\theta = 0.50 \qquad 1 - \beta = 1 - (0.0002 + 0.0011 + 0.0046) = 0.9941$

12.19 $x_i - \mu_0 = (x_i - \bar{x}) + (\bar{x} - \mu_0)$

$$\begin{aligned}\sum (x_i - \mu_0)^2 &= \sum (x_i - \bar{x})^2 + 2\sum (x_i - \bar{x})(\bar{x} - \mu_0) + \sum (\bar{x} - \mu_0)^2 \\ &= \sum (x_i - \bar{x})^2 + 2\sum (\bar{x} - \mu_0)\sum (x_i - \bar{x}) + \sum (\bar{x} - \mu_0)^2 \\ &= \sum (x_i - \bar{x})^2 + \sum (\bar{x} - \mu_0)^2\end{aligned}$$

Therefore $\lambda = e^{-1/2\sigma^2}\left[\sum (x_i - \mu_0)^2 - \sum (x_i - x)^2\right]$

$$= e^{-(1/2\sigma^2)}\sum (\bar{x} - \mu_0)^2$$

$$= e^{-(n/2\sigma^2)(\bar{x} - \mu_0)^2}$$

12.20 (a) $L = \binom{n}{x}\theta^x (1-\theta)^{n-x} \qquad L_0 = \binom{n}{x}\left(\frac{1}{2}\right)^n$

$$\ln L = \ln\binom{n}{x} + x\ln\theta + (n-x)\ln(1-\theta)$$

$$\frac{d\ln L}{d\theta} = \frac{x}{\theta} - \frac{n-x}{1-\theta} = 0 \text{ yields } \theta = \frac{x}{n}$$

$$\max L = \binom{n}{x}\left(\frac{x}{n}\right)^x\left(\frac{n-x}{n}\right)^{n-x}$$

and $\lambda = \dfrac{\left(\frac{1}{2}\right)^n}{\left(\frac{x}{n}\right)^x\left(\frac{n-x}{n}\right)^{n-x}} = \dfrac{(n/2)^n}{x^x(n-x)^{n-x}} \le k$

(b) $-n\ln 2 + n\ln n - x\ln x - (n-x)\ln(n-x) \le \ln k$

$-x\ln x - (n-x)\ln(n-x) \le k'$

$x\ln x + (n-x)\ln(n-x) \ge K$

(c) $f(x) = x\ln x + (n-x)\ln(n-x)$

$$\frac{df(x)}{dx} = \ln x + 1 - \ln(n-x) - 1 = 0$$

$x = n - x$ and $x = \frac{n}{2}$ is minimum

Since $f(n-x) = f(x)$, symmetrical about $x = \frac{n}{2}$. Therefore critical region is

$\left|x - \frac{n}{2}\right| \ge c$.

12.21 (a) $L = \frac{1}{\theta^n} e^{-(1/\theta)\sum x}$ $\qquad$ $\max L_0 = \frac{1}{\theta_0^n} e^{-(1/\theta_0)\sum x}$

$$\ln L = -n \ln \theta - \frac{1}{\theta}\sum x$$

$$\frac{d \ln L}{d\theta} = -\frac{n}{\theta} + \frac{\sum x}{\theta^2} = 0 \qquad \theta = \overline{x}$$

$$\lambda = \frac{\frac{1}{\theta_0^n} e^{-(1/\theta_0)\sum x}}{\frac{1}{\overline{x}^n} e^{-(1/\overline{x})\sum x}} = \left(\frac{\overline{x}}{\theta_0}\right)^n e^{-(n\overline{x}/\theta_0)+n}$$

(b) $\left(\frac{\overline{x}}{n}\right)^n e^{-(n\overline{x}/\theta_0)} \le \frac{k}{e^n} = k'$

$$\frac{\overline{x}}{n} e^{-\overline{x}/\theta_0} \le \sqrt[n]{k}$$

$$\overline{x} e^{-\overline{x}/\theta_0} \le n\sqrt[n]{k} = K$$

$$\overline{x} e^{-\overline{x}/\theta_0} \le K$$

12.22 Over Ω maximum likelihood estimates are $\hat{\mu} = \overline{x}$ and $\hat{\sigma}^2 = \frac{\sum (x-\overline{x})^2}{n}$

Over w maximum likelihood estimates are $\hat{\mu}_0 = \mu_0$ and $\hat{\sigma}_0^2 = \frac{\sum (x-\mu_0)^2}{n}$

$$\lambda = \frac{\frac{1}{(\sqrt{2\pi})^n \hat{\sigma}_0^n} e^{-(1/2\hat{\sigma}^2)}}{\frac{1}{(\sqrt{2\pi})^n \hat{\sigma}^2} e^{-(1/2\hat{\sigma}^2)}} = \left(\frac{\hat{\sigma}^2}{\hat{\sigma}_0^2}\right)^{n/2} = \left(\frac{\hat{\sigma}_0^2}{\hat{\sigma}^2}\right)^{-n/2}$$

$$\lambda^{-2/n} = \frac{\sum (x-\mu_0)^2}{\sum (x-\overline{x})^2} = \frac{\sum (x-\overline{x})^2 + n(\overline{x}-\mu_0)^2}{\sum (x-\overline{x})^2} = 1 + \frac{n(\overline{x}-\mu_0)}{\sum (x-\overline{x})^2}$$

$$= 1 + \frac{t^2}{n-1} \text{ where } t = \frac{\sqrt{n}(\overline{x}-\mu_0)^2}{s}$$

$$\lambda = 1 + \frac{t^2}{n-1}, \text{ where } t = \frac{\sqrt{n}(\overline{x}-\mu_0)}{s}$$

12.23 Use $\ln(1+\lambda) = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 \ldots$

$$\lambda^{-2} = \left(1 + \frac{t^2}{n-1}\right)^n$$

$$-2\ln\lambda = n\ln\left(1 + \frac{t^2}{n-1}\right) = n\left[\frac{t^2}{n-1} - \frac{1}{2}\left(\frac{t^2}{n-1}\right)^2 + \frac{1}{3}\left(\frac{t^2}{n-1}\right)^3 - \ldots\right]$$

$$\rightarrow \quad t^2$$

12.24 $\max L_0 = \dfrac{1}{(\sqrt{2\pi})^n \sigma_0^n} e^{-(1/2\sigma_0^2)\sum(x-\bar{x})^2}$

$$\max L = \frac{1}{(\sqrt{2\pi})^n \hat{\sigma}_0^n} e^{-(1/2\hat{\sigma}^2)\sum(x-\bar{x})^2}$$

$$\lambda = \left[\frac{\sum(x-\bar{x})^2}{n\sigma_0^2}\right]^{n/2} e^{-(1/2)\sum(x-\bar{x})^2(1/\sigma_0^2 - 1/\hat{\sigma}^2)}$$

$$\frac{1}{\sigma_0^2} - \frac{1}{\hat{\sigma}^2} = \frac{1}{\sigma_0^2} - \frac{n}{\sum(x-\bar{x})^2}$$

$$\lambda = \left[\frac{\sum(x-\bar{x})^2}{n\sigma_0^2}\right]^{n/2} e^{-(1/2)\left\{\left[\sum(x-\bar{x})^2/\sigma_0^2\right]-n\right\}}$$

12.25 (a) $L = \prod_{i=1}^{k} = \dfrac{1}{(\sqrt{2\pi})^{n_i} \sigma_i^{n_i}} e^{-\left[\frac{1}{2\sigma_i^2}\sum_{j=1}^{n_i}(x_{ij}-\mu_i)^2\right]}$

proceed as in Example 10.17

(b) $\max L_0 = \prod_{i=1}^{k} = \dfrac{1}{(\sqrt{2\pi})^{n_i} tc\hat{\sigma}_i^{n_i}} e^{-(1/2\hat{\sigma}_i^2)\sum_j (x_{ij}-\bar{x}_i)^2}$

$$\max L = \prod_{i=1}^{k} = \frac{1}{(\sqrt{2\pi})^{n_i} \hat{\hat{\sigma}}_i^{n_i}} e^{-(1/2\hat{\sigma}_i^2)\sum_j (x_{ij}-\bar{x}_i)^2}$$

$$\hat{\sigma}_i^2 = \sum_i \frac{(n_i-1)s_i^2}{\sum n_i} \qquad \hat{\hat{\sigma}}_i^2 = \frac{(n_i-1)s_i^2}{n_i}$$

$$\lambda = \frac{\prod_i \left[\frac{(n_i-1)s_i^2}{n_i}\right]^{n_i/2}}{\left[\sum_i \frac{(n_i-1)s_i^2}{n}\right]^{n/2}}$$

12.26 Dividing numerator and denominator by $\left(s_1^2\right)^{(n_1+n_2)/2}$ yields

$$\lambda = \frac{\left(\dfrac{n_1-1}{n_1}\right)^{n_1/2}\left(\dfrac{n_2-1}{n_2}\cdot\dfrac{s_2^2}{s_1^2}\right)^{n_2/2}}{\left(\dfrac{n_1-1}{n}+\dfrac{n_2-1}{n}\cdot\dfrac{s_2^2}{s_1^2}\right)^{n_2-2}} \qquad \text{QED}$$

12.27 $L = 1+\theta^2\left(\frac{1}{2}-x\right)$

$$\pi(0) = \int_0^\alpha 1dx = \sigma$$

$$\beta = \int_\alpha^1 \left[1+\theta^2\left(\frac{1}{2}-x\right)\right]dx = 1-\alpha-\frac{1}{2}\theta^2\alpha(1-\alpha)$$

$$1-\beta = \alpha+\frac{1}{2}\theta^2\alpha(1-\alpha)$$

$$\pi(\theta) = \alpha+\frac{1}{2}\theta^2\alpha(1-\alpha)$$

Since $\frac{1}{2}\theta^2\alpha(1-\alpha) > 0$ for $0<\alpha<1$

$\pi(0)$ has minimum at $\theta = 0$

12.28 They would be committing a type I error if they erroneously reject the null hypothesis that 60% of their passengers object to smoking inside the plane.

They would be committing a type I error if they erroneously accept this null hypothesis.

12.29 The doctor would commit a type I error if he/she erroneously rejects the null hypothesis that the executive is able to take on additional responsibilities. The doctor would commit a type II error if he/she erroneously accepts this null hypothesis.

12.30 **(a)** The manufacturer should use the alternative hypothesis $\mu < 20$ and make the modification only if the null hypothesis can be rejected.

(b) The manufacturer should use the alternative hypothesis $\mu > 20$ and make the modification unless the null hypothesis can be rejected.

12.31 **(a)** $H_1 : \mu_2 > \mu_1$

(b) $H_1 : \mu_1 > \mu_2$

(c) $H_1 : \mu_1 \neq \mu_2$

12.32 With $\mu = 9.6,\ \bar{x} = 10.2,$ and $n = 80$

(a) Decision: reject H_0: since H_0 is true, decision is in error.

(b) Decision: reject H_0: since H_0 is false, decision is not in error.

(c) Decision: reject H_0: since H_0 is true, decision is in error.

(d) Decision: reject H_0: since H_0 is true, decision is not in error.

12.33 (a) $H_0 : \mu_1 = \mu_2$

(b) $H_1 : \mu_2 > \mu_1$

(c) $H_1 : \mu_2 < \mu_1$

12.34 (a) H_0: the antipollution device is effective. A type I error would be made if the device is effective and H_0 is rejected. A type II error would be made if the device is not effective and H_0 is not rejecte4d.

(b) H_0: The antipollution device is not effective.

12.35 (a) She will correctly reject the null hypothesis.
(b) She will erroneously reject the null hypothesis.

12.36 (a) He will erroneously accept the null hypothesis.
(b) He will correctly accept the null hypothesis.

12.37 (a) $-\sqrt{n} + 1.645 + -1.88$

$\sqrt{n} = 3.525 \qquad n = 12.43 \qquad n = 13$ rounded up to nearest integer

(b) $-\sqrt{n} + 1.645 = -2.33$

$\sqrt{n} = 3.975 \qquad n = 15.80 \qquad n = 16$ rounde4d up to nearest integer

12.38 (a) Yes; **(b)** Yes

12.39 (a)
$$1 - \int_8^{12} \frac{1}{10} e^{-x/10} dx = 1 + e^{-x/10}\Big|_8^{12} + 1 + e^{-1.2} - e^{-0.8}$$
$$= 1 + 0.3012 - 0.4493 = 0.852$$

(b)
$$\int_8^{12} \frac{1}{2} e^{-x/2} dx = -e^{-x/2}\Big|_8^{12} = e^{-4} - e^{-6} = 0.0183 - 0.0025 = 0.016$$
$$\int_8^{12} \frac{1}{4} e^{-x/4} dx = -e^{-x/4}\Big|_8^{12} = e^{-2} - e^{-3} = 0.1353 - 0.0448 = 0.086$$
$$\int_8^{12} \frac{1}{6} e^{-x/6} dx = -e^{-x/6}\Big|_8^{12} = e^{-1.33} - e^{-2} = 0.2645 - 0.1353 = 0.129$$
$$\int_8^{12} \frac{1}{8} e^{-x/8} dx = -e^{-x/8}\Big|_8^{12} = e^{-1} - e^{-1.5} = 0.3679 - 0.2231 = 0.145$$

$$\int_8^{12} \frac{1}{12} e^{-x/12} dx = -e^{-x/12}\Big|_8^{12} = e^{-0.67} - e^{-1} = 0.5117 - 0.3679 = 0.144$$

$$\int_8^{12} \frac{1}{16} e^{-x/16} dx = -e^{-x/16}\Big|_8^{12} = e^{-0.50} - e^{-0.75} = 0.6065 - 0.4724 = 0.134$$

$$\int_8^{12} \frac{1}{20} e^{-x/20} dx = -e^{-x/20}\Big|_8^{12} = e^{-0.40} - e^{-0.60} = 0.6703 - 0.5488 = 0.122$$

12.40 Reject if $\bar{x} > 43.5$ $\qquad \sigma_{\bar{x}} = \sqrt{\frac{265}{64}} = 2$

(a) $z = \frac{43.5-37}{2} = 3.25,\ \ P(\bar{X} > 43.5 | \mu = 37) = P(Z > 3.25) = 0.00058$

$z = \frac{43.5-38}{2} = 2.75,\ \ P(\bar{X} > 43.5 | \mu = 38) = P(Z > 2.75) = 0.003$

$z = \frac{43.5-39}{2} = 2.25,\ \ P(\bar{X} > 43.5 | \mu = 39) = P(Z > 2.25) = 0.0122$

$z = \frac{43.5-40}{2} = 1.75,\ \ P(\bar{X} > 43.5 | \mu = 40) = P(Z > 1.75) = 0.04$

(b) $z = \frac{43.5-41}{2} = 1.25,\ \ P(\bar{X} \le 43.5 | \mu = 41) = P(Z \le 1.25) = 0.8944$

$z = \frac{43.5-42}{2} = 0.75,\ \ P(\bar{X} \le 43.5 | \mu = 42) = P(Z \le 0.75) = 0.7734$

$z = \frac{43.5-43}{2} = 0.25,\ \ P(\bar{X} \le 43.5 | \mu = 43) = P(Z \le 0.25) = 0.5987$

$z = \frac{43.5-44}{2} = -0.25,\ \ P(\bar{X} \le 43.5 | \mu = 44) = P(Z \le -0.25) = 0.4103$

$z = \frac{43.5-45}{2} = -0.75,\ \ P(\bar{X} \le 43.5 | \mu = 45) = P(Z \le -0.75) = 0.2266$

$z = \frac{43.5-46}{2} = -1.25,\ \ P(\bar{X} \le 43.5 | \mu = 46) = P(Z \le -1.25) = 0.1056$

$z = \frac{43.5-47}{2} = -1.75,\ \ P(\bar{X} \le 43.5 | \mu = 47) = P(Z \le -1.75) = 0.04$

$z = \frac{43.5-48}{2} = -2.25,\ \ P(\bar{X} \le 43.5 | \mu = 48) = P(Z \le 2.25) = 0.0122$

12.41 (a) Reject if $\sum x \le 5$ Use Table II

$\lambda = 11$	$p = 0.0375$	$\lambda = 12$	$p = 0.0203$
$\lambda = 13$	$p = 0.0107$	$\lambda = 14$	$p = 0.0055$
$\lambda = 15$	$p = 0.0027$		

(b) $\lambda = 10,\ \ 1-0.0671 = 0.9329,\ \ \lambda = 7.5,\ \ 1-0.2415 = 0.7585$

$\lambda = 5,\ \ 1-0.6160 = 0.3840,\ \ \lambda = 2.5,\ \ 1-0.9580 = 0.0420$

12.42 $\mu = 50, \quad \sigma = 5, \quad z = \dfrac{56.6 - 50}{5} = 1.3$

Probability of 57 or more heads is $0.500 - 0.4032 = 0.0968$
Since $0.0968 > 0.05$ null hypothesis cannot be rejected.

12.43 $$\lambda = \frac{\left(\dfrac{7 \cdot 16}{8}\right)^4 \left(\dfrac{9 \cdot 25}{10}\right)^5 \left(\dfrac{5 \cdot 12}{6}\right)^3 \left(\dfrac{7 \cdot 24}{8}\right)^4}{\left[(112 + 225 + 60 + 168)/32\right]^{16}}$$
$$= \frac{14^4 \cdot 22.5^5 \cdot 10^3 \cdot 21^4}{17.656^{16}}$$

$$\ln \lambda = 4(2.63906) + 5(3.11352) + 3(2.30259) + 4(3.04452) - 16(2.8711)$$
$$= -0.712 \qquad\qquad -2 \ln \lambda = 1.424$$

Since this is less than $\chi^2_{0.05,3} = 7.815$, the null hypothesis cannot be rejected.

12.44 From Exercise 12.21
$$\lambda = \left(\frac{\bar{x}}{\theta_0}\right)^n e^{-(n\bar{x}/\theta_0)+n}$$
$$\ln \lambda = n \ln \frac{\bar{x}}{\theta_0} - \frac{n\bar{x}}{\theta_0} + n = 20 \ln \frac{529}{300} - \frac{529}{15} + 20$$
$$= 20(0.5670) - 15.27 = -3.93 \qquad -2 \ln \lambda = 2(3.93) = 7.86$$

Since 7.86 exceeds $\chi^2_{0.05,1} = 3.841$, the null hypothesis must be rejected.