Chapter 15

15.1 $$\frac{n\sum_{i=1}^{a}(\overline{x}_i - \overline{x}_{..})^2}{a-1} = \frac{n}{a-1}\sum_{i=1}^{a}[\overline{x}_{i.} - 2\overline{x}_{i.}\overline{x}_{..} + \overline{x}_{..}^2]$$

$$= \frac{n}{a-1}\sum_{i=1}^{a}\overline{x}_{i.} - \frac{an}{a-1}\overline{x}_{..}^2$$

$$E\left[\frac{n\sum_{i=1}^{a}(\overline{x}_{i.} - \overline{x}_{..})^2}{a-1}\right] = \frac{n}{a-1}\sum_{i=1}^{a}\left\{\frac{\sigma^2}{n} + (\mu + \sigma_i)^2\right\} - \frac{an}{a-1}\left(\frac{\sigma^2}{na} + \mu^2\right)$$

$$= \sigma^2 + \frac{n}{a-1}\sum_{i=1}^{a}\alpha_i^2$$

15.2 $$SST = \sum_{i=1}^{a}\sum_{j=1}^{n}(x_{ij} - \overline{x}_{..})^2$$

$$= \sum_{i=1}^{a}\sum_{j=1}^{n}x_{ij}^2 - 2\overline{x}_{..}\sum_{i=1}^{a}\sum_{j=1}^{n}x_{ij} + na\overline{x}_{..}^2$$

$$= \sum_{i=1}^{a}\sum_{j=1}^{n}x_{ij}^2 - 2\cdot\frac{T_{..}}{na}\cdot T_{..} + \frac{naT_{..}^2}{n^2a^2}$$

$$= \sum_{i=1}^{a}\sum_{j=1}^{n}x_{ij}^2 - \frac{1}{na}T_{..}^2$$

$$SS(Tr) = n\sum_{i=1}^{a}(\overline{x}_{i.} - \overline{x}_{..})^2$$

$$= n\sum_{i=1}^{a}\overline{x}_{i.}^2 - 2n\sum_{i=1}^{a}\overline{x}_{i.}\overline{x}_{..} + n\sum_{i=1}^{a}\overline{x}_{..}^2$$

$$= n\sum_{i=1}^{a}\frac{T_{i.}^2}{n^2} - 2n\overline{x}_{..}(a\overline{x}_{..}) + nax_{..}^2$$

$$= \frac{1}{n}\sum_{i=1}^{a}T_{i.}^2 - \frac{1}{na}T_{..}^2$$

15.3 $\sum_{i=1}^{a}\sum_{j=1}^{n_i}(x_{ij}-\bar{x}_{..})^2 = \sum_{i=1}^{a}\sum_{j=1}^{n_i}[(\bar{x}_{i.}-\bar{x}_{..})+(x_{ij}-\bar{x}_{i.}]^2$

$$= \sum_{i=1}^{a}\sum_{j=1}^{n_i}(\bar{x}_{i.}-\bar{x}_{..})^2 + \sum_{i=1}^{a}\sum_{j=1}^{n_i}(x_{ij}-\bar{x}_{i.})^2$$

Since $\sum_{i=1}^{a}\sum_{j=1}^{n_i}(\bar{x}_j-\bar{x}_{..})(x_{ij}-\bar{x}_{..}) = \sum_{j=1}^{a_i}(\bar{x}_{i.}-\bar{x}_{..})\sum_{j=1}^{n_i}(x_{ij}-\bar{x}_{i.}) = 0$

$$\sum_{i=1}^{a}\sum_{j=1}^{n_i}(x_{ij}-\bar{x}_{..})^2 \sum_{i=1}^{a} n(\bar{x}_{i.}-\bar{x}_{..})^2 + \sum_{i=1}^{a}\sum_{j=1}^{n_i}(x_{ij}-\bar{x}_{i.})^2$$

SST is such that $\frac{SST}{\sigma^2}$ is value of random variable having χ^2 distribution with $\sum_{i=1}^{a} n_i - 1 = N-1$ degrees of freedom. For each i, $\frac{1}{\sigma^2}\sum_{j=1}^{n_i}(x_{ij}-\bar{x}_{i.})^2$ is value of random variable having χ^2 distribution with $n_i - 1$ degrees of freedom, so that $\frac{1}{\sigma^2}SSE$ is value of random variable having $\sum_{i=1}^{a}(n_i-1) = N-a$ degrees of freedom. Also $\frac{SST}{\sigma^2}$ is value of random variable having χ^2 distribution with $a-1$ degrees of freedom.

15.4 $SST = \sum_{i=1}^{a}\sum_{j=1}^{n_i}(x_{ij}-\bar{x}_{..})^2$

$$= \sum_{i=1}^{a}\sum_{j=1}^{n_i}x_{ij}^2 - 2\bar{x}_{..}\sum_{i=1}^{a}\sum_{j=1}^{n_i}x_{ij} + \sum_{i=1}^{a}\sum_{j=1}^{n_i}.\bar{x}_{..}^2$$

$$= \sum_{i=1}^{a}\sum_{j=1}^{n_i}x_{ij}^2 - \frac{2}{N}T_{..}^2 + \frac{1}{N}T_{..}^2 = \sum_{i=1}^{a}\sum_{j=1}^{n_i}x_{ij}^2 - \frac{1}{N}T_{..}^2$$

$$SS(Tr) = \sum_{i=1}^{n_i} n_i(\bar{x}_{i.}-\bar{x}_{..})^2 = \sum_{i=1}^{n_i} n_i\bar{x}_{i.}^2 - 2\bar{x}_{..}\sum_{i=1}^{n_i} n_i\bar{x}_i + \sum_{i=1}^{n_i} n_i\bar{x}_{..}^2$$

$$= \sum_{i=1}^{n_i}\frac{T_{i.}^2}{n_i} - 2N\bar{x}_{..}^2 + N\bar{x}_{..}^2 = \sum_{i=1}^{n_i}\frac{T_{i.}^2}{n_i} - \frac{1}{N}T_{..}^2$$

$SSE = SST - SS(Tr)$ from identities of Exercise 15.3

15.5 $SS(Tr) = n_1(\overline{x}_{1.} - \overline{x}_{..})^2 + n_2(\overline{x}_{2.} - \overline{x}_{..})^2 \qquad \overline{x}_{..} = \dfrac{n_1\overline{x}_{1.} + n_2\overline{x}_{2.}}{n_1 + n_2}$

$$= n_1\left(\overline{x}_{1.} - \frac{n_1\overline{x}_{1.} - n_2\overline{x}_{2.}}{n_1+n_2}\right)^2 + n_2\left(\overline{x}_{2.} - \frac{n_1\overline{x}_{1.} - n_2\overline{x}_{2.}}{n_1+n_2}\right)^2$$

$$= n_1\left(\frac{n_2\overline{x}_{1.} - n_2\overline{x}_{1.}}{n_1+n_2}\right)^2 + n_2\left(\frac{n_1\overline{x}_{2.} - n_1\overline{x}_{1.}}{n_1+n_2}\right)^2$$

$$= \frac{n_1 n_2^2}{(n_1+n_2)^2}(\overline{x}_{1.} - \overline{x}_{2.})^2 + \frac{n_1^2 n_2}{(n_1+n_2)^2}(\overline{x}_{1.} - \overline{x}_{2.})^2$$

$$= \frac{n_1 n_2}{(n_1+n_2)}(\overline{x}_{1.} - \overline{x}_{2.})^2 = \frac{(\overline{x}_{1.} - \overline{x}_{2.})^2}{\frac{1}{n_1} + \frac{1}{n_2}}$$

$$SSE = \sum_{j=1}^{n_1}(x_{1j} - \overline{x}_{1.})^2 + \sum_{j=1}^{n_2}(x_{2j} - \overline{x}_{2.})^2 = (n_1-1)s_1^2 + (n_2-1)s_2^2$$

$$= (n_1 + n_2 - 2)s^2 p$$

$$F = \frac{(\overline{x}_1. - \overline{x}_2.)^2}{\frac{1}{n_1} + \frac{1}{n_2}} + \frac{(n_1+n_2-2)s^2 p}{n_1+n_2-2}$$

$$= \frac{(\overline{x}_1. - x_2.)^2}{s^2 p\left(\frac{1}{n_1} + \frac{1}{n_2}\right)} = t^2 \qquad \text{QED}$$

15.6 $u = \sum_{i=1}^{a}\sum_{j=1}^{n_i}[x_{ij} - (\mu + \alpha_i)]^2 + \lambda\sum\alpha_i$

$$\frac{\partial u}{\partial \mu} = 2\sum_{i=1}^{a}\sum_{j=1}^{n_i}[x_{ij} - (\mu+\alpha_i)](-1) = 0$$

$$= \sum_{i=1}^{a}\sum_{j=1}^{n_i}x_{ij} - a\left(\sum_{i=1}^{a}n_i\right)\mu = 0 \qquad \hat{\mu} = \overline{x}_{..}$$

$$\frac{\partial u}{\partial \alpha_i} = 2\sum_{j=1}^{n_i}[x_{ij} - (\mu+\alpha_i)](-1) + \lambda = 0$$

sum over i; $= -N\overline{x}_{..} + N\overline{x}_{..} + \lambda = 0 \qquad \lambda = 0$

$$\sum_{j=1}^{n_i}[x_{ij} - (\overline{x}_{..} + \alpha_i)] = 0$$

$$n_i x_{i.} - n_i\overline{x}_{..} - n_i\alpha_i = 0 \qquad \hat{\alpha} = \overline{x}_{i.} - \overline{x}_{..}$$

15.7 $\sum_{i=1}^{a}\sum_{j=1}^{n}(x_{ij}-\bar{x}_{..})^2$

$$= \sum_{i=1}^{a}\sum_{j=1}^{n}[(x_{i.}-\bar{x}_{..})+(\bar{x}_{.j}-\bar{x}_{..})+(x_{ij}-\bar{x}_{i.}-\bar{x}_{.j}+\bar{x}_{..})]^2$$

$$= n\sum_{i=1}^{a}(\bar{x}_{i.}-\bar{x}_{..})^2 + a\sum_{j=1}^{n}(x_{.j}-\bar{x}_{..})^2$$

$$+\sum_{i=1}^{a}\sum_{j=1}^{n}(x_{ij}-\bar{x}_{i.}-\bar{x}_{.j}+\bar{x}_{..})^2$$

$$+2\left[\sum_{i=1}^{a}(x_{i.}-\bar{x}_{..})\sum_{j=1}^{n}(x_{.j}-\bar{x}_{..})\right]$$

$$+2\sum_{i=1}^{a}\left[(x_{i.}-\bar{x}_{..})\sum_{j=1}^{n}(x_{ij}-\bar{x}_{i.}-\bar{x}_{.j}+\bar{x}_{..})\right]$$

$$+2\sum_{j=1}^{n}\left[(x_{.j}-\bar{x}_{..})\sum_{i=1}^{a}(x_{ij}-\bar{x}_{i.}-\bar{x}_{.j}+\bar{x}_{..})\right]$$

$$= \sum_{i=1}^{a}\sum_{j=1}^{n}(x_{ij}-\bar{x}_{..})^2 + n\sum_{i=1}^{a}(x_{ij}-\bar{x}_{..})^2 + a\sum_{j=1}^{n}[(x_{.j}-\bar{x}_{..})^2$$

$$+\sum_{i=1}^{a}\sum_{j=1}^{n}(x_{ij}-\bar{x}_{i.}-\bar{x}_{.j}+\bar{x}_{..})^2 \qquad \text{QED}$$

15.8 $\mu_{ij} = \mu + \alpha_i + \beta_j$

$$\frac{1}{na}\sum_{i=1}^{a}\sum_{j=1}^{n}(\mu+\alpha_i+\beta_j) = \frac{1}{na}\sum_{i=1}^{a}\sum_{j=1}^{n}\mu + \sum_{i=1}^{a}\sum_{j=1}^{n}\alpha_i + \sum_{i=1}^{a}\sum_{j=1}^{n}\beta_j$$

then since $\sum_{i=1}^{a}\alpha_i = 0$ and $\sum_{j=1}^{n}\beta_j = 0$

$$\frac{1}{na}\sum_{i=1}^{a}\sum_{j=1}^{n}\mu_{ij} = \frac{1}{na}\sum_{i=1}^{a}\sum_{j=1}^{n}\mu = \frac{1}{na}\cdot na\mu = \mu$$

15.9 $$\frac{a}{n-1}\sum_{j=1}^{n}(\overline{x}_{.j}-\overline{x}_{..})^2=\frac{a}{n-1}\sum_{i=1}^{n}[\overline{x}_{.j}^2-2\overline{x}_{.j}x_{..}+\overline{x}_{..}^2]$$

$$=\frac{a}{n-1}\sum_{j=1}^{n}\overline{x}_{.j}^2-\frac{an}{n-1}\overline{x}_{..}^2$$

$$E\left[\frac{a}{n-1}\sum_{j=1}^{n}(\overline{x}_{ij}-\overline{x}..)^2\right]=\frac{a}{n-1}\sum_{i=1}^{n}\left\{\frac{\sigma^2}{a}-(\mu+\beta_j)^2\right\}=\frac{an}{n-1}\left(\frac{\sigma^2}{na}+\mu^2\right)$$

$$=\sigma^2\frac{na}{(n-1)a}-\sigma^2\frac{1}{n-1}+\frac{a}{n-1}\sum_{j=1}^{n}\beta_j^2$$

$$=\sigma^2+\frac{a}{n-1}\sum_{j=1}^{n}\beta_j^2 \qquad \text{(see also 15.1)}$$

15.10 $$SSB=a\sum_{j=1}^{n}(\overline{x}_{.j}-\overline{x}_{..})^2$$

$$=a\sum_{j=1}^{n}\overline{x}_{.j}^2-2a\sum_{j=1}^{n}\overline{x}_{.j}\overline{x}_{..}+a\sum_{j=1}^{n}\overline{x}_{..}^2$$

$$=a\sum_{j=1}^{n}\frac{T_{.j}^2}{k^2}-2a\overline{x}_{..}(n\overline{x}_{..})+na\overline{x}_{..}^2$$

$$=\frac{1}{a}\sum_{j=1}^{n}T_{.j}^2-\frac{1}{na}(T_{..})^2 \qquad \text{QED}$$

15.11 $\mu_{ijr}=\mu+\alpha_i+\beta_j+\rho_r+(\alpha\beta)_{ij}$

$$\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{r}\mu_{ijk}=rba\mu+rb\sum_{i=1}^{a}\alpha_i+ar\sum_{j=1}^{b}\beta_j+ab\sum_{k=1}^{r}\rho_r+r\sum_{i=1}^{a}\sum_{j=1}^{b}(\alpha\beta)_{ij}$$

But

$$\sum_{i=1}^{a}\alpha_i=\sum_{j=1}^{b}\beta_j=\sum_{k=1}^{r}\rho_k=0 \quad \text{also} \quad \sum_{j=1}^{b}(\alpha\beta)_{ij}=0; \quad \therefore +r\sum_{i=1}^{a}\sum_{j=1}^{b}(\alpha\beta)_{ij}=0$$

Finally

$$\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{r}\mu_{ijk}=rba\mu; \quad \mu=\frac{\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{r}\mu_{ijk}}{rba}$$

15.12 Dropping the indexes of summation for simplicity, we have

$$\sum\sum\sum(x_{ijk}-\bar{x}_{...})^2 = \sum\sum\sum(\bar{x}_{i..}-\bar{x}_{...})^2 + \sum\sum\sum(\bar{x}_{.j.}-\bar{x}_{...})^{2.} + \sum\sum\sum(\bar{x}_{..k}-\bar{x}_{...})^2 + \sum\sum\sum(\bar{x}_{ij.}-\bar{x}_{i..}-\bar{x}_{.j.}+\bar{x}_{...})^2 = \sum\sum\sum(\bar{x}_{ijk}-\bar{x}_{ij.}-\bar{x}_{..k}+\bar{x}_{...})^2 +$$

six cross-product terms.

To indicate the proof that all cross-product terms sum to zero, we take the following example:

$$2\sum_i\sum_j\sum_k(\bar{x}_{ij.}-\bar{x}_{i..}-\bar{x}_{.j.}+\bar{x}_{...})(x_{ijk}-\bar{x}_{ij.}-\bar{x}_{..k}+\bar{x}_{...})^2 =$$

$$2\sum_i\sum_j(\bar{x}_{ij.}-\bar{x}_{i..}-\bar{x}_{.j.}+\bar{x}_{...})\sum_k(\bar{x}_{ijk}-\bar{x}_{ij.}-\bar{x}_{..k}+\bar{x}_{...})$$

The summation on k equals zero, which completes the proof.

15.13 By Theorem 15.5,

$$SSA = rb\sum_{i=1}^{a}(\bar{x}_{i..}-\bar{x}_{...})^2 = rb\left[\sum_{i=1}^{a}\bar{x}_{i..}^2 - a\bar{x}_{...}^2\right] = rb\sum_{i=1}^{a}\bar{x}_{i..}^2 - rba\bar{x}_{..}^2$$

Now,

$$\bar{x}_{i..} = \frac{T_{i..}}{rb} \text{ and } \bar{x}_{...} = \frac{T_{...}}{rba}$$

Thus,

$$SSA = rb\sum_{i=1}^{a}\frac{T_{i..}^2}{(rb)^2} - rba\frac{T_{...}^2}{(rba)^2} = \frac{\sum_{i=1}^{a}T_{i..}^2}{rb} - C$$

The proofs for SSB and SSR are analogous. For *SSI*, we have

$$SSI = r\sum_{i=1}^{a}\sum_{j=1}^{b}(\bar{x}_{ij.}-\bar{x}_{i..}-\bar{x}_{.j.}+\bar{x}_{...})^2$$

Using the identity

$$\bar{x}_{ij.}-\bar{x}_{i..}-\bar{x}_{.j.}-\bar{x}_{...} = (\bar{x}_{ij.}-\bar{x}_{...})-(\bar{x}_{i..}-\bar{x}_{...})-(\bar{x}_{.j.}-\bar{x}_{...})$$

we can write

$$SSI = r\sum\sum\left[(\bar{x}_{ij.}-\bar{x}_{...})^2+(\bar{x}_{i..}-\bar{x}_{...})^2+(\bar{x}_{.j.}-\bar{x}_{...})^2\right]$$
$$-2r\sum\sum\left[(\bar{x}_{ij.}-\bar{x}_{...})(\bar{x}_{i..}-\bar{x}_{...})+(\bar{x}_{ij.}-\bar{x}_{...})^2(x_{.j.}-x_{...})+(\bar{x}_{i..}-\bar{x}_{...})(\bar{x}_{.j.}-\bar{x}_{...})\right]$$

$$= r\sum\sum(\bar{x}_{ij.}-\bar{x}_{...})^2 + SSA + SSB - 2SSA - 2SSB - 0$$

$$= \frac{\sum_{i=1}^{a}\sum_{j=1}^{b}T_{ij.}^2}{r} - C - SSA - SSB$$

15.14 First we write the identity

$$x_{ij(k)} - \bar{x}_{..} = (\bar{x}_{i..} - \bar{x}_{..}) + (\bar{x}_{.j} - \bar{x}_{..}) + (\bar{x}_{(k)} - \bar{x}_{..}) + (x_{ij(k)} - \bar{x}_{i.} - \bar{x}_{.j} - \bar{x}_{(k)} + 2\bar{x}_{..})$$

Then we square each side of the equation and sum each term on i and j from 1 to n. Recognizing that each of the cross-product terms sums to zero, we are left with

$$\sum_{i=1}^{n}\sum_{j=1}^{n}(x_{ij(k)} - \bar{x}_{..})^2 = n\sum_{i=1}^{n}(\bar{x}_{i.} - \bar{x}_{..})^2 + n\sum_{j=1}^{n}(\bar{x}_{.j} - \bar{x}_{..})^2 + n\sum_{k=1}^{n}(\bar{x}_{(k)} - \bar{x}_{..})^2$$
$$+\sum_{i=1}^{n}\sum_{j=1}^{n}(x_{ij(k)} - \bar{x}_{i.} - \bar{x}_{.j} - \bar{x}_{(k)} + 2\bar{x}_{..})^2 \qquad \text{QED}$$

15.15 The left-hand side of the identity in Exercise 15.14 is the total sum of squares, *SST*; the terms on the right-hand side are, respectively, the row sum of squares, *SSR*, the column sum of squares, *SSC*, the treatment sum of squares, *SS*(*Tr*) and the error sum of squares, *SSE*. Thus, we can write the following analysis-of-variance table for the Latin square of size n.

Source of Variation	Degrees of Freedom	Sum of Squares	Mean Square	F
Rows	$n-1$	SSR	$SSR/(n-1)$	MSR/MSE
Columns	$n-1$	SSC	$SSC/(n-1)$	MSC/MSE
Treatments	$n-1$	$SS(Tr)$	$SS(Tr)/(n-1)$	$MS(Tr)/MSE$
Error	$(n-1)(n-2)$	SSE	$SSE/(n-1)(n-2)$	
Total	n^2-1	SST		

where $SSR = \frac{1}{n}\left(\sum_{i=1}^{n}\bar{x}_{i.}\right)^2 - C$; $\quad SSC = \frac{1}{2}\left(\sum \bar{x}_{.j}\right)^2 - C$;

$SS(Tr) = \frac{1}{n}\left(\sum_{k=1}^{n}\bar{x}_{(k)}\right)^2 - C$ where $C = \frac{1}{n^2}\sum_{j=1}^{n}\sum_{i=1}^{n}x_{ij(k)}^2$

15.16 $a = 3,\ n = 8,\ T_{1.} = 456.8,\ T_{2..} = 473.4,\ T_{3..} = 547.6,\ T_{..} = 1477.8,$

and $\sum\sum x^2 = 91{,}939.96$

$SST = 91{,}939.96 - \frac{1}{24}(14.77.8)^2 = 944.425$ (d.f. = 23)

$SS(Tr) = \frac{1}{8}(732{,}639.56) - 90{,}995.535 = 584.41$ (d.f. = 2)

$SSE = 994.425 - 584.41 = 360.015$ (d.f. = 21)

$F = \frac{584.41/2}{360.015/21} = 17.0$

Since F = 17.0 exceeds $F_{0.01,2,21} = 5.78$, null hypothesis must be rejected. The difference in effectiveness are significant.

15.17 $a = 4,\ n = 5,\ T_{1.} = 70,\ T_{2.} = 75,\ T_{3.} = 79,\ T_{4.} = 69,\ T_{..} = 293,$

and $\sum\sum x^2 = 4407$

$$SST = 4407 - \frac{1}{20}(293)^2 = 4407 - 4292.45 = 114.45 \text{ (d.f.} = 19)$$

$$SS(Tr) = \frac{1}{5}(21{,}527) - 4292.45 = 12.95 \text{ (d.f.} = 3)$$

$$SSE = 114.55 - 12.95 = 101.6 \text{ (d.f.} = 16)$$

$$F = \frac{12.95/3}{101.6/16} = 0.68$$

Since $F = 0.68$ does not exceed $F_{0.05,3,16} = 3.24$, mull hypothesis cannot be rejected. Differences among the sample means are not significant.

15.18 $a = 3,\ n = 6,\ T_{1.} = 135,\ T_{2.} = 120,\ T_{3.} = 78,\ T_{..} = 333,\ \sum\sum x^2 = 6507$

$$SST = 6507 - \frac{1}{18}(333)^2 = 6507 - 6160.5 = 346.5 \text{ (d.f.} = 17)$$

$$SS(Tr) = \frac{1}{6}(38{,}709) - 6160.5 = 291.0 \text{ (d.f.} = 2)$$

$$SSE = 346.5 - 291.0 = 55.5 \text{ (d.f.} = 15)$$

$$F = \frac{291.0/2}{55.5/15} = 39.3$$

Since $F = 39.3$ exceeds $F_{0.05,2,15} = 3.68$, mull hypothesis must be rejected. Differences in dosage have an effect.

$$\hat{\mu} = \frac{133}{18} = 18.5$$

$$\hat{\alpha}_1 = \frac{135}{6} - 18.5 = 4.0, \quad \hat{\alpha}_2 == \frac{120}{6} - 18.5 = 1.5,$$

$$\alpha_3 = \frac{78}{6} - 18.5 = -5.5$$

15.19 $a = 4,\ n_1 = 8,\ n_2 = 8,\ n_3 = 6,\ n_4 = 9,\ N = 31,\ T_{1.} = 574,\ T_{2.} = 547,$

$T_{3.} = 449,\ T_{4.} = 584,\ T_{..} = 2154$

$$\sum\sum x^2 = 41{,}386 + 37{,}491 + 33{,}683 + 38{,}064 = 150{,}624$$

$$SST = 150{,}624 - \frac{1}{31}(2154)^2 = 150{,}624 - 149{,}668.26 = 955.74$$

$$SS(Tr) = (41{,}184.5 + 37{,}401.125 + 33{,}600.17 + 37{,}895.11) - 149.668.26 = 412.645$$

$$SSE = 955.74 - 412.645 = 543.095$$

$$F = \frac{412{,}645/3}{543.095/27} = 6.84 \qquad F_{0.05,3,27} = 2.99$$

Differences cannot be attributed to chance.

15.20 $a=3,\ n_1=4,\ n_2=2,\ n_3=3,\ N=9,\ T_{1.}=1908,\ T_{2.}=990,\ T_{3.}=1445,$
$T_{..}=4343$

$\sum\sum x^2 = 910,662+490,068+696,725 = 2,097,455$

$SST = 2,097,45-\frac{1}{9}(4343)^2 = 2,097,415-2,095,738.8 = 1676.2$ (d.f. $=8$)

$SS(Tr) = 910,116+490,050+696,008.3-2,095,738.8 = 435.5$ (d.f. $=2$)

$SSE = 1676.2-435.5 = 1240.7$ (d.f. $=6$)

$F=\frac{435.5/2}{1240.7/6}=1.05 \qquad F_{0.05,2,6}=5.14$

Null hypothesis cannot be rejected; differences can be attributed to chance.

15.21 $a=3,\ n_1=400,\ n_2=500,\ n_3=400,\ N=1300,\ T_{1.}=81,\ T_{2.}=72,$
$T_{3.}=43,\ T_{..}=196, \sum\sum x^2 = 840$

$SST = 840-\frac{1}{1300}(196)^2 = 840-29.95 = 810.45$ (d.f. $=1299$)

$SS(Tr) = (16.40+10.37+4.62)-29.95 = 1.84$ (d.f. $=2$)

$SSE = 808.61$ (d.f. $=1297$)

$F=\frac{1.84/2}{808.61/1297}=1.48 \qquad F_{0.05,2,1297}=3.00$

Null hypothesis cannot be rejected. Result same as in Ex. 13.74.

15.22

	−1	0	1	
A	12	23	89	124
B	8	12	62	82
C	21	30	119	170
	41	65	270	

$k=3,\ n_1=124,\ n_2=82$
$n_3=170,\ N=376$
$T_{1.}=77,\ T_{2.}=54,\ T_{3.}=98$
$T_{..}=229,\ \sum\sum x^2 = 311$

$SST = 311-\frac{1}{376}(229)^2 = 311-139.47 = 171.53$ (d.f. = 375)

$SS(Tr) = (47.81+35.56+56.49)-139.47 = 0.39$ (d.f.) = 2

$SSE = 171.35-0.39 = 170.96$ (d. f. = 373)

$F=\frac{0.39/2}{170.96/373}=0.43 \qquad F_{0.01,2,373}=4.61$

Null hypothesis cannot be rejected. Result same as in Ex. 13.73.

15.23 $a = 3,\ n = 4,\ \mathrm{T}_{1.} = 197.4,\ T_{2.} = 185.9\ T_{3.} = 206.0,\ \mathrm{T}_{.1} = 137.6,$

$T_{.2} = 165.5,\ T_{.3} = 157.6,\ T_{.4} = 128.6,\ T_{...} = 589.3$

$\sum\sum x^2 = 9{,}888.3 + 8{,}732.45 + 10.697.8 = 29{,}318.55$

$SST = 29{,}318.55 - \frac{1}{12}(589.3)^2 = 29{,}318{,}55 - 28{,}939.54 = 379.01\ \ (\text{d.f.} = 11)$

$SS(Tr) = \frac{1}{4}(115{,}961.57 - 28{,}939.54 = 50.85\ (\text{d.f.} = 2)$

$SSB = \frac{1}{3}(87{,}699.73) - 28{,}939.54 = 293.70\ \ (\text{d.f.} = 3)$

$SSE = 379.01 - 50.85 - 293.70 = 34.46\ (\text{d.f.} = 6)$

$F_{Tr} = \frac{50.85/2}{34.46/6} = 4.43 \qquad F_B = \frac{293.70/3}{34.46/6} = 17.05$

$F_{0.01,2,6} = 10.9 \qquad F_{0.01,3,6} = 9.78$

Since $F = 4.43 < 10.9$, null hypothesis for launchers cannot be rejected. Since $F = 17.05 > 9.78$, mull hypothesis for fuels must be rejected. Difference among fuels is significant.

15.24 $a = 4,\ n = 3,\ \mathrm{T}_{1.} = 8.8,\ T_{2.} = 8.8,\ T_{3.} = 9.7,\ T_{4.}$=10.3, $\mathrm{T}_{.1} = 13.2,$

$T_{.2} = 11.4,\ T_{.3} = 13.0,\ T_{...} = 37.16$

$\sum\sum x^2 = 26.16 + 25.9 + 31.45 + 35.55 = 119.06$

$SST = 119.06 - \frac{1}{12}(37.6)^2 = 119.06 - 117.818 = 1.25\ (\text{d.f.} = 11)$

$SS(Tr) = \frac{1}{3}(355.06) - 117.81 = 0.54\ (\text{d.f.} = 3)$

$SSB = \frac{1}{4}(473.2) - 117.81 = 0.49\ (\text{d.f.} = 2)$

$SSE = 1.25 - 0.54 - 0.49 = 0.22\ (\text{d.f.} = 6)$

$F_{Tr} = \frac{0.54/3}{0.22/6} = 4.91 \qquad F_B = \frac{0.49/2}{0.22/6} = 6.68$

$F_{0.05,3,6,} = 4.76 \qquad F_{0.05,2,6} = 5.14$

Since 4.91 > 4.76, null hypothesis for laboratories must be rejected.
Since 6.68 > 5.14, null hypothesis for diet foods must be rejected.

15.25 $a = 5,\ n = 4,\ \mathrm{T}_{1.} = 83.1,\ T_{2.} = 103,\ T_{3.} = 94.5,\ \mathrm{T}_{4.} = 95.2,\ T_{5.} = 85,$

$T_{.1} = 115.8,\ T_{.2} = 112.1,\ T_{.3} = 114,\ T_{.4} = 118.9,\ T_{...} = 460.8$

$\sum\sum x^2 = 17.28.59 + 2655.48 + 2241.47 + 2277.22 + 1810.42 = 10{,}713.18$

$SST = 10{,}713.18 - \frac{1}{20}(460.8)^2 = 10{,}713.18 - 10{,}616.83 = 96.35\ (\text{d.f.} = 19)$

$SS(Tr) = \frac{1}{4}(42{,}732.9) - 10{,}616.83 = 66.40\ (\text{d.f.} = 4)$

$SSB = \frac{1}{5}(53{,}109.26) - 10{,}616.83 = 5.02\ (\text{d.f.} = 3)$

$SSE = 96.35 - 66.40 - 5.02 = 24.93\ (\text{d.f.} = 12)$

$F_{Tr} = \dfrac{66.40/4}{24.93/12} = 7.99$ $\qquad F_B = \dfrac{5.02/3}{24.93/12} = 0.81$

$F_{0.05,4,12,} = 3.26$ $\qquad F_{0.05,3,12} = 3.49$

$F_{Tr} = 7.99$ (for threads) is significant. $F_B = 0.81$ (for measuring instruments) is not significant.

15.26

	Teacher	Lawyer	Doctor
East	I	R	D
South	R	D	I
West	D	I	R

I = independent
R = Republican
D = Democrat

Completing the Latin Square, we find that
Doctor who is a Western is a *Republican.*

15.27 Summing the observations in each replicate, we have $T_{..1} = 589.3$, $T_{..2} = 595.8$.
Summing over the two replicates, we obtain the following two-way table:

	Fuels				
Launchers	1	2	3	4	Totals
X	92.0	113.5	104.8	86.0	396.3
Y	92.3	103.1	101.5	78.4	375.3
Z	91.5	114.8	111.5	95.7	413.5
Totals	275.8	331.4	317.8	260.1	1,185.1

$$C = \frac{(1{,}185.1)^2}{24} = 58{,}519.25$$

$$SS(\text{Total}) = (45.9)^2 + (57.6)^2 + \ldots + (47.6)^2 - C = 721.04$$

$$SS(\text{Launchers}) = [(396.3)^2 + (375.3)^2 + (413.4)^2]/8 - C = 91.50$$

$$SS(\text{Fuels}) = [(275.8)^2 + (331.4)^2) + (317.8)^2 + (260.1)^2]/6 - C = 570.83$$

$$SS(\text{Replicates}) = [(589.3)^2 + (595.8)^2]/12 - C = 1.76$$

$$SS(\text{Interaction}) = [(92.0)^2 + (113.5)^2 + \ldots + (95.7^2)]/2 - C - SS(\text{Launchers}) - SS(\text{Fuels}) = 50.94$$

$$SS(\text{Error}) = SST - SS(\text{Launchers}) - SS(\text{Fuels}) - SS(\text{Replicates}) - SS(\text{Interaction}) = 6.01$$

ANALYSIS OF VARIANCE

Source of Variation	Degrees of Freedom	Sum of Squares	Mean Square	F	Critical $F_{0.01}$
Launchers	2	91.40	45.75	83.2	7.21
Fuels	3	570.83	190.28	346.0	6.22
Replicates	1	1.76	1.76	3.2	9.65
Interaction	6	50.94	8.49	15.4	5.07
Error	11	6.01	0.55		
Total	23	721.04			

Thus, the Launchers, Fuels, and Interaction means are significantly different at the 0.01 level of significance.

15.28 Summing the observations in each replicate, we have $T_{..1} = 37.6,\ T_{..2} = 39.0$. Summing over the two replicates, we obtain the following two-way table:

	Foods			
Laboratories	A	B	C	Totals
1	6.9	5.1	5.7	17.7
2	6.0	5.6	6.3	17.9
3	6.9	6.4	7.2	20.5
4	6.8	6.6	7.1	20.5
Totals	26.6	23.7	26.3	76.6

$$C = \frac{(76.6)^2}{24} = 244.48$$

$$SS(\text{Total}) = 247.28 - C = 2.80$$

$$SS(\text{Laboratories}) = 245.70 - C = 1.22$$

$$SS(\text{Foods}) = 245.12 - C = 0.64$$

$$SS(\text{Replicates}) = 244.56 - C = 0.08$$

$$SS(\text{Interaction}) = 246.89 - C - SS(\text{Laboratories}) - SS(\text{Foods}) = 0.55$$

$$SS(\text{Error}) = SST - SS(\text{Laboratories}) - SS(\text{Foods}) - SS(\text{Replicates}) - SS(\text{Interaction}) = 0.31$$

ANALYSIS OF VARIANCE

Source of Variation	Degrees of Freedom	Sum of Squares	Mean Square	F	Critical $F_{0.05}$
Laboratories	3	1.22	0.41	13.7	3.59
Foods	2	0.64	0.32	10.7	3.98
Replicates	1	0.08	0.08	2.7	4.84
Interaction	6	0.55	0.09	3.0	3.09
Error	11	0.31	0.03		
Total	23	2.80			

Thus, the Laboratories and Foods means are significantly different at the 0.05 level of significance.

15.29 Summing the observations in each replicate, we have $T_{..1} = 122.8,\ T_{..2} = 122.7$. Summing over the two replicates, we obtain the following two-way table:

	Bonders				
Operators	A	B	C	D	Totals
1	22.4	21.5	22.4	20.1	86.4
2	22.4	22.7	21.0	22.1	88.2
3	21.4	20.1	20.5	8.9	70.9
Totals	66.2	64.3	63.9	51.1	245.5

$$C = \frac{(245.5)^2}{24} = 2{,}511.26$$

$SS(\text{Total}) = 2{,}609.51 - C = 98.25$

$SS(\text{Operators}) = 2{,}533.88 - C = 22.62$

$SS(\text{Bonders}) = 2{,}535.23 - C = 23.97$

$SS(\text{Replicates}) = 2{,}511.26 - C = 0.00$

$SS(\text{Interaction}) = 2{,}588.84 - C - SS(\text{Operators}) - SS(\text{Bonders}) = 30.99$

$SS(\text{Error}) = SST - SS(\text{Operators}) - SS(\text{Bonders}) - SS(\text{Replicates}) - SS(\text{Interaction}) = 20.67$

ANALYSIS OF VARIANCE

Source of Variation	Degrees of Freedom	Sum of Squares	Mean Square	F	Critical $F_{0.05}$
Operators	2	22.62	11.31	6.02	3.98
Bonders	3	23.97	7.99	4.25	3.59
Replicates	1	0.00	0.00	0.00	4.84
Interaction	6	30.99	5.17	2.75	3.09
Error	11	20.67	1.88		
Total	23	98.25			

Thus, the Operators and Bonders means are significantly different at the 0.05 level of significance.

15.30 Summing the observations in each replicate, we have $T_{..1} = 266.6$, $T_{..2} = 267.0$, $T_{..3} = 262.5$, $T_{..4} = 270.6$.
Summing over the two replicates, we obtain the following two-way table:

	DSS				
Time	0	50	100	150	Totals
1	138.1	140.3	141.9	144.1	564.4
2	112.8	123.4	131.5	134.6	502.3
Totals	250.9	263.7	273.4	278.7	1,066.7

$C = \frac{(1{,}066.7)^2}{24} = 35{,}557.78$

$SS(\text{Total}) = 35{,}765.15 - C = 207.37$

$SS(\text{DSS}) = 35{,}613.72 - C = 55.94$

$SS(\text{Time}) = 35{,}678.29 - C = 120.51$

$SS(\text{Replicates}) = 35{,}561.90 - C = 4.12$

$SS(\text{Interaction}) = 35{,}754.23 - C - SS(DSS) - SS(\text{Time}) = 20.00$

$SS(\text{Error}) = SST - SS(DSS) - SS(\text{Time}) - SS(\text{Replicates}) - SS(\text{Interaction}) = 6.80$

ANALYSIS OF VARIANCE

Source of Variation	Degrees of Freedom	Sum of Squares	Mean Square	F	Critical $F_{0.05}$
DSS Level	3	55.94	18.65	58.28	3.07
Time	1	120.51	120.51	376.59	4.32
Replicates	3	4.12	1.37	4.28	3.07
Interaction	3	20.00	6.67	20.84	3.07
Error	21	6.80	0.32		
Total	31	207.37			

Thus, the DSS, Time, Replicates, and Interaction means are all significantly different at the 0.05 level of significance.

15.31 The three detergent means are: A: 77.0 B: 68.0 C: 80.0

$$s_{\bar{x}} = \sqrt{\frac{MSE}{n}} = \sqrt{\frac{23}{5}} = 2.14$$

For Table IX, with $\alpha = 0.01$ and 12 d.f. and using $R_p = r_p \cdot s_{\bar{x}}$ we get

p	2	3
r_p	4.32	4.50
R_p	9.24	9.63

We obtain

Detergents:	B	A	C
Means:	68.0	<u>77.0</u>	<u>80.0</u>

and we conclude that detergents A and C do not give rise to significantly different means at the 0.01 level so significance.

15.32 The five block means are: 24.75, 27.50, 28.25, 27.75, and 30.75. Proceeding as in Exercise 15.31 with

$$s_{\bar{x}} = \sqrt{\frac{2.27}{4}} = 0.75$$

we obtain from Table IX, with $\alpha = 0.05$ and 12 d.f.

p	2	3	4	5
r_p	3.08	3.23	3.31	3.37
R_p	2.31	2.42	2.48	2.53

Thus,

Blocks:	Monday	Tuesday	Thursday	Wednesday	Friday
Means:	24.75	<u>27.50</u>	<u>27.75</u>	<u>28.25</u>	30.75

and we conclude that there is no significant difference among the means for Tuesday, Wednesday, and Thursday at the 0.05 level of significance.

15.33 The four compressor-design means are: 46.50, 22.63, 61.25, and 48.00. The four region means are: 52.88, 40.50, 52.88, and 32.13. With

$$s_{\bar{x}} = \sqrt{\frac{65}{8}} = 2.65$$

For both designs and regions, and from Table IX with $\alpha = 0.05$ and 15 d. f., we get

p	2	3	4
r_p	3.01	3.16	3.25
R_p	8.58	9.01	9.26

Thus,

Designs:	B	A	D	C
Means:	22.63	46.50	48.00	61.25

Regions:	Southwest	Southeast	Northwest	Northeast
Means:	32.13	40.50	52.88	52.88

We conclude, at the 0.05 level of significance, that designs A and D do not give rise to significantly different means and that the same is true for the Southwest and Southeast and for the Northwest and northeast regions.

15.34 The three diet-food means are: 3.33, 2.96, and 3.29. The four laboratory means are 2.95, 2.98, 3.42, and 3.42. With

$$\textit{Diet foods}: \ s_{\bar{x}} = \sqrt{\frac{0.03}{8}} = 0.06; \qquad \textit{Laboratories}: \ s_{\bar{x}} = \sqrt{\frac{0.03}{6}} = 0.07$$

and using Table IX with $\alpha = 0.05$ and 11 d.f., we get

	Diet Foods		Laboratories		
p	2	3	2	3	4
r_p	3.11	3.26	3.11	3.26	3.34
R_p	0.19	0.20	0.22	0.23	0.23

Thus

	B	C	A	1	2	3	4
Means:	2.96	3.29	3.33	2.95	2.98	3.42	3.42

We conclude, at the 0.05 level of significance, that diet foods A and C, laboratories 1 and 2, and laboratories 3 and 4 do not give rise to significantly different means.

15.35 The three launcher means are: 49.54, 46.91, and 51.69. The four fuel means are: 45.97, 55.23, 52.97, and 43.35. With

$$\textit{Launchers}:\ s_{\bar{x}} = \sqrt{\frac{0.55}{8}} = 0.26; \qquad \textit{Fuels}:\ s_{\bar{x}} = \sqrt{\frac{0.55}{6}} = 0.30$$

and using Table IX with $\alpha = 0.01$ and 11 d.f., we get

	Launchers		Fuels		
p	2	3	2	3	4
r_p	4.39	4.58	4.39	4.58	4.70
R_p	1.14	1.19	1.32	1.37	1.41

Thus

	Y	X	Z	4	1	3	2
Means:	46.91	49.54	51.69	43.35	45.97	52.97	55.23

We conclude, at the 0.01 level of significance, that fuels 2 and 3 are not associated with significantly different means.

15.36 The DSS means are: 31.36, 32.96, 34.18, and 34.84. With

$$\textit{DSS Level}:\ s_{\bar{x}} = \sqrt{\frac{1.37}{8}} = 0.41; \qquad \textit{Time}:\ s_{\bar{x}} = \sqrt{\frac{1.37}{16}} = 0.29$$

and using Table IX with $\alpha = 0.05$ and 21 d.f., we get

	DSS Level			Time
p	2	3	4	2
r_p	2.95	3.10	3.19	2.95
R_p	1.21	1.27	1.31	0.86

Thus

	0	50	100	150	28	7
Means:	31.36	32.96	34.18	34.84	31.89	35.28

We conclude, at the 0.05 level of significance, that the means associated with DSS levels 100 and 150 are significantly different.

15.37 The Bonder means are: 11.03, 10.72, 10.65, and 8.52. The Operator means are: 10.80, 11.03, and 8.85. With

$$\textit{Bonders: } s_{\bar{x}} = \sqrt{\frac{1.88}{6}} = 0.56; \qquad \textit{Operators: } s_{\bar{x}} = \sqrt{\frac{1.88}{8}} = 0.48$$

And using Table IX with $\alpha = 0.05$ and 11 d.f., we get

	Bonders			Operators	
P	2	3	4	2	3
r_p	3.11	3.26	3.34	3.11	3.26
R_p	1.74	1.83	1.87	1.49	1.56

Thus,

	D	C	B	A	3	1	2
Means:	8.52	10.65	10.72	11.03	8.86	10.80	11.03

We conclude, at the 0.05 level of significance, that the mean bonding strengths for bonders A, B, and C are not significantly different, nor are those for operators 1 and 2.

15.38 $m = 3,\ T_{1.} = 230,\ T_{2.} = 260,\ T_{3.} = 246,\ T_{.1} = 240,\ T_{.2} = 248,$

$T_{.3} = 248,\ T_A = 244,\ T_B = 274,\ T_C = 218,\ T_{...} = 736$

$\sum\sum x^2 = 17{,}782 + 22{,}662, + 20{,}438 = 60{,}882$

$SST = 60{,}882 - \frac{1}{9}(736)^2 = 60{,}882 - 60{,}188.44 = 693.56$ (d.f.= 8)

$SSR = \frac{1}{3}(181{,}016) - 60{,}188.4 = 150.23$ (d.f. = 2)

$SSC = \frac{1}{3}(180.608) - 60{,}188.44 = 14.23$ (d.f. = 2)

$SS(Tr) = \frac{1}{3}(182{,}136) - 60{,}188.44 = 523.56$ (d.f. = 2)

$SSE = 693.56 - 150.23 - 14.23 - 523.56 = 554$ (d.f. = 2)

$F_R = \frac{150.23/2}{5.54/2} = 27.12,\ \ F_C = \frac{14.23/2}{5.54/2} = 2.57,\ \ F_{Tr} = \frac{5.23.56/2}{5.54/2} = 94.51$

$F_{0.05,2,2} = 19.0$

(a) F_{Tr} (for instructor) = 94.5 is significant

(b) F_C = 2.57 (for ethnic background) is not significant

(c) F_R = 27.12 (for professional interest) is significant

15.39 (a) First we calculate the following totals: $T_{..} = 2{,}030,\ T_{1.} = 645,\ T_{2.} = 771,\ T_{3.} = 614$, $T_{.1} = 913,\ T_{.2} = 380,\ T_{.3} = T_{(1)} = 680,\ T_{(2)} = 646,\ T_{(3)} = 704$. The correction term is $C = T_{..}^2 / 9 = 457{,}878$. The total sum of squares is the sum of the squares of the nine observations, minus the correction term. The sums of squares for rows, columns, and treatments, respectively, is the sum of the squares of the corresponding totals, divided by 3, minus the correction term. For example, the sum of squares for rows is $(645^2 + 771^2 + 614^2)/3 - C = 4{,}609$. We then get the following analysis-of-variance table:

Source of Variability	Degrees of Freedom	Sum of Squares	Mean Square	f
Rows	2	4,609	2,305	10.4
Columns	2	49,168	24,584	111
Treatments	2	566	283	1.28
Error	2	441	221	
Total	8	54,784		

(b) No. With only 2 degrees of freedom for error, the f-tests have very little power.

15.40 (a) First we calculate the following totals: $T_{..} = 763.5,\ T_{1.} = 154.2,\ T_{2.} = 151.7,\ T_{3.} = 143.2$ $T_{4.} = 154.3,\ T_{5.} = 150.1,\ T_{.1} = 161.4,\ T_{.2} = 164.8,\ T_{.3} = 152.1,\ T_{.4} = 124.1,\ T_{.5} = 161.1,$ $T_{(1)} = 156.8,\ T_{(2)} = 150.9,\ T_{(3)} = 152.2,\ T_{(4)} = 154.1,\ \mathrm{T}_{(5)} = 149.5,$ and $T_{..} = 763.5$. The correction term is $C = T_{..}^2 / 9 = 457{,}878$. The total sum of squares is the sum of the squares of the nine observations, minus the correction term. The sums of squares for rows, columns, and treatments, respectively, is the sum of the squares of the corresponding totals, divided by 5, minus the correction term. For example, the sum of squares for rows is $(154.2^2 + 151.7^2 + ... + 150.1^2)/5 - C = 244{,}76$. We then get the following analysis-of-variance table:

Source of Variability	Degrees of Freedom	Sum of Squares	Mean Square	f
Rows	4	2.56	0.64	<1
Columns	4	222.20	55.55	49.4
Treatments	4	6.50	1.63	1.45
Error	12	13.50	1.125	
Total	24	54,784		

15.41 (a)

Factor	Level 1	Level 2	Level 3	Level 4
A	1	2		
B	1	2	3	
C	1	2	3	4

(b) For r replicates, the total degrees of freedom is $24r - 1$. This leaves $24r - 1 - 23 - (r-1) = 23(r-1)$ degrees of freedom for error. For there to be at least 30 degrees of freedom for error, r must be at least 3 replicates.

(c) The only three-factor interaction is ABC, with 6 degrees of freedom. Without replication, and assuming ABC = 0, there would be only 6 degrees of freedom for error.

15.42 The analysis of variance shows the following significant effects (effects having P-values less than or equal to 0.05).

Effect	df	Mean Square	f	P
A	1	270.28	12.45	0.003
B	1	205.03	9.45	0.007
C	1	124.03	5.71	0.029
E	1	357.78	16.49	0.001
CE	1	157.53	7.26	0.016

15.43 There are 16 three-factor and higher-order interactions. If it is assumed that they do not exist, there will be 16 degrees for freedom for error.

15.44 MINITAB software provides a table of means for the main effects. Here are the means for the significant main effects.

Level	N	A	Level	N	B	Level	N	C	Level	N	E
1	16	44.063	1	16	38.625	1	16	43.125	1	16	37.812
2	16	38.250	2	16	43.688	2	16	39.188	2	16	44.500

Each main effect is the difference between its mean at level 2 and at level 1. Thus, the significant main effects are:

$$A = -5.813, \ B = 5.063, \ C = -3.937, \ E = 6.688$$

15.45. No. The effects C and E interact with each other.

15.47 Increasing temperature from 68° to 74°F decreases the gain by 5.813. Increasing the partial pressure from 10^{-15} to 10^{-4} increases the gain by 5.063. While there was only a negligible change in the gain when the relative humidity was increased in the laboratory from 1% to 30% (an increase of 0.5), the gain decreased by 20%, from 42.000 to 33.625, on the production line. (Confidence intervals should be constructed for these estimates.)