Chapter 16

16.1 **(a)** $t = \frac{\bar{x}}{s/\sqrt{2}} = \frac{\sqrt{2}(x_1 + x_2)}{2\frac{\sqrt{(x_1 - x_2)^2 + (x_2 - x_1)^2}}{4}} = \frac{x_1 + x_2}{x_1 - x_2}$

(b) Since the function $f(x) = \frac{x_1 + x_2}{x_1 - x_2}$ is decreasing for $x > x_2 > 0$

it follows that $\lim_{x \to \infty} f(x) = 1 < t' = f(10x_1) < t = f(x_1)$

16.2 When $T^+ = k$ then $T^- = \frac{n(n+1)}{2} - k$ and then

$$P(T^+ = k) = P\left(T^- = \frac{n(n+1)}{2} - k\right)$$
$$= P\left(T^+ = \frac{n(n+1)}{2} - k\right)$$

So that distribution is symmetrical about $\frac{n(n+1)}{4}$.

$$P\left(T^+ = \frac{n(n+1)}{4} + c\right) = P\left(T^- = \frac{n(n+1)}{4} - c\right)$$
$$= P\left(T^+ = \frac{n(n+1)}{4} - c\right)$$

16.3 $T^+ - T^- = T^+ - \left[\frac{n(n+1)}{2} - T^+\right] = 2T^+ - \frac{n(n+1)}{2} = X$

$E(X) = 2 \cdot \frac{n(n+1)}{4} - \frac{n(n+1)}{2} = 0$ by Theorem 16.1

$\text{var}(X) = 4 \cdot \frac{n(n+1)(2n+1)}{24}$ by Theorem 16.1

$= \frac{n(n+1)(2n+1)}{6}$

16.4 $n = 5$, $P(T = 0) = [P(x = 0)]^5 = (0.5)^5 = 0.031 > 0.02$, where x is a Bernoulli variable. Therefore, $T_{0.02}$ does not exist for $n = 5$.

16.5 **(a)** $U_1 + U_2 = W_1 - \frac{n_1(n_1+1)}{2} + W_2 - \frac{n_2(n_2+1)}{2}$

$$= \frac{(n_1 + n_2)(n_1 + n_2 + 1)}{2} - \frac{n_1(n_1+1)}{2} - \frac{n_2(n_2+1)}{2}$$
$$= n_1 n_2$$

(b) $\min U_1 = \dfrac{n_1(n_1+1)}{2} - \dfrac{n_1(n_1+1)}{2} = 0$

$$\max U_1 = \frac{n_1}{2}(n_1+1+n_2+n_1) - \frac{n_1(n_1+1)}{2} - \frac{n_2(n_2+1)}{2}$$
$$= n_1 n_2$$

Same proofs for U_2.

16.6

r_i

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$n_1+n_2+1-r_i$

From left to right we get $W_1 = \sum r_i$ and right to left we get $W_1 = n_1(n_1+n_2+1) - \sum r_i$.
Probabilities are the same.

$P(W_1) = P(n_1\{n_1+n_2+1\} - W_2)$ $\quad\therefore$ symmetrical about $\dfrac{n_1(n_1+n_2+1)}{2}$

when $W_1 = \dfrac{n_1(n_1+n_2+1)}{2}$

$$U_1 = \frac{n_1(n_1+n_2+1)}{2} - \frac{n_1(n_1+1)}{2} = \frac{n_1 n_2}{2}$$

16.7 $U_1 = W_1 - \dfrac{n_1(n_1+1)}{2} = \dfrac{(n_1+n_2)(n_1+n_2+1)}{2} - W_2 - \dfrac{n_1(n_1+1)}{2}$

$$= \frac{n_1 n_2}{2} + \frac{n_2(n_1+n_2+1)}{2} - W_2$$
$$= n_1 n_2 + \frac{n_2(n_2+n_1)}{2} - W_2$$

Proof is same for U_2.

16.8 Ranking of x's are $r_1 < r_2 < r_3 < \ldots < r_{n_1}$

r_1 $\quad r_2$ $\quad r_3$ $\quad r_{n_1}$

y's ---------|--------------|----------------|----------------|----------------|----

$r_1 - 1$ $\quad r_2 - r_1 - 1$ $\quad r_3 - r_2 - 1$ $\quad \ldots$ $\quad r_{n_1} - r_{n_1-1} - 1$

Number of y's preceding r_1 is $r_1 - 1$
Number of y's preceding r_2 is $(r_1-1)+(r_2-r_1-1) = r_2 - 2$
Number of y's preceding r_3 is $(r_1-1)+(r_2-r_1-1)+(r_3-r_2-1) = r_3 - 3$
$\vdots$
Number of y's preceding $r_{n_1} = r_{n_1} - n_1$

$$\sum\sum d_{ij}^2 = \sum_{i=1}^{n_1} r_i - (1+2+3+\ldots n_1) = W_2 - \frac{n_1(n_1+1)}{2}$$

16.9 $$H = \frac{12}{n(n+1)} - \sum_{i=1}^{k} n_1 \left[\frac{R_i}{n_i} - \frac{n+1}{2} \right]^2$$
$$= \frac{12}{n(n+1)} \left[\sum_{i=1}^{k} \frac{R_i^2}{n_i} - (n+1) \sum_{i=1}^{k} R_i + \left(\frac{n+1}{2} \right)^2 \sum_{i=1}^{k} n_i \right]$$
$$= \frac{12}{n(n+1)} \sum_{i=1}^{k} \frac{R_i^2}{n_i} - \frac{12}{n} \sum_{i=1}^{k} R_i + \frac{12}{n(n+1} \cdot \frac{(n+1)^2}{4} \cdot n$$
$$= \frac{12}{n(n+1)} \sum_{i=1}^{k} \frac{R_i^2}{n_i} - \frac{12}{n} \cdot \frac{n(n+1)}{2} + 3(n+1)$$
$$= \frac{12}{n(n+1)} \sum_{i=1}^{k} \frac{R_i^2}{n_i} - 6(n+1) + 3(n+1)$$
$$= \frac{12}{n(n+1)} \sum_{i=1}^{k} \frac{R_i^2}{n_i} - 3(n+1) \qquad \text{QED}$$

16.10 $T_i = R_i, \quad \sum n_i = n, \quad T_{..} = \dfrac{n(n+1)}{2}$

$$\sum \sum x^2 = \frac{n(n+1)(2n+1)}{6}$$
$$SST = \frac{n(n+1)(2n+1)}{6} - \frac{1}{n} \left[\frac{n(n+1)}{2} \right]^2 = \frac{n(n^2-1)}{12} \qquad (\text{d.f.} = n-1)$$
$$SST_r = \sum_{i=1}^{k} \frac{R_i^2}{n_i} - \frac{1}{n} \left[\frac{n(n+1)}{`2} \right]^2 = \sum_{i=1}^{k} \frac{R_i^2}{n_i} - \frac{n(n+1)^2}{4} \qquad (\text{d.f.} = k-1)$$
$$SSE = SST - SST_r = \frac{n(n+1)(2n+1)}{6} - \sum_{i=1}^{k} \frac{R_i^2}{n_i} \qquad (\text{d.f.} = n-k)$$

Since $\dfrac{n(n+1)}{12} H = \displaystyle\sum_{i=1}^{k} \frac{R_i}{n_i} - \frac{n(n+1)^2}{4}$

$$SST_r = \frac{n(n+1)}{12} H$$
$$SSE = \frac{n(n+1)(2n+1)}{6} - \frac{n(n+1)}{12} H - \frac{n(n+1)^2}{4} = \frac{n(n^2-1)}{12} - \frac{n(n+1)}{12} H$$
$$F = \frac{\dfrac{n(n+1)}{12(k-1)} H}{\dfrac{n(n^2-1)}{12(n-k)} - \dfrac{n(n+1)}{12(n-k)} H} = \frac{\dfrac{n-k}{k-1} H}{(n-1) - H}$$

The test based on F is equivalent to test based on H.

16.11 $k+1$ runs of first kind and k

runs of second kind in $\binom{n_1-1}{k}\binom{n_2-1}{k-1}$ ways

k runs of first kind and

$k+1$ runs of second kind in $\binom{n_1-1}{k-1}\binom{n_2-1}{k}$ ways

In total $\binom{n_1-1}{k}\binom{n_2-1}{k-1}+\binom{n_1-1}{k-1}\binom{n_2-1}{k}$ ways

So $f(2k+1)=\dfrac{\binom{n_1-1}{k}\binom{n_2-1}{k-1}+\binom{n_1-1}{k-1}\binom{n_2-1}{k}}{\binom{n_1+n_2}{n_1}}$ QED

16.12 $n_1=7,\ n_2=3$

$$f(2)=\frac{2\binom{6}{0}\binom{2}{0}}{\binom{10}{7}}=\frac{2}{120}=\frac{1}{60};\ f(3)=\frac{\binom{6}{1}\binom{2}{0}+\binom{6}{0}\binom{2}{1}}{120}=\frac{8}{120}=\frac{4}{60}$$

$$f(4)=\frac{2\binom{6}{1}\binom{2}{1}}{120}=\frac{24}{120}=\frac{12}{60};\ f(5)=\frac{\binom{6}{2}\binom{2}{1}+\binom{6}{1}\binom{2}{2}}{120}=\frac{36}{120}=\frac{18}{60}$$

$$f(6)=\frac{2\binom{6}{2}\binom{2}{2}}{120}=\frac{30}{120}=\frac{15}{16};\ f(7)=\frac{\binom{6}{3}\binom{2}{2}+\binom{6}{2}\binom{2}{3}}{120}=\frac{20}{120}=\frac{10}{60}$$

16.13 $f(8)=\dfrac{2\binom{5}{3}\binom{4}{3}}{\binom{11}{6}}=\dfrac{2\cdot10\cdot4}{462}=\dfrac{80}{462}$

$$f(9)=\frac{\binom{5}{4}\binom{4}{3}+\binom{5}{3}\binom{4}{4}}{462}=\frac{5\cdot4+10\cdot1}{462}=\frac{30}{462}$$

$$f(10)=\frac{2\binom{5}{4}\binom{4}{4}}{462}=\frac{2\cdot5\cdot1}{462}=\frac{10}{462}$$

$$f(11)=\frac{\binom{5}{5}\binom{4}{4}+\binom{5}{4}\binom{4}{5}}{462}=\frac{1}{462}$$

$$f(8)+f(9)+f(10)+f(11)=\frac{121}{462}=\frac{11}{42}$$

16.14 $f(2) = \dfrac{\binom{7}{0}\binom{7}{0}}{\binom{16}{8}} = \dfrac{2}{12{,}870} = 0.000155$

$$f(3) = \frac{\binom{7}{1}\binom{7}{0} + 2\binom{7}{0}\binom{7}{1}}{12{,}870} = \frac{14}{12{,}870} = 0.001088$$

$$f(4) = \frac{\binom{7}{1}\binom{7}{1}}{12{,}870} = \frac{98}{12{,}870} = 0.007615$$

$f(2) + f(3) = 0.001243$

$f(2) + f(3) + f(4) = 0.008858$

$$f(16) = \frac{\binom{7}{7}\binom{7}{7}}{12{,}870} = \frac{2}{12{,}870} = 0.000155$$

$$f(15) = \frac{\binom{7}{7}\binom{7}{6} + \binom{7}{6}\binom{7}{7}}{12{,}870} = \frac{14}{12{,}870} = 0.001088$$

$$f(14) = \frac{\binom{7}{6}\binom{7}{6}}{12{,}870} = \frac{98}{12{,}870} = 0.007615$$

Reject null hypothesis for $U = 2$, 3, 15, and 16

16.15 $W = 0$ makes $R_i = \dfrac{k(n+1)}{2}$ for each value of i; it reflects a complete lack of association.

There is complete agreement, for instance, when $R_i = ki$ and

$$\begin{aligned}
W &= \frac{12}{n(n^2-1)} \sum_{i=1}^{n} \left[i - \frac{n+1}{2} \right]^2 \\
&= \frac{12}{n(n^2-1)} \left[\sum_{i=1}^{n} i^2 - (n+1)\sum_{i=1}^{n} i + \frac{n(n+1)^2}{4} \right] \\
&= \frac{12}{n(n^2-1)} \left[\frac{n(n+1)(2n+1)}{6} - \frac{(n+1)n(n+1)}{2} + \frac{n(n+1)^2}{4} \right] \\
&= \frac{1}{n-1} \{ 2(2n+1) - 6(n+1) + 3(n+1) \} \\
&= 1
\end{aligned}$$

16.16 $\mu = 20 \cdot \dfrac{1}{2} = 10 \qquad \sigma = \sqrt{20 \dfrac{1}{2} \cdot \dfrac{1}{2}} = 2.236$

$$z = \frac{4-10}{2.236} = -2.68 \text{ or } z = \frac{4.5-10}{2.236} = -2.46$$

Since $z = -2.68$ (and $z = -2.46$) is less than -1.96, null hypothesis must be rejected.

16.17 Differences are

1.3	0.9	1.1	3.8	3.1	2.6	1.8	2.5	1.2	2.4
−	+	−	−	+	−	−	−	−	−
11	7.5	9	20	19	17	13	16	10	15

0.1	2.9	0.1	0.8	0.6	0.6	0.3	1.9	0.9	1.4
−	−	+	−	+	−	−	−	−	−
1.5	18	1.5	6	4.5	4.5	3	14	7.5	12

$T^+ = 7.5 + 19 + 1.5 + 4.5 = 32.5$ less than T^-

$T = 32.5$ $\quad T_{0.05} = 52$ for $n = 20$

Since 32.5 < 52, null hypothesis must be rejected.

16.18 There are $x = 12$ plus signs among $n = 16$ $\quad \alpha = 0.05$

$p = 0.5$ against $p > 0.50$, p-value $p(x \geq 12) = 0.0381$

Since p-value is less than 0.05, reject the null hypothesis.

16.19

1.15	0.85	4.75	−0.37	2.09	6.63	−2.35	0.27
8	6	14	4	11	16	12	3
−0.20	2.45	1.29	0.51	4.80	1	−1.52	0.11
2	13	9	5	15	1	10	1

$T^- = 28,\ T^+ = \dfrac{16 \cdot 17}{2} - 28 = 108,\ T = 28 \qquad \alpha = 0.05$

Reject if $T^- \leq T_{0.10} = 36$

Since $T^{-1} = 28 < 36$; null hypothesis must be rejected.

16.20 $n = 10,\ \alpha = 0.05$

(a) based on T; reject if $T \leq T_{0.05} = 8 \qquad T \leq 8$

(b) based on T^-; reject if $T^- \leq T_{0.10} = 11 \quad T^- \leq 11$

(c) based on T^+; reject if $T^+ \leq T_{0.10} = 11 \quad T^+ \leq 11$

16.21 $n = 10,\ \alpha = 0.01$

(a) based on T; reject if $T \leq T_{0.01} = 3$

(b) based on T^-; reject if $T^- \leq T_{0.02} = 5$

(c) based on T^+; reject if $T^+ \leq T_{0.02} = 5$

16.22 $\mu_0 = 35$ against $\mu \neq 35,\ \alpha = 0.05$, $n = 11$

3	8	1	−6	9	−7	5	15	4	12	−2
3	8	1	6	9	7	5	11	4	10	2

$T^- = 15,\ T^+ = 51,\ T = 15,\ T_{0.05} = 11$

Since $T = 15$ is not ≤ 11, null hypothesis cannot be rejected.

16.23

15	18	20	22	25	27	28	29	32	35	36	38
2	2	2	2	1	2	1	2	1	1	1	1
1	2	3	4	5	6	7	8	9	10	11	12

$W_2 = 1+2+3+4+6+8 = 24$

$u_1 = 24 - \dfrac{6 \cdot 7}{2} = 3 \qquad \mu_1 > \mu_2 \qquad \alpha = 0.01 \qquad U_{0.02} = 3$

Since $U_2 = 3 = U_{0.02}$, null hypothesis must be rejected.

16.24 $\alpha = 0.05 \qquad \mu_1 < \mu_2 \qquad U_1 \le U_{0.01} = 10$

$W_1 = 8+1+3.5+5+2+7 = 26.5$

$U_1 = 26.5 - \dfrac{6 \cdot 7}{2} = 5.5$

Since $U_1 = 5.5 < 10$, null hypothesis must be rejected.

16.25 $\alpha = 0.05 \qquad \mu_1 \ne \mu_2 \qquad n_1 = 10,\ n_2 = 12,\ U \le U_{0.05} = 49$

$W_1 = 18+2+9+10+5+16+27+11+9+20+14+23+6+25+23+3 = 208$

$U_1 = 208 - \dfrac{15 \cdot 16}{2} = 88,\ U_2 = 15 \cdot 12 - 88 = 92,\ U = 88$

Since $U = 88$ exceeds 49, null hypothesis cannot be rejected.

16.26 $\mu = \dfrac{15 \cdot 12}{2} = 90,\ \sigma^2 = \dfrac{15 \cdot 12 \cdot 28}{12} = 420,\ \sigma = 20.5,\ z = \dfrac{88-90}{20.5} = -0.10$

Since $z = -0.10$ falls between -1.96 and 1.96, null hypothesis cannot be rejected.

16.27 A B A A A B A B B A B A A A B B B B B

$\sum\sum d = 1+4+5+5+6+9+9+9+9+9 = 66 = U_2$

$\sum\sum d = 0+1+1+1+2+4+5+5+5 = 24 = U_1$

16.28 B B B B A B A B A A A A

$U = 0+0+0+0+1+2 = 3$

16.29 $n_1 = 14,\ n_2 = 8,\ u = 5,\ \alpha = 0.05$

$u'_{0.025} = 6$ Since $u = 5 < 6$, null hypothesis of randomness must be rejected.

16.30 $n_1 = 12,\ n_2 = 10,\ u = 17,\ \alpha = 0.05$

Since $u = 17$ and $u_{0.025} = 17$, null hypothesis of randomness must be rejected.

16.31 $n_1 = 5,\ n_2 = 8,\ u = 4,\ \alpha = 0.05$

Since $u = 4$ falls between $u'_{0.025} = 3$ and $u_{0.025} = 11$, null hypothesis of randomness cannot be rejected.

16.32 $n_1 = 38,\ n_2 = 22,\ u = 28,\ \alpha = 0.05$

$$\mu = \frac{2 \cdot 38 \cdot 22}{60} + 1 = 28.87$$

$$\sigma^2 = \frac{2 \cdot 38 \cdot 22(2 \cdot 38 \cdot 22 - 60)}{60^2 \cdot 59} = \frac{1672 \cdot 1612}{212{,}400} = 12.69 \qquad \alpha = 3.56$$

$$z = \frac{28 - 28.87}{3.56} = 0.24 \text{ or } z = \frac{28.5 - 28.87}{3.56} = -0.10$$

Since $z = 0.24$ (or -0.10 with continuity correction) falls between -1.96 and 1.96, null hypothesis cannot be rejected.

16.33 $n_1 = 24,\ n_2 = 24,\ u = 30,\ \alpha = 0.01$

$$\mu = \frac{2 \cdot 24 \cdot 24}{48} + 1 = 25$$

$$\sigma^2 = \frac{2 \cdot 24 \cdot 24(2 \cdot 24 \cdot 24 - 48)}{48^2 \cdot 47} = \frac{1152 \cdot 1104}{108{,}288} = 11.74 \qquad \alpha = 3.43$$

$$z = \frac{30 - 25}{3.43} = 1.46 \text{ or } z = \frac{29.5 - 25}{3.43} = 1.31 \quad \text{(with continuity correction)}$$

Since $z = 1.46$ falls between -2.575 and 2.575, null hypothesis cannot be rejected.

16.35 Median is 30.5 and we get

b b a b b b a a a b a a a b a b a b b b a a a b

$n_1 = 12,\ n_2 = 12,\ u = 13,\ \alpha = 0.01$

Since $u = 13$ falls between 6 and 20, null hypothesis cannot be rejected.

16.36 Median is 99.7

b a a b a a a b b b a b a b a b a b a b a b a b

$n_1 = 12,\ n_2 = 12,\ u = 19,\ c = 0.05$

$$\mu = \frac{2 \cdot 12 \cdot 12}{24} + 1 = 13$$

$$\sigma^2 = \frac{2 \cdot 12 \cdot 12(2 \cdot 12 \cdot 12 - 24)}{24^2 \cdot 23} = \frac{288 \cdot 264}{13{,}248} = 5.739 \quad \sigma = 2.40$$

$$z = \frac{18.5 - 13}{2.40} = 2.29 \quad \text{(with continuity correction)}$$

Since 2.29 exceeds 1.645, null hypothesis must be rejected. There is a definite cyclical pattern.

16.37

11.3	12.2	13.0	13.2	14.1	14.7	14.9	15.2	15.3	15.4
A	B	A	A	A	B	A	B	B	A
16.2	16.6	16.9	17.0	18.3	18.9	19.4	19.8	21.2	
B	A	A	A	B	B	B	B	B	

$n_1 = 9,\ n_2 = 10,\ u = 10,\ \alpha = 0.05$

$$\mu = \frac{2 \cdot 9 \cdot 10}{19} + 1 = 10.47 \qquad \sigma^2 = \frac{180 \cdot 161}{19^2 \cdot 18} = \frac{28{,}980}{6498} = 4.46 \qquad \sigma = 2.11$$

$$z = \frac{10 - 10.47}{2.11} = -0.22$$

Since -0.22 is greater than -1.645, null hypothesis cannot be rejected.

16.38

R_x	R_y	d
13	12	1
14	11	3
1	2	−1
16.5	14.5	2
2.5	1	1.5
15	16	−1
16.5	17.5	−1
8	13	−5
6.5	8.5	−2
18	17.5	0.5
10.5	14.5	−4.0
2.5	8.5	−6.0
4	4.5	−0.5
5	3	2
10.5	6	4.5
10.5	8.5	2
6.5	4.5	2
10.5	8.5	2

$\sum d^2 = 137$

$$r_s = 1 - \frac{6(137)}{18.323} = 1 - 0.14 = 0.86$$

16.39 $z = \dfrac{0.86 - 0}{1/\sqrt{18-1}} = 0.86(4.423) = 3.55$

Since 3.55 exceeds 1.96, the value of r_s is significant.

16.40 $\sum d^2 = 138$ $\qquad r_s = 1 - \dfrac{6(138)}{15 \cdot 224} = 1 - 0.25 = 0.75$

16.41 $\sum d^2 = 130.5$ $\qquad r_s = 1 - \dfrac{6(130.5)}{12 \cdot 143} = 1 - 0.46 = 0.54$

$$z = \frac{0.54 - 0}{1/\sqrt{11}} = 0.54(3.3166) = 1.79$$

Since $z = 1.79$ falls between -1.96 and 1.96, null hypothesis cannot be rejected; $r_s = 0.54$ is not significant.

16.42 $R_1 = 15,\ R_2 = 12,\ R_3 = 7,\ R_4 = 15,\ R_5 = 29,\ R_6 = 10,\ R_7 = 11,\ R_8 = 25,$

$R_9 = 25,\ R_{10} = 15,\ \dfrac{k(n+1)}{2} = \dfrac{3 \cdot 11}{2} = 16.5$

$$W = \frac{12}{9 \cdot 10 \cdot 99}[(-1.5)^2 + (-4.5)^2 + (-9.5)^2 + (-1.5)^2 + (12.5)^2 + (-6.5)^2 + (-5.5)^2 + (9.5)^2 + (8.5)^2 + (-1.5)^2$$

$$= \frac{12}{90 \cdot 99}[508.5) = 0.685$$

16.43 A and B $\quad \sum d^2 = 86 \quad r_s = 1 - \frac{6(86)}{10 \cdot 99} = 1 - 0.521 = 0.479$

A and C $\quad \sum d^2 = 40 \quad r_s = 1 - \frac{6(40)}{990} = 1 - 0.242 = 0.758$

B and C $\quad \sum d^2 = 108 \quad r_s = 1 - \frac{6(108)}{990} = 1 - 0.655 = 0.345$

$\overline{r}_s = 0.527; \ \frac{kW - 1}{k - 1} = \frac{3(0.685) - 1}{2} = 0.5275$

16.44 Number of plus signs = 25 out of $n = 36 \qquad \alpha = 0.01$

$\mu = 35(0.5) = 18, \ \sigma = \sqrt{36(0.5)(0.5)} = 3, \ z = \frac{24.5 - 18}{3} = 2.16$ using continuity correction.

Since 2.16 is less than $z_{0.01} = 2.33$, null hypothesis cannot be rejected.

16.45

0.1	1.1	0.3	1.1	1.0	0.7	0.6	0.4	0.8	1.0
−	+	+	+	+	+	+	+	+	−
3.5	33.5	12.5	33.5	30.5	25	23	17	26	30.5
0.3	0.2	0.1	0.1	0.1	0.6	0.6	0.2	0.4	0.5
+	+	−	−	+	+	−	−	+	+
12.5	8.5	3.5	3.5	3.5	23	23	8.5	17	20.5
1.6	1.4	1.0	0.1	0.3	0.4	0.2	0.1	0.3	0.5
+	+	+	+	+	−	−	−	+	+
36	35	30.5	3.5	12.5	17	8.5	3.5	12.5	20.5
0.2	0.4	1.0	0.4	0.9	0.9				
+	−	+	+	+	−				
8.5	17	30.5	17	27.5	27.5				

$$T^- = 3.5 + 30.5 + 3.5 + 3.5 + 23 + 8.5 + 17 + 8.5 + 3.5 + 17 + 27.5$$
$$= 146$$

$$\mu = \frac{36 \cdot 37}{4} = 333, \ \sigma^2 = \frac{36 \cdot 37 \cdot 73}{24} = 4051.5, \ \sigma = 63.65, \ z = \frac{146 - 333}{63.65} = -2.94$$

$\alpha = 0.01$

Since $-2.94 < -2.33$, null hypothesis must be rejected.

16.46 + + + − + + + − − + + − $\qquad x = 8$

For $n = 12$ and $p = 0.5 \qquad P(x \geq 8) = 0.1937 \quad \alpha = 0.01$

Since 0.1937 > 0.01, null hypothesis cannot be rejected.

16.47

43	35	13	11	6	18	12	6	2	7	3	10
+	+	+	−	+	+	+	−	−	+	+	−
12	11	9	7	3.5	10	8	3.5	1	5	2	6

$T^- = 7 + 3.5 + 1 + 6 = 17.5 \qquad T_{0l02} = 10$

Since 17.5 > 10, null hypothesis cannot be rejected.

16.48 Number of plus signs $x = 7$ $n = 24$ $\alpha = 0.05$

$\mu = 24(0.5) = 12$ and $\sigma = \sqrt{24(0.5)(0.5)} = 2.45$

$$z = \frac{7-12}{2.45} = -2.04 \text{ or } z = \frac{7.5-12}{2.45} = -1.84 \text{ (with continuity correction)}$$

Since $-1.84 < -1.64$, null hypothesis must be rejected.

16.49

−5	−13	−6	−7	9	−8	−1	6	−7	7	−11		
9	24	12	15	20	18	1.5	12	15	15	21		
−1	−8	−3	4	−12	−3	6	−5	12	−8	−3	2	−5
1.5	18	5	7	22.5	5	12	9	22.5	18	5	3	9

$T^+ = 20 + 12 + 15 + 7 + 12 + 22.5 + 3 = 91.5$

$n = 24$ $T_{0.10} = 92$

Since 91.5 < 92, null hypothesis must be rejected.

16.50

−5	9.4	11.1	−9.3	−1.5	15.6	29	4.3	12.9	−0.9
11	16	17	15	4	22	24	9	19	2
13	7.7	11.2	−0.1	3.8	−1.9	26.3		5.5	15.4
20	14	18	1	7	6	23		12	21
3.9	1.6	6.2	4.7	−1.4					
8	5	13	10	3					

$T^- = 11 + 15 + 4 + 2 + 1 + 6 + 3 = 42,\ T^+ = \frac{24 \cdot 25}{2} = 42 = 258$

$T = 42$

(a) $T_{0.05} = 81$ Since 42 < 81, null hypothesis must be rejected.

(b) $\mu = \frac{24 \cdot 25}{4} = 150$ $\sigma^2 = \frac{24 \cdot 25 \cdot 49}{24} = 1225$ $\sigma = 35$

$z = \frac{258-150}{35} = 3.09$

Since 3.09 exceeds 1.96, null hypothesis must be rejected.

16.51

5	−12	−3	8	11	−8	−16	13
7.5	16	4	11.5	15	11.5	19	17
3	5	−2	−10	−15	1	9	7
4	7.5	2	14	18	1	13	10
6	4	−3					
9	6	4					

(a) $T^+ = 7.5 + 11.5 + 15 + 17 + 4 + 7.5 + 1 + 13 + 10 + 9 + 6 = 101.5$

$T^- = \frac{19 \cdot 20}{2} - 101.5 = 98.5$, $\quad T = 98.5 \quad T_{0.05} = 45$

Since 98.5 is not $\leq$ 46, null hypothesis cannot be rejected.

(b) $\mu = \frac{19 \cdot 20}{4} = 95 \qquad \sigma^2 = \frac{19 \cdot 20 \cdot 39}{24} = 617.5 \qquad \sigma = 24.85$

$z = \frac{101.5 - 95}{24.85} = 0.26$

Since 0.26 falls between -1.96 and 1.96, null hypothesis cannot be rejected.

16.52 $\alpha = 0.05 \qquad \mu_1 \neq \mu_2 \qquad n_1 = n_2 = 20$

$\mu = \frac{20 \cdot 20}{2} = 200,\ \sigma^2 = \frac{20 \cdot 20 \cdot 41}{12} = 1366.7,\ \sigma = 36.97$

$W_1 = 499,\ U_1 = 499 - \frac{20 \cdot 21}{2} = 289,\ z = \frac{289 - 200}{36.97} = 2.41$

Since $z = 2.41$ exceeds 1.96, null hypothesis must be rejected.

16.53 $\alpha = 0.05 \qquad \mu_1 > \mu_2 \qquad n_1 = n_2 = 16 \qquad W_1 = 307$

$U_1 = 307 - \frac{16 \cdot 17}{2} = 171,\ \mu = \frac{16 \cdot 16}{2} = 128,\ \sigma^2 = \frac{16 \cdot 16 \cdot 33}{12} = 704$

and $\sigma = 26.53$

$z = \frac{171 - 128}{26.53} = 1.62$

Since $z = 1.62$ is less than 1.645, null hypothesis cannot be rejected.

16.54 $\alpha = 0.05 \qquad \chi^2_{0.05,3} = 7.815$

$R_1 = 4 + 7 + 10 + 14 + 18 = 53$

$R_2 = 5 + 12 + 15 + 16 + 20 = 68$

$R_3 = 1 + 3 + 6 + 9 + 11 = 30$

$R_4 = 2 + 8 + 13 + 17 + 19 = 59$

$$H = \frac{12}{20 \cdot 21}\left(\frac{53^2}{5} + \frac{68^2}{5} + \frac{30^2}{5} + \frac{59^2}{5}\right) - 3.21 = 4.51$$

Since $\chi^2 = 4.51$ is less than 7.815, null hypothesis cannot be rejected.

16.55 $n_1 = n_2 = n_3 = 10 \qquad \alpha = 0.05 \qquad \text{d.f.} = 2 \qquad \chi^2_{0.05,2} = 5.991$

$R_1 = 1.5 + 5 + 7.5 + 10.5 + 12 + 13 + 15.5 + 18 + 25 + 28 = 136$

$R_2 = 3 + 5 + 7.5 + 9 + 10.5 + 20 + 21 + 22.5 + 28 + 30 = 156.5$

$R_3 = 1.5 + 5 + 14 + 15.5 + 18 + 18 + 22.5 + 25 + 25 + 28 = 172.5$

$$H = \frac{12}{30 \cdot 31}\left[\frac{136^2}{10} + \frac{156.5^2}{10} + \frac{172.5^2}{10}\right] - 3.31 = 93.86 - 93 = 0.86$$

Since $H = 0.86$ is less than 5.991, null hypothesis cannot be rejected.

16.56 $n_1 = 8,\ n_2 = 10,\ n_3 = 8$ $\alpha = 0.01$ $\chi^2_{0.01,2} = 9.210$

$R_1 = 3+6+12+13+15+21+25+26 = 121$

$R_2 = 2+4+8+11+14+16+20+22+23+24 = 144$

$R_3 = 1+5+7+9+10+17+18+19 = 86$

$$H = \frac{12}{26 \cdot 27}\left[\frac{121^2}{8} + \frac{144^2}{10} + \frac{86^2}{8}\right] - 3(27) = (0.017094)(4828.225)$$

$$= 82.53 - 81 = 1.53$$

Since H = 1.53 is less than 9.210, null hypothesis cannot be rejected.

16.57 Median = 21.5

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a b a a a a a a a a a a a a

$n_1 = 25,\ n_2 = 25,\ u = 12,\ \alpha = 0.025$

$$\mu = \frac{2 \cdot 25 \cdot 25}{50} + 1 = 26 \qquad \sigma^2 = \frac{2 \cdot 25 \cdot 25(2 \cdot 25 \cdot 25 - 50)}{50 \cdot 50 \cdot 49} = 12.24$$

$$\sigma = 3.50 \qquad z = \frac{12 - 26}{3.50} = -4 \ \ (-3.86 \text{ with continuity correction})$$

Since $z = -4$ (or -3.86 with continuity correction) is less than -1.645, null hypothesis must be rejected; there is a trend.

16.58 Median is 5

$n_1 = 14,\ n_2 = 13,\ u = 5,\ \alpha = 0.01$

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Since u = 5 is less than 7, the null hypothesis must be rejected.

16.59 Median = 138 $\alpha = 0.05$

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$n_1 = 16,\ n_2 = 16,\ u = 12$

$$\mu = \frac{2 \cdot 16 \cdot 16}{32} + 1 = 17$$

$$\sigma^2 = \frac{2 \cdot 16 \cdot 16(2 \cdot 16 \cdot 16 - 32)}{32^2 \cdot 31} = \frac{512 \cdot 480}{31{,}744} = 7.742 \qquad \sigma = 2.78$$

$$z = \frac{12 - 27}{2.78} = -1.80$$

Since $z = -1.80$ is less than -1.645; the null hypothesis of randomness must be rejected; there seems to be a trend.