

Chapter 13

13.1 Test statistic $z = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}}$

Then by Theorem 8.7 $\left(\frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}} \right)^2$ is random variable having χ^2 distribution with $\nu = 1$. So

rejection criterion becomes $\frac{n(\bar{x} - \mu_0)^2}{\sigma^2} \geq \chi_{\alpha,1}^2$

13.2 $K = \mu_0 + z_\alpha \frac{\sigma}{\sqrt{n}}$ and $K = \mu_1 - z_\beta \frac{\sigma}{\sqrt{n}}$

$$\mu_0 + z_\alpha \frac{\sigma}{\sqrt{n}} = \mu_1 - z_\beta \frac{\sigma}{\sqrt{n}}$$

$$\mu_1 - \mu_0 = (z_\alpha + z_\beta) \frac{\sigma}{\sqrt{n}} \rightarrow \sqrt{n} = \frac{\sigma(z_\alpha + z_\beta)}{\mu_1 - \mu_0}$$

and $n = \frac{\sigma^2(z_\alpha + z_\beta)^2}{(\mu_1 - \mu_0)^2}$

13.3 $n = \frac{9^2(1.645 + 2.33)^2}{5^2} = \frac{81(3.975)^2}{25} = 51.19 \quad n = 52$

13.4 $K = \delta + z_\alpha \sqrt{\frac{\sigma_1^2 + \sigma_2^2}{n}} \quad K = \delta' - z_\beta \sqrt{\frac{\sigma_1^2 + \sigma_2^2}{n}}$

$$\delta + z_\alpha \sqrt{\frac{\sigma_1^2 + \sigma_2^2}{n}} = \delta' - z_\beta \sqrt{\frac{\sigma_1^2 + \sigma_2^2}{n}}$$

$$\delta - \delta' = (z_\alpha + z_\beta) \sqrt{\frac{\sigma_1^2 + \sigma_2^2}{n}}$$

$$\sqrt{n} = \frac{(z_\alpha + z_\beta) \sqrt{\sigma_1^2 + \sigma_2^2}}{\delta - \delta'} \text{ and } n = \frac{(\sigma_1^2 + \sigma_2^2)(z_\alpha + z_\beta)^2}{(\delta - \delta')^2}$$

13.5 $n = \frac{(81 + 169)(2.33 + 2.33)^2}{6^2} = \frac{(250)(21.7156)}{36} = 150.80 = 151$

13.6 $\frac{(n-1)s^2}{\sigma_0^2}$ has chi square distribution with $(n-1)$ degrees of freedom, so that according to

corollary 2 to Theorem 6.3

$$\mu = n-1 \text{ and } \sigma = \sqrt{2(n-1)}$$

Using normal approximation, critical region is

$$\frac{(n-1)s^2}{\sigma_0^2} \geq n-1 + z_\alpha \sqrt{2(n-1)}$$

$$\text{or } s^2 \geq \sigma_0^2 \left[1 + z_\alpha \sqrt{\frac{2}{n-1}} \right]$$

$$\text{For } H_1 : \sigma^2 < \sigma_0^2 \text{ critical region is } s^2 \leq \sigma_0^2 \left[1 - z_\alpha \sqrt{\frac{2}{n-1}} \right]$$

$$\text{For } H_1 : \sigma^2 \neq \sigma_0^2 \text{ critical region is } s^2 \leq \sigma_0^2 \left[1 - z_{\alpha/2} \sqrt{\frac{2}{n-1}} \right] \text{ or } s^2 \geq \sigma_0^2 \left[1 + z_{\alpha/2} \sqrt{\frac{2}{n-1}} \right]$$

13.7 If x has χ^2 distribution with $n-1$ degrees of freedom, then according to Example 8.42 $\sqrt{2x} - \sqrt{2(n-1)} \rightarrow$ standard normal distribution.

Since $\frac{(n-1)s^2}{\sigma_0^2}$ has chi square distribution with $n-1$ degrees of freedom.

$$\sqrt{\frac{2(n-1)s^2}{\sigma_0^2}} - \sqrt{2(n-1)} \text{ has approximately standard normal distribution}$$

$$\frac{s}{\sigma_0} \sqrt{2(n-1)} - \sqrt{2(n-1)} \text{ has approximately standard normal distribution}$$

$$\left(\frac{s}{\sigma_0} - 1 \right) \sqrt{2(n-1)} \text{ has approximately standard normal distribution}$$

13.8 $e_{i1} = n_i \hat{\theta}$, $e_{i2} = n_i(1 - \hat{\theta})$, $f_{i1} = x_i$, $f_{i2} = n_i - x_i$

$$\begin{aligned} \chi^2 &= \sum_{i=1}^k \sum_{j=1}^2 \frac{(f_{ij} - e_{ij})^2}{e_{ij}} = \sum_{i=1}^k \frac{(x_i - n_i \hat{\theta})^2}{n_i \hat{\theta}} + \frac{[n_i - x_i - n_i(1 - \hat{\theta})]^2}{n_i(1 - \hat{\theta})} \\ &= \sum_{i=1}^k \frac{(x_i - n_i \hat{\theta})^2 + \hat{\theta}(x_i - n_i \hat{\theta})^2}{n_i \hat{\theta}(1 - \hat{\theta})} \\ &= \sum_{i=1}^k \frac{(x_i - n_i \hat{\theta})^2}{n_i \hat{\theta}(1 - \hat{\theta})} \quad \text{QED} \end{aligned}$$

13.9 $H_1 : \lambda > \lambda_0$, Reject null hypothesis if $\sum_{i=1}^n x_i \geq k_\alpha$, where k_α is smallest integer for which

$$\sum_{y=k_\alpha}^{\infty} p(y; n\lambda_0) \leq \alpha.$$

$H_1 : \lambda < \lambda_0$, Reject null hypothesis if $\sum_{i=1}^n x_i \leq k'_\alpha$, where k'_α is smallest integer for which

$$\sum_{y=0}^{k'_\alpha} p(y; n\lambda_0) \leq \alpha.$$

$H_1 : \lambda \neq \lambda_0$, Reject null hypothesis if $\sum x \leq k'_{\alpha/2}$ or $\sum x \geq k_{\alpha/2}$

13.10 From Table II with $\lambda = 5(3.6) = 18$

$k_{0.025} = 25$ (Probability $X \geq 28 = 0.0173$, $x \geq 27 = 0.0282$)

$k'_{0.025} = 9$ (Probability $X \leq 9 = 0.0153$, $x \leq 10 = 0.0303$)

13.11 Substitute $e_{11} = \frac{n_1(x_1 + x_2)}{n_1 + n_2}$, $f_{11} = e_{21} = \frac{n_2(x_1 + x_2)}{n_1 + n_2}$

$$f_{21} = x_2, \quad e_{12} = \frac{n_1[(n_1 + n_2) - (x_1 + x_2)]}{n_1 + n_2}, \quad f_{12} = n_1 - x_1$$

$$e_{22} = \frac{n_2[(n_1 + n_2) - (x_1 + x_2)]}{n_1 + n_2}, \quad f_{22} = n_2 - x_2 \text{ into}$$

$$\chi^2 = \sum_{i=1}^k \sum_{j=1}^2 \frac{(f_{ij} - e_{ij})^2}{e_{ij}} \text{ and simplify algebraically}$$

13.12 $E\left(\frac{x_1}{n_1} - \frac{x_2}{n_2}\right) = \theta_1 - \theta_2 = 0$

$$\begin{aligned} \text{var}\left(\frac{x_1}{n_1} - \frac{x_2}{n_2}\right) &= \text{var}\left(\frac{x_1}{n_1}\right) + \text{var}\left(\frac{x_2}{n_2}\right) \\ &= \frac{\theta_2(1-\theta_2)}{n_1} + \frac{\theta_2(1-\theta_2)}{n_2} \end{aligned}$$

$$\theta_1 = \theta_2 = \theta \text{ estimated by } \hat{\theta} = \frac{x_1 + x_2}{n_1 + n_2}$$

$$\hat{\theta}(1-\hat{\theta})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)$$

$$\text{Thus, } z = \frac{\frac{x_1}{n_1} - \frac{x_2}{n_2} - 0}{\sqrt{\hat{\theta}(1-\hat{\theta})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{\frac{x_1}{n_1} - \frac{x_2}{n_2}}{\sqrt{\hat{\theta}(1-\hat{\theta})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

has approximately standard normal distribution.

$$\begin{aligned}
13.13 \quad \chi^2 &= \frac{(x_1 - n_1 \hat{\theta})^2}{n_1 \hat{\theta}(1 - \hat{\theta})} + \frac{(x_2 - n_2 \hat{\theta})^2}{n_2 \hat{\theta}(1 - \hat{\theta})} \\
&= \frac{\left[x_1 - \frac{n_1(x_1 + x_2)^2}{n_1 + n_2} \right]^2}{n_1 \hat{\theta}(1 - \hat{\theta})} + \frac{\left[x_2 - \frac{n_2(x_1 + x_2)^2}{n_1 + n_2} \right]^2}{n_2 \hat{\theta}(1 - \hat{\theta})} \\
&= \frac{\left[\frac{x_1 n_2}{n_1 + n_2} - \frac{n_1 x_2}{n_1 + n_2} \right]^2}{n_1 \hat{\theta}(1 - \hat{\theta})} + \frac{\left[\frac{x_2 n_1}{n_1 + n_2} - \frac{n_2 x_1}{n_1 + n_2} \right]^2}{n_2 \hat{\theta}(1 - \hat{\theta})} \\
&= \frac{\frac{n_1^2 \cdot n_2}{n_1^2 (n_1 + n_2)^2} \left(\frac{x_1}{n_1} - \frac{x_2}{n_2} \right)^2 + \frac{n_2^2 \cdot n_1}{n_1^2 (n_1 + n_2)^2} \left(\frac{x_1}{n_1} - \frac{x_2}{n_2} \right)^2}{\frac{(n_1 + n_2) \hat{\theta}(1 - \hat{\theta})}{n_1 n_2}} = \frac{\left(\frac{x_1}{n_1} - \frac{x_2}{n_2} \right)^2}{\frac{(n_1 + n_2) \hat{\theta}(1 - \hat{\theta})}{n_1 n_2}} \\
&= \frac{\left(\frac{x_1}{n_1} - \frac{x_2}{n_2} \right)^2}{\left(\frac{1}{n_1} + \frac{1}{n_2} \right) \hat{\theta}(1 - \hat{\theta})} = Z^2 \quad \text{QED}
\end{aligned}$$

$$\begin{aligned}
13.14 \quad e_{ij} &= \frac{\sum_i f_{ij} \sum_j f_{ij}}{n} \\
\sum_i e_{ij} &= \frac{\sum_i f_{ij} \cdot \sum_i \sum_j f_{ij}}{n} = \frac{\sum_i f_{ij} \cdot n}{n} = \sum_i f_{ij} \\
\sum_j e_{ij} &= \frac{\sum_j f_{ij} \cdot \sum_i \sum_j f_{ij}}{n} = \frac{\sum_j f_{ij} \cdot n}{n} = \sum_j f_{ij}
\end{aligned}$$

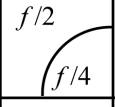
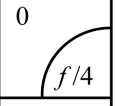
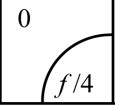
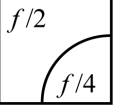
$$\begin{aligned}
13.15 \quad \text{Under } H_o : e_{1j} = \theta_{2j} = \dots = \theta_{nj} \text{ for } j = 1, 2, \dots \\
= \theta_j
\end{aligned}$$

$$\hat{\theta}_j = \frac{\sum_i f_{ij}}{n} \quad e_{ij} = \frac{\sum_i f_{ij}}{n} \cdot \sum_j f_{ij} = \frac{\sum_i f_{ij} \cdot \sum_j f_{ij}}{n}$$

$$\begin{aligned}
13.16 \quad \chi^2 &= \sum_i \sum_j \frac{(f_{ij} - e_{ij})^2}{e_{ij}} = \sum_i \sum_j \frac{f_{ij}^2}{e_{ij}} - 2 \sum_i \sum_j f_{ij} + \sum_i \sum_j e_{ij} \\
&= \sum_i \sum_j \frac{f_{ij}^2}{e_{ij}} - 2f + f \quad (\text{see Ex 13.14}) \\
&= \sum_i \sum_j \frac{f_{ij}^2}{e_{ij}} - f \quad \text{QED}
\end{aligned}$$

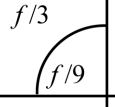
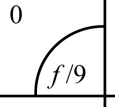
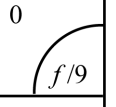
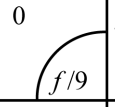
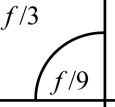
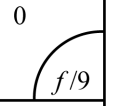
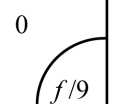
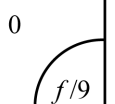
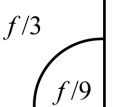
$$\begin{aligned}
 13.17 \quad \chi^2 &= \frac{232^2}{212} + \frac{260^2}{265} + \frac{197^2}{212} + \frac{168^2}{188} + \frac{240^2}{235} + \frac{203^2}{188} - 1300 \\
 &= 253.887 + 255.094 + 183.061 + 150.128 + 245.106 + 219.197 - 1300 \\
 &= 6.473 \quad (\text{differs due to rounding})
 \end{aligned}$$

13.18 (a)

$f/2$  $f/4$	0  $f/4$
0  $f/4$	$f/2$  $f/4$

$$\begin{aligned}
 \chi^2 &= \frac{(f/4)^2}{f/4} + \frac{(f/4)^2}{f/4} + \frac{(f/4)^2}{f/4} + \frac{(f/4)^2}{f/4} \\
 &= f \\
 C &= \sqrt{\frac{f}{f+f}} = \sqrt{\frac{1}{2}} = \frac{\sqrt{2}}{2}
 \end{aligned}$$

(b)

$f/3$  $f/9$	0  $f/9$	0  $f/9$
0  $f/9$	$f/3$  $f/9$	0  $f/9$
0  $f/9$	0  $f/9$	$f/3$  $f/9$

$$\begin{aligned}
 \chi^2 &= 3 \cdot \frac{\left(\frac{2f}{9}\right)^2}{f/9} + 6 \cdot \frac{\left(\frac{f}{9}\right)^2}{f/9} \\
 &= \frac{4}{3}f + \frac{2}{3}f = 2f \\
 C &= \sqrt{\frac{2f}{2f+f}} = \sqrt{\frac{2}{3}} = \frac{1}{3}\sqrt{6}
 \end{aligned}$$

13.19 (a) not necessarily; (b) yes

- 13.20 (a) No, since $0.0316 > 0.01$
 (b) Yes, since $0.0316 < 0.05$
 (c) Yes, since $0.0316 < 0.10$

13.21 Normal curve area corresponding to $z = 2.84$ is 0.4977
 p -value is $2(0.5000 - 0.4977) = 0.0046$

13.22 Normal curve area corresponding to 1.40 is 0.4192
 p -value is $0.5000 - 0.4192 = 0.0808$

13.23 p -value is $\frac{1 - 0.3502}{2} = 0.3249$. As it exceeds 0.05, null hypothesis *cannot* be rejected.

13.24 $H_0: \mu = 10$; $H_1: \mu < 10$

$$z = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}} = \frac{8.4 - 10}{3.2 / \sqrt{16}} = -2.0$$

Since $z_{0.05} = 1.645$, we reject H_0 in favor of H_1 .

13.25 1. $H_0 : \mu = 84.3, H_1 : \mu > 84.3, \alpha = 0.01$

2. Reject null hypothesis if $z \geq 2.33$

3. $z = \frac{87.5 - 84.3}{8.6 / \sqrt{45}} = 2.73$

4. Since 2.73 exceeds 2.33, null hypothesis must be rejected.

13.26 2. $z = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}}$

3. $z = \frac{87.5 - 84.3}{8.6 / \sqrt{45}} = 2.73, p\text{-value} = 0.5000 - 0.4968 = 0.0032$

4. Since $0.0032 < 0.01$, null hypothesis must be rejected.

13.27 1. $H_0 : \mu = 30, H_1 : \mu \neq 30, \alpha = 0.01$

2. Reject null hypothesis if $z \leq -2.575$ or $z \geq 2.575$

3. $z = \frac{30.8 - 30}{1.5 / \sqrt{32}} = \frac{0.8\sqrt{32}}{1.5} = 3.02$

4. Since $3.02 > 2.575$, null hypothesis must be rejected.

13.28 2. $z = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}}$

3. $z = \frac{30.8 - 30}{1.5 / \sqrt{32}} = 3.02, p\text{-value} = 2(0.5 - 0.4987) = 0.0026$

4. Since 0.0026 is less than 0.005, null hypothesis must be rejected.

13.29 1. $H_0 : \mu = 35, H_1 : \mu < 35, \alpha = 0.05$

2. Reject null hypothesis if $t \leq -t_{0.05,11} = -1.796$

3. $t = \frac{33.6 - 35}{2.3 / \sqrt{12}} = \frac{-1.4}{2.3\sqrt{12}} = -2.11$

4. Since $-2.11 < -1.796$, the null hypothesis must be rejected.

13.30 $n = 5, \bar{x} = 14.4, s = 0.158$

1. $H_0 : \mu = 14, H_1 : \mu \neq 14, \alpha = 0.05$

2. Reject null hypothesis if $t \leq -2.776$ or $t \geq 2.776$

3. $t = \frac{14.4 - 14}{0.158 / \sqrt{5}} = 5.66$

4. Since 5.66 exceeds 2.776, null hypothesis must be rejected.

13.31 $n = 5, \bar{x} = 14.7, s = 0.742$

3. $t = \frac{14.7 - 14}{0.742 / \sqrt{5}} = 2.11$

4. Since $t = 2.11$ falls between -2.776 and 2.776 , null hypothesis cannot be rejected.

$\bar{x} - \mu_0$ has increased from 14.4 to 14.7 but s has increased from 0.158 to 0.742.

13.32 $t = 5.66$, d.f. = 4

$$p\text{-value} = 1 - 0.9952 = 0.0048$$

Since $0.0048 < 0.05$, null hypothesis must be rejected.

13.33 (a) $P(\text{reject } H_0 | H_0 \text{ is true}) = 0.05$ (by definition)

(b) $P(\text{reject } H_0 \text{ on experiment 1 or experiment 2 (or both)} | H_0 \text{ is true}) =$
 $0.05 + 0.05 - 0.0025 = 0.0975$

(c) $P(\text{Reject } H_0 \text{ on one or more of 30 experiments} | H_0 \text{ is true}) =$
 $1 - P(\text{do not reject } H_0 \text{ on any experiment} | H_0 \text{ is true}) = 1 - (0.95)^{30} = 0.79.$

13.34 (a) $P(\text{reject } H_0 \text{ on exactly one factor} | H_0 \text{ is true for all 48 factors}) =$

$$\binom{48}{1} (0.01)^1 (0.99)^{47} = 0.30$$

(b) $P(\text{reject } H_0 \text{ on more than one factor} | H_0 \text{ is true for all 48 factors}) = 1 - 0.30 = 0.70.$

13.35 $\frac{(\bar{x}_1 - \bar{x}_2) - 0.20}{\sqrt{\frac{(0.12)^2}{50} + \frac{(0.14)^2}{40}}} \leq -1.96 \text{ or } \geq 1.96$

$$\frac{(\bar{x}_1 - \bar{x}_2) - 0.20}{0.0279} \leq -1.96 \text{ or } \geq 1.96$$

$$\bar{x}_1 - \bar{x}_2 \leq 0.20 - 0.0547 = 0.145$$

$$\text{or } \bar{x}_1 - \bar{x}_2 \geq 0.20 + 0.0547 = 0.255$$

(a) $z = \frac{0.145 - 0.12}{0.0279} = 0.90$ and $z = \frac{0.255 - 0.12}{0.0279} = 4.84$
 $\beta = 0.5 - 0.3159 = 0.1841 = 0.18$

(b) $z = \frac{0.145 - 0.16}{0.0279} = -0.54$ and $z = \frac{0.255 - 0.16}{0.0279} = 3.405$
 $\beta = 0.2054 + 0.5 = 0.7054 = 0.71$

(c) $z = \frac{0.145 - 0.24}{0.0279} = -3.40$ and $z = \frac{0.255 - 0.24}{0.0279} = 0.54$
 $\beta = 0.2054 + 0.5 = 0.7054 = 0.71$

(d) $z = \frac{0.145 - 0.28}{0.0279} = -4.84$ and $z = \frac{0.255 - 0.28}{0.0279} = -0.90$
 $\beta = 0.5 - 0.3159 = 0.1841 = 0.18$

- 13.36** 1. $H_0 : \mu_1 - \mu_2 = 0, H_1 : \mu_1 - \mu_2 \neq 0, \alpha = 0.05$
 2. Reject null hypothesis if $z \leq -1.96$ or $z \geq 1.96$
 3.
$$z = \frac{9.1 - 8}{\sqrt{\frac{1.9^2}{40} + \frac{2.1^2}{50}}} = \frac{1.1}{0.4224} = 2.60$$

 4. Since $2.60 > 1.96$, null hypothesis must be rejected.

13.37 $z = 2.60, p\text{-value} = 2(0.5 - 0.4953) = 0.0094$
 Since $0.0094 < 0.05$, null hypothesis must be rejected.

- 13.38** 1. $H_0 : \mu_1 - \mu_2 = -0.05, H_1 : \mu_1 - \mu_2 < -0.05, \alpha = 0.05$
 2. Reject null hypothesis if $z \leq -1.645$
 3.
$$z = \frac{(53.8 - 54.5) + 0.05}{\sqrt{\frac{2.4^2}{400} + \frac{2.5^2}{500}}} = \frac{-0.20}{0.164} = -1.22$$

 4. Since $-1.22 > -1.645$, null hypothesis cannot be rejected.

13.39 $z = -1.22, p\text{-value} = 0.5 - 0.3888 = 0.1112$
 Since $0.1112 > 0.05$, null hypothesis cannot be rejected.

- 13.40** 1. $H_0 : \mu_1 - \mu_2 = 0, H_1 : \mu_1 - \mu_2 \neq 0, \alpha = 0.01$
 2. Reject null hypothesis if $t \leq -t_{0.005} = -3.169$ or $t > t_{0.005} = 3.169$
 3.
$$s_p^2 = \frac{5(3.3)^2 + 5(2.1)^2}{10} = 7.65 \text{ and } s_p = 2.766$$

$$t = \frac{77.4 - 72.2}{2.766 \sqrt{\frac{1}{6} + \frac{1}{6}}} = \frac{5.2}{(2.766)(0.577)} = 3.26$$

 4. Since $3.26 > 3.169$, null hypothesis must be rejected.

13.41 $t = 2.67, \text{d.f.} = 6, \alpha = 0.05$

$$p\text{-value} = \frac{1}{2}(1 - 0.9630) = 0.0185$$

- 13.42** $\bar{x}_1 = 144, s_1 = 19.06, \bar{x}_2 = 149, s_2 = 14.21$
 1. $H_0 : \mu_1 = \mu_2, H_1 : \mu_1 \neq \mu_2, \alpha = 0.01$
 2. Reject null hypothesis if $t \leq -3.169$ or $t \geq 3.169$
 3.
$$s_p^2 = \frac{5(19.06)^2 + 5(14.21)^2}{10} = 282.604 \text{ and } s_p = 16.802$$

$$t = \frac{144 - 149}{16.802 \sqrt{\frac{1}{6} + \frac{1}{6}}} = \frac{-5}{(16.802)(0.577)} = -0.52$$

 4. Since -0.52 falls between -3.169 and 3.169 , null hypothesis cannot be rejected.

13.43 $t = -0.52$, d.f. = 10

$$p\text{-value} = 1 - 0.3856 = 0.61$$

Since $0.61 > 0.01$, null hypothesis cannot be rejected.

13.44 13, 7, -1, 5, 3, 2, -1, 0, 6, 1, 4, 3, 2, 6, 12, 4

$$\bar{x} = 4.125, s = 4.064, n = 16$$

$$1. \quad H_0 : \mu = 0, H_1 : \mu > 0, \alpha = 0.05$$

$$2. \quad \text{Reject null hypothesis if } t \geq t_{0.05, 15} = 1.753$$

$$3. \quad t = \frac{4.125 - 0}{4.064 / \sqrt{16}} = 4.06$$

$$4. \quad \text{Since } 4.06 > 1.753, \text{ null hypothesis must be rejected. Exercises are effective in reducing weight.}$$

13.45 9, 13, 2, 5, -2, 6, 6, 5, 2, 6

$$n = 10, \bar{x} = 5.2, s = 4.08$$

$$1. \quad H_0 : \mu = 0, H_1 : \mu > 0, \alpha = 0.05$$

$$2. \quad \text{Reject null hypothesis if } t > t_{0.05, 9} = 1.833$$

$$3. \quad t = \frac{5.2 - 0}{4.08 / \sqrt{10}} = 4.03$$

$$4. \quad \text{Since } 4.03 > 1.833, \text{ null hypothesis must be rejected. Safety program is effective.}$$

13.46 $t = 4.03$, d.f. = 9

$$p\text{-value} = \frac{1}{2}(1 - 0.997) = 0.0015$$

13.47 1. $H_0 : \sigma = 0.0100, H_1 : \sigma < 0.0100, \alpha = 0.05$

$$2. \quad \text{Reject null hypothesis if } \chi^2 \leq \chi_{0.95, 8}^2 = 2.733$$

$$3. \quad \chi^2 = \frac{8(0.0086)^2}{(0.0100)^2} = 5.92$$

$$4. \quad \text{Since } 5.92 > 2.733, \text{ null hypothesis cannot be rejected.}$$

13.48 $s = 238, n = 24$

$$1. \quad H_0 : \sigma = 250, H_1 : \sigma \neq 250, \alpha = 0.01$$

$$2. \quad \text{Reject null hypothesis if } \chi^2 \leq \chi_{0.995, 23}^2 = 9.260 \text{ or } \chi^2 \geq \chi_{0.005, 23}^2 = 44.181$$

$$3. \quad \chi^2 = \frac{23(238)^2}{(250)^2} = 20.84$$

$$4. \quad \text{Since } 9.260 < 20.84 < 44.181, \text{ null hypothesis cannot be rejected.}$$

13.49 $s = 2.53, n = 30, \alpha = 0.05$

$$1. \quad H_0 : \sigma = 2.85, H_1 : \sigma < 2.85, \alpha = 0.05$$

$$2. \quad \text{Reject null hypothesis if } \chi^2 \leq \chi_{0.95, 29}^2 = 17.708$$

$$3. \quad \chi^2 = \frac{29(2.53)^2}{(2.85)^2} = 22.85$$

$$4. \quad \text{Since } 22.85 > 17.708, \text{ null hypothesis cannot be rejected.}$$

- 13.50**
1. $H_0 : \sigma = \sigma_0, H_1 : \sigma < \sigma_0, \alpha = 0.05$
 2. Reject null hypothesis if $z \leq -z_{0.05} = -1.645$
 3. $z = \left(\frac{2.53}{2.85} - 1 \right) \sqrt{2 \cdot 29} = -0.1123(7.616) = -0.85$
 4. Since $-0.85 > -1.645$, null hypothesis cannot be rejected.

- 13.51** $n = 50, s = 0.49$
1. $H_0 : \sigma = 0.41, H_1 : \sigma > 0.41, \alpha = 0.05$
 2. Reject null hypothesis if $z \geq z_{0.05} = 1.645$
 3. $z = \left(\frac{0.49}{0.41} - 1 \right) \sqrt{2 \cdot 49} = (0.1951)(9.8995) = 1.93$
 4. Since $1.93 > 1.645$, null hypothesis must be rejected.

- 13.52** $p\text{-value} = 0.5 - 0.4732 = 0.0268$
 Since $0.0268 < 0.05$, null hypothesis must be rejected.

- 13.53** $n_1 = 4, s_1 = 31, n_2 = 4, s_2 = 26, \alpha = 0.05$
1. $H_0 : \sigma_1 - \sigma_2 = 0, H_1 : \sigma_1 - \sigma_2 > 0, \alpha = 0.05$
 2. Reject null hypothesis if $\frac{s_1^2}{s_2^2} \geq F_{0.05,3,3} = 9.28$
 3. $\frac{s_1^2}{s_2^2} = 1.42$
 4. Since 1.42 does not exceed 9.28, null hypothesis cannot be rejected.

- 13.54**
1. $H_0 : \sigma_1 - \sigma_2 = 0, H_1 : \sigma_1 - \sigma_2 \neq 0, \alpha = 0.10$
 2. Reject null hypothesis if $\frac{s_1^2}{s_2^2} \geq F_{0.05,5,5} = 5.05$
 3. $\frac{s_1^2}{s_2^2} = \frac{3.3^2}{2.1^2} = 2.47$
 4. Since $2.47 < 5.05$, null hypothesis cannot be rejected. Assumption was reasonable.

- 13.55** $s_1 = 19.06, s_2 = 14.21, n_1 = n_2 = 6$
1. $H_0 : \sigma_1 - \sigma_2 = 0, H_1 : \sigma_1 - \sigma_2 \neq 0, \alpha = 0.02$
 2. Reject null hypothesis if $\max \left(\frac{s_1^2}{s_2^2}, \frac{s_2^2}{s_1^2} \right) \geq F_{0.01,5,5} = 11.0$
 3. $\frac{s_1^2}{s_2^2} = 1.80$
 4. Since $1.80 < 11.0$, null hypothesis cannot be rejected.

13.56 $n = 20$, $\theta = 0.5$ against $\theta \neq 0.50$, $\alpha = 0.05$

$$\begin{aligned} p(x \leq 5) &= 0.0207 && \text{Critical region is } x \leq 5 \text{ or } x \geq 15 \\ p(x \leq 6) &= 0.0507 && \alpha = 0.0207 + 0.0207 = 0.0414 \\ p(x \geq 15) &= 0.0207 \\ p(x \geq 14) &= 0.0507 \end{aligned}$$

13.57 1. $H_0 : \theta = 0.40$, $H_1 : \theta > 0.40$, $\alpha = 0.05$

2. Observed number of successes in $n = 18$ trials

3. $x = 10$ $P(X \geq 10) = 0.1348$ $p\text{-value} = 0.1348$

4. Since $0.1348 > 0.05$, null hypothesis cannot be rejected.

13.58 $p(X \geq 12) = 0.0203$ Critical region is $x \geq 12$

$$p(X \geq 11) = 0.0577 \quad \alpha = 0.0203$$

13.59 1. $H_0 : \theta = 0.30$, $H_1 : \theta < 0.30$, $\alpha = 0.05$

2. Observed number of successes in $n = 19$ trials

3. $x = 1$ $p\text{-value} = 0.0011 + 0.0093 = 0.0104$

4. Since $0.0104 < 0.05$, null hypothesis must be rejected.

13.60 $p(x \leq 2) = 0.0462$ Critical region is $x \leq 2$

$$p(x \leq 3) = 0.1331 \quad \alpha = 0.0462$$

13.61 1. $H_0 : \theta = 0.40$, $H_1 : \theta \neq 0.40$, $\alpha = 0.01$

2. Observed number of successions in $n = 14$ trials

3. $p(x \geq 12) = 0.0006$, $p\text{-value} = 0.0012$

4. Since $0.0012 < 0.01$, null hypothesis must be rejected.

13.62 $P(x \leq 0) = 0.0008$, $P(x \geq 11) = 0.0039$, Critical region is $x = 0$, or $x \geq 11$

$$P(x \leq 1) = 0.0081, P(x \geq 10) = 0.0175, \alpha = 0.008 + 0.0039 = 0.0047$$

13.63 $H_0 : \theta = 0.35$; $H_1 : \theta < 0.35$. Using the normal approximation

$$z = \frac{x - n\theta_0}{\sqrt{n\theta_0(1 - \theta_0)}} = \frac{290 - 350}{\sqrt{(350)(0.65)}} = -3.98$$

Since $z_{0.05} = 1.645$, we reject H_0 at the 0.05 level of significance and conclude that $\theta < 0.35$; thus, the statement can be refuted.

13.64 1. $H_0 : \theta = 0.20$, $H_1 : \theta > 0.20$, $\alpha = 0.01$

2. Number of successes in $n = 12$ trials

3. $x = 6$, $p(X \geq 6) = 0.0194 = p\text{-value}$

4. Since $0.0194 > 0.01$, null hypothesis cannot be rejected.

- 13.65** 1. $H_0 : \theta = 0.60, H_1 : \theta \neq 0.60, \alpha = 0.05$
 2. Number of failures in $n = 18$ trials
 3. $x = 7, n - x = 18 - 7 = 11$
 $P(X \geq 11; \theta = 0.40) = 0.0577$
 p -value is $2(0.0577) = 0.1154$
 4. Since $0.1154 > 0.05$, null hypothesis cannot be rejected.

- 13.66** 1. $H_0 : \theta = 0.30, H_1 : \theta \neq 0.30, \alpha = 0.05$
 2. Reject if $z \leq -1.96$ or $z \geq 1.96$
 3. $z = \frac{157 - 600(0.30)}{\sqrt{600(0.3)(0.7)}} = -2.05$
 4. Since $-2.05 < -1.96$, null hypothesis must be rejected.

- 13.67** 1. $H_0 : \theta = 0.90, H_1 : \theta < 0.90, \alpha = 0.05$
 2. Reject if $z < -1.645$
 3. $z = \frac{174 - 200(0.9)}{\sqrt{200(0.9)(0.1)}} = -\frac{6}{4.2426} = -1.41$
 4. Since $-1.41 > -1.645$, null hypothesis cannot be rejected.

- 13.68** 1. $H_0 : \theta_1 = \theta_2, H_1 : \theta_1 \neq \theta_2, \alpha = 0.01$
 2. Reject null hypothesis if $\chi^2 \geq \chi_{0.01,1}^2 = 6.635$

74	92	166
83	83	
176	158	334
167	167	
250	250	500

$$e_{11} = \frac{166 \cdot 250}{500} = 83, \text{ others by subtraction}$$

$$\chi^2 = \frac{9^2}{83} + \frac{9^2}{83} + \frac{9^2}{167} + \frac{9^2}{167} = 2.92$$

4. Since $2.92 < 6.635$, null hypothesis cannot be rejected.

- 13.69** 1. $H_0 : \theta_1 = \theta_2, H_1 : \theta_1 \neq \theta_2, \alpha = 0.01$
 2. Reject null hypothesis if $z \leq -z_{0.005}$ or $z \geq z_{0.005}$

$$\hat{\theta} = \frac{74 + 92}{500} = 0.332$$





3. $z = \frac{\frac{74}{250} - \frac{92}{250}}{\sqrt{(0.332)(0.668)(0.008)}} = -\frac{0.072}{0.04212} = -1.71$

4. Since -1.71 falls between -2.575 and 2.575 , null hypothesis cannot be rejected.

13.70 1. $H_0 : \theta_1 = \theta_2, H_1 : \theta_1 \neq \theta_2, \alpha = 0.05$

2. Reject null hypothesis if $\chi^2 \geq \chi_{0.05,1}^2 = 3.841$

3.

46  42.7	18  21.3	64
354  357.3	182  178.7	536
400	400	600

$$e_{11} = \frac{64 \cdot 400}{600} = 42.7, \text{ others by subtraction}$$

$$\chi^2 = \frac{3.3^2}{42.7} + \frac{3.3^2}{21.3} + \frac{3.3^2}{357.3} + \frac{3.3^2}{178.7}$$

$$= 0.255 + 0.511 + 0.030 + 0.061$$

$$= 0.86$$

4. Since $0.86 < 3.841$, null hypothesis cannot be rejected.

13.71 1. $H_0 : \theta_1 = \theta_2, H_1 : \theta_1 \neq \theta_2, \alpha = 0.05$

2. Reject null hypothesis if $z \leq -1.96$ or $z \geq 1.96$

$$\hat{\theta} = \frac{74 + 92}{500} = 0.332$$

3.

$$z = \frac{\frac{46}{400} - \frac{18}{200}}{\sqrt{(0.107)(0.893)(0.0075)}} = \frac{0.025}{0.0268} = 0.93$$

$$z^2 = (0.93)^2 = 0.8649 = 0.86 = \chi^2$$

13.72 $H_0 : \theta_1 = \theta_2, H_1 : \theta_1 > \theta_2, \alpha = 0.05$

2. Reject null hypothesis if $z \geq 1.645$

3.









$$\hat{\theta} = \frac{169}{500} = 0.338 \quad z = \frac{\frac{82}{200} - \frac{87}{300}}{\sqrt{(0.338)(0.662)(0.00833)}} = 2.78$$

4. Since $2.78 > 1.645$, null hypothesis must be rejected.

13.73 $H_0 : \theta_1 = \theta_2 = \theta_3 = \theta_4, H_1 : \text{not all equal}, \alpha = 0.05$

2. Reject null hypothesis if $\chi^2 \geq \chi_{0.05,3}^2 = 7.815$

3.

26  24	23  24	15  24	32  24	96
174  176	177  176	185  176	168  176	704
200	200	200	200	800

$$e_{11} = \frac{96 \cdot 200}{800} = 24 \text{ etc.}$$

$$\chi^2 = \frac{4+1+81+64}{24} + \frac{4+1+81+64}{24} = 7.10$$

4. Since $7.10 < 7.818$, null hypothesis cannot be rejected.

13.74 $H_0 : \theta_1 = \theta_2 = \theta_3$, $H_1 : \text{not all equal}$, $\alpha = 0.05$

2. Reject null hypothesis if $\chi^2 \geq \chi_{0.05,2}^2 = 5.991$

3.

155 150	118 120	87 90	360
95 100	82 80	63 60	240
250	200	150	600

$$e_{11} = \frac{360 \cdot 250}{600}$$

$$\chi^2 = \frac{25}{100} + \frac{4}{120} + \frac{9}{90} + \frac{25}{100} + \frac{4}{80} + \frac{9}{60} = 0.75$$

4. Since $0.75 < 5.991$, null hypothesis cannot be rejected.

13.75 In the following contingency table, the expected frequency is given below the observed frequency in each cell.

	TOTALS		
	45 45.0	58 49.8	49 57.3
	21 21.0	15 23.2	35 26.7
TOTALS	66	73	84
			223

The expected frequencies were calculated as $\frac{152 \times 66}{223} = 45.0$, etc.

$$\begin{aligned} \text{Thus, } \chi^2 &= \frac{(45 - 45.0)^2}{45.0} + \frac{(58 - 49.8)^2}{49.8} + \dots + \frac{(35 - 26.7)^2}{26.7} \\ &= 0.00 + 1.35 + 1.20 + 0.00 + 2.90 + 2.58 = 8.03 \end{aligned}$$

Since $\chi_{0.01}^2 = 9.210$, we cannot reject H_0 , and we have no reason to conclude that the three processes have different probabilities of passing the strength standard.

13.76

48 44.4	40 38.6	12 17.0	100
55 60.9	53 52.9	29 23.2	137
57 54.7	46 47.5	20 20.8	123
160	139	61	360

1. $H_0 : \text{independent}$, $H_1 : \text{not independent}$, $\alpha = 0.05$

2. Reject null hypothesis, if $\chi^2 \geq \chi_{0.05,4}^2 = 9.488$

$$\begin{aligned} \chi^2 &= 0.292 + 0.051 + 1.471 + 0.572 \\ &\quad + 0.000 + 1.450 + 0.097 \\ &\quad + 0.047 + 0.031 \\ &= 4.01 = 4.0 \end{aligned}$$

4. Since $4.0 < 9.488$, null hypothesis cannot be rejected.

13.77

7	12	31	50
15	22.1	12.9	
35	59	18	112
33.6	49.5	28.9	
15	13	0	28
8.4	12.4	7.2	
57	8.4	49	190

1. H_0 : independent, H_1 : not independent, $\alpha = 0.01$
2. Reject null hypothesis, if $\chi^2 \geq \chi_{0.01,4}^2 = 13.277$
3. $\chi^2 = 4.27 + 4.62 + 25.40 + 0.06 + 1.82 + 4.11 + 5.19 + 0.029 + 7.2 = 52.7$
4. Since $52.7 > 13.277$, null hypothesis must be rejected.

13.78

12	23	89	124
13.5	21.4	89.1	
8	12	62	82
8.9	14.2	58.9	
21	30	119	170
18.6	29.4	122.0	
41	65	270	376

1. H_0 : Venders ship equal quantities
 H_1 : Venders do not ship equal quantities; $\alpha = 0.01$
2. Reject null hypothesis, if $\chi^2 \geq \chi_{0.01,4}^2 = 13.277$
3. $\chi^2 = 0.17 + 0.12 + 0.00 + 0.09 + 0.34 + 0.16 + 0.31 + 0.01 + 0.07 = 1.27 = 1.3$
4. Since $1.3 < 13.277$, null hypothesis cannot be rejected.

13.79

174	93	133	400
159.4	99.1	141.5	
196	124	180	500
199.2	123.8	177.0	
148	105	147	400
159.4	99.1	141.5	
518	322	460	1300

1. H_0 : percentages same for three cities
 H_1 : percentages *not* same for three cities $\alpha = 0.05$
2. Reject null hypothesis, if $\chi^2 \geq \chi_{0.05,4}^2 = 9.488$
3. $\chi^2 = 1.34 + 0.38 + 0.51 + 0.05 + 0.00 + 0.05 + 0.82 + 0.35 + 0.21 = 3.71$
4. Since $3.71 < 9.488$, null hypothesis cannot be rejected.

13.80

	f	prob	e
0	19	1/16	10
1	54	4/16	40
2	58	10/16	60
3	23	4/16	40
4	6	1/16	10

1. H_0 : coins are balanced
 H_1 : coins are *not* balanced
 $\alpha = 0.05$
2. Reject null hypothesis if $\chi^2 \geq \chi_{0.05,4}^2 = 9.488$
3. $\chi^2 = \frac{81}{10} + \frac{196}{40} + \frac{4}{60} + \frac{289}{40} + \frac{16}{10} = 8.1 + 4.9 + 0.1 + 7.2 + 1.6 = 21.9$
4. Since $21.9 > 9.488$, null hypothesis must be rejected.

13.81

	f	prob	e	
0	19	0.0907	27.2	1. H_0 : Poisson distribution with $\lambda = 2.4$
1	48	0.2177	65.3	H_1 : not Poisson distribution with
2	66	0.2613	78.4	$\lambda = 2.4$
3	74	0.2090	62.7	$\alpha = 0.05$
4	44	0.1254	37.6	
5	35	0.0602	18.1	2. Reject null hypothesis if
6	10	0.0241	7.2	$\chi^2 \geq \chi^2_{0.05,6} = 12.592$
7 or more	4	0.0117	3.5	
	300			

$$3. \quad \chi^2 = 2.47 + 4.58 + 1.96 + 2.04 + 1.09 + 15.78 + 1.02 = 28.9$$

4. Since $28.9 > 12.592$, null hypothesis must be rejected.

$$13.82 \quad \bar{x} = \frac{0 \cdot 1 + 1 \cdot 16 + 2 \cdot 55 + 3 \cdot 228}{300} = \frac{810}{300} = 2.7 \quad \hat{\theta} = \frac{2.7}{3} = 0.9$$

	f	prob	e	
0	1	0.001	0.3	1. H_0 : binomial distribution
1	16	0.027	8.1	H_1 : not binomial distribution
2	55	0.243	72.9	$\alpha = 0.05$
3	228	0.729	218.7	2. Reject null hypothesis if
				$\chi^2 \geq \chi^2_{0.05,1} = 3.841$

$$3. \quad \chi^2 = 8.80 + 4.40 + 0.40 = 13.6$$

4. Since $13.6 > 3.841$, null hypothesis must be rejected.

13.83 (a) $\bar{x} = 20$ and $s = 5.025 = 5$

$$\text{using } \bar{x} = \frac{\sum xf}{n} \text{ and } s = \sqrt{\frac{n(\sum x^2 f) - (\sum xf)^2}{n(n-1)}}$$

where x 's are the class marks (midpoints)

(b)

	z		e	
9.5	-2.1	0.4821	0.0179	1.8
14.5	-1.1	0.3643	0.1178	11.8
19.5	-0.1	0.0398	0.3245	32.4
24.5	0.9	0.3159	0.3557	35.5
29.5	1.9	0.4713	0.1554	15.5
34.5	2.9	0.4981	0.0268	2.7
			0.0019	0.2

Probabilities are 0.0179, 0.1178, 0.3245, 0.3557, 0.1554, 0.0268, 0.0019.

(c) Expected frequencies are 1.8, 11.8, 32.4, 35.6, 15.5, 2.7, 0.2

1. H_0 : normally distributed random variables
 H_A : not normally distributed random variables, $\alpha = 0.05$

f	e
11	13.6
37	32.4
36	35.6
16	18.4

2. Reject null hypothesis if $\chi^2 \geq \chi_{0.05,1}^2 = 3.841$
3. $\chi^2 = 0.50 + 0.65 + 0.00 + 0.31 = 1.46$
4. Since $1.46 < 3.841$, null hypothesis cannot be rejected.

13.84 $H_0: \mu = 300$; $H_1: \mu < 300$. Using MINITAB:

MTB> Ttest 300 C1;
 SUBC> Alternative -1.

we get

N	MEAN	ST DEV	SEMEAN	T	P VALUE
38	284.553	104.220	16.907	-0.91	0.18

With a P -value of 0.18, the mean failure time is not significantly less than 300 hours at the 0.01 level of significance.

13.85 $H_0: \mu_1 = \mu_2$; $H_1: \mu_1 \neq \mu_2$ Using MINITAB:

MTB> TwosampleT for C1 vs C2

we get

	N	MEAN	ST DEV	SEMEAN
C1	20	57.76	3.66	0.82
C2	20	52.75	5.01	1.1

TTEST MUC1=MUC2: T = 3.61 P = 0.0009 DF = 38

With a P -value of 0.0009, we conclude that the difference between the mean drying times is significant at the 0.05 level of significance.

13.86 Using MINITAB, we enter the three columns in this table into C1, C2, and C3, respectively.

MTB> Chisquare C1 C2 C3

Expected counts are printed below observed counts.

	C1	C2	C3	Total
1	36	22	18	76
	35.32	23.91	16.77	
2.	63	45	29	137
	63.68	43.09	30.23	
Total	99	67	47	213

$\text{Chisq} = 0.013 + 0.152 + 0.090 + 0.007 + 0.084 + 0.050 = 0.397$

From Table V with $df = 2$, $\chi_{0.05,2}^2 = 5.991$, and we cannot reject the null hypothesis that the three materials have the same probability of leaking at the 0.05 level of significance.