Chapter 11

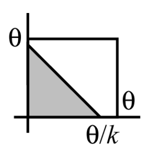
11.1 $P(0 < \theta < kx) = 1 - \alpha$

$$= p\left(x > \frac{\theta}{k} \right)$$

$$\int_{\theta/k}^{\infty} \frac{1}{\theta} e^{-x/\theta} dx = -e^{-x/\theta}\Big|_{\theta/k}^{\infty} = e^{-1/k} = 1 - \alpha$$

$$-\frac{1}{k} = \ln(1-\alpha) \text{ and } k = \frac{-1}{\ln(1-\alpha)}$$

11.2 **(a)**



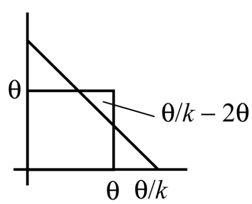
$$p[0 < \theta < k(x_1 + x_2)] = 1 - \alpha$$

$$p\left[(x_1 + x_2) > \frac{\theta}{k} \right] = 1 - \alpha$$

$$p\left[(x_1 + x_2) < \frac{\theta}{k} \right] = \alpha$$

$$\frac{1}{2} \cdot \frac{\theta^2}{k^2} \cdot \frac{1}{\theta^2} = \alpha \qquad \frac{1}{2k^2} = \alpha \qquad k^2 = \frac{1}{2\alpha} \qquad k = \frac{1}{\sqrt{2\alpha}}$$

(b)



$$p\left(x_1 + x_2 > \frac{\theta}{k} \right) = 1 - \alpha$$

$$\frac{1}{2}\left(\frac{\theta}{k} - 2\theta \right)^2 \frac{1}{\theta^2} = 1 - \alpha$$

$$\left(\frac{1}{k} - 2 \right)^2 = 2(1-\alpha), \ \frac{1}{k} - 2 = \pm\sqrt{2(1-\alpha)}$$

$$k = \frac{1}{2 \pm \sqrt{2(1-\alpha)}}$$

$$k = \frac{1}{2 - \sqrt{2(1-\alpha)}}$$

11.3 $p(R < \theta < cR) = 1 - \alpha$

$$p\left(\frac{\theta}{c} < R < \theta\right) = 1 - \alpha$$

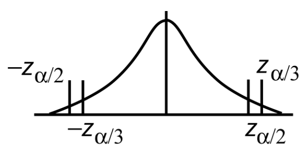
$$\frac{2}{\theta^2}\int_{\theta/c}^{\theta}(\theta - R)dR = 1 - \alpha \qquad \frac{2}{\theta^2}\left[\theta R - \frac{R^2}{2}\right]\Bigg|_{\theta/c}^{\theta}$$

$$\frac{2}{\theta^2}\left(\theta^2 - \frac{\theta^2}{2} - \frac{\theta^2}{c} + \frac{\theta^2}{2c^2}\right) = 1 - \alpha$$

$$1 - \frac{2}{c} + \frac{1}{2c^2} = 1 - \alpha,\ c^2 - 2c + 1 = (1-\alpha)c^2$$

$$ac^2 - 2c + 1 = 0 \text{ and } c = \frac{2 \pm \sqrt{4 - 4\alpha}}{2\alpha} = \frac{1 \pm \sqrt{1-\alpha}}{\alpha}$$

11.4

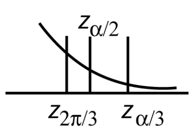


By inspection

$$z_{\alpha/3} - z_{\alpha/2} > z_{\alpha/2} - z_{2\alpha/3}$$

$$2z_{\alpha/2} < z_{\alpha/3} + z_{2\alpha/3}$$

length of first confidence interval is less than that of 2nd confidence interval



11.5 Length of confidence interval:

$$L = \bar{X} + z_{(1-k)\alpha}\cdot\frac{\sigma}{\sqrt{n}} - \left(\bar{X} - z_{k\alpha}\cdot\frac{\sigma}{\sqrt{n}}\right)$$

$$= (z_{(1-k)\alpha} + z_{k\alpha})\cdot\frac{\sigma}{\sqrt{n}}$$

If $k = 1/2$, $\qquad L_{1/2} = 2z_{\alpha/2}\cdot\dfrac{\sigma}{\sqrt{n}}$

If $k < 1/2$,

$$z_{k\alpha} = z_{a/2} + \delta_1 > z_{\alpha/2} \qquad \delta_1 > 0;\ z_{(1-k)\alpha} < z_{(1-k)\alpha} + \delta_2 = z_{\alpha/2} \text{ where } \delta_2 > 0$$

and $\qquad L_k = [2z_{\alpha/2} + (\hat{\delta}_1 - \hat{\delta}_2)]\cdot\dfrac{\sigma}{\sqrt{n}}$

Since the normal density function $f(x)$ is decreasing for $x > 0$, $\delta_2 < \delta_1$, thus

$$L_k > 2z_{\alpha/2}\cdot\frac{\sigma}{\sqrt{n}}$$

By the symmetry of $f(x)$, for $k > 1/2$, $L_k > 2z_{\alpha/2}\cdot\dfrac{\sigma}{\sqrt{n}}$

11.6 $p\left[|\bar{x}-\mu| < z_{\alpha/2}\cdot\frac{\sigma}{\sqrt{n}} = 1-\alpha\right]$

$z_{\alpha/2}\cdot\frac{\sigma}{\sqrt{n}} = E$ and $\sqrt{n} = z_{\alpha/2}\cdot\frac{\sigma}{E}$

$n = \left[z_{\alpha/2}\cdot\frac{\sigma}{E}\right]^2$

11.7 Substitute $t_{\alpha/2,n-1}\cdot\frac{s}{\sqrt{n}}$ for $= z_{\alpha/2}\cdot\frac{\sigma}{\sqrt{n}}$

If $\bar{x}$, the mean of a random sample of size n from a formal population with the mean μ, is used as an estimate of μ, we can assert with $(1-\alpha)100\%$ confidence that the error is less than $t_{\alpha/2,n-1}\cdot\frac{s}{\sqrt{n}}$.

11.8 If $\bar{x}_1$ and $\bar{x}_2$ are the means of independent random samples of size n_1 and n_2 from normal populations with μ_1, μ_2, σ_1, and σ_2, and $\bar{x}_1-\bar{x}_2$ is to be used as an estimate if $\mu_1-\mu_2$, the probability is $1-\alpha$ that error will be less than

$$z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}}$$

11.9 $E(S_p^2) = \frac{n_1-1}{n_1+n_2-2}\cdot\sigma^2 + \frac{n_2-1}{n_1+n_2-2}\cdot\sigma^2 = \frac{n_1+n_2-2}{n_1+n_2-2}\cdot\sigma^2 = \sigma^2$

therefore unbiased

$\frac{(n_1-1)s_1^2}{\sigma^2} \to \chi^2(n_1-1)$ $\qquad$ $\frac{(n_2-1)s_2^2}{\sigma^2} \to \chi^2(n_2-1)$

$\frac{(n_1-1)s_1^2}{\sigma^2} + \frac{(n_2-1)s_2^2}{\sigma^2} \to \chi^2(n_1+n_2-2)$ $\qquad$ var is $2(n_1+n_2-2)$

$(n_1-1)s_1^2 + (n_2-1)s_2^2$ $\qquad$ var is $2\sigma^4(n_1+n_2-2)$

$\frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}$ $\qquad$ var is $\frac{2\sigma^4}{(n_1+n_2-2)}$

11.10 $T = \frac{Z}{\sqrt{\frac{Y}{n_1+n_2-2}}} = \frac{(\bar{x}_1-\bar{x}_2)-(\mu_2-\mu_1)}{\sigma\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}}\cdot\frac{1}{\sqrt{\frac{(n_1+n_2-2)S_p^2}{n_1+n_2-2}}}$

$= \frac{(\bar{x}_1-\bar{x}_2)-(\mu_1-\mu_2)}{S_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}}$

11.11 $-z_{\alpha/2}\sqrt{n\theta(1-\theta)} = x - np$ and $z_{\alpha/2}\sqrt{n\theta(1-\theta)} = x - np$

$${z^2}_{\alpha/2} n\theta(1-\theta) = (x-n\theta)^2 = x^2 - 2xn\theta + n^2\theta^2$$

$$n^2\theta^2 + nz^2_{\alpha/2} - 2xn\theta - nz^2_{\alpha/2}\theta + x^2 = 0$$

$$(n + z^2_{\alpha/2})\theta^2 - (2x + z^2_{\alpha/2})\theta + \frac{x^2}{n} = 0$$

by quadratic formula

$$\theta = \frac{2x + z^2_{a/2} \pm \sqrt{(2x + z^2_{\alpha/2})^2 - 4(n + z^2_{\alpha/2})\left(\frac{x^2}{n}\right)}}{2(\left(n + z^2_{\alpha/2}\right)}$$

11.13 $-z_{\alpha/2} < \dfrac{x - n\theta'}{\sqrt{n\theta'(1-\theta'')}}$; $\qquad \dfrac{x - n\theta''}{\sqrt{n\theta''(1-\theta'')}} < z_{\alpha/2}$

Let $\theta*$ = value of θ with $\theta' < \theta < \theta''$ closest to $\frac{1}{2}$. By Theorem 11.7,

$e < z_{\alpha/2}\sqrt{\dfrac{\theta*(1-\theta*)}{n}}$ and $n = \theta*(1-\theta*)\dfrac{z^2_{\alpha/2}}{e^2}$

11.15 By Theorem 11.8 with probability approximately $1-\alpha$

$$E < z_{\alpha/2}\sqrt{\frac{\hat{\theta}_1(1-\hat{\theta}_1)}{n_1} + \frac{\hat{\theta}_2(1-\hat{\theta}_2)}{n_2}}$$

11.16 If $n_1 = n_2 = n$, then $E < z_{\alpha/2}\sqrt{\dfrac{\hat{\theta}_1(1-\hat{\theta}_1) + \hat{\theta}_2(1-\hat{\theta}_2)}{n}}$

The right-hand side of this inequality is maximized when $\theta_1 = \theta_2 = \dfrac{1}{2}$.

Thus, $E < z_{\alpha/2}\sqrt{\dfrac{1}{2n}}$, $\qquad E^2 < \dfrac{z^2_{\alpha/2}}{2n}$, $\qquad$ and $n = \dfrac{z^2_{\alpha/2}}{2E^2}$.

11.17 $\dfrac{1}{2n}\chi^2_{\alpha,2(x+1)} = \dfrac{1}{400}\chi^2_{0.01,8} = 0.050$

11.18 $\dfrac{1}{f_{1-\alpha/2,n_1-1,n_2-1}} > \dfrac{\sigma_1^2 s_2^2}{\sigma_2^2 s_1^2} > \dfrac{1}{f_{\alpha/2,n_1-1,n_2-1}}$

$$\frac{s_1^2}{s_2^2}\cdot\frac{1}{f_{\alpha/2,n_1-1,n_2-1}} < \frac{\sigma_1^2}{\sigma_2^2} < \frac{s_1^2}{s_2^2}\cdot\frac{1}{f_{1-\alpha/2,n_1-1,n_2-1}}$$

$$\frac{s_1^2}{s_2^2}\cdot\frac{1}{f_{\alpha/2,n_1-1,n_2-1}} < \frac{\sigma_1^2}{\sigma_2^2} < \frac{s_1^2}{s_2^2}\cdot f_{\alpha/2,n_2-1,n_1-1}$$

11.19 $\sigma - z_{\alpha/2}\dfrac{\sigma}{\sqrt{2n}} < s < \sigma < z_{\alpha/2}\dfrac{\sigma}{\sqrt{2n}}$

$$\sigma\left(1-\frac{z_{\alpha/2}}{\sqrt{2n}}\right) < s < \sigma\left(1+\frac{z_{\alpha/2}}{\sqrt{2n}}\right)$$

$$\frac{1}{\sigma\left(1-\frac{z_{\alpha/2}}{\sqrt{2n}}\right)} > \frac{1}{s} > \frac{1}{\sigma\left(1+\frac{z_{\alpha/2}}{\sqrt{2n}}\right)}$$

$$\frac{s}{1+\frac{z_{\alpha/2}}{\sqrt{2n}}} < \sigma < \frac{s}{1-\frac{z_{\alpha/2}}{\sqrt{2n}}}$$

11.20 $n = 150 \qquad \sigma = 9.4 \qquad E = 1.96\dfrac{9.4}{\sqrt{150}} = \dfrac{1.96(9.4)}{12.247} = 1.50$

11.21 $61.8 \pm 2.575\cdot\dfrac{9.4}{\sqrt{150}} = 61.8 \pm 1.98,\ 59.82 < \mu < 63.78$

11.22 $E = 2.575\cdot\dfrac{10.5}{\sqrt{120}} = 2.575\cdot\dfrac{10.5}{10.955} = 2.47$ mm

11.23 $141.8 \pm 2.33\cdot\dfrac{10.5}{\sqrt{120}} = 141.8 \pm 2.33\dfrac{10.5}{10.955} = 141.8 \pm 2.23$

$$139.57 < \mu < 144.03$$

11.24 $\bar{x} \pm z_{0.005}\dfrac{s}{\sqrt{n}}; \qquad 52.80 \pm 2.575\dfrac{45}{\sqrt{64}}$, or (51.35, 54.25).

11.25 $e < z_{0.025}\dfrac{s}{\sqrt{n}} = 1.96\dfrac{2.68}{\sqrt{40}} = 0.83$ min.

11.26 $e < z_{0.025}\dfrac{\sigma}{\sqrt{n}}\sqrt{\dfrac{N-n}{N-1}} = 1.96\dfrac{9.4}{\sqrt{150}}\sqrt{\dfrac{900-150}{900-1}} = 1.37.$

11.27 $\bar{x} \pm z_{0.005}\dfrac{\sigma}{\sqrt{n}}\sqrt{\dfrac{N-n}{N-1}};\ \ 61.8 \pm 2.575\dfrac{9.4}{\sqrt{150}}\sqrt{\dfrac{900-150}{900-1}}$, or (60.01, 63.61).

11.28 $n = \left[z_{0.025}\dfrac{\sigma}{e}\right]^2 = \left[1.96\dfrac{12.2}{2.5}\right]^2 = 91.48$ or 92, rounded up to the nearest integer.

11.29 $n = \left[z_{\alpha/2}\dfrac{\sigma}{e}\right]^2 = 1.96\left[\dfrac{3.2}{1/3}\right]^2 = 354.04$ or 355, rounded up to the nearest integer.

11.30 $\bar{x} \pm t_{0.025,n-1}\dfrac{s}{\sqrt{n}}$; $5.68 \pm 2.262\dfrac{0.29}{\sqrt{10}}$, or (5.47, 5.89)

11.31 $\bar{x} \pm t_{0.005,17}\dfrac{s}{\sqrt{n}}$; $63.84 \pm 2.898\dfrac{2.75}{\sqrt{18}}$; or (61.96, 65.72).

11.32 $e < t_{0.025,11}\dfrac{s}{\sqrt{n}} = 2.201\dfrac{0.625}{\sqrt{12}} = 0.40$

11.33 $(\bar{x}_1 - \bar{x}_2) \pm z_{0.05}\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}$; $-5.2 \pm 1.645\sqrt{\dfrac{4.8^2}{16} + \dfrac{3.5^2}{25}}$, or $(-7.49, -2.91)$.

11.34 $(\bar{x}_1 - \bar{x}_2) \pm z_{0.05}\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}$; $-7.4 \pm 2.575\sqrt{\dfrac{19.4^2 + 18.8^2}{61}}$, or $(-16.31, 1.51)$.

11.35 $s_p^2 = \dfrac{11(1.2)^2 + 14(1.5)^2}{25} = 1.8936$ $s_p = 1.376$

$(13.8 - 12.9) \pm 2.060(1.376)\sqrt{\dfrac{1}{12} + \dfrac{1}{15}}$

$0.9 \pm 2.8346(0.387)$, 0.9 ± 1.098

$-0.198 < \mu_1 - \mu_2 < 1.998$ feet

11.36 $\bar{x}_1 = 8260$, $s_1 = 251.89$, $\bar{x}_2 = 7930$, $s_2 = 206.52$

$s_p^2 = \dfrac{4(251.89)^2 + 4(206.52)^2}{8} = 53{,}049.54$ $s_p = 230.32$

$8260 - 7930 \pm 3.355(230.32)\sqrt{\dfrac{1}{5} + \dfrac{1}{5}}$

330 ± 488.75

$-158.75 < \mu_1 - \mu_2 < 818.75$ million calorie per ton

11.37 $E = 2.33\sqrt{\dfrac{(0.004)^2}{35} + \dfrac{(0.005)^2}{45}}$

$= 2.33(0.001) = 0.0023$ ohm

11.38 $\hat{\theta} = \dfrac{204}{300} = 0.68$

$0.68 \pm 1.96\sqrt{\dfrac{(0.68)(0.32)}{300}}$ 0.68 ± 0.053

$0.627 < \theta < 0.733$

11.39 $e = 2.575\sqrt{\dfrac{(0.68)(0.32)}{300}} = 0.069$

11.40 **(a)** $\frac{190}{250} = 0.76 \qquad 0.76 \pm 2.575\sqrt{\frac{(0.76)(0.24)}{250}}$

$0.76 \pm 0.070 \qquad 0.690 < \theta < 0.830$

(b) $$\frac{190 + \frac{1}{2}(2.575)^2 \pm 2.575\sqrt{\frac{190(60)}{250} + \frac{1}{4}(2.575)^2}}{250 + (2.575)^2}$$

$$\frac{190 + 3.315 \pm 2.575\sqrt{45.6 + 1.658}}{250 + 6.631}$$

$$\frac{193.315 \pm 17.702}{256.631} \qquad 0.684 < \theta < 0.822$$

11.41 $e = 1.96\sqrt{\frac{(0.76)(0.24)}{250}} = 0.053$

11.42 $0.18 \pm 2.575\sqrt{\frac{(0.18)(0.82)}{100}} \qquad 0.18 \pm 0.099$

$0.081 < \theta < 0.279$

11.43 $\frac{54}{120} = 0.45 \qquad e = 1.645\sqrt{\frac{(0.45)(0.55)}{120}} = 0.075$

11.44 $0.05 = z\sqrt{\frac{(0.34)(0.66)}{300}} \qquad 0.05 = 0.02735z \qquad z = 1.83$

confidence is $2(0.4664) \cdot 100 = 93.3\%$

11.45 $n = \frac{(1.96)^2}{4(0.02)^2} = 2401$

11.46 $n = (0.03)(0.70)\left(\frac{1.96}{0.02}\right)^2 = (0.21)(9604) = 2017$

11.47 $n = \frac{(2.575)^2}{4(0.04)^2} = 1037$ rounded up

11.48 $n = (0.65)(0.35)\left(\frac{2.575}{0.04}\right)^2 = 943$

11.49 $\frac{84}{250} = 0.336$ $\qquad \frac{156}{250} = 0.624$

$(0.336 - 0.624) \pm 1.96\sqrt{\frac{(0.336)(0.664)}{250} + \frac{(0.624)(0.376)}{250}}$

$-0.288 \pm 0.084 \qquad -0.372 < \theta_1 - \theta_2 < -0.204$

11.50 $\frac{48}{500} = 0.096, \quad \frac{68}{400} = 0.170$

$0.096 - 0.170 \pm 2.575\sqrt{\frac{(0.096)(0.904)}{500} + \frac{(0.170)(0.830)}{400}}$

$-0.074 \pm 0.059 \qquad -0.133 < \theta_1 - \theta_2 < -0.015$

11.51 $e = 2.33\sqrt{\frac{(0.096)(0.904)}{500} + \frac{(0.170)(0.830)}{400}}$

$= 2.33(0.022939) = 0.053$

11.52 $n = \frac{(1.96)^2}{2(0.05)^2} = 769$

11.53 $\frac{9(0.29)^2}{19.023} < \sigma^2 < \frac{9(0.29)^2}{2.700}$

$0.04 < \sigma^2 < 0.28$

11.54 $\frac{11(0.625)^2}{19.675} < \sigma^2 < \frac{11(0.625)^2}{4.575}$

$0.2184 < \sigma^2 < 0.939 \qquad 0.47 < \sigma < 0.97$

11.55 $\frac{4.5}{1 + \frac{2.575}{\sqrt{128}}} < \sigma < \frac{4.5}{1 - \frac{2.575}{\sqrt{128}}} \qquad 3.67 < \sigma < 5.83$

11.56 $\frac{2.68}{1 + \frac{2.33}{\sqrt{80}}} < \sigma < \frac{2.68}{1 - \frac{2.33}{\sqrt{80}}} \qquad 2.13 < \sigma < 3.62$

11.57 $\frac{19.4^2}{18.8^2} \cdot \frac{1}{f_{0.01,60,60}} < \frac{\sigma_1^2}{\sigma_2^2} < \frac{19.4^2}{18.8^2} \cdot f_{0.01,60,60}$

$\frac{19.4^2}{18.8^2} \cdot \frac{1}{1.84} < \frac{\sigma_1^2}{\sigma_2^2} < \frac{19.4^2}{18.8^2} \cdot 1.84 \qquad 0.58 < \frac{\sigma_1^2}{\sigma_2^2} < 1.96$

11.58 $\frac{(1.2)^2}{(1.5)^2} \cdot \frac{1}{f_{0.01,11,14}} < \frac{\sigma_1^2}{\sigma_2^2} < \frac{(1.2)^2}{(1.5)^2} \cdot f_{0.01,14,11}$

$\frac{0.64}{3.87} < \frac{\sigma_1^2}{\sigma_2^2} < (0.64)(4.30)$ $\quad 0.165 < \frac{\sigma_1^2}{\sigma_2^2} < 2.752$

11.59 $\frac{(251.89)^2}{(206.52)^2} \cdot \frac{1}{f_{0.05,4,4}} < \frac{\sigma_1^2}{\sigma_2^2} < \frac{(251.89)^2}{(206.52)^2} \cdot f_{0.05,4,4}$

$\frac{1.4876}{6.39} < \frac{\sigma_1^2}{\sigma_2^2} < 1.4876(6.39)$ $\quad 0.233 < \frac{\sigma_1^2}{\sigma_2^2} < 9.506$

11.60 Using MINITAB we enter the data into C1 and we give the command

MTB> Tinterval 95.0 C1

Obtaining

N	MEAN	STDEV	SEMEAN	95.0 PERCENT C.I.
20	6.145	1.467	0.328	(5.458, 6.832)

11.61 Using MINITAB we enter the data into C1 and C2 and we give the command

MTB> St Dev C1 obtaining

ST DEV = 275.87

Then, with $\chi^2_{0.05,29} = 42.557$ and $\chi^2_{0.95,29} = 17.70$, we have

$$\frac{29(275.87)^2}{42.557} < \sigma^2 < \frac{29(275.87)^2}{17.78}$$

or $227.7 < \sigma < 352.3$ with 90% confidence.