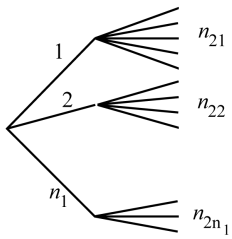
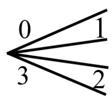
Chapter 1

1.1



(a) $\sum_{i=1}^{n_1} n_{2i}$

(b)



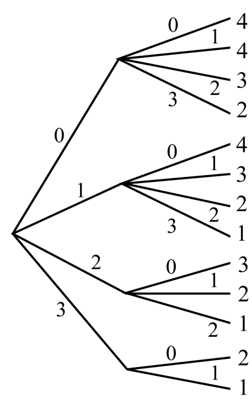
0 1 2 3
0 1 2 3
0 1 2
0 1

$\sum = 13$

1.2 $\sum_{i=1}^{n_1} n_2 i = \sum_{i=1}^{n_1} n_2 = n_1 n_2$

1.3 (a)

$n_{300} = 4$ $\quad n_{320} = 3$
$n_{301} = 4$ $\quad n_{321} = 2$
$n_{302} = 3$ $\quad n_{322} = 1$
$n_{303} = 2$ $\quad n_{330} = 2$
$n_{310} = 4$ $\quad n_{331} = 1$
$n_{311} = 3$
$n_{312} = 2$
$n_{313} = 1$



(b) $\sum = 4 + 4 + 3 + \ldots + 2 + 1 = 32$

1.4 $\sum_{i=1}^{n_1} \sum_{j=1}^{n_2} n_2 = n_1 n_2 n_3$

1.5 (b) 6, 20, and 70

"2 out of 3" $m = 2$ $\quad 2\left[\binom{1}{1}+\binom{2}{1}\right] = 2(1+2) = 6$

"3 out of 5" $m = 3$ $\quad 2\left[\binom{2}{2}+\binom{3}{2}+\binom{4}{2}\right] = 2(1+3+6) = 20$

"4 out of 7" $m = 4$ $\quad 2\left[\binom{3}{3}+\binom{4}{3}+\binom{5}{3}+\binom{6}{3}\right] = 2(1+4+10+20) = 70$

1.6 (a) $10! \approx \sqrt{20\pi}\left(\frac{10}{e}\right)^{10} = (7.92665)(3.678797)^{10} = (7.92665)(454{,}002.49) = 3{,}598{,}719$

$$\% \text{ error } = \frac{3.6288 - 3.5987}{3.6288} \cdot 100 = 0.83\%$$

$$12! \approx \sqrt{24\pi}\left(\frac{12}{e}\right)^{12} = (8.683215)(4.41455)^{12} = 475{,}683{,}224$$

$$\% \text{ error} = \frac{4.7800 - 4.7568}{4.7900} \cdot 100 = 0.69\%$$

(b) $$\binom{52}{13} = \frac{52!}{13!\ 39!} = \frac{\sqrt{104\pi}\left(\frac{52}{e}\right)^{52}}{\sqrt{26\pi}\sqrt{78\pi}\left(\frac{13}{e}\right)^{13}\left(\frac{39}{e}\right)^{39}}$$

$$= \frac{13^{52} \cdot 4^{52}}{\sqrt{19.5\pi}\ 13^{13} \cdot 13^{39} \cdot 3^{39}} = \frac{4^{52}}{\sqrt{19.5\pi}\ 3^{39}} = \text{ 639 billion}$$

1.7 Using Stirling's formula in $\binom{2n}{n} = \frac{2n!}{n!\ n!}$ yields

$$\frac{\binom{2n}{n}\sqrt{\pi n}}{2^{2n}} = \frac{\sqrt{4\pi n}\left(\frac{2n}{e}\right)^{2n}}{\left[\sqrt{2\pi n}\left(\frac{\pi}{e}\right)^{n}\right]^{2}} \cdot \frac{\sqrt{\pi n}}{2^{2n}} = 1$$

1.8 n^r and $12^3 = 1{,}728$

1.9 $\binom{r+n-1}{r}$ and $\binom{5+3-1}{5} = \binom{7}{5} = 21$

1.10 Substitute $r - n$ for r into result of 1.9

$$\binom{r-n+n-1}{r-n} = \binom{r-1}{r-n} \text{ and } \binom{5-1}{5-3} = \binom{4}{2} = 6$$

1.11 (b) Seventh row is 1, 6, 15, 20, 15, 6, 1

Eighth row is 1, 7, 21, 35, 35, 21, 7, 1

$$(x+y)^6 = x^6 + 6x^5y + 15x^4y^2 + 20x^2y^3 + 15x^2y^4 + 6xy^5 + y^6$$

$$(x+y)^7 = x^7 + 7x^6y + 21x^5y^2 + 35x^4y^3 + 35x^3y^4 + 21x^2y^5 + 7xy^6 + y^7$$

1.14 (a) Set $x = 1$ and $y = 1$
(b) Set $x = 1$ and $y = -1$
(c) Set $x = 1$ and $y = a - 1$

1.19 (a) $$\frac{\frac{1}{2}\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right)}{24} = -\frac{15}{384} \text{ and } \frac{(-3)(-4)(-5)}{6} = -10$$

(b) $$\sqrt{5} = 2\left(1+\frac{1}{4}\right)^{\frac{1}{2}} = 2\left[1+\frac{1}{2}\left(\frac{1}{4}\right)+\frac{1}{2}\left(-\frac{1}{2}\right)\left(\frac{1}{4}\right)^2+\frac{1}{2}\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(\frac{1}{4}\right)^3\right]$$
$$= 2\left[1+\frac{1}{8}-\frac{1}{64}+\frac{3}{512}\cdots\right] = 2\cdot\frac{512+64-8+3}{512}$$
$$= 2\cdot\frac{571}{512} = 2.23$$

$$\frac{1142}{512} = 2.230$$

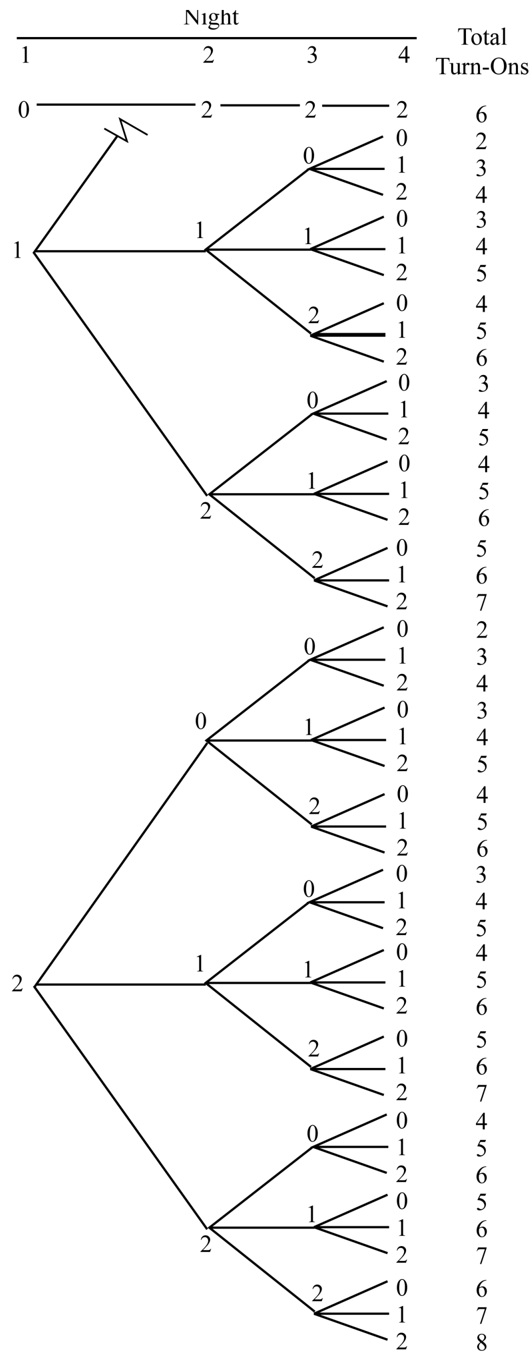
1.20 (a) $$\frac{(-1)(-2)\ldots(-r)}{r!} = (-1)^r$$

(b) $$\binom{-n}{r} = \frac{(-n)(-n-1)\ldots(-n-r+1)}{r!} = (-1)^r\frac{n(n+1)\ldots(n+r-1)}{r!}$$
$$= (-1)^r\frac{(n+r-1)\ldots(n+1)n}{r!} = (-1)^r\binom{n+r-1}{r}$$

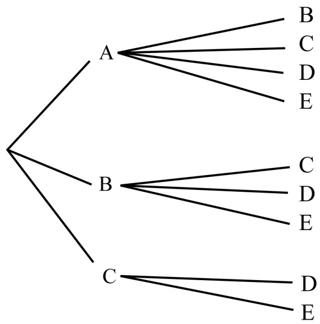
1.21 $$\frac{8!}{2!\,3!\,3!} = \frac{8\cdot7\cdot6\cdot5\cdot4}{2\cdot6} = 560$$

1.22 $$\frac{9!}{3!\,2!\,3!}\cdot2^3\cdot3^2\cdot(-4)^3 = \frac{9\cdot8\cdot7\cdot6\cdot5\cdot4}{12}\cdot8\cdot9\cdot64 = -23{,}224{,}320$$

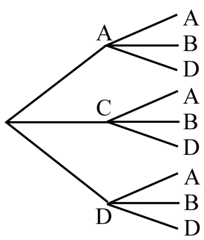
1.24 Note: If there are 0 turn-ons the first night, 6 turn-ons in four nights can only occur if there are 2 turn-ons on each of the subsequent three nights. Thus, we need to show only that part of the tree following this event.



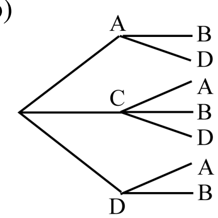
1.25



1.26 (a)



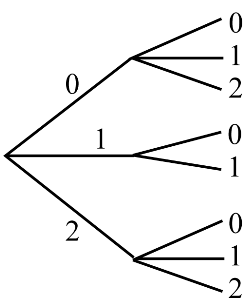
(b)



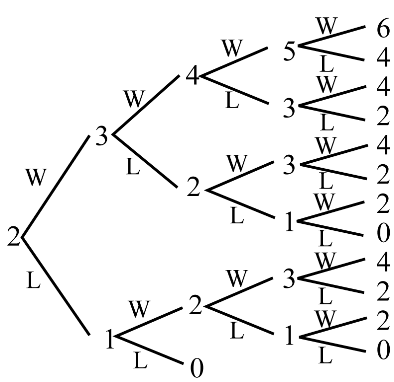
1.27 (a) 5

(b) 4

1.28



1.29



1.30 (a) $6 \cdot 5 = 30$; (b) $6 \cdot 6 = 36$;

1.31 (a) 6; (b) $6 \cdot 5 = 30$; (c) $5 \cdot 4 = 20$ first one fixed; (d) $6 + 30 + 20 = 56$

1.32 (a) $4 \cdot 5 \cdot 2 = 40$; (b) $5 \cdot 6 \cdot 3 = 90$

1.33 (a) $5 \cdot 4 = 20$; (b) $5 \cdot 4 \cdot 3 = 60$

1.34 $3^{15} = 14{,}348{,}907$

1.35 $\frac{15 \cdot 14}{2 \cdot 1} = 105$

1.36 (a) $10 \cdot 9 \cdot 8 \cdot 7 = 5040$; (b) $\frac{5040}{24} = 210$

1.37 (a) $\frac{14 \cdot 13}{2 \cdot 1} = 91$; (b) $\frac{14 \cdot 13 \cdot 12}{3 \cdot 2 \cdot 1} = 364$

1.38 $6! = 720$

1.39 $\frac{6!}{2!\,2!\,2!} = \frac{720}{8} = 90$

1.40 $5! = 120$ and $120 - 2 \cdot 4! = 72$

1.41 $7! = 5040$

1.42 (a) $5! = 120$; (b) $\frac{5!}{2!} = 60$

1.43 $\frac{10!}{3!\,3!\,2!} = \frac{3628800}{72} = 50{,}400$ and $\frac{8!}{3!\,2!} = \frac{40320}{12} = 3360$

1.44 $\frac{10!}{5!\,4!} = \frac{3628800}{120 \cdot 24} = 1{,}260$

1.45 $\frac{8!}{3!\,4!} = \frac{40320}{6 \cdot 24} = 280$

1.46 (a) $\binom{20}{7} = 77{,}520$; (b) $\binom{20}{10} = 184{,}755$

(c) $\binom{20}{17} + \binom{20}{18} + \binom{20}{19} + \binom{20}{20} = 1140 + 190 + 20 + 1 = 1351$

1.47 (a) $\binom{7}{2} = 21$; (b) $\binom{4}{2} = 6$; (c) $3 \cdot 4 = 12$

1.48 $\binom{3}{2}\binom{7}{2}+\binom{3}{3}\binom{7}{1}=3\cdot 21+1\cdot 7=63+7=70$

1.49 $\binom{4}{2}\binom{7}{3}\binom{3}{1}=6\cdot 35\cdot 3=630$

1.50 $\binom{13}{5}\binom{13}{3}\binom{13}{3}\binom{13}{2}=1287\cdot 286\cdot 286\cdot 78=8,211,173,256$

1.51 $\frac{7!}{3!\ 2!}=\frac{5040}{12}=420$

1.52 $3^{10}=59,049$

1.53 $5^{5}=15,625$

1.54 $\binom{12+6-1}{12}=\binom{17}{12}=\binom{17}{5}=6,188$

1.55 $\binom{12-1}{6}=\binom{11}{6}=462$

1.56 $\binom{14+3-1}{14}=\binom{16}{14}=120$

1.57 $\binom{r-2n+n-1}{n-1}=\binom{r-n-1}{n-1}$

$$\binom{r-n-1}{n-1}=\binom{10}{2}=45$$