Ass #2

Q1) $P(A) = 0.1$, $P(B) = 0.2$

$P(C) = 0.3$

(i) $P(A^c \cap B^c \cap C^c) = 0.504$

(ii) $P(A \cap B \cap C) = 6 \times 10^{-3}$

(iii) $P = P(A \cap B \cap C^c) + P(A^c \cap B \cap C^c) + P(A^c \cap B^c \cap C)$

$P = 0.398$ (exactly one crashes)

(iv) $P = 1 - P(\text{none crash})$

$P = 1 - 0.504$

$\boxed{P = 0.496}$

(v) $P = P(\text{None crashes}) + P(\text{exactly one crashes})$

$= 0.504 + 0.398 = 0.902$

$\boxed{P = 0.902}$

(iii) $P = P(A \cap B \cap C^c) + P(A \cap B^c \cap C) + P(A^c \cap B \cap C)$

$\boxed{P = 0.092}$

(2) $f(x) = \begin{cases} x & 0 < x < 1 \\ 2-x & 1 \le x < 2 \\ 0 & \text{else} \end{cases}$

$$F(x) = \int_0^x t\,dt = \frac{x^2}{2} \qquad 0 < x < 1$$

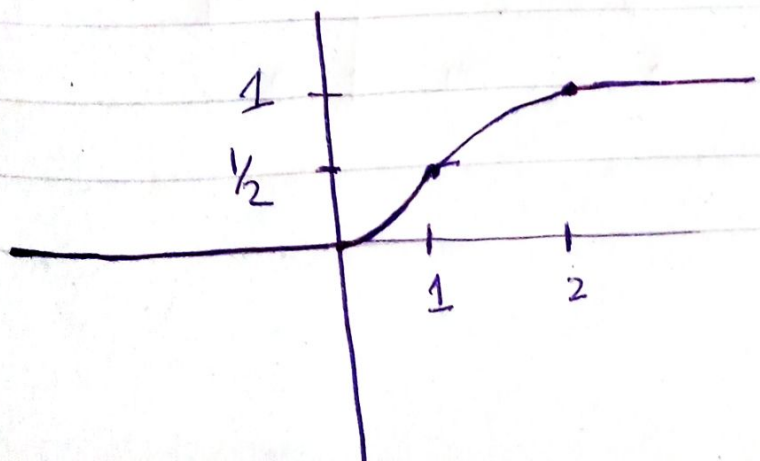
$$F(x) = \int_0^1 t\,dt + \int_1^x (2-t)\,dt \qquad 1 \le x < 2$$

$$= \frac{1}{2} + 2x - 2 - \frac{x^2}{2} + \frac{1}{2}$$

$$= 2x - \frac{x^2}{2} - 1 \qquad 1 \le x < 2$$

$$F(x) = 1 \qquad x \ge 2$$

$$F(x) = \begin{cases} 0 & x \le 0 \\ x^2/2 & 0 < x < 1 \\ 2x - x^2/2 - 1 & 1 \le x < 2 \\ 1 & x \ge 2 \end{cases}$$



Q3) $A = \left[\begin{array}{ccc|c} K & 0 & m & 2 \\ K & K & 4 & 4 \\ 0 & K & 2 & m \end{array}\right]$

$R_2 = R_2 - R_1$

$\left[\begin{array}{ccc|c} K & 0 & m & 2 \\ 0 & K & 4-m & 2 \\ 0 & K & 2 & m \end{array}\right]$

$R_3 = R_3 - R_2$

$\left[\begin{array}{ccc|c} K & 0 & m & 2 \\ 0 & K & 4-m & 2 \\ 0 & 0 & m-2 & m-2 \end{array}\right]$

(i) $m \neq 2$, $K \neq 0$

(ii) $m = 2$, $K \neq 0$

(iii) $m = 2$, $K = 0$

(iv) $K = 0$, $m \neq 2$

Q4) $$f(x) = \begin{cases} \frac{1}{19-14} & 14 \le x \le 19 \\ 0 & \text{else} \end{cases}$$

$$f(x) = \begin{cases} \frac{1}{5} & 14 \le x \le 19 \\ 0 & \text{else} \end{cases}$$

$$F(x) = \int_{14}^{x} \frac{1}{5}\, dt = \frac{1}{5}\left[x - 14\right]$$

$$F(x) = \begin{cases} \frac{x-14}{5} & 14 \le x \le 19 \\ 1 & x > 19 \\ 0 & \text{else} \end{cases}$$

(ii) $$P(X \le 16) = \int_{14}^{16} \frac{1}{5}\, dt$$

$$P(X \le 16) = 0.4$$

(iii) $$P(X \ge 15) = -\int_{14}^{15} \frac{1}{5}\, dt + 1$$

$$P(X \ge 15) = 0.8$$

(iv) $P(X = 18) = 0$

(v) $P(15 < X < 17) = \int_{15}^{17} \frac{1}{5}\, dt$

$P(15 < X < 17) = 0.4$

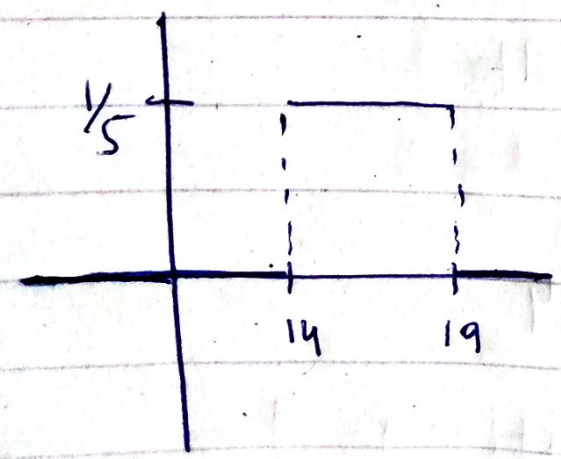
(v) $E(X) = \frac{a+b}{2} = \frac{14+19}{2}$

$$\boxed{E(X) = 16.5}$$

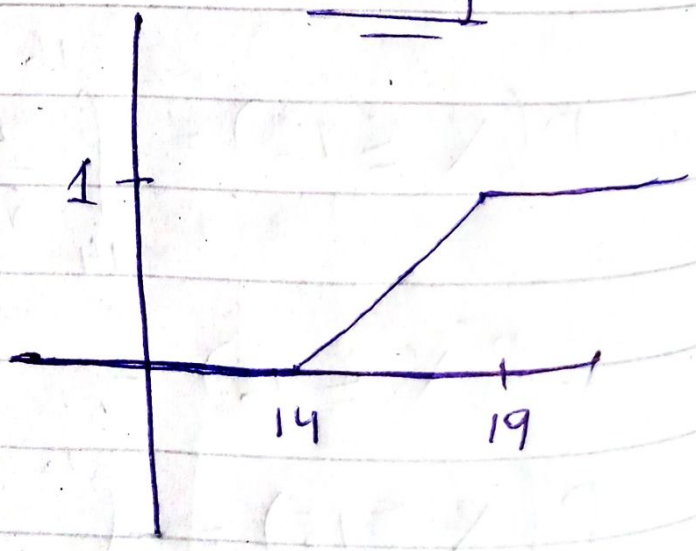
(vi) $V(X) = \frac{(b-a)^2}{12} = 2.08$

Graph

P.d.f



C.d.f



Q5) $A = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

C_1 and C_2 are pivot

(ii) $B = \begin{bmatrix} 2 & 5 & 0 \\ 0 & 0 & 0 \\ 1 & 3 & 1 \end{bmatrix}$

by row reduction

$\Rightarrow \begin{bmatrix} 1 & 0 & -5 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$

C_1 and C_2 are pivot