02_Components

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0.0.1 1. What is Time Series Decomposition?

Time series decomposition is the process of breaking down a time series into **3 main components**:

- **Trend-Cycle (T_t)**: The long-term progression of the series (e.g., employment gradually increasing over years).
- **Seasonal (S_t)**: Repeating short-term patterns (e.g., holiday hiring spikes every December).
- **Remainder/Residual (R_t)**: The noise or irregular fluctuations not explained by trend or seasonality.

0.0.2 2. Additive vs. Multiplicative Decomposition

Additive Decomposition

$$y_t = S_t + T_t + R_t$$

- Seasonal and irregular components are **constant in magnitude** (do not grow/shrink as the overall level of the series changes).
- Use when the seasonal effect is **independent** of the overall level.

Multiplicative Decomposition

$$y_t = S_t \times T_t \times R_t$$

- Seasonal and irregular components are **proportional** to the trend.
- Common in **economic data** where seasonal effects grow with the trend.

0.0.3 3. When to Use Log Transformation?

If you want to use an additive model on a series where the multiplicative model is more appropriate, you can **log-transform the data**. Why? Because:

$$\log(y_t) = \log(S_t) + \log(T_t) + \log(R_t)$$

This converts the **multiplicative** model into an **additive** one.

0.0.4 4. Example: US Retail Employment

- The example uses monthly U.S. retail employment data from 1990 onward.
- They use the **STL method** (Seasonal-Trend decomposition using Loess) to decompose the series into its components.

0.0.5 1. Simulate a Time Series Dataset

[2]:
```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
from statsmodels.tsa.seasonal import STL

# Set seed for reproducibility
np.random.seed(42)

# Create monthly date range
date_range = pd.date_range(start='2010-01-01', periods=120, freq='M')  # 10␣
↪years

# Create components
trend = np.linspace(100, 200, 120)                          # gradually␣
↪increasing trend
seasonality = 10 * np.sin(2 * np.pi * date_range.month / 12)  # yearly␣
↪seasonality
noise = np.random.normal(0, 3, 120)                         # random noise

# Combine them into an additive series
employed = trend + seasonality + noise

# Build DataFrame
df = pd.DataFrame({
    'Month': date_range,
    'Employed': employed
})
df.set_index('Month', inplace=True)

# Preview
df.head()
```

```
/tmp/ipykernel_75732/2804562724.py:10: FutureWarning: 'M' is deprecated and will
be removed in a future version, please use 'ME' instead.
  date_range = pd.date_range(start='2010-01-01', periods=120, freq='M')  # 10
years
```

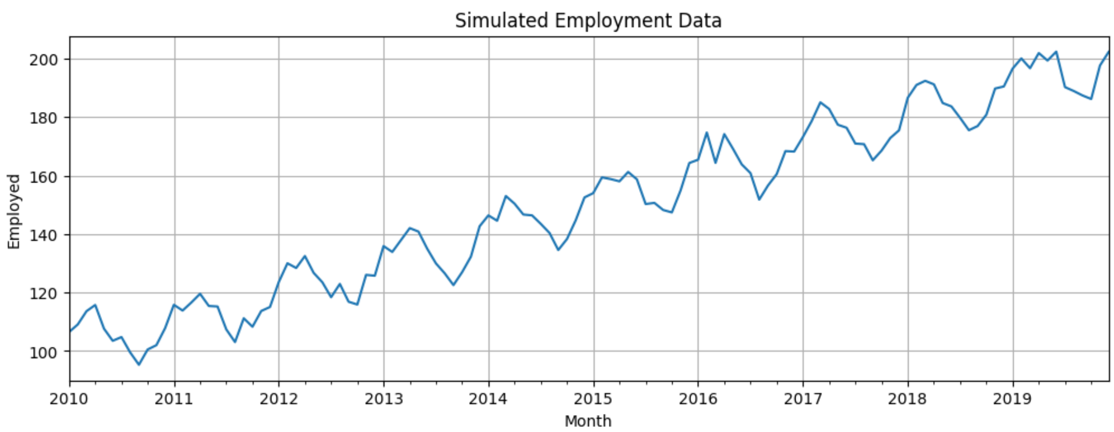
[2]:
```
              Employed
Month
2010-01-31  106.490142
```

```
2010-02-28   109.085797
2010-03-31   113.623738
2010-04-30   115.750352
2010-05-31   107.658884
```

0.0.6 2. Plot Simulated Data

[3]:
```python
# plotting simulated data

df['Employed'].plot(figsize=(12, 4), title='Simulated Employment Data',␣
 ↪ylabel='Employed')
plt.grid(True)
plt.show()
```



0.0.7 3. Apply STL Decomposition

[4]:
```python
stl = STL(df['Employed'], period=12)  # monthly data
res = stl.fit()

# Create component DataFrame
components = pd.DataFrame({
    'Employed': df['Employed'],
    'Trend': res.trend,
    'Seasonal': res.seasonal,
    'Remainder': res.resid
}, index=df.index)
```

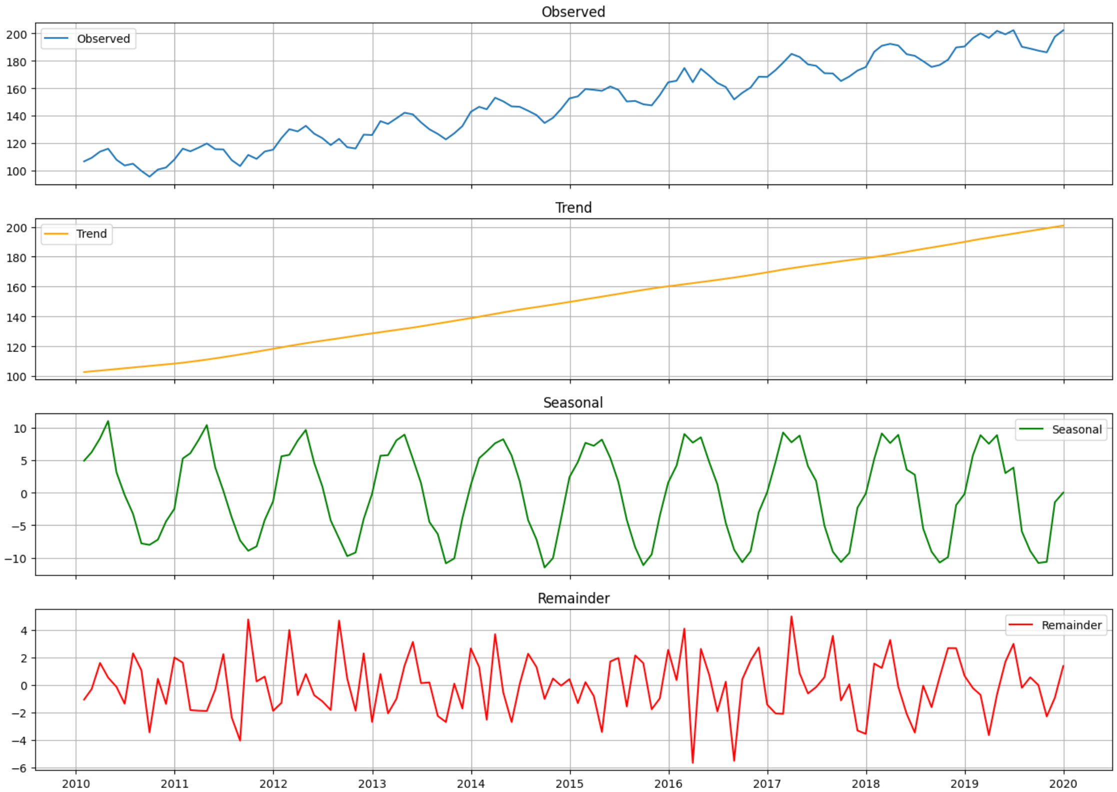
0.0.8 4. Plot All Components Together

[5]:
```
fig, axs = plt.subplots(4, 1, figsize=(14, 10), sharex=True)

axs[0].plot(components.index, components['Employed'], label='Observed')
axs[0].set_title('Observed')
axs[1].plot(components.index, components['Trend'], color='orange',
 ↪label='Trend')
axs[1].set_title('Trend')
axs[2].plot(components.index, components['Seasonal'], color='green',
 ↪label='Seasonal')
axs[2].set_title('Seasonal')
axs[3].plot(components.index, components['Remainder'], color='red',
 ↪label='Remainder')
axs[3].set_title('Remainder')

for ax in axs:
    ax.legend()
    ax.grid(True)

plt.tight_layout()
plt.show()
```



0.0.9 Interpretation of the Components

Component	What it shows
Observed	The actual data we generated.
Trend	The long-term increase in the number of employed people.
Seasonal	Recurring ups and downs within a year.
Remainder	What's left (randomness or noise) after removing trend + seasonality.

0.0.10 Additive vs Multiplicative Decomposition

Example with Log Transformation: Suppose you have multiplicative data:

$$y_t = T_t \times S_t \times R_t$$

You can **log-transform** it:

$$\log(y_t) = \log(T_t) + \log(S_t) + \log(R_t)$$

Now you can apply **additive decomposition** on the log-transformed data.

0.1 Summary

- Use `STL` to extract **trend**, **seasonal**, and **remainder** from a time series.
- Additive decomposition is good when seasonality is stable in size.
- Multiplicative decomposition is used when fluctuations grow/shrink with the trend.
- You can always log-transform multiplicative time series to make them additive.