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Assignment 2

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Determinent:-

Determinant is a special number that can be calculated from a square matrix using method. It is also function of entries of a square matrix.

Properties:-

There are 10 properties of determinant.

1. Reflection:

The determinant remains unchanged if it rows are change into columns & vice versa **i.e.**

$$\det A^T = \det A$$

2. All Zero:

In case, all elements of rows or collumns are zero Then determinant will be ze **.i.e:**

$$\text{let } A = \begin{bmatrix} 7 & 6 & 8 \\ 0 & 0 & 0 \\ 4 & 5 & 8 \end{bmatrix}$$

$$\det = 1(8(0) - 5(0)) - 6(8(0) - 4(0)) - 8(5(0) - 4(0)) = 0$$

3. Repitition:

If the elements of a row or column are identical to the elements of same other row or column then the determinant is zero **i.e:**

$$\text{let } A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 5 & 1 \\ 1 & 2 & 3 \end{bmatrix}$$

$$|A| = 1((5)(3) - (2)(1)) - 2((1)(3) - (1)(1)) + 3((1)(2) - (5)(1))$$

$$|A| = 13 - 4 + 6 = 0$$

4. Switching:

The interchange of any two rows or columns of determinant of A will change its sign i.e:

let $\det A = -\det B$

$A = \begin{bmatrix} 5 & 6 \\ 2 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 4 \\ 5 & 6 \end{bmatrix}$

$|A| = (5)(4) - (6)(2) = 20 - 12 = 8$

$B = (6)(2) - (5)(4) = 12 - 20 = -8$

5. Scalar Multiple:

If all the elements of a row or column of a det are multiplied by a non zero constant then the det gets multiplied by the same constant.

i.e. Let $A = \begin{bmatrix} 1 & 1 & 1 \\ \frac{2}{2} & \frac{2}{2} & \frac{2}{2} \end{bmatrix}$ $\det A = 1$

$\det A = 1(2) = 2$

6. Sum:

The sum of the product of the elements of any row or column with the cofactors of the corresponding elements is zero i.e:

$$\begin{bmatrix} a_1 & c_1 & d_1 \\ a_2 & c_2 & d_2 \\ a_3 & c_3 & d_3 \end{bmatrix} + \begin{bmatrix} b_1 & c_1 & d_1 \\ b_2 & c_2 & d_2 \\ b_3 & c_3 & d_3 \end{bmatrix} = \begin{bmatrix} a_1+b_1 & c_1 & d_1 \\ a_2+b_2 & c_2 & d_2 \\ a_3+b_3 & c_3 & d_3 \end{bmatrix}$$

7. Invarience:

The determinant remains unchanged under an operation of the form i.e:

$$\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix} = \begin{bmatrix} a_1+\alpha b_1+\beta c_1 & b_1 & c_1 \\ a_2+\alpha b_2+\beta c_2 & b_2 & c_2 \\ a_3+\alpha b_3-\beta c_3 & b_3 & c_3 \end{bmatrix}$$

8. Triangle:-

If all the element of a determinant above or below the main diagonal consists of zeros then the determinant is equal to product of diagonal elements.

i.e.
$$\begin{vmatrix} a_1 & a_2 & a_3 \\ 0 & b_2 & b_3 \\ 0 & 0 & c_3 \end{vmatrix} = \begin{vmatrix} a_1 & 0 & 0 \\ a_2 & b_2 & 0 \\ a_3 & b_3 & c_3 \end{vmatrix} = a_1 b_2 c_3$$

9. Factor:-

If a determinant is polynomial in x, Then $x-\alpha$ is a factor of determinant if its value is zero when we put $x = \alpha$

10 Determinant of Inverse :-

Let A be a $n \times n$ matrix, Then A is inversible iff $\det(A) \neq 0$ **i.e:**

Let

$$A \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \Rightarrow A = 4(1) - 3(2)$$

$|A| = -2 \neq 0$, A is inversible.

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