CIS521 HW3: Boyang Zhang

2.1.1

The constraints are as follows:

- $Revenue = 5x_1 + 1.5x_2 + x3 + 2x4 + 7x5$
- $x_2 \geq 10$
- $x_4 \geq 15$
- $x_1, x_2, x_3, x_4, x_5 \leq 24$
- $8x_1 + 5x_2 + 3x_3 + 6x_4 + 10x_5 \leq 400$

Since x_5 has the highest price, my intuition is to try and max it and min all the other values. So we have:
$x_5 = 24$
$x_4 = 15$
$x_2 = 10$

This leaves us with just x_1 and x_3 to try and evaluate. If we plug everything into the power constraint. We get:
$8x_1 + 50 + 3x_3 + 90 + 240 \leq 400$
$8x_1 + 3x_3 \leq 20$
Since x_1 is the highest price, I will max it and min x_3. So we get from the power equation:
$x_1 = 2.5$
$x_3 = 0$

Then from our revenue function:
$Revenue = 12.5 + 15 + 0 + 30 + 168 = 225.5$ as the max revenue

2.1.2 The previous solution is not valid because we have $x_1 = 2.5$ and with the new constraints, we cannot have non-integer numbers. Still we can follow the general intution from before and max x_5. So we will have:
$x_5 = 24$
$x_4 = 15$
$x_3 =?$
$x_2 = 10$
$x_1 =?$

Now we need to re-evaluate the power function simplified to:
$8x_1 + 3x_3 \leq 20$
We have options that satisfy this constraint.
First: $x_1 = 1, x_3 = 4$

This will produce $revenue = 222$. However this is not the max value. Rather:
$x_1 = 2, x_3 = 0$ will give a $revenue = 223$ which is higher, but we need to look further.
Max is actually:
$x_5 = 22$
$x_4 = 15$
$x_3 = 0$
$x_2 = 10$
$x_1 = 5$
Which gives a max revenue of 224.

2.1.3 Since in problem 1, $x_1 = 2.5$, we will round this to $x_1 = 3$. This will increase the max revenue to 228. However this will invalidate the power constraint set above. Thus we would need to add a slack of 4 to the total power in order to not invalidate the constraint since:
$8(3) + 5(10) + 0 + 6(15) + 10(24) = 404kW/hr$

2.1.4 Problem 1 is the easiest to solve because it's an Linear programming problem so it can be solved in polynomial time. However, question 2 is harder to solve because it's set up as an integer programming problem and without relaxing constraints would need a complex potentially exponential time algorithm to solve.

2.2.1 Let $m_i =$ number of meetings with person i for people from $1, 2, ...i$
Let $s_i =$ start of meeting
Let $e_i =$ end of meeting

Maximize $\sum_{i=1}^{n} x_i$ for n people.
We must set the constraint that separate timeslots cannot overlap between people. So:
For the randomly chosen person p_i, s_i cannot be between s_j and e_j for a person p_j and the tuple (s_i, e_i) must be in the set of available time slots, $S_i^{(av)}$.

2.2.2

In Sudoku we had 27 constraints that contained the numbers 1-9. We must go through the set of 27 constraints and check to see if each variable satisfies the constraint or not through containing integers 1-9. In order to do this let us see what this would look like as an integer program:

Let $\delta_i j = 0 or 1$ where the number j from $1 - 9$ is within one of the 27 constraints, i.
Since the integer programming question is asking us to convert the logical constraint of 0 or 1 into an arithmetic constraint. We can say that the $\sum_i \sum_j \delta_i j \leq 243$ because there are 27 constraints and 9 numerical constraints.

2.3.1

a)2, 9, 9

b)

N	α	β
A	4	∞
B	-∞	4
C	4	3
D	4	1
E	4	4
F	8	8
G	6	6
H	7	7
I	3	3
J	N/A	N/A
K	1	1
L	N/A	N/A
M	N/a	N/A

2.3.2

a) The best option would be to go left (toward node B) because the sum of the expected value of the left path is $(0.8)(2) + (0.2)(1) = 1.8$. This is greater than the sum of the expected value of the right path which is $(0.6)(-2) + (0.4)(2) = -0.4$.

b) If we only know the first six leaves then we need to evaluate the seventh and eight nodes because the first 6 nodes are in distinct subtrees from the 7th and 8th node. Because of this, we do not know the range of possible values in the 7th and 8th nodes. This is because, while we know that the right path is 1.8, the left path is $(0.6)(-2) + (0.4) * x$ where x is the smaller value of 7 and 8.
However if we have seen the first seven nodes then we do not have to evaluate the 8th node because we know that $B = 1.8$ and C is currently evaluated to $C = -1.2 + 0.4(2) = -0.4$. Since our α value carries over from the β B of 1.8 We know that the $C = -0.4$ is a smaller β than the α therefore we do not need to evaluate the 8th node.

c) The value range of the left-hand node is between 1.2 and 2.0 (1.6 + or - 4).

d) If we know the range of the leaf nodes is between [-2,2] then we can prune looking at node 6, 7, 8 because we know that when we reach node 5, we know that the expected value cannnot be greater than $(0.6)(-2) + (0.4)x$ and this value cannot be greater than the value of the left path which is 1.8.