CIS 391/521: HW 5 - Logic

This homework consists of only a written portion. Please submit your written responses at the **beginning** of class on **Tuesday, February 28**.

Let us know if you have any questions, check the discussion board, and remember that you can always come to Office Hours, even if only to keep the TAs company.

1 Written Portion

1. (14 points) Represent the following sentences in first-order logic. (You will need to define your own objects, relations and predicates.)

   a. Only one student failed Economics.

   b. Only one student failed both Economics and Psychology.

   c. The best score in Mathematics was better than the best score in Physics.

   d. Every person who dislikes all vegetarians is smart.

   e. There is a woman who likes all men who are not vegetarians.

   f. There is a barber who shaves all men in town who do not shave themselves.

   g. Musicians can be loved by some people all of the time, and they can be loved by all people some of the time, but they can't be loved by all people all of the time.

2. (6 points) A popular children's riddle is "Brothers and sisters have I none, but that man's father is my father's son." Use the rules of the family domain (Chapter 8) to show who that man is. You may apply any of the inference methods described in class or in the book.

3. (20 points) You are given the following facts:

   (a) $\forall x \ (\forall y \ Eats(x,y) \Rightarrow FastFood(y)) \Rightarrow (\exists y \ HasHealthProblem(x,y))$

   (b) $\forall x \ (\exists y \ HasHealthProblem(x,y)) \Rightarrow HighCholesterol(x) \lor HighBloodSugar(x)$

   (c) $\forall x \ \neg DrinksCoke(x) \Rightarrow \neg HighBloodSugar(x)$

   (d) $\forall x \ (HighCholesterol(x) \land \neg WorksOut(x)) \Rightarrow ShortLife(x)$

   (e) $\neg \exists x Lazy(x) \land WorksOut(x)$

   (f) $\forall x \ Eats(Donald, x) \Rightarrow FastFood(x)$

   (g) $\neg DrinksCoke(Donald)$

   (h) $Lazy(Donald)$

   a. (10 points) Restate the above statements in conjunctive normal form.

b. (10 points) Given the facts above, use resolution to prove the statement ShortLife(Donald). Show each step of the resolution.

4. (5 points) Use Forward Chaining to solve the following problem. Given:

   (a) A
   (b) B
   (c) C
   (d) $A \wedge B \Rightarrow D$
   (e) $B \wedge D \Rightarrow F$
   (f) $F \Rightarrow G$
   (g) $A \wedge E \Rightarrow H$
   (h) $A \wedge C \Rightarrow E$

   Is H true? (You might want to draw the tree, but you don't have to hand it in.) Show the sequence of rules (i.e. write a list of the rule letters that you applied during your proof.

5. (5 points) Use Backward Chaining on the following knowledge base to prove Q:

   (a) $P \Rightarrow Q$
   (b) $E \Rightarrow B$
   (c) $R \Rightarrow Q$
   (d) $M \wedge N \Rightarrow Q$
   (e) $A \wedge B \Rightarrow P$
   (f) $A \Rightarrow M$
   (g) $C \Rightarrow M$
   (h) $D \Rightarrow N$
   (i) D
   (j) A

   Is Q true? (You might want to draw the tree, but you don't have to hand it in.) Show the sequence of rules you followed.

6. (5 points) Suppose you are given the following axioms:

   (a) $\log 100 = 2$
   (b) $\log 632 = 2.8$
   (c) $632 \times 100 = 63200$
   (d) $2.8 + 2 = 4.8$
   (e) $\forall x, y \; x = y \Rightarrow \log x = \log y$
   (f) $\forall x, y \; \log xy = \log x + \log y$
   (g) $\forall x, y, z \; x = y \wedge y = z \Rightarrow x = z$
   (h) $\forall x, y, z, w \; \log x = y \wedge \log z = w \Rightarrow \log x + \log z = y + w$

   Give a backward-chaining proof of the sentence: $\log(63200) = 4.8$. Be sure to use only axioms given here, not anything else you may know about arithmetic. Show only the steps that lead to a success, not all the steps you try. (I.e., what sequence of rules leads.