CIS 391/521: HW 3 - Linear Programming, Adversarial Search

1 Instructions

This homework only has a written portion. Please upload your **pdf** formated file to Blackboard by the beginning of class on **Tuesday, Feb 14**.

Let us know if you have any questions, check the piazza discussion board, and remember that you can always come to office hours, even if only to keep the TAs company.

2 Written Portion

2.1 [14 points] Linear Programming (LP) & Integer Programming (IP) Question 1

Suppose you work for α company, which is a intelligent scheduling startup in Philadelphia. Here is a schedule problem from a local flour factory: The factory have five machines, $M_1, ..., M_5$, that produce different kinds of flour. Each kilogram of flour produced by those machines has a fixed price; Those prices are given in Table 1. Assume that each machine produces 1 kg of flour per hour that it is running.. According to the market research department, there are two special products, those produced by M_2 and M_4, taht need to have a minimum amount (See Table 1 for specific values) to satisfy daily market needs. Also, your total electric power used should be below $400kW \cdot h$ per day; the power consumption of the machines is listed in Table 1. Your objective (duh!) is to maximize daily revenue.

1. [5 points] Cast this problem into an LP problem with variables $x_1, ..., x_5$ to indicate the weight of flour produced by the corresponding machine. What is the maximum revenue and the corresponding weight of flour produced?

2. [3 points] Now, the strategy of the factory has changed; They decided to sell their product by the package. Eeach package weighs 1 kilogram (which means they don't sell an amount like 1.25 kg any more). Is the solution given in previous question still valid? If yes, please give the reason and if no, please give a new maximum revenue and corresponding weight of flour produced by each machine.

3. [3 points] If we solve question 2 by **rounding** the solution of question 1, is this a valid solution? If yes, how much better is the true optimal solution compared with the rounded one? If no, which

Machine	Product Price	Minimum Amount	Power Consumption
M1	5.00	0	8
M2	1.50	10	5
M3	1.00	0	3
M4	2.00	15	6
M5	7.00	0	10

Table 1: Price, Minimum amount of products and Power consumption

constraint is causing the problem? How much "slack" (meaning the value that we add to the right of the inequality) do we need to make the solution valid?

4. [3 points] Which question (1 or 2) do you think is easier to solve? Why?

2.2 More Linear Programming (LP) & Integer Programming (IP)

1. [10 points] **Time Slot Scheduling Problem** Suppose you want to meet with several people separately, and that each person has a set of available time slots. You want to meet with as many people as possible. More formally, you want to schedule to meet with n people, $p_1, ..., p_n$. Each of them has a set of available time slots, I.e., person p_i is associated with a set of available time slots $S_i^{(av)} = \{(s_{ik}, e_{ik}) | k \in 1, ..., K_i\}$. You only need to meet with each of those people at most once. You want to maximize the number of people you meet. Cast this problem into a IP problem, describe your variables, and give an objective function and constraints.

2. [10 points] **Sudoku as Integer Programming** Cast solving the Sudoku problem as a integer programming problem. Describe your variables, objective function and constraints. How many variables are there in your formulation? How many constraints?

2.3 Adversarial Search

1. [8 points] **Alpha-Beta Pruning** Perform the alpha-beta pruning algorithm in Figure 1. The utility estimates of the bottom layer nodes are given in the figure.

   For each of the nodes that is visited (including those nodes with given utility value) during the algorithm, label the node in the order the node is visited (I.e. first node is 1, second node is 2, ...). Then do alpha-beta search. You don't need to hand this part in.

   (a) [3 points] Which nodes will be pruned? (list their numbers)

   (b) [5 points] Write down the α and β values of all fo the nodes that have visited at the termination of the algorithm. I.e., produce table with 3 columns; the first is the node number, the second is the alpha value at the end of the search, the thrid is the beta at the end of the search.

   (Note: All nodes are evaluated in left to right order.)

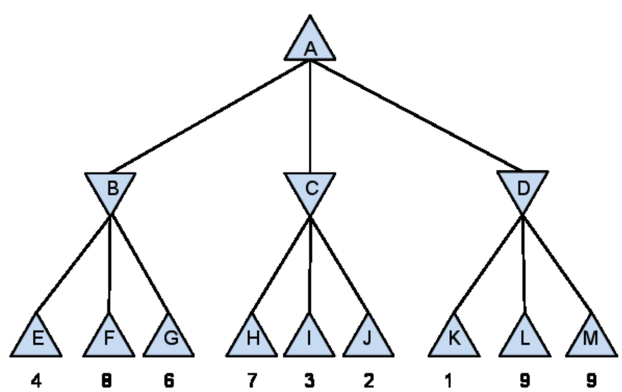

Figure 1:

2. [14 points] **Pruning in Games with Chance Nodes** This question considers pruning in games with chance nodes. Figure 2 shows the complete game tree for a trivial game. Assume that the leaf nodes are to be evaluated in left- to-right order, and that before a leaf node is evaluated, we know nothing about its value and the range of possible values is $(-\infty, \infty)$.

   (a) [3 points] What is the best move at the root? explain.

   (b) [4 points] Given the values of the first six leaves, do we need to evaluate the seventh and eighth leaves? Given the values of the first seven leaves, do we need to evaluate the eighth leaf? Explain your answers.

   (c) [3 points] Suppose the leaf node values are known to lie between -2 and 2 inclusive. After the first two leaves are evaluated, what is the value range for the left-hand chance node?

   (d) [4 points] List all the leaves (numbered 1-8 from left to right) that *do not* need to be evaluated under the assumption in (c).

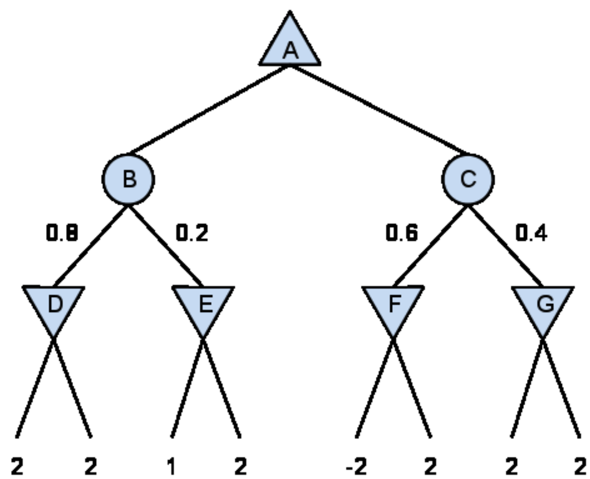

Figure 2: