02244 Logic for Security
Information Flow
Week 10: Volpano's Approach

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Challenges for Denning's

We add arrays and procedures to our language

- D ::= ... | **integer array** C A[n] | ...
  ... | **proc** p(**in** T_1 C_1 $\mathrm{var}_{\mathrm{in}}$, **out** T_2 C_2 $\mathrm{var}_{\mathrm{out}}$) is S_{body}
- E ::= ... | A[E_1]
- S ::= ... | A[E_1] := E_2 | **call** p(E,var)

Define the new constraints that we need to impose.

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Rules

D	**Security Class**
integer C A[n]	$\underline{A} =$
proc p(**in** T_1 C_1 var_{in}, **out** T_2 C_2 var_{out}) is S_{body}	$\underline{p} =$ $\underline{var_{in}} =$ $\underline{var_{out}} =$

E	**Security Class**
A[E_1]	$\underline{E} =$

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- D ::= ... | **integer array** C A[n] | ...
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Rules

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proc p(**in** T_1 C_1 var_{in}, **out** T_2 C_2 var_{out}) is S_{body}	$\underline{p} =$ $\underline{var_{in}} =$ $\underline{var_{out}} =$

E	**Security Class**
A[E_1]	$\underline{E} =$

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integer C A[n]	$\underline{A} = C$
proc p(**in** T_1 C_1 var_{in}, **out** T_2 C_2 var_{out}) is S_{body}	$\underline{p} = \underline{S_{body}}$ $\underline{var_{in}} =$ $\underline{var_{out}} =$

E	**Security Class**
A[E_1]	$\underline{E} =$

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Rules

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integer C A[n]	$\underline{A} = C$
proc p(**in** T_1 C_1 var_{in}, **out** T_2 C_2 var_{out}) is S_{body}	$\underline{p} = \underline{S_{body}}$ $\underline{var_{in}} = C_1$ $\underline{var_{out}} = C_2$

E	**Security Class**
A[E_1]	$\underline{E} =$

Challenges for Denning's

We add arrays and procedures to our language

- D ::= ... | **integer array** C A[n] | ...
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Rules

D	**Security Class**
integer C A[n]	$\underline{A} = C$
proc p(**in** T_1 C_1 var_{in}, **out** T_2 C_2 var_{out}) is S_{body}	$\underline{p} = \underline{S_{body}}$ $\underline{var_{in}} = C_1$ $\underline{var_{out}} = C_2$

E	**Security Class**
A[E_1]	$\underline{E} = \underline{A} \sqcup \underline{E_1}$

Challenges for Denning's

We add arrays and procedures to our language

- D ::= ... | **integer array** C A[n] | ...
  ... | **proc** p(**in** T_1 C_1 var_{in}, **out** T_2 C_2 var_{out}) is S_{body}
- E ::= ... | A[E_1]
- S ::= ... | A[E_1] := E_2 | **call** p(E,var)

Rules

S	**security class** of S	**constraint**
A[E_1] := E_2	$\underline{S}$ =	
call p(E,var)	$\underline{S}$ =	

Challenges for Denning's

We add arrays and procedures to our language

- D ::= ... | **integer array** C A[n] | ...
  ... | **proc** p(**in** T_1 C_1 var_{in}, **out** T_2 C_2 var_{out}) is S_{body}
- E ::= ... | A[E_1]
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Rules

S	**security class** of S	**constraint**
A[E_1] := E_2	$\underline{S} =$	$\underline{E_1} \sqcup \underline{E_2} \sqsubseteq \underline{A}$
call p(E,var)	$\underline{S} =$	

We add arrays and procedures to our language

- D ::= ... | **integer array** C A[n] | ...
  ... | **proc** p(**in** T_1 C_1 var_{in}, **out** T_2 C_2 var_{out}) is S_{body}
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Rules

S	**security class** of S	**constraint**
A[E_1] := E_2	$\underline{S} = \underline{A}$	$\underline{E_1} \sqcup \underline{E_2} \sqsubseteq \underline{A}$
call p(E,var)	$\underline{S} =$	

Challenges for Denning's

We add arrays and procedures to our language

- D ::= ... | **integer array** C A[n] | ...
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call p(E,var)	$\underline{S} = \underline{p}$	

Challenges for Denning's

We add arrays and procedures to our language

- D ::= ... | **integer array** C A[n] | ...
  ... | **proc** p(**in** T_1 C_1 var_{in}, **out** T_2 C_2 var_{out}) is S_{body}
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Rules

S	**security class** of S	**constraint**
A[E_1] := E_2	$\underline{S} = \underline{A}$	$\underline{E_1} \sqcup \underline{E_2} \sqsubseteq \underline{A}$
call p(E,var)	$\underline{S} = \underline{p}$	$\underline{E} \sqsubseteq \underline{var_{in}}$

Challenges for Denning's

We add arrays and procedures to our language

- D ::= ... | **integer array** C A[n] | ...
  ... | **proc** p(**in** T_1 C_1 var_{in}, **out** T_2 C_2 var_{out}) is S_{body}
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Rules

S	**security class** of S	**constraint**
A[E_1] := E_2	$\underline{S} = \underline{A}$	$\underline{E_1} \sqcup \underline{E_2} \sqsubseteq \underline{A}$
call p(E,var)	$\underline{S} = \underline{p}$	$\underline{E} \sqsubseteq \underline{var_{in}}$
		$\underline{var_{out}} \sqsubseteq \underline{var}$

Challenge for Myers'

```
coronatest = record [ subject : cpr  {shs:shs},
                      testdate: date {shs:shs},
                      positive: bool {shs:shs} ]
publish_stat(db:array[coronatest:{⊥}]{⊥},
             day:date{shs:shs})
returns infections:int{⊥}

lookup_result(db:array[coronatest:{⊥}]{⊥},
               today:date{⊥},
               client:cpr{⊥})
returns result:bool{client:client}
```

`shs` is for sundhedsstyrelsen

Implement the functions `publish_stat` and `lookup_result`

- What authority/declassifications do the functions need?
- Prove that this is indeed safe.

So What?

Up to this point, we (and Denning and Denning) have only...

- defined a set of security classes
- decorated variables with these security classes
- decorated programs with constraints on security classes

So?

- If a program satisfies the constraints, then what guarantees do we really obtain from that?
- Given a set of rules for information flow (like the ones we just gave for arrays and procedures):
  how can one judge if they are "correct" or "wrong"?

Volpano's Approach to IFC

Dennis M. Volpano and Geoffrey Smith and Cynthia E. Irvine: *A Sound Type System for Secure Flow Analysis*, Journal of Computer Security, 4(2/3), pp. 167–188, 1996. [2]

- Essentially the same approach as Denning and Denning [1]
- ... but represented as a type system
  - ★ Security classes like Low and High are considered as types
  - ★ The ordering of security classes $\sqsubseteq$ is considered as a subtype relation
  - ★ Information Flow Analysis is described as a set of type-inference rules
- They give a natural semantics for the programming language.
- Type system and semantics allows for proving a precise statement about the security of programs that fulfill the information flow policy: a Non-Interference Result.

Volpano's Approach to IFC

Dennis M. Volpano and Geoffrey Smith and Cynthia E. Irvine: *A Sound Type System for Secure Flow Analysis*, Journal of Computer Security, 4(2/3), pp. 167–188, 1996. [2]

We simplify the paper a bit:

- Volpano et al. distinguish *variables* and *memory locations*, we treat them here all as variables (dropping the concept of introducing local variables).
- The corresponding symbol tables γ and λ are merged to just γ.
- We directly work with the syntax-directed form and do not need to distinguish the syntactic roles of variables, expressions and commands in the type system (i.e., we do not have *var* τ etc.)

Syntax

Syntax

$$\begin{array}{llll} \text{Expressions} & e & ::= & x \mid n \mid e + e' \mid \ldots \mid e = e' \mid \ldots \\ \text{Commands} & c & ::= & x := e \mid c; c' \mid \mathsf{if}\ e\ \mathsf{then}\ c\ \mathsf{else}\ c' \mid \mathsf{while}\ e\ \mathsf{do}\ c \end{array}$$

where x is for identifiers (for variables) and n is for constants.

Typing Rules

We define type inference rules where:

- γ is a type environment: it gives for every variable its type.
- The types are the security classes SC (e.g. High and Low) and $\sqsubseteq$ is the comparison relation on the security classes.
- Type judgements for expressions are of the form

$$\gamma \vdash e : \tau$$

Read: under γ, the expression e can be typed as τ.

Typing Rules (Expressions)

$$\text{(CONST)}\ \frac{}{\gamma \vdash n : \tau}$$

$$\text{(VAR)}\ \frac{}{\gamma \vdash x : \tau}\ \gamma(x) \sqsubseteq \tau$$

$$\text{(ARITH)}\ \frac{\gamma \vdash e : \tau \quad \gamma \vdash e' : \tau}{\gamma \vdash e\ op\ e' : \tau}$$

Typing Rules

- Type judgements for commands are of the form

$$\gamma \vdash c : \tau$$

Read: under γ, the command c can be typed as τ.

Typing Rules (Commands)

$$\text{(ASSIGN)} \quad \frac{\gamma \vdash e : \tau}{\gamma \vdash x := e : \tau'} \; \gamma(x) = \tau, \tau' \sqsubseteq \tau$$

$$\text{(COMPOSE)} \quad \frac{\gamma \vdash c : \tau \quad \gamma \vdash c' : \tau}{\gamma \vdash c; c' : \tau}$$

$$\text{(IF)} \quad \frac{\gamma \vdash e : \tau \quad \gamma \vdash c : \tau \quad \gamma \vdash c' : \tau}{\gamma \vdash \text{if } e \text{ then } c \text{ else } c' : \tau'} \; \tau' \sqsubseteq \tau$$

$$\text{(WHILE)} \quad \frac{\gamma \vdash e : \tau \quad \gamma \vdash c : \tau}{\gamma \vdash \text{while } e \text{ do } c : \tau'} \; \tau' \sqsubseteq \tau$$

Example

- Let $\gamma(x) = H$ and $\gamma(y) = L$

$$\frac{TODO}{\gamma \vdash \text{if } x > 1000 \text{ then } x := 5 \text{ else } y := y + x * 3 : \tau_1}$$

Constraints:

Fitting rule

$$\text{(IF)} \quad \frac{\gamma \vdash e : \tau \quad \gamma \vdash c : \tau \quad \gamma \vdash c' : \tau}{\gamma \vdash \text{if } e \text{ then } c \text{ else } c' : \tau'} \; \tau' \sqsubseteq \tau$$

Example

- Let $\gamma(x) = H$ and $\gamma(y) = L$

$$\dfrac{\dfrac{TODO}{\gamma \vdash x > 1000 : \tau_2} \quad \dfrac{TODO}{\gamma \vdash x := 5 : \tau_2} \quad \dfrac{TODO}{\gamma \vdash y := y + x * 3 : \tau_2}}{\gamma \vdash \text{if } x > 1000 \text{ then } x := 5 \text{ else } y := y + x * 3 : \tau_1}$$

Constraints: $\tau_1 \sqsubseteq \tau_2$

Fitting rule

$$\text{(ARITH)} \quad \dfrac{\gamma \vdash e : \tau \quad \gamma \vdash e' : \tau}{\gamma \vdash e \ op\ e' : \tau}$$

Example

- Let $\gamma(x) = H$ and $\gamma(y) = L$

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Constraints: $\tau_1 \sqsubseteq \tau_2$

Fitting rule

(VAR) $\dfrac{}{\gamma \vdash x : \tau} \; \gamma(x) \sqsubseteq \tau$

Example

- Let $\gamma(x) = H$ and $\gamma(y) = L$

$$\dfrac{\dfrac{}{\gamma \vdash x : H} \quad \dfrac{TODO}{\gamma \vdash 1000 : H}}{\gamma \vdash x > 1000 : H} \quad \dfrac{TODO}{\gamma \vdash x := 5 : H} \quad \dfrac{TODO}{\gamma \vdash y := y + x * 3 : H}$$
$$\overline{\gamma \vdash \text{if } x > 1000 \text{ then } x := 5 \text{ else } y := y + x * 3 : \tau_1}$$

Constraints: $\tau_1 \sqsubseteq \tau_2 \wedge \gamma(x) = H \sqsubseteq \tau_2$ Thus: $\tau_2 = H$.

Fitting rule

(CONST) $\dfrac{}{\gamma \vdash n : \tau}$

Example

- Let $\gamma(x) = H$ and $\gamma(y) = L$

$$\dfrac{\dfrac{\overline{\gamma \vdash x : H} \quad \overline{\gamma \vdash 1000 : H}}{\gamma \vdash x > 1000 : H} \quad \dfrac{TODO}{\gamma \vdash x := 5 : H} \quad \dfrac{TODO}{\gamma \vdash y := y + x * 3 : H}}{\gamma \vdash \text{if } x > 1000 \text{ then } x := 5 \text{ else } y := y + x * 3 : \tau_1}$$

Constraints:

Fitting rule

$$\text{(ASSIGN)} \quad \frac{\gamma \vdash e : \tau}{\gamma \vdash x := e : \tau'} \; \gamma(x) = \tau, \tau' \sqsubseteq \tau$$

Example

- Let $\gamma(x) = H$ and $\gamma(y) = L$

$$\dfrac{\dfrac{\dfrac{}{\gamma \vdash x : H} \quad \dfrac{}{\gamma \vdash 1000 : H}}{\gamma \vdash x > 1000 : H} \quad \dfrac{\dfrac{}{\gamma \vdash 5 : H}}{\gamma \vdash x := 5 : H} \quad \dfrac{TODO}{\gamma \vdash y := y + x * 3 : H}}{\gamma \vdash \text{if } x > 1000 \text{ then } x := 5 \text{ else } y := y + x * 3 : \tau_1}$$

Constraints:

Fitting rule

$$(\text{ASSIGN}) \quad \dfrac{\gamma \vdash e : \tau}{\gamma \vdash x := e : \tau'} \; \gamma(x) = \tau, \tau' \sqsubseteq \tau$$

Example

- Let $\gamma(x) = H$ and $\gamma(y) = L$

$$\dfrac{\dfrac{\dfrac{}{\gamma \vdash x : H} \quad \dfrac{}{\gamma \vdash 1000 : H}}{\gamma \vdash x > 1000 : H} \quad \dfrac{\dfrac{}{\gamma \vdash 5 : H}}{\gamma \vdash x := 5 : H} \quad \dfrac{\color{red}{\text{requires } \gamma(y) \sqsupseteq H!}}{\gamma \vdash y := y + x * 3 : H}}{\gamma \vdash \text{if } x > 1000 \text{ then } x := 5 \text{ else } y := y + x * 3 : \tau_1}$$

Constraints:

Fitting rule

$$\text{(ASSIGN)} \quad \frac{\gamma \vdash e : \tau}{\gamma \vdash x := e : \tau'} \; \gamma(x) = \tau, \tau' \sqsubseteq \tau$$

There is no way to finish the red part of the proof. It does not type!

Example

- Let $\gamma(x) = L$ and $\gamma(y) = H$

$$\frac{TODO}{\gamma \vdash \text{if } x > 1000 \text{ then } x := 5 \text{ else } y := y + x * 3 : \tau_1}$$

Constraints:

Fitting Rule

$$(\text{IF}) \quad \frac{\gamma \vdash e : \tau \quad \gamma \vdash c : \tau \quad \gamma \vdash c' : \tau}{\gamma \vdash \text{if } e \text{ then } c \text{ else } c' : \tau'} \; \tau' \sqsubseteq \tau$$

Example

- Let $\gamma(x) = L$ and $\gamma(y) = H$

$$\dfrac{\dfrac{TODO}{\gamma \vdash x > 1000 : \tau_2} \quad \dfrac{TODO}{\gamma \vdash x := 5 : \tau_2} \quad \dfrac{TODO}{\gamma \vdash y := y + x * 3 : \tau_2}}{\gamma \vdash \text{if } x > 1000 \text{ then } x := 5 \text{ else } y := y + x * 3 : \tau_1}$$

Constraints: $\tau_1 \sqsubseteq \tau_2$

Fitting Rule

$$(\text{ARITH}) \quad \dfrac{\gamma \vdash e : \tau \quad \gamma \vdash e' : \tau}{\gamma \vdash e \; op \; e' : \tau}$$

Example

- Let $\gamma(x) = L$ and $\gamma(y) = H$

$$\dfrac{\dfrac{\dfrac{TODO}{\gamma \vdash x : \tau_2} \quad \dfrac{}{\gamma \vdash 1000 : \tau_2}}{\gamma \vdash x > 1000 : \tau_2} \quad \dfrac{TODO}{\gamma \vdash x := 5 : \tau_2} \quad \dfrac{TODO}{\gamma \vdash y := y + x * 3 : \tau_2}}{\gamma \vdash \text{if } x > 1000 \text{ then } x := 5 \text{ else } y := y + x * 3 : \tau_1}$$

Constraints: $\tau_1 \sqsubseteq \tau_2$

Fitting Rule

(VAR) $\dfrac{}{\gamma \vdash x : \tau} \; \gamma(x) \sqsubseteq \tau$

Example

- Let $\gamma(x) = L$ and $\gamma(y) = H$

$$\dfrac{\dfrac{\dfrac{}{\gamma \vdash x : \tau_2} \quad \dfrac{}{\gamma \vdash 1000 : \tau_2}}{\gamma \vdash x > 1000 : \tau_2} \quad \dfrac{TODO}{\gamma \vdash x := 5 : \tau_2} \quad \dfrac{TODO}{\gamma \vdash y := y + x * 3 : \tau_2}}{\gamma \vdash \text{if } x > 1000 \text{ then } x := 5 \text{ else } y := y + x * 3 : \tau_1}$$

Constraints: $\tau_1 \sqsubseteq \tau_2 \qquad L \sqsubseteq \tau_2$ (trivially true)

Fitting Rule

(ASSIGN) $\dfrac{\gamma \vdash e : \tau}{\gamma \vdash x := e : \tau'} \; \gamma(x) = \tau, \tau' \sqsubseteq \tau$

Example

- Let $\gamma(x) = L$ and $\gamma(y) = H$

$$\dfrac{\dfrac{\dfrac{}{\gamma \vdash x : L} \quad \dfrac{}{\gamma \vdash 1000 : L}}{\gamma \vdash x > 1000 : L} \qquad \dfrac{\dfrac{}{\gamma \vdash 5 : L}}{\gamma \vdash x := 5 : L} \quad \dfrac{TODO}{\gamma \vdash y := y + x * 3 : L}}{\gamma \vdash \text{if } x > 1000 \text{ then } x := 5 \text{ else } y := y + x * 3 : L}$$

Constraints: $\tau_1 \sqsubseteq \tau_2 \wedge \tau_2 \sqsubseteq L = \gamma(x)$. Thus $\tau_1 = \tau_2 = L$.

Fitting Rule

$$(\text{ASSIGN}) \ \dfrac{\gamma \vdash e : \tau}{\gamma \vdash x := e : \tau'} \ \gamma(x) = \tau, \tau' \sqsubseteq \tau$$

Example

- Let $\gamma(x) = L$ and $\gamma(y) = H$

$$\frac{\dfrac{\ldots}{\gamma \vdash x > 1000 : L} \quad \dfrac{\ldots}{\gamma \vdash x := 5 : L} \quad \dfrac{TODO}{\gamma \vdash y := y + x * 3 : L}}{\gamma \vdash \text{if } x > 1000 \text{ then } x := 5 \text{ else } y := y + x * 3 : L}$$

Constraints:

Fitting Rule

$$(\text{ASSIGN}) \ \frac{\gamma \vdash e : \tau}{\gamma \vdash x := e : \tau'} \ \gamma(x) = \tau, \tau' \sqsubseteq \tau$$

Example

- Let $\gamma(x) = L$ and $\gamma(y) = H$

$$
\dfrac{\dfrac{\dots}{\gamma \vdash x > 1000 : L} \quad \dfrac{\dots}{\gamma \vdash x := 5 : L} \quad \dfrac{\dfrac{\dfrac{}{\gamma \vdash y : H} \quad \dfrac{\dfrac{}{\gamma \vdash x : H} \quad \dfrac{}{\gamma \vdash 3 : H}}{\gamma \vdash x * 3 : H}}{\gamma \vdash y + x * 3 : H}}{\gamma \vdash y := y + x * 3 : L}}{\gamma \vdash \text{if } x > 1000 \text{ then } x := 5 \text{ else } y := y + x * 3 : L}
$$

Constraints: $\gamma(y) \sqsupseteq L$

Fitting Rule

(VAR) $\dfrac{}{\gamma \vdash x : \tau} \; \gamma(x) \sqsubseteq \tau$

It types!

Semantics

- To make a meaningful statement about the language, we must define what programs mean.
- Since we are reading and writing variables in our program, we need a concept of memory μ:
  - ★ $\mu(x)$ means reading the memory location for variable x (yields an integer if x is defined)
  - ★ $\mu[x := n]$ means writing the integer n into the memory location of variable x
- We define the semantics as two relations:
  - ★ For expressions e we define

    $$\mu \vdash e \Rightarrow n$$

    to mean: at memory state μ, the expression e gets evaluated to value n.
  - ★ For commands c we define

    $$\mu \vdash c \Rightarrow \mu'$$

    to mean: starting at memory state μ, the execution of command c changes the memory state to μ'.

Semantics

Definition (Semantics of Expressions)

$$\text{(BASE)} \quad \frac{}{\mu \vdash n \Rightarrow n}$$

$$\text{(CONTENT)} \quad \frac{}{\mu \vdash x \Rightarrow \mu(x)}$$

$$\text{(ADD)} \quad \frac{\mu \vdash e \Rightarrow n \quad \mu \vdash e' \Rightarrow n'}{\mu \vdash e + e' \Rightarrow n + n'}$$

Similar for other operators

Example

$$\underbrace{\boxed{\begin{array}{l} x := 3 \\ y := -1 \end{array}}}_{\mu} \vdash \underbrace{3 * x + y}_{e} \Rightarrow 8$$

Semantics

Definition (Semantics of Commands)

$$\text{(UPDATE)}\quad \frac{\mu \vdash e \Rightarrow n}{\mu \vdash x := e \Rightarrow \mu[x := n]}$$

$$\text{(SEQUENCE)}\quad \frac{\mu \vdash c_1 \Rightarrow \mu' \quad \mu' \vdash c_2 \Rightarrow \mu''}{\mu \vdash c_1; c_2 \Rightarrow \mu''}$$

$$\text{(IFTRUE)}\quad \frac{\mu \vdash e \Rightarrow 1 \quad \mu \vdash c_1 \Rightarrow \mu'}{\mu \vdash \text{if } e \text{ then } c_1 \text{ else } c_2 \Rightarrow \mu'}$$

$$\text{(IFFALSE)}\quad \frac{\mu \vdash e \Rightarrow 0 \quad \mu \vdash c_2 \Rightarrow \mu'}{\mu \vdash \text{if } e \text{ then } c_1 \text{ else } c_2 \Rightarrow \mu'}$$

Definition (Semantics of Commands)

$$\text{(WHILETRUE)} \quad \frac{\mu \vdash e \Rightarrow 1 \quad \mu \vdash c; \text{while } e \text{ do } c \Rightarrow \mu'}{\mu \vdash \text{while } e \text{ do } c \Rightarrow \mu'}$$

$$\text{(WHILEFALSE)} \quad \frac{\mu \vdash e \Rightarrow 0}{\mu \vdash \text{while } e \text{ do } c \Rightarrow \mu}$$

Example

$$\underbrace{\boxed{\begin{array}{l} x := 2 \\ y := 3 \end{array}}}_{\mu} \vdash \underbrace{\begin{array}{l} z := 1; \\ \text{while } y > 0 \text{ do} \\ \quad z := z * x; \quad y := y - 1 \end{array}}_{c} \Rightarrow \underbrace{\boxed{\begin{array}{l} x := 2 \\ y := 0 \\ z := 8 \end{array}}}_{\mu'}$$

Result

Theorem (Non-Interference instantiated for $L \sqsubseteq H$-security)

- *Suppose a program c satisfies information flow policy γ:*

  $$\gamma \vdash c : \tau \qquad \textit{(for any type } \tau\textit{, does not matter)}$$

- *Suppose μ_1 and μ_2 are memories that are equal on all low variables:*

  $$\mu_1(x) = \mu_2(x) \textit{ for every } x \textit{ with } \gamma(x) = L$$

- *If we run the program on these memories:*
  - $\mu_1 \vdash c \Rightarrow \mu_1'$
  - $\mu_2 \vdash c \Rightarrow \mu_2'$

  (and they both terminate)

- *...then the result will be the same on all low variables:*

  $$\mu_1'(x) = \mu_2'(x) \textit{ for every } x \textit{ with } \gamma(x) = L$$

Illustration

$$\begin{array}{l} \mu_1 \\ x := 42 \\ y := 5 \\ z := 3 \end{array} \xRightarrow[c]{} \begin{array}{l} \mu_1' \\ x := 12 \\ y := 0 \\ z := 7 \end{array}$$

$$\begin{array}{l} \mu_2 \\ x := 24 \\ y := 2 \\ z := 3 \end{array} \xRightarrow[c]{} \begin{array}{l} \mu_2' \\ x := 7 \\ y := 3 \\ z := 7 \end{array}$$

- If the low-variables (in the example: z) of μ_1 and μ_2 agree,
- then they still agree after running any command c that satisfies information flow.

Illustration

$$\begin{array}{l}\mu_1\\ \boxed{\begin{array}{l}x := 42\\ y := 5\end{array}}\\ z := 3\end{array} \xRightarrow[c]{} \begin{array}{l}\mu_1'\\ \boxed{\begin{array}{l}x := 12\\ y := 0\end{array}}\\ z := 7\end{array}$$

$$\begin{array}{l}\mu_2\\ \boxed{\begin{array}{l}x := 24\\ y := 2\end{array}}\\ z := 3\end{array} \xRightarrow[c]{} \begin{array}{l}\mu_2'\\ \boxed{\begin{array}{l}x := 7\\ y := 3\end{array}}\\ z := 7\end{array}$$

- If the low-variables (in the example: z) of μ_1 and μ_2 agree,
- then they still agree after running any command c that satisfies information flow.
- Thus, from observing the low variables we get no information about the high variables.

Relation to Secrecy and Privacy

- Note that we do not assume that the values in the high variables are strong secrets. They may contain...
  - personal information like name, age, CPR, medical data...
  - a guessable password
  - a credit card number and expiration information
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  - a guessable password
  - a credit card number and expiration information
  - a vote
- Information Flow/Noninterference guarantees that the intruder does not learn anything about it as long as he can only read low variables.
- It is thus...
  - ... not like secrecy in OFMC
  - ... a bit like guessable secrecy in OFMC
  - ... close to α-β-privacy!

Noninterference vs. α-β-privacy

$$\begin{array}{c} \mu_1 \\ \boxed{\begin{array}{l} x := 42 \\ y := 5 \end{array}} \\ z := 3 \end{array} \xrightarrow[c]{\quad} \begin{array}{c} \mu_1' \\ \boxed{\begin{array}{l} x := 12 \\ y := 0 \end{array}} \\ z := 7 \end{array}$$

- Idea for encoding:
  - ★ α says that initially x and y are *some* integer values
  - ★ β contains the values of z and how the algorithm manipulates x, y, and z.[1]

[1] This does not directly work on while-loops with conditions of type High.

Noninterference vs. α-β-privacy

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x := 42 \\
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- It is a violation of privacy if the intruder can tell anything about the variables x and y.
- α-β privacy allows to release some information:
  - ★ E.g. the intruder may learn an encrypted message containing a High value (as long as he cannot decrypt)
  - ★ E.g. we may release the total result of an election

[1] This does not directly work on while-loops with conditions of type High.

Noninteference for Authentication / Integrity

μ_1		μ'_1
$x := 42$		$x := 12$
$y := 5$	$\xRightarrow{c}$	$y := 0$
$z := 3$		$z := 7$

μ_2		μ'_2
$x := 24$		$x := 7$
$y := 2$	$\xRightarrow{c}$	$y := 3$
$z := 3$		$z := 7$

- Suppose we define the variables x and y as type Untrusted and z as type Trusted.
- Suppose *Trusted* $\sqsubseteq$ *Untrusted*.

Noninteference for Authentication / Integrity

$$\begin{array}{c} \mu_1 \\ \begin{array}{|l|} \hline x := 42 \\ y := 5 \\ \hline z := 3 \\ \hline \end{array} \end{array} \xRightarrow[c]{} \begin{array}{c} \mu_1' \\ \begin{array}{|l|} \hline x := 12 \\ y := 0 \\ \hline z := 7 \\ \hline \end{array} \end{array}$$

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- Suppose we define the variables x and y as type Untrusted and z as type Trusted.
- Suppose *Trusted* $\sqsubseteq$ *Untrusted*.
- Suppose further the intruder can manipulate the untrusted variables, but not the trusted ones.
- Then $\gamma \vdash c : \tau$ guarantees us that the intruder cannot do anything that has any influence on the trusted variables.

Result

Theorem (Non-Interference instantiated for T$\sqsubseteq$U)

- *Suppose a program c satisfies information flow policy γ:*

$$\gamma \vdash c : \tau \qquad \textit{(for any type } \tau\textit{, does not matter)}$$

- *Suppose μ_1 and μ_2 are memories that are equal on all trusted variables:*

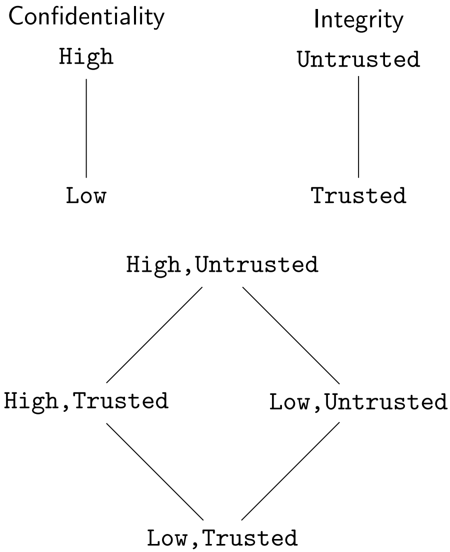
$$\mu_1(x) = \mu_2(x) \text{ for every } x \text{ with } \gamma(x) = T$$

- *If we run the program on these memories:*
  - ★ $\mu_1 \vdash c \Rightarrow \mu_1'$
  - ★ $\mu_2 \vdash c \Rightarrow \mu_2'$

  (and they both terminate)
- *…then the result will be the same on all trusted variables:*

$$\mu_1'(x) = \mu_2'(x) \text{ for every } x \text{ with } \gamma(x) = T$$

Confidentiality and Integrity in One Go?



Question

How can we become more general?

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Answer

Security Policy Frameworks
for Mandatory Access Control.

Security Policy Framework

A **security policy framework** is a 4-tuple $(S, \sqsubseteq, \sqcup, \sqcap)$ where

- S is a **finite** and **non-empty** set of **security labels**.

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- $\sqsubseteq$: $S \times S$ is a **binary relation**
  - (a) $\sqsubseteq$ is **reflexive** : for all $s \in S : s \sqsubseteq s$
  - (b) $\sqsubseteq$ is **transitive**: for all $s_1,\ s_2,\ s_3 \in S$:

    $$s_1 \sqsubseteq s_2 \wedge s_2 \sqsubseteq s_3 \Rightarrow s_1 \sqsubseteq s_3$$
  - (c) $\sqsubseteq$ is **anti-symmetric**: for all $s_1,\ s_2 \in S$:

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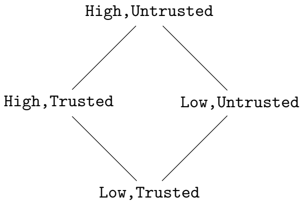
    $$s_1 \sqsubseteq s_2 \wedge s_1 \sqsupseteq s_2 \implies s_1 = s_2$$
- $\sqcup : S \times S \to S$ and $\sqcap : S \times S \to S$ are two operations for **combining labels** such that

  for all $s_1, s_2 \in S$:
  $$s_1 \sqsubseteq s_1 \sqcup s_2 \text{ and } s_2 \sqsubseteq s_1 \sqcup s_2$$
  $$s_1 \sqcap s_2 \sqsubseteq s_1 \text{ and } s_1 \sqcap s_2 \sqsubseteq s_2$$

Security Lattice

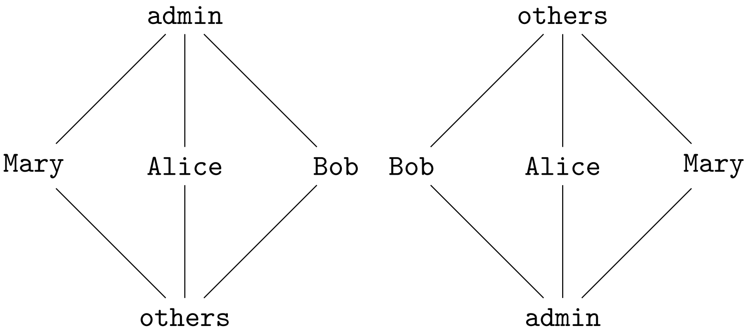
$(S, \sqsubseteq)$ is a *lattice*. This allows to depict it nicely by just denoting the direct successors of security classes:



In a security framework $(S,\sqsubseteq,\sqcup,\sqcap)$

- there is the bottom security label $\perp$ that can be obtained as $s_1 \sqcap\ s_2 \ldots \sqcap s_n$ (if $S = \{s_1, \ldots, s_n\}$).
- there is the top security label $\top$ that can be obtained as $s_1 \sqcup\ s_2 \ldots \sqcup s_n$ (if $S = \{s_1, \ldots, s_n\}$).

Example



The set of security labels

$$S = \{\texttt{others}, \texttt{Mary}, \texttt{Alice}, \texttt{Bob}, \texttt{admin}\}$$

What about $\sqsubseteq$, $\sqcup$ and $\sqcap$?

Typical Security Policy Frameworks
Components

Start with

- A finite and non-empty set C of **security categories**.

Typical Security Policy Frameworks
Components

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- A finite and non-empty set C of **security categories**.

Security labels can be all subsets of C:

- $(\mathrm{PowerSet}(C),\supseteq,\cap,\cup)$ for **confidentiality policies**.
- $(\mathrm{PowerSet}(C),\subseteq,\cup,\cap)$ for **integrity policies**.

A security label $s \in \mathrm{Powerset}(C)$ is called a **component**.

Recall $\mathrm{Powerset}(C) = \{C_0 \mid C_0 \subseteq C\}$, i.e., the set of subsets of C.

Example (Confidentiality): (PowerSet(C),$\supseteq$,$\cap$,$\cup$)

$$
\begin{tikzcd}[column sep=small]
 & \{\} \arrow[dl, dash] \arrow[dr, dash] & \\
\{\texttt{medical}\} \arrow[dr, dash] & & \{\texttt{finance}\} \arrow[dl, dash] \\
 & \{\texttt{finance}, \texttt{medical}\} &
\end{tikzcd}
$$

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- The **more departments** can **read** to the data, the **less sensitive** it is.

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  $\{\texttt{finance}, \texttt{medical}\} \sqsubseteq \{\texttt{medical}\}$, $\{\texttt{medical}\} \not\sqsubseteq \{\texttt{finance}\}$,
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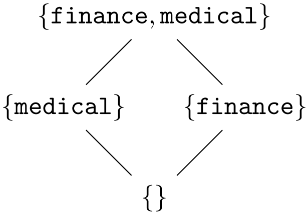
- Combining labels examples ($\sqcup = \cap$, $\sqcap = \cup$):
  $\{\mathtt{finance}, \mathtt{medical}\} \sqcup \{\mathtt{medical}\} = \{\mathtt{medical}\}$
  $\{\mathtt{medical}\} \sqcap \{\mathtt{finance}\} = \{\mathtt{finance}, \mathtt{medical}\}$

Example (Integrity): (PowerSet(C),$\subseteq$,$\cup$,$\cap$)

$$\begin{tikzcd}[column sep=small]
 & \{\texttt{finance}, \texttt{medical}\} \arrow[dl, dash] \arrow[dr, dash] & \\
\{\texttt{medical}\} \arrow[dr, dash] & & \{\texttt{finance}\} \arrow[dl, dash] \\
 & \{\} &
\end{tikzcd}$$

Example (Integrity): (PowerSet(C),$\subseteq$,$\cup$,$\cap$)



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Example (Integrity): (PowerSet(C),$\subseteq$,$\cup$,$\cap$)

$$\begin{tikzcd}[column sep=small]
 & \{\mathtt{finance}, \mathtt{medical}\} \arrow[dl, no head] \arrow[dr, no head] & \\
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- Combining labels examples ($\sqcup = \cup$, $\sqcap = \cap$):
  $\{\mathtt{finance}\} \sqcup \{\mathtt{medical}\} = \{\mathtt{finance}, \mathtt{medical}\}$
  $\{\mathtt{medical}\} \sqcap \{\mathtt{finance}\} = \{\}$

Combining Security Policy Frameworks
Product Construction

Whenever $(S_1,\sqsubseteq_1,\sqcup_1,\sqcap_1)$ and $(S_2,\sqsubseteq_2,\sqcup_2,\sqcap_2)$ are two security frameworks, we can construct the framework $(S,\sqsubseteq,\sqcup,\sqcap)$ where

- $S = S_1 \times S_2$
- for $(s_{11}, s_{12}), (s_{21}, s_{22}) \in S$:

$$(s_{11}, s_{12}) \sqsubseteq (s_{21}, s_{22}) \text{ iff } s_{11} \sqsubseteq_1 s_{21} \wedge s_{12} \sqsubseteq_2 s_{22}$$

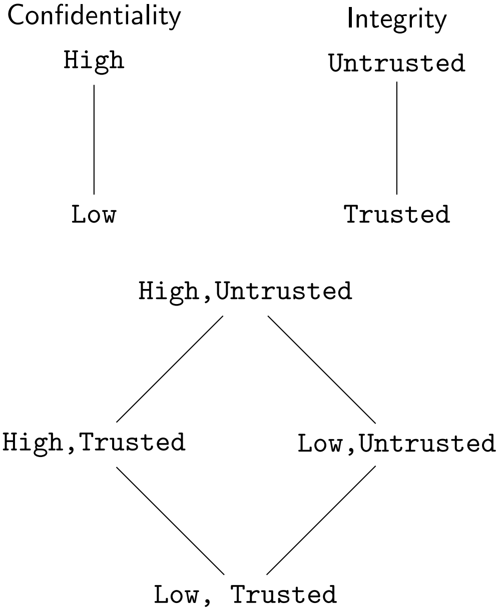
- for $(s_{11}, s_{12}), (s_{21}, s_{22}) \in S$:

$$(s_{11}, s_{12}) \sqcup (s_{21}, s_{22}) = (s_{11} \sqcup_1 s_{21}, s_{12} \sqcup_2 s_{22})$$

  and

$$(s_{11}, s_{12}) \sqcap (s_{21}, s_{22}) = (s_{11} \sqcap_1 s_{21}, s_{12} \sqcap_2 s_{22})$$

Example (Combining Integrity and Confidentiality):



Result

Theorem (Non-Interference for an arbitrary Security Policy Framework)

- *Suppose a program c satisfies information flow policy γ:*

$$\gamma \vdash c : \tau \qquad \textit{(for any type } \tau\textit{, does not matter)}$$

- *Suppose μ_1 and μ_2 are memories that are equal on all variables up to level τ_0:*

$$\mu_1(x) = \mu_2(x) \text{ for every } x \text{ with } \gamma(x) \sqsubseteq \tau_0$$

- *If we run the program on these memories:*
  - ★ $\mu_1 \vdash c \Rightarrow \mu_1'$
  - ★ $\mu_2 \vdash c \Rightarrow \mu_2'$

  (and they both terminate)

- *…then the result will be the same on all variables up to level τ_0:*

$$\mu_1'(x) = \mu_2'(x) \text{ for every } x \text{ with } \gamma(x) \sqsubseteq \tau_0$$

Review: Myers' Security Labels

We have the security framework ($P \hookrightarrow \text{PowerSet}(P)$, $\sqsubseteq$, $\sqcup$, $\sqcap$) where

- $P \hookrightarrow \text{PowerSet}(P)$ is the set of all **partial mappings** from P to PowerSet(P).
- For a label s we define $\text{Owners}(s) = \text{Domain}(s)$
- For a security label s and principal p define
  $\text{Readers}(s, p) = \begin{cases} s(p) & \text{if } p \in \text{Owners}(s) \\ P & \text{if } p \notin \text{Owners}(s) \end{cases}$
- Alternative notation (bit easier to read):

$$\{(A : A, C), (B : B, C)\} \text{ for } \{A : \{A, C\}, B : \{B, C\}\}$$

Review: Myers' Security Labels/Ordering

- for two security labels s_1, s_2 we have
  - ★ $s_1 \sqsubseteq s_2$ iff
    $\mathrm{Owners}(s_1) \subseteq \mathrm{Owners}(s_2)$ and
    $\mathrm{Readers}(s_1, o) \supseteq \mathrm{Readers}(s_2, o)$ for every $o \in \mathrm{Owners}(s_1)$
  - ★ $s_1 \sqcup s_2$ such that
    $$\mathrm{Owners}(s_1 \sqcup s_2) = \mathrm{Owners}(s_1) \cup \mathrm{Owners}(s_2)$$
    $$\mathrm{Readers}(s_1 \sqcup s_2, o) = \mathrm{Readers}(s_1, o) \cap \mathrm{Readers}(s_2, o) \quad \text{for every } o \in \mathrm{Owners}(s_1 \sqcup s_2)$$
  - ★ $s_1 \sqcap s_2$ such that
    $$\mathrm{Owners}(s_1 \sqcap s_2) = \mathrm{Owners}(s_1) \cap \mathrm{Owners}(s_2)$$
    $$\mathrm{Readers}(s_1 \sqcap s_2, o) = \mathrm{Readers}(s_1, o) \cup \mathrm{Readers}(s_2, o) \quad \text{for every owner } o \in \mathrm{Owners}(s_1 \sqcap s_2)$$

Integrity

The Myers-Liskov approach also allows for integrity –
For integrity one can use the dual definitions:

- Instead of Readers we have Writers
- $s_1 \sqsubseteq s_2$ iff $\quad$ $\mathrm{Owners}(s_1) \supseteq \mathrm{Owners}(s_2)$ and
  $\forall o \in \mathrm{Owners}(s_2) : \mathrm{Writers}(s_1, o) \subseteq \mathrm{Writers}(s_2, o)$
- $s_1 \sqcup s_2$ s.t. $\quad$ $\mathrm{Owners}(s_1 \sqcup s_2) = \mathrm{Owners}(s_1) \cap \mathrm{Owners}(s_2)$
  $\mathrm{Writers}(s_1 \sqcup s_2) = \mathrm{Writers}(s_1, o) \cup \mathrm{Writers}(s_2, o)$
- $s_1 \sqcap s_2$ s.t. $\quad$ $\mathrm{Owners}(s_1 \sqcap s_2) = \mathrm{Owners}(s_1) \cup \mathrm{Owners}(s_2)$
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Declassification for Integrity?

- A core feature of decentralized label model is declassification for confidentiality.
- For integrity, declassification does not really make sense.
- Since everything in integrity "mirrors" confidentiality, is there something like declassification for integrity?

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Yes:

Endorsement

- Declassification: we make the exception that something confidential is allowed to become public.
- Endorsement: we make the exception that something untrusted is now considered trusted.

Example: two untrusted but independent sources tell the same story.

Security Policy for the Assignment

Remember that for the assignment you are asked to use Myers-Liskov decentralized label model:

- You can use decentralized confidentiality labels (owners and readers)
- You can use decentralized integrity labels (owners and writers)
- You can use the product lattice of the two
- You can use declassification and endorsement, but only in the way defined by Myers-Liskov – an owner can relax or remove their own constraint.

Challenge

Let $\gamma = [x \mapsto \mathtt{Low}, y \mapsto \mathtt{High}]$ and the context $\mathsf{ctx} = \mathtt{Low}$

- Is the program $\mathsf{S} = (\textbf{while } y > 0 \textbf{ do } y := y + 1)\ ;\ x := 2$ secure?

Use Volpano's typing rules to determine if S is secure

- Do you agree with the result of the type system?

References I

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