

DATA SCIENCE METHODS

Assignment 02

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Group 01:

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Clean environment

```
rm(list=ls())
```

Load the required libraries

```
library(foreign)
library(data.table)
library(dplyr)
library(fastDummies)
library(stringr)
library(readxl)
library(leaps)
library(glmnet)
library(pls)
library(fastDummies)
library(randomForest)
```

QUESTION 01:

Not to have scientific notation

```
options(scipen = 999)
```

Load and inspect the data set

```
soccer <- read_excel("soccer.xlsx")
summary(soccer)
```

```
##      Item      ID      Name      Age
##  Min.   :    0  Min.   :   16  Length:18207  Min.   :16.00
## 1st Qu.: 4552 1st Qu.:200316  Class :character 1st Qu.:21.00
## Median : 9103 Median :221759  Mode  :character  Median :25.00
## Mean   : 9103 Mean   :214298             Mean   :25.12
## 3rd Qu.:13654 3rd Qu.:236530             3rd Qu.:28.00
## Max.   :18206 Max.   :246620             Max.   :45.00
##
## Nationality      Overall      Potential      Value
## Length:18207      Min.   :46.00  Min.   :48.00  Length:18207
```

```

## Class :character 1st Qu.:62.00 1st Qu.:67.00 Class :character
## Mode :character Median :66.00 Median :71.00 Mode :character
## Mean :66.24 Mean :71.31
## 3rd Qu.:71.00 3rd Qu.:75.00
## Max. :94.00 Max. :95.00
##
## Wage Preferred Foot International Reputation Weak Foot
## Length:18207 Length:18207 Min. :1.000 Min. :1.000
## Class :character Class :character 1st Qu.:1.000 1st Qu.:3.000
## Mode :character Mode :character Median :1.000 Median :3.000
## Mean :1.113 Mean :2.947
## 3rd Qu.:1.000 3rd Qu.:3.000
## Max. :5.000 Max. :5.000
## NA's :48 NA's :48
## Skill Moves Work Rate Body Type Position
## Min. :1.000 Length:18207 Length:18207 Length:18207
## 1st Qu.:2.000 Class :character Class :character Class :character
## Median :2.000 Mode :character Mode :character Mode :character
## Mean :2.361
## 3rd Qu.:3.000
## Max. :5.000
## NA's :48
## Height Weight Crossing Finishing
## Length:18207 Length:18207 Min. : 5.00 Min. : 2.00
## Class :character Class :character 1st Qu.:38.00 1st Qu.:30.00
## Mode :character Mode :character Median :54.00 Median :49.00
## Mean :49.73 Mean :45.55
## 3rd Qu.:64.00 3rd Qu.:62.00
## Max. :93.00 Max. :95.00
## NA's :48 NA's :48
## HeadingAccuracy ShortPassing Volleys Dribbling
## Min. : 4.0 Min. : 7.00 Min. : 4.00 Min. : 4.00
## 1st Qu.:44.0 1st Qu.:54.00 1st Qu.:30.00 1st Qu.:49.00
## Median :56.0 Median :62.00 Median :44.00 Median :61.00
## Mean :52.3 Mean :58.69 Mean :42.91 Mean :55.37
## 3rd Qu.:64.0 3rd Qu.:68.00 3rd Qu.:57.00 3rd Qu.:68.00
## Max. :94.0 Max. :93.00 Max. :90.00 Max. :97.00
## NA's :48 NA's :48 NA's :48 NA's :48
## Curve FKAccuracy LongPassing BallControl
## Min. : 6.00 Min. : 3.00 Min. : 9.00 Min. : 5.00
## 1st Qu.:34.00 1st Qu.:31.00 1st Qu.:43.00 1st Qu.:54.00
## Median :48.00 Median :41.00 Median :56.00 Median :63.00
## Mean :47.17 Mean :42.86 Mean :52.71 Mean :58.37
## 3rd Qu.:62.00 3rd Qu.:57.00 3rd Qu.:64.00 3rd Qu.:69.00
## Max. :94.00 Max. :94.00 Max. :93.00 Max. :96.00
## NA's :48 NA's :48 NA's :48 NA's :48
## Acceleration SprintSpeed Agility Reactions Balance
## Min. :12.00 Min. :12.00 Min. :14.0 Min. :21.00 Min. :16.00
## 1st Qu.:57.00 1st Qu.:57.00 1st Qu.:55.0 1st Qu.:56.00 1st Qu.:56.00
## Median :67.00 Median :67.00 Median :66.0 Median :62.00 Median :66.00
## Mean :64.61 Mean :64.73 Mean :63.5 Mean :61.84 Mean :63.97
## 3rd Qu.:75.00 3rd Qu.:75.00 3rd Qu.:74.0 3rd Qu.:68.00 3rd Qu.:74.00

```

##	Max.	:97.00	Max.	:96.00	Max.	:96.0	Max.	:96.00	Max.	:96.00
##	NA's	:48	NA's	:48	NA's	:48	NA's	:48	NA's	:48
##	ShotPower		Jumping		Stamina		Strength			
##	Min.	: 2.00	Min.	:15.00	Min.	:12.00	Min.	:17.00		
##	1st Qu.:	45.00	1st Qu.:	58.00	1st Qu.:	56.00	1st Qu.:	58.00		
##	Median	:59.00	Median	:66.00	Median	:66.00	Median	:67.00		
##	Mean	:55.46	Mean	:65.09	Mean	:63.22	Mean	:65.31		
##	3rd Qu.:	68.00	3rd Qu.:	73.00	3rd Qu.:	74.00	3rd Qu.:	74.00		
##	Max.	:95.00	Max.	:95.00	Max.	:96.00	Max.	:97.00		
##	NA's	:48	NA's	:48	NA's	:48	NA's	:48		
##	LongShots		Aggression		Interceptions		Positioning		Vision	
##	Min.	: 3.00	Min.	:11.00	Min.	: 3.0	Min.	: 2.00	Min.	:10.0
##	1st Qu.:	33.00	1st Qu.:	44.00	1st Qu.:	26.0	1st Qu.:	38.00	1st Qu.:	44.0
##	Median	:51.00	Median	:59.00	Median	:52.0	Median	:55.00	Median	:55.0
##	Mean	:47.11	Mean	:55.87	Mean	:46.7	Mean	:49.96	Mean	:53.4
##	3rd Qu.:	62.00	3rd Qu.:	69.00	3rd Qu.:	64.0	3rd Qu.:	64.00	3rd Qu.:	64.0
##	Max.	:94.00	Max.	:95.00	Max.	:92.0	Max.	:95.00	Max.	:94.0
##	NA's	:48	NA's	:48	NA's	:48	NA's	:48	NA's	:48
##	Penalties		Composure		Marking		StandingTackle		SlidingTackle	
##	Min.	: 5.00	Min.	: 3.00	Min.	: 3.00	Min.	: 2.0	Min.	: 3.00
##	1st Qu.:	39.00	1st Qu.:	51.00	1st Qu.:	30.00	1st Qu.:	27.0	1st Qu.:	24.00
##	Median	:49.00	Median	:60.00	Median	:53.00	Median	:55.0	Median	:52.00
##	Mean	:48.55	Mean	:58.65	Mean	:47.28	Mean	:47.7	Mean	:45.66
##	3rd Qu.:	60.00	3rd Qu.:	67.00	3rd Qu.:	64.00	3rd Qu.:	66.0	3rd Qu.:	64.00
##	Max.	:92.00	Max.	:96.00	Max.	:94.00	Max.	:93.0	Max.	:91.00
##	NA's	:48	NA's	:48	NA's	:48	NA's	:48	NA's	:48
##	GKDividing		GKHandling		GKKicking		GKPositioning			
##	Min.	: 1.00	Min.	: 1.00	Min.	: 1.00	Min.	: 1.00		
##	1st Qu.:	8.00	1st Qu.:	8.00	1st Qu.:	8.00	1st Qu.:	8.00		
##	Median	:11.00	Median	:11.00	Median	:11.00	Median	:11.00		
##	Mean	:16.62	Mean	:16.39	Mean	:16.23	Mean	:16.39		
##	3rd Qu.:	14.00	3rd Qu.:	14.00	3rd Qu.:	14.00	3rd Qu.:	14.00		
##	Max.	:90.00	Max.	:92.00	Max.	:91.00	Max.	:90.00		
##	NA's	:48	NA's	:48	NA's	:48	NA's	:48		
##	GKReflexes		Release Clause							
##	Min.	: 1.00	Length:	18207						
##	1st Qu.:	8.00	Class	:character						
##	Median	:11.00	Mode	:character						
##	Mean	:16.71								
##	3rd Qu.:	14.00								
##	Max.	:94.00								
##	NA's	:48								

Drop the missing and duplicated values

```
dta <- na.omit(soccer)
dta <- dta[!duplicated(dta$Name),]
dta <- dta %>% select(c(4:20))
```

Transform variables if necessary

Work Rate

```
unique(dta$`Work Rate`)
dta$`Work Rate` <- ordered(dta$`Work Rate`, levels =
```

```

c("Low/ Low", "Low/ Medium", "Low/ High",
  "Medium/ Low", "Medium/ Medium", "Medium/ High",
  "High/ Low", "High/ Medium", "High/ High"))
dta$`Work Rate` <- unclass(dta$`Work Rate`)

# Body Types
unique(dta$`Body Type`)
table(dta$`Body Type`)
dta$`Body Type`[dta$`Body Type` == "PLAYER_BODY_TYPE_25"] <- "Other"
dta <- dummy_cols(dta, select_columns = "Body Type")
dta <- dta %>% select(-c("Body Type"))

# Position
unique(dta$Position)
table(dta$Position)
dta <- dummy_cols(dta, select_columns = "Position")
dta <- dta %>% select(-c("Position"))

# Preferred Foot
unique(dta$`Preferred Foot`)
table(dta$`Preferred Foot`)
dta <- dummy_cols(dta, select_columns = "Preferred Foot")
dta <- dta %>% select(-c("Preferred Foot"))

# Convert Wage into Numeric
sum(!str_detect(dta$Wage, "€"))
sum(!str_detect(dta$Wage, "K"))
dta$Wage <- str_remove(dta$Wage, "€")
dta$Wage <- str_remove(dta$Wage, "K") %>% as.numeric()
dta$Wage <- dta$Wage * 1000

# Convert Value into Numeric
money_convert <- function(x) {
  value = as.numeric(str_sub(x, 2, -2))
  suffix = str_sub(x, nchar(x), -1)
  if (suffix == "K"){
    value = value*1000
  } else if ((suffix == "M")) {
    value = value*1000000
  }
}
sum(!str_detect(dta$Value, "€"))
sum(!str_detect(dta$Value, "K"))
sum(!str_detect(dta$Value, "M"))
dta$Value <- sapply(dta$Value, money_convert)

# Nationality
dta <- dummy_cols(dta, select_columns = "Nationality")
dta <- dta %>% select(-c("Nationality"))

# Weight
dta$Weight <- str_remove(dta$Weight, "lbs") %>% as.numeric()

```

```
# Height
dta$Height <- str_replace(dta$Height, "'", ".") %>% as.numeric()
head(dta)

data <- lapply(dta, as.numeric) %>% as.data.frame()
```

1a.

Linear Regression

```
# Split data into two parts
set.seed(21) # for reproducibility
sum(is.na(data)) # check whether there are missing values

## [1] 0

train.size = dim(data)[1] / 2 # Learn how many observations we need to sample
train = sample(1:dim(data)[1], train.size) # get indexes of observations for
training set
test = -train # get indexes of observations for test set
data.train = data[train, ] # extract training observations
data.test = data[test, ] # extract test observations

# Linear Regression
lm.fit = lm(Wage~., data=data.train) # apply linear regression
lm.pred = predict(lm.fit, data.test) %>% as.numeric() # get predicted outcomes
testerror.ls = mean((data.test[, "Wage"] - lm.pred)^2) # find MSE
testerror.ls

## [1] 118531610
```

Lasso

```
# Convert data to matrix format
train.mat = model.matrix(Wage~., data=data.train)
test.mat = model.matrix(Wage~., data=data.test)

# Create a grid of Lambdas from a large range
grid = 10^seq(2, -3, length=100)

# Lasso
mod.lasso = cv.glmnet(train.mat, data.train[, "Wage"], alpha=1, lambda=grid,
thresh=1e-12, nfolds = 5)
lambda.best.lasso = mod.lasso$lambda.min
lambda.best.lasso

## [1] 100

lasso.pred = predict(mod.lasso, newx=test.mat, s=lambda.best.lasso)
testerror.lasso=mean((data.test[, "Wage"] - lasso.pred)^2)
testerror.lasso

## [1] 117873828
```

```
# estimate lasso regression with full data and present coefficients
mod.lasso = glmnet(model.matrix(Wage~., data=data), data[, "Wage"], alpha=1)
coef.lasso=predict(mod.lasso, s=lambda.best.lasso, type="coefficients")
coef.lasso
```

```
## 213 x 1 sparse Matrix of class "dgCMatrix"
##                                     s1
## (Intercept)                      -19529.221253574
## (Intercept)                        .
## Age                               184.514478608
## Overall                           57.402264242
## Potential                         3.936971196
## Value                             0.002810146
## International.Reputation          9819.432044598
## Weak.Foot                         .
## Skill.Moves                      -373.758869558
## Work.Rate                         16.320494118
## Height                           100.509695415
## Weight                            9.093830527
## Crossing                         20.846424846
## Finishing                         .
## Body.Type_Akinfenwa               .
## Body.Type_C..Ronaldo             136711.375069051
## Body.Type_Courtois               43948.549484346
## Body.Type_Lean                   24.170958905
## Body.Type_Messi                  200632.048838682
## Body.Type_Neymar                 -73487.004001360
## Body.Type_Normal                 .
## Body.Type_Other                  25247.210082757
## Body.Type_Shaqiri                43257.607186249
## Body.Type_Stocky                 -459.668585589
## Position_CAM                     -759.437695977
## Position_CB                      636.574460170
## Position_CDM                     357.976658925
## Position_CF                      -474.419856295
## Position_CM                      .
## Position_GK                      -7.049049916
## Position_LAM                     .
## Position_LB                      881.822147549
## Position_LCB                     -397.253669401
## Position_LCM                     -1857.434185634
## Position_LDM                     -398.717921965
## Position_LF                      -8798.191668584
## Position_LM                      -682.388138278
## Position_LS                      -2817.097231881
## Position_LW                      396.833401371
## Position_LWB                     .
## Position_RAM                     .
## Position_RB                      605.438113714
## Position_RCB                     .
## Position_RCM                     -830.522807105
## Position_RDM                     .
```

## Position_RF	-2124.104959230
## Position_RM	-523.446917780
## Position_RS	-1040.099499173
## Position_RW	508.059795713
## Position_RWB	.
## Position_ST	92.965176392
## Preferred.Foot_Left	.
## Preferred.Foot_Right	.
## Nationality_Afghanistan	.
## Nationality_Albania	.
## Nationality_Algeria	.
## Nationality_Andorra	.
## Nationality_Angola	.
## Nationality_Antigua...Barbuda	.
## Nationality_Argentina	-4.853373043
## Nationality_Armenia	1954.764732489
## Nationality_Australia	.
## Nationality_Austria	524.188063263
## Nationality_Azerbaijan	.
## Nationality_Barbados	.
## Nationality_Belarus	.
## Nationality_Belgium	551.566871237
## Nationality_Belize	.
## Nationality_Benin	.
## Nationality_Bermuda	.
## Nationality_Bolivia	.
## Nationality_Bosnia.Herzegovina	.
## Nationality_Botswana	.
## Nationality_Brazil	766.171614043
## Nationality_Bulgaria	-1037.098388635
## Nationality_Burkina.Faso	.
## Nationality_Burundi	.
## Nationality_Cameroon	.
## Nationality_Canada	.
## Nationality_Cape.Verde	-1118.551797137
## Nationality_Central.African.Rep.	-7134.146835703
## Nationality_Chad	.
## Nationality_Chile	.
## Nationality_China.PR	235.266717132
## Nationality_Colombia	-921.635328289
## Nationality_Comoros	.
## Nationality_Congo	.
## Nationality_Costa.Rica	189.548374268
## Nationality_Croatia	679.904781659
## Nationality_Cuba	.
## Nationality_Curacao	.
## Nationality_Cyprus	.
## Nationality_Czech.Republic	-2197.956835381
## Nationality_Denmark	.
## Nationality_Dominican.Republic	31751.867279432
## Nationality_DR.Congo	.
## Nationality_Ecuador	3290.314436635

## Nationality_Egypt	.
## Nationality_El.Salvador	.
## Nationality_England	3833.217440922
## Nationality_Equatorial.Guinea	540.793834508
## Nationality_Eritrea	.
## Nationality_Estonia	.
## Nationality_Ethiopia	.
## Nationality_Faroe.Islands	.
## Nationality_Fiji	.
## Nationality_Finland	.
## Nationality_France	.
## Nationality_FYR.Macedonia	.
## Nationality_Gabon	1745.865955640
## Nationality_Gambia	.
## Nationality_Georgia	-884.544520423
## Nationality_Germany	.
## Nationality_Ghana	.
## Nationality_Greece	-3131.479792121
## Nationality_Grenada	.
## Nationality_Guam	.
## Nationality_Guatemala	.
## Nationality_Guinea	.
## Nationality_Guinea.Bissau	.
## Nationality_Guyana	.
## Nationality_Haiti	.
## Nationality_Honduras	.
## Nationality_Hong.Kong	.
## Nationality_Hungary	.
## Nationality_Iceland	.
## Nationality_Indonesia	.
## Nationality_Iran	.
## Nationality_Iraq	.
## Nationality_Israel	.
## Nationality_Italy	.
## Nationality_Ivory.Coast	.
## Nationality_Jamaica	.
## Nationality_Japan	.
## Nationality_Jordan	.
## Nationality_Kazakhstan	.
## Nationality_Kenya	.
## Nationality_Korea.DPR	.
## Nationality_Korea.Republic	-871.027516634
## Nationality_Kosovo	.
## Nationality_Kuwait	.
## Nationality_Latvia	.
## Nationality_Lebanon	.
## Nationality_Libya	.
## Nationality_Liechtenstein	.
## Nationality_Lithuania	.
## Nationality_Luxembourg	.
## Nationality_Madagascar	.
## Nationality_Mali	.

## Nationality_Mauritania	.
## Nationality_Mauritius	.
## Nationality_Mexico	944.400342818
## Nationality_Moldova	.
## Nationality_Montenegro	.
## Nationality_Montserrat	.
## Nationality_Morocco	.
## Nationality_Mozambique	.
## Nationality_Namibia	.
## Nationality_Netherlands	-406.977552111
## Nationality_New.Caledonia	.
## Nationality_New.Zealand	.
## Nationality_Nicaragua	.
## Nationality_Niger	.
## Nationality_Nigeria	.
## Nationality_Northern.Ireland	1542.644349819
## Nationality_Norway	-25.480712754
## Nationality_Oman	.
## Nationality_Palestine	.
## Nationality_Panama	.
## Nationality_Paraguay	.
## Nationality_Peru	-787.525050394
## Nationality_Philippines	.
## Nationality_Poland	.
## Nationality_Portugal	-4451.264602794
## Nationality_Puerto.Rico	.
## Nationality_Qatar	.
## Nationality_Republic.of.Ireland	935.613742321
## Nationality_Romania	.
## Nationality_Russia	-8312.310309193
## Nationality_Rwanda	.
## Nationality_São.Tomé...Príncipe	.
## Nationality_Saudi.Arabia	2249.233566503
## Nationality_Scotland	2214.321129065
## Nationality_Senegal	.
## Nationality_Serbia	-650.040071472
## Nationality_Sierra.Leone	.
## Nationality_Slovakia	-1250.412913662
## Nationality_Slovenia	-4631.582762604
## Nationality_South.Africa	-931.431610064
## Nationality_South.Sudan	.
## Nationality_Spain	.
## Nationality_St.Kitts.Nevis	.
## Nationality_Sudan	.
## Nationality_Suriname	.
## Nationality_Sweden	-171.645174726
## Nationality_Switzerland	.
## Nationality_Syria	.
## Nationality_Tanzania	.
## Nationality_Thailand	.
## Nationality_Togo	.
## Nationality_Trinidad...Tobago	.

```
## Nationality_Tunisia .
## Nationality_Turkey 2000.812420090
## Nationality_Uganda .
## Nationality_Ukraine -6890.255602278
## Nationality_United.Arab.Emirates .
## Nationality_United.States -888.980175804
## Nationality_Uruguay .
## Nationality_Uzbekistan .
## Nationality_Venezuela -814.177982834
## Nationality_Wales 3965.252059828
## Nationality_Zambia .
## Nationality_Zimbabwe .
```

```
# the number of non-zero coefficient (of predictors)
length(which(coef.lasso[-(1:2),] != 0))
```

```
## [1] 79
```

```
which(coef.lasso[-(1:2),] != 0)
```

```
## Age Overall
## 1 2
## Potential Value
## 3 4
## International.Reputation Skill.Moves
## 5 7
## Work.Rate Height
## 8 9
## Weight Crossing
## 10 11
## Body.Type_C..Ronaldo Body.Type_Courtois
## 14 15
## Body.Type_Lean Body.Type_Messi
## 16 17
## Body.Type_Neymar Body.Type_Other
## 18 20
## Body.Type_Shaqiri Body.Type_Stocky
## 21 22
## Position_CAM Position_CB
## 23 24
## Position_CDM Position_CF
## 25 26
## Position_GK Position_LB
## 28 30
## Position_LCB Position_LCM
## 31 32
## Position_LDM Position_LF
## 33 34
## Position_LM Position_LS
## 35 36
## Position_LW Position_RB
## 37 40
## Position_RCM Position_RF
```

##	42	44
##	Position_RM	Position_RS
##	45	46
##	Position_RW	Position_ST
##	47	49
##	Nationality_Argentina	Nationality_Armenia
##	58	59
##	Nationality_Austria	Nationality_Belgium
##	61	65
##	Nationality_Brazil	Nationality_Bulgaria
##	72	73
##	Nationality_Cape.Verde	Nationality_Central.African.Rep.
##	78	79
##	Nationality_China.PR	Nationality_Colombia
##	82	83
##	Nationality_Costa.Rica	Nationality_Croatia
##	86	87
##	Nationality_Czech.Republic	Nationality_Dominican.Republic
##	91	93
##	Nationality_Ecuador	Nationality_England
##	95	98
##	Nationality_Equatorial.Guinea	Nationality_Gabon
##	99	108
##	Nationality_Georgia	Nationality_Greece
##	110	113
##	Nationality_Korea.Republic	Nationality_Mexico
##	137	150
##	Nationality_Netherlands	Nationality_Northern.Ireland
##	157	163
##	Nationality_Norway	Nationality_Peru
##	164	169
##	Nationality_Portugal	Nationality_Republic.of.Ireland
##	172	175
##	Nationality_Russia	Nationality_Saudi.Arabia
##	177	180
##	Nationality_Scotland	Nationality_Serbia
##	181	183
##	Nationality_Slovakia	Nationality_Slovenia
##	185	186
##	Nationality_South.Africa	Nationality_Sweden
##	187	193
##	Nationality_Turkey	Nationality_Ukraine
##	201	203
##	Nationality_United.States	Nationality_Venezuela
##	205	208
##	Nationality_Wales	
##	209	

```

coef_lasso <- coef.lasso[-(1:2),] %>% as.data.frame()
names(coef_lasso) <- "Coef.Est."
coef_lasso <- coef_lasso %>% arrange(abs(coef_lasso$Coef.Est.))

```

Ridge

Ridge

```
mod.ridge = cv.glmnet(train.mat, data.train[, "Wage"], alpha=0, lambda=grid,
thresh=1e-12, nfolds = 5)
lambda.best.ridge = mod.ridge$lambda.min
lambda.best.ridge
```

```
## [1] 79.24829
```

```
ridge.pred = predict(mod.ridge, newx=test.mat, s=lambda.best.ridge)
testerror.ridge = mean((data.test[, "Wage"] - ridge.pred)^2)
testerror.ridge
```

```
## [1] 118420695
```

estimate ridge regression with full data and present coefficients

```
mod.ridge = glmnet(model.matrix(Wage~., data=data), data[, "Wage"], alpha=1)
coef.ridge=predict(mod.ridge, s=lambda.best.ridge, type="coefficients")
coef.ridge
```

```
## 213 x 1 sparse Matrix of class "dgCMatrix"
```

```
##                                     s1
## (Intercept)                -21155.2291835064970655
## (Intercept)                  .
## Age                        199.9653290308133364
## Overall                     51.3595832598032729
## Potential                   18.9460054265721354
## Value                       0.0028116445117049
## International.Reputation    9791.2657512245987164
## Weak.Foot                   .
## Skill.Moves                 -509.3747380644920781
## Work.Rate                   30.9667784291440675
## Height                     151.7361546265980223
## Weight                      11.4707167430354584
## Crossing                    24.0739856165341379
## Finishing                   .
## Body.Type_Akinfenwa         .
## Body.Type_C..Ronaldo       139287.1939886672771536
## Body.Type_Courtois         46246.8231726534650079
## Body.Type_Lean              73.0586547217811955
## Body.Type_Messi            204205.6455278349167202
## Body.Type_Neymar           -76415.5472667725989595
## Body.Type_Normal           .
## Body.Type_Other            27853.7464074500458082
## Body.Type_Shaqiri          46188.5273870402670582
## Body.Type_Stocky           -554.4959708901798194
## Position_CAM               -801.9967335756008424
## Position_CB                652.4815201591922005
## Position_CDM               413.2547788757198646
## Position_CF                -724.4447933512340114
## Position_CM                61.1174956758651646
## Position_GK                -192.5527991420316880
## Position_LAM               .
```

## Position_LB	938.3588936769497195
## Position_LCB	-587.0891176696935645
## Position_LCM	-2018.2975113316492752
## Position_LDM	-591.1499650035382274
## Position_LF	-9422.8278330104185443
## Position_LM	-728.2449920498286247
## Position_LS	-3031.5392498330647868
## Position_LW	633.7011527776261346
## Position_LWB	.
## Position_RAM	.
## Position_RB	630.5788759742354159
## Position_RCB	.
## Position_RCM	-991.3837415691752994
## Position_RDM	.
## Position_RF	-3138.5475913640334511
## Position_RM	-576.6380572854802722
## Position_RS	-1248.5459137256286795
## Position_RW	723.9669908511991707
## Position_RWB	.
## Position_ST	163.2551533179317573
## Preferred.Foot_Left	-43.2716340465480584
## Preferred.Foot_Right	0.000000002150304
## Nationality_Afghanistan	.
## Nationality_Albania	.
## Nationality_Algeria	.
## Nationality_Andorra	.
## Nationality_Angola	.
## Nationality_Antigua...Barbuda	.
## Nationality_Argentina	-104.6827042824260730
## Nationality_Armenia	2835.5386577184294765
## Nationality_Australia	.
## Nationality_Austria	695.4131107244789973
## Nationality_Azerbaijan	.
## Nationality_Barbados	.
## Nationality_Belarus	.
## Nationality_Belgium	713.3336085736502810
## Nationality_Belize	.
## Nationality_Benin	15.5060601241888474
## Nationality_Bermuda	.
## Nationality_Bolivia	.
## Nationality_Bosnia.Herzegovina	.
## Nationality_Botswana	.
## Nationality_Brazil	848.8358889060687034
## Nationality_Bulgaria	-1768.3553338266272021
## Nationality_Burkina.Faso	.
## Nationality_Burundi	.
## Nationality_Cameroon	.
## Nationality_Canada	.
## Nationality_Cape.Verde	-1754.8522053831013636
## Nationality_Central.African.Rep.	-8660.7248105068501900
## Nationality_Chad	.
## Nationality_Chile	-52.0711165889367322

## Nationality_China.PR	400.1442957552147845
## Nationality_Colombia	-1016.7383176647890650
## Nationality_Comoros	.
## Nationality_Congo	.
## Nationality_Costa.Rica	709.9658393454252518
## Nationality_Croatia	879.5100340692451937
## Nationality_Cuba	.
## Nationality_Curacao	.
## Nationality_Cyprus	.
## Nationality_Czech.Republic	-2534.2905730154711819
## Nationality_Denmark	.
## Nationality_Dominican.Republic	33499.4597449186549056
## Nationality_DR.Congo	11.4456028963948668
## Nationality_Ecuador	3813.4340850079265692
## Nationality_Egypt	.
## Nationality_El.Salvador	.
## Nationality_England	3908.4720066626714470
## Nationality_Equatorial.Guinea	1713.4894873529246979
## Nationality_Eritrea	.
## Nationality_Estonia	.
## Nationality_Ethiopia	.
## Nationality_Faroe.Islands	.
## Nationality_Fiji	.
## Nationality_Finland	.
## Nationality_France	.
## Nationality_FYR.Macedonia	.
## Nationality_Gabon	2512.7851754799744413
## Nationality_Gambia	.
## Nationality_Georgia	-1359.3071483257608634
## Nationality_Germany	.
## Nationality_Ghana	.
## Nationality_Greece	-3444.8205715860890450
## Nationality_Grenada	.
## Nationality_Guam	.
## Nationality_Guatemala	.
## Nationality_Guinea	.
## Nationality_Guinea.Bissau	.
## Nationality_Guyana	.
## Nationality_Haiti	.
## Nationality_Honduras	-303.6912997164046715
## Nationality_Hong.Kong	.
## Nationality_Hungary	-496.1294968885154049
## Nationality_Iceland	26.6639850632590623
## Nationality_Indonesia	.
## Nationality_Iran	.
## Nationality_Iraq	.
## Nationality_Israel	.
## Nationality_Italy	.
## Nationality_Ivory.Coast	.
## Nationality_Jamaica	.
## Nationality_Japan	-29.4997787859302960
## Nationality_Jordan	.

## Nationality_Kazakhstan	.
## Nationality_Kenya	.
## Nationality_Korea.DPR	.
## Nationality_Korea.Republic	-1015.8015353838673036
## Nationality_Kosovo	.
## Nationality_Kuwait	.
## Nationality_Latvia	.
## Nationality_Lebanon	.
## Nationality_Libya	.
## Nationality_Liechtenstein	.
## Nationality_Lithuania	.
## Nationality_Luxembourg	.
## Nationality_Madagascar	.
## Nationality_Mali	.
## Nationality_Mauritania	.
## Nationality_Mauritius	.
## Nationality_Mexico	1101.5498550642232658
## Nationality_Moldova	.
## Nationality_Montenegro	.
## Nationality_Montserrat	.
## Nationality_Morocco	.
## Nationality_Mozambique	.
## Nationality_Namibia	.
## Nationality_Netherlands	-533.6034747162877920
## Nationality_New.Caledonia	.
## Nationality_New.Zealand	.
## Nationality_Nicaragua	.
## Nationality_Niger	.
## Nationality_Nigeria	.
## Nationality_Northern.Ireland	1883.0015745612774936
## Nationality_Norway	-177.7034338836225800
## Nationality_Oman	.
## Nationality_Palestine	.
## Nationality_Panama	.
## Nationality_Paraguay	.
## Nationality_Peru	-1272.5334076209082923
## Nationality_Philippines	1258.2173982549272750
## Nationality_Poland	-86.1080363032450151
## Nationality_Portugal	-4638.8155428014943027
## Nationality_Puerto.Rico	.
## Nationality_Qatar	.
## Nationality_Republic.of.Ireland	1112.4137976484134924
## Nationality_Romania	.
## Nationality_Russia	-8664.4519571483415348
## Nationality_Rwanda	.
## Nationality_São.Tomé...Príncipe	.
## Nationality_Saudi.Arabia	2463.3129163874455116
## Nationality_Scotland	2401.2434199851413723
## Nationality_Senegal	122.3181147333869490
## Nationality_Serbia	-947.9531552762744013
## Nationality_Sierra.Leone	.
## Nationality_Slovakia	-1650.1593556799791713

```
## Nationality_Slovenia -5051.3065033215716539
## Nationality_South.Africa -1249.6666357632636846
## Nationality_South.Sudan .
## Nationality_Spain .
## Nationality_St.Kitts.Nevis 776.7396036760649167
## Nationality_Sudan .
## Nationality_Suriname .
## Nationality_Sweden -305.5958207569042884
## Nationality_Switzerland .
## Nationality_Syria 0.4542967748221321
## Nationality_Tanzania .
## Nationality_Thailand .
## Nationality_Togo .
## Nationality_Trinidad...Tobago .
## Nationality_Tunisia .
## Nationality_Turkey 2162.1795497841599172
## Nationality_Uganda .
## Nationality_Ukraine -7241.7979829341866207
## Nationality_United.Arab.Emirates .
## Nationality_United.States -1044.4198572383943429
## Nationality_Uruguay .
## Nationality_Uzbekistan .
## Nationality_Venezuela -1204.7617073567621446
## Nationality_Wales 4216.1231917654749850
## Nationality_Zambia .
## Nationality_Zimbabwe -54.4907238864165677
```

the number of non-zero coefficient (of predictors)

```
length(which(coef.ridge[-(1:2),]!=0))
```

```
## [1] 95
```

```
which(coef.ridge[-(1:2),]!=0)
```

```
## Age Overall
## 1 2
## Potential Value
## 3 4
## International.Reputation Skill.Moves
## 5 7
## Work.Rate Height
## 8 9
## Weight Crossing
## 10 11
## Body.Type_C..Ronaldo Body.Type_Courtois
## 14 15
## Body.Type_Lean Body.Type_Messi
## 16 17
## Body.Type_Neymar Body.Type_Other
## 18 20
## Body.Type_Shaqiri Body.Type_Stocky
## 21 22
## Position_CAM Position_CB
```


##	23	24
##	Position_CDM	Position_CF
##	25	26
##	Position_CM	Position_GK
##	27	28
##	Position_LB	Position_LCB
##	30	31
##	Position_LCM	Position_LDM
##	32	33
##	Position_LF	Position_LM
##	34	35
##	Position_LS	Position_LW
##	36	37
##	Position_RB	Position_RCM
##	40	42
##	Position_RF	Position_RM
##	44	45
##	Position_RS	Position_RW
##	46	47
##	Position_ST	Preferred.Foot_Left
##	49	50
##	Preferred.Foot_Right	Nationality_Argentina
##	51	58
##	Nationality_Armenia	Nationality_Austria
##	59	61
##	Nationality_Belgium	Nationality_Benin
##	65	67
##	Nationality_Brazil	Nationality_Bulgaria
##	72	73
##	Nationality_Cape.Verde	Nationality_Central.African.Rep.
##	78	79
##	Nationality_Chile	Nationality_China.PR
##	81	82
##	Nationality_Colombia	Nationality_Costa.Rica
##	83	86
##	Nationality_Croatia	Nationality_Czech.Republic
##	87	91
##	Nationality_Dominican.Republic	Nationality_DR.Congo
##	93	94
##	Nationality_Ecuador	Nationality_England
##	95	98
##	Nationality_Equatorial.Guinea	Nationality_Gabon
##	99	108
##	Nationality_Georgia	Nationality_Greece
##	110	113
##	Nationality_Honduras	Nationality_Hungary
##	121	123
##	Nationality_Iceland	Nationality_Japan
##	124	132
##	Nationality_Korea.Republic	Nationality_Mexico
##	137	150
##	Nationality_Netherlands	Nationality_Northern.Ireland

```
##          157          163
##      Nationality_Norway      Nationality_Peru
##          164          169
##      Nationality_Philippines      Nationality_Poland
##          170          171
##      Nationality_Portugal      Nationality_Republic.of.Ireland
##          172          175
##      Nationality_Russia      Nationality_Saudi.Arabia
##          177          180
##      Nationality_Scotland      Nationality_Senegal
##          181          182
##      Nationality_Serbia      Nationality_Slovakia
##          183          185
##      Nationality_Slovenia      Nationality_South.Africa
##          186          187
##      Nationality_St.Kitts.Nevis      Nationality_Sweden
##          190          193
##      Nationality_Syria      Nationality_Turkey
##          195          201
##      Nationality_Ukraine      Nationality_United.States
##          203          205
##      Nationality_Venezuela      Nationality_Wales
##          208          209
##      Nationality_Zimbabwe
##          211
```

```
coef_ridge <- coef.ridge[-(1:2),] %>% as.data.frame()
names(coef_ridge) <- "Coef.Est."
coef_ridge<- coef_ridge %>% arrange(abs(coef_ridge$Coef.Est.))
```

Random Forest

Random Forest

```
mod.rf = randomForest(Wage~., data=data, subset=train, mtry=7, importance=TRUE)
yhat.rf = predict(mod.rf, newdata=data[-train,])
rf.test = data[-train, "Wage"]
testerror.rf = mean((yhat.rf-rf.test)^2)
importance(mod.rf)
```

```
##          %IncMSE  IncNodePurity
## Age          13.13315366  59074339390.6
## Overall      21.76741215  577335448331.9
## Potential    17.58954910  349085154703.4
## Value        21.61971732  585453742853.4
## International.Reputation  17.44845456  338084313088.2
## Weak.Foot     1.97390397  22662946783.0
## Skill.Moves   7.34469033  91301489671.1
## Work.Rate     2.80482875  30772209982.7
## Height        6.49212315  21727608937.2
## Weight        5.06804327  37555339688.1
## Crossing     12.45831779  156938470465.7
## Finishing    10.39944420  139354309445.0
## Body.Type_Akinfenwa  0.00000000  2667381.5
## Body.Type_C..Ronaldo  0.00000000  0.0
```

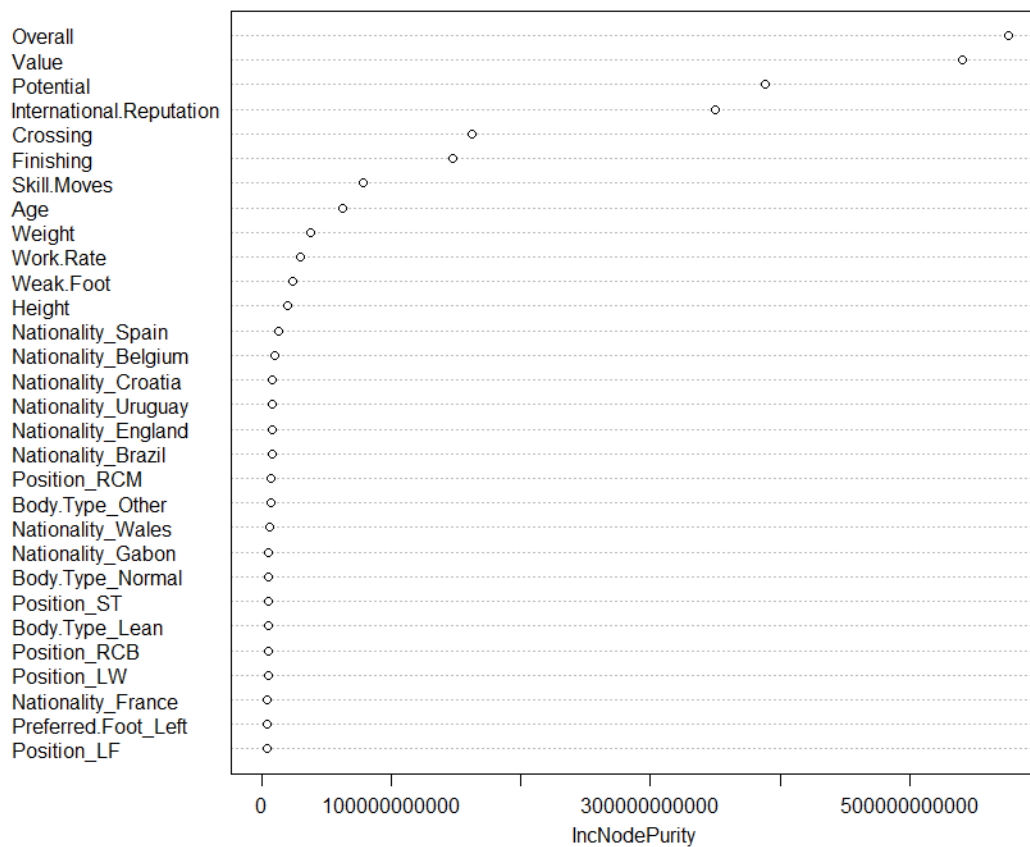
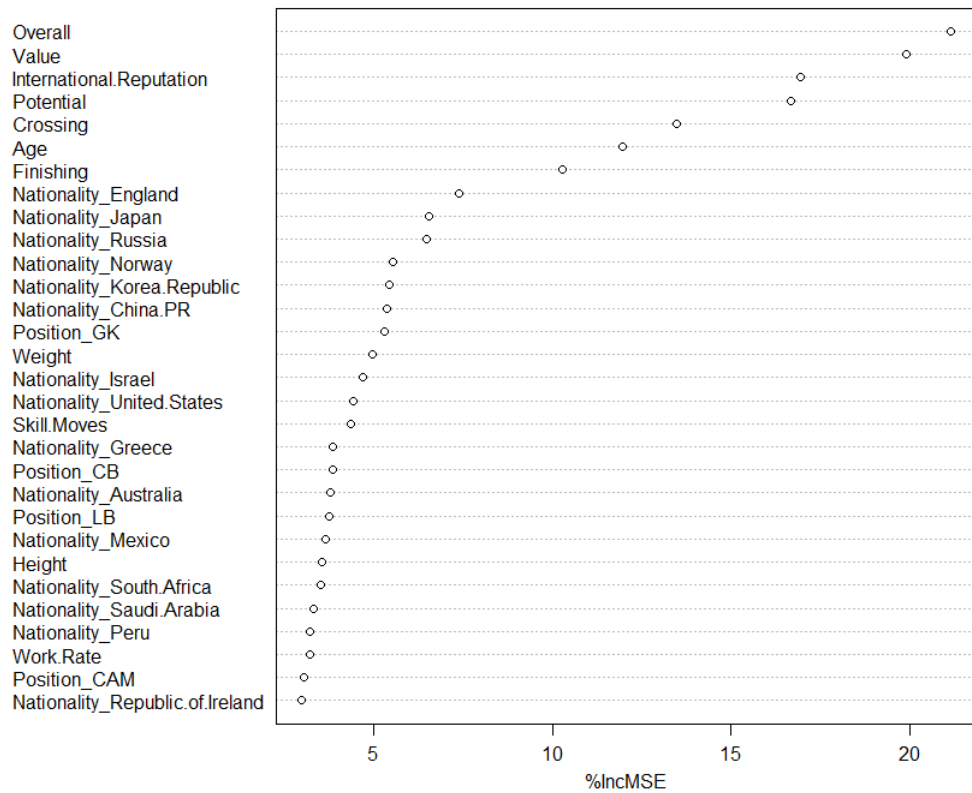
## Body.Type_Courtois	0.00000000	0.0
## Body.Type_Lean	2.06775695	6706864127.5
## Body.Type_Messi	0.00000000	0.0
## Body.Type_Neymar	0.00000000	0.0
## Body.Type_Normal	-1.66374312	5575828022.9
## Body.Type_Other	0.00000000	9244821988.2
## Body.Type_Shaqiri	0.00000000	0.0
## Body.Type_Stocky	0.17294342	2931583006.7
## Position_CAM	-0.18466021	2138303920.8
## Position_CB	4.28114441	2496368675.4
## Position_CDM	0.14898222	6214842317.2
## Position_CF	0.97901282	439381512.9
## Position_CM	1.57377426	3097211594.9
## Position_GK	4.29778497	3312562371.3
## Position_LAM	0.41265076	153340440.4
## Position_LB	2.89625098	1818593802.7
## Position_LCB	-0.47427207	3164383886.3
## Position_LCM	0.49493778	1517822699.2
## Position_LDM	-0.94678385	2625104785.3
## Position_LF	-2.27994322	5590494018.0
## Position_LM	-0.83659707	3062775309.9
## Position_LS	2.60022362	1394241032.8
## Position_LW	-1.04947057	3365315095.7
## Position_LWB	-0.21370470	479842015.9
## Position_RAM	-1.13658896	1021376743.7
## Position_RB	2.62565268	2606470794.9
## Position_RCB	1.52337464	4701865838.3
## Position_RCM	-0.65235482	5407684368.2
## Position_RDM	0.35464131	1700365294.3
## Position_RF	-1.38698631	596630129.4
## Position_RM	-0.97319820	3062700621.2
## Position_RS	1.28955581	7234775572.9
## Position_RW	-2.58828394	3607200597.5
## Position_RWB	0.80612463	450049235.5
## Position_ST	2.64991076	6457888801.5
## Preferred.Foot_Left	0.95285286	4858824245.2
## Preferred.Foot_Right	-0.01336665	4456260577.7
## Nationality_Afghanistan	0.00000000	0.0
## Nationality_Albania	0.23405248	64753312.6
## Nationality_Algeria	-1.06813509	2234592071.9
## Nationality_Andorra	0.00000000	2333381.2
## Nationality_Angola	0.98938492	8270901.1
## Nationality_Antigua...Barbuda	-0.43520278	3611817.6
## Nationality_Argentina	1.38063353	2347022713.5
## Nationality_Armenia	-0.93360992	2127555566.7
## Nationality_Australia	3.41946738	647894193.3
## Nationality_Austria	-0.04165737	710091822.4
## Nationality_Azerbaijan	-1.00100150	7445168.1
## Nationality_Barbados	0.00000000	1589637.2
## Nationality_Belarus	1.28731818	9310479.5
## Nationality_Belgium	-1.66452912	8781890532.4
## Nationality_Belize	0.00000000	3644492.6

## Nationality_Benin	1.35799268	10999125.2
## Nationality_Bermuda	0.00000000	0.0
## Nationality_Bolivia	-0.94536857	8001445.9
## Nationality_Bosnia.Herzegovina	1.00409858	1905635681.8
## Nationality_Botswana	0.00000000	0.0
## Nationality_Brazil	0.62973322	7835687319.8
## Nationality_Bulgaria	1.08079306	55514722.7
## Nationality_Burkina.Faso	1.01506925	514649570.1
## Nationality_Burundi	-2.11554560	44133086.9
## Nationality_Cameroon	-1.58715675	773512973.5
## Nationality_Canada	2.25414803	201447278.2
## Nationality_Cape.Verde	-0.25800943	428793625.7
## Nationality_Central.African.Rep.	-2.04566011	229994801.8
## Nationality_Chad	0.00000000	1427012.0
## Nationality_Chile	2.46462643	888499670.2
## Nationality_China.PR	4.23405682	1317000952.6
## Nationality_Colombia	2.22166878	2242323164.0
## Nationality_Comoros	1.24578114	9403697.4
## Nationality_Congo	1.15224161	44788933.3
## Nationality_Costa.Rica	-0.01540030	210061533.4
## Nationality_Croatia	-0.56092284	9631500857.4
## Nationality_Cuba	0.00000000	0.0
## Nationality_Curacao	0.70319136	44941873.1
## Nationality_Cyprus	-0.85529295	16621107.6
## Nationality_Czech.Republic	2.35468384	1008807102.2
## Nationality_Denmark	-1.12643210	1685112164.2
## Nationality_Dominican.Republic	-0.55464800	3999602871.7
## Nationality_DR.Congo	4.86523126	237289921.3
## Nationality_Ecuador	0.71991015	2011123692.5
## Nationality_Egypt	0.86464298	3500714765.0
## Nationality_El.Salvador	0.00000000	923717.8
## Nationality_England	6.86834500	8136087335.1
## Nationality_Equatorial.Guinea	-0.85050259	4932342.0
## Nationality_Eritrea	0.00000000	1641780.7
## Nationality_Estonia	-2.57789745	56385815.0
## Nationality_Ethiopia	0.00000000	5498630.6
## Nationality_Faroe.Islands	0.00000000	0.0
## Nationality_Fiji	0.00000000	3917047.1
## Nationality_Finland	2.16002352	117565203.3
## Nationality_France	-1.89925849	5803809708.5
## Nationality_FYR.Macedonia	0.06687352	91586334.4
## Nationality_Gabon	-2.04984498	5451557983.3
## Nationality_Gambia	0.60495531	115425259.4
## Nationality_Georgia	1.99959924	98900670.7
## Nationality_Germany	1.74865236	2397169987.1
## Nationality_Ghana	2.36713022	277588404.2
## Nationality_Greece	3.18935656	1014710223.2
## Nationality_Grenada	0.00000000	906345.8
## Nationality_Guam	0.00000000	1108791.6
## Nationality_Guatemala	-0.53823229	4469337.9
## Nationality_Guinea	-1.56429223	208734632.9
## Nationality_Guinea.Bissau	1.26979722	56829195.0

## Nationality_Guyana	1.01476177	3900378.0
## Nationality_Haiti	1.04804728	48451989.0
## Nationality_Honduras	-0.26691513	109791675.9
## Nationality_Hong.Kong	0.00000000	0.0
## Nationality_Hungary	0.59473529	217265534.3
## Nationality_Iceland	-1.05935933	294924754.7
## Nationality_Indonesia	0.00000000	1952568.8
## Nationality_Iran	-1.95880317	47979733.7
## Nationality_Iraq	-1.66418107	4977804.7
## Nationality_Israel	2.97329976	456397591.0
## Nationality_Italy	-1.72904525	3175882984.7
## Nationality_Ivory.Coast	0.48972540	974072782.3
## Nationality_Jamaica	0.09649999	273528502.5
## Nationality_Japan	5.12682932	2214944113.6
## Nationality_Jordan	0.00000000	515350.9
## Nationality_Kazakhstan	0.95631527	6358305.6
## Nationality_Kenya	0.81867546	11879018.2
## Nationality_Korea.DPR	-1.10124169	4437200.4
## Nationality_Korea.Republic	6.50918812	2078233854.1
## Nationality_Kosovo	-1.07474241	45548608.4
## Nationality_Kuwait	0.00000000	0.0
## Nationality_Latvia	1.24598140	5962007.5
## Nationality_Lebanon	0.00000000	563305.6
## Nationality_Libya	0.00000000	24720952.3
## Nationality_Liechtenstein	-0.22585378	3323249.8
## Nationality_Lithuania	0.28347215	22941869.9
## Nationality_Luxembourg	-1.03302560	4165749.8
## Nationality_Madagascar	-2.21029283	252257262.9
## Nationality_Mali	-1.06270793	212709872.3
## Nationality_Mauritania	0.00000000	2017292.1
## Nationality_Mauritius	0.00000000	998823.9
## Nationality_Mexico	4.19557220	1400165613.4
## Nationality_Moldova	1.75994765	28779486.4
## Nationality_Montenegro	0.84778478	39773476.4
## Nationality_Montserrat	0.00000000	0.0
## Nationality_Morocco	1.00820856	494905783.1
## Nationality_Mozambique	1.01706895	56220380.1
## Nationality_Namibia	0.00000000	4112796.0
## Nationality_Netherlands	2.52000029	1737186725.0
## Nationality_New.Caledonia	0.00000000	1623255.2
## Nationality_New.Zealand	1.14382020	81301886.6
## Nationality_Nicaragua	0.00000000	2404996.5
## Nationality_Niger	1.24604735	1844525.1
## Nationality_Nigeria	0.73014389	456956111.7
## Nationality_Northern.Ireland	0.61622067	531602761.8
## Nationality_Norway	5.08606035	1556371904.8
## Nationality_Oman	0.00000000	7384648.9
## Nationality_Palestine	0.00000000	0.0
## Nationality_Panama	-0.38271957	31137643.0
## Nationality_Paraguay	-0.43203964	809912694.2
## Nationality_Peru	1.01092809	230432607.0
## Nationality_Philippines	0.00000000	0.0

## Nationality_Poland	0.76203322	1193507728.3
## Nationality_Portugal	1.58671633	3275814146.5
## Nationality_Puerto.Rico	0.00000000	0.0
## Nationality_Qatar	0.00000000	981990.6
## Nationality_Republic.of.Ireland	3.54381876	1148482764.1
## Nationality_Romania	-1.51382869	301052291.5
## Nationality_Russia	4.88560490	1103435117.3
## Nationality_Rwanda	0.00000000	756789.3
## Nationality_São.Tomé...Príncipe	0.00000000	8163242.8
## Nationality_Saudi.Arabia	3.15872159	526490803.0
## Nationality_Scotland	1.40225407	1006159049.7
## Nationality_Senegal	-1.41317804	1676312493.9
## Nationality_Serbia	0.71479664	1323273298.0
## Nationality_Sierra.Leone	-1.00100150	5934458.0
## Nationality_Slovakia	0.50436168	199627602.5
## Nationality_Slovenia	1.63216108	1216728586.7
## Nationality_South.Africa	2.47113071	298466113.1
## Nationality_South.Sudan	0.00000000	2807769.1
## Nationality_Spain	-3.86779796	11954561696.2
## Nationality_St.Kitts.Nevis	0.00000000	3752809.9
## Nationality_Sudan	1.12632412	8431421.0
## Nationality_Suriname	1.41577932	6734468.3
## Nationality_Sweden	4.15260873	1831696486.3
## Nationality_Switzerland	1.32749486	1061472722.9
## Nationality_Syria	-2.03033997	167656032.0
## Nationality_Tanzania	-0.51621077	27505871.2
## Nationality_Thailand	0.00000000	2710287.1
## Nationality_Togo	1.56743549	180776152.0
## Nationality_Trinidad...Tobago	1.37884512	11820406.9
## Nationality_Tunisia	1.07416343	85380647.1
## Nationality_Turkey	0.96779346	1110453433.8
## Nationality_Uganda	-1.71587764	971674.1
## Nationality_Ukraine	-0.69032073	889269027.2
## Nationality_United.Arab.Emirates	0.00000000	115200971.6
## Nationality_United.States	4.63515260	1292140879.2
## Nationality_Uruguay	-1.21461895	5787149516.2
## Nationality_Uzbekistan	-2.50140842	17690866.1
## Nationality_Venezuela	-0.86687176	376631572.2
## Nationality_Wales	0.56279480	8314717096.4
## Nationality_Zambia	1.45507106	10444495.1
## Nationality_Zimbabwe	-0.84652380	123865674.7

```
varImpPlot(mod.rf)
```



Overview

Overview of test errors

```
testerror.ls
## [1] 118531610

testerror.lasso
## [1] 117873828

testerror.ridge
## [1] 118420695

testerror.rf
## [1] 171586047

MSE <- c(testerror.ls, testerror.lasso, testerror.ridge, testerror.rf)
names(MSE) <- c("Linear Regression", "Lasso", "Ridge", "Random Forest")
MSE

## Linear Regression      Lasso      Ridge      Random Forest
##      118531610      117873828      118420695      171586047

which.min(MSE)

## Lasso
##      2
```

Conclusion: Lasso performs the best. Lasso tends to select one regressor and not the other.

Since we have perfect multicollinearity among our dummy variables,

Lasso performs the best by setting perfectly correlated variables to zero.

1b.

Linear Regression

The best 10 covariates from linear regression are: Value, International.Reputation, Age, Nationality_Dominican.Republic, Position_LS, Position_CAM, Skill.Moves, Position_LF, Position_RF, Body.Type_Other.

Lasso

The best 10 covariates from Lasso are: Body.Type_Messi, Body.Type_C..Ronaldo, Body.Type_Neymar, Body.Type_Shaqiri, Body.Type_Courtois, Nationality_Dominican.Republic, Body.Type_Other, Nationality_Central.African.Rep. Position_LF, Nationality_Russia

Ridge

The best 10 covariates from Ridge are: Body.Type_Messi, Body.Type_C..Ronaldo, Body.Type_Neymar, Body.Type_Shaqiri, Body.Type_Courtois, Nationality_Dominican.Republic, Body.Type_Other, Nationality_Central.African.Rep. Position_LF, Nationality_Russia

Random Forest

The best 10 covariates from random forest are: Age, Overall, Potential, Value, International.Reputation, Weak.Foot, Skill.Moves, Work.Rate, Height, Weight.

1c.

Messi, Ronaldo and Neymar are the represent players. They have the highest coefficients in the Lasso estimation meaning that they have the highest importance in explaining the wages.

QUESTION 02:

$$a) \min_{\beta} \sum_{i=1}^N (y_i - \beta_i)^2 + \lambda \sum_{i=3}^N [(\beta_i - \beta_{i-1}) - (\beta_{i-1} - \beta_{i-2})]^2$$

$$= \min_{\beta} \sum_{i=1}^N (y_i - \beta_i)^2 + \lambda \sum_{i=3}^N [\beta_i - 2\beta_{i-1} + \beta_{i-2}]^2$$

$$b) y = (y_1, \dots, y_n)', \beta = (\beta_1, \dots, \beta_n)'$$

$$\min_{\beta} (y - \beta)'(y - \beta) + \lambda (Q\beta)'(Q\beta)$$

$$= \min_{\beta} (y' - \beta') (y - \beta) + \lambda \beta' Q' Q \beta$$

$$= \min_{\beta} y'y - \underbrace{y'\beta}_{=} - \underbrace{\beta'y}_{=} + \underbrace{\beta'\beta}_{=} + \lambda \beta' Q' Q \beta$$

$$= \min_{\beta} \beta' (I + \lambda Q' Q) \beta - 2y'\beta + y'y$$

$$\text{where } Q = \begin{bmatrix} 1 & -2 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & -2 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -2 & 1 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 1 & -2 & 1 \end{bmatrix}$$

c) FOC wrt β :

$$\frac{\partial}{\partial \beta} \left[\beta' (I + \lambda Q' Q) \beta - 2y'\beta + y'y \right] \bigg|_{\beta = \hat{\beta}} = 0$$

$$\Rightarrow (I + \lambda Q' Q) \hat{\beta} - y = 0$$

$$\Rightarrow \hat{\beta} = (I + \lambda Q' Q)^{-1} y$$

It looks similar to the Ridge estimator, which is given by $\hat{\beta}_{\text{Ridge}} = (X'X + \lambda I_p)^{-1} X'y$

$$d) \hat{\beta} = (I + \lambda Q'Q)^{-1} y$$

$$\begin{aligned} \text{Expectation} \\ E[\hat{\beta} | x] &= E[(I + \lambda Q'Q)^{-1} (\beta + \varepsilon) | x] \pm (Q'Q\lambda + I)^{-1} \lambda Q'Q \rho \\ &= E[(I + \lambda Q'Q)^{-1} \beta + (I + \lambda Q'Q)^{-1} \varepsilon | x] \\ &= E[\underbrace{(I + \lambda Q'Q)^{-1} \beta} + \underbrace{(I + \lambda Q'Q)^{-1} \lambda Q'Q \rho} \\ &\quad + (I + \lambda Q'Q)^{-1} \varepsilon - (I + \lambda Q'Q)^{-1} \lambda Q'Q \rho | x] \end{aligned}$$

$$\begin{aligned}
 &= E[(I + \lambda Q'Q)^{-1}(\beta + \lambda Q'Q\beta) + (I + \lambda Q'Q)^{-1}(\varepsilon + \lambda Q'Q\varepsilon) | x] \\
 &= E[(I + \lambda Q'Q)^{-1}(I + \lambda Q'Q)\beta + \underbrace{(I + \lambda Q'Q)^{-1}(\varepsilon + \lambda Q'Q\varepsilon)}_{w(\lambda)} | x] \\
 &= \beta + w(\lambda) E(\varepsilon | x) + w(\lambda) \lambda Q'Q\beta = \beta + w(\lambda) \lambda Q'Q\beta
 \end{aligned}$$

Variance

$$\begin{aligned}\hat{\beta} &= (I + \lambda Q'Q)^{-1} y = W(\lambda) y & \text{Var}(\beta + \varepsilon) &= I \\ \text{Var}(\hat{\beta} | x) &= \text{Var}(W(\lambda) y | x) = W'(\lambda) \overset{II}{\text{Var}(y | x)} W(\lambda) \\ &= W(\lambda)' W(\lambda) = W(\lambda) W(\lambda)\end{aligned}$$

e) If $\lambda = 0$, the expectation & variance will be as if we didn't have penalty.

$$\lambda = 0 : E(\hat{\beta} | x) = \beta$$

QUESTION 03:

The objective function for lasso for a linear regression model:

$$\hat{\beta}_{lasso} = \arg \min_{\beta} \left\| y - \sum_{j=1}^p x_j \beta_j \right\|^2 + \lambda \sum_{j=1}^p |\beta_j|$$

The objective function for adaptive lasso for a linear regression model:

$$\hat{\beta}_{a.lasso} = \arg \min_{\beta} \left\| y - \sum_{j=1}^p x_j \beta_j \right\|^2 + \lambda \sum_{j=1}^p w_j |\beta_j|$$

where $w_j = \frac{1}{|\beta_j|^\gamma}$, $\gamma \in [1,2]$ is a known weights vector.

A researcher would prefer to use adaptive lasso when the lasso is inconsistent for variable selection. The adaptive lasso where adaptive weights w_j are introduced to penalize different coefficients in the ℓ_1 penalty (i.e., the second term in the objective function for lasso) enjoys the oracle properties, which means that it performs as well as if the true underlying model was given in advance. Nevertheless, the adaptive lasso entails a finite sample bias in the non-zero coefficient estimates. As sample size grows, the data-dependent weights for zero-coefficient predictors get inflated (to infinity), whereas the weights for nonzero coefficient predictors converge to a finite constant. The adaptive lasso should therefore be adopted so the researcher could select the true zeros only, unbiasedly (asymptotically) re-estimate large coefficients and small threshold estimates in finite samples.