Project Stage 1

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November 2022

1 Part A

1.1 Assumptions

1. A coal factory in one region can service itself and/or other regions through utilized lines on condition that it is decided to build utilized lines between the “supplier” region and the “receiver” region.

2. If a region does not have a coal factory, then it can be either serviced by one or more coal factories in different regions or its own solar-energy panels.

3. If a region has a coal factory, then it can’t get serviced from different coal factories at other regions or use solar energy panels.

4. Ideally, a region using coal energy from other regions close to it should get its demand met by as few coal factories as possible.

5. The company wants to minimize the cost of meeting the energy demands of each region.

6. The company can use either solar or coal energy at a region to meet its energy demand.

7. The company prefers using coal energy over solar energy therefore it is assumed that the total number of regions satisfying their demand by coal energy should be greater than the total number of regions using solar panels.

8. A region with solar energy panels can’t service regions other than itself

9. The total cost due to a coal factory built at a certain region is the sum of building the factory and the cost of building utilized lines to other regions if any utilized line from the region with coal factory to another receiver region is chosen to be built, and the cost of overcapacity work if the factory is decided to be working on overcapacity.

10. The cost due to a solar panel is the construction cost plus the maintenance cost proportional to the maximum energy the solar panel at a certain region produces.

11. The amount of energy produced by each coal factory is limited, and the maximum production capacity of each coal factory is the same.

12. Each coal factory does not have to produce the same amount of energy.

13. Overcapacity energy production in coal factories is possible but subject to a cost.

14. If overcapacity in a coal factory is enabled, the amount of energy produced increases by a certain percentage determined by our parameter v_c.

15. If the second and fifth districts both have coal factories and utilize the overcapacity option, then neither of the third and fourth districts utilizes the overcapacity function if they have coal factories.

16. If the first district has a coal factory, then it should either supply both the second and third districts a similar amount of energy (parameter) or not supply energy to them at all.

17. If the districts are "close" (as defined in the 7th parameter "d") to each other, then they should be using the same type of energy.

1.2 English Description of Constraints

1. In total all of the k regions should have their energy demand satisfied. (see (1))

2. If a district has solar panels, it can only use the solar panels to satisfy its own demand. (see (2))

3. A region can only use one type of energy source. (see (3))

4. If Region 2 and Region 5 go overcapacity Region 3 and 4 can't go overcapacity. (see (4))

5. If a Region i uses coal energy then it either has a coal factory or gets supplied by another region. (see (5))

6. If a region i is "close" to another region j then the distance between region i and j should be less than or equal to the the maximum distance "d" in kilometers between two regions before they are no longer considered "close" to each other. (see (6))

7. If a region i is close to another region j then the type of energy they use should be the same. (see (7))

8. The total number of regions satisfying their demand by coal energy should be greater than the total number of regions using solar panels. (see (8))
9. If a coal factory does not use overcapacity then it's energy production can not be more than the maximum production capacity of all coal factories M. ($M \epsilon \mathcal{R}$) (see (9))
10. The amount of energy supplied to the second and third districts from the first district is similar therefore less than s. (see (10))
11. If a coal factory uses overcapacity then the amount of energy production at that coal factory is the product of maximum production capacity of all coal factories (M) and the percentage of production increase in case of overcapacity usage (v_c). (see (11))
12. A Region i gets its energy demand satisfied in three cases: either it has a coal factory that supplies all its demand, or it uses coal energy from one or more other coal factories that supplies it, or it uses solar energy panels. (see (12))
13. If a region is close to one or more regions with coal factories then it should get its energy demand met by getting coal energy supplies from those factories. (see (13))
14. If the cost of building a solar panel at Region i is less than or equal to building the utility lines between Region i and all the nearby districts with coal factories j, then Region i should use solar energy. (see (14))
15. If the amount of regions with solar panels close to a district is greater than or equal to the amount of regions that use coal energy then that region should use coal energy. (see (15))
16. If a coal factory supplies other regions than itself, then the amount of energy it supplies in addition to its own demand has to be less than the maximum amount of energy it can produce with or without overcapacity used. (see (16))

1.3 Why this choice of variables:

1. We need to know the amount of production of coal factories and solar energy panels (CFP's and SP's) because it is needed to ensure that the demands get met and that we do not exceed the maximum amount energy coal factories can produce. Also SP's are necessary to calculate the cost of building solar panels at a region so it is also in the objective function.
2. We need to know if a region has a coal factory (CF's) to be able to differentiate between whether a region using coal energy produces it itself or gets it from other regions. It is also in the objective function because building a coal factory comes with a construction cost.

3. We need to know if a coal factory at a region works at overcapacity (O's) to determine its maximum production amount and, it is also counted in the objective function because it affects the total cost.

4. We need to know whether a region gets its demand met (S's) to ensure that no region is left without its energy demand satisfied.

5. We need to know what type of energy a region uses (X's and Y's) to make sure that each region only utilizes one type of energy resource.

6. We need to know if a coal factory supplies another region (CS's) to be able to calculate the cost of constructing utility lines between those regions which is in our objective function. We also need it to ensure that a region using coal energy satisfies its demand by either a coal factory at that region or from coal factories in other regions.

7. We need to know the amount of energy a region gets from coal factories at different districts (CD's) is necessary to make sure that it gets it energy demand satisfied.

1.4 Parameters:

1. C_o = Cost of enabling overcapacity at a coal factory. ($C_o \epsilon \mathcal{R}$)
2. C_B = Cost of building a coal factory. ($C_B \epsilon \mathcal{R}$)
3. C_P = Cost per kilometer of building a utility line between a coal factory and the region it supplies. ($C_P \epsilon \mathcal{R}$)
4. v_S = The proportion of the production for cost when a solar factory is built. ($v_S \epsilon \mathcal{R}$)
5. M = The maximum production capacity of all coal factories. ($M \epsilon \mathcal{R}$)
6. v_C = The percentage of the production increase in case of overcapacity usage at a coal factory. ($v_C \epsilon \mathcal{R}$)
7. d = The maximum distance in kilometers between two regions before they are no longer considered "close" to each other. ($d \epsilon \mathcal{Z}$)
8. s = The maximum amount of energy between two regions that can be considered a similar amount. ($s \epsilon \mathcal{R}$)
9. $$N_{ij} = \begin{cases} 1, & \text{if a Region i is close to a region j (i=1,\ldots k) and (j = 1,...k)} \\ 0, & \text{otherwise} \end{cases} \tag{1}$$
10. W_{ij} = The distance between Region i and j. ($W_{ij} \epsilon \mathcal{R}$) (i=1,... k) and (j = 1,...k)
11. D_i = Energy demand of each Region i. ($D_i \epsilon \mathcal{R}$)(i = 1,...k)

1.5 Decision Variables:

1.

$$\text{CFP}_i = \text{The amount of production of a coal factory at Region i. (i = 1,...k)} \quad (2)$$

2.

$$SP_i = \text{The amount of energy production of a solar panel at Region i.(i = 1,...k)} \quad (3)$$

3.

$$CF_i = \begin{cases} 1, & \text{if Region i has a coal factory (i = 1,...k)} \\ 0, & \text{otherwise} \end{cases} \quad (4)$$

4.

$$O_i = \begin{cases} 1, & \text{if coal factory at Region i works at overcapacity (i=1,..k)} \\ 0, & \text{otherwise} \end{cases} \quad (5)$$

5.

$$S_j = \begin{cases} 1, & \text{if a Region j gets its demand satisfied. (j = 1,...k)} \\ 0, & \text{otherwise} \end{cases} \quad (6)$$

6.

$$X_j = \begin{cases} 1, & \text{if a Region j uses coal energy. (j = 1,...k)} \\ 0, & \text{otherwise} \end{cases} \quad (7)$$

7.

$$Y_j = \begin{cases} 1, & \text{if a Region j uses solar panel energy. (j = 1,...k)} \\ 0, & \text{otherwise} \end{cases} \quad (8)$$

8.

$$CS_{ij} = \begin{cases} 1, & \text{if a Coal Factory at Region i Supplies Region j. (i=1,...k) and (j = 1,...k)} \\ 0, & \text{otherwise} \end{cases} \quad (9)$$

9.

$$CD_{ij} = \text{the amount of energy Region j gets from a coal factory at Region(i=1,...k) and (j = 1,...k)} \quad (10)$$

1.6 Model:

- minimize $\sum_{i=1}^{k}[(SP_i * v_s) + (Y_i * C_s) + (CF_i * C_B) + (C_P * CS_{ij} * W_{ij}) + (O_i * C_O)]$

- subject to:
  $\sum_{j=1}^{k} S_j = k$ (j = 1,...k)

  $Y_i - SP_i + D_i \geq 1$ i=1,. . . k)

  $X_j + Y_j \geq 1$ (j = 1,...k)

  $O_3 \leq 2 - O_2 - O_5$

  $O_4 \leq 2 - O_2 - O_5$

  $CD_{13} - CD_{12} \leq s$

  $CD_{13} - CD_{12} - s$

  $(\sum_{j=1}^{k} CD_{ij}) + D_i \leq 0$ (i=1,. . . k) (j = 1,...k)

  $CS_{ij} \geq N_{ij}$ (i=1,. . . k) (j = 1,...k)

  $\sum_{i=1}^{k}(D_j = N_i j * Dj)$ (i=1,. . . k) (j = 1,...k)

  $1 - X_i \leq zandCF_i + \sum_{j=1}^{k} CS_{ji} - 2 \leq (1 - z)z\epsilon\{0,1\}$ (i=1,. . . k) (j = 1,...k)

  $ifO_i = 1thenCFP_i = M * v_c$

  $CF_i\epsilon\{0,1\}$ (i=1,. . . k)

  $O_i\epsilon\{0,1\}$ (i=1,. . . k)

  $S_j\epsilon\{0,1\}$ (j = 1,...k)

  $X_j\epsilon\{0,1\}$ (j = 1,...k)

  $Y_j\epsilon\{0,1\}$ (j = 1,...k)

  $CS_{ij}\epsilon\{0,1\}$ (i=1,. . . k) (j = 1,...k)

2 Part B:

We would need to introduce a new parameter Z = the maximum amount of solar panels we can build.
Then we would add a new constraint, $\sum_{j=1}^{k} Y_j \leq Z$.
The objective function would remain the same however, the total cost of the entire operation would go up in the end because the installation of solar panels is when it is cheaper to build than a coal factory or utility lines to satisfy the demand. Therefore, in some regions we would've had to choose the more expensive option because of the constraint on the number of solar panels we can build.