IE202 Group 23 Stage 3

Group 24
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1 Problem:

1.1 English Description of Constraints:

1. The total amount of fuel bought from the three companies should exceed the total amount of fuel that will be used by each car in each team during every race in a season.
2. The sum of double the amount of fuel bought from company 1, triple the amount from company 2 and five times the amount from company 3 should not exceed four million gallons.
3. The proportions of amount of fuel bought from company 1, 2 and 3 should be 3, 5 and it should be more than 1000 gallons per race. The total number of races in a season is 20.
4. The sum of the labor capacity for all companies should not exceed 20000 minutes.
5. The amount of fuel bought from a company has to be non-negative.

1.2 Why this choice of variables?

The only variable we need is the amount of fuel bought from each company since the rest of the data is fixed and are not variables.

1.3 Decision Variables:

X_i = The amount of fuel bought from company i, in gallons. i = {1,2,3}

1.4 Parameters:

1. cars = number of cars in a team $cars \epsilon \mathcal{R}$
2. teams = number of teams $teams \epsilon \mathcal{R}$
3. races = number of races in a season $races \epsilon \mathcal{R}$
4. labor = labor capacity in minutes $labor \epsilon \mathcal{R}$
5. $price_i$: fuel price per gallon of company i i = {1, 2, 3} $price_i \epsilon \mathcal{R}$

1.5 Data:

1. cars = 2
2. teams = 10
3. races = 20
4. labor = 20000
5. $price_1 = \$2$
6. $price_2 = \$1,5$
7. $price_3 = \$3$

1.6 Model:

- $$minimize 2 * X_1 + 1.5 * X_2 + 3 * X_3$$
- subject to:

$$X_1 + X_2 + X_3 \geq cars * teams * races * 20$$

$$2X_1 + 3X_2 + 5X_3 \leq 4000000$$

$$3X_1 + 5X_2 + 2X_3 \geq 1000 * races$$

$$2X_1 + 7X_2 + 9X_3 \leq 20000$$

$$X_1 \geq 0$$
$$X_2 \geq 0$$
$$X_3 \geq 0$$

2 Excel Report and Sensitivity Analysis

The excel sheet is as follows:

Decision Variables		Parameters				
"X_1="	7200	"cars="	2,00			
"X_2="	800	"teams="	10,00			
"X_3="	0	"races="	20,00			
		"labor="	20000,00			
		"price_1="	2,00			
		"price_2="	1,50			
		"price_3="	3,00			
		"max="	4000000,00			
Constraints						
"X_1 + X_2 + X_3"	8000	"≥"	"cars*teams*races*20"		≥ 8000	
"2*X_1 + 3*X_2 + 5*X_3"	16800	"≤"	"max"		≤ 4000000,00	
"3*X_1 + 5*X_2 + 2*X_3"	25600	"≥"	"1000*races"		≥ 20000	
"2*X_1 + 7*X_2 + 9*X_3"	20000	"≤"	"labor"		≤ 20000,00	
"X_1"	7200	"≥"	0,00		≥ 0	
"X_2"	800	"≥"	0,00		≥ 0	
"X_3"	0	"≥"	0,00		≥ 0	
"minimize"	"price_1 * X_1 + price_2 * X_2 + price_3 * X_3"	min 15600				

Figure 1: Excel Sheet

2.1 What we can interpret from the results?

- The objective function value is 15600.
- The first constraint X_1: how many gallons of fuel is bought from Company 1 is 7200 gallons.
- The first constraint X_2: how many gallons of fuel is bought from Company 2 is 800 gallons.
- The first constraint X_3: how many gallons of fuel is bought from Company 3 is 0 gallons.
- The parameter values from D2:D9 are fixed values that we determine.
- We relate the constraint values to each other during the modeling stage in the Open Solver.

The sensitivity analysis is as follows:

OpenSolver Sensitivity Report - CBC
Worksheet: [model irem copy.xlsx] Sayfa2 Sensitivity
Report Created: 21.12.2022 23:43:27

Decision Variables

Cells	Name	Final Value	Reduced Costs	Objective Value	Allowable Increase	Allowable Decrease
B2	"X_1="	7200	0	2	1E+100	0,500000056
B3	"X_2="	800	0	1,5	0,500000056	1E+100
B4	"X_3="	0	1,7	3	1E+100	1,7

Constraints

Cells	Name	Final Value	Shadow Price	RHS Value	Allowable Increase	Allowable Decrease
B14>=F14	"X_1 + X_2 + X_3"	8000	2,2	8000	2000	2545,454545
B15<=F15	"2*X_1 + 3*X_2 + 5*X_3"	16800	0	4000000	1E+100	3983200
B16>=F16	"3*X_1 + 5*X_2 + 2*X_3"	25600	0	20000	5600	1E+100
B18>=F18	"X_1"	7200	0	0	7200	1E+100
B19>=F19	"X_2"	800	0	0	800	1E+100
B20>=F20	"X_3"	0	0	0	0	1E+100
B17<=F17	"2*X_1 + 7*X_2 + 9*X_3"	20000	-0,1	20000	36000	4000

Figure 2: Sensitivity Analysis

2.2 Sensitivity Analysis of the Excel Report

2.2.1 For the Decision Variables:

Final Value:

- Final value for decision variables is the value of each decision variable in order to obtain the solution.
- Final value of X_1 is 7200.
- Final value of X_2 is 800.
- Final value of X_3 is 0.

Reduced Costs:

- Reduced cost for decision variable X_i is the amount by which the objective function will change if variable X_i is increased by 1 unit, it will decrease for maximization (hence it is negative) and increase for minimization (hence it is positive).
- It will decrease for maximization (hence it is negative) increase for minimization (hence it is positive).
- X_1 has reduced cost 0.
- X_2 has reduced cost 0.

- X_3 has reduced cost 1.7.

Objective Value:

- It is the coefficients of the decision variables in the objective function.
- X_1 has objective coefficient 2.
- X_2 has objective coefficient 1,5.
- X_3 has objective coefficient 3.

Allowable Increase:

- It is how much we can increase the final value of a variable without leaving the optimal basis and keeping the solution optimal.
- X_1 can be increased as much as we want without losing optimality.
- X_2 can be increased as much as 0.5 units and if it is increased more than that the solution will no longer be optimal.
- X_3 can be increased as much as we want without losing optimality.

Allowable Decrease:

- It is how much we can decrease the final value of a variable without leaving the optimal basis and keeping the solution optimal.
- X_1 can be decreased as much as 0.5 units and if it is decreased more than that the solution will no longer be optimal.
- X_2 can be decreased as much as we want without losing optimality.
- X_3 can be decreased as much as 1,7 units and if it is decreased more than that the solution will no longer be optimal.

2.2.2 For the Constraints:

Final Value:

- Final value is the value of the left-hand side when you plug in the final values for the variables.
- For the first constraint, the final value is 8000.
- For the second constraint, the final value is 16800.
- For the third constraint, the final value is 25600.
- For the fourth constraint, the final value is 7200.
- For the fifth constraint, the final value is 800.
- For the sixth constraint, the final value is 0.
- For the seventh constraint, the final value is 20000.
- Notice how the fourth constraint gives the final value of variable X_1 , the fifth constraint gives the final value of variable X_2 , the sixth constraint gives the final value of variable X_3 .

Shadow Price:

- Shadow prices are the Row 0 coefficients of the slack variables in Simplex Tableau.
- Shadow (dual) price gives the improvement in the optimal value if the rhs of the constraint is increased by 1 unit within the allowable range.
- When increasing the RHS value of a constraint, choosing the highest shadow price will increase the objective function value the most. The amount paid would be (the amount of increase)*(shadow price).
- If the shadow price of a constraint is zero, changes to the value of the constraint won't change the optimal value. The optimal solution will change to $B^{-1}\bar{b}$.

- In our model if we increase the right hand side value of the first constraint by 1, the objective function value will change to 15602,2.
- As long as the increase or decrease are within the bounds of the available increase and decrease the solution will remain optimal.
- Changing the RHS value of the 2nd, 3rd, 4th, 5th and 6th constraints won't change the optimal value as their shadow prices are equal to 0.
- Changing the RHS value of the 7th constraint will increase or decrease the optimal value by the (amount changed)*(-0,1).

RHS Value:

- The constraint R.H. side is the right hand side of the constraint which is, what value for each constraint was given in the beginning of the problem.

Allowable Increase:

- It is how much we can increase the RHS value of the constraint without leaving the optimal basis and keeping the solution optimal.
- First constraint can be increased as much as 2000 units and if it is increased more than that the solution will no longer be optimal.
- Second constraint can be increased as much as we want without losing optimality.
- Third constraint can be increased by as much as 5600 units and if it is increased more than that the solution will no longer be optimal.
- Fourth constraint can be increased as much as 7200 units and if it is increased more than that the solution will no longer be optimal.
- Fifth constraint can be increased as much as 800 units and if it is increased more than that the solution will no longer be optimal.
- Sixth constraint cannot be increased at all without losing optimality.
- Seventh constraint can be increased as much as 36000 units and if it is increased more than that the solution will no longer be optimal.

Allowable Decrease:

- It is how much we can decrease the RHS value of the constraint without leaving the optimal basis and keeping the solution optimal.
- First constraint can be decreased as much as 2545,45 units and if it is decreased more than that the solution will no longer be optimal.
- Second constraint can be decreased 3983200 units and if it is decreased more than that the solution will no longer be optimal.
- Third constraint can be decreased as much as we want without losing optimality.
- Fourth constraint can be decreased as much as we want without losing optimality.
- Fifth constraint can be decreased as much as we want without losing optimality.
- Sixth constraint can be decreased as much as we want without losing optimality.
- Seventh constraint can be decreased as much as 4000 units and if it is decreased more than that the solution will no longer be optimal.

The Excel Solver pop-up page is as follows:

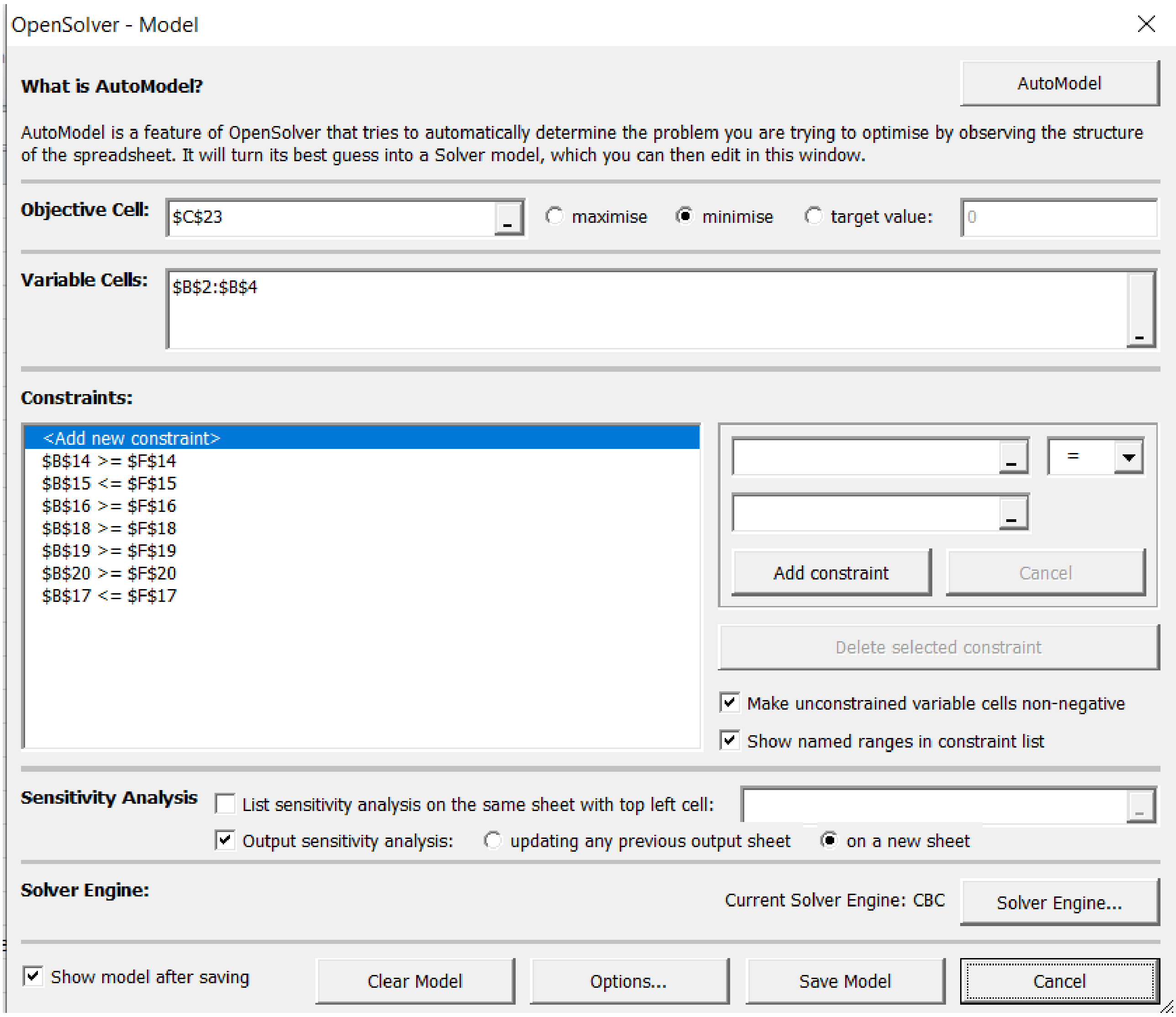

Figure 3: Model Pop-up Page

3 Gurobi Solution:

We see from the results that they are the same as the data we have obtained from doing the Excel report.

```
In [45]: print(x1)

         <gurobi.Var x1 (value 7200.0)>

In [46]: print(x2)

         <gurobi.Var x2 (value 800.0)>

In [47]: print(x3)

         <gurobi.Var x3 (value -0.0)>

In [49]: m.getObjective().getValue()

Out[49]: 15600.0
```

Figure 4: Model Pop-up Page

In [41]:
```
m.optimize()
```

```
Gurobi Optimizer version 10.0.0 build v10.0.0rc2 (win64)

CPU model: Intel(R) Core(TM) i5-3470 CPU @ 3.20GHz, instruction set [SSE2|AVX]
Thread count: 4 physical cores, 4 logical processors, using up to 4 threads

Optimize a model with 21 rows, 3 columns and 45 nonzeros
Model fingerprint: 0x9cf593dc
Variable types: 0 continuous, 3 integer (0 binary)
Coefficient statistics:
  Matrix range     [1e+00, 9e+00]
  Objective range  [2e+00, 3e+00]
  Bounds range     [0e+00, 0e+00]
  RHS range        [8e+03, 4e+06]

MIP start from previous solve did not produce a new incumbent solution
MIP start from previous solve violates constraint R10 by 36000.000000000

Presolve removed 19 rows and 0 columns
Presolve time: 0.00s
Presolved: 2 rows, 3 columns, 5 nonzeros
Variable types: 0 continuous, 3 integer (0 binary)
Found heuristic solution: objective 15600.000000

Root relaxation: cutoff, 2 iterations, 0.05 seconds (0.00 work units)

    Nodes    |    Current Node    |     Objective Bounds      |     Work
 Expl Unexpl |  Obj  Depth IntInf | Incumbent    BestBd   Gap | It/Node Time

     0     0     cutoff    0      15600.0000 15600.0000  0.00%     -    0s

Explored 1 nodes (2 simplex iterations) in 0.32 seconds (0.00 work units)
Thread count was 4 (of 4 available processors)

Solution count 1: 15600

Optimal solution found (tolerance 1.00e-04)
Best objective 1.560000000000e+04, best bound 1.560000000000e+04, gap 0.0000%
```

Figure 5: Model Pop-up Page