# CSC343 A3 Part2

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# 1 Part 2

## 1.1 (a)

BCNF requires that the LHS of an FD be a superkey.
$AC^+ = ABCDEFG$ . Therefore $AC \rightarrow D$ does not violate BCNF.
$BG^+ = BEFG$ Since $BG$ is not a super key so $BG \rightarrow E$ violates BCNF.
$D^+ = BCDEFG$ Since $D$ is not a super key so $D \rightarrow CFG$ violates BCNF.
$DG^+ = BCDEFG$ So $DG$ is not a super key therefore $DG \rightarrow B$ violates BCNF.
$G^+ = FG$ Since $G$ is not a super key so $G \rightarrow F$ violates BCNF.
FDs violate BCNF: $\{BG \rightarrow E,\ D \rightarrow CFG,\ DG \rightarrow B,\ G \rightarrow F\ \}$

## 1.2 (b)

We start our decomposition from $D \rightarrow CFG$:

R1: $AD$
R2: $BCDEFG$
Shared Part: $D$
Project FDs onto R1 and R2:

| Table $R1$:$AD$ | |
|---|---|
| Closure | FD |
| $A^+ = A$ | |
| $D^+ = D$ | |
| $AD^+ = AD$ | |

| Table R2: $BCDEFG$ | |
|---|---|
| Closure | FD in R2 |
| $B^+ = B$ | |
| $C^+ = C$ | |
| $D^+ = BCDEFG$ | $D \rightarrow BCDEFG$ |
| $E^+ = E$ | |
| $F^+ = F$ | |
| $G^+ = FG$ | $G \rightarrow F$ (violates BCNF, stop the process) |

Decompose R2 on $G \to F$:

R3: $FG$
R4: $BCDEG$
Shared Part: $G$

| Table R3:$FG$ | |
|---|---|
| Closure | FD |
| $F^+ = F$ | |
| $G^+ = FG$ | $G \to F$ (G is a superkey in R3) |

| Table R4: $BCDEG$ | |
|---|---|
| Closure | FD |
| $B^+ = B$ | |
| $C^+ = C$ | |
| $D^+ = BCDEFG$ | $D \to BCEG$ |
| $E^+ = E$ | |
| $G^+ = FG$ | |
| $BG^+ = BEFG$ | $BG \to E$ (BG not a superkey in R4, stop) |

Decompose R4 on $BG \to E$:

$R5$: $BEG$
$R6$: $BCDG$
Shared Part: $BG$

| Table $R5$:$BEG$ | |
|---|---|
| Closure | FD |
| $B^+ = B$ | |
| $E^+ = E$ | |
| $G^+ = FG$ | |
| $BE^+ = BE$ | |
| $EG^+ = EFG$ | |
| $BG^+ = BGEF$ | $BG \to E$ |

| Table $R6$:$BCDG$ | |
|---|---|
| Closure | FD |
| $B^+ = B$ | |
| $G^+ = FG$ | |
| $C^+ = C$ | |
| $D^+ = BCDEFG$ | $D \to BCG$ (D is a superkey in R6, so every combination with D will satisfy BCNF) |
| $BC^+ = BC$ | |
| $BG^+ = BEFG$ | |
| $CG^+ = CFG$ | |
| $BCG^+ = BCGEF$ | |

Therefore, the final decomposition is : $AD$ with no FDs , $BCDG$ $\{D \to BCG\}$, $BEG$ $\{BG \to E\}$, $FG$ $\{G \to F\}$,

### 1.3 (c)

No, it is not. The FDs in the final decomposition ensure that $BG \to E$ and $G \to F$ are preserved. However, at least one FD is not: $AC \to D$. Here is an counter example:

| A | D |
|---|---|
| 1 | 3 |
| 1 | 4 |

| B | E | G |
|---|---|---|
| $b_1$ | $e_1$ | $g_1$ |
| b | e | g |

| B | C | D | G |
|---|---|---|---|
| $b_1$ | 2 | 3 | $g_1$ |
| b | 2 | 4 | g |

| F | G |
|---|---|
| $f_1$ | $g_1$ |
| f | g |

Rejoining these tables together, we violates the original FD $AC \to D$:

| A | B | C | D | E | F | G |
|---|---|---|---|---|---|---|
| 1 | $b_1$ | 2 | 3 | $e_1$ | $f_1$ | $g_1$ |
| 1 | $b$ | 2 | 4 | $e$ | $f$ | $g$ |

### 1.4 (d)

Set up $t = abcdefg$
Initial Tableau:

| Initial Tableau | | | | | | |
|---|---|---|---|---|---|---|
| A | B | C | D | E | F | G |
| $a$ | $b_1$ | $c_1$ | $d$ | $e_1$ | $f_1$ | $g_1$ |
| $a_2$ | $b$ | $c_2$ | $d_2$ | $e$ | $f_2$ | $g$ |
| $a_3$ | $b$ | $c$ | $d$ | $e_3$ | $f_3$ | $g$ |
| $a_4$ | $b_4$ | $c_4$ | $d_4$ | $e_4$ | $f$ | $g$ |

According to FD $AC \to D$, nothing is changed, then proceed $BG \to E$:

| $BG \to E$ | | | | | | |
|---|---|---|---|---|---|---|
| A | B | C | D | E | F | G |
| $a$ | $b_1$ | $c_1$ | $d$ | $e_1$ | $f_1$ | $g_1$ |
| $a_2$ | $b$ | $c_2$ | $d_2$ | $e$ | $f_2$ | $g$ |
| $a_3$ | $b$ | $c$ | $d$ | $e$ | $f_3$ | $g$ |
| $a_4$ | $b_4$ | $c_4$ | $d_4$ | $e_4$ | $f$ | $g$ |

According to FD $D \to CFG$, we change the table to:

| $D \to CFG$ | | | | | | |
|---|---|---|---|---|---|---|
| A | B | C | D | E | F | G |
| $a$ | $b_1$ | $c$ | $d$ | $e_1$ | $f_1$ | $g$ |
| $a_2$ | $b$ | $c_2$ | $d_2$ | $e$ | $f_2$ | $g$ |
| $a_3$ | $b$ | $c$ | $d$ | $e$ | $f_1$ | $g$ |
| $a_4$ | $b_4$ | $c_4$ | $d_4$ | $e_4$ | $f$ | $g$ |

According to FD $DG \rightarrow B$, we change the table to:

| $DG \rightarrow B$ | | | | | | |
|---|---|---|---|---|---|---|
| A | B | C | D | E | F | G |
| $a$ | $b$ | $c$ | $d$ | $e_1$ | $f_1$ | $g$ |
| $a_2$ | $b$ | $c_2$ | $d_2$ | $e$ | $f_2$ | $g$ |
| $a_3$ | $b$ | $c$ | $d$ | $e$ | $f_1$ | $g$ |
| $a_4$ | $b_4$ | $c_4$ | $d_4$ | $e_4$ | $f$ | $g$ |

According to FD $G \rightarrow F$, we change the table to:

| $G \rightarrow F$ | | | | | | |
|---|---|---|---|---|---|---|
| A | B | C | D | E | F | G |
| $a$ | $b$ | $c$ | $d$ | $e_1$ | $f$ | $g$ |
| $a_2$ | $b$ | $c_2$ | $d_2$ | $e$ | $f$ | $g$ |
| $a_3$ | $b$ | $c$ | $d$ | $e$ | $f$ | $g$ |
| $a_4$ | $b_4$ | $c_4$ | $d_4$ | $e_4$ | $f$ | $g$ |

Since no completely unsubscribed row ($= t$) appears, proceed FDs again. According to $AC \rightarrow D$ nothing changed, then proceed $BG \rightarrow E$, we changed the table to:

| $BG \rightarrow E$ | | | | | | |
|---|---|---|---|---|---|---|
| A | B | C | D | E | F | G |
| $a$ | $b$ | $c$ | $d$ | $e$ | $f$ | $g$ |
| $a_2$ | $b$ | $c_2$ | $d_2$ | $e$ | $f$ | $g$ |
| $a_3$ | $b$ | $c$ | $d$ | $e$ | $f$ | $g$ |
| $a_4$ | $b_4$ | $c_4$ | $d_4$ | $e_4$ | $f$ | $g$ |

Notice that the first row is a completely unsubscribed row ($= t$), which proves that this is a lossless-join decomposition.

## 2 Question2

### 2.1 (a)

**Step1: Split RHS**

1. $N \to M$
2. $NO \to L$
3. $NO \to R$
4. $Q \to M$
5. $Q \to P$
6. $P \to R$
7. $Q \to N$
8. $Q \to O$

**Step2: Try to reduce LHS**

1. no simplification needed since there is only 1 attribute on the LHS.
2. Nothing is changed since $N^+ = NM$ and $O^+ = O$.
3. no simplification needed since there is only 1 attribute on the LHS.
4. Since $Q^+ = QNOLRMP$, we change this to $Q \to M$.
5. Since $Q^+ = QNOLRMP$, we change this to $Q \to P$.
6. no simplification needed since there is only 1 attribute on the LHS.
7.no simplification needed since there is only 1 attribute on the LHS.
8. no simplification needed since there is only 1 attribute on the LHS.

**Step3: Eliminate Entire FDs**

| Table Step3 | | | |
|---|---|---|---|
| FD | Exclude these from S1 when computing closure | Closure | Decision |
| 1 | 1 | $N^+ = N$ | keep |
| 2 | 2 | $NO^+ = NOMR$ | keep |
| 3 | 3 | $NO^+ = NOML$ | keep |
| 4 | 4 | $Q^+ = QNOPLRM$ | discard |
| 5 | 4,5 | $Q^+ = QNOMLR$ | keep |
| 6 | 4,6 | $P^+ = P$ | keep |
| 7 | 4,7 | $Q^+ = QOPR$ | keep |
| 8 | 4,8 | $Q^+ = QMNPR$ | keep |

So we keep 1,2,3,5,6,7,8

The final answer is $\{N \to M,\ NO \to L,\ NO \to R,\ P \to R,\ Q \to O,\ Q \to P,\ Q \to N\}$

### 2.2 (b)

From the minimal basis in part (a), we summarized the occurrence of attributes as follow:

| Attribute | Appears on LHS | Appears on RHS | Conclusion |
|---|---|---|---|
| S | – | – | must be in every key |
| Q | ✓ | – | must be in every key |
| L, M, R | – | ✓ | is not in any key |
| N, O, P | ✓ | ✓ | must check |

The keys for P is { SQ }, since from the summary table, we know that SQ must in every key, before compute the closure of $P + SQ$ or $O + SQ$ or $N + SQ$, the closure of SQ gives $SQ^+ = SQOPNMLR$, therefore, SQ is already the key for this relation, no need to check any set of attributes contains SQ but greater that it.

### 2.3 (c)

Define four relations by 3NF:
R1: NM where NM is not a superkey since $NM^+ = NM$
R2: NOLR where NOLR is not a superkey since $NOLR^+ = NOLRM$
R3: NOPQ where NOPQ is not a superkey since $NOPQ^+ = NOPQLRM$
R4: PR where PR is not a superkey since $PR^+ = PR$
Since none of above 4 relations is superkey for B, we added one more relation that is a key as computed in the previous part:
R5: SQ
Therefore, we get our relations after 3NF synthesis: R1: NM, R2: NOLR, R3: NOPQ, R4: PR, R5: SQ.

### 2.4 (d)

There is no redundancy in this relation that satisfies 3NF, since R1, R4, R5 are 2-attribute table, satisfy BCNF; for R2, and R3, each FD is superkey when being projected on them, so no redundancy in R2 and R3.

| Table $R2$:$NOLR$ | |
|---|---|
| Closure | FDs in R2 |
| $N^+ = NM$<br>$NO^+ = NOLR$<br>$L^+ = L$<br>$R^+ = R$<br>$LR^+ = LR$ | $NO \rightarrow LR$ (NO is the superkey for this relation) |

| Table $R3$:$NOPQ$ | |
|---|---|
| Closure | FDs in R3 |
| $N^+ = NM$<br>$O^+ = O$<br>$P^+ = PR$<br>$Q^+ = QOPN$<br>$NO^+ = NOLR$<br>$NP^+ = NPMR$<br>$OP^+ = OPR$<br>$NOP^+ = NOPLRM$ | $Q \rightarrow NOP$ (Q is the superkey for this relation) |