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5.2.3 Transposing a Product of Matrices

5.2.3 Transposing a Product of Matrices

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Homework 5.2.3.1

66/66 points (graded)

Let $A = \begin{pmatrix} 2 & 0 & 1 \\ -1 & 1 & 0 \\ 1 & 3 & 1 \\ -1 & 1 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & 1 & 2 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{pmatrix}$. Compute

- $A^T A =$

7 ✔ Answer: 7	1 ✔ Answer: 1	2 ✔ Answer: 2
1 ✔ Answer: 1	11 ✔ Answer: 11	4 ✔ Answer: 4
2 ✔ Answer: 2	4 ✔ Answer: 4	3 ✔ Answer: 3

- $AA^T =$

5 ✔ Answer: 5	-2 ✔ Answer: -2	3 ✔ Answer: 3	-1 ✔ Answer: -1
-2 ✔ Answer: -2	2 ✔ Answer: 2	2 ✔ Answer: 2	2 ✔ Answer: 2
3 ✔ Answer: 3	2 ✔ Answer: 2	11 ✔ Answer: 11	3 ✔ Answer: 3
-1 ✔ Answer: -1	2 ✔ Answer: 2	3 ✔ Answer: 3	3 ✔ Answer: 3

- $(AB)^T =$

5	-2	3	-1
Answer: 5	Answer: -2	Answer: 3	Answer: -1
2	0	4	0
Answer: 2	Answer: 0	Answer: 4	Answer: 0
5	-2	3	-1
Answer: 5	Answer: -2	Answer: 3	Answer: -1
2	0	4	0
Answer: 2	Answer: 0	Answer: 4	Answer: 0

- $A^T B^T =$

4	-2	3
Answer: 4	Answer: -2	Answer: 3
8	2	3
Answer: 8	Answer: 2	Answer: 3
5	1	2
Answer: 5	Answer: 1	Answer: 2

- $B^T A^T =$

5	-2	3	-1
Answer: 5	Answer: -2	Answer: 3	Answer: -1
2	0	4	0
Answer: 2	Answer: 0	Answer: 4	Answer: 0
5	-2	3	-1
Answer: 5	Answer: -2	Answer: 3	Answer: -1
2	0	4	0
Answer: 2	Answer: 0	Answer: 4	Answer: 0

- $A^T A = \begin{pmatrix} 7 & 1 & 2 \\ 1 & 11 & 4 \\ 2 & 4 & 3 \end{pmatrix}$
- $AA^T = \begin{pmatrix} 5 & -2 & 3 & -1 \\ -2 & 2 & 2 & 2 \\ 3 & 2 & 11 & 3 \\ -1 & 2 & 3 & 3 \end{pmatrix}$
- $(AB)^T = \begin{pmatrix} 5 & -2 & 3 & -1 \\ 2 & 0 & 4 & 0 \\ 5 & -2 & 3 & -1 \\ 2 & 0 & 4 & 0 \end{pmatrix}$
- $A^T B^T = \begin{pmatrix} 4 & -2 & 3 \\ 8 & 2 & 3 \\ 5 & 1 & 2 \end{pmatrix}$
- $B^T A^T = \begin{pmatrix} 5 & -2 & 3 & -1 \\ 2 & 0 & 4 & 0 \\ 5 & -2 & 3 & -1 \\ 2 & 0 & 4 & 0 \end{pmatrix}$

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Homework 5.2.3.2

1/1 point (graded)

Let $A \in \mathbb{R}^{m \times k}$ and $B \in \mathbb{R}^{k \times n}$. $(AB)^T = B^T A^T$.

Always ▾ **Answer:** Always

Answer:

Proof 1:

In an example in the previous unit, we partitioned C into elements (scalars) and A and B by rows and columns, respectively, before performing the partitioned matrix-matrix multiplication $C = AB$. This insight forms the basis for the following proof:

$$
\begin{array}{l}
(AB)^T \quad = \quad < \text{Partition } A \text{ by rows and } B \text{ by columns} > \\
\left(\left(\begin{array}{c} \tilde{a}_0^T \\ \hline \tilde{a}_1^T \\ \hline \vdots \\ \hline \tilde{a}_{m-1}^T \end{array} \right) \left(\begin{array}{c|c|c|c} b_0 & b_1 & \cdots & b_{n-1} \end{array} \right) \right)^T \\
\quad = \quad < \text{Partitioned matrix-matrix multiplication} > \\
\left(\begin{array}{c|c|c|c} \tilde{a}_0^T b_0 & \tilde{a}_0^T b_1 & \cdots & \tilde{a}_0^T b_{n-1} \\ \hline \tilde{a}_1^T b_0 & \tilde{a}_1^T b_1 & \cdots & \tilde{a}_1^T b_{n-1} \\ \hline \vdots & \vdots & \ddots & \vdots \\ \hline \tilde{a}_{m-1}^T b_0 & \tilde{a}_{m-1}^T b_1 & \cdots & \tilde{a}_{m-1}^T b_{n-1} \end{array} \right)^T \\
\quad = \quad < \text{Transpose the matrix} > \\
\left(\begin{array}{c|c|c|c} \tilde{a}_0^T b_0 & \tilde{a}_1^T b_0 & \cdots & \tilde{a}_{m-1}^T b_0 \\ \hline \tilde{a}_0^T b_1 & \tilde{a}_1^T b_1 & \cdots & \tilde{a}_{m-1}^T b_1 \\ \hline \vdots & \vdots & \ddots & \vdots \\ \hline \tilde{a}_0^T b_{n-1} & \tilde{a}_1^T b_{n-1} & \cdots & \tilde{a}_{m-1}^T b_{n-1} \end{array} \right) \\
\quad = \quad < \text{dot product commutes} > \\
\left(\begin{array}{c|c|c|c} b_0^T \tilde{a}_0 & b_0^T \tilde{a}_1 & \cdots & b_0^T \tilde{a}_{m-1} \\ \hline b_1^T \tilde{a}_0 & b_1^T \tilde{a}_1 & \cdots & b_1^T \tilde{a}_{m-1} \\ \hline \vdots & \vdots & \ddots & \vdots \\ \hline b_{n-1}^T \tilde{a}_0 & b_{n-1}^T \tilde{a}_1 & \cdots & b_{m-1}^T \tilde{a}_{m-1} \end{array} \right) \\
\quad = \quad < \text{Partitioned matrix-matrix multiplication} > \\
\left(\begin{array}{c} b_0^T \\ \hline b_1^T \\ \hline \vdots \\ \hline b_{n-1}^T \end{array} \right) \left(\begin{array}{c|c|c|c} \tilde{a}_0 & \tilde{a}_1 & \cdots & \tilde{a}_{m-1} \end{array} \right) \\
\quad = \quad < \text{Partitioned matrix transposition} > \\
\left(\begin{array}{c|c|c|c} b_0 & b_1 & \cdots & b_{n-1} \end{array} \right)^T \left(\begin{array}{c} \tilde{a}_0^T \\ \hline \tilde{a}_1^T \\ \hline \vdots \end{array} \right)^T = B^T A^T.
\end{array}
$$

$$\left(\begin{array}{c} \vdots \\ \hline \tilde{a}_{m-1}^T \end{array}\right)$$

Proof 2:

Let $C = AB$ and $D = B^T A^T$. We need to show that $\gamma_{i,j} = \delta_{j,i}$.

But

$\gamma_{i,j}$

$\quad = \quad <$ Earlier observation $>$

$e_i^T C e_j$

$\quad = \quad < C = AB >$

$e_i^T (AB) e_j$

$\quad = \quad <$ Associativity of multiplication; e_i^T and e_j are matrices $>$

$(e_i^T A)(B e_j)$

$\quad = \quad <$ Property of multiplication; $\widetilde{a}_i^T$ is ith row of A, b_j is jth column of B $>$

$\widetilde{a}_i^T b_j$

$\quad = \quad <$ Dot product commutes $>$

$b_j^T \widetilde{a}_i$

$\quad = \quad <$ Property of multiplication $>$

$(e_j^T B^T)(A^T e_i)$

$\quad = \quad <$ Associativity of multiplication; e_i^T and e_j are matrices $>$

$e_j^T (B^T A^T) e_i$

$\quad = \quad < C = AB >$

$e_j^T D e_i$

$\quad = \quad <$ earlier observation $>$

$\delta_{j,i}$

Proof 3:

(I vaguely recall that somewhere we proved that $(Ax)^T = x^T A^T$ ... If not, one should prove that first...)

$$
\begin{array}{l}
(AB)^T \\
\quad = \quad < \text{ Partition } B \text{ by columns } > \\
(A \left(\begin{array}{cccc} b_0 & b_1 & \cdots & b_{n-1} \end{array} \right))^T \\
\quad = \quad < \text{ Partitioned matrix-matrix multiplication } > \\
\left(\begin{array}{cccc} Ab_0 & Ab_1 & \cdots & Ab_{n-1} \end{array} \right)^T \\
\quad = \quad < \text{ Transposing a partitioned matrix } > \\
\left(\begin{array}{c} (Ab_0)^T \\ (Ab_1)^T \\ \vdots \\ (Ab_{n-1})^T \end{array} \right) \\
\quad = \quad < (Ax)^T = x^T A^T > \\
\left(\begin{array}{c} b_0^T A^T \\ b_1^T A^T \\ \vdots \\ b_{n-1}^T A^T \end{array} \right) \quad = \quad < \text{ Partitioned matrix-matrix multiplication } > \\
\left(\begin{array}{c} b_0^T \\ b_1^T \\ \vdots \\ b_{n-1}^T \end{array} \right) A^T \\
\quad = \quad < \text{ Partitioned matrix transposition } > \\
\left(\begin{array}{cccc} b_0 & b_1^T & \cdots & b_{n-1} \end{array} \right)^T A^T \\
\quad = \quad < \text{ Partition } B \text{ by columns } > \\
B^T A^T
\end{array}
$$

Proof 4: (For those who don't like the $\cdots$ in arguments...)

Proof by induction on n, the number of columns of B.

(I vaguely recall that somewhere we proved that $(Ax)^T = x^T A^T$... If not, one should prove that first...)

Base case: $n = 1$. Then $B = (b_0)$. But then $(AB)^T = (Ab_0)^T = b_0^T A^T = B^T A^T$.

Inductive Step: The inductive hypothesis is: Assume that $(AB)^T = B^T A^T$ for all matrices B with $n = N$ columns. We now need to show that, assuming this, $(AB)^T = B^T A^T$ for all matrices B with $n = N + 1$ columns.

Assume that B has $N+1$ columns. Then

$$
\begin{array}{l}
(AB)^T \\
\quad = \quad < \text{Partition } B \ > \\
(A \left(\begin{array}{cc} B_0 & b_1 \end{array} \right))^T \\
\quad = \quad < \text{Partitioned matrix-matrix multiplication} > \\
(\left(\begin{array}{cc} AB_0 & Ab_1 \end{array} \right))^T \\
\quad = \quad < \text{Partitioned matrix transposition} > \\
(\left(\begin{array}{c} (AB_0)^T \\ (Ab_1)^T \end{array} \right)) \\
\quad = \quad < \text{I.H. and } (Ax)^T = x^T A^T > \\
\left(\begin{array}{c} B_0^T A^T \\ b_1^T A^T \end{array} \right) \\
\quad = \quad < \text{Partitioned matrix-matrix multiplication} > \\
\left(\begin{array}{c} B_0^T \\ b_1^T \end{array} \right) A^T \\
\quad = \quad < \text{Transposing a partitioned matrix} > \\
\left(\begin{array}{cc} B_0 & b_1 \end{array} \right)^T A^T \\
\quad = \quad < \text{Partitioning of } B > \\
B^T A^T
\end{array}
$$

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Homework 5.2.3.3

1/1 point (graded)

Let $A, B,$ and C be conformal matrices so that ABC is well-defined. Then $(ABC)^T = C^T B^T A^T$

Always

Answer: Always

Explanation

Answer: Always

$$(ABC)^T = (A(BC))^T = (BC)^T A^T = (C^T B^T) A^T = C^T B^T A^T$$

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