6.2.5 Towards an Algorithm

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Dr. Robert van de Geijn: So now we're ready to talk about an algorithm

for performing the computations that we illustrated

with this concrete example that was just three equations and three unknowns

Here is our appended system.

We talk about how we can work first with the matrix,

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Homework 6.2.5.1

1/1 point (graded)

Algorithm: $A := \text{GAUSSIAN\_ELIMINATION}(A)$

Partition $A \rightarrow \left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right)$

where A_{TL} is 0×0

while $m(A_{TL}) < m(A)$ **do**

Repartition

$$\left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c|c|c} A_{00} & a_{01} & A_{02} \\ \hline a_{10}^T & \alpha_{11} & a_{12}^T \\ \hline A_{20} & a_{21} & A_{22} \end{array}\right)$$

$$a_{21} := a_{21}/\alpha_{11} \qquad (= l_{21})$$
$$A_{22} := A_{22} - a_{21}a_{12}^T \qquad (= A_{22} - l_{21}a_{12}^T)$$

Continue with

$$\left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c|c|c} A_{00} & a_{01} & A_{02} \\ \hline a_{10}^T & \alpha_{11} & a_{12}^T \\ \hline A_{20} & a_{21} & A_{22} \end{array}\right)$$

endwhile

Algorithm: $b := \text{FORWARD\_SUBSTITUTION}(A, b)$

Partition $A \rightarrow \left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right)$, $b \rightarrow \left(\begin{array}{c} b_T \\ \hline b_B \end{array}\right)$

where A_{TL} is 0×0, b_T has 0 rows

while $m(A_{TL}) < m(A)$ **do**

Repartition

$$\left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c|c|c} A_{00} & a_{01} & A_{02} \\ \hline a_{10}^T & \alpha_{11} & a_{12}^T \\ \hline A_{20} & a_{21} & A_{22} \end{array}\right), \left(\begin{array}{c} b_T \\ \hline b_B \end{array}\right) \rightarrow \left(\begin{array}{c} b_0 \\ \hline \beta_1 \\ \hline b_2 \end{array}\right)$$

$$b_2 := b_2 - \beta_1 a_{21} \qquad (= b_2 - \beta_1 l_{21})$$

Continue with

$$\left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c|c|c} A_{00} & a_{01} & A_{02} \\ \hline a_{10}^T & \alpha_{11} & a_{12}^T \\ \hline A_{20} & a_{21} & A_{22} \end{array}\right), \left(\begin{array}{c} b_T \\ \hline b_B \end{array}\right) \leftarrow \left(\begin{array}{c} b_0 \\ \hline \beta_1 \\ \hline b_2 \end{array}\right)$$

endwhile

Implement the described Gaussian Elimination and Forward Substitution algorithms

- `[ A_out ] = GaussianElimination( A )`
- `[ b_out ] = ForwardSubstitution( A, b )`

You can check that they compute the right answers with the following script:

- test_GausianElimination.m (In `LAFF-2.0xM/Programming/Week06/`)

Unfortunately, PictureFLAME may not work for this problem, since a zero may appear on the diagonal, causing a divide by zero.

This script exercises the functions by factoring the matrix

```
A = [
     2     0     1     2
    -2    -1     1    -1
     4    -1     5     4
    -4     1    -3    -8
]
```

by calling

```
LU = GaussianElimination( A )
```

Next, solve $Ax = b$ where

```
b = [
     2
     2
    11
    -3
]
```

by first apply forward substitution to b, using the output matrix LU:

```
bhat = ForwardSubstitution( LU, b )
```

extracting the upper triangular matrix U from LU:

```
U = triu( LU )
```

and then solving $Ux = \hat{b}$ (which is equivalent to backward substitution) with the intrinsic function:

```
x = U \ bhat
```

Finally, check that you got the right answer:

```
b - A * x
```

(the result should be a zero vector with four elements).

Done/Skip

- `GausianElimination.m`
- `ForwardSubstitution.m`

Submit

Answers are displayed within the problem