6.3.1 LU factorization (Gaussian elimination)

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Homework 6.3.1.1

1 point possible (graded)

Algorithm: $A := \text{LU\_UNB\_VAR5}(A)$

Partition $A \rightarrow \left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right)$

where A_{TL} is 0×0

while $m(A_{TL}) < m(A)$ **do**

Repartition

$$\left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c|c|c} A_{00} & a_{01} & A_{02} \\ \hline a_{10}^T & \alpha_{11} & a_{12}^T \\ \hline A_{20} & a_{21} & A_{22} \end{array}\right)$$

$a_{21} := a_{21}/\alpha_{11}$ $(= l_{21})$

$A_{22} := A_{22} - a_{21}a_{12}^T$ $(= A_{22} - l_{21}a_{12}^T)$

Continue with

$$\left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c|c|c} A_{00} & a_{01} & A_{02} \\ \hline a_{10}^T & \alpha_{11} & a_{12}^T \\ \hline A_{20} & a_{21} & A_{22} \end{array}\right)$$

endwhile

Implement the above LU factorization algorithm.

- `[ A_out ] = LU_unb_var5( A )`

You can check that they compute the right answers with the following script:

- `test_LU_unb_var5.m` (In `LAFF-2.0xM/Programming/Week06/`)

Unfortunately, PictureFLAME may not work for this problem, since a zero may be encountered on the diagonal causing a divide by zero.

This script exercises the functions by factoring the matrix

```
A = [
     2     0     1     2
    -2    -1     1    -1
     4    -1     5     4
    -4     1    -3    -8
]
```

by calling

```
LU = LU_unb_var5( A )
```

Next, it extracts the unit lower triangular matrix and upper triangular matrix:

```
L = tril( LU, -1 ) + eye( size( A ) )

U = triu( LU )
```

and checks if the correct factors were computed:

```
A - L * U
```

which should yield a 4×4 zero matrix.

Done/Skip ✔

Here is our implementations of the function:

- LU_unb_var5.m

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Answers are displayed within the problem

Homework 6.3.1.2

18/18 points (graded)

What is the LU factorization of $\begin{bmatrix} 1 & -2 & 2 \\ 5 & -15 & 8 \\ -2 & -11 & -11 \end{bmatrix}$?

$L =$

1	0	0
✔ Answer: 1	✔ Answer: 0	✔ Answer: 0
5	1	0
✔ Answer: 5	✔ Answer: 1	✔ Answer: 0
-2	3	1
✔ Answer: -2	✔ Answer: 3	✔ Answer: 1

$U =$

1	-2	2
✔ Answer: 1	✔ Answer: -2	✔ Answer: 2
0	-5	-2
✔ Answer: 0	✔ Answer: -5	✔ Answer: -2
0	0	-1
✔ Answer: 0	✔ Answer: 0	✔ Answer: -1

Answer:

$$\begin{pmatrix} 1 & -2 & 2 \\ 5 & -15 & 8 \\ -2 & -11 & -11 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 5 & 1 & 0 \\ -2 & 3 & 1 \end{pmatrix} \begin{pmatrix} 1 & -2 & 2 \\ 0 & -5 & -2 \\ 0 & 0 & -1 \end{pmatrix}.$$

Here are the details when executing the algorithm:

Iteration	Before	After
1	$\left(\begin{array}{c\|cc} 1 & -2 & 2 \\ \hline 5 & -15 & 8 \\ -2 & -11 & -11 \end{array}\right)$	$\left(\begin{array}{c\|cc} 1 & -2 & 2 \\ \hline 5 & -5 & -2 \\ -2 & -15 & -7 \end{array}\right)$
2	$\left(\begin{array}{c\|cc} 1 & -2 & 2 \\ \hline 5 & -5 & -2 \\ \hline -2 & -15 & -7 \end{array}\right)$	$\left(\begin{array}{c\|cc} 1 & -2 & 2 \\ \hline 5 & -5 & -2 \\ \hline -2 & 3 & -1 \end{array}\right)$
3	$\left(\begin{array}{cc\|c} 1 & -2 & 2 \\ 5 & -5 & -2 \\ \hline -2 & 3 & -1 \end{array}\right)$	$\left(\begin{array}{cc\|c} 1 & -2 & 2 \\ 5 & -5 & -2 \\ \hline -2 & 3 & -1 \end{array}\right)$

The unit lower triangular matrix L and upper triangular matrix U can then be read off as:

$$L = \begin{pmatrix} 1 & 0 & 0 \\ 5 & 1 & 0 \\ -2 & 3 & 1 \end{pmatrix} \qquad \begin{pmatrix} 1 & -2 & 2 \\ 0 & -5 & -2 \\ 0 & 0 & -1 \end{pmatrix}$$

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Answers are displayed within the problem