6.3.3 Solving Ux = b (Back substitution)

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Homework 6.3.3.1

3/3 points (graded)

Algorithm: $[b] := \text{UTRSV\_UNB\_VAR1}(U, b)$

Partition $U \rightarrow \left(\begin{array}{c|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right)$, $b \rightarrow \left(\begin{array}{c} b_T \\ \hline b_B \end{array}\right)$

where U_{BR} is 0×0, b_B has 0 rows

while $m(U_{BR}) < m(U)$ **do**

Repartition

$$\left(\begin{array}{c|c} U_{TL} & U_{TR} \\ \hline 0 & U_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c|c|c} U_{00} & u_{01} & U_{02} \\ \hline 0 & \upsilon_{11} & u_{12}^T \\ \hline 0 & 0 & U_{22} \end{array}\right), \left(\begin{array}{c} b_T \\ \hline b_B \end{array}\right) \rightarrow \left(\begin{array}{c} b_0 \\ \hline \beta_1 \\ \hline b_2 \end{array}\right)$$

$\beta_1 := \beta_1 - u_{12}^T b_2$

$\beta_1 := \beta_1 / \upsilon_{11}$

Continue with

$$\left(\begin{array}{c|c} U_{TL} & U_{TR} \\ \hline 0 & U_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c|c|c} U_{00} & u_{01} & U_{02} \\ \hline 0 & \upsilon_{11} & u_{12}^T \\ \hline 0 & 0 & U_{22} \end{array}\right), \left(\begin{array}{c} b_T \\ \hline b_B \end{array}\right) \leftarrow \left(\begin{array}{c} b_0 \\ \hline \beta_1 \\ \hline b_2 \end{array}\right)$$

endwhile

With pencil and paper, side-by-side, solve the upper triangular linear system

$$\begin{aligned} -2\chi_0 - \chi_1 + \chi_2 &= 6 \\ -3\chi_1 - 2\chi_2 &= 9 \\ \chi_2 &= 3 \end{aligned}$$

via back substitution and by executing the above algorithm with inputs

$$U = \begin{pmatrix} -2 & -1 & 1 \\ 0 & -3 & -2 \\ 0 & 0 & 1 \end{pmatrix} \text{ and } b = \begin{pmatrix} 6 \\ 9 \\ 3 \end{pmatrix}.$$

$\chi_0 =$ 1 ✔ **Answer:** 1

$\chi_1 =$ -5 ✔ **Answer:** -5

$\chi_2 =$ 3 ✔ **Answer:** 3

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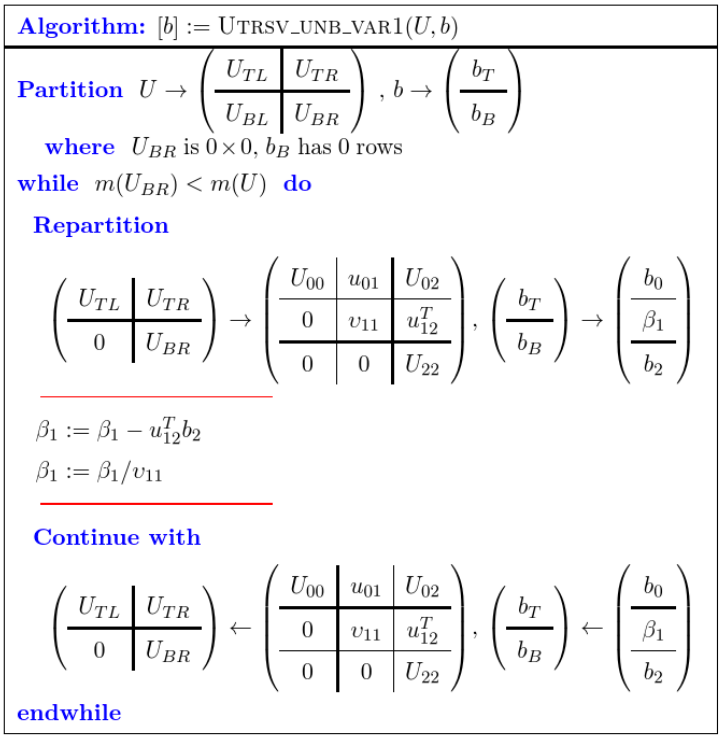
Answers are displayed within the problem

Homework 6.3.3.2

1 point possible (graded)



Implement the algorithm in the above figure.

- `[ b_out ] = Utrsv_unb_var1( U, b )`

You can check that it computes the right answer with the following script:

- `test_Utrsv_unb_var1.m` (In `LAFF-2.0xM/Programming/Week06/`)

Unfortunately, PictureFLAME does not work for this problem.

This script exercises the function by starting with matrix

```
U = [
     2     0     1     2
     0    -1     2     1
     0     0     1    -1
     0     0     0    -2
]
```

Next, it solves $Ux = b$ with the right-hand size vector

```
b = [
     2
     4
     3
     2
]
```

by calling

```
x = Utrsv_unb_var1( U, b )
```

Finally, it checks if x indeed solves $Ux = b$ by computing

```
b - U * x
```

which should yield a zero vector of size four.

Done/Skip ✔

Here is our implementations of the function:

- Utrsv_unb_var1.m

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