edX

Course › Week... › 5.2 O... › 5.2.4 ...

5.2.4 Matrix-Matrix Multiplication with Special Matrices

5.2.4 Matrix-Matrix Multiplication with Special Matrices

No introductory video

Discussion

Hide Discussion

Topic: Week 5 / 5.2.4

Add a Post

Show all posts ▼ by recent activity ▼

There are no posts in this topic yet.

×

Homework 5.2.4.1

21/21 points (graded)

Compute

- $\begin{pmatrix} 1 & -2 & -1 \\ 2 & 0 & 2 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} =$
  1 ✔
  2 ✔

- $\begin{pmatrix} 1 & -2 & -1 \\ 2 & 0 & 2 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} =$ -2 ✔ 0 ✔

- $\begin{pmatrix} 1 & -2 & -1 \\ 2 & 0 & 2 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} =$ -1 ✔ 2 ✔

- $\begin{pmatrix} 1 & -2 & -1 \\ 2 & 0 & 2 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} =$

1 ✔	-2 ✔	-1 ✔
2 ✔	0 ✔	2 ✔

- $\begin{pmatrix} 1 & -2 & -1 \\ 2 & 0 & 2 \\ -1 & 3 & -1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} =$

1 ✔	-2 ✔	-1 ✔
2 ✔	0 ✔	2 ✔
-1 ✔	3 ✔	-1 ✔

Submit

Homework 5.2.4.2

18/18 points (graded)

Compute

- $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} =$

1 ✔
2 ✔
-1 ✔

- $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} -2 \\ 0 \\ 3 \end{pmatrix} =$

-2 ✔
0 ✔
3 ✔

- $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} -1 \\ 2 \\ -1 \end{pmatrix} =$

-1 ✔
2 ✔
-1 ✔

- $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & -2 & -1 \\ 2 & 0 & 2 \\ -1 & 3 & -1 \end{pmatrix} =$

1	-2	-1
2	0	2
-1	3	-1

Submit

Homework 5.2.4.3

1/1 point (graded)

Let $A \in \mathbb{R}^{m \times n}$ and let I denote the identity matrix of appropriate size.

$AI = IA = A$

Always

Submit

Homework 5.2.4.4

12/12 points (graded)

Compute

- $\begin{pmatrix} 1 & -2 & -1 \\ 2 & 0 & 2 \end{pmatrix} \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix} =$

2

4

- $\begin{pmatrix} 1 & -2 & -1 \\ 2 & 0 & 2 \end{pmatrix} \begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix} =$ 2 ✓ 0 ✓

- $\begin{pmatrix} 1 & -2 & -1 \\ 2 & 0 & 2 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ -3 \end{pmatrix} =$ 3 ✓ -6 ✓

- $\begin{pmatrix} 1 & -2 & -1 \\ 2 & 0 & 2 \end{pmatrix} \begin{pmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -3 \end{pmatrix} =$

2 ✓	2 ✓	3 ✓
4 ✓	0 ✓	-6 ✓

Submit

Homework 5.2.4.5

18/18 points (graded)

- $\begin{pmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -3 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} =$ 2 ✔ -2 ✔ 3 ✔

- $\begin{pmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -3 \end{pmatrix} \begin{pmatrix} -2 \\ 0 \\ 3 \end{pmatrix} =$ -4 ✔ 0 ✔ -9 ✔

- $\begin{pmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -3 \end{pmatrix} \begin{pmatrix} -1 \\ 2 \\ -1 \end{pmatrix} =$ -2 ✔ -2 ✔ 3 ✔

- $\begin{pmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -3 \end{pmatrix} \begin{pmatrix} 1 & -2 & -1 \\ 2 & 0 & 2 \\ -1 & 3 & -1 \end{pmatrix} =$

2 ✔	-4 ✔	-2 ✔
-2 ✔	0 ✔	-2 ✔
3 ✔	-9 ✔	3 ✔

Submit

Homework 5.2.4.6

1/1 point (graded)

Let $A \in \mathbb{R}^{m \times n}$ and let D denote the diagonal matrix with diagonal elements $\delta_0, \delta_1, \cdots, \delta_{n-1}$. Partition A by columns :

$$A = \left(\begin{array}{c|c|c|c} a_0 & a_1 & \ldots & a_{n-1} \end{array} \right).$$

$$AD = \left(\begin{array}{c|c|c|c} \delta_0 a_0 & \delta_1 a_1 & \ldots & \delta_{n-1} a_{n-1} \end{array} \right).$$

Always ✔

Submit

Homework 5.2.4.7

1/1 point (graded)

Let $A \in \mathbb{R}^{m \times n}$ and let D denote the diagonal matrix with diagonal elements $\delta_0, \delta_1, \cdots, \delta_{m-1}$. Partition A by rows :

$$A = \left(\begin{array}{c} \tilde{a}_0^T \\ \hline \tilde{a}_1^T \\ \hline \vdots \\ \hline \tilde{a}_{m-1}^T \end{array} \right).$$

$$DA = \left(\begin{array}{c} \delta_0 \tilde{a}_0^T \\ \hline \delta_1 \tilde{a}_1^T \\ \hline \vdots \\ \hline \delta_{m-1} \tilde{a}_{m-1}^T \end{array} \right).$$

Always ✔

Submit

Homework 5.2.4.8

9/9 points (graded)

Compute

$$\begin{pmatrix} 1 & -1 & -2 \\ 0 & 2 & 3 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} -2 & 1 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix} =$$

-2	0	-5
0	2	7
0	0	1

Submit

Homework 5.2.4.9

9/9 points (graded)

Compute the following, using what you know about partitioned matrix-matrix multiplication:

$$\left(\begin{array}{cc|c} 1 & -1 & -2 \\ 0 & 2 & 3 \\ \hline 0 & 0 & 1 \end{array}\right) \left(\begin{array}{cc|c} -2 & 1 & -1 \\ 0 & 1 & 2 \\ \hline 0 & 0 & 1 \end{array}\right) =$$

-2 (Answer: -2)	0 (Answer: 0)	-5 (Answer: -5)
0 (Answer: 0)	2 (Answer: 2)	7 (Answer: 7)
0 (Answer: 0)	0 (Answer: 0)	1 (Answer: 1)

Answer:

$$
\begin{aligned}
&\left(\begin{array}{cc|c} 1 & -1 & -2 \\ 0 & 2 & 3 \\ \hline 0 & 0 & 1 \end{array}\right)\left(\begin{array}{cc|c} -2 & 1 & -1 \\ 0 & 1 & 2 \\ \hline 0 & 0 & 1 \end{array}\right) \\
&= \left(\begin{array}{c|c} \begin{pmatrix} 1 & -1 \\ 0 & 2 \end{pmatrix}\begin{pmatrix} -2 & 1 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} -2 \\ 3 \end{pmatrix}\begin{pmatrix} 0 & 0 \end{pmatrix} & \begin{pmatrix} 1 & -1 \\ 0 & 2 \end{pmatrix}\begin{pmatrix} -1 \\ 2 \end{pmatrix} + \begin{pmatrix} -2 \\ 3 \end{pmatrix}(1) \\ \hline \begin{pmatrix} 0 & 0 \end{pmatrix}\begin{pmatrix} -2 & 1 \\ 0 & 1 \end{pmatrix} + (1)\begin{pmatrix} 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 \end{pmatrix}\begin{pmatrix} -1 \\ 2 \end{pmatrix} + (1)(1) \end{array}\right) \\
&= \left(\begin{array}{c|c} \begin{pmatrix} -2 & 0 \\ 0 & 2 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} -3 \\ 4 \end{pmatrix} + \begin{pmatrix} -2 \\ 3 \end{pmatrix} \\ \hline \begin{pmatrix} 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 \end{pmatrix} & (0+0)+(1) \end{array}\right) = \left(\begin{array}{cc|c} -2 & 0 & -5 \\ 0 & 2 & 7 \\ \hline 0 & 0 & 1 \end{array}\right)
\end{aligned}
$$

Submit

Answers are displayed within the problem

Homework 5.2.4.10

1/1 point (graded)

Let $U, R \in \mathbb{R}^{n\times n}$ be uppertriangular matrices.

The product UR is an upper triangular matrix.

Always ✔

Submit

✔ Correct (1/1 point)

Homework 5.2.4.11

1/1 point (graded)

The product of an n x n lower triangular matrix times an n x n lower triangular matrix is a lower triangular matrix.

Always ✔

Submit

✔ Correct (1/1 point)

Homework 5.2.4.12

0/1 point (graded)

The product of an n x n lower triangular matrix times an n x n upper triangular matrix is a diagonal matrix.

Always ✗ **Answer:** Sometimes

Explanation

Diagonal matrices are both upper and lower triangular. Multiply them together, and you get a diagonal matrix. But take any lower triangular matrix that is not diagonal and multiply it by an upper triangular matrix (diagonal or not), and you don't get a diagonal matrix.

Submit

Answers are displayed within the problem

Homework 5.2.4.13

1/1 point (graded)

Let $A \in \mathbb{R}^{m \times n}$.

$A^T A$ is symmetric.

Always ✓ **Answer:** Always

Explanation

Answer: Always

$$(A^T A)^T = A^T (A^T)^T = A^T A.$$

Hence, $A^T A$ is symmetric.

Submit

Answers are displayed within the problem

Homework 5.2.4.14

45/45 points (graded)

Compute

- $\begin{pmatrix} -1 \\ 1 \\ 2 \end{pmatrix} \begin{pmatrix} -1 & 1 & 2 \end{pmatrix} =$

1 ✔ Answer: 1	-1 ✔ Answer: -1	-2 ✔ Answer: -2
-1 ✔ Answer: -1	1 ✔ Answer: 1	2 ✔ Answer: 2
-2 ✔ Answer: -2	2 ✔ Answer: 2	4 ✔ Answer: 4

- $\begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix} \begin{pmatrix} 2 & 0 & -1 \end{pmatrix} =$

4 ✔ Answer: 4	0 ✔ Answer: 0	-2 ✔ Answer: -2
0 ✔ Answer: 0	0 ✔ Answer: 0	0 ✔ Answer: 0
-2 ✔ Answer: -2	0 ✔ Answer: 0	1 ✔ Answer: 1

- $\left(\begin{array}{r|r} -1 & 2 \\ 1 & 0 \\ 2 & -1 \end{array}\right) \left(\begin{array}{rrr} -1 & 1 & 2 \\ \hline 2 & 0 & -1 \end{array}\right) =$

5 ✔ Answer: 5	-1 ✔ Answer: -1	-4 ✔ Answer: -4
-1 ✔ Answer: -1	1 ✔ Answer: 1	2 ✔ Answer: 2
-4 ✔ Answer: -4	2 ✔ Answer: 2	5 ✔ Answer: 5

- $\begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix} \begin{pmatrix} 1 & -2 & 2 \end{pmatrix} =$

1 ✔ Answer: 1	-2 ✔ Answer: -2	2 ✔ Answer: 2
-2 ✔ Answer: -2	4 ✔ Answer: 4	-4 ✔ Answer: -4
2 ✔ Answer: 2	-4 ✔ Answer: -4	4 ✔ Answer: 4

- $\left(\begin{array}{rr|r} -1 & 2 & 1 \\ 1 & 0 & -2 \\ 2 & -1 & 2 \end{array}\right) \left(\begin{array}{rrr} -1 & 1 & 2 \\ 2 & 0 & -1 \\ \hline 1 & -2 & 2 \end{array}\right) =$

6 ✔ Answer: 6	-3 ✔ Answer: -3	-2 ✔ Answer: -2
-3 ✔ Answer: -3	5 ✔ Answer: 5	-2 ✔ Answer: -2
-2 ✔ Answer: -2	-2 ✔ Answer: -2	9 ✔ Answer: 9

- $\begin{pmatrix} -1 \\ 1 \\ 2 \end{pmatrix} \begin{pmatrix} -1 & 1 & 2 \end{pmatrix} = \begin{pmatrix} 1 & -1 & -2 \\ -1 & 1 & 2 \\ -2 & 2 & 4 \end{pmatrix}.$
- $\begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix} \begin{pmatrix} 2 & 0 & -1 \end{pmatrix} = \begin{pmatrix} 4 & 0 & -2 \\ 0 & 0 & 0 \\ -2 & 0 & 1 \end{pmatrix}.$
- $\left(\begin{array}{r|r} -1 & 2 \\ 1 & 0 \\ 2 & -1 \end{array}\right) \left(\begin{array}{rrr} -1 & 1 & 2 \\ \hline 2 & 0 & -1 \end{array}\right) = \begin{pmatrix} 1 & -1 & -2 \\ -1 & 1 & 2 \\ -2 & 2 & 4 \end{pmatrix} + \begin{pmatrix} 4 & 0 & -2 \\ 0 & 0 & 0 \\ -2 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 5 & -1 & -4 \\ -1 & 1 & 2 \\ -4 & 2 & 5 \end{pmatrix}.$
- $\begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix} \begin{pmatrix} 1 & -2 & 2 \end{pmatrix} = \begin{pmatrix} 1 & -2 & 2 \\ -2 & 4 & -4 \\ 2 & -4 & 4 \end{pmatrix}.$
- $\left(\begin{array}{rr|r} -1 & 2 & 1 \\ 1 & 0 & -2 \\ 2 & -1 & 2 \end{array}\right) \left(\begin{array}{rrr} -1 & 1 & 2 \\ 2 & 0 & -1 \\ \hline 1 & -2 & 2 \end{array}\right) =$

Answer:

$$\begin{pmatrix} -1 & 2 \\ 1 & 0 \\ 2 & -1 \end{pmatrix} \begin{pmatrix} -1 & 1 & 2 \\ 2 & 0 & -1 \end{pmatrix} + \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix} \begin{pmatrix} 1 & -2 & 2 \end{pmatrix}$$
$$= \begin{pmatrix} 5 & -1 & -4 \\ -1 & 1 & 2 \\ -4 & 2 & 5 \end{pmatrix} + \begin{pmatrix} 1 & -2 & 2 \\ -2 & 4 & -4 \\ 2 & -4 & 4 \end{pmatrix} = \begin{pmatrix} 6 & -3 & -2 \\ -3 & 5 & -2 \\ -2 & -2 & 9 \end{pmatrix}.$$

Submit

Answers are displayed within the problem

Homework 5.2.4.15

1/1 point (graded)

Let $x \in \mathbb{R}^n$.

xx^T is symmetric.

Always ▾ ✔ **Answer:** Always

Explanation

Answer: Always

Proof 1: Since A^TA is symmetric for any matrix $A \in \mathbb{R}^{m\times n}$ and vector $A = x^T \in \mathbb{R}^n$ is just the special case where the matrix is a vector.

Proof 2:

$$xx^T = \begin{pmatrix} \chi_0 \\ \chi_1 \\ \vdots \\ \chi_{n-1} \end{pmatrix} \begin{pmatrix} \chi_0 \\ \chi_1 \\ \vdots \\ \chi_{n-1} \end{pmatrix}^T = \begin{pmatrix} \chi_0 \\ \chi_1 \\ \vdots \\ \chi_{n-1} \end{pmatrix} \begin{pmatrix} \chi_0 & \chi_1 & \cdots & \chi_{n-1} \end{pmatrix}$$
$$= \begin{pmatrix} \chi_0\chi_0 & \chi_0\chi_1 & \cdots & \chi_0\chi_{n-1} \\ \chi_1\chi_0 & \chi_1\chi_1 & \cdots & \chi_1\chi_{n-1} \\ \vdots & \vdots & & \vdots \\ \chi_{n-1}\chi_0 & \chi_{n-1}\chi_1 & \cdots & \chi_{n-1}\chi_{n-1} \end{pmatrix}.$$

Since $\chi_i\chi_j = \chi_j\chi_i$, the (i,j) element of xx^T equals the (j,i) element of xx^T. This means xx^T is symmetric.

Proof 3: $(xx^T)^T = (x^T)^Tx^T = xx^T$.

Submit

Answers are displayed within the problem

Homework 5.2.4.16

1/1 point (graded)

Let $A \in \mathbb{R}^{n\times n}$ be symmetric and $x \in \mathbb{R}^n$.

$A + xx^T$ is symmetric.

Always ▾ **Answer:** Always

Explanation

If matrices $A, B \in \mathbb{R}^{nxn}$ are symmetric, then $A + B$ is symmetric since $(A+B)^T = A^T + B^T = A + B$. In this case, $B = xx^T$.

Submit

Answers are displayed within the problem

Homework 5.2.4.17

1/1 point (graded)

Let $A \in \mathbb{R}^{m\times n}$.

AA^T is symmetric.

Always ▾ **Answer:** Always

Explanation

Answer: Always
Proof 1: $(AA^T)^T = (A^T)^T A^T = AA^T$.

Proof 2: We know that $A^T A$ is symmetric. Take $B = A^T$. Then $AA^T = B^T B$ and hence AA^T is symmetric.

Proof 3:

$$\begin{aligned} AA^T &= \left(a_0 \,\middle|\, a_1 \,\middle|\, \cdots \,\middle|\, a_{n-1} \right) \left(a_0 \,\middle|\, a_1 \,\middle|\, \cdots \,\middle|\, a_{n-1} \right)^T \\ &= \left(a_0 \,\middle|\, a_1 \,\middle|\, \cdots \,\middle|\, a_{n-1} \right) \begin{pmatrix} a_0^T \\ \hline a_1^T \\ \hline \vdots \\ \hline a_{n-1}^T \end{pmatrix} \\ &= a_0 a_0^T + a_1 a_1^T + \cdots + a_{n-1} a_{n-1}^T. \end{aligned}$$

But each $a_j a_j^T$ is symmetric (by a previous exercise) and adding symmetric matrices yields a symmetric matrix. Hence, AA^T is symmetric.

Proof 4:
Proof by induction on n.

Base case: $A = \left(a_0 \right)$, where a_0 is a vector. Then $AA^T = a_0 a^T$. But we saw in an earlier homework that if x is a vector, then xx^T is symmetric.

Induction Step: Assume that AA^T is symmetric for matrices with $n = N$ columns, where $N \geq 1$.
We will show that AA^T is symmetric for matrices with $n = N + 1$ columns.
Let A have $N + 1$ columns.

$$\begin{array}{l} AA^T \\ \quad = \quad < \text{Partition } A > \\ \left(A_0 \,\middle|\, a_1 \right) \left(A_0 \,\middle|\, a_1 \right)^T \\ \quad = \quad < \text{Transpose partitioned matrix} > \\ \left(A_0 \,\middle|\, a_1 \right) \begin{pmatrix} A_0^T \\ \hline a_1^T \end{pmatrix} \\ \quad = \quad < \text{Partitioned matrix-matrix multiplication} > \\ A_0 A_0^T + a_1 a_1^T \end{array}$$

Now, by the I.H. $A_0 A_0^T$ is symmetric. From a previous exercise we know that xx^T is symmetric and hence $a_1 a_1^T$ is. From another exercise we know that adding symmetric matrices yields a symmetric matrix.

By the Principle of Mathematical Induction (PMI), the result holds for all n.

Submit

Answers are displayed within the problem

Homework 5.2.4.18

1/1 point (graded)
Let $A, B \in \mathbb{R}^{n \times n}$ be symmetric matrices.

AB is symmetric.

Sometimes ✔ **Answer:** Sometimes

Explanation
Examples when this is true
• $B = A$ so that $AB = AA$. Then $(AA)^T = A^T A^T = AA$.
• $A = I$ or $B = I$. $IB = B$ and hence IB is symmetric. Similarly, $AI = A$ and hense AI is symmetric.
An example when this is false

$A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ $B = \begin{pmatrix} 1 & -1 \\ -1 & 2 \end{pmatrix}$. Then $AB = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & -1 \\ -1 & 2 \end{pmatrix} = \begin{pmatrix} -1 & 2 \\ 1 & -1 \end{pmatrix}$, which is not a symmetric matrix.

Submit

Answers are displayed within the problem