6.3.5 Cost

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Homework 6.3.5.1

5/5 points (graded)

Consider the following algorithm for solving $Ux = b$ where U is an $n \times n$ upper triangular matrix and b is overwritten with the solution x.

Algorithm: $[b] := \text{UTRSV\_UNB\_VAR1}(U, b)$

Partition $U \rightarrow \left(\begin{array}{c|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right)$, $b \rightarrow \left(\begin{array}{c} b_T \\ \hline b_B \end{array}\right)$

where U_{BR} is 0×0, b_B has 0 rows

while $m(U_{BR}) < m(U)$ **do**

Repartition

$$\left(\begin{array}{c|c} U_{TL} & U_{TR} \\ \hline 0 & U_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c|c|c} U_{00} & u_{01} & U_{02} \\ \hline 0 & \upsilon_{11} & u_{12}^T \\ \hline 0 & 0 & U_{22} \end{array}\right), \left(\begin{array}{c} b_T \\ \hline b_B \end{array}\right) \rightarrow \left(\begin{array}{c} b_0 \\ \hline \beta_1 \\ \hline b_2 \end{array}\right)$$

$\beta_1 := \beta_1 - u_{12}^T b_2$

$\beta_1 := \beta_1 / \upsilon_{11}$

Continue with

$$\left(\begin{array}{c|c} U_{TL} & U_{TR} \\ \hline 0 & U_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c|c|c} U_{00} & u_{01} & U_{02} \\ \hline 0 & \upsilon_{11} & u_{12}^T \\ \hline 0 & 0 & U_{22} \end{array}\right), \left(\begin{array}{c} b_T \\ \hline b_B \end{array}\right) \leftarrow \left(\begin{array}{c} b_0 \\ \hline \beta_1 \\ \hline b_2 \end{array}\right)$$

endwhile

Assume that during the kth iteration U_{BR} is $k \times k$. (Notice we are purposely saying that U_{BR} is $k \times k$ because this algorithm moves in the opposite direction!)

Answer the following questions:

• What is the size of submatrix U_{22}?

- 1×1
- $k \times 1$

- $1 \times k$
- $k \times k$ ✔

• What is the size of row vector u_{12}^T?

- 1
- $1 \times k$ ✔
- $1 \times n$
- $1 \times (n-k)$

• What is the size of vector b_2?

- 1
- k ✔
- n
- $n-k$

Approximately how many flops does the dot operation $\beta_1 := \beta_1 - u_{12}^T b_2$ requires?

- ◉ $2k$ ✔
- ○ $2n$

We need to sum this over all iterations $k = 0, \ldots, (n-1)$ (You may ignore the divisions)

Compute how many floating point operations this equals. What is the approximite result?

- ○ 1
- ○ n
- ◉ n^2 ✔
- ○ n^3

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ℹ Answers are displayed within the problem