5.5.1 Homework

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For all of the below homeworks, only consider matrices that have real valued elements.

Homework 5.5.1.1

1/1 point (graded)

Let A and B be matrices and AB be well-defined.

$(AB)^2 = A^2B^2$

Sometimes ▾ ✔ **Answer:** Sometimes

Explanation

Answer: Sometimes

The result is obviously true if $A = B$. (There are other examples. E.g., if A or B is a zero matrix, or if A or B is an identity matrix.)

If $A \neq B$, then the result is not well defined unless A and B are both square. (Why?). Let's assume A and B are both square. Even then, generally $(AB)^2 \neq A^2B^2$. Let

$$A = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}.$$

Then

$$(AB)^2 = ABAB = \begin{pmatrix} 2 & 3 \\ 3 & 5 \end{pmatrix}$$

and

$$A^2B^2 = AABB = \begin{pmatrix} 1 & 2 \\ 2 & 5 \end{pmatrix}.$$

(I used Python to check some possible matrices. There was nothing special about my choice of using triangular matrices.)

This may be counter intuitive since if α and β are scalars, then $(\alpha\beta)^2 = \alpha^2\beta^2$.

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Answers are displayed within the problem

Homework 5.5.1.2

1/1 point (graded)

Let A be symmetric.

A^2 is symmetric.

Always ▾ ✔ **Answer:** Always

Explanation

Answer: Always

$$(AA)^T = A^T A^T = AA.$$

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Homework 5.5.1.3

1/1 point (graded)

Let $A, B \in \mathbb{R}^{n \times n}$ both be symmetric.

AB is symmetric.

Sometimes ▾ ✔ **Answer:** Sometimes

Explanation

Answer: Sometimes Simple examples of when it is *true*: $A = I$ and/or $B = I$. $A = 0$ and/or $B = 0$. All cases where $n = 1$.

Simple example of where it is NOT true:

$$A = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} 3 & 0 \\ 0 & -2 \end{pmatrix}$$

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Homework 5.5.1.4

1/1 point (graded)

Let $A, B \in \mathbb{R}^{n \times n}$ both be symmetric.

$A^2 - B^2$ is symmetric.

Always ▾ **Answer:** Always

Explanation

Answer: Always We just saw that AA is always symmetric. Hence AA and BB are symmetric. But adding two symmetric matrices yields a symmetric matrix, so the resulting matrix is symmetric.

Or:

$$(A^2 - B^2)^T = (A^2)^T - (B^2)^T = A^2 - B^2.$$

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Homework 5.5.1.5

1/1 point (graded)

Let $A, B \in \mathbb{R}^{n \times n}$ both be symmetric.

$(A + B)(A - B)$ is symmetric.

Sometimes ▾ **Answer:** Sometimes

Answer: Sometimes

Examples of when it IS symmetric: $A = B$ or $A = 0$ or $A = I$.

Examples of when it is NOT symmetric: Create random 2x2 matrices A and B in MATLAB. Then set $A := A^T A$ and $B = B^T B$ to make them symmetric. With probability 1 you will see that $(A+B)(A-B)$ is not symmetric. Here is an example:

$$A = \begin{pmatrix} 2 & 1 \\ 1 & 3 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}.$$

BUT, what we really want you to notice is that if you multiply out

$$(A+B)(A-B) = A^2 + BA - AB - B^2$$

the middle terms do NOT cancel. Compare this to the case where you work with real scalars:

$$(\alpha+\beta)(\alpha-\beta) = \alpha^2 + \beta\alpha - \alpha\beta - \beta^2 = \alpha^2 - \beta^2.$$

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Homework 5.5.1.6

1/1 point (graded)

Let $A, B \in \mathbb{R}^{n \times n}$ both be symmetric.

ABA is symmetric.

Always ▾ ✔ **Answer:** Always

Explanation

Answer: Always

$$(ABA)^T = A^T B^T A^T = ABA.$$

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Answers are displayed within the problem

Homework 5.5.1.7

1/1 point (graded)

Let $A, B \in \mathbb{R}^{n \times n}$ both be symmetric.

$ABAB$ is symmetric.

Sometimes ▾ ✔ **Answer:** Sometimes

Explanation

Answer: Sometimes It is *true* for, for example, $A = B$. But is is, for example, *false* for

$$A = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} 3 & 0 \\ 0 & -2 \end{pmatrix}$$

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Homework 5.5.1.8

1/1 point (graded)

Let A be symmetric.

$A^T A = A A^T$.

Always ▾ ✔ **Answer:** Always

Explanation

Answer: Always Trivial, since $A = A^T$.

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Answers are displayed within the problem

5.5.1.9 Homework

0/1 point (graded)

If $A = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}$ then $A^T A = A A^T$

True ▾ ✖ **Answer:** False

Explanation

Answer: False

$$\begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}^T \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix} = 2 \text{ and } \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}^T = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

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Answers are displayed within the problem

Homework 5.5.1.10

1/1 point (graded)

Propose an algorithm for computing $C := UR$ where C, U, and R are all upper triangular matrices by completing the below algorithm.

Algorithm: $[C] := \text{TRTRMM\_UU\_UNB\_VAR1}(U, R, C)$

Partition $U \rightarrow \left(\begin{array}{c|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right)$, $R \rightarrow \left(\begin{array}{c|c} R_{TL} & R_{TR} \\ \hline R_{BL} & R_{BR} \end{array}\right)$, $C \rightarrow \left(\begin{array}{c|c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array}\right)$

where U_{TL} is 0×0, R_{TL} is 0×0, C_{TL} is 0×0

while $m(U_{TL}) < m(U)$ **do**

Repartition

$$\left(\begin{array}{c|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c|c|c} U_{00} & u_{01} & U_{02} \\ \hline u_{10}^T & \upsilon_{11} & u_{12}^T \\ \hline U_{20} & u_{21} & U_{22} \end{array}\right), \left(\begin{array}{c|c} R_{TL} & R_{TR} \\ \hline R_{BL} & R_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c|c|c} R_{00} & r_{01} & R_{02} \\ \hline r_{10}^T & \rho_{11} & r_{12}^T \\ \hline R_{20} & r_{21} & R_{22} \end{array}\right),$$

$$\left(\begin{array}{c|c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c|c|c} C_{00} & c_{01} & C_{02} \\ \hline c_{10}^T & \gamma_{11} & c_{12}^T \\ \hline C_{20} & c_{21} & C_{22} \end{array}\right)$$

where υ_{11} is 1×1, ρ_{11} is 1×1, γ_{11} is 1×1

Continue with

$$\left(\begin{array}{c|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c|c|c} U_{00} & u_{01} & U_{02} \\ \hline u_{10}^T & \upsilon_{11} & u_{12}^T \\ \hline U_{20} & u_{21} & U_{22} \end{array}\right), \left(\begin{array}{c|c} R_{TL} & R_{TR} \\ \hline R_{BL} & R_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c|c|c} R_{00} & r_{01} & R_{02} \\ \hline r_{10}^T & \rho_{11} & r_{12}^T \\ \hline R_{20} & r_{21} & R_{22} \end{array}\right),$$

$$\left(\begin{array}{c|c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c|c|c} C_{00} & c_{01} & C_{02} \\ \hline c_{10}^T & \gamma_{11} & c_{12}^T \\ \hline C_{20} & c_{21} & C_{22} \end{array}\right)$$

endwhile

Hint: consider Homework 5.2.4.10.

Write the routine

- `[ C_out ] = Trtrmm_unb_var1( U, R, C )` that computes $C := UR$ where U, and R are upper triangular, using the above algorithm. (You will want to write your algorithm so as not to use any data below the diagonal of C, U, or R

Some links that will come in handy:

- Spark (alternatively, open the file `LAFFSpring2015 -> Spark -> index.html`)
- PictureFLAME (alternatively, open the file `LAFFSpring2015 -> PictureFLAME -> PictureFLAME.html`)

To implement this routine, you will want add the function

```
laff_trmv( uplo, trans, diag, A, x )
```

which, when called as

```
laff_trmv( 'Upper triangular', 'No transpose', 'Nonunit diag', U, x )
```

overwrites *x* with *U x* where *U* is upper triangular, stored in the upper triangular part of *U* . Download laff_trmv.m and place it in `LAFFSpring2015 -> laff -> matvec` .

You may want to use the following script to test your implementations:

- test_Trtrmm_unb_var1.m

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✔

Explanation

Answer: (continued)

Algorithm: $[C] := \text{TRTRMM\_UU\_UNB\_VAR1}(U, R, C)$

Partition $U \rightarrow \left(\begin{array}{c|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right)$, $R \rightarrow \left(\begin{array}{c|c} R_{TL} & R_{TR} \\ \hline R_{BL} & R_{BR} \end{array}\right)$, $C \rightarrow \left(\begin{array}{c|c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array}\right)$

where U_{TL} is 0×0, R_{TL} is 0×0, C_{TL} is 0×0

while $m(U_{TL}) < m(U)$ **do**

Repartition

$$\left(\begin{array}{c|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c|c|c} U_{00} & u_{01} & U_{02} \\ \hline u_{10}^T & \upsilon_{11} & u_{12}^T \\ U_{20} & u_{21} & U_{22} \end{array}\right), \left(\begin{array}{c|c} R_{TL} & R_{TR} \\ \hline R_{BL} & R_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c|cc} R_{00} & r_{01} & R_{02} \\ \hline r_{10}^T & \rho_{11} & r_{12}^T \\ R_{20} & r_{21} & R_{22} \end{array}\right),$$

$$\left(\begin{array}{c|c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c|c|c} C_{00} & c_{01} & C_{02} \\ \hline c_{10}^T & \gamma_{11} & c_{12}^T \\ \hline C_{20} & c_{21} & C_{22} \end{array}\right)$$

where υ_{11} is 1×1, ρ_{11} is 1×1, γ_{11} is 1×1

$c_{01} := U_{00} r_{01}$

$c_{01} := \rho_{11} u_{01} + c_{01}$

$\gamma_{11} := \rho_{11} \upsilon_{11}$

Continue with

$$\left(\begin{array}{c|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c|c|c} U_{00} & u_{01} & U_{02} \\ \hline u_{10}^T & \upsilon_{11} & u_{12}^T \\ \hline U_{20} & u_{21} & U_{22} \end{array}\right), \left(\begin{array}{c|c} R_{TL} & R_{TR} \\ \hline R_{BL} & R_{BR} \end{array}\right) \leftarrow \left(\begin{array}{cc|c} R_{00} & r_{01} & R_{02} \\ r_{10}^T & \rho_{11} & r_{12}^T \\ \hline R_{20} & r_{21} & R_{22} \end{array}\right),$$

$$\left(\begin{array}{c|c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array}\right) \leftarrow \left(\begin{array}{cc|c} C_{00} & c_{01} & C_{02} \\ \hline c_{10}^T & \gamma_{11} & c_{12}^T \\ \hline C_{20} & c_{21} & C_{22} \end{array}\right)$$

endwhile

Trtrmm_unb_var1.m

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Answers are displayed within the problem

Challenge 5.5.1.11

There is another challenge question in the notes. Skipped here.

Challenge 5.5.1.12

Propose many algorithms for computing $C := UR$ where C, U, and R are all upper triangular matrices. This time, derive all algorithm systematically by following the methodology in

The Science of Programming Matrix Computations (You will want to read Chapters 2-5.) You may want to use this PDF for a partially filled out worksheet.

(No credit for this one. It is just a challenge!)