edX

Course › Week... › 2.2 Li... › 2.2.2 ...

2.2.2 What is a linear transformation?

2.2.2 What is a linear transformation?

result is equal to

that result.

Clearly they are.

So we're in good shape.

And therefore we conclude that both f of alpha x and alpha times f of x

evaluate to the same expression.

And therefore they are equal.

A lot of people like to present this as this is equal to that is equal to

that is equal to that.

And then they would like to keep going.

Now this is just the second argument on the earlier slide but backwards.

This is equal to that is equal to that is equal to that.

0:00 / 11:35 1.0x And if you put the two

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Topic: Week 2 / 2.2.2

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? Homework 2.2.2.7. didn't get it
why the choice f(x)=||x||2 is wrong? 3

2.2.2.5
That's a very interesting solution that shows me I need to brush up on my logic! 2

? L(0) 2

Homework 2.2.2.1

1/1 point (graded)

The vector function $f\left(\begin{pmatrix}\chi\\ \psi\end{pmatrix}\right) = \begin{pmatrix}\chi\psi\\ \chi\end{pmatrix}$ is a linear transformation.

FALSE ▾ ✔ **Answer:** FALSE

After you answer, try to prove your response. Be sure to check the solution, since part of what we want you to learn is often in the solution to a problem. (This is the last time we repeat this.)

Explanation

Transcripted in final section of this week

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Answer: FALSE The first check should be whether $f(0) = 0$. The answer in this case is *yes*. However,

$$f(2\begin{pmatrix} 1 \\ 1 \end{pmatrix}) = f(\begin{pmatrix} 2 \\ 2 \end{pmatrix}) = \begin{pmatrix} 2 \times 2 \\ 2 \end{pmatrix} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$$

and

$$2f(\begin{pmatrix} 1 \\ 1 \end{pmatrix}) = 2\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \end{pmatrix}.$$

Hence, there is a vector $x \in \mathbb{R}^2$ and $\alpha \in \mathbb{R}$ such that $f(\alpha x) \neq \alpha f(x)$. We conclude that this function is *not* a linear transformation.
(Obviously, you may come up with other examples that show the function is not a linear transformation.)

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Homework 2.2.2.2

1/1 point (graded)

$$f(\begin{pmatrix} \chi_0 \\ \chi_1 \\ \chi_2 \end{pmatrix}) = \begin{pmatrix} \chi_0 + 1 \\ \chi_1 + 2 \\ \chi_2 + 3 \end{pmatrix}$$ is a linear transformation.

FALSE ▾ **Answer:** FALSE

Explanation

Answer: FALSE
In Homework 1.4.6.1 you saw a number of examples where $f(\alpha x) \neq \alpha f(x)$.

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Homework 2.2.2.3

1/1 point (graded)

$f\left(\begin{pmatrix} \chi_0 \\ \chi_1 \\ \chi_2 \end{pmatrix}\right) = \begin{pmatrix} \chi_0 \\ \chi_0 + \chi_1 \\ \chi_0 + \chi_1 + \chi_2 \end{pmatrix}$ is a linear transformation.

TRUE ▾ ✔ **Answer:** TRUE

Explanation

Answer: TRUE

Pick arbitrary $\alpha \in \mathbb{R}$, $x = \begin{pmatrix} \chi_0 \\ \chi_1 \\ \chi_2 \end{pmatrix}$, and $y = \begin{pmatrix} \psi_0 \\ \psi_1 \\ \psi_2 \end{pmatrix}$. Then

- Show $f(\alpha x) = \alpha f(x)$:

$$f(\alpha x) = f(\alpha \begin{pmatrix} \chi_0 \\ \chi_1 \\ \chi_2 \end{pmatrix}) = f(\begin{pmatrix} \alpha\chi_0 \\ \alpha\chi_1 \\ \alpha\chi_2 \end{pmatrix}) = \begin{pmatrix} \alpha\chi_0 \\ \alpha\chi_0 + \alpha\chi_1 \\ \alpha\chi_0 + \alpha\chi_1 + \alpha\chi_2 \end{pmatrix}$$

and

$$\alpha f(x) = \alpha f(\begin{pmatrix} \chi_0 \\ \chi_1 \\ \chi_2 \end{pmatrix}) = \alpha \begin{pmatrix} \chi_0 \\ \chi_0 + \chi_1 \\ \chi_0 + \chi_1 + \chi_2 \end{pmatrix} = \begin{pmatrix} \alpha\chi_0 \\ \alpha(\chi_0 + \chi_1) \\ \alpha(\chi_0 + \chi_1 + \chi_2) \end{pmatrix} = \begin{pmatrix} \alpha\chi_0 \\ \alpha\chi_0 + \alpha\chi_1 \\ \alpha\chi_0 + \alpha\chi_1 + \alpha\chi_2 \end{pmatrix}.$$

Thus, $f(\alpha x) = \alpha f(x)$.

- Show $f(x + y) = f(x) + f(y)$:

$$f(x+y) = f(\begin{pmatrix} \chi_0 \\ \chi_1 \\ \chi_2 \end{pmatrix} + \begin{pmatrix} \psi_0 \\ \psi_1 \\ \psi_2 \end{pmatrix}) = f(\begin{pmatrix} \chi_0 + \psi_0 \\ \chi_1 + \psi_1 \\ \chi_2 + \psi_2 \end{pmatrix}) = \begin{pmatrix} \chi_0 + \psi_0 \\ (\chi_0 + \psi_0) + (\chi_1 + \psi_1) \\ (\chi_0 + \psi_0) + (\chi_1 + \psi_1) + (\chi_2 + \psi_2) \end{pmatrix}$$

and

$$\begin{aligned} f(x) + f(y) &= f(\begin{pmatrix} \chi_0 \\ \chi_1 \\ \chi_2 \end{pmatrix}) + f(\begin{pmatrix} \psi_0 \\ \psi_1 \\ \psi_2 \end{pmatrix}) = \begin{pmatrix} \chi_0 \\ \chi_0 + \chi_1 \\ \chi_0 + \chi_1 + \chi_2 \end{pmatrix} + \begin{pmatrix} \psi_0 \\ \psi_0 + \psi_1 \\ \psi_0 + \psi_1 + \psi_2 \end{pmatrix} \\ &= \begin{pmatrix} \chi_0 + \psi_0 \\ (\chi_0 + \chi_1) + (\psi_0 + \psi_1) \\ (\chi_0 + \chi_1 + \chi_2) + (\psi_0 + \psi_1 + \psi_2) \end{pmatrix} = \begin{pmatrix} \chi_0 + \psi_0 \\ (\chi_0 + \psi_0) + (\chi_1 + \psi_1) \\ (\chi_0 + \psi_0) + (\chi_1 + \psi_1) + (\chi_2 + \psi_2) \end{pmatrix} \end{aligned}$$

Hence $f(x + y) = f(x) + f(y)$.

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Homework 2.2.2.4

1/1 point (graded)

If $L : \mathbb{R}^n \to \mathbb{R}^m$ is a linear transformation, then $L(0) = 0$. (Recall that here 0 represents vectors of appropriate size whose components are all 0.)

Always ✔ **Answer:** Always

Explanation

Transcripted in final section of this week

Answer: Always. We know that for all scalars α and vector $x \in \mathbb{R}^n$ it is the case that $L(\alpha x) = \alpha L(x)$. Now, pick $\alpha = 0$. We know that for this choice of α it has to be the case that $L(\alpha x) = \alpha L(x)$. We conclude that $L(0x) = 0L(x)$. But $0x = 0$. (Here the first 0 is the scalar 0 and the second is the vector with n components all equal to zero.) Similarly, regardless of what vector $L(x)$ equals, multiplying it by the scalar zero yields the vector 0 (with m zero components). So, $L(0x) = 0L(x)$ implies that $L(0) = 0$.

A typical mathematician would be much more terse, writing down merely: Pick $\alpha = 0$. Then

$$L(0) = L(0x) = L(\alpha x) = \alpha L(x) = 0L(x) = 0.$$

There are actually many ways of proving this:

$$L(0) = L(x - x) = L(x + (-x)) = L(x) + L(-x) = L(x) + (-L(x)) = L(x) - L(x) = 0.$$

Alternatively, $L(x) = L(x + 0) = L(x) + L(0)$, hence $L(0) = L(x) - L(x) = 0$.

Typically, it is really easy to evaluate $f(0)$. Therefore, if you think a given vector function f is *not* a linear transformation, then you may want to first evaluate $f(0)$. If it does not evaluate to the zero vector, then you know it is not a linear transformation.

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Homework 2.2.2.5

1/1 point (graded)

Let $f : \mathbb{R}^n \to \mathbb{R}^m$ and $f(0) \neq 0$. Then f is not a linear transformation.

TRUE ▾ ✔ **Answer:** TRUE

Explanation

Transcripted in final section of this week

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Homework 2.2.2.6

1/1 point (graded)

If $f : \mathbb{R}^n \to \mathbb{R}^m$ and $f(0) = 0$.

Then f is a linear transformation.

Sometimes ▾ **Answer:** Sometimes

Explanation

Transcripted in final section of this week

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Homework 2.2.2.7

0/1 point (graded)

For which of the following is $f(\alpha x) = \alpha f(x)$ for all α and all x but there are examples for x and y such that $f(x+y) \neq f(x) + f(y)$

- $f\left(\begin{pmatrix} \chi_0 \\ \chi_1 \end{pmatrix}\right) = \begin{cases} \chi_0 & if\; \chi_0 = \chi_1 \\ 0 & otherwise \end{cases}$ ✔
- $f(x) = \|x\|_2$
- $f(x) = x$

- $f(x) = 3x$
- None of the above

Explanation

Answer: $f(\begin{pmatrix} 0 \\ 1 \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \end{pmatrix}) = f(\begin{pmatrix} 1 \\ 1 \end{pmatrix}) = 1$ but $f(\begin{pmatrix} 0 \\ 1 \end{pmatrix}) + f(\begin{pmatrix} 1 \\ 0 \end{pmatrix}) = 0 + 0 = 0.$

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Homework 2.2.2.8

1/1 point (graded)

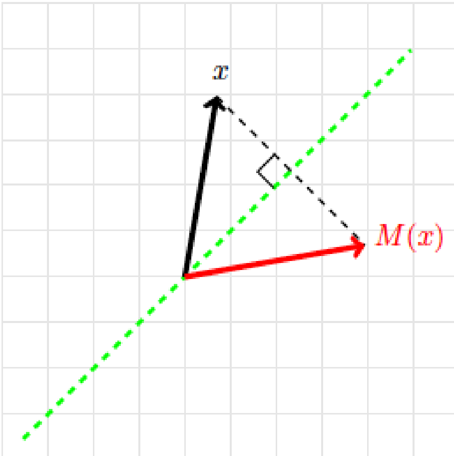
$f(\begin{pmatrix} \chi_0 \\ \chi_1 \end{pmatrix}) = \begin{pmatrix} \chi_1 \\ \chi_0 \end{pmatrix}$ is a linear transformation.

TRUE ▾ ✔ **Answer:** TRUE

Explanation

Answer: TRUE

This is actually the reflection with respect to 45 degrees line that we talked about earlier:



Pick arbitrary $\alpha \in \mathbb{R}$, $x = \begin{pmatrix} \chi_0 \\ \chi_1 \end{pmatrix}$, and $y = \begin{pmatrix} \psi_0 \\ \psi_1 \end{pmatrix}$. Then

- Show $f(\alpha x) = \alpha f(x)$:

$$f(\alpha x) = f(\alpha \begin{pmatrix} \chi_0 \\ \chi_1 \end{pmatrix}) = f(\begin{pmatrix} \alpha\chi_0 \\ \alpha\chi_1 \end{pmatrix}) = \begin{pmatrix} \alpha\chi_1 \\ \alpha\chi_0 \end{pmatrix} = \alpha \begin{pmatrix} \chi_1 \\ \chi_0 \end{pmatrix} = \alpha f(\begin{pmatrix} \chi_0 \\ \chi_1 \end{pmatrix}).$$

- Show $f(x+y) = f(x) + f(y)$:

$$f(x+y) = f(\begin{pmatrix} \chi_0 \\ \chi_1 \end{pmatrix} + \begin{pmatrix} \psi_0 \\ \psi_1 \end{pmatrix}) = f(\begin{pmatrix} \chi_0+\psi_0 \\ \chi_1+\psi_1 \end{pmatrix}) = \begin{pmatrix} \chi_1+\psi_1 \\ \chi_0+\psi_0 \end{pmatrix}$$

  and

$$\begin{aligned} f(x)+f(y) &= f(\begin{pmatrix} \chi_0 \\ \chi_1 \end{pmatrix}) + f(\begin{pmatrix} \psi_0 \\ \psi_1 \end{pmatrix}) = \begin{pmatrix} \chi_1 \\ \chi_0 \end{pmatrix} + \begin{pmatrix} \psi_1 \\ \psi_0 \end{pmatrix} \\ &= \begin{pmatrix} \chi_1+\psi_1 \\ \chi_0+\psi_0 \end{pmatrix}. \end{aligned}$$

  Hence $f(x+y) = f(x) + f(y)$.

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Answers are displayed within the problem