

A System maintenance is scheduled for Wednesday, August 29, 2018 from 14:30-15:30 UTC. Courses might not be available during this time.

Course > Week... > 4.6 W... > 4.6.2 ...

4.6.2 Summary

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Partitioned matrix-vector multiplication

$$\left(\begin{array}{c|c|c} A_{0,0} & A_{0,1} & \cdots & A_{0,N-1} \\ \hline A_{1,0} & A_{1,1} & \cdots & A_{1,N-1} \\ \hline \vdots & \vdots & \ddots & \vdots \\ \hline A_{M-1,0} & A_{M-1,1} & \cdots & A_{M-1,N-1} \end{array}\right) \left(\begin{array}{c} x_0 \\ \hline x_1 \\ \hline \vdots \\ \hline x_{N-1} \end{array}\right) = \left(\begin{array}{c|c|c} A_{0,0}x_0 + A_{0,1}x_1 + \cdots + A_{0,N-1}x_{N-1} \\ \hline A_{1,0}x_0 + A_{1,1}x_1 + \cdots + A_{1,N-1}x_{N-1} \\ \hline \vdots \\ \hline A_{M-1,0}x_0 + A_{M-1,1}x_1 + \cdots + A_{M-1,N-1}x_{N-1} \end{array}\right)$$

Transposing a partitioned matrix

$$\left(\begin{array}{c|c|c|c|c} A_{0,0} & A_{0,1} & \cdots & A_{0,N-1} \\ \hline A_{1,0} & A_{1,1} & \cdots & A_{1,N-1} \\ \hline \vdots & \vdots & & \vdots \\ \hline A_{M-1,0} & A_{M-1,1} & \cdots & A_{M-1,N-1} \end{array}\right)^T = \left(\begin{array}{c|c|c|c|c|c} A_{0,0}^T & A_{1,0}^T & \cdots & A_{M-1,0}^T \\ \hline A_{0,1}^T & A_{1,1}^T & \cdots & A_{M-1,1}^T \\ \hline \vdots & \vdots & & \vdots \\ \hline A_{0,N-1}^T & A_{1,N-1}^T & \cdots & A_{M-1,N-1}^T \end{array}\right).$$

Composing linear transformations

Let $L_A:\mathbb{R}^k o\mathbb{R}^m$ and $L_B:\mathbb{R}^n o\mathbb{R}^k$ both be linear transformations and, for all $x\in\mathbb{R}^n$, define the function $L_C:\mathbb{R}^n o\mathbb{R}^m$ by $L_C(x)=L_A(L_B(x))$. Then $L_C(x)$ is a linear transformations.

Matrix-matrix multiplication

$$AB = A \left(\begin{array}{c|c} b_0 & b_1 & \cdots & b_{n-1} \end{array} \right) = \left(\begin{array}{c|c} Ab_0 & Ab_1 & \cdots & Ab_{n-1} \end{array} \right).$$

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$$C = egin{pmatrix} \gamma_{0,0} & \gamma_{0,1} & \cdots & \gamma_{0,n-1} \ \gamma_{1,0} & \gamma_{1,1} & \cdots & \gamma_{1,n-1} \ dots & dots & dots & dots \ \gamma_{m-1,0} & \gamma_{m-1,1} & \cdots & \gamma_{m-1,n-1} \end{pmatrix}, \quad A = egin{pmatrix} lpha_{0,0} & lpha_{0,1} & \cdots & lpha_{0,k-1} \ lpha_{1,0} & lpha_{1,1} & \cdots & lpha_{1,k-1} \ dots & dots & dots & dots \ lpha_{m-1,0} & lpha_{m-1,1} & \cdots & lpha_{m-1,k-1} \ \end{pmatrix}, \quad ext{and} \quad B = egin{pmatrix} eta_{0,0} & eta_{0,1} & \cdots & eta_{0,n-1} \ eta_{1,0} & eta_{1,1} & \cdots & eta_{1,n-1} \ dots & dots & dots \ eta_{k-1,0} & eta_{k-1,1} & \cdots & eta_{k-1,n-1} \ \end{pmatrix}.$$

then C=AB means that $\gamma_{i,j}=\sum_{p=0}^{k-1}lpha_{i,p}eta_{p,j}$.

A table of matrix-matrix multiplications with matrices of special shape is given at the end of this unit.

Outer product

Let $x \in \mathbb{R}^m$ and $y \in \mathbb{R}^n$. Then the *outer product* of x and y is given by xy^T . Notice that this yields an $m \times n$ matrix:

$$egin{array}{lll} oldsymbol{x} oldsymbol{x}^T &=& egin{pmatrix} \chi_0 \ \chi_1 \ dots \ \chi_{m-1} \end{pmatrix} egin{pmatrix} \psi_0 \ \psi_1 \ dots \ \psi_{m-1} \end{pmatrix}^T &=& egin{pmatrix} \chi_0 \ \chi_1 \ dots \ \chi_{m-1} \end{pmatrix} (\psi_0 \quad \psi_1 \quad \cdots \quad \psi_{n-1}) \ dots \ \chi_{m-1} \psi_0 & \chi_0 \psi_1 & \cdots & \chi_0 \psi_{n-1} \ \chi_1 \psi_0 & \chi_1 \psi_1 & \cdots & \chi_1 \psi_{n-1} \ dots \ \chi_{m-1} \psi_0 & \chi_{m-1} \psi_1 & \cdots & \chi_{m-1} \psi_{n-1} \end{pmatrix}. \end{array}$$

m	n	k	Shape	Comment
1	1	1	$1 \downarrow \overline{C} = 1 \downarrow \overline{A} \qquad 1 \downarrow \overline{B}$	Scalar multiplication
m	1	1	$ \begin{array}{c c} 1 & 1 \\ \hline C & = m \\ A \end{array} $	Vector times scalar = scalar times vector
1	n	1	$1 \updownarrow $	Scalar times row vector
1	1	k	$1 \updownarrow \boxed{C} = 1 \updownarrow \boxed{A}$ $k \downarrow \boxed{B}$	Dot product (with row and column)
m	n	1	$ \begin{array}{c c} & n & & 1 & & n \\ \hline & & & & & \\ & & & & & \\ & & & & & \\ & & & &$	Outer product
m	1	k	$ \begin{array}{c c} 1 & k & 1 \\ \hline C & = m & k & B \end{array} $	Matrix-vector multiplication
1	n	k	$ \begin{array}{c c} $	Row vector times matrix multiply

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