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## **5.2.3 Transposing a Product of Matrices**

# 5.2.3 Transposing a Product of Matrices

No introductory video



### Discussion

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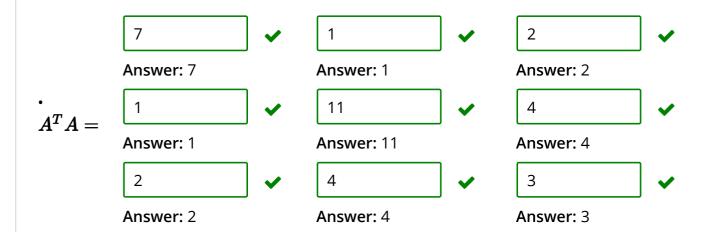
**Topic:** Week 5 / 5.2.3

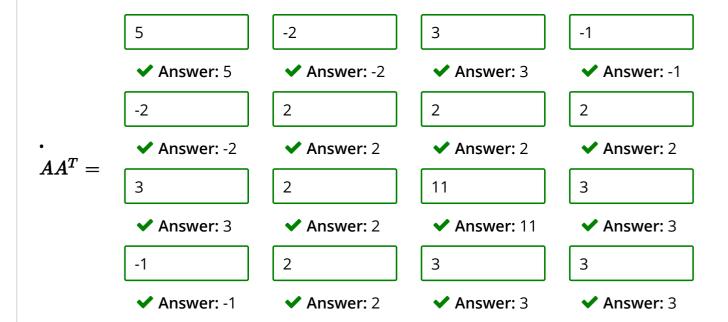
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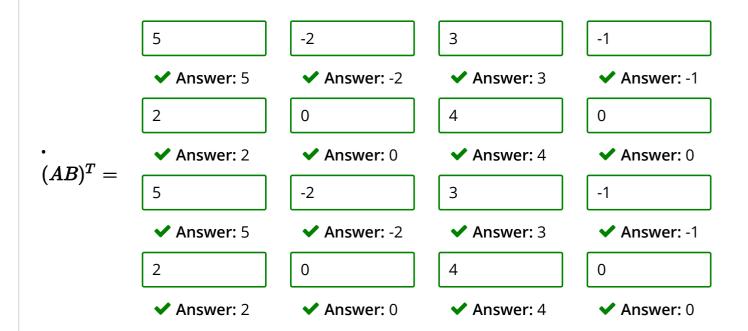
### Homework 5.2.3.1

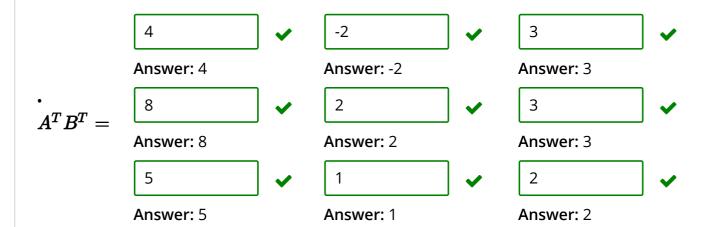
66/66 points (graded)

Let 
$$A=egin{pmatrix} 2&0&1\\-1&1&0\\1&3&1\\-1&1&1 \end{pmatrix}$$
 and  $B=egin{pmatrix} 2&1&2&1\\0&1&0&1\\1&0&1&0 \end{pmatrix}$  . Compute









$\dot{B}^TA^T=$	5	-2	3	-1
	<b>✓</b> Answer: 5	<b>✓</b> Answer: -2	<b>✓ Answer:</b> 3	✓ Answer: -1
	2	0	4	0
	✓ Answer: 2	<b>✓ Answer:</b> 0	✓ Answer: 4	✓ Answer: 0
	5	-2	3	-1
	<b>✓</b> Answer: 5	<b>✓</b> Answer: -2	<b>✓ Answer:</b> 3	✓ Answer: -1
	2	0	4	0
	✓ Answer: 2	✓ Answer: 0	✓ Answer: 4	<b>✓ Answer:</b> 0

$$\bullet \ A^T A = \left( \begin{array}{ccc} 7 & 1 & 2 \\ 1 & 11 & 4 \\ 2 & 4 & 3 \end{array} \right)$$

$$\bullet \ AA^T = \left( \begin{array}{cccc} 5 & -2 & 3 & -1 \\ -2 & 2 & 2 & 2 \\ 3 & 2 & 11 & 3 \\ -1 & 2 & 3 & 3 \end{array} \right)$$

$$\bullet \ (AB)^T = \left( \begin{array}{cccc} 5 & -2 & 3 & -1 \\ 2 & 0 & 4 & 0 \\ 5 & -2 & 3 & -1 \\ 2 & 0 & 4 & 0 \end{array} \right)$$

$$\bullet \ A^T B^T = \left( \begin{array}{ccc} 4 & -2 & 3 \\ 8 & 2 & 3 \\ 5 & 1 & 2 \end{array} \right)$$

$$\bullet \ B^T A^T = \left( \begin{array}{cccc} 5 & -2 & 3 & -1 \\ 2 & 0 & 4 & 0 \\ 5 & -2 & 3 & -1 \\ 2 & 0 & 4 & 0 \end{array} \right)$$

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**1** Answers are displayed within the problem

### Homework 5.2.3.2

1/1 point (graded)

Let  $A \in \mathbb{R}^{m imes k}$  and  $B \in \mathbb{R}^{k imes n}$ .  $(AB)^T = B^T A^T$ .

Always •

✓ **Answer:** Always

#### Answer:

#### Proof 1:

In an example in the previous unit, we partitioned C into elements (scalars) and A and B by rows and columns, respectively, before performing the partitioned matrix-matrix multiplication C = AB. This

insight forms the basis for the following proof:

$$(AB)^T = \langle \text{Partition } A \text{ by rows and } B \text{ by columns} \rangle$$

$$\left( \left( \frac{\tilde{a}_0^T}{\frac{\tilde{a}_1^T}{\vdots}} \right) \left( b_0 \mid b_1 \mid \dots \mid b_{n-1} \right) \right)^T$$

= < Partitioned matrix-matrix multiplication >

= < Transpose the matrix >

$$\begin{pmatrix}
\tilde{a}_{0}^{T}b_{0} & \tilde{a}_{1}^{T}b_{0} & \cdots & \tilde{a}_{m-1}^{T}b_{0} \\
\tilde{a}_{0}^{T}b_{1} & \tilde{a}_{1}^{T}b_{1} & \cdots & \tilde{a}_{m-1}^{T}b_{1} \\
\vdots & \vdots & \ddots & \vdots \\
\tilde{a}_{0}^{T}b_{n-1} & \tilde{a}_{1}^{T}b_{n-1} & \cdots & \tilde{a}_{m-1}^{T}b_{n-1}
\end{pmatrix}$$

= < dot product commutes >

$$\begin{pmatrix} b_0^T \tilde{a}_0 & b_0^T \tilde{a}_1 & \cdots & b_0^T \tilde{a}_{m-1} \\ \hline b_1^T \tilde{a}_0 & b_1^T \tilde{a}_1 & \cdots & b_1^T \tilde{a}_{m-1} \\ \hline \vdots & \vdots & \ddots & \vdots \\ \hline b_{n-1}^T \tilde{a}_0 & b_{n-1}^T \tilde{a}_1 & \cdots & b_{m-1}^T \tilde{a}_{m-1} \end{pmatrix}$$

= < Partitioned matrix-matrix multiplication >

$$\left( \frac{b_0^T}{b_1^T} \right) \left( \tilde{a}_0 \mid \tilde{a}_1 \mid \dots \mid \tilde{a}_{m-1} \right) \\
 \overline{b_{n-1}^T} \right)$$

= < Partitioned matrix transposition >

$$\left(\begin{array}{c|c}b_0\mid b_1\mid\cdots\mid b_{n-1}\end{array}\right)^T \left(\begin{array}{c} \tilde{a}_0^T\\ \hline \tilde{a}_1^T \end{array}\right)^T = B^TA^T.$$

$$\left(\frac{:}{\tilde{a}_{m-1}^T}\right)$$

#### Proof 2:

Let C = AB and  $D = B^TA^T$ . We need to show that  $\gamma_{i,j} = \delta_{j,i}$ .

But

$$\begin{split} \gamma_{i,j} &= &< \text{Earlier observation} > \\ e_i^T C e_j &= &< C = AB > \\ e_i^T (AB) e_j &= &< \text{Associativity of multiplication; } e_i^T \text{ and } e_j \text{ are matrices} > \\ &(e_i^T A) (B e_j) &= &< \text{Property of multiplication; } \widetilde{a}_i^T \text{ is } i \text{th row of } A, b_j \text{ is } j \text{th column of } B > \\ \widetilde{a}_i^T b_j &= &< \text{Dot product commutes} > \\ b_j^T \widetilde{a}_i &= &< \text{Property of multiplication} > \\ &(e_j^T B^T) (A^T e_i) &= &< \text{Associativity of multiplication; } e_i^T \text{ and } e_j \text{ are matrices} > \\ e_j^T (B^T A^T) e_i &= &< C = AB > \\ e_j^T D e_i &= &< \text{earlier observation} > \\ \delta_{j,i} &= &< \text{earlier observation} > \end{split}$$

#### Proof 3:

(I vaguely recall that somewhere we proved that $(Ax)^T = x^T A^T$ If not, one should prove that first)

$$(AB)^{T} = \langle \text{Partition } B \text{ by columns} \rangle$$

$$(A \left( \begin{array}{cccc} b_{0} & b_{1} & \cdots & b_{n-1} \end{array} \right))^{T} \\ = \langle \text{Partitioned matrix-matrix multiplication} \rangle$$

$$\left( \begin{array}{cccc} Ab_{0} & Ab_{1} & \cdots & Ab_{n-1} \end{array} \right)^{T} \\ = \langle \text{Transposing a partitioned matrix} \rangle$$

$$\left( \begin{array}{cccc} (Ab_{0})^{T} \\ (Ab_{1})^{T} \\ \vdots \\ (Ab_{n-1})^{T} \end{array} \right)$$

$$= \langle (Ax)^{T} = x^{T}A^{T} \rangle$$

$$= \langle (Ax)^{T} = x^{T}A^{T} \rangle$$

$$= \langle (b_{0}^{T}A^{T}) \\ b_{1}^{T}A^{T} \\ \vdots \\ b_{n-1}^{T}A^{T} \rangle$$

$$= \langle (b_{0}^{T}A^{T}) \\ b_{1}^{T} \\ \vdots \\ b_{n-1}^{T} \rangle$$

$$= \langle (b_{0}^{T}A^{T}) \\ - \langle ($$

**Proof 4:** (For those who don't like the · · · in arguments...)

Proof by induction on n, the number of columns of B.

(I vaguely recall that somewhere we proved that  $(Ax)^T = x^T A^T$ ... If not, one should prove that first...)

**Base case:** n = 1. Then  $B = (b_0)$ . But then  $(AB)^T = (Ab_0)^T = b_0^T A^T = B^T A^T$ .

**Inductive Step:** The inductive hypothesis is: Assume that  $(AB)^T = B^T A^T$  for all matrices B with n = N columns. We now need to show that, assuming this,  $(AB)^T = B^T A^T$  for all matrices B with n = N + 1columns.

#### Assume that B has N+1 columns. Then

$$(AB)^{T}$$

$$= \langle Partition B \rangle$$

$$(A \begin{pmatrix} B_{0} & b_{1} \end{pmatrix})^{T}$$

$$= \langle Partitioned matrix-matrix multiplication \rangle$$

$$(\begin{pmatrix} AB_{0} & Ab_{1} \end{pmatrix})^{T}$$

$$= \langle Partitioned matrix transposition \rangle$$

$$(\begin{pmatrix} (AB_{0})^{T} \\ (Ab_{1})^{T} \end{pmatrix})$$

$$= \langle I.H. \text{ and } (Ax)^{T} = x^{T}A^{T} \rangle$$

$$\begin{pmatrix} B_{0}^{T}A^{T} \\ b_{1}^{T}A^{T} \end{pmatrix}$$

$$= \langle Partitioned matrix-matrix multiplication \rangle$$

$$\begin{pmatrix} B_{0}^{T} \\ b_{1}^{T} \end{pmatrix} A^{T}$$

$$= \langle Transposing a partitioned matrix \rangle$$

$$\begin{pmatrix} B_{0} & b_{1} \end{pmatrix}^{T}A^{T}$$

$$= \langle Partitioning of B \rangle$$

$$B^{T}A^{T}$$

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**1** Answers are displayed within the problem

### Homework 5.2.3.3

1/1 point (graded)

Let  $A,B,\ {
m and}\ C$  be conformal matrices so that ABC is well-defined. Then  $(ABC)^{T} = C^{T}B^{T}A^{T}$ 

Always

✓ Answer: Always

### **Explanation**

Answer: Always

$$(ABC)^T = (A(BC))^T = (BC)^TA^T = (C^TB^T)A^T = C^TB^TA^T$$

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**1** Answers are displayed within the problem

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