# 2.3.2 Examples

2.3.2 Examples

Now, remember, the sum i equals 0 to n minus 1 of i is just 0 plus 1 plus dot

dot dot plus n minus 1.

So we're just summing the first n non-negative integers.

I'm picking this particular example, not only because it's relatively

simple, but also because we're going to see this particular summation when

we analyze the cost of various operations next week.

How do you prove this?

Well, you start with the base case.

You set n equals 1, and you state what is it that I need to show.

And what it is that you need to show, you get from simply taking this

0:00 / 6:43 1.0x

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Start of transcript. Skip to the end.

Dr. Robert van de Geijn: Now, an alternative proof that actually helps

you remember this particular result--

and you should remember this particular result, because it is going

to come up in our analyses of cost of various algorithms.

Now notice that the sum from i equal to 0 to n minus 1 of i is

0:00 / 2:02 1.0x

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HW 2.3.2.4
Just out of interest, what method would you normally use for (k+1)(2k+1)? You said you don't normally u... 4

# Homework 2.3.2.1

1/1 point (graded)

Let $n \geq 1$. Then $\sum_{i=1}^{n} i = n(n+1)/2$

Always ▼ **Answer:** Always

## Explanation

Transcripted in final section of this week

Click to see PDF for answer in video

**Answer:** We can prove this in three different ways:

1. By mathematical induction, carefully mimicing the proof that $\sum_{i=0}^{n-1} i = (n-1)n/2$; or
2. Using a trick similar to the one used in the alternative proof given for $\sum_{i=0}^{n-1} i = (n-1)n/2$; or
3. Using the fact that $\sum_{i=0}^{n-1} i = n(n-1)/2$.

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Answers are displayed within the problem

# Homework 2.3.2.2

1/1 point (graded)

Let $n \geq 1$

$\sum_{i=0}^{n-1} 1 = n$

Always ▾ **Answer:** Always

**Explanation**

**Answer:** Always.

**Base case:** $n = 1$. For this case, we must show that $\sum_{i=0}^{1-1} 1 = 1$.

$\sum_{i=0}^{1-1} 1$

$=$ (Definition of summation)

$1$

This proves the base case.

**Inductive step:** Inductive Hypothesis (IH): Assume that the result is true for $n = k$ where $k \geq 1$:

$$\sum_{i=0}^{k-1} 1 = k.$$

We will show that the result is then also true for $n = k + 1$:

$$\sum_{i=0}^{(k+1)-1} 1 = (k + 1).$$

Assume that $k \geq 1$. Then

$\sum_{i=0}^{(k+1)-1} 1$

$=$ (arithmetic)

$\sum_{i=0}^{k} 1$

$=$ (split off last term)

$\left(\sum_{i=0}^{k-1} 1\right) + 1$

$=$ (I.H.)

$k + 1$.

This proves the inductive step.

**By the Principle of Mathematical Induction** the result holds for all $n$.

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## Homework 2.3.2.3

1/1 point (graded)

Let $n \geq 1$ and $x \in \mathbb{R}$.

$$\sum_{i=0}^{n-1} x = \underbrace{x + x + \cdots + x}_{n} = nx$$

Always ▾ ✔ **Answer:** Always

## Explanation

**Answer:** Always.

$$\sum_{i=0}^{n-1} x = \left(\sum_{i=0}^{n-1} 1\right) x = nx.$$

However, we want you to prove this with mathematical induction:

**Base case:** $n = 1$. For this case, we must show that $\sum_{i=0}^{1-1} x = x$.

$\sum_{i=0}^{1-1} x$

$=$ $\quad$ < Definition of summation>

$x$

This proves the base case.

**Inductive step:** Inductive Hypothesis (IH): Assume that the result is true for $n = k$ where $k \geq 1$:

$$\sum_{i=0}^{k-1} x = kx.$$

We will show that the result is then also true for $n = k + 1$:

$$\sum_{i=0}^{(k+1)-1} x = (k+1)x.$$

Assume that $k \geq 1$. Then

$\sum_{i=0}^{(k+1)-1} x$

$=$ $\quad$ < arithmetic>

$\sum_{i=0}^{k} x$

$=$ $\quad$ < split off last term>

$\sum_{i=0}^{k-1} x + x$

$=$ $\quad$ < I.H.>

$kx + x$

$=$ $\quad$ < algebra>

$(k+1)x$

This proves the inductive step.

**By the Principle of Mathematical Induction** the result holds for all $n$.

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Answers are displayed within the problem

## Homework 2.3.2.4

1/1 point (graded)

Let $n \geq 1$

$\sum_{i=0}^{n-1} i^2 = (n-1)n(2n-1)/6$

Always

**Answer:** Always

**Explanation**

Transcripted in final section of this week

Click to see PDF of answer in video

Always

**Base case:** $n = 1$. For this case, we must show that $\sum_{i=0}^{1-1} i^2 = (1-1)(1)(2(1)-1)/6$. But $\sum_{i=0}^{1-1} i^2 = 0 = (1-1)(1)(2(1)-1)/6$. This proves the base case.

**Inductive step:** Inductive Hypothesis (IH): Assume that the result is true for $n = k$ where $k \geq 1$:

$$\sum_{i=0}^{k-1} i^2 = (k-1)k(2k-1)/6.$$

We will show that the result is then also true for $n = k+1$:

$$\sum_{i=0}^{(k+1)-1} i^2 = ((k+1)-1)(k+1)(2(k+1)-1)/6 = (k)(k+1)(2k+1)/6.$$

Assume that $k \geq 1$. Then

$$\begin{array}{ll} \sum_{i=0}^{(k+1)-1} i^2 & \\ = & \text{(arithmetic)} \\ \sum_{i=0}^{k} i^2 & \\ = & \text{(split off last term)} \\ \sum_{i=0}^{k-1} i^2 + k^2 & \\ = & \text{(I.H.)} \\ (k-1)k(2k-1)/6 + k^2 & \\ = & \text{(algebra)} \\ [(k-1)k(2k-1) + 6k^2]/6. & \end{array}$$

Now,

$$(k)(k+1)(2k+1) = (k^2+k)(2k+1) = 2k^3 + 2k^2 + k^2 + k = 2k^3 + 3k^2 + k$$

and

$$(k-1)k(2k-1) + 6k^2 = (k^2-k)(2k-1) + 6k^2 = 2k^3 - 2k^2 - k^2 + k + 6k^2 = 2k^3 + 3k^2 + k.$$

Hence

$$\sum_{i=0}^{(k+1)-1} i^2 = (k)(k+1)(2k+1)/6$$

This proves the inductive step.

**By the Principle of Mathematical Induction** the result holds for all $n$.

Submit

Answers are displayed within the problem