

MACHINE LEARNING, Section A Final Exam

TOTAL POINTS

92 / 100

QUESTION 1

1 SVM 14 / 14

- ✓ - 0 pts Correct
- 5 pts 1a incorrect
- 4 pts 1b incorrect
- 5 pts 1c incorrect

QUESTION 2

2 Neural Networks 14 / 14

- ✓ + 14 pts Correct
- 14 pts Incorrect
- 3 pts Missing Plot
- 3 pts Incomplete plot
- 3 pts Missing Code for part (c)

QUESTION 3

3 Backpropagation 14 / 14

- ✓ - 0 pts Correct
- 2 pts a) Missing summations
- 2.5 pts c) du/dz is not shown
- 2.5 pts c) du/dz is incorrect
- 2.5 pts du/dz is not always 0

QUESTION 4

4 CNN dimensions 8 / 14

- 0 pts Everything is correct
- 1 pts a) wrong answer for maxpool output
- 2 pts (a) no answer for conv1 input and wrong answer for maxpool output
- ✓ - 3 pts (a) wrong answers for conv1 output, maxpool1 input and maxpool1 output
- 4 pts (a) wrong answers for conv1 and maxpool1 dimensions
- 2 pts (b) wrong answer for conv1 params
- 2 pts (b) wrong answer for maxpool1 params

- 4 pts (b) wrong answers for conv1 and maxpool1 layers

- 6 pts (c) wrong answers for conv1 and maxpool1 layers

✓ - 3 pts (c) wrong answers for maxpool1 layer

- (a) wrong answers for conv1 output, maxpool1 input and maxpool1 output (c) wrong answers for maxpool1 layer

QUESTION 5

5 Transfer learning 15 / 15

- ✓ - 0 pts Correct
- 3 pts Part a incorrect
- 1 pts Minor Code Issues
- 2 pts Click here to replace this description.
- 5 pts Click here to replace this description.
- 15 pts Did not submit

QUESTION 6

6 PCA 12 / 14

- + 4 pts (a) Correct
- ✓ + 2 pts Part of (a) wrong
- + 0 pts (a) is wrong.
- ✓ + 5 pts (b) correct
- + 3 pts Should use the value of question.
- + 2.5 pts Part of (b) wrong
- + 0 pts (b) is wrong
- ✓ + 5 pts (c) correct
- + 4.5 pts PCs are orthonormal, or norm of basis should be one
- + 4 pts No v_1
- + 3 pts Should use the value of question
- + 2.5 pts Part of (c) is wrong
- + 0 pts (c) is wrong
- + 4 pts Not minus, should be add.

QUESTION 7

7 K-means 15 / 15

✓ + 15 pts Correct

- 15 pts Incorrect
- 4 pts Part a incorrect
- 6 pts Part b incorrect
- 5 pts Part c incorrect
- 2 pts Part a partial credit
- 3 pts Part b partial credit
- 2 pts Part c partial credit
- 1 pts Part a partial credit
- 1 pts Part b partial credit
- 1 pts Part c partial credit
- 3 pts Part a partial credit
- 4 pts Part b partial credit
- 4 pts Part c partial credit

1.

(a)

$$d_1 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -0.5 & 0.5 & 0 & 0 \\ -1 & 1 & 0 & 0 \end{bmatrix}^2$$

$$\| \quad \|$$

$$(0.5)^2 + (-1)^2 + (0.5)^2 + (1)^2$$

$$\| \quad \|$$

$$2.5$$

$$d_2 = \begin{bmatrix} 1 & -1 & 0 & 0 \\ 0.5 & -0.5 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}^2$$

$$\| \quad \|$$

$$1^2 + (-1)^2 + 0.5^2 + (-0.5)^2$$

$$\| \quad \|$$

$$4.5$$

$$d_3 = \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0.5 & 0 & -1 & 0 \\ 0 & 0.5 & -1 & 0 \end{bmatrix}^2$$

$$\| \quad \|$$

$$1^2 + 1^2 + 0.5^2 + 0.5^2$$

$$\| \quad \|$$

$$7.5$$

$$d = \begin{bmatrix} 2.5 \\ 4.5 \\ 7.5 \end{bmatrix}$$

(b)

for $y_i = 1 \quad K(x, x_i) > 0$

$$1 - d_1 r > 0$$

$$1 - d_2 r > 0 \Rightarrow \begin{cases} 1 - 2.5r > 0 \\ 1 - 4.5r > 0 \end{cases} \Rightarrow r < \frac{1}{4.5} = \frac{2}{9} \quad r < \frac{2}{9}$$

for $y_i = -1 \quad K(x, x_i) \leq 0$

$$1 - d_3 r \leq 0$$

$$r \geq \frac{2}{15}$$

$$\frac{2}{15} \leq r < \frac{2}{9}$$

$$r \geq \frac{2}{15}$$

(c)

```
def predict(X, Xtr, ytr, b, alpha, gamma):
    D = np.sum((X[:, None, :] - Xtr[None, :, :])**2, axis = 2) * gamma
    K = np.maximum((0, 1-D))
    z = K.dot(ytr * alpha) + b
    yhat = 2*(z > 0)-1
    return yhat
```

1 SVM 14 / 14

✓ - 0 pts Correct

- 5 pts 1a incorrect

- 4 pts 1b incorrect

- 5 pts 1c incorrect

2.

$$a) z_j^H = \begin{bmatrix} x_{j1} - 0.5 \\ x_{j2} - 1 \\ -x_{j1} + x_{j2} + 3 \end{bmatrix}$$

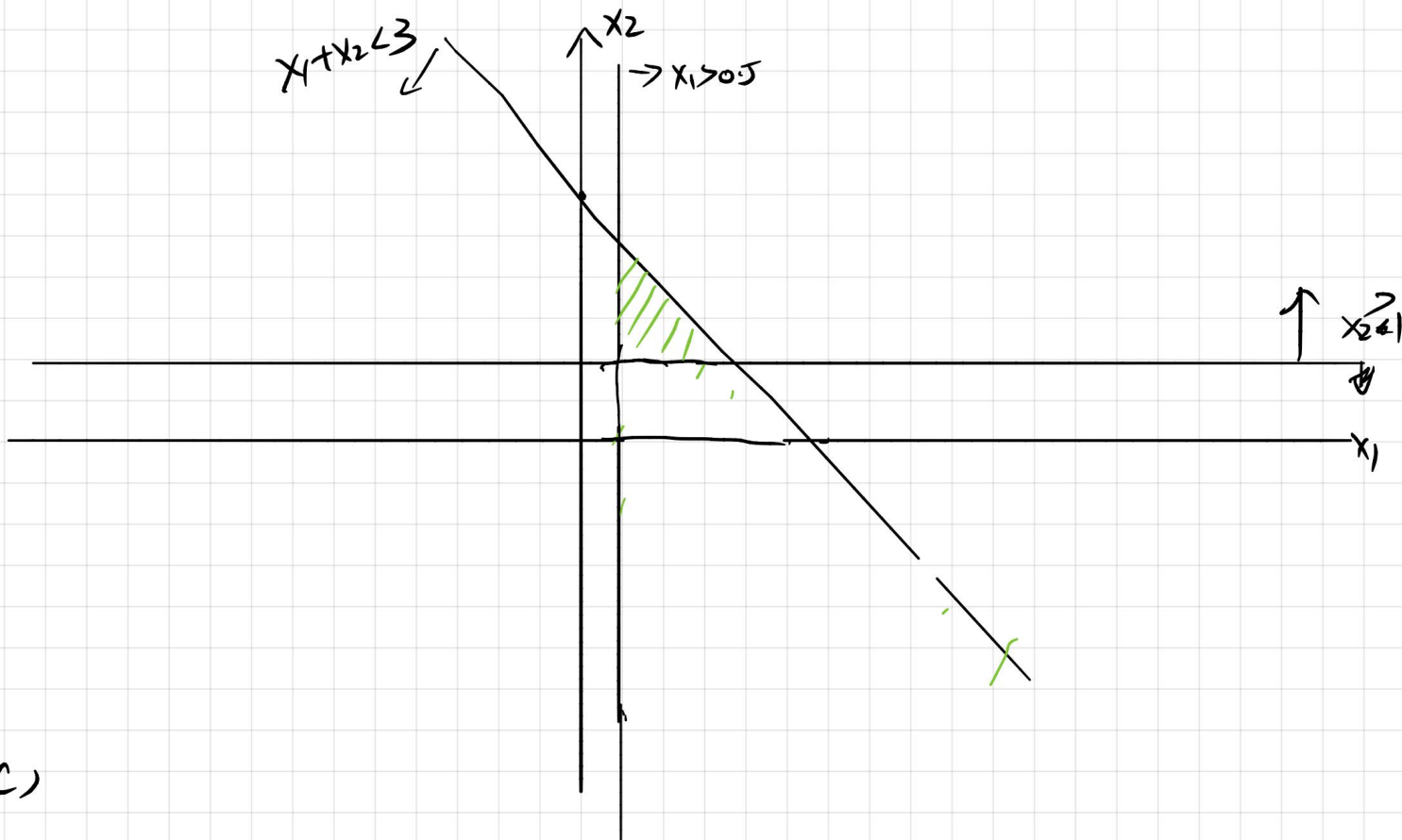
b) input (x_1, x_2)

$$z^H = \begin{bmatrix} x_1 - 0.5 \\ x_2 - 1 \\ -x_1 - x_2 + 3 \end{bmatrix} \Rightarrow$$

if $w^H > 0$

$$\begin{cases} x_1 - 0.5 > 0 \\ x_2 - 1 > 0 \\ -x_1 - x_2 + 3 > 0 \end{cases} \Rightarrow \begin{cases} x_1 > 0.5 \\ x_2 > 1 \\ x_1 + x_2 < 3 \end{cases}$$

$x_1 + x_2 < 3$
 $x_1 > 0.5$



(c)

```
zh = wh.dot(X) + bh[None,:]
zh[zh < 0] = 0
uh = np.maximum((0, zh))
beta = lstsq(uh, y)
b = beta[:, 0]
w =[:, 1:]
```

2 Neural Networks 14 / 14

✓ + 14 pts Correct

- 14 pts Incorrect

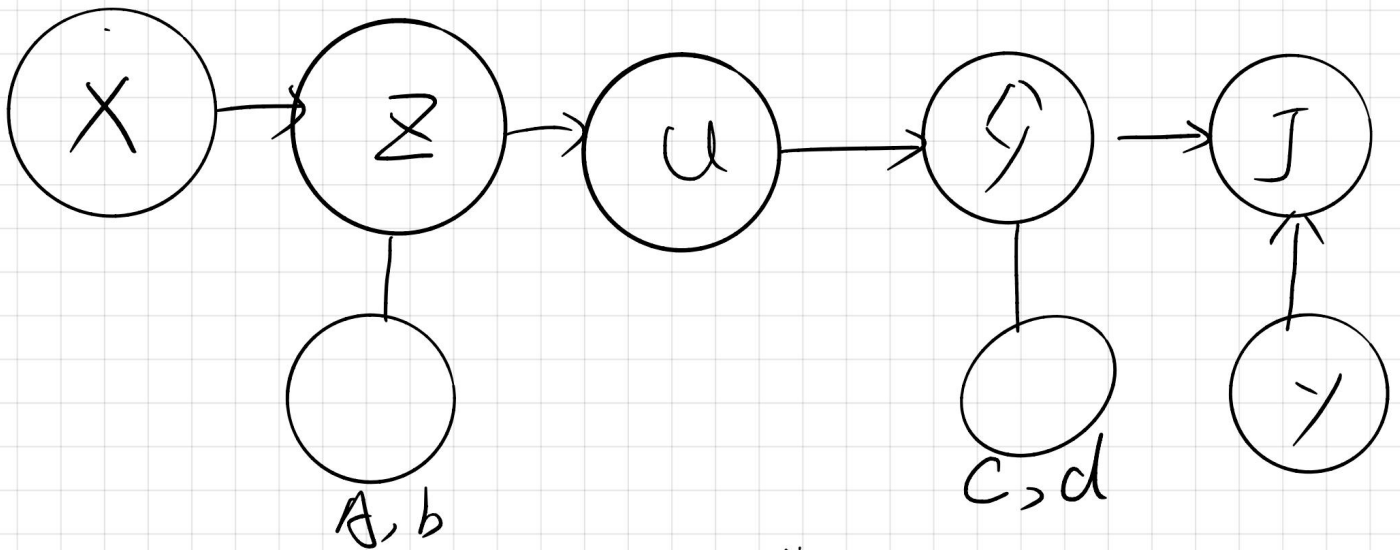
- 3 pts Missing Plot

- 3 pts Incomplete plot

- 3 pts Missing Code for part (c)

3. (a)
$$\hat{y}_i = \sum_{k=1}^K \sum_{j=1}^M c_{kj} x_{ij} u_{ik} + \sum_{k=1}^K d_k u_{ik}$$

(b)



(c)

$$z_{ik} = \sum_{j=1}^M A_{kj} x_{ij} + b_k \quad u_{ik} = \frac{e^{z_{ik}}}{\sum_{l=1}^K e^{z_{il}}} \quad k=1 \dots K$$

$$\frac{\partial J}{\partial z_{ik}} = \frac{\partial J}{\partial u} \frac{\partial u_{ik}}{\partial z_{ik}}$$

$$\frac{\partial u_{ik}}{\partial z_{ik}} = \frac{e^{z_{ik}} \cdot \sum_{l=1}^K e^{z_{il}} - e^{2z_{ik}}}{\left(\sum_{l=1}^K e^{z_{il}} \right)^2}$$

$$\therefore \frac{\partial J}{\partial z_{ik}} = \frac{\partial J}{\partial u} \cdot \frac{\partial u_{ik}}{\partial z_{ik}} = \frac{\partial J}{\partial u} \cdot \frac{e^{z_{ik}} \cdot \sum_{l=1}^K e^{z_{il}} - e^{2z_{ik}}}{\left(\sum_{l=1}^K e^{z_{il}} \right)^2}$$

3 Backpropagation 14 / 14

✓ - 0 pts Correct

- 2 pts a) Missing summations
- 2.5 pts c) du/dz is not shown
- 2.5 pts c) du/dz is incorrect
- 2.5 pts du/dz is not always 0

4. ca

$$\text{conv} \begin{cases} \text{input: } 32 \times 128 \times 256 \times 3 \\ \text{output: } 32 \times 128 \times 256 \times 3 \end{cases}$$

$$\text{maxpooling} \begin{cases} \text{input: } 32 \times 128 \times 256 \times 3 \\ \text{output: } 32 \times 63 \times 127 \times 64 \end{cases}$$

(b)

$$\text{conv} = 9 \times 9 \times 3 \times 64 + 64 = 15616$$

$$\text{maxpooling} = 0$$

(c)

$$\text{conv} : 9 \times 9 \times 64 = 5184$$

$$\text{maxpooling} : 6 \times 6 \times 64 = 2304$$

4 CNN dimensions 8 / 14

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- ☞ (a) wrong answers for conv1 output, maxpool1 input and maxpool1 output (c) wrong answers for maxpool1 layer

5.

(a)

$X (N, 5, 128, 128, 3)$
 $y (N, 1)$

(b) concat these 5×2000 output as an input for $X_{tr}(i)$ + fit

```
def classify(X, pretrained):
    yhat = []
    for i in range(len(X)): #Traverse all samples
        index = 0
        for j in range(len(X[i])): #Traverse 5 pics in one sample X[i]
            Z = pretrained_base.predict(X[i][j])
            if (np.argmax(Z) == 49):
                index = 1
                break
        yhat.append(index)
```

(c)

```
def fit(Xtr, ytr):
    K = np.ones(len(Xtr), 5*2000)
    for i in range(len(Xtr)):
        k = np.ones(5, 2000)
        for j in range(5):
            z = pretrained.predict(Xtr[i][j])
            k[j] = z
        k.reshape(2000*5, 1)
        K[i] = k
    reg = LogisticRegression()
    reg.fit(K, ytr)

def predict(X):
    K = np.ones(len(X), 5*2000)
    for i in range(len(X)):
        k = np.ones(5, 2000)
        for j in range(5):
            z = pretrained.predict(X[i][j])
            k[j] = z
        k.reshape(2000*5, 1)
        K[i] = k
    reg.predict(K)
```

5 Transfer learning 15 / 15

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- **1 pts** Minor Code Issues

- **2 pts** Click here to replace this description.

- **5 pts** Click here to replace this description.

- **15 pts** Did not submit

$$6. \quad \begin{array}{ll} \text{(a)} & U \quad 100 \times 3 \\ & V \quad 3 \times 200 \end{array}$$

$$\text{(b)} \quad X_{\text{hat}} = \bar{y}_n U \cdot \text{diag}(s) \cdot V^T$$

$$X_1 = 10 \begin{bmatrix} 0.5 & 0.3 & 0.1 \end{bmatrix} \begin{bmatrix} 10 \\ 8 \\ 2 \end{bmatrix} \begin{bmatrix} v_1^T \\ v_2^T \\ v_3^T \end{bmatrix} + \mu$$

$$X_1 = 50v_1^T + 24v_2^T + 2v_3^T + \mu$$

$$\text{(c)} \quad \hat{X}_1 - X_1 = \sum_{j=d+1}^p U[j:] \cdot s \cdot v^T = \|24v_2^T + 2v_3^T\|^2$$

$$v_i \cdot v_j = \begin{cases} 0 & \text{if } i \neq j \\ 1 & \text{if } i = j \end{cases}$$

$$\|\hat{X}_1 - X_1\|^2 = 24^2 + 2^2 = 580$$

6 PCA 12 / 14

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7.

(a) because it's far from other centers

So $\|x - \mu_k\|^2$ is large so $e^{-\gamma\|x - \mu_k\|^2}$ is very small

So it can only have small influence on the result

We can just focus on the K we're near with

the equation becomes

$$\hat{y} \approx \frac{\bar{y}_k e^{-\gamma\|x - \mu_k\|^2}}{e^{-\gamma\|x - \mu_k\|^2}} \approx \bar{y}_k$$

$$\hat{y} \approx 2 + 4 + 5 + 6 / 4 = 4.75$$

(b)

```
def fit(Xtr, ytr, nc):
    yk = np.ones(nc)
    km = KMeans(n_cluster=nc)
    km.fit(X)
    pre = km.predict(X)
    centers = km.cluster_centers_
    for i in range(nc):
        idx = np.where(pre == i)
        yk[i] = np.sum(ytr[idx]) / len(idx)

    return centers, yk
```

(c)

```
def predict(X, centers, yk, gamma):
    dis = np.sum((X[:, None] - centers[None, :])**2, axis = 1)
    portion = np.sum(yk[None, :] * exp(-gamma * dis), axis = 1) #the upper
    sums = np.sum(exp(-gamma * dis), axis = 1)
    yhat = portion / sums
    return yhat
```

7 K-means 15 / 15

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