This homework was done by Tianwei Mo (tm3929).

1.

a)

$$b = -0.2, w1 = -1, w2 = 2.4$$

b)

$$\gamma \geq 0.2$$

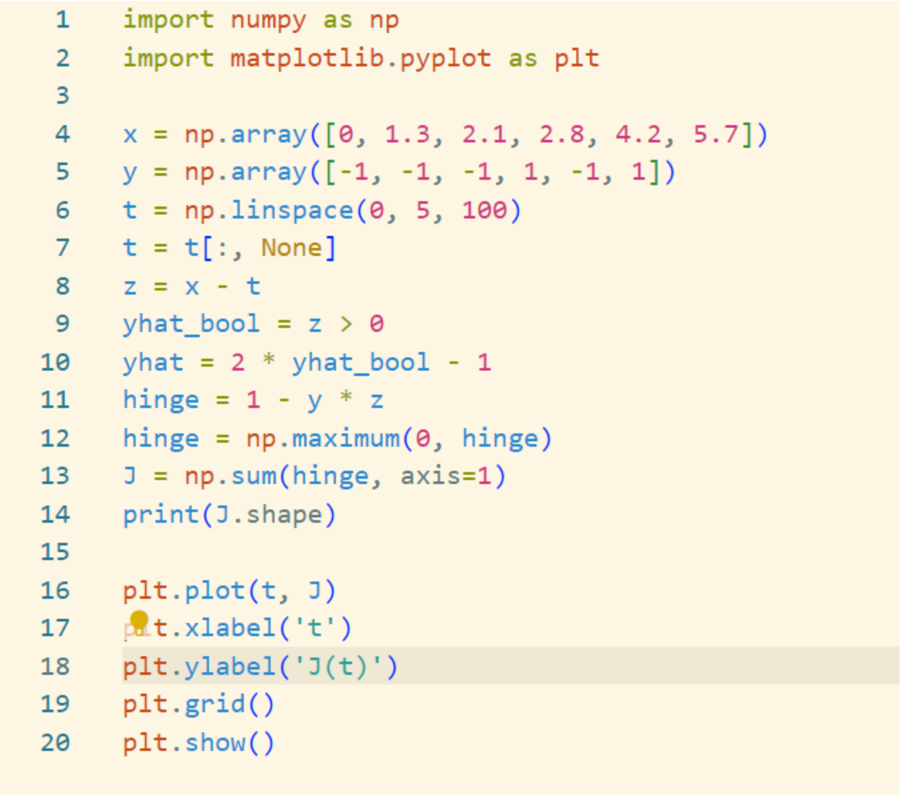
c)

$$m = \frac{0.2}{\sqrt{1 + 2.4^2}} = 0.0296$$

d) Samples 1 and 4 are on the margin.

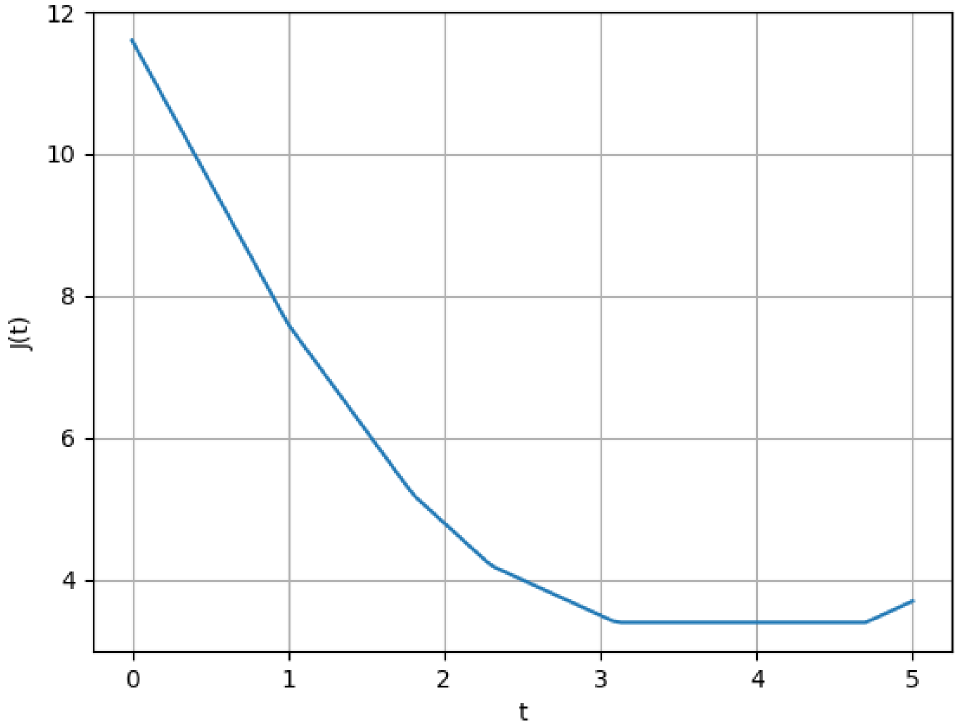
2.

a) Python code:



```python
import numpy as np
import matplotlib.pyplot as plt

x = np.array([0, 1.3, 2.1, 2.8, 4.2, 5.7])
y = np.array([-1, -1, -1, 1, -1, 1])
t = np.linspace(0, 5, 100)
t = t[:, None]
z = x - t
yhat_bool = z > 0
yhat = 2 * yhat_bool - 1
hinge = 1 - y * z
hinge = np.maximum(0, hinge)
J = np.sum(hinge, axis=1)
print(J.shape)

plt.plot(t, J)
plt.xlabel('t')
plt.ylabel('J(t)')
plt.grid()
plt.show()
```

The plot:



b) One of the values is $t = 4$.

c)

$$z = [-4, -2.7, -1.9, -1.2, 0.2, 1.7]$$
$$\epsilon_i = [0, 0, 0, 2.2, 1.2, 0]$$

d) No samples violate the margin. Sample 4 and 5 violate are misclassified.

3.

a)

$$x = [0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 1, 0, 0, 0, 0]^T$$
$$w = [0, 0, 0, 0, 0, 1, 1, 0, 0, 1, 1, 0, 0, 0, 0, 0]^T$$

b)

$$z = 2$$

c)

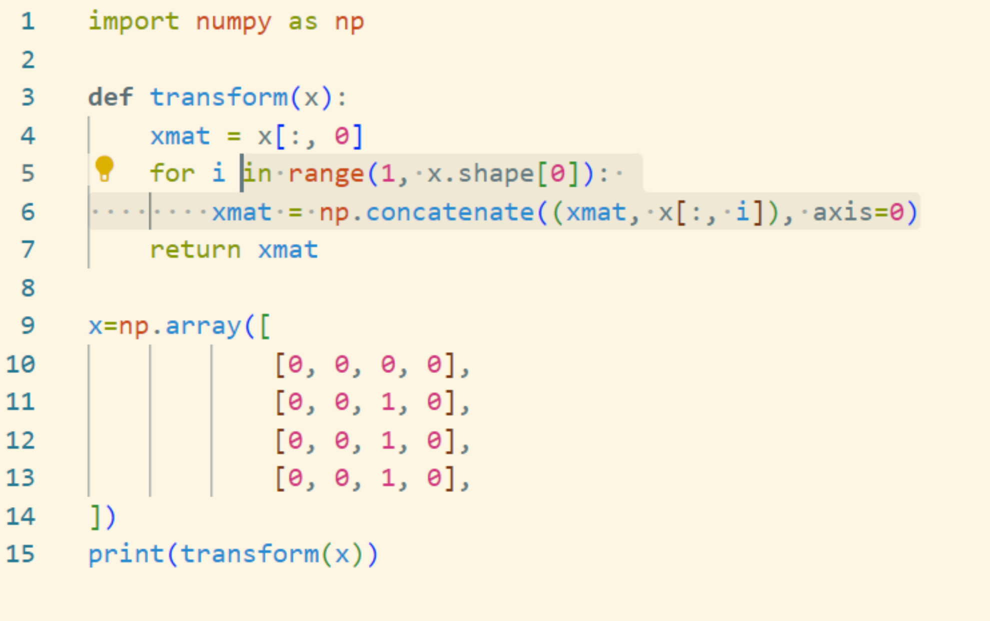
$$x_{right} = [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 1]^T$$

$$z = 0$$

d)

$$x = [0, 0, 0, 0, 0, 1, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0]^T$$
$$z = 2$$

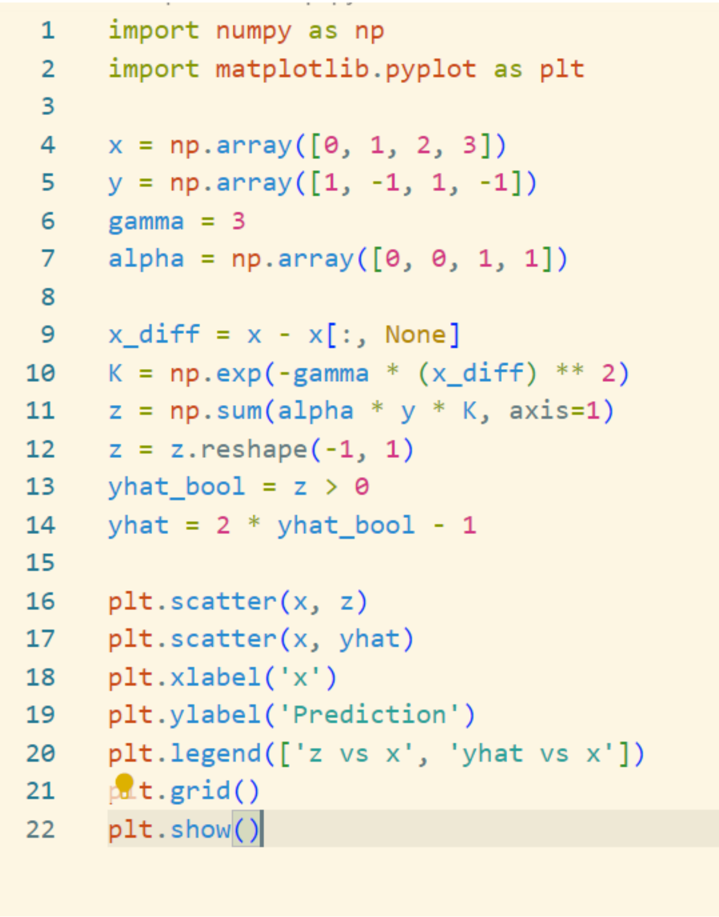
e) Python command:



```python
import numpy as np

def transform(x):
    xmat = x[:, 0]
    for i in range(1, x.shape[0]):
        xmat = np.concatenate((xmat, x[:, i]), axis=0)
    return xmat

x=np.array([
            [0, 0, 0, 0],
            [0, 0, 1, 0],
            [0, 0, 1, 0],
            [0, 0, 1, 0],
])
print(transform(x))
```

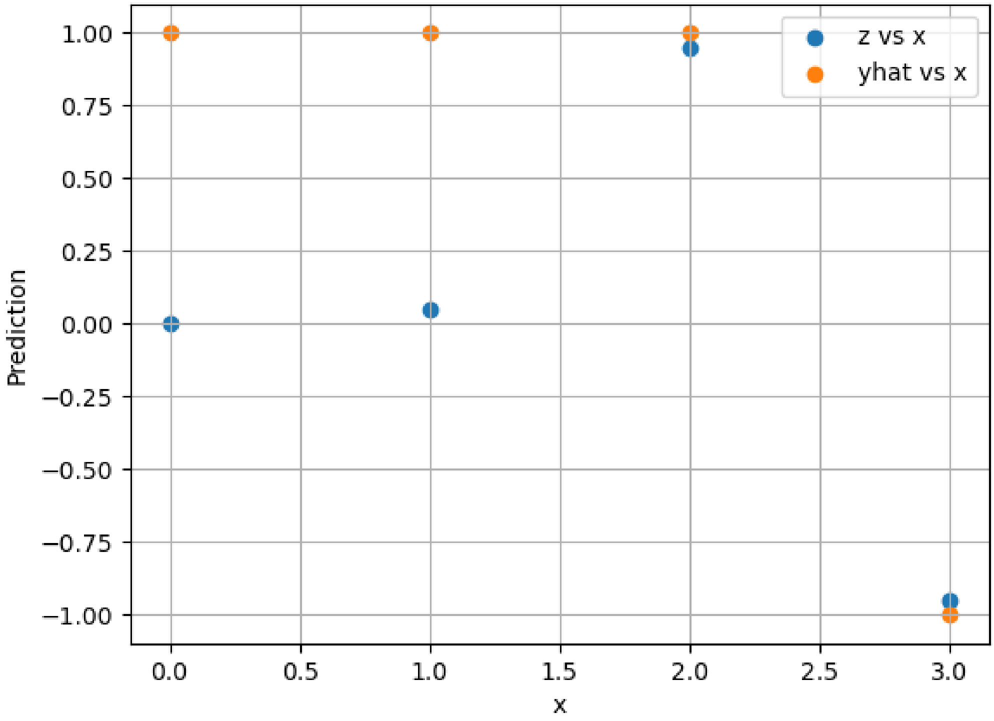
4.

a) Python code:



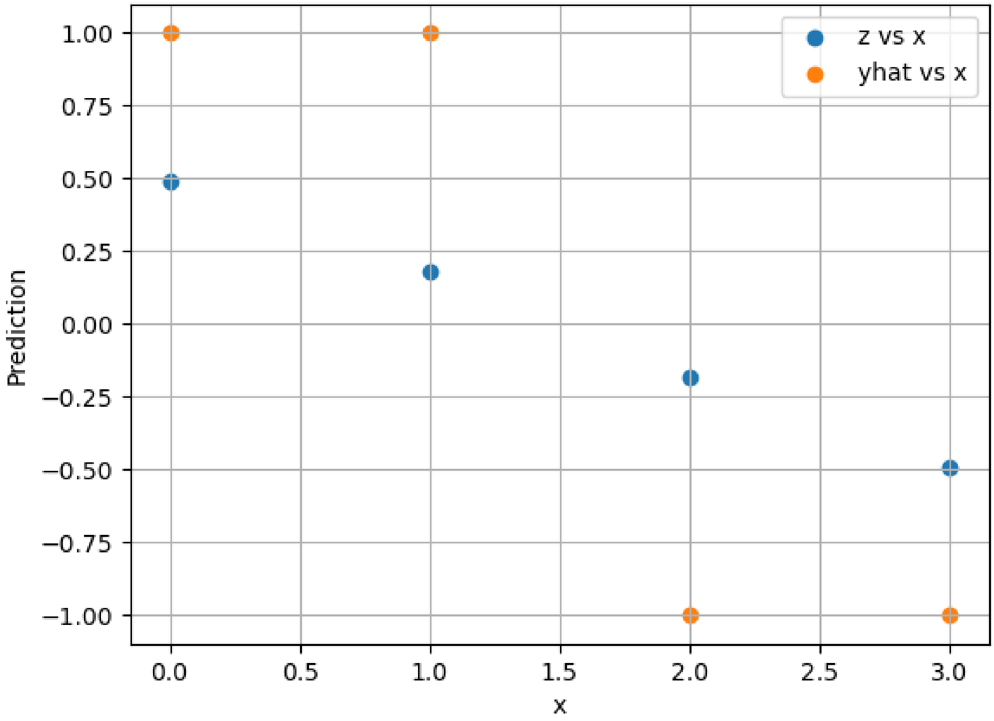
```python
import numpy as np
import matplotlib.pyplot as plt

x = np.array([0, 1, 2, 3])
y = np.array([1, -1, 1, -1])
gamma = 3
alpha = np.array([0, 0, 1, 1])

x_diff = x - x[:, None]
K = np.exp(-gamma * (x_diff) ** 2)
z = np.sum(alpha * y * K, axis=1)
z = z.reshape(-1, 1)
yhat_bool = z > 0
yhat = 2 * yhat_bool - 1

plt.scatter(x, z)
plt.scatter(x, yhat)
plt.xlabel('x')
plt.ylabel('Prediction')
plt.legend(['z vs x', 'yhat vs x'])
plt.grid()
plt.show()
```

Plot:



z vs x
yhat vs x
Prediction
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
0.0
0.5
1.0
1.5
2.0
2.5
3.0
x

b) Plot:



z vs x
yhat vs x
Prediction
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
0.0
0.5
1.0
1.5
2.0
2.5
3.0
x

c) The second classifier makes more errors.