First, we find out which lily pads are accessible to the frog.

Claim: If the frog can access three consecutive lily pads, then it can access all lily pads after the three lily pads. That means, if the frog can access lily pad i, $i+1$ and $i+2$ for $i \in (1, n-2)$, the frog can access to lily pad $i+j$ for integer $j \geq 0$ and $(i+j) \in (1, n)$.

Proof: We only need to prove if the frog can access three consecutive lily pads, then it can access another three consecutive lily pads after the three lily pads. That means, if the frog can access lily pad i, $i+1$ and $i+2$ for $i \in (1, n-2)$, the frog can access to lily pad $i+3$, $i+4$ and $i+5$ if lily pads $i+3$, $i+4$ and $i+5$ exist. Because for any lily pad after lily pad i, we can prove it is accessible by recursively proof the following three consecutive lily pads are accessible. Since the frog can jump from lily pad i to either lily pad $i+3$ or lily pad $i+4$, lily pad $i+3$ and $i+4$ can be accessible from lily pad i. Lily pad $i+5$ can be accessible from lily pad $i+1$ by jumping forward 4 pads. Thus, we proved our claim.

Now we can find the first three consecutive lily pads that are accessible. Starting from lily pad 1, the frog can only access to lily pad 4 and 5. From lily pad 4 the frog can access to lily pad 7 and 8, and from lily pad 5 the frog can access to lily pad 9. Thus, we found the first three consecutive accessible lily pads: lily pad 7, 8 and 9. **Therefore, the frog can access to all lily pads except lily pad 2, 3 and 6.**

Next, we try to find and solve the subproblem.

In order to consider the question in general cases, we assume all lily pads are accessible, which means we consider lily pads after lily pad 7. To reach lily pad i, the frog has to jump forward either three lily pads from lily pad $i-3$ or four lily pads from lily pad $i-4$. If the frog come from lily pad $i-3$, the largest number of flies the frog can catch is the largest number of flies the frog can catch when it reaches lily pad $i-3$ plus numbers of flies on lily pad i. Similarly, if the frog come from lily pad $i-4$, the largest number of flies the frog can catch is the largest number of flies the frog can catch when it reaches lily pad $i-4$ plus numbers of flies on lily pad i. **Our subproblem is: Find the largest number of flies that the frog can catch when it reaches lily pad $\boldsymbol{i}$. The recursion is:**

$$opt(i) = \{\max(opt(i-3), opt(i-4)) + f_i : 11 \leq i \leq n\}.$$

Note that the minimum value of i is 11. Because we only consider lily pads after lily pad 7 for the sake of take only considering general cases. And the smallest variable in our recursion is $i-4$, which should greater or equal to 7.

Now let us consider special cases When $\boldsymbol{1 \leq i < 11}$. Since lily pad 2, 3 and 6 are not accessible, there is no $opt(2)$, $opt(3)$ and $opt(6)$.

$$opt(1) = f_1, opt(4) = opt(1) + f_4 = f_1 + f_4, opt(5) = opt(1) + f_5 = f_1 + f_5,$$

$$opt(7) = opt(4) + f_7 = f_1 + f_4 + f_7, opt(8) = \max\big(opt(4), opt(5)\big) + f_8,$$

$$opt(9) = opt(5) + f_9 = f_1 + f_5 + f_9, opt(10) = opt(7) + f_{10} = f_1 + f_4 + f_7 + f_{10}.$$

Now we have considered all cases. The answer is the largest optimal solution among them.