

We find and solve the subproblem.

For the sake of considering only general cases later, we first consider squares in the first row and column. Since there is only one way to reach them. We need to manually find out optimal solutions to reach them, which are $opt(1, R)$, $opt(1, R - 1), \dots, opt(1, 1)$ and $opt(2, R)$, $opt(3, R), \dots, opt(C, R)$. Calculating them is straightforward, because for $(1, R)$, $(1, R - 1), \dots, (1, 1)$, they do not have square above them. The only way to access them is from the square on their left. Similarly, for $(2, R)$, $(3, R), \dots, (C, R)$, they do not have squares on the left. The only way to access them is from the square above them.

Now we consider general cases. According to the question, to reach the grid (C_i, R_j) , the last step should be either from (C_i, R_{j+1}) to (C_i, R_j) or from (C_{i-1}, R_j) to (C_i, R_j) . If the last step is from (C_i, R_{j+1}) to (C_i, R_j) , and

$(C_i, R_{j+1}) < (C_i, R_j)$, which means we move from lower elevation to higher elevation, the optimal solution is the optimal solution from $(1, R)$ to (C_i, R_{j+1}) plus one. Otherwise, the optimal solution is optimal solution from $(1, R)$ to (C_i, R_{j+1}) . Similarly, If the last step is from (C_{i-1}, R_j) to (C_i, R_j) ,

and $(C_{i-1}, R_j) < (C_i, R_j)$, which means we move from lower elevation to higher elevation, the optimal solution is the optimal solution from $(1, R)$ to (C_{i-1}, R_j) plus one. Otherwise, the optimal solution is optimal solution from $(1, R)$ to (C_{i-1}, R_j) .

We aim to solve the following subproblem: What is the optimal path from $(1, R)$ to (C_i, R_j) so that the number of moves from lower elevation to higher elevation along such a path is as small as possible? The recursion is:

$$opt(C_i, R_j) = \min \left\{ \begin{array}{l} opt(C_i, R_{j+1}) + 1 \text{ if } (C_i, R_{j+1}) < (C_i, R_j), \\ opt(C_i, R_{j+1}) \text{ otherwise} \end{array} \right\},$$

$$\left(\begin{array}{l} opt(C_{i-1}, R_j) + 1 \text{ if } (C_{i-1}, R_j) < (C_i, R_j), \\ opt(C_{i-1}, R_j) \text{ otherwise} \end{array} \right) : 2 \leq i \leq C, 2 \leq j \leq R$$

The final solution is $opt(C, 1)$.