

We find and solve the subproblem.

In order to explain, we define $E(i, j)$ to represent the total enjoyment of optimal solution for i days that choose to do activity j on the last day. According to the question, for $2 \leq i \leq N$, if we choose to do activity 1 in day i , we must do one of the rest two activities, let us say activity 2 and 3, in day $i - 1$. The total enjoyment of optimal solution for $i - 1$ days that choose to do activity 2 is $E(i - 1, 2)$, and the total enjoyment of optimal solution for $i - 1$ days that choose to do activity 3 is $E(i - 1, 3)$. If $E(i - 1, 2) > E(i - 1, 3)$, the optimal solution for i days is $E(i - 1, 2) + e(i, 1)$. Otherwise, the optimal solution for i days is $E(i - 1, 3) + e(i, 1)$. Consequently, the total enjoyment of optimal solution for i days that choose to do activity 1 is $\max\{E(i - 1, 2), E(i - 1, 3)\} + e(i, 1)$. Similarly, if we choose to do activity 2 in day i instead of activity 1, the total enjoyment of optimal solution is $\max\{E(i - 1, 1), E(i - 1, 3)\} + e(i, 2)$. If we choose to do activity 3 in day i instead of activity 2, the total enjoyment of optimal solution is $\max\{E(i - 1, 1), E(i - 1, 2)\} + e(i, 3)$. The optimal solution for i days is $\max\{\max\{E(i - 1, 2), E(i - 1, 3)\} + e(i, 1), \max\{E(i - 1, 1), E(i - 1, 3)\} + e(i, 2), \max\{E(i - 1, 1), E(i - 1, 2)\} + e(i, 3)\}$.

Our subproblem is: Determining the maximum total enjoyment possible over the entire stay of i days and which activity to do in day i . The base case is $opt(1) = \max\{e(1, 1), e(1, 2), e(1, 3)\}$. The recursion is:

$$\begin{aligned} opt(i) &= \max\{E(i, 1), E(i, 2), E(i, 3)\} \\ &= \max\{\max\{E(i - 1, 2), E(i - 1, 3)\} + e(i, 1), \\ &\quad \max\{E(i - 1, 1), E(i - 1, 3)\} + e(i, 2), \\ &\quad \max\{E(i - 1, 1), E(i - 1, 2)\} + e(i, 3)\}, 2 \leq i \leq N. \end{aligned}$$

The final solution is $opt(N)$.

Note that to calculate $E(i - 1, 1), E(i - 1, 2)$ and $E(i - 1, 3)$ is the same as calculating $opt(i - 1)$, since

$$opt(i - 1) = \max\{E(i - 1, 1), E(i - 1, 2), E(i - 1, 3)\}.$$

Also, every time we calculate $opt(i)$, we have to determine the activity in day i . **Thus, we simply record the activity each time we choose to get the sequence of activities we should do at each day.**