To prove the f(n) = O(g(n)), we can prove “There exists positive constants c and such that 0 ≤ f(n) ≤ cg(n) for all ”. If f(n) is 0, then f(n) = O(g(n)) if . If f(n) is not 0, previous statement can be rewrite to there exists positive constants c and such that for all .

1. By applying l L’Hˆopital’s rule, we have

So, we get , which means . So, there exist a , let c = 1, for all . Thus, f(n)=O(g(n)).

Let c = 1 and , . Because is monotonically increasing, there exist c = 1 and such that for all . Therefore, f(n) = O(g(n)).

. since n should always be integer, , . Thus, there let c = 1 and , and for all . Therefore, both f(n) = O(g(n)) and g(n) = O(f(n)).