1. The sub-problems are: For every sub-tree T(i), rooted at a vertex i of T, finds the length of the longest possible path through the sub-tree T(i) which starts at the root i and such that all vertices on this path are of the same colour as the root i.
2. The recursion is:

opt(i) = max{(opt(j) + 1, if j has the same colour as i, 0 otherwise.), for every child vertex j}

1. The final answer is opt(i).
2. Time complexity is O(n), since to find opt(i), we need to recursively visit every vertex in T(i) once.
3. The sub-problems are part a.
4. The recursion is:

opt(i) = max{(opt(j) + 1, if j has the same colour as i, 0 otherwise.), for every child vertex j}

1. The final answer is max{opt(i), for every vertex i in T}.
2. Time complexity is O(n^2), since to find a opt(i), we need to recursively visit every vertex in T(i) once. To find maximum opt(i), we need to compare n numbers.