From the corollary we learned from the course, if we pick uniformly at random a point from the unit ball, the probability that its coordinate x_1 satisfies $|x_1| > \frac{c}{\sqrt{d-1}}$ is less than $\frac{2}{c}e^{-\frac{c^2}{2}}$. If we want to prove the probability that such a point will be at a distance at most $\frac{1}{d}$ from the surface of such a ball is larger than $1-\frac{1}{e}$, which means $|x_1| < \frac{1}{d}$ is less than $1-\frac{1}{e}$, we can simply replace $\frac{c}{\sqrt{d-1}}$ with $\frac{1}{d}$ and prove $\frac{2}{c}e^{-\frac{c^2}{2}}$ is less or equal than $1-\frac{1}{e}$. let

$$\frac{c}{\sqrt{d-1}} = \frac{1}{d},$$

We get $c = \frac{(d-1)^{\frac{3}{2}}}{d}$, which is approximately bigger than 31. Then we want to prove

$$1-\frac{1}{e} \geq \frac{2}{c}e^{-\frac{c^2}{2}},$$

Which is $e-1 \geq \frac{2}{c}e^{1-\frac{c^2}{2}}$. Because $c > 31$, $0 < \frac{2}{c} < 1$, and $0 < e^{1-\frac{c^2}{2}} < 1$. Thus $0 < \frac{2}{c}e^{1-\frac{c^2}{2}} < 1$. Therefore, we get

$$e-1 > 1 > \frac{2}{c}e^{1-\frac{c^2}{2}}.$$

Thus, $P\left(|x_1| < \frac{1}{d}\right) < 1-\frac{1}{e}$.