

(a) The probability of getting i consecutive heads when flipping a coin i times is

$\frac{1}{3^i}$. Thus, an element has links on levels $0 - i$ (and possibly also on higher levels) with probability $\frac{1}{3^i}$.

(b) If n elements belong to a set with a probability p each, then the expected size of that set is np . Thus, an n element Skip List has on average $\frac{n}{3^i}$ elements with links on level i .

(c) Let $\#(i)$ denote the number of elements on level i . By Markov inequality, $P(\#(i) \geq 1) \leq \frac{E[\#(i)]}{1} = \frac{n}{3^i}$. Thus, the probability to have at least one element on level $k \log_3 n$ is smaller than $\frac{n}{3^{k \log_3 n}} = \frac{1}{n^{k-1}}$.

(d) $E \leq \sum_{k=1}^{\infty} \frac{k}{n^{k-1}} = \left(\frac{n}{n-1}\right)^2$. Thus, the expected number of levels is barely larger than $\log n$.

(e) If an element has a link at a level i then with probability $\frac{1}{3}$ it also has a link at level $i + 1$.

(f) The expected number of elements between any two consecutive elements with a link on level $i + 1$ which have links only up to level i is smaller than

$$0 \frac{2}{3} + 1 \left(\frac{2}{3}\right)^2 + 2 \left(\frac{2}{3}\right)^3 + \dots = 4.$$

So once on level i , on average we will have to inspect 5 elements on that level before going to a lower level.

(g) On average, there will be fewer than $5 \log n$ levels to go down, with visiting on average 5 elements per each level. Consequently, on average, the search will be in time $O(\log n)$.

(h) Searching with a skip list built with such a biased coin is probably slower.

Because with this biased coin, we have less chance to build high layers, so there is less element with high layers, which means on average we used higher layers less for searching.