Cross Dominance:
A Shared-Interest Parallel to Strict Dominance

Shiko Kudo
National Taiwan University
b12303095@ntu.edu.tw

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Abstract

We describe *cross dominance*—a bilateral strengthening of weak dominance: switching $B \to A$ is never worse for either player. Cross dominance is strictly stronger than weak dominance yet orthogonal to strict dominance; within the *Pareto-monotone* slice we have SD $\Rightarrow$ CD $\Rightarrow$ WD. This yields a shared-interest ladder (weak $\to$ cross $\to$ strict-cross) that runs in parallel to the classical self-interest ladder (weak $\to$ strict), offering a simple, outcome-agnostic rationale for pruning strategies like B in the motivating 2×2 game presented.

1 Introduction

Consider the following simple 2-player game:

	L	**R**
A	2, 5	**0, 6**
B	1, 2	0, 4

At first glance, strategy B for Player 1 appears utterly pointless. Strategy A weakly dominates B, giving Player 1 equal or better payoffs regardless of Player 2's choice. However at this point, properties of weakly dominating moves tell us that it might be possible still for this weakly dominated move to have some purpose, if we keep it and do IEDS.

Upon closer inspection however, there is something more striking here that is not obvious on first glance: *Player 2 is also strictly better off whenever Player 1 chooses A over B, no matter which response Player 2 chooses.*

Intuitively, this feels like it should be a slam-dunk case for elimination. And yet, the standard toolkit of game theory does not quite provide us with an answer that captures the unique reasoning this game seems to evoke:

- **Iterated Elimination of Strictly Dominated Strategies (IEDS)** refuses to touch it, as A doesn't *strictly* dominate B. This is caused by the tie when Player 2 plays R.
- **Iterated Elimination of Weakly Dominated Strategies (IEWDS)** will eliminate B; however IEWDS comes with caveats. First and foremost, it is order-dependent and can discard valid Nash equilibria reliant on weakly dominated strategies. This issue is discussed in depth in [1]. Nevertheless, we are left wondering: is there something *stronger* justifying this elimination beyond mere weak dominance?

- **Trembling Hand Perfect Equilibrium (THPE)** [2] would also eliminate the (B, R) equilibrium, but its rationale is one of robustness against mistakes and irrationality. That is not quite what is happening here. There is something fundamentally irrational about playing the move B itself that is difficult to as yet articulate.

We are able to resolve this puzzle by considering the key components that define this game's mystery empirically:

- First, P1 prefers A (or at least weakly prefers A).
- But second, and crucially, *P2 also prefers that P1 plays A.*
- Therefore, a rational P1 should never play B, and a rational P2 should never *expect* a rational P1 to play B. The strategy is pointless from every rational angle. It has no strategic value as a threat, a bluff, or a rational choice.
- Thus, both players have a ***shared interest*** in P1 playing A over B.

What should be present here is elimination justified not by self-interest alone, but by both players agreeing that A is the right choice over B. A rational Player 2 should recognize that strategy B is strictly worse for both of them in all circumstances. Why would Player 2 ever anticipate Player 1 playing a strategy that is both self-detrimental (due to weak dominance) *and* mutually harmful (Pareto-worsening)?

From the perspective of Player 1, if A cross-dominates B, then a rational P1 never plays B, and a rational P2 never expects P1 to play B: for Player 1, the switch from playing move B to A is *Pareto-safe*, as it never harms either player no matter how Player 2 responds.

Here we formalize this intuition. We describe and formalize **cross dominance**, a concept that captures exactly this scenario, and show that it sits in a parallel ladder to the familiar strict dominance hierarchy. Where strict dominance represents "elimination by self-interest," cross dominance represents "elimination by shared interest."

Principle (Pareto-safety): A replacement $B \to A$ is *Pareto-safe* if $u_1(A, y) \geq u_1(B, y)$ and $u_2(A, y) \geq u_2(B, y)$ for every reply y, with at least one strict inequality for P1. Pareto-safety underwrites *cross dominance* (WD + PM): elimination by *shared interest*. Within the Pareto-safe slice we obtain the ladder SD $\Rightarrow$ CD $\Rightarrow$ WD; globally, CD is orthogonal to SD.

2 Setup

We study finite, two-player normal-form games with Player 1 (row) payoffs u_1 and Player 2 (column) payoffs u_2. Let $\mathcal{Y}$ denote Player 2's set of pure responses. For Player 1's strategies A and B, comparisons are taken pointwise in $y \in \mathcal{Y}$.

3 Definitions

Definition 1 (Weak Dominance (WD), Strict Dominance (SD))**.** *A weakly dominates B* for Player 1 if $u_1(A, y) \geq u_1(B, y)$ for all $y \in \mathcal{Y}$ and $u_1(A, y) > u_1(B, y)$ for some y.
A strictly dominates B for Player 1 if $u_1(A, y) > u_1(B, y)$ for all $y \in \mathcal{Y}$.

Definition 2 (Pareto-monotone (PM))**.** The pair (A, B), representing the replacement $B \to A$ is *Pareto-monotone* for Player 2 if $u_2(A, y) \geq u_2(B, y)$ for all $y \in \mathcal{Y}$.

Definition 3 (Cross Dominance (CD))**.** *A cross-dominates B* (*cross/Pareto-safe dominance*, denoted CD) if WD holds and (A, B) is PM. In other words: CD = WD + PM.

Definition 4 (Strict-Cross Dominance ($\mathrm{CD}^{\mathrm{str}}$))**.** *A strict-cross-dominates B* if SD holds and (A, B) is PM. In other words: $\mathrm{CD}^{\mathrm{str}}$ = SD + PM.

Remark 1 (Beliefs)**.** Because the comparisons are pointwise, by linearity the relations extend to mixed responses: if an inequality holds for all pure y, it holds for all beliefs over $\mathcal{Y}$.

3.1 The Parallel Ladders

These definitions give rise to two parallel hierarchies:

1. **Self-Interest Ladder:** SD $\Rightarrow$ WD (as sets: SD $\subset$ WD)

   This is the classical dominance hierarchy. If a strategy is strictly better for Player 1 no matter what Player 2's move is, it must certainly also be weakly better than the alternative. The justification is purely from Player 1's perspective.

2. **Shared-Interest Ladder:** $\mathrm{CD}^{\mathrm{str}} \Rightarrow \mathrm{CD} \Rightarrow \mathrm{WD}$ (as sets: $\mathrm{CD}^{\mathrm{str}} \subset \mathrm{CD} \subset \mathrm{WD}$)

   If switching from B to A is strictly better for Player 1 while being harmless to Player 2 ($\mathrm{CD}^{\mathrm{str}}$), then it must be so that switching from B to A must be by definition also weakly better for Player 1 while being harmless to Player 2 (CD); and if switching from B to A is weakly better for Player 1 while being harmless to Player 2 (CD), then it must be so that switching from B to A is by definition also weakly better for Player 1 (WD).

These ladders are *parallel*, as one can be true while the other is not. Globally, CD and SD are **incomparable**—each contains cases the other misses.

Consider our motivating example for instance, which already demonstrates one instance where CD holds but SD fails. Conversely, there are games where SD holds but CD fails, as we will see in Section 5. Intuitively this occurs when Player 1 strictly prefers A, but Player 2 loses at some response to that choice, violating PM and making it so that Player 2 does not always prefer Player 1 to play A over B, but rather only sometimes (or never) prefers it when their own response can make use of it.

Notably though, when we **condition on Pareto-monotonicity (the "PM slice")**, the ladders align: SD $\Rightarrow$ CD $\Rightarrow$ WD.

4 Basic relations

Proposition 1 (Cross dominance is strictly stronger than weak dominance)**.** CD $\Rightarrow$ WD, and the implication is strict: there are games with WD but not CD.

Proof. Immediate from Definition 3. If for instance $u_1(A, \mathrm{L}) = 2,\ u_1(B, \mathrm{L}) = 1$ and $u_1(A, \mathrm{R}) = u_1(B, \mathrm{R}) = 0$, WD holds; but if $u_2(A, \mathrm{L}) = 0 < 1 = u_2(B, \mathrm{L})$ then PM fails, hence not CD. $\square$

Proposition 2 (Global incomparability with strict dominance)**.** CD neither contains SD nor vice versa: CD $\not\subseteq$ SD and SD $\not\subseteq$ CD.

Witnesses. (CD $\not\Rightarrow$ SD) Example 1 below: ties for Player 1 block SD, while PM holds, so CD also holds.
(SD $\not\Rightarrow$ CD) Example 2 below: Player 1 strictly prefers A, but Player 2 loses at some reply, violating PM. $\square$

Proposition 3 (Inside the Pareto-monotone slice)**.** Conditioned on PM, we have SD $\Rightarrow$ CD $\Rightarrow$ WD.

Proof. If PM holds and SD holds which then implies WD also holding, then Definition 3 gives CD. Trivially CD $\Rightarrow$ WD. $\square$

Parallel ladders. The self-interest chain is SD $\Rightarrow$ WD (set-wise SD $\subset$ WD). The shared-interest chain is CD$^{\text{str}}$ $\Rightarrow$ CD $\Rightarrow$ WD (set-wise CD$^{\text{str}}$ $\subset$ CD $\subset$ WD). Globally, CD and SD are orthogonal per Proposition 2.

5 Examples

Payoffs are displayed as (u_1, u_2).

Example 1 (CD holds, SD fails)**.**

	L	**R**
A	2, 5	**0, 6**
B	1, 2	0, 4

For Player 1, $A \geq B$ pointwise with a strict improvement at L, and for Player 2, $A \geq B$ pointwise. Thus CD holds. However, the tie at R prevents SD.

Example 2 (SD holds, CD fails)**.**

	L	**R**
A	2, 5	**1, 3**
B	1, 2	0, 4

Player 1 strictly prefers A everywhere (SD), but at R Player 2 has $3 < 4$, so PM fails and hence not CD.

Example 3 (Variant highlighting the belief issue)**.**

	L	**R**
A	2, 5	**0, 4.5**
B	1, 2	0, 4

As in Example 1 but with $u_2(A, \text{R}) < u_2(A, \text{L})$.

6 The Rationality of Cross Dominance

Let's return to a slightly modified version of our motivating example, which should illuminate better what makes cross dominance compelling.

6.1 The Argument from Rationality

Consider Example 3:

	L	**R**
A	2, 5	**0, 4.5**
B	1, 2	0, 4

This example is, in effect, a slightly modified version of Example 1. Notice that Player 2's payoffs have been adjusted so that $u_2(A, \text{R}) < u_2(A, \text{L})$. The significance of this will become clear shortly.

We will begin with finding pure Nash Equilibria using the standard best-response marking method.

	L	**R**
A	$\underline{\mathbf{2}}, \overline{\mathbf{5}}$	$\underline{0}, 4.5$
B	$1, 2$	$\underline{\mathbf{0}}, \overline{\mathbf{4}}$

We immediately find two pure Nash Equilibria this way: (A, L) and (B, R).

But a question presents itself immediately: Is the (B, R) equilibrium reasonable? A point can be made that it is an unreasonable solution:

Firstly, the equilibrium relies on Player 1 being indifferent between A and B *only if* Player 2 commits to playing R. However, consider the game from Player 2's perspective. A rational Player 2 should recognize that strategy B is strictly worse in this case for both them and their opponent in all circumstances; in which case, why would Player 2 ever anticipate Player 1 playing a strategy that is both self-detrimental (due to WD) *and* malicious (due to Pareto-worsening)?

Therefore, the belief that "Player 1 will play B" is **not a reasonable belief for a rational Player 2 to hold.** Which brings us to the modified payoff $u_2(A, \mathrm{R})$ for Player 2. With the newly held belief that Player 1 will only ever play A, in Example 3 it is clearly visible that Player 2 will thus play L at all times since in the reduced form game from Player 2's perspective:

	L	**R**
A	2, 5	0, 4.5
~~**B**~~	~~1, 2~~	~~0, 4~~

This contradicts and undermines the (B, R) equilibrium's core assumption that Player 2 will commit to R; thus, the stability of the equilibrium unravels.

7 Dead Weight in the Game Tree

When the conditions of WD + PM are met, the dominated strategy B is in essence dead-weight in the game tree.

Game-theoretic reasoning arises when there is strategic tension. One move by Player A might make them better off, but at the cost of making Player B worse off; and vice versa. Without this tension, there is no Game Theory.

But Cross Dominated moves have no such strategic tension.

Crucially, there is no need to even invoke repeated games (as is the case for punishment/reward strategies in games with multiple rounds) or welfare maximization on both sides to justify their elimination. Unlike repeated games, there is no need for the potential for future payoff changes to justify the elimination of Cross Dominated moves. Unlike welfare optimization, there is no tension of common welfare vs individual interest for Cross Dominated moves to debate on its removal. With the simplest, most fundamentally shaved-down one-shot games, with pure-self-interested players, Cross Dominated moves are already pointless strategically. There "is a more interesting game to be played" that has a fundamentally higher "range" (as in, potential payoff output) of potential higher outcomes for both players when it is pruned (conceptually, lifting the minimum of payoff range for both players up higher). This is why it is close to dominance. Dominance is fundamentally erasing moves that isolate the strategically interesting parts of the game tree. Cross Dominance allows us to do the same, one step further.

The language of shared-interest is useful to describe *the lack of strategic tension* of these moves in this way, but it is easy to misinterpret it due to how it is used usually, which is why this section has been included for clarification.

8 Iterated cross elimination (ICE)

One round of *cross elimination* can be done by deleting any strategy B for which there exists an A with $\mathrm{CD}(A, B)$. We may then iterate to a reduced game. ICE is a stricter justification than IEWDS, and thus may eliminate fewer strategies. We leave order-independence of ICE open.

9 Conclusion

Cross dominance supplies a compact, justifiable principle for pruning strategies: *elimination by shared interest*. It strictly strengthens weak dominance, is orthogonal to strict dominance, and aligns with refinement intuitions while remaining elementary.

This concept benefits from a clear name as it identifies an unnamed axis in the elimination literature: not self-interest, not robustness to mistakes, but *rational shared interest*. When both players agree a strategy is not beneficial, it can serve as an equally powerful justification for acknowledging its elimination, when traditional self-interest criteria might preserve it.

References

[1] Martin J. Osborne and Ariel Rubinstein. *A Course in Game Theory*. MIT Press, Cambridge, MA, 1994.

[2] Reinhard Selten. Reexamination of the perfectness concept for equilibrium points in extensive games. *International Journal of Game Theory*, 4:25–55, 1975.

A Mixed strategies lemma

Lemma 1 (Extension to mixed strategies)**.** We state inequalities pointwise in the opponent's pure replies; by linearity they extend to all beliefs (mixed strategies). If $u_1(A, y) \geq u_1(B, y)$ for all pure y, then for every belief σ over $\mathcal{Y}$, $\mathbb{E}_{y\sim\sigma}[u_1(A, y)] \geq \mathbb{E}_{y\sim\sigma}[u_1(B, y)]$, and the same can be said for u_2.

B THP lemma

Lemma 2 (Why (B, R) isn't THP in Ex. 1/3)**.** For any mixed σ_2 with $\sigma_2(L) > 0$, Player 1's expected payoff from A exceeds that from B (since $u_1(A, L) > u_1(B, L)$ and $u_1(A, R) = u_1(B, R)$). Thus B is not a best response to any such perturbation; hence (B, R) is not trembling-hand perfect.

C List of all definitions for reference

Weak dominance (WD): $(u_1(A, y) \geq u_1(B, y))\ \forall y$, and $(>)$ for some y.

Strict dominance (SD): $(u_1(A, y) > u_1(B, y))\ \forall y$.

Pareto-monotone (PM) ("Harmless to P2 from P1 switching"): $(u_2(A, y) \geq u_2(B, y))\ \forall y$.

Cross dominance (CD) ("A win-win situation implies rational pruning"): WD + PM.

Strict-cross (CD$^{\mathbf{str}}$): SD + PM.

Self-interest ladder: SD $\Rightarrow$ WD (sets: SD $\subset$ WD).

Shared-interest ladder: CD$^{\text{str}}$ $\Rightarrow$ CD $\Rightarrow$ WD (sets: CD$^{\text{str}}$ $\subset$ CD $\subset$ WD).

Global relation: CD $\not\subset$ SD and SD $\not\subset$ CD.

Within the PM slice: SD $\Rightarrow$ CD $\Rightarrow$ WD.