CS 440 - Homework 5

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Problem 1

1. Is this network a polytree? If so, how do you know? If not, why not?

Yes, because if you replace the directed edges with undirected edges, the resulting graph is both connected and acylic.

2. How many parameters are necessary to fully express the joint distribution? How many parameters are necessary in the Bayesian network?

- Joint Distribution: $2^n - 1 = 2^5 - 1 = 31$ parameters
- Bayesian Network: $P(L) + P(U) + P(R|L) + P(G|R) + P(T|U,R) = 1 + 1 + 2 + 2 + 4 = 10$ parameters

3. You checked Google Maps and there is traffic. Would finding out if it is also raining affect your belief about whether a UFO has landed on the highway? Justify your answer using conditional independence rules.

Yes it would decrease my belief of the UFO landing on the highway. Per the Bayesian Net, $P(T)$ is conditionally dependent on $P(U)$ and $P(R)$. Whether or not it rains does not directly influence whether there is a ufo (as there is no connection between them in the net, hence they are independent). However, both of them influence traffic, and if it is raining, I am more likely to assume that rain is the reason for the traffic. If I do not know whether it is raining, I would follow conditional probabilities. Hence $P(U), P(R)$ are not conditionally independent.

4. You check your barometer and you see that there is a low pressure outside. Does this give you any information about whether a UFO landed on the highway? Justify your answer using conditional independence rules.

Yes. L influences R, ($P(R|L)$) in the Bayesian Net, and R influences T, $P(T|U,R)$. The rest of the argument follows 3 (above).

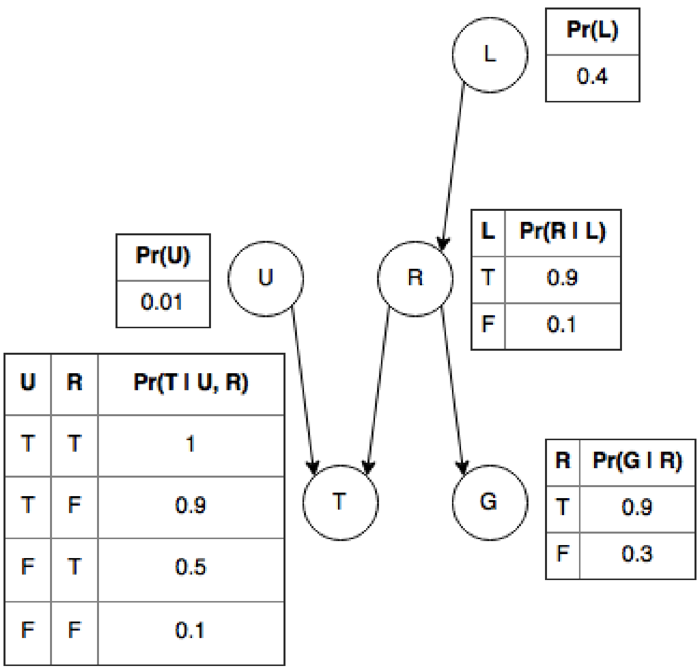
5. Does knowing there is a low pressure give you any information about whether there is traffic? Justify your answer using conditional independence rules.

Yes. L influences R, ($P(R|L)$) in the Bayesian Net, and R influences T, $P(T|U,R)$. Hence, knowing there is low pressure means it is more likely to have rain, and thus more likely to have traffic.

6. Assume that we observe the data on the right, and we want to estimate the parameters of this network:

- Compute the most likely distribution *P(U)* with no smoothing
  - $P(U) = \frac{0}{6} = 0$
  - $P(\neg U) = \frac{6}{6} = 1$
- Compute *P(U)* with Laplace smoothing with a smoothing constant 1
  - $P(U) = \frac{0+1}{6+1+1} = 0.125$
  - $P(\neg U) = \frac{6+1}{6+1+1} = 0.875$
- Which of these would you expect to be a better model of the real world? Explain.
  I would expect the distribution with laplace smoothing to be a better model. We can not say for certain that there are no aliens, and thus can not conclusively state that it is impossible for a UFO to land on the highway (as the distributio with no smoothing states $P(U) = 0$).

7. For the next parts, use the following as the parameters of the network



Pr(L)
0.4

L	Pr(R I L)
T	0.9
F	0.1

Pr(U)
0.01

U	R	Pr(T I U, R)
T	T	1
T	F	0.9
F	T	0.5
F	F	0.1

R	Pr(G I R)
T	0.9
F	0.3

(a) Compute $P(G = 1)$

$$\begin{aligned} P(G = 1) &= P(R) * 0.9 + P(\neg R) * 0.3 \\ &= [P(L) * 0.9 + P(\neg L) * 0.1] * 0.9 + [P(L) * 0.1 + P(\neg L) * 0.9] * 0.3 \\ &= [0.4 * 0.9 + 0.6 * 0.1] * 0.9 + [0.4 * 0.1 + 0.6 * 0.9] * 0.3 \\ &= 0.552 \end{aligned}$$

(b) Compute $P(G = 1|L = 0)$

$$\begin{aligned}
P(G = 1|L = 0) &= P(G|\neg L) \\
&= P(R|\neg L) * 0.9 + P(\neg R|\neg L) * 0.3 \\
&= 0.1 * 0.9 + 0.9 * 0.3 \\
&= 0.36
\end{aligned}$$

(c) Computer the joint probability $P(L = 1, R = 1, G = 0, T = 0, U = 0)$

$$\begin{aligned}
P(\text{L=1, R=1, G=0, T=0, U=0}) &= P(L, R, \neg G, \neg T, \neg U) \\
&= P(L) * P(R|L) * P(\neg G|R, L) * P(\neg T|R, \neg U) * P(\neg U) \\
&= 0.4 * [0.4 * 0.9] * [P(R|L) * 0.1] * [P(R|L) * P(\neg U) * 0.5] * 0.99 \\
&= 0.4 * 0.36 * [(0.4 * 0.9) * 0.1] * [(0.4 * 0.9) * (0.99) * 0.5] * 0.99 \\
&= 0.4 * 0.36 * [(0.4 * 0.9) * 0.1] * [(0.4 * 0.9) * (0.99) * 0.5] * 0.99 \\
&= 0.000915
\end{aligned}$$

8. Specify the Markov blanket for each random variable in this Bayesian network.

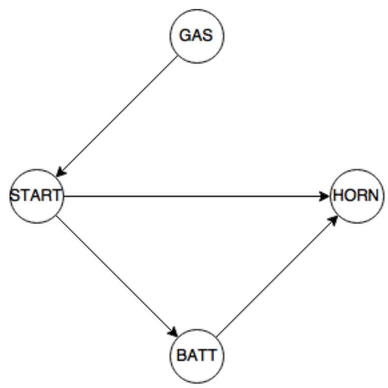
- $MB(L)$: {R}
- $MB(R)$: {L, G, U, T}
- $MB(G)$: {R}
- $MB(U)$: {R, T}
- $MB(T)$: {U, R}

Problem 2

1. How many parameters are required to represent a full joint distribution?

$3 * 2 * 2 * 3 - 1 = 2^2 * 3^2 - 1 = 35$ parameters

2. Give the ordering: GAS, START, HORN, BATT, draw the Bayesian network. You may optionally explain any non-intuitive decisions.



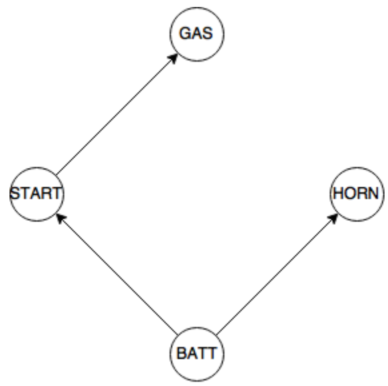
3. How many parameters are required to represent the distribution using your Bayesian network of part 2?

$$\begin{aligned}\text{Parameters} &= P(\text{GAS}) + P(\text{START}|\text{GAS}) + P(\text{BATT}|\text{START}) + P(\text{HORN}|\text{START,BATT}) \\ &= 2 + 3 + 3 + 6 \\ &= 14\end{aligned}$$

4. What is the best Bayesian network ordering you can think of for these random variables?

BATT, HORN, START, GAS

5. Draw another Bayesian network using the ordering you came up with in part 4. You may optionally explain any non-intuitive decisions.



6. How many parameters are required to represent the distribution using your network of part 5?

$$\begin{aligned}\text{Parameters} &= P(\text{BATT}) + P(\text{HORN}|\text{BATT}) + P(\text{START}|\text{BATT}) + P(\text{GAS}|\text{START}) \\ &= 2 + 3 + 3 + 3 \\ &= 11\end{aligned}$$