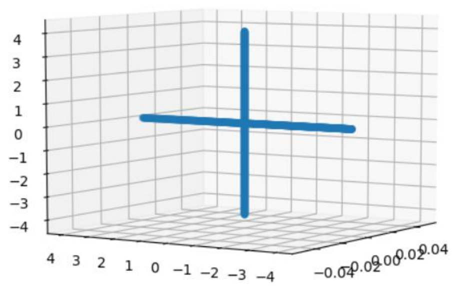
Report 2

We first try a few experiment in the cross-line model.

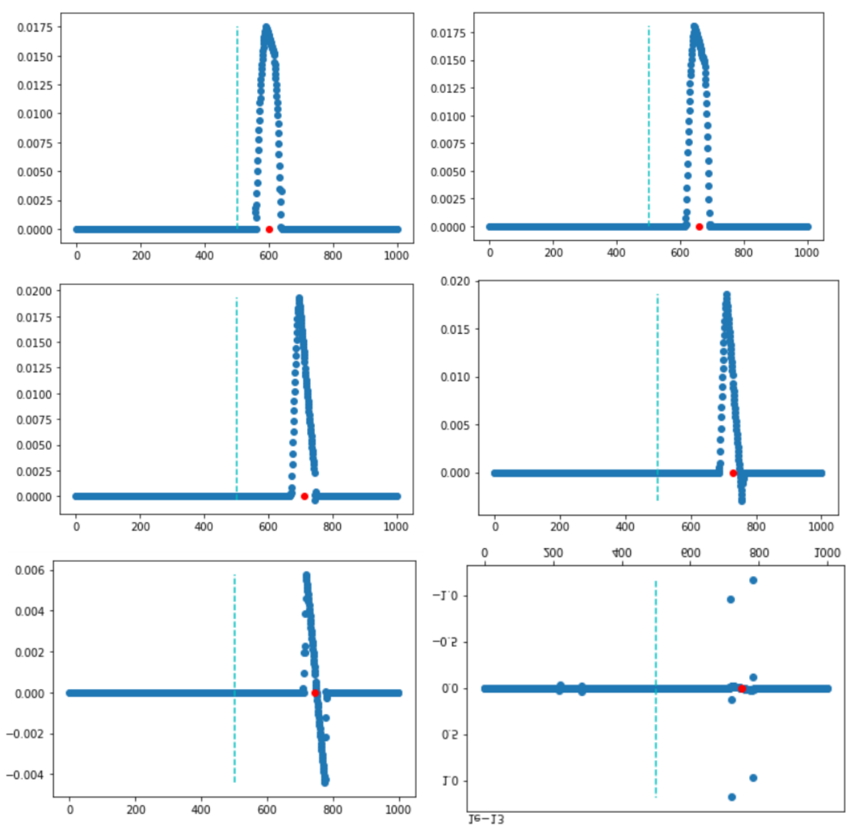
Original plot:

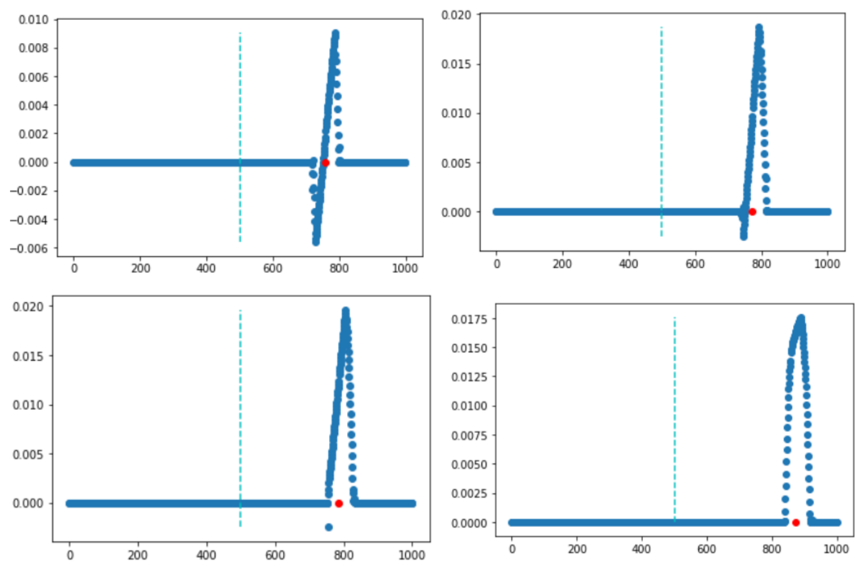


In the original plot, two orthogonal lines interact at the origin. There are 500 points in each line.

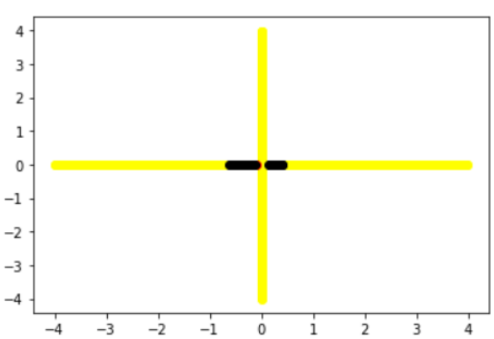
1. $\min \frac{1}{2}||Y - X\beta||^2 + (\lambda_1 \left||\beta|\right|_1 +)\lambda_2 \sum_{i=1}^{n} |\beta_i| ||Y - x_i||_2^2$

   We let every point be response Y, the rest point be X and solve the optimization problem above. We obtain coefficients for corresponding points. Let x-asis be index for points and y-asis be values for coefficients. The blue dashed line is the separating line for points from two lines and the red point is the central point(response).





According to the plots above, there exist interesting patterns: coefficients get larger when points closer to the central point and roughly symmetry about vertical line through central point. When the central point is far from origin, the peak of coefficients are similar and nearly all coefficients are positive. When central point moves to the origin, some negative coefficients gradually appear. At the origin, all coefficients tend to zero. When moving pass the origin, it shows similar patterns as the left side. Checking all points on a line, no points from the other line are connected.

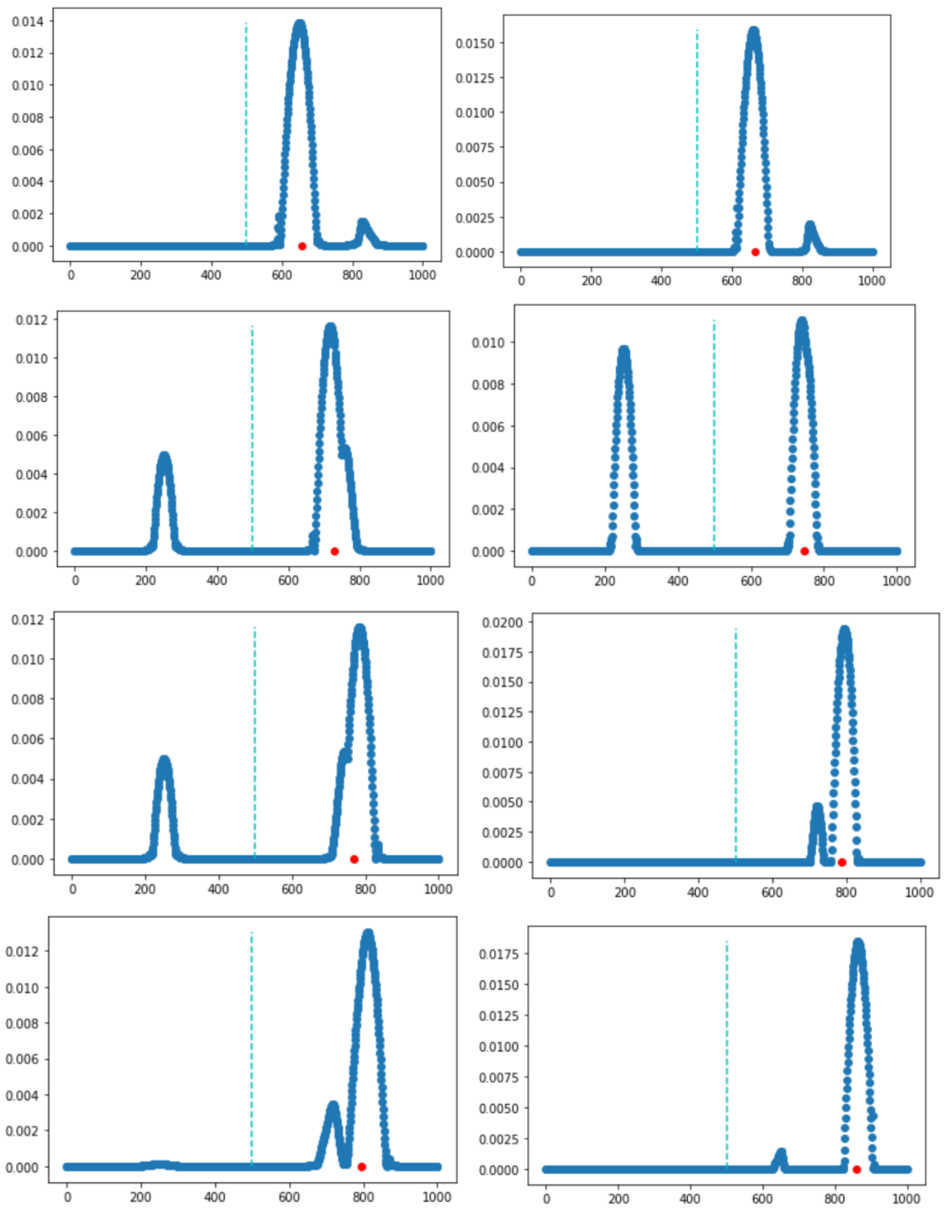


Left plot is the actual plot of two lines. The red point is the central point and black ones are points with first 5 largest coefficients. From this plot, even central point is really close to the intersection point, none of points from the vertical line is connected to it.

Note: the L1 penalty could be erased.

2. $$\min_{\begin{cases}\sum\beta_i=1\\ \beta_i\geq 0\end{cases}} \frac{1}{2}||Y-X\beta||^2+\lambda_1\sum_{i=1}^{n}|\beta_i|\min\{\big||y-x_i|\big|,||-y-x_i||\}$$

We simulate same data set as in section 1. And do similar things, pick every point as response Y and other points as variables X. Solve the above optimization problem and obtain coefficients. But the plot we obtained is different.

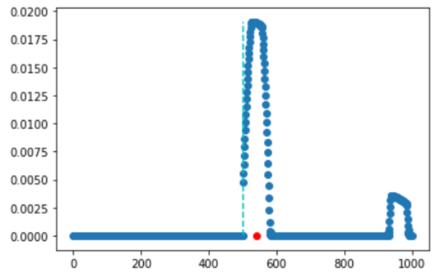


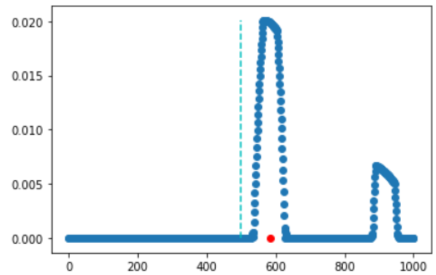
Central points also move from left bottom to the right. When response is far from intersection point(origin), there exist two peaks both of which are seems to be symmetry and their centers are symmetry about the origin. Though two peaks are on the same side in the plot which means they are on the same line, peak above the red point(response) is far higher than the other one. When central point moves closer to the origin, two peaks on the same side gradually merge and another peak on the other side appears. This means some points from the orthogonal line are connected. But fortunately, this only happens when response are very close to the origin. And when passing the origin, the pattern is similar to the previous pattern.

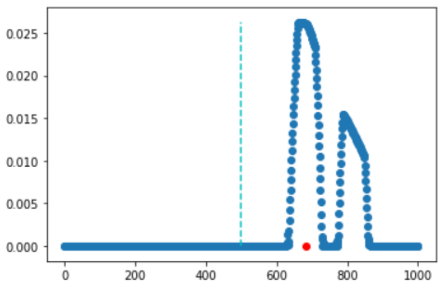
Note: the penalty we add allow us to connect the central point with points not only around it but also far away on the other side of this line. We add the constraints in order to avoid all-0 coefficients on the origin. But the constraints have a few drawbacks: points near the intersection point(here is origin) will be connected to both lines.

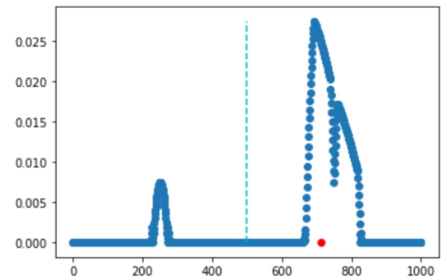
3. $\min\limits_{\beta_i \geq 0} \frac{1}{2}||Y - X\beta||^2 + \lambda_1 \sum_{i=1}^{n} |\beta_i| \min\{\big||y - x_i|\big|, ||-y - x_i||\}$

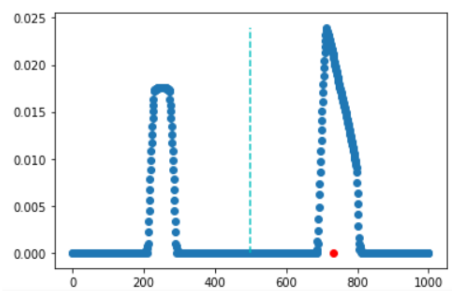
   We remove the constraints $\sum \beta_i = 1$ and redo the experiment.

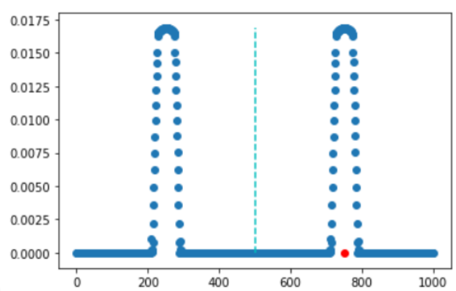


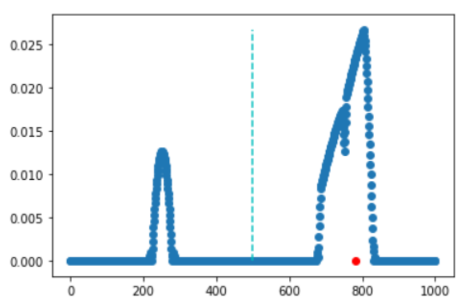


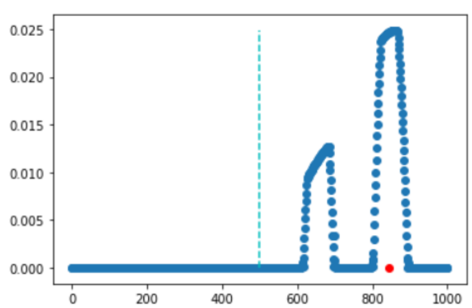








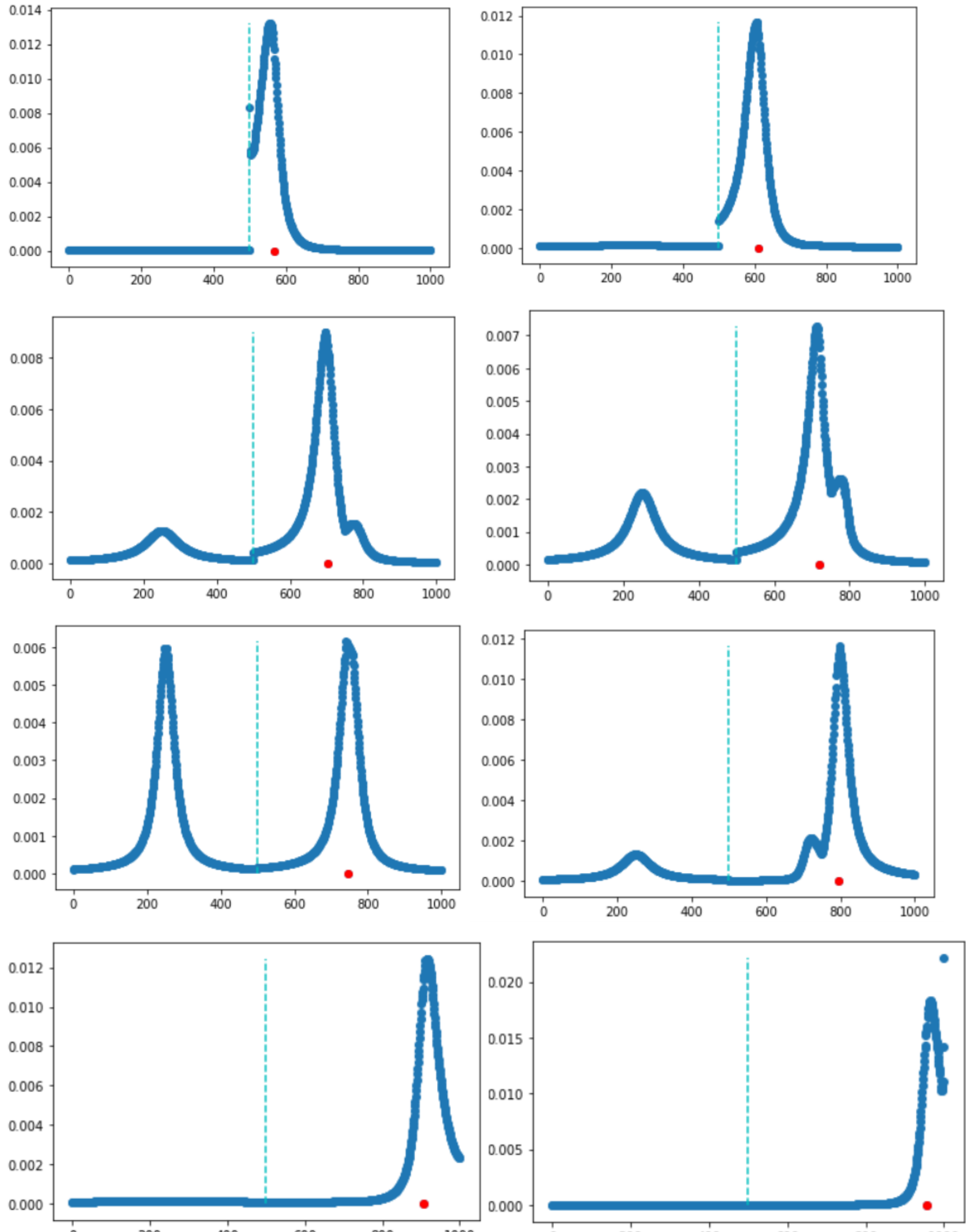




   The results are quite similar with section 2. The difference is the height of peaks. Obviously, the difference between two peaks is smaller.

4. $\min\limits_{\begin{cases}\sum \beta_i = 1 \\ \beta_i \geq 0\end{cases}} \frac{1}{2}||Y - X\beta||^2 + \lambda_1 \sum_{i=1}^{n} |\beta_i|^2 \min\{\big||y - x_i|\big|^2, ||-y - x_i||^2\}$

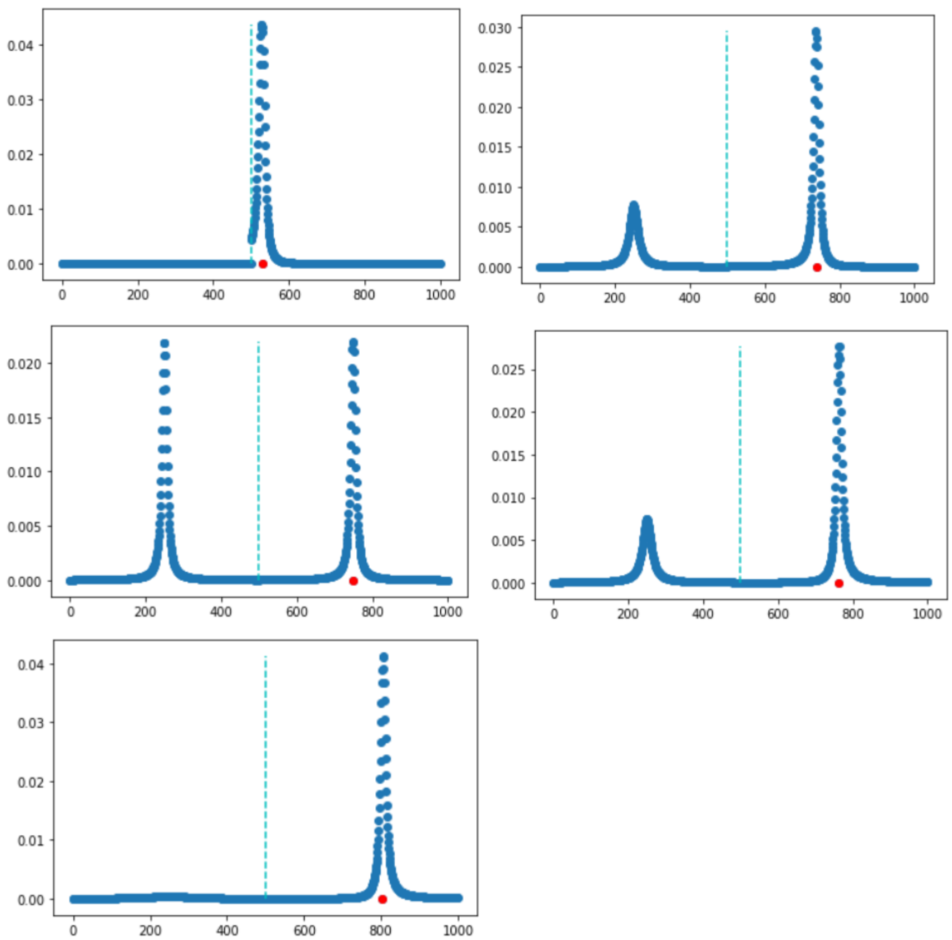
   We change the penalty, using $|\beta_i|^2$ but not $|\beta_i|$. This penalty is like a modification of ridge regression. Still on the orthogonal lines.



After we change the penalty, though the new penalty indicates there might be two similar patterns on the same line in the plot, the results only shows one. When the central point is far from the intersection point(the origin), only one peak exists. If the point is close to the central point, two peaks may exist but one peak is from the other line. And when point moves to the right side, similar pattern as the left side.

5. $\min_{\begin{cases}\sum\beta_i=1 \\ \beta_i\geq 0\end{cases}} \frac{1}{2}||Y-X\beta||^2+\lambda_1\sum_{i=1}^{n}|\beta_i|^2\,||Y-x_i||_2^2$

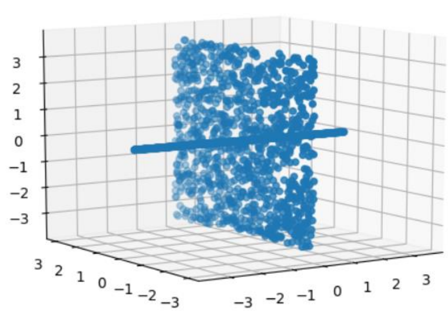
We change the penalty again and redo the experiment.



This looks similar as the section 4.

Then we try similar experiments on a new model: a line and a plane, orthogonal. Both have 1000 points.

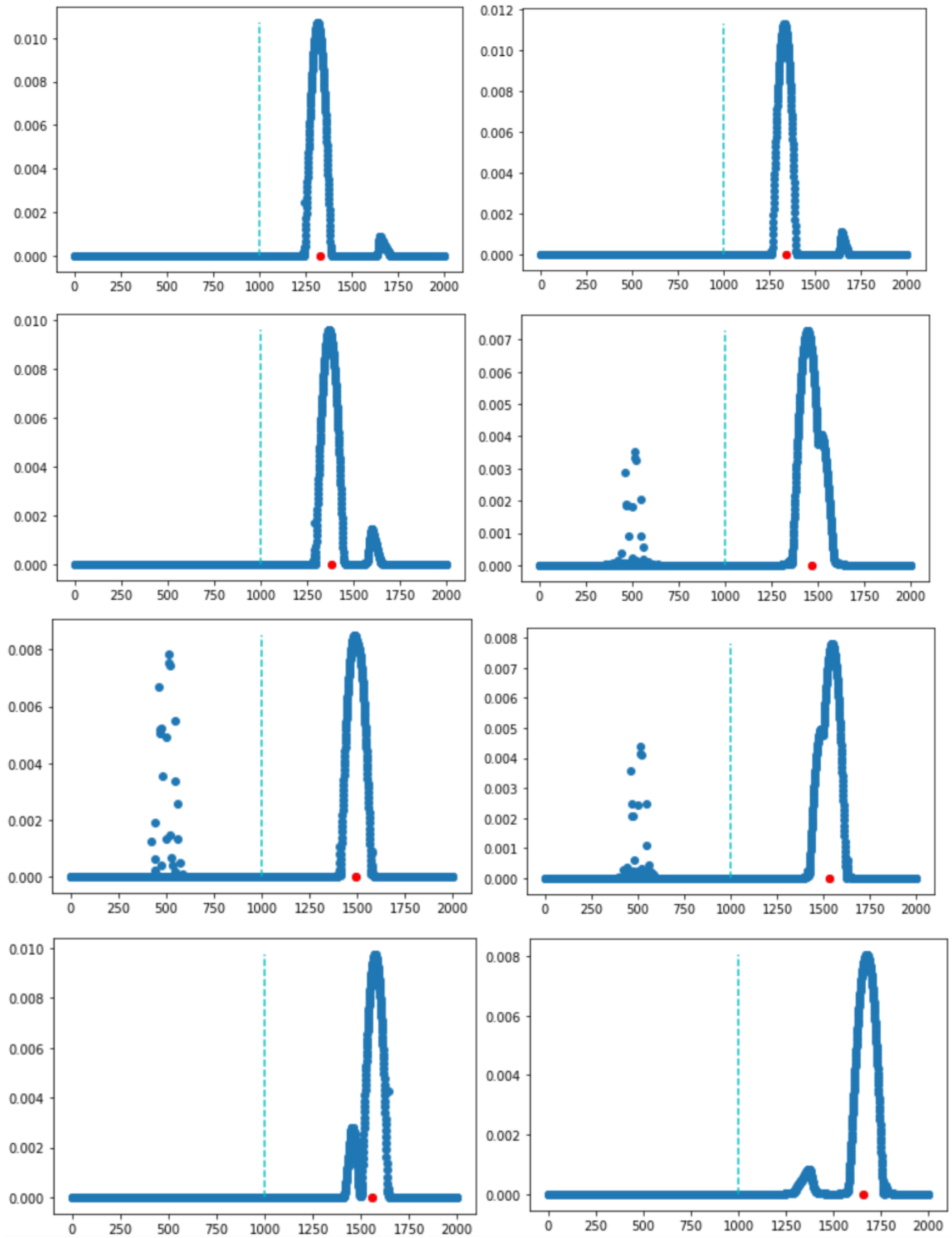
Original plot:



This plot consists of a line and an orthogonal plane. Both have 1000 points and intersect at the origin. We do same thing and obtain the coefficients.

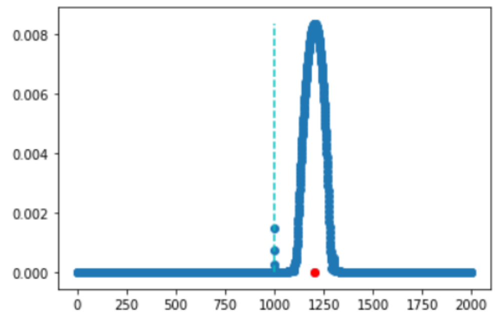
6. $\min_{\begin{cases}\sum \beta_i = 1 \\ \beta_i \geq 0\end{cases}} \frac{1}{2}||Y - X\beta||^2 + \lambda_1 \sum_{i=1}^{n} |\beta_i| \min\{\left||y - x_i|\right|, || -y - x_i||\}$

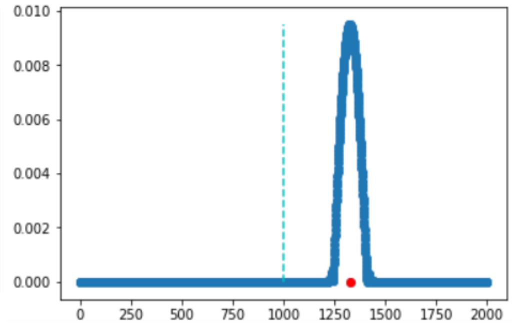
   Original plot:

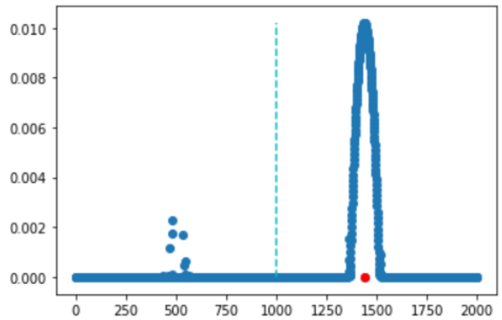


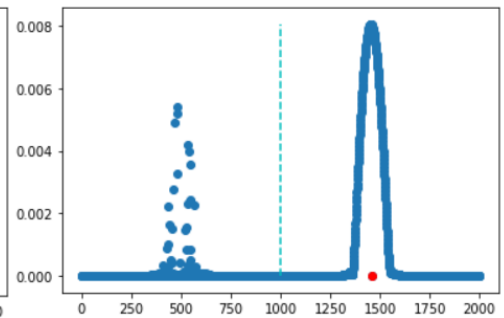
Also, these plots show what the coefficients behave like when the central points(red point) on the line move from one side to the other. The patterns look a little bit similar, but at the bottom of two side, there are some difference. 4th to 6th plot shows how coefficients behave when the central point moves through the intersection plot.

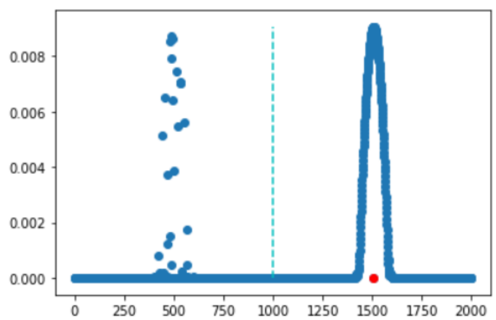
7. $\min_{\begin{cases}\sum \beta_i = 1 \\ \beta_i \geq 0\end{cases}} \frac{1}{2}||Y - X\beta||^2 + \lambda_1 \sum_{i=1}^{n} |\beta_i| ||Y - x_i||_2^2$

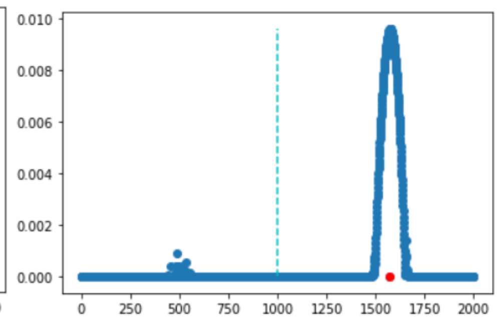


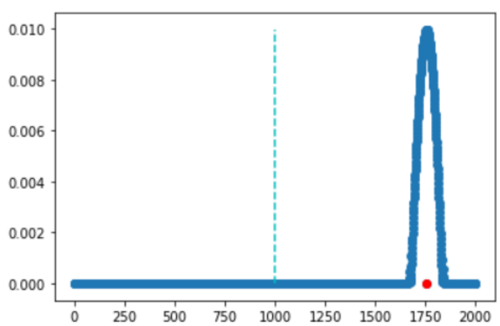






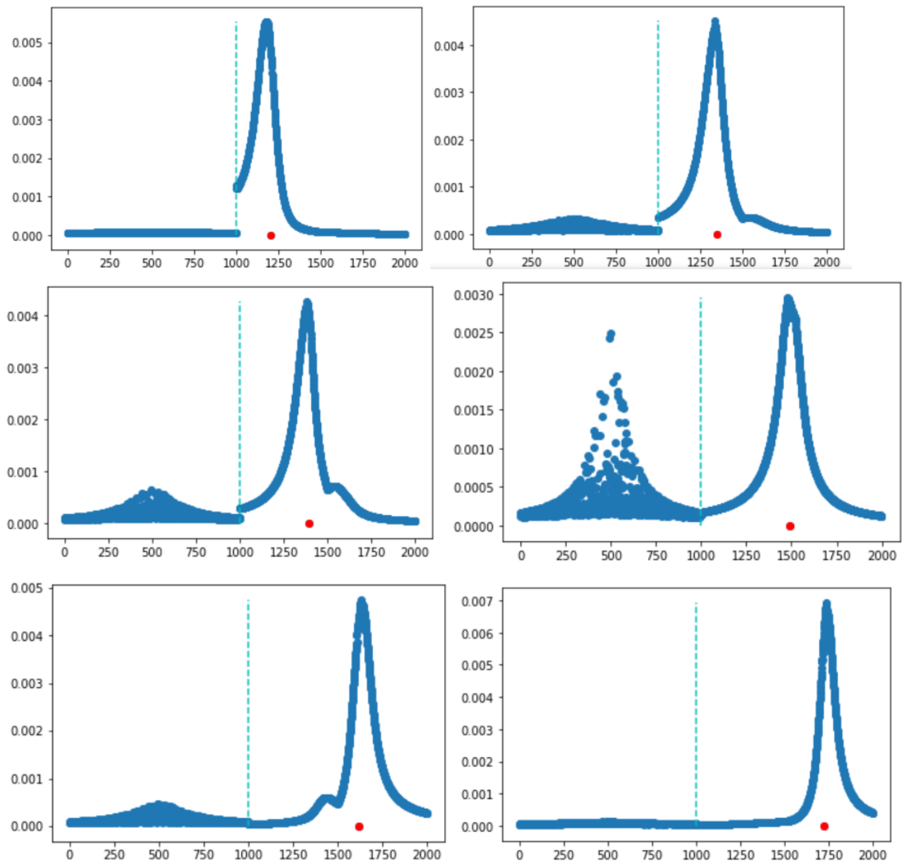






When central is far from the intersection point, there exists one peak. When the point is closer to intersection point as 3rd to 6th plot shows, another peak appears on the other side which means some points in the plane are connected. When the point moves forward, this peak gets smaller and finally disappears.

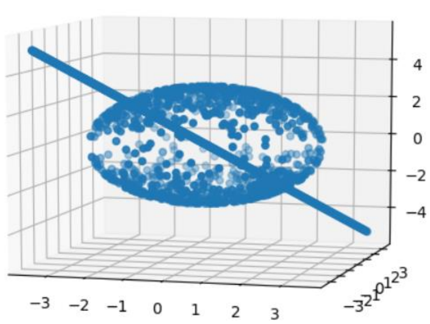
8. $$\min_{\begin{cases}\sum \beta_i=1\\ \beta_i\geq 0\end{cases}} \frac{1}{2}||Y-X\beta||^2+\lambda_1\sum_{i=1}^{n}|\beta_i|^2\min\{||y-x_i||,||-y-x_i||\}$$



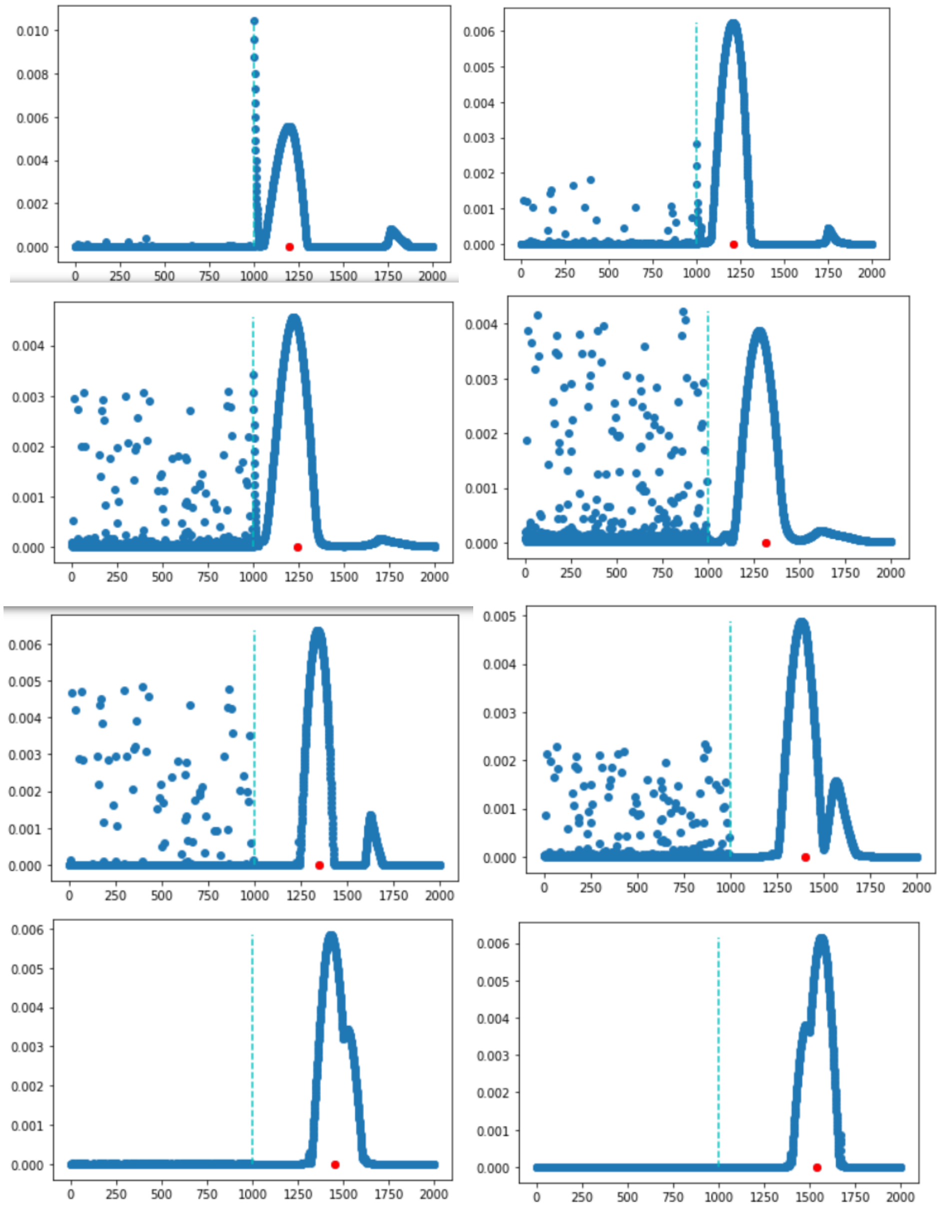
These plots show how coefficients change when central point moves from one side to the other. Most plots show one peak, but when close to intersection point, another peak appears.

We will try the series of experiments on a new model: a sphere and a line, also orthogonal. Both have 1000 points.

Original plot:

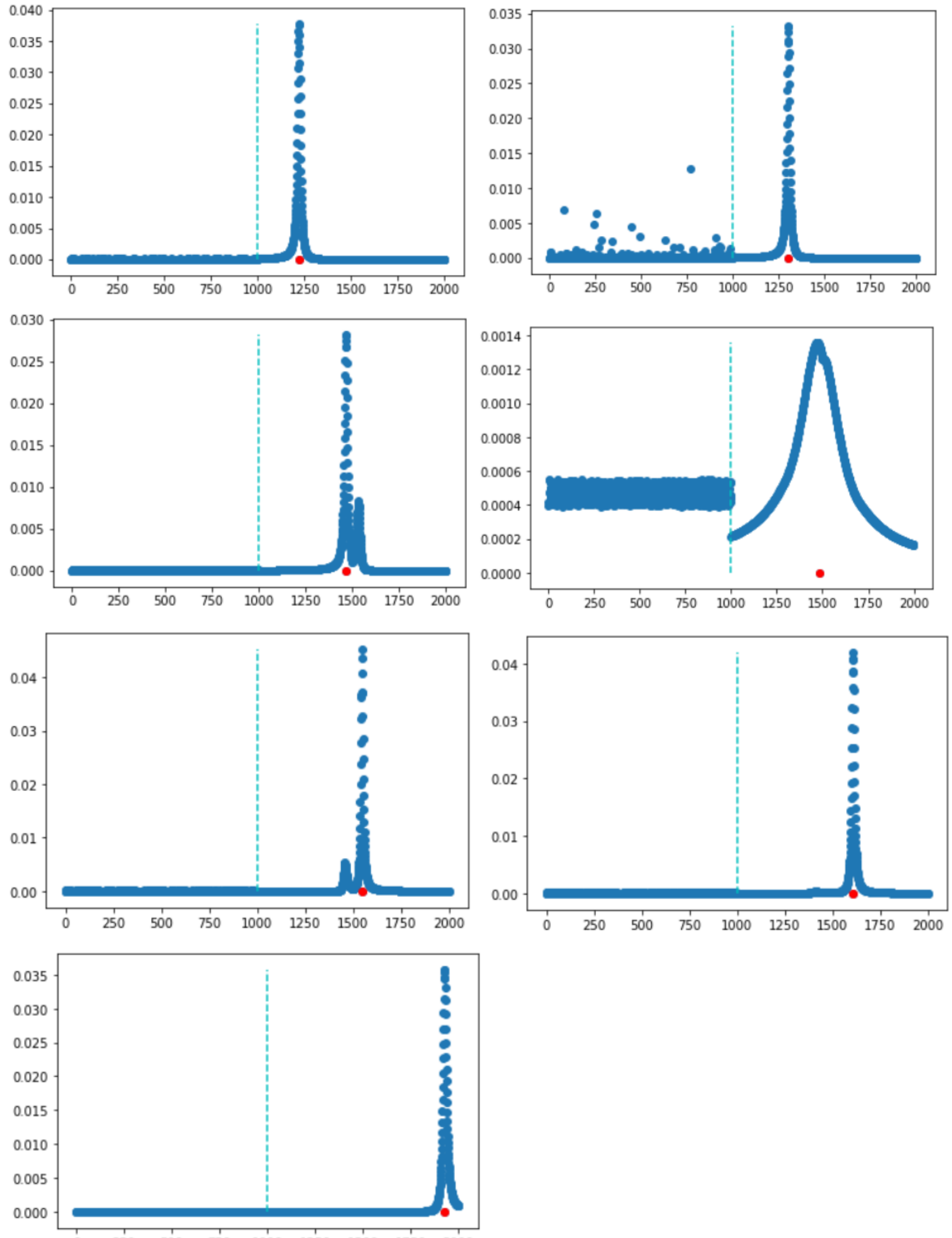


9. $\min_{\substack{\sum \beta_i=1 \\ \beta_i \geq 0}} \frac{1}{2}||Y - X\beta||^2 + \lambda_1 \sum_{i=1}^{n} |\beta_i| \min\{||y - x_i||, ||-y - x_i||\}$

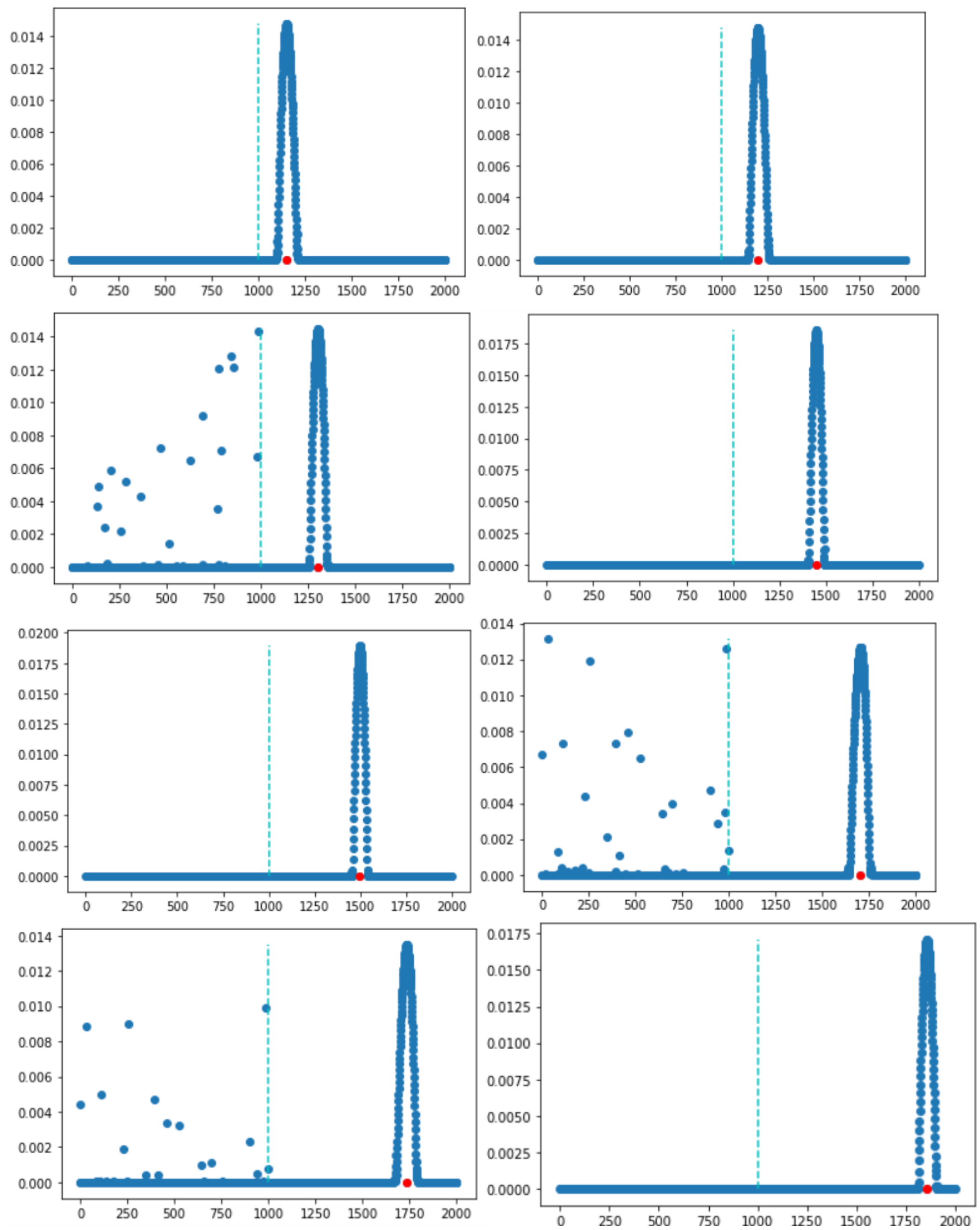


We do the regression globally and let central points move on the line from one side to the other. When the central point is close to the intersection point(not origin), some points from the sphere are connected.

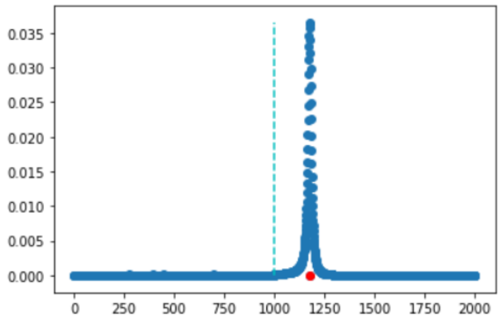
10. $$\min_{\begin{cases}\sum \beta_i = 1 \\ \beta_i \geq 0\end{cases}} \frac{1}{2}||Y - X\beta||^2 + \lambda_1 \sum_{i=1}^{n} |\beta_i|^2 \min\{||y - x_i||, ||-y - x_i||\}$$

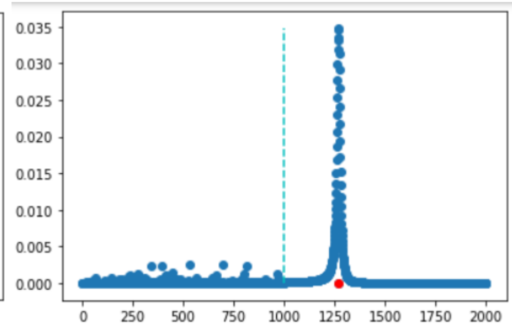


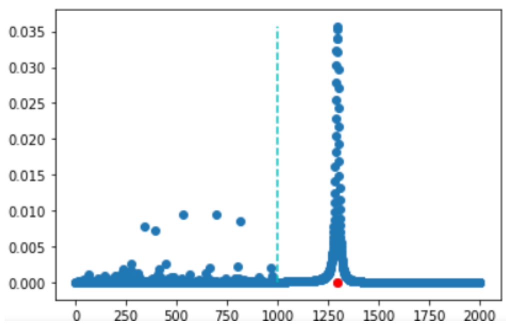
11. $$\min_{\begin{cases}\sum \beta_i = 1 \\ \beta_i \geq 0\end{cases}} \frac{1}{2}||Y - X\beta||^2 + \lambda_1 \sum_{i=1}^{n} |\beta_i| ||Y - x_i||_2^2$$

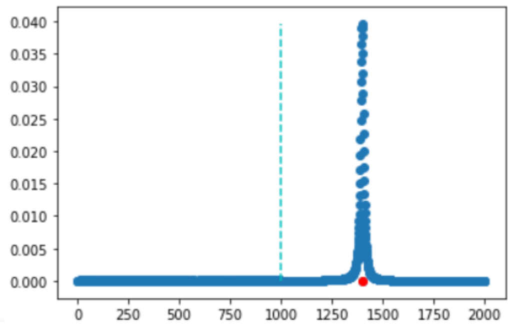


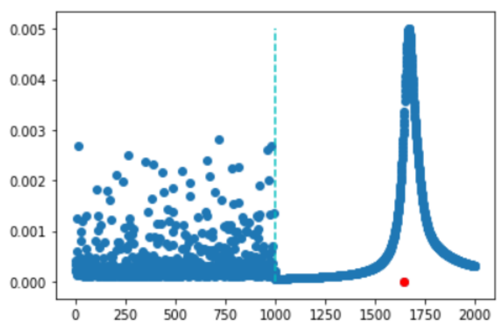
12. $\min_{\left\{\substack{\sum\beta_i=1\\ \beta_i\geq 0}\right.} \frac{1}{2}||Y-X\beta||^2+\lambda_1\sum_{i=1}^{n}|\beta_i|^2||Y-x_i||_2^2$

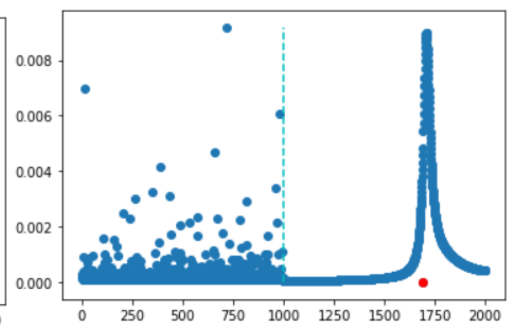


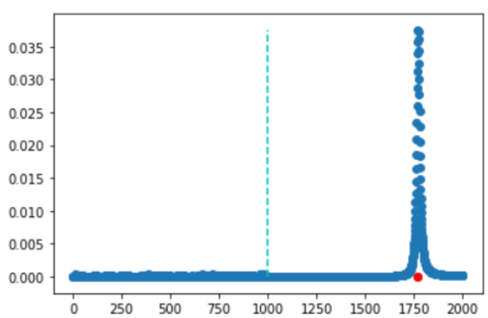












Notice there are two intersection points in this model. So there is some noise in the plot above, as we could see in the 2^{nd} ,5^{th} and 6^{th} plot.