A Hybrid Benders Decomposition of the Assembly Line

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AMSI Optimise Workshop

Masters Thesis

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Overview

1. The Problem: Assembly Line Balancing
2. The Method: Benders Decomposition
3. Results

Assembly Lines through History

Ford Motor Company, Detroit 1915

Volkswagen, Wolfsburg 1951

Assembly Lines through History

Kia Motors, Žilina 2012

The Assembly Line Balancing Problem

Production system designed to *efficiently* manufacture a product

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1. Tasks
2. Work stations
3. Precedence Relations

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Production system designed to *efficiently* manufacture a product

1. Tasks
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Def^{n}: The cycle time of the line is the largest workload among all stations

$$c = \max\{\, l_k \mid k \in K \,\}$$

where K is the set of stations and l_k is the load of station k

The Type-2 ALBP

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  - Fixed number of work stations
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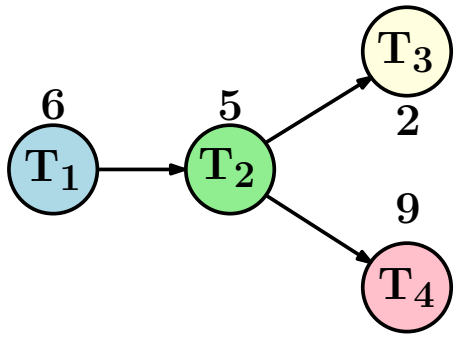
- Aim: Maximize production rate $\iff$ Minimize cycle time
  - By balancing workload across the stations
- What are our decisions?
  - Assigning tasks to stations

Simple Assembly Line Example

4 Tasks

3 Stations



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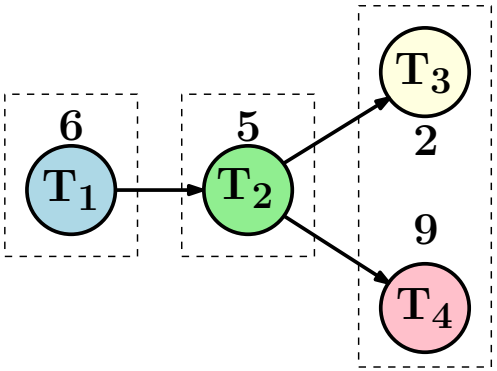

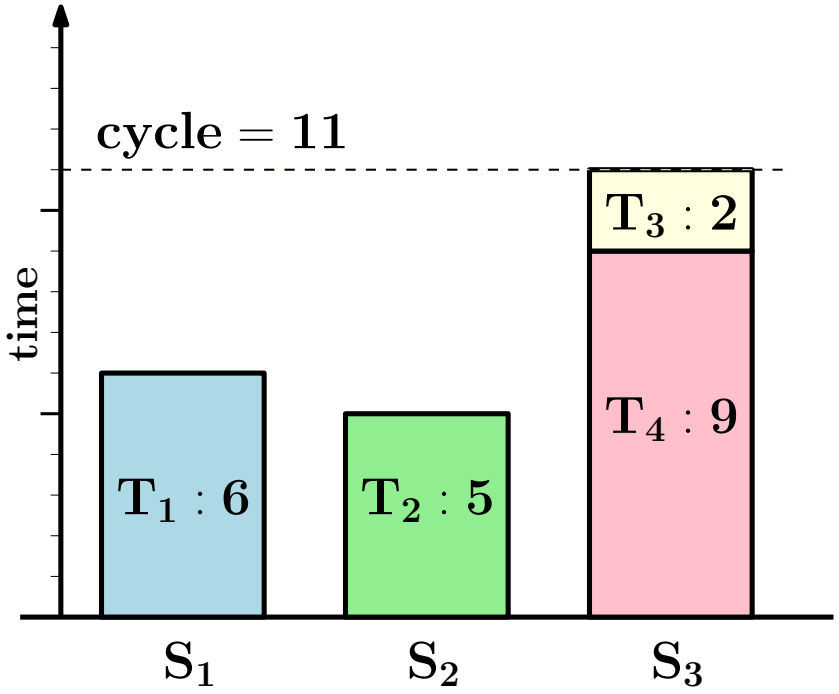

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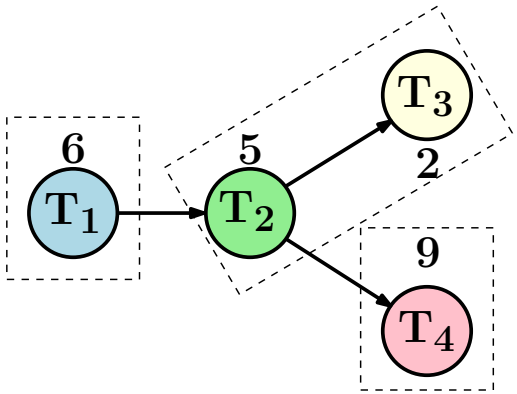

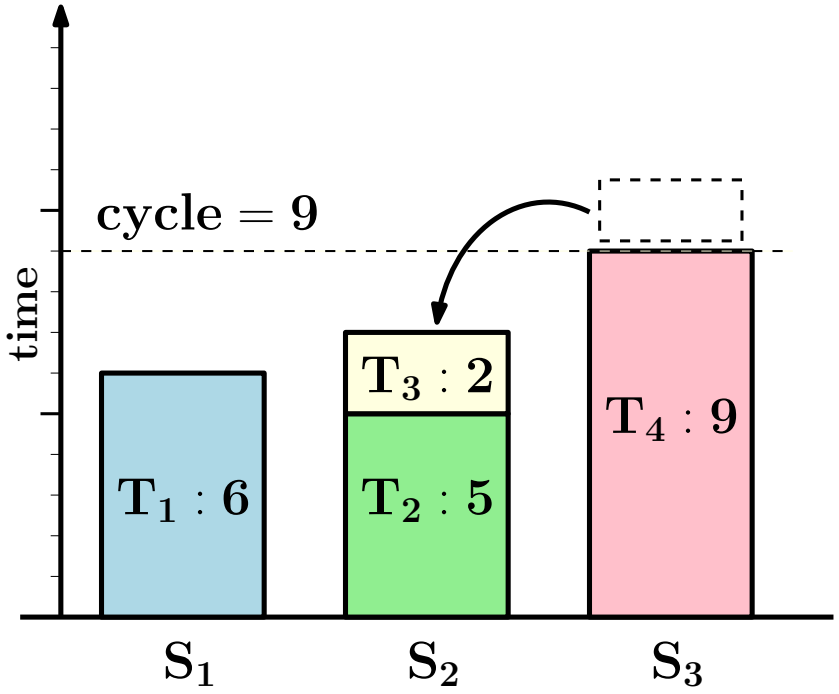

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- Theory usually assumes task sequence within a station can be arbitrary
  - Not always the case in practice!

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  - Walking distances
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  - Cooling periods

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- Theory usually assumes task sequence within a station can be arbitrary
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- Setup times between tasks can vary a lot
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  - Cooling periods
- Considering setups leads to the SetUp Assembly Line Balancing and Scheduling Problem, or SUALBSP
  - Type-2 problem is denoted SUALBSP-2

The SUALBSP-2

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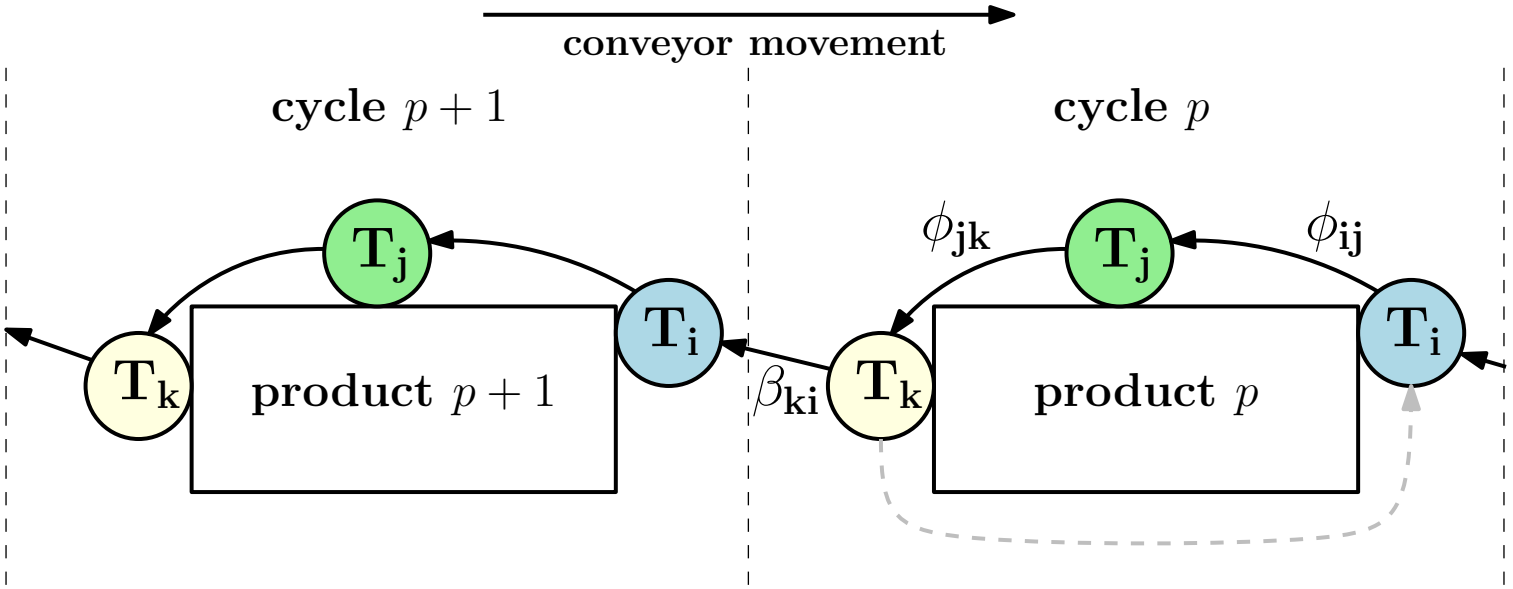
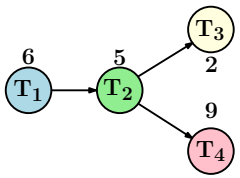
- Consider one station with tasks T_i, T_j and T_k:



Example with Setup Times



ϕ_{ij}	T_1	T_2	T_3	T_4
T_1	–	3	3	3
T_2	–	–	2	3
T_3	–	–	–	1
T_4	–	–	2	–

β_{ij}	T_1	T_2	T_3	T_4
T_1	0	–	–	–
T_2	3	0	–	–
T_3	3	5	0	3
T_4	3	3	1	0

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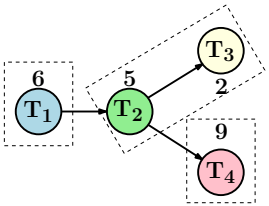

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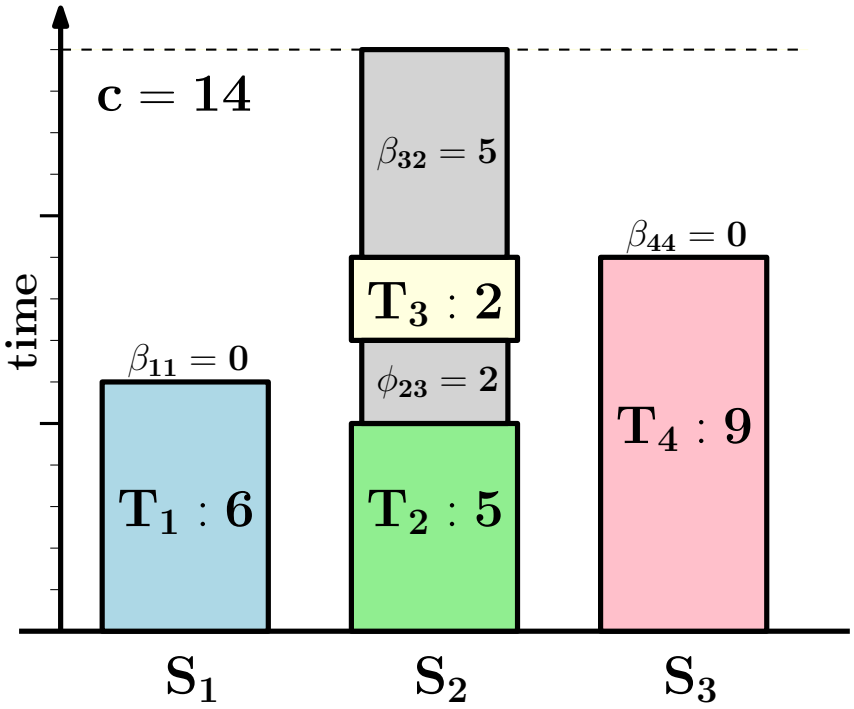

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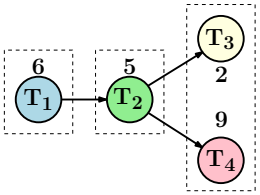

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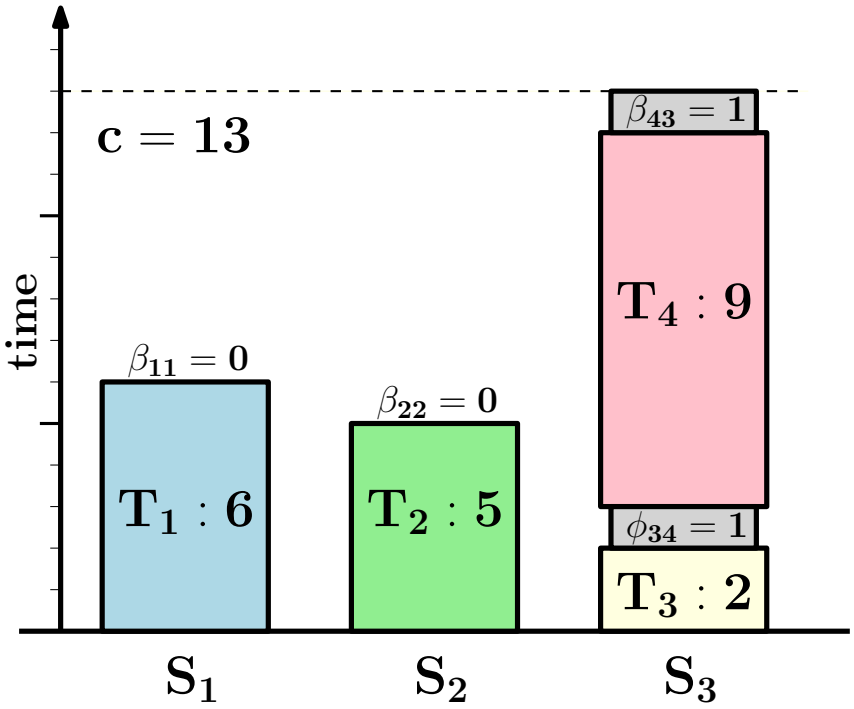

Existing Approaches

Data

- Huge data library for ALBPs
  - `http://assembly-line-balancing.mansci.de/`
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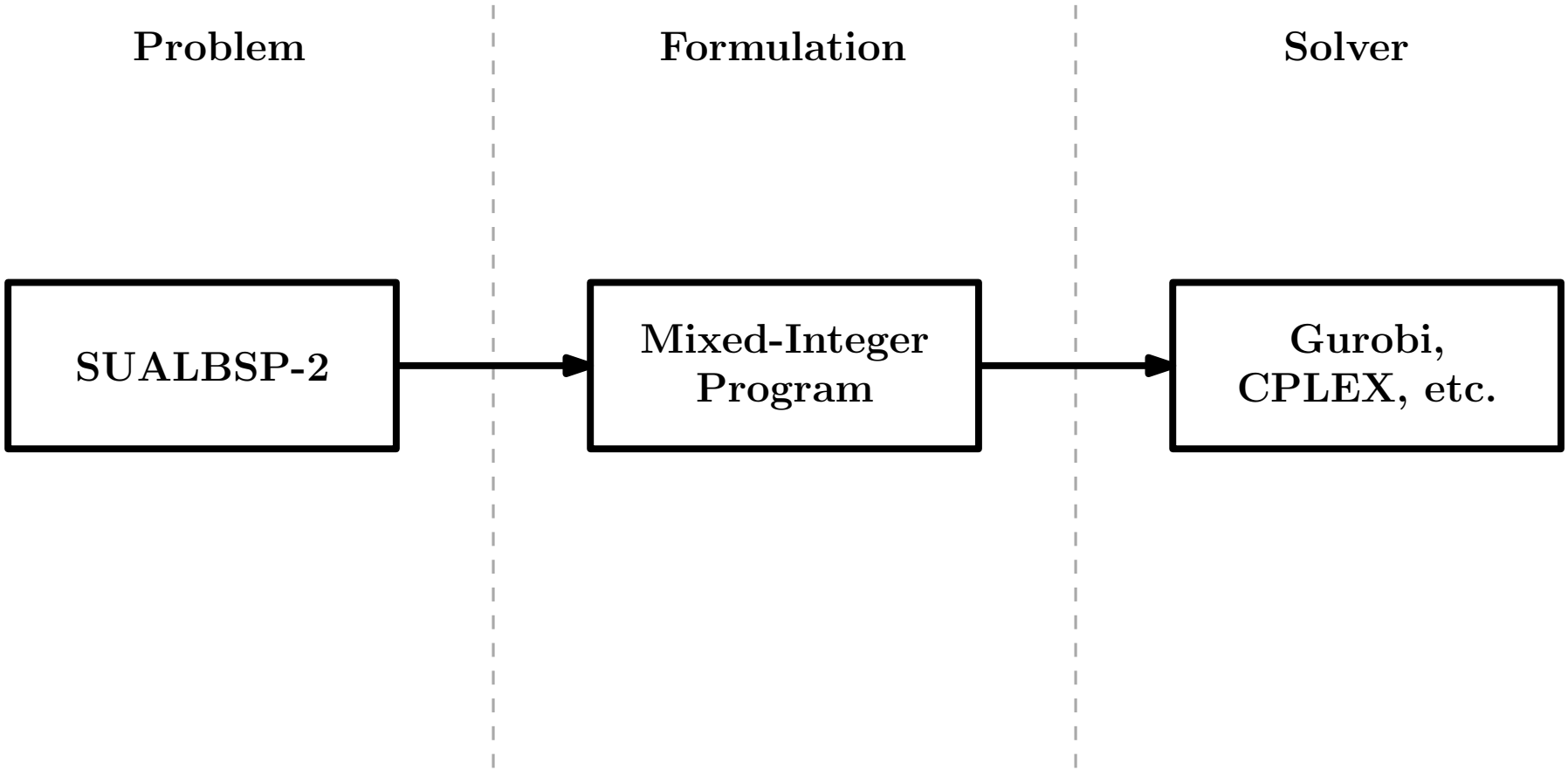
Data set	# Instances	# Tasks	# Stations	# Precedences
1	108	7-21 (small)	2-8	6-27
2	112	25-30 (medium)	3-14	32-40
3	176	32-58 (large)	3-31	38-62

Exact Solution Procedure



Best Known MIPs for SUALBSP-2

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SCBF-2: $\min c$ (33)

$$\sum_{j \in F_i^F} y_{ij} + \sum_{j \in F_i^B} w_{ij} = 1 \qquad \forall i \in V \qquad (4)$$

$$\sum_{i \in P_j^F} y_{ij} + \sum_{i \in P_j^B} w_{ij} = 1 \qquad \forall j \in V \qquad (5)$$

$$s_i + t_i + \sum_{j \in F_i^B} \mu_{ij} \cdot w_{ij} \leq c \qquad \forall i \in V \qquad (30)$$

$$r_i + 1 \leq r_j \qquad \forall (i, j) \in E \qquad (44)$$

$$q_{ij} + q_{ji} = 1 \qquad \forall i \in V \text{ and } j \in V \backslash (P_i^* \cup F_i^*) \text{ with } i < j \qquad (54)$$

$$r_i + 1 + (n - |F_i^*| - |P_j^*|) \times (q_{ij} - 1) \leq r_j \qquad \forall i \in V \text{ and } j \in V \backslash (P_i^* \cup F_i^*) \text{ with } i \neq j \qquad (55)$$

$$r_j - 1 + (n - |F_j^*| - |P_i^*| - 2) \times (y_{ij} - 1) \leq r_i \qquad \forall i \in V \text{ and } j \in F_i^F \qquad (56)$$

$$y_{ij} \leq q_{ij} \qquad \forall i \in V \text{ and } j \in F_i^F \backslash F_i^* \qquad (57)$$

$$w_{ji} \leq q_{ij} \qquad \forall i \in V \text{ and } j \in P_i^B \backslash F_i^* \text{ with } i \neq j \qquad (58)$$

$$q_{ij} \in \{0, 1\} \qquad \forall i \in V \text{ and } j \in V \backslash (P_i^* \cup F_i^*) \text{ with } i \neq j \qquad (59)$$

$$s_i + (t_i + \tau_{ij}) + (\bar{c} + \tau_{ij}) \cdot (y_{ij} - 1) \leq s_j \qquad \forall i \in V \text{ and } j \in F_i^F \qquad (60)$$

$$\sum_{i \in V} \sum_{j \in F_i^B} w_{ij} = m \qquad (61)$$

$$|P_i^*| + 1 \leq r_i \leq n - |F_i^*| \qquad \forall i \in V \qquad (51)$$

$$\underline{c} \leq c \leq \bar{c} \qquad (37)$$

$$y_{ij} \in \{0, 1\} \qquad \forall i \in V \text{ and } j \in F_i^F \qquad (13)$$

$$w_{ij} \in \{0, 1\} \qquad \forall i \in V \text{ and } j \in F_i^B \qquad (14)$$

$$s_i \geq 0 \qquad \forall i \in V \qquad (32)$$

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SCBF-2: min c (33)

$$\sum_{j \in F_i^F} y_{ij} + \sum_{j \in F_i^B} w_{ij} = 1 \qquad \forall i$$

$$\sum_{i \in P_j^F} y_{ij} + \sum_{i \in P_j^B} w_{ij} = 1 \qquad \forall j$$

$$s_i + t_i + \sum_{j \in F_i^B} \mu_{ij} \cdot w_{ij} \le c$$

$$r_i + 1 \le r_j \qquad \forall (i,j) \in E$$

$$q_{ij} + q_{ji} = 1 \qquad \forall i \in V \text{ and } j$$

$$r_i + 1 + (n - |F_i^*| - |P_j^*|) \times (q_{ij} - 1) \le r_j \qquad \forall i \in V \text{ and } j$$

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$$s_i + (t_i + \tau_{ij}) + (\bar{c} + \tau_{ij}) \cdot (y_{ij} - 1) \le s_j$$

$$\sum_{i \in V} \sum_{j \in F_i^B} w_{ij} = m$$

$$|P_i^*| + 1 \le r_i \le n - |F_i^*| \qquad \forall i \in V$$

$$\underline{c} \le c \le \bar{c}$$

$$y_{ij} \in \{0,1\}$$

$$w_{ij} \in \{0,1\}$$

$$s_i \ge 0$$

FSBF-2: Min c (3.1)

s.t.

$$\sum_{k \in FS_i} x_{ik} = 1 \qquad \forall i \in V \qquad (3.2)$$

$$\sum_{k \in FS_i} k \cdot x_{ik} = w_i \qquad \forall i \in V \qquad (3.3)$$

$$\sum_{j \in F_i^\phi} y_{ij} + \sum_{j \in F_i^\beta} z_{ij} = 1 \qquad \forall i \in V \qquad (3.4)$$

$$\sum_{i \in P_j^\phi} y_{ij} + \sum_{i \in P_j^\beta} z_{ij} = 1 \qquad \forall j \in V \qquad (3.5)$$

$$o_{ik} \le x_{ik} \qquad \forall i \in V,\ k \in FS_i \qquad (3.6)$$

$$\sum_{j \in F_i^\beta} z_{ij} \le \sum_{k \in FS_i} o_{ik} \qquad \forall i \in V \qquad (3.7)$$

$$\sum_{i \in FT_k} o_{ik} \le 1 \qquad \forall k \in K \qquad (3.8)$$

$$w_j - w_i \le M \cdot (1 - y_{ij}) \text{ and } w_i - w_j \le M \cdot (1 - y_{ij}) \qquad \forall i \in V,\ j \in F_i^\phi \qquad (3.9)$$

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$$\sum_{i \in FT_k} t_i \cdot x_{ik} \le c \qquad \forall k \in K \qquad (3.11)$$

$$s_i + t_i + \sum_{j \in F_i^\beta} \beta_{ij} \cdot z_{ij} \le c \qquad \forall i \in V \qquad (3.12)$$

$$\sum_{i \in V} \sum_{j \in F_i^\beta} z_{ij} \ge m \qquad (3.13)$$

$$s_i + c \cdot (w_i - w_j) + t_i + \phi_{ij} \cdot y_{ij} \le s_j \qquad \forall (i,j) \in E \qquad (3.14)$$

$$s_i + (t_i + \phi_{ij}) + (c + \phi_{ij}) \cdot (y_{ij} - 1) \le s_j \qquad \forall i \in V,\ j \in F_i^\phi \setminus F_i \qquad (3.15)$$

$$\underline{c} \le c \le \bar{c} \qquad (3.16)$$

$$s_i \ge 0 \qquad \forall i \in V \qquad (3.17)$$

$$x_{ik} \in \{0,1\} \qquad \forall i \in V,\ k \in FS_i \qquad (3.18)$$

$$y_{ij} \in \{0,1\} \qquad \forall i \in V,\ j \in F_i^\phi \qquad (3.19)$$

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# Tasks	Model	Gap (%)	No Sol. (%)	Optimal (%)	Runtime (s)
7–21	MIP-1	0.16	0.0	93.5	166.57
	MIP-2	1.62	0.0	86.1	374.88
25–30	MIP-1	13.76	0.0	**0.0**	1800.66
	MIP-2	18.01	**25.0**	**0.0**	1800.33
32–58	MIP-1	19.56	5.1	**3.9**	1759.67
	MIP-2	10.09	**61.9**	**0.0**	1800.88

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"There is a need for more efficient exact methods."

— Esmaeilbeigi *et al.* (2016)

Our Method

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- Largely neglected by the literature for ~ 40 years
- Recently re-emerged showing it's applicability
  - See surveys by Costa (2005) and Rahmaniani *et al.* (2016)
- Generalised to *logic-based* Benders decomposition by Hooker (2003)
  - Can now be applied to a much wider range of problems

Benders Decomposition: Crash Course

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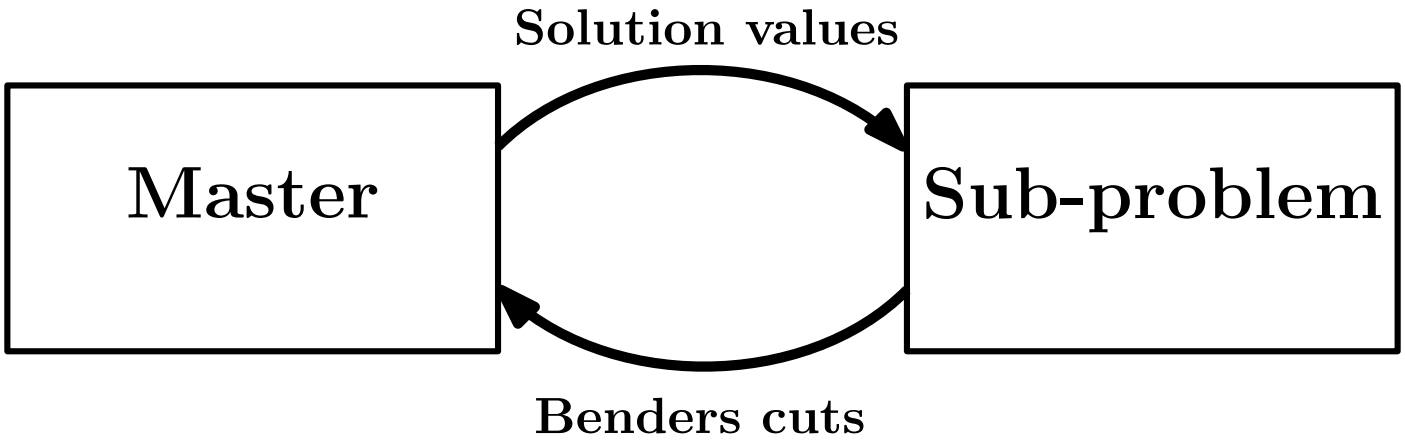

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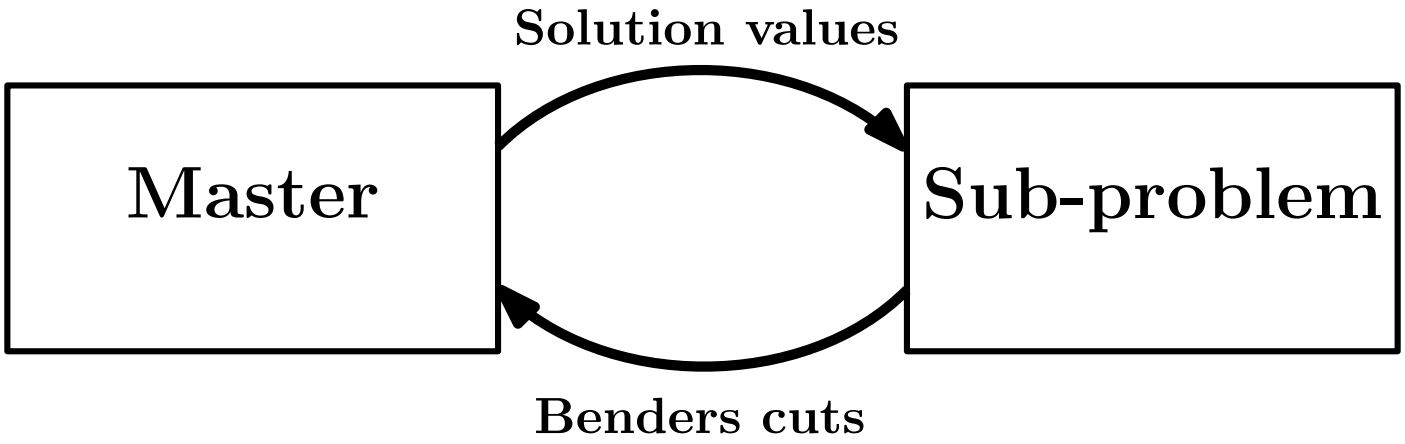

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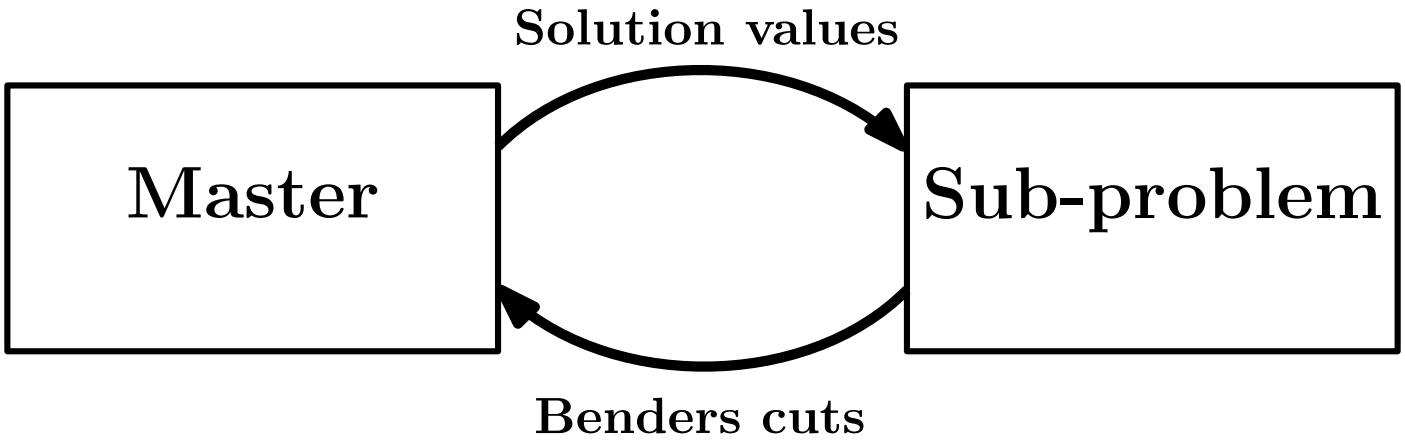

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- Sub-problem tells Master where the most important infeasibilities are

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  2. Sub-problem (SP) formulation
  3. Benders cuts $\Rightarrow$ How these problems communicate
- Logic-based Benders decompositon allows lots of flexibility in our formulations

Decomposing the Assembly Line

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RMP:
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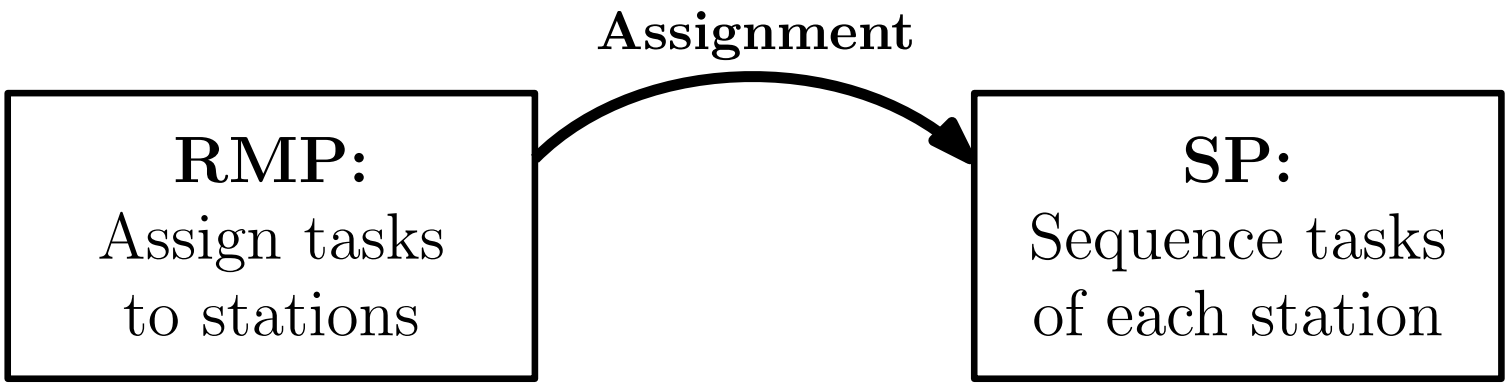

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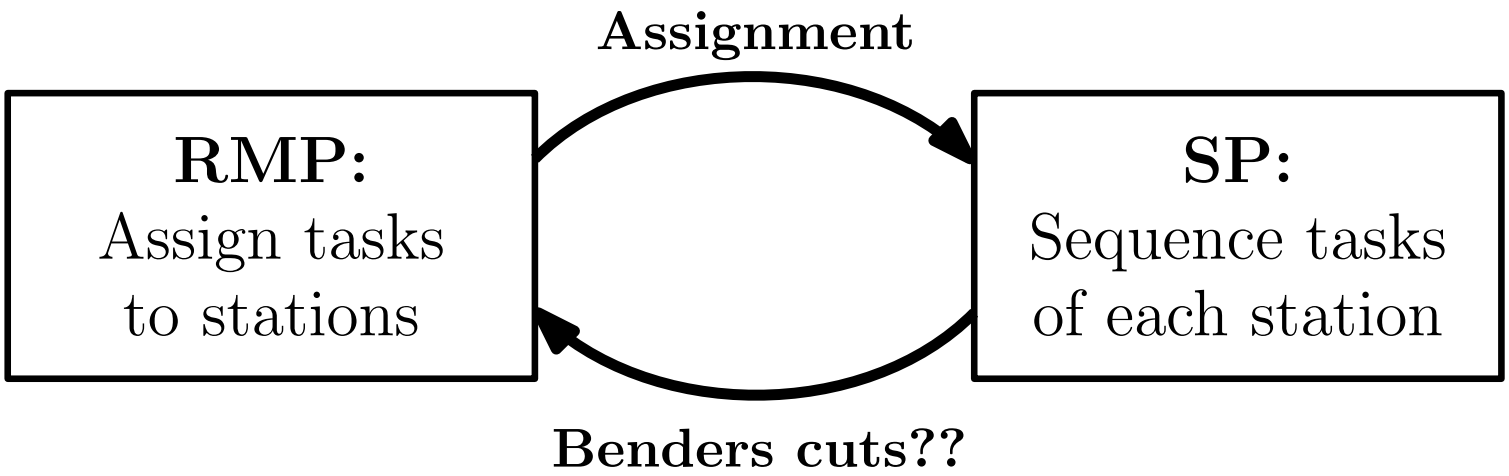
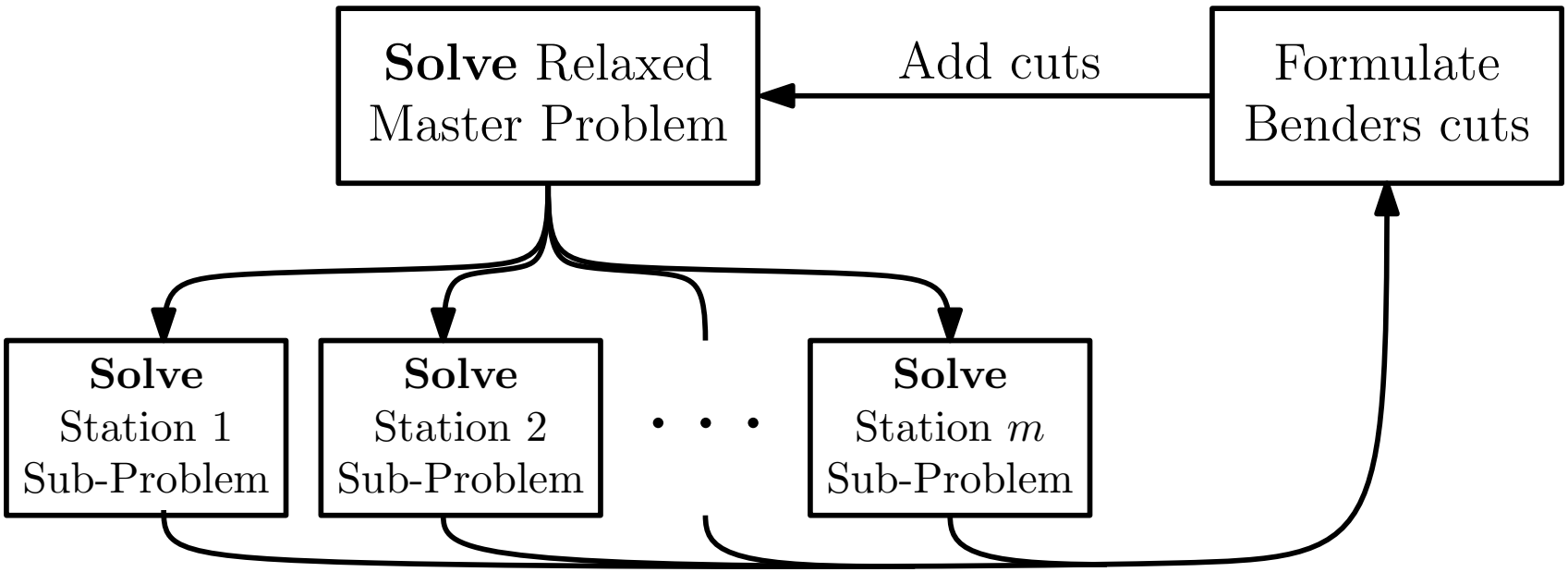
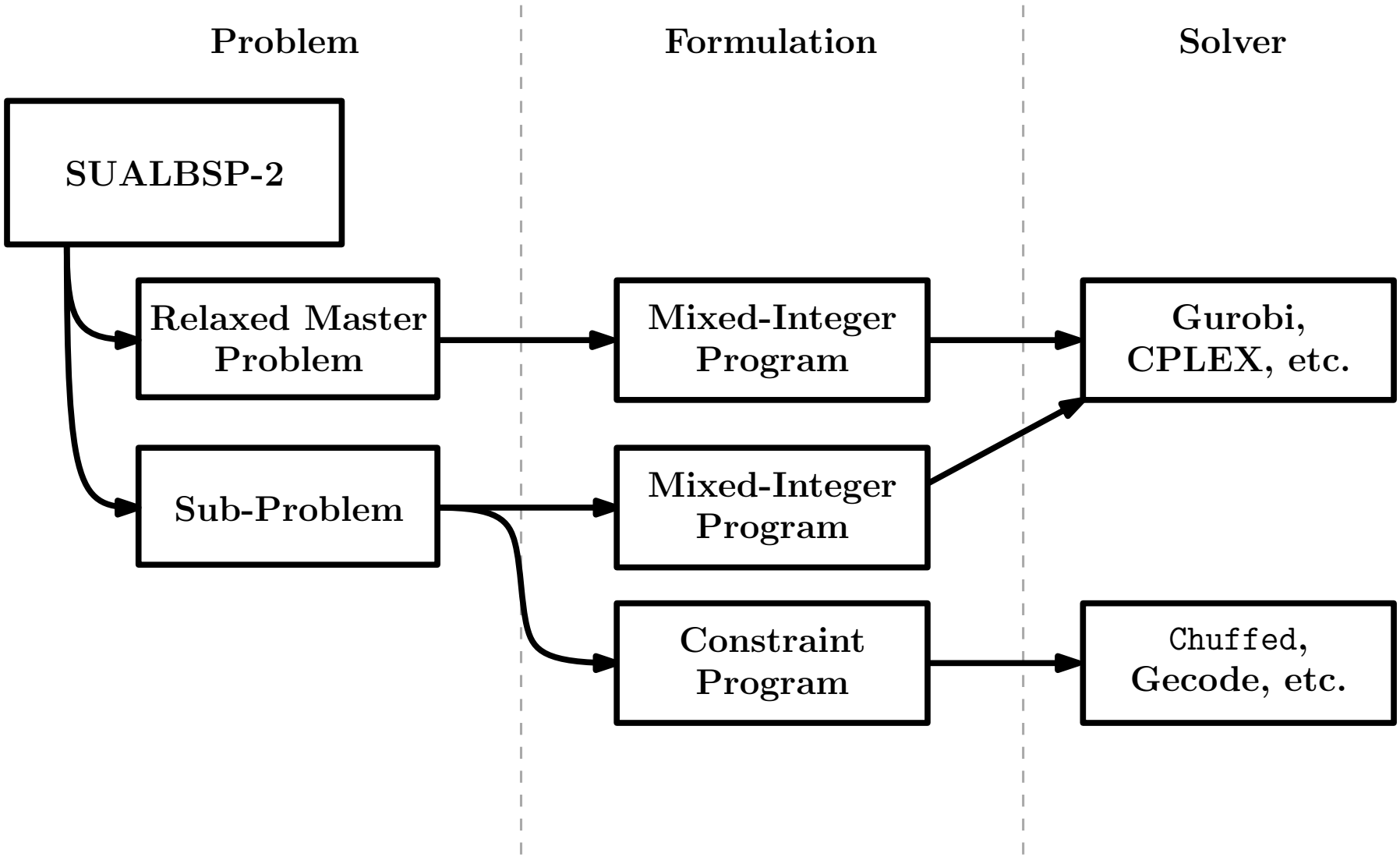
Assignment of tasks to stations	Sequencing of tasks within each station
Master problem	Sub-problem



Framework of our Decomposition



Solution Methodology: Overview



Master Problem Formulation

Assignment problem $\Rightarrow$ Formulate a MIP, solve with Gurobi

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RMP(μ): Min c^{μ}

$$
\begin{aligned}
\text{s.t.} \quad & \sum_{k \in FS_i} x_{ik}^{\mu} = 1 && \forall i \in V \\
& \sum_{k \in FS_i} k \cdot x_{ik}^{\mu} \leq \sum_{k \in FS_j} k \cdot x_{jk}^{\mu} && \forall (i,j) \in E \\
& \sum_{i \in V} t_i \cdot x_{ik}^{\mu} \leq c^{\mu} && \forall k \in K \\
& \mathcal{C}^{\nu} && \nu \in \{1, 2, \ldots, \mu - 1\} \\
& \underline{c}^{\mu} \leq c^{\mu} \leq \bar{c}^{\mu} \\
& x_{ik}^{\mu} \in \{0, 1\} && \forall i \in V,\ k \in FS_i
\end{aligned}
$$

Sub-Problem Formulation

- 'Kind of' similar to the asymmetric TSP $\Rightarrow$ highly combinatorial

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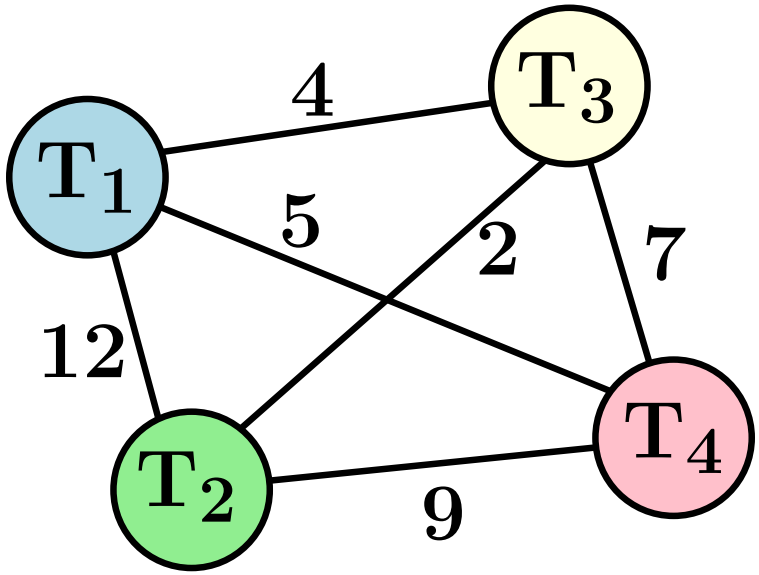

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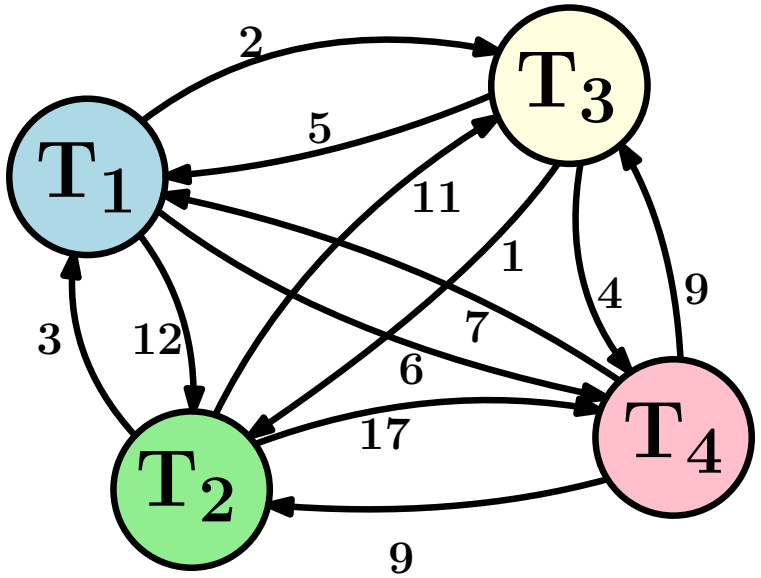

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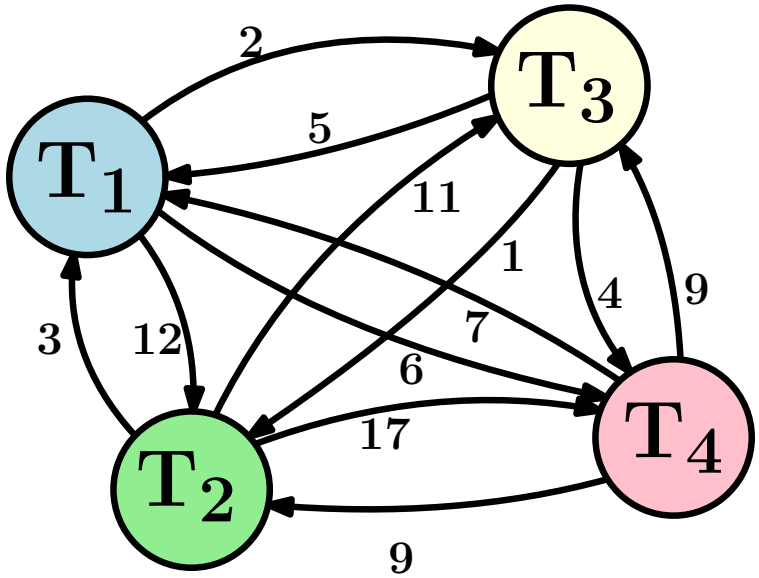

- Possible approaches
  - Mixed-Integer Problem
  - Constraint Problem
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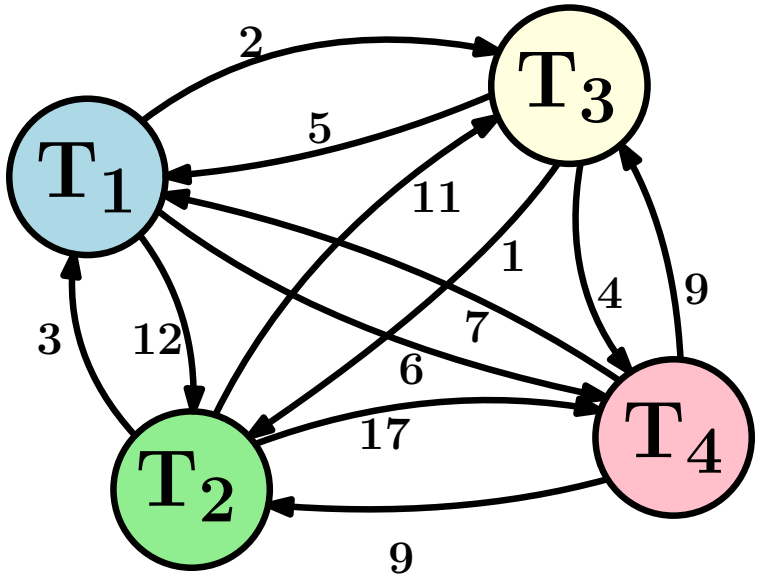

- Possible approaches
  - Mixed-Integer Problem
  - Constraint Problem
  - Travelling Salesperson Problem (requires some adaptation)
    - ★ Pro: Use dedicated TSP solvers (Concorde, `tsp_solve` etc.)
    - ★ Con: Leads to multiplying number of tasks by $2n^2$ ⇒ Did not explore

Sub-Problem: Mixed Integer Program

SP-MIP(μ): Min I_k^μ

s.t.

$$\sum_{j \in F_i^\phi} y_{ij} + \sum_{j \in F_i^\beta} z_{ij} = 1 \qquad \forall i \in V_k^\mu$$

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$$\sum_{i \in V_k^\mu} \sum_{j \in F_i^\beta} z_{ij} = 1$$

$$s_i + t_i + \phi_{ij} \cdot y_{ij} \leq s_j \qquad \forall (i,j) \in E_k^\mu$$

$$s_i + t_i + \phi_{ij} \leq s_j + M(1 - y_{ij}) \qquad \forall i \in V_k^\mu,\ j \in F_i^\phi$$

$$s_i + t_i + \sum_{j \in F_i^\beta} \beta_{ij} \cdot z_{ij} \leq I_k^\mu \qquad \forall i \in V_k^\mu$$

$$I_k^\mu \geq 0,\ s_i \geq 0 \qquad \forall i \in V_k^\mu$$

$$y_{ij}^\mu\ (z_{ij}^\mu) \in \{0,1\} \qquad \forall i \in V_k^\mu,\ j \in F_i^\phi (F_i^\beta)$$

Sub-Problem: Constraint Program

- Contraint Programming handles scheduling problems well

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Sub-Problem: Constraint Program

- Contraint Programming handles scheduling problems well
- Expressive nature of CP removes need for big-M constraints
- `Chuffed` is a state-of-the-art solver for many scheduling problems
- 'Learning' solver which combines
  - Lazy clause generation
  - Boolean satisfiability solving

CP Solver Chuffed: Global Constraints

- First main advantage of CP
- Capture common combinatorial structures across problems
- Provide effective propagation for CP solvers

CP Solver Chuffed: Global Constraints

- First main advantage of CP
- Capture common combinatorial structures across problems
- Provide effective propagation for CP solvers
- Some include:
  - `knapsack()`
  - `disjunctive()`
  - `cumulative()`
  - `bin_packing()`
  - `int_set_channel()`
  - `diffn()`
  - `geost()`
  - `all_different()`
  - ⋮

CP Solver Chuffed: Search Strategy

- Second main advantage of CP
- Combinatorial problem $\Rightarrow$ Constant battle against huge solution space

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`Chuffed`'s search language:

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- Basic searches
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  - new! Priority: `priority_search`

Benders Cuts

- Communicate information from sub-problems to master problem
- Richer information leads to faster convergence

Benders Cuts

- Communicate information from sub-problems to master problem
- Richer information leads to faster convergence
- We proposed a range of cutting procedures:

Feasibility Cuts	Optimality Cuts
Nogood cut	Logic cut
	Inference cut #1 (simple)
	Inference cut #2 (smarter)
	Inference cut #3 (smartest)
	Global cut/bound

Benders Cuts: Inference Cut Example

Inference cut #1 (simple):

$$I_k^\nu - M \sum_{i \in V_k^\nu} (1 - x_{ik}^\mu) \leq c^\mu$$

- If same assignment as before $\Rightarrow$ Lower bound on cycle time

Benders Cuts: Inference Cut Example

Inference cut #1 (simple):

$$l_k^\nu - M \sum_{i \in V_k^\nu} (1 - x_{ik}^\mu) \leq c^\mu$$

- If same assignment as before $\Rightarrow$ Lower bound on cycle time

Inference cut #3 (smartest):

$$l_k^\nu - \sum_{i \in V_k^\nu} B_{ik}^\nu (1 - x_{ik}^\mu) + \sum_{i \in V \setminus V_k^\nu} b_{ik}^\nu \cdot x_{ik}^\mu \leq c^\mu$$

- If *similar* assignment as before $\Rightarrow$ Lower bound on cycle time

Results

Benders Results: Sub-Problem Formulations

- Medium sized instances
- Time limit: 30 minutes on each instance

#Tasks	Type	#SP Nodes	%Gap	%Opt.	SP Runtime(s)
25–30	SP-MIP	639k	11.7	82.1	226.2
	SP-CP	1,692k	13.0	**83.9**	229.4

Benders Results: Comparison with MIPs

- All 3 data sets
- Time limit: 30 minutes on each instance

	Benders			MIP-1			MIP-2		
#Tasks	%Gap	%NoSol.	%Opt.	%Gap	%NoSol.	%Opt.	%Gap	%NoSol.	%Opt.
7–21	0.0	0.0	100.0	0.1	0.0	93.5	1.6	0.0	77.8
25–30	13.0	0.0	83.9	13.8	0.0	0.0	18.0	25.0	0.0
32–58	35.8	0.0	32.4	19.6	5.1	3.9	10.1	61.9	0.0

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- Solved the small instances in 0.1 seconds on average
- "...more efficient exact methods."

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- Solved the small instances in 0.1 seconds on average
- "...more efficient exact methods." Done ✓

Benders Results: Runtime Distribution

- All 3 data sets
- Time limit: 30 minutes on each instance

#Tasks	# Iters.	# Cuts	Master Runtime (s)	SP Runtime(s)
7–21	3	9	0.03 (27.3%)	0.08 (72.7%)
25–30	71	262	269.9 (54.1%)	229.4 (45.9%)
32–58	21	113	832.5 (65.4%)	440.9 (34.6%)

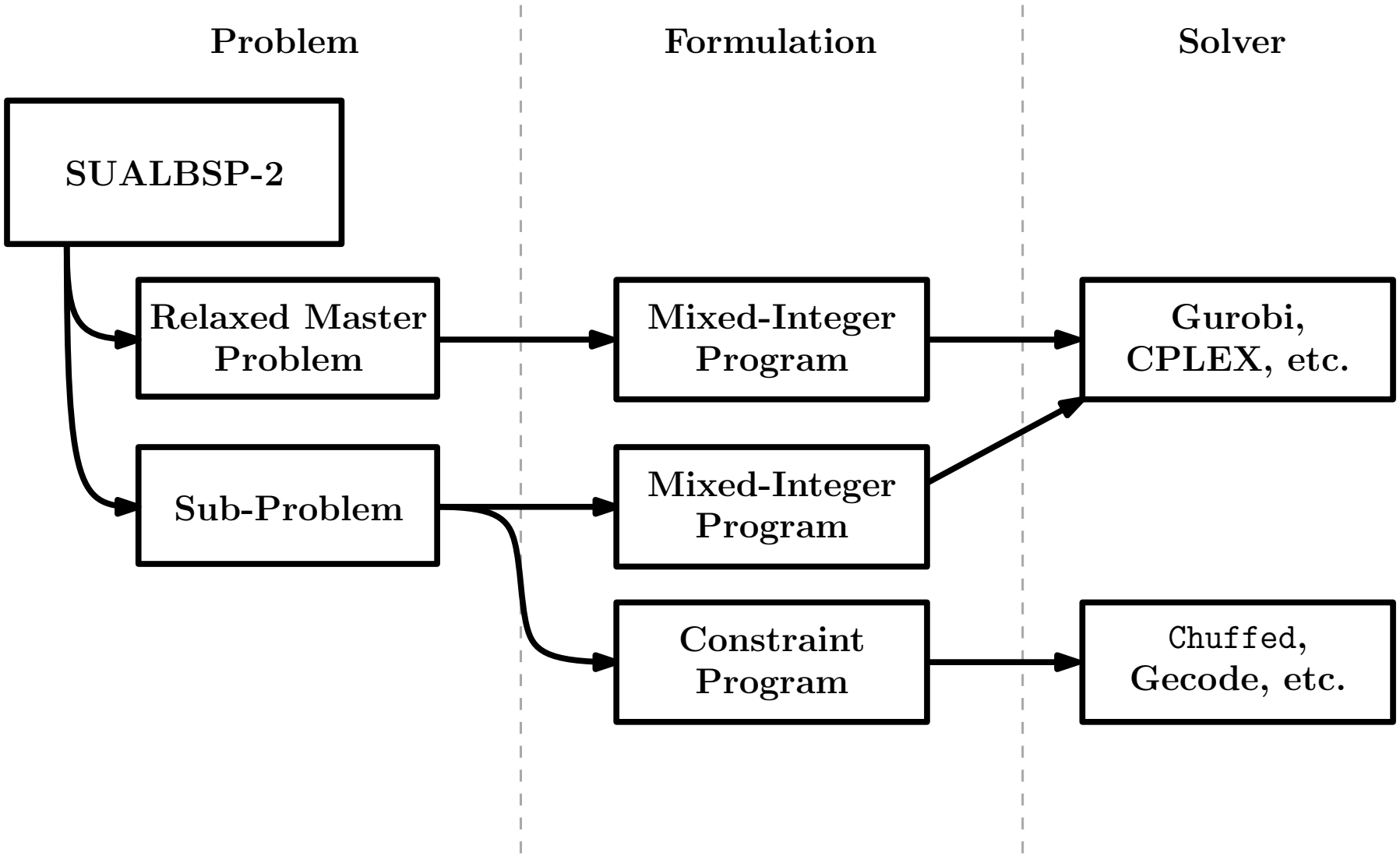
Conclusion

- Proposed a logic-based Benders decomposition framework
  - Together with some Benders cuts

Conclusion

- Proposed a logic-based Benders decomposition framework
  - Together with some Benders cuts
- Optimally solved 257 of the 396 instances in 30 minute time limit
  - Closed 149 open instances

Future Work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Solver
SUALBSP-2
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Gurobi, CPLEX, etc.
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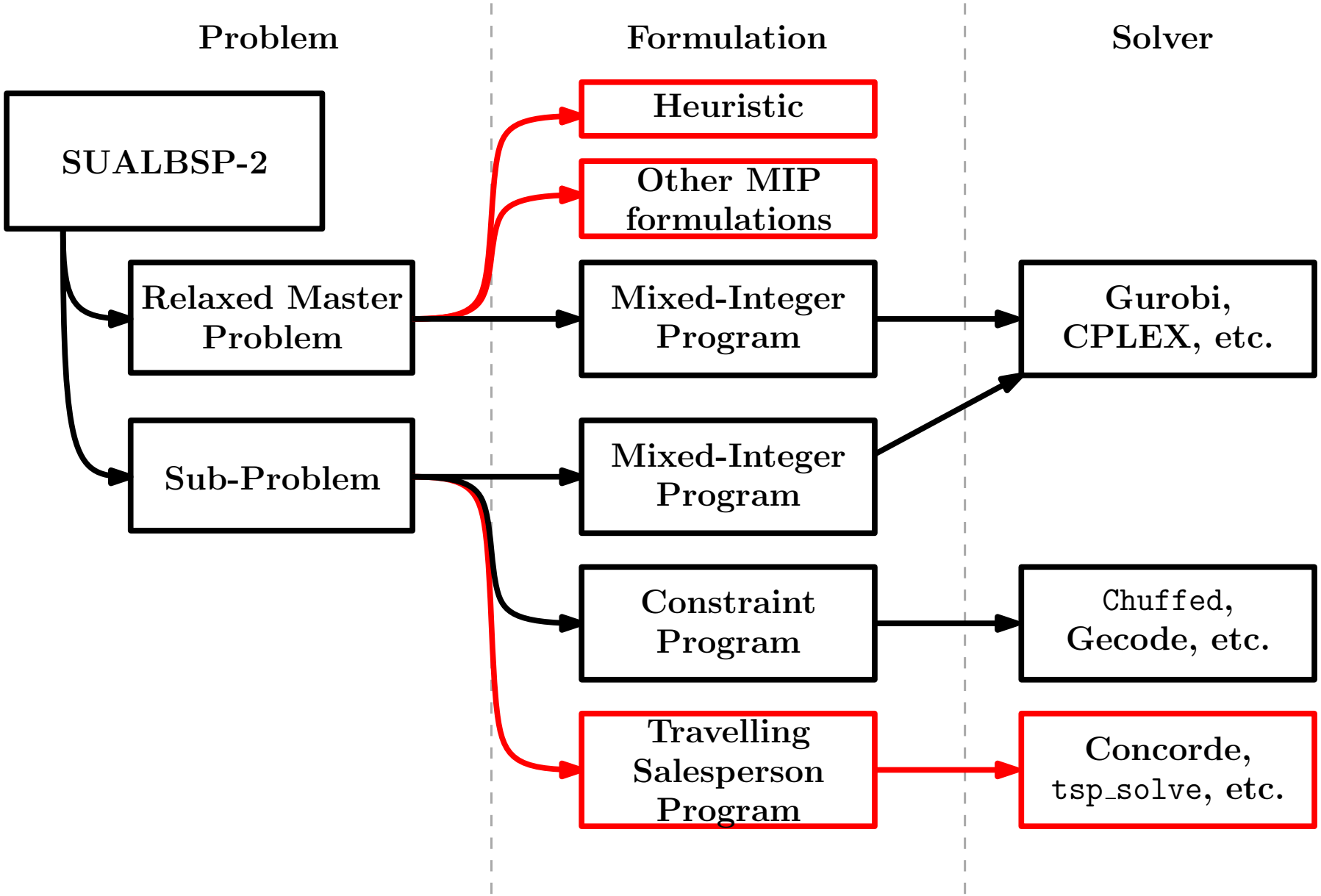
Future Work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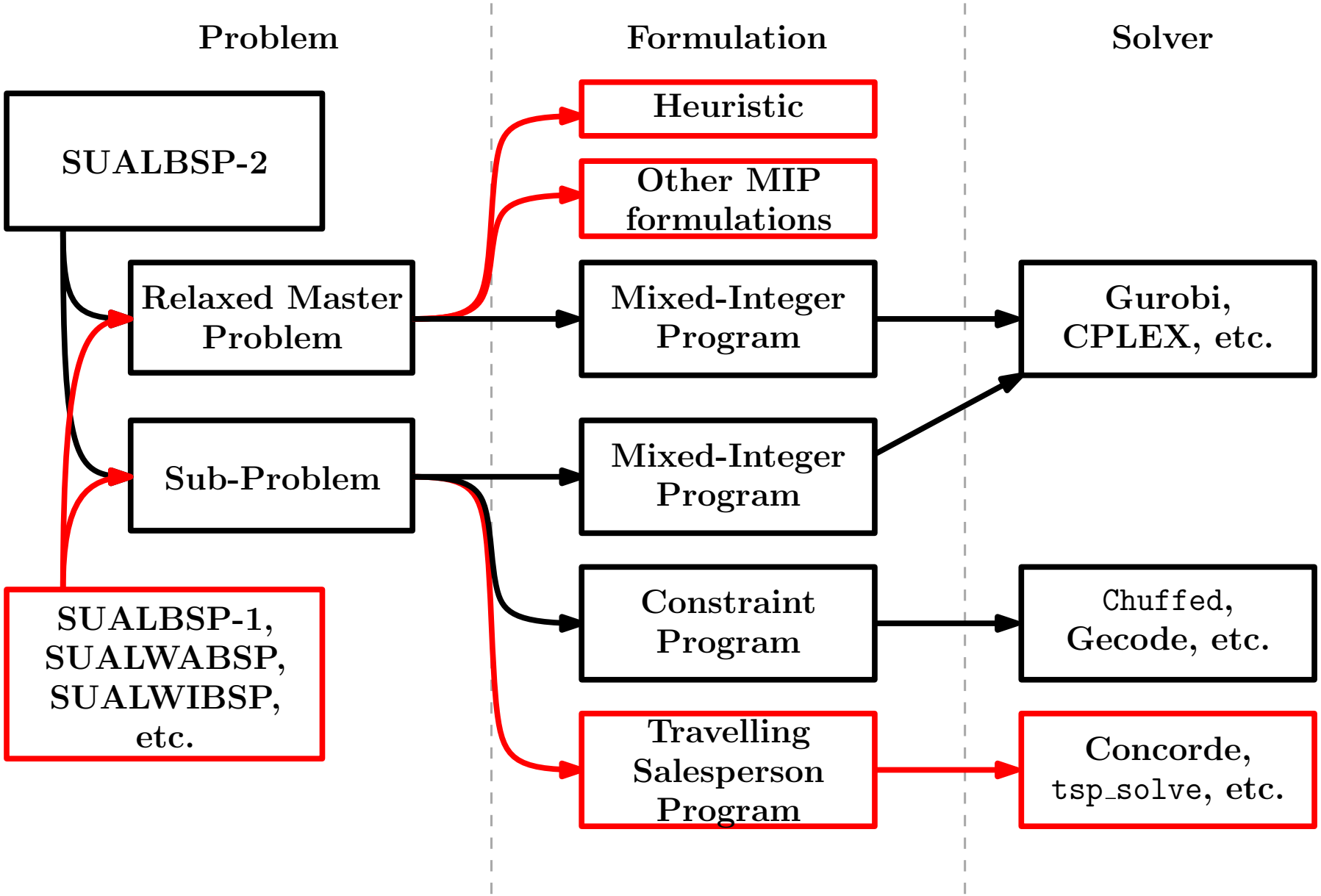
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Thanks for listening!

Any questions?