Fast Scheduling of Robot Teams Performing Tasks With Temporospatial Constraints

Matthew C. Gombolay, Ronald J. Wilcox, and Julie A. Shah

***Abstract*—The application of robotics to traditionally manual manufacturing processes requires careful coordination between human and robotic agents in order to support safe and efficient coordinated work. Tasks must be allocated to agents and sequenced according to temporal and spatial constraints. Also, systems must be capable of responding on-the-fly to disturbances and people working in close physical proximity to robots. In this paper, we present a centralized algorithm, named "Tercio," that handles tightly intercoupled temporal and spatial constraints. Our key innovation is a fast, satisficing multi-agent task sequencer inspired by real-time processor scheduling techniques and adapted to leverage a hierarchical problem structure. We use this sequencer in conjunction with a mixed-integer linear program solver and empirically demonstrate the ability to generate near-optimal schedules for real-world problems an order of magnitude larger than those reported in prior art. Finally, we demonstrate the use of our algorithm in a multirobot hardware testbed.**

***Index Terms*—Human–robot teaming, scheduling.**

I. Introduction

ROBOTIC systems are increasingly entering into domains previously occupied exclusively by humans. In the manufacturing field, for example, there is a strong economic motivation to enable human and robotic agents to cooperatively perform traditionally manual work. This integration requires a choreography of human and robotic work that meets upper and lower bound temporal deadlines for task completion (e.g., the assigned work must be completed within a single shift) and spatial restrictions on agent proximity (e.g., robots must maintain at least 4 m of separation from other agents) in order to support safe and efficient human–robot cooperation. Multi-agent coordination problems with temporospatial constraints can be readily formulated as a mixed-integer linear program (MILP) [6], [49]; however, the problem of optimally scheduling $n \geq 3$ tasks (each with a sequence of n_i subtasks) on a set of m machines is NP-hard [73] and is computationally intractable for problems of interest to large-scale factory operations.

Various decentralized or distributed approaches have achieved fast computation and favorable scalability characteristics [8], [14], [16], [68], [76]. Rapid computation is desirable because it enables the system to respond on-the-fly to schedule disturbances, such as an inaccurately performed job, a machine breakdown, or changing temporal constraints [1], [8], [70]. Some approaches boost computational performance by decomposing plan constraints and contributions to the objective function among agents [14]. However, these methods break down when agents' schedules become tightly intercoupled, such as scenarios wherein multiple agents are maneuvering in close physical proximity to one another. While distributed coordination approaches are necessary for field operations in which environment and geography affect communication among agents, factory operations allow sufficient connectivity and bandwidth for either centralized or distributed approaches to task assignment and scheduling.

In this paper, we present Tercio,[1] a centralized task assignment and scheduling algorithm that scales to multi-agent, factory-size problems and supports on-the-fly scheduling in the presence of temporal and spatial proximity constraints. We empirically demonstrate that this capability enables human and robotic agents to perform manufacturing tasks effectively and in close proximity to one another.

Tercio takes as input a set of tasks composed of precedence-related subtasks; a set of interval temporal constraints relating the start and finish times of subtasks; two-dimensional coordinates specifying the spatial locations where subtasks are performed; physical constraints restricting agent proximity; a set of agent capabilities specifying the tasks and subtasks each agent may perform and the minimum, maximum, and expected time for each agent to complete each task; and an objective function to optimize. Tercio provides a solution consisting of the assignment of agents (i.e., humans or robots) to tasks and a schedule for each agent's jobs (i.e., start and finish times for each job) such that all temporal and spatial proximity constraints are satisfied and the objective function is empirically within 10% of optimal the majority of the time (see Section X).

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[1] "Tercio" is named for the Spanish military formation used during the Renaissance period, which consisted of several different types of troops, each with their own strengths, working together as a single unit.

Tercio is made efficient through a fast, satisficing multi-agent task sequencer that is inspired by real-time processor scheduling techniques but adapted to leverage a hierarchical problem structure. Our task sequencer computes, in polynomial time, a multi-agent schedule satisfying upper and lower bound temporal deadlines, as well as spatial restrictions of agent proximity. Although the sequencing algorithm is satisficing and incomplete, we empirically show that it produces task schedules within 10% of optimal. We also demonstrate that it produces solutions for nearly all problem instances solvable by a complete solution technique. We use this fast task sequencer in conjunction with an MILP solver and indicate that Tercio is able to generate task assignments and schedules for up to 10 agents and 1000 subtasks in approximately one minute, on average. In this regard, Tercio scales better than representative state-of-the-art benchmarks in heuristic (i.e., [58]), metaheuristic (i.e., [88]), and exact (i.e., [35]) approaches. Tercio also returns flexible time windows for execution [84], enabling agents to adapt to small disturbances online without the need for a full recomputation of the schedule.

Tercio was first introduced by Gombolay *et al.* [34].[2] In this paper, we present two advancements for this work, proofs of correctness, and further analysis of the algorithm's performance. First, we generalize the types of task sets that Tercio can efficiently schedule. Second, we present an improved sequencer that produces less-conservative schedules than the method originally presented [34]. Third, we include proofs of correctness for Tercio's subroutines. Finally, we expand the empirical evaluation of Tercio to demonstrate its significance over the closest available benchmarks in prior literature [35], [58], [88].

II. Related Work

There is a wealth of prior work related to task assignment and scheduling within the manufacturing field and other applications. Korsah *et al.* provided a comprehensive taxonomy [43] for the multirobot task allocation and scheduling problem. According to this taxonomy, our problem fits within the cross-schedule dependencies [XD] category, with single-task robots [ST], single-robot tasks [SR], and the time-extended allocation [TA] problem (XD [ST-SR-TA]). Cross-schedule dependencies exist when the utility of one agent is directly affected by the scheduling commitment of another. As noted by [42], these dependencies occur in the form of various temporal constraints (e.g., precedence constraints) or of finite-capacity resources that must be shared by agents to complete their tasks. We consider both temporal and resource-based cross-schedule dependencies in this work. We note that prior works often seek to find a feasible schedule with the shortest duration, although other application-specific cost functions are sometimes considered.

While in many cases the problem of task allocation with cross-schedule dependencies can be readily formulated and solved as an MILP [6], [49], the complexity MILP-based solution techniques (e.g., branch-and-bound search) are exponential, leading to computational intractability for large-scale factory operations. Various algorithms have been proposed with the goal of achieving favorable scalability characteristics. Here, we survey solution techniques for scheduling problems similar to the problem we present in this work—including MILP formulations, auction- and market-based methods, and other hybrid approaches—and discuss the applicability of these prior techniques to our problem. Table 1 depicts a summary of the relevant approaches.

[2]Parts of this work have been published in Proceedings of Robotics: Science and Systems (RSS 2013).

	ST vs. MT	SR vs. MR	TA	XD	Heterogeneous Agents	Precedence/Wait Constraints	Absolute Task Deadlines	Task-Task Deadlines	Shared Resources	Optimal	Complete
Tercio, 2017	ST	SR	✓	✓	✓	✓	✓	✓	✓		
McIntire et al., 2016	ST	SR	✓	✓	✓	✓	✓				
Nunes & Gini, 2015	ST	SR	✓		✓		✓				
Sung et al., 2015	ST	MR		✓		✓					
Zhang & Wong, 2015	ST	SR	✓	✓						Probabilistic	
Liu & Shell, 2013	ST	SR			✓					✓	✓
Erdem et al., 2013	MT	MR	✓	✓	✓	✓	✓		✓	✓	✓
Castro & Petrovic, 2012	ST	SR	✓	✓	✓	✓	✓	✓			
Ren & Tang, 2009	ST	SR	✓	✓	✓	✓	✓				
Choi et al., 2009b	ST	SR			✓					≤ 50%	Yes
Chen & Askin, 2009	MT	MR	✓	✓	✓	✓	✓				
Li & Womer, 2008	ST	MR	✓	✓	✓	✓	✓				
Zavlano et al., 2008	ST	SR	✓		✓					✓	✓
Mercier et al., 2005	MT	MR	✓				✓	✓	✓	✓	✓
Hooker, 2004	MT	SR	✓	✓	✓	✓	✓			✓	✓
Lemaire et al., 2004	ST	SR	✓	✓	✓	✓	✓	✓			
Tan, 2004	ST	SR	✓	✓	✓	✓	✓				
Cesta et al., 2002	ST	SR	✓				✓			Probabilistic	
Harujunkowski & Grossman, 2002	ST	SR	✓	✓	✓	✓	✓			✓	✓
Cordeau et al., 2001	ST	SR	✓	✓	✓	✓	✓	✓			
Chien et al., 2000	ST	SR	✓	✓		✓	✓	✓	✓		
Sycara et al., 1991	MT	MR	✓	✓	✓	✓	✓		✓		

Fig. 1. This table provides a summary of relevant prior work.

A. MILP/CP Solution Techniques

One promising approach to solving this class of problems has been the development of a hybrid algorithm incorporating MILP and constraint programming (CP) methods along with decomposition. Techniques based on Benders decomposition [4], [30], [53] are among the most widely used. Benders decomposition works by iteratively updating a function $f(\bar{y})$, which provides a lower bound on the optimal solution for an optimization problem where y is a subset of the decision variables and $\bar{y}$ indicates a specific assignment to those decision variables. For a scheduling problem, y might represent the assignment of agents to jobs, and the remaining variables might represent potential sequences of those jobs. Through bifurcation of the decision variables, it is possible to improve the computation time for a solution to the optimization problem [38].

Various scheduling applications have incorporated Benders decomposition. Logic-based Benders decomposition in particular has been applied to solve job shop scheduling problems where n tasks must be scheduled at m facilities [38]. Benders cuts (i.e., a lower bound constraint on the optimal solution provided by $f(\bar{y})$) are generated by separating the allocation of tasks from the sequencing of those tasks. Reported results have demonstrated optimal solutions for problems involving approximately 10 facilities and 50 tasks.

Cordeau *et al.* [15] and Mercier *et al.* [55] employed Benders decomposition for aircraft routing and crew scheduling [15]. The authors heuristically decomposed aircraft routing from

crew assignment and generated Benders cuts for the resulting assignment subproblem. They empirically demonstrated that this method produced more cost-efficient schedules than prior art. Rekik *et al.* similarly employed Benders decomposition to schedule personnel shift-work [66]. The authors applied hand-tailored *forward* and *backward constraints* to cut the search space and proved the correctness of these constraints. Rekik *et al.* showed that Benders decomposition can be used to solve particularly challenging problems in which the *forward* and *backward constraints* do not sufficiently prune the search space.

While Benders decomposition has served as the basis for many state-of-the-art scheduling algorithms, several alternative techniques have also successfully combined MILP and CP approaches. Jain and Grossmann [39] presented an iterative method that first solves a relaxed MILP formulation and then searches for the complete solution using CP. When applied to the scheduling of jobs for machines, the MILP relaxation solves for the assignment of jobs and the CP solves for the schedule. If a solution is identified, the algorithm returns the optimal solution; otherwise, the algorithm infers cuts based on the solution of the MILP relaxation and solves the new MILP relaxation using these cuts. The authors reported results for problems involving up to 20 tasks and 5 machines. Li and Womer later improved on this work by employing a hybrid Benders decomposition algorithm with MILP and CP solver subroutines [50] and reported that their method can solve problems involving 30 tasks and eight different agent types up to four times faster than a standard MILP. Ren and Tang [67] took a similar approach but employed heuristic strategies to generate informative cuts in the event that the CP solver was unable to identify a feasible task sequence. A related method proposed by Harjunkoski and Grossman [37] utilized an iterative approach to producing task assignments and schedules. Both Ren and Tang and Harjunkoski and Grossman empirically demonstrated that their algorithms can solve problems involving up to eight machines and 36 jobs; however, these works did not address problems associated with cross-schedule dependencies.

Although not mathematical programs, some prior works have incorporated planning domain definition language (PDDL)-style problem formulations. For example, Erdem *et al.* [23], [24] developed distributed and semidistributed techniques for scheduling problems involving precedence and absolute temporal constraints, as well as other resource and spatial proximity constraints. These works utilize a formulation that supports causality-based reasoning. The Erdem *et al.* [23], [24] technique yielded optimal solutions for problems with absolute deadlines and complex geometric constraints and readily scaled to problems involving up to eight agents and 80 tasks.

B. Auction and Market-Based Solution Techniques

Auction methods and other market-based approaches to scheduling problems, such as those developed by Ponda *et al.*, Choi *et al.*, and Liu and Shell, also frequently rely upon decomposition of the problem structure [14], [52], [64]. For example, Ponda *et al.* [64] developed a decentralized, market-based solution technique for allocating tasks to agents, given that tasks are constrained by time windows. Ponda *et al.* [64] employed the consensus-based bundle algorithm (CBBA) [14], in which the objective function and constraints are decomposed by agent so that each agent can quickly solve for the value of its bid on each task. Thus, while the work by [64] represents an added capability for the CBBA, the scalability of the algorithm is strictly worse than the CBBA.

Recently, Nunes and Gini [58] developed the temporal sequential single-item (TeSSI) auction algorithm for decentralized scheduling, which has been shown to outperform the CBBA. TeSSI takes as input a task set in the form of a simple temporal problem (STP); each task has an earliest start and latest finish time (absolute wait and deadline constraints). Constraints relating tasks comprise travel time constraints; resource constraints are not included. Nunes and Gini [58] empirically demonstrated their approach yields improved performance compared with the CBBA [14]. Furthermore, their approach has since been extended to incorporate precedence relations among tasks [54]. However, TeSSI and its variants [54], [58] solve a narrower class of problem than Tercio, in that while they handle some cross-schedule dependencies in the form of precedence relations, they do not handle subtask-to-subtask deadlines (i.e., deadlines constraining the maximum time between the start time of one subtask and the finish time of the second) or resource capacity constraints.

Other techniques solve the task allocation problem efficiently [3], [5], [52], [74], [85], but do not address the sequencing problem. For example, Sung *et al.* addressed multirobot task allocation where each agent maintains a queue of tasks and partial information about other agents' queues [74]. During execution, agents communicate when possible and choose to exchange tasks using a heuristic approach. Sung *et al.* empirically demonstrated that their algorithm can solve problems involving up to six agents and up to 250 tasks; however, the problems did not involve cross-schedule dependencies or task deadlines.

Liu and Shell recently proposed a novel distributed method for task allocation via strategic pricing [52]. Their work builds upon prior approaches to distributed auction algorithms [5], [85], runs in polynomial time, and produces globally optimal solutions. However, this technique does not consider coupling constraints—for example, a problem in which one agent's assignment directly affects the domain of feasible assignments for other agents, as is the case when agents are performing tasks subject to temporal and resource constraints. Chien *et al.* proposed planning methods for a team of robotic rovers to accomplish a set of scientific objectives [13]. The rovers needed to complete a set of tasks where each task required the use of shared, single-access resources (i.e., "shared resources"). The approach by Chien *et al.* [13] uses an iterative-repair centralized planner coupled with an auction algorithm to perform centralized goal allocation and decentralized route planning and goal sequencing. Chien *et al.* benchmarked against a set of randomized problems with three rovers and 12 goals; however, thorough empirical evaluation with an optimal benchmark was not reported.

Lemaire *et al.* approached the problem of allocating UAVs to tasks, represented by a bipartite graph [48]. Here, one set of tasks (UAV navigation) was required to be completed before

the second set (target sensing). The authors first presented a centralized auction solution and reported empirical results for a problem involving 50 tasks and four agents. Next, they described a distributed approach wherein an auctioneer agent assigns the first set of tasks to the multirobot team, and then the second set of tasks is auctioned. This method supports rescheduling in light of dynamic disturbances occurring during task execution; however, the authors did not report empirical results for their distributed method. Sycara *et al.* [75] explored the problem of task allocation and sequencing for a set of agents. In this work, agents were required to share a finite set of resources necessary for task execution. The authors' formulation was distributed in nature and relied upon multiple heuristics to sequentially construct an effective schedule; however, their approach was suboptimal and did not consider deadline constraints relating tasks.

C. Hybrid Solution Techniques

Other hybrid approaches have integrated heuristic schedulers within the MILP solver to achieve better scalability characteristics. For example, Chen *et al.* incorporated depth-first search (DFS) with heuristic scheduling [12]. In this approach, a DFS algorithm sequentially assigns tasks to agents, and a heuristic scheduling algorithm sequences the tasks according to a minimum slack priority. The algorithm also employs heuristics to guide the order in which tasks are assigned to resources during the search. Chen benchmarked their work on problems involving approximately 50 tasks and 10 resources (or agents) using a standard problem database [21], [60].

Alternatively, Castro and Petrovic [9] used a heuristic scheduler to seed a feasible schedule for the MILP with regard to patient procedures conducted within a hospital. This method incorporates a tiered approach to minimize a three-term objective function: First, a heuristic scheduling algorithm generates an initial feasible solution. Next, an MILP is solved for the first term of the objective function using the heuristic solution as a seed schedule. The MILP is solved again to optimize the second objective function term, using the solution from the first MILP as a constraint. This process repeats, but for the third objective function term. The solution time is reduced by sequentially optimizing the objective function terms; however, this approach sacrifices global optimality.

Other approaches perform cooperative scheduling by incorporating Tabu search within an MILP solver [77], or by applying heuristics to abstract the problem to groupings of agents [44]. These hybrid methods are able to solve scheduling problems involving 5 agents (or groups of agents) and 50 tasks in minutes or seconds and address problems incorporating multiple resources, task precedence, and temporal constraints relating task start and end times to the plan epoch time. However, these approaches do not take more general, task–task temporal constraints into consideration.

Cesta *et al.* [10] addressed the problem of project scheduling with time windows by formulating it as a constraint satisfaction problem. In their work, candidate (potentially infeasible) solutions are initially generated using heuristic methods; random and deterministic heuristic techniques are then used to iteratively repair any infeasibilities in the problem. Cesta *et al.* [10] noted that randomization is essential to counteract the bias of greedy scheduling heuristics; however, they did not consider wait constraints that create cross-schedule dependencies, nor did they consider shared resources.

D. Metaheuristic Techniques

Successful metaheuristic methods have included simulated annealing (SA) and genetic algorithms (GAs). Davis [18] produced one of the early works applying GAs to job shop scheduling, although many researchers have since followed suit [25], [26], [31], [87]. Recently, Zhang and Wong [88] developed a GA to perform process planning for single-task machines, single-machine tasks, and time-extended scheduling with precedence constraints; however, the formulation did not consider deadline constraints or shared resource constraints.

Researchers have also sought to apply SA techniques to specific scheduling problems [56], [59], [81]. Prior works have combined GAs and SA to further improve upon solution quality [17], [83]. These techniques rely upon a random walk through the space of possible schedules; as the number of steps in the walk (i.e., algorithm iterations) approaches ∞, the optimal solution will be found.

These solution techniques are typically applied to job shop scheduling problems in which tasks are related through neither tightly intercoupled upper and lower bound temporal constraints nor shared resource constraints [25], [26], [87], [88]. As they are only probabilistically optimal and rely upon random search, metaheuristics can require a large amount of computation time in order to identify and improve schedules when tight upper bound constraints exist.

E. Application to Task Assignment and Scheduling With Temporospatial Constraints

The problem of scheduling—allocating agents to tasks and sequencing those tasks—has been studied in a wide array of works incorporating various solution techniques. However, prior research does not address the need to quickly solve large-scale problems involving tight dependencies among agents' schedules, which can make decomposition problematic. Typically, allowing multiple robots to work closely within the same physical space produces dependencies among the agents' temporal and spatial constraints, leading to uninformative decompositions. We are unaware of prior work that has yielded a solution technique for time-extended scheduling of heterogeneous agents in which each unit of work (i.e., subtask) is related through upper and lower bound temporal constraints without restriction on problem structure (e.g., only some subtasks can be related by certain constraints), where agents must share access to resources (e.g., physical locations) when performing their work.

While we are not aware of prior work focused on our problem definition, we are able to adapt heuristic, metaheuristic, and exact solution techniques to provide an informative empirical validation. In particular, we benchmarked our solution technique, Tercio, against the following techniques:

1) We adapted the insertion-heuristic-based TeSSI algorithm [58] to accommodate our class of problems.

Insertion heuristics are commonly used in vehicle routing and scheduling [7], [27], [72].

2) A state-of-the-art GA proposed by Zhang and Wong [88] for integrated process planning and scheduling.
3) An exact, MILP-based solution technique, Gurobi [35], which uses state-of-the-art branch-and-bound search and cutting plane heuristics, for the mathematical formulation presented in Section III.

In Section X, we show that our technique has the singular ability to balance computational efficiency with solution quality to provide fast, empirically near-optimal solutions for large-scale scheduling problems.

III. Problem Statement

In this section, we formulate a task assignment and scheduling problem for multiple robots working within a shared physical space.

Tercio takes the following as input:

1) $\boldsymbol{\tau}$*:* The set of all tasks to be performed. Each task $\tau_i \in \boldsymbol{\tau}$ is composed of m_i precedence-constrained subtasks, denoted τ_i^j, for $j = [1, m_i]$, where a subtask τ_i^{j+1} may not be released to execute until its predecessor τ_i^j, finishes executing. Note $|\boldsymbol{\tau}|$ is the number of subtasks in $\boldsymbol{\tau}$.
2) $\boldsymbol{A}$*:* The set of all agents that perform subtasks.
3) $\boldsymbol{TC}$*:* The set of *interval temporal constraints* [19] relating subtasks' start and finish times. Lower bound "wait constraints" are denoted as $W_{\langle \tau_i^j, \tau_x^y \rangle}$ and specify that τ_x^y starts at least $W \geq 0$ time units after τ_i^j ends. Upper bound "deadline constraints" are denoted as $D^{s2s}_{\langle \tau_i^j, \tau_x^y \rangle}$ and specify that τ_x^y must finish within $D^{s2s} > 0$ time units of the start of τ_i^j. "Absolute" deadline constraints $D^{\text{abs}}_{\tau_i^j}$ limit the finish time of τ_i^j to within $D^{\text{abs}} > 0$ time units of the epoch start time.
4) $\boldsymbol{\tau_R}$*:* The set of all subtask pairs separated by less than the allowable spatial proximity, derived from spatial locations on the factory floor where subtasks are performed (specified via two-dimensional Cartesian coordinates).
5) $\boldsymbol{AC}$*:* The set of *agent capabilities* specifying the subtasks each agent $a \in \boldsymbol{A}$ may perform, as well as that agent's minimum $lb^a_{\tau_i^j}$, maximum $ub^a_{\tau_i^j}$, and expected completion time $C^a_{\tau_i^j}$ for each subtask τ_i^j.
6) z*:* An *objective function* to minimize that includes the makespan and possibly other application-specific terms.

A solution to the problem consists of an assignment (i.e., $\forall \tau_i^j, \exists! a \in \boldsymbol{A}$ s.t. $A^a_{\tau_i^j} = 1$) of each subtask $\tau_i^j \in \boldsymbol{\tau}$ to one agent $a \in \boldsymbol{A}$ and the assignment of the start $s_i^j \in \mathbb{R}$ and finish time $f_i^j \in \mathbb{R}$ of each subtask such that all constraints [see (2)–(12)] are satisfied and the objective function [see (1)] is minimized.

Remark (On Complexity): Sotskov and Shakhlevich [73] proved that the following problem (denoted SS'95) is NP-hard: Find a minimum makespan schedule for a set of machines $M_l \in M$ (i.e., agents $a \in A$) completing a set of jobs $J_i \in J$ (i.e., tasks $\tau_i \in \boldsymbol{\tau}$) consisting of operations $l_i^j \in J_i$ (i.e., $\tau_i^j \in \boldsymbol{\tau}$) with duration t_i^j such that l_i^j must be completed before l_i^{j+1} (i.e., $\boldsymbol{TC} = \{W_{\langle \tau_i^j, \tau_i^{j+1} \rangle} | \tau_i^j, \tau_i^{j+1} \in \boldsymbol{\tau}\}$), and operation $l_{i,j}$ must be completed on machine M_j (i.e., a fixed agent assignment such that $(A^a_{\tau_i^j} = 1 | j \neq a) \rightarrow$ infeasible), where $|M|, |J| \geq 3$. In this paper, we consider a problem specification (denoted GWS'17) that is a superset of SS'95 by including general deadline, wait, and resource constraints, and a variable agent assignment. One can map in polynomial time a problem in SS'95 to one in GWS'17 by specifying the fixed agent assignment (i.e., $C^a_{\tau_i^j} \leftarrow \infty \quad \forall j \neq a$ and $C^a_{\tau_i^j} \leftarrow t_i^j$ otherwise) in $O(|\boldsymbol{\tau}||A|)$ time. TC consists only of the wait constraints between consecutive subtasks (i.e., $\boldsymbol{TC} = \{W_{\langle \tau_i^j, \tau_i^{j+1} \rangle} | \tau_i^j, \tau_i^{j+1} \in \boldsymbol{\tau}\}$) and $\boldsymbol{\tau_R} = \emptyset$. As such, there exists a mapping in P from a problem in SS'95 to one in GWS'17. Therefore, we can say that SS'95 $\leq_P$ GWS'17. As problems in SS'95 are NP-hard, the problem we address are at least NP-hard provided that P $\neq$ NP.

The multi-agent coordination problem (with temporospatial constraints) can be readily formulated as an MILP as follows. In this formulation, $A^a_{\tau_i^j} \in \{0, 1\}$ is a binary decision variable for the assignment of agent a to subtask τ_i^j, $x_{\langle \tau_i^j, \tau_x^y \rangle} \in \{0, 1\}$ is a binary decision variable specifying whether τ_i^j comes after or before τ_x^y, and $s_i^j, f_i^j \in [0, \infty)$ are the start and finish times of τ_i^j. $\boldsymbol{A}$ is the set of all agents a, $\boldsymbol{\tau}$ is the set of all tasks and subtasks, and $\boldsymbol{\tau_R}$ is the set of all subtask pairs $\langle \tau_i^j, \tau_x^y \rangle$ that are separated by less than the allowable spatial proximity. M is an artificial variable set to a large positive number and is used to encode conditional constraints.

Equation (2) ensures that each task is assigned to one agent. Equation (3) encodes the explicit ordering of subtasks according to the lower bound temporal constraints. Equations (4) and (5) encode the minimum wait times and upper bound deadline constraints, respectively, between pairs of subtasks. Equation (6) enforces the absolute deadlines constraining subtask finish times. Equations (7) and (8) ensure that agents are not required to complete tasks any faster or slower than they are capable of. Note that human and robotic workers inherently have varying capabilities. For example, a robot may be able to place composite material, whereas a human might be able to perform a more dexterous task, such as wire-laying. Also, two humans are likely to perform the same task at different rates. In this work, we assume that workers are heterogeneous agents and that each subtask has an agent-specific completion time. Equations (9) and (10) sequence actions to ensure that agents maintain safe buffer distances from one another while performing tasks. Equations (11) and (12) ensure that each agent only performs one task at a time. Note that (9) and (10) couple the variables relevant to sequencing and spatial proximity constraints and to task start and end times and produce tight dependencies among agents' schedules

$$\text{Objective: } \min(z) \tag{1}$$

subject to

$$\sum_{a \in \boldsymbol{A}} A^a_{\tau_i^j} = 1 \quad \forall \tau_i^j \in \boldsymbol{\tau} \tag{2}$$

$$x_{\langle \tau_i^j, \tau_x^y \rangle} = 1 \quad \forall W_{\langle \tau_i^j, \tau_x^y \rangle} \in \boldsymbol{TC} \tag{3}$$

$$W_{\langle \tau_i^j, \tau_x^y \rangle} \leq s_x^y - f_i^j \quad \forall W_{\langle \tau_i^j, \tau_x^y \rangle} \in \boldsymbol{TC} \tag{4}$$

$$D^{s2s}_{\langle \tau_i^j, \tau_x^y \rangle} \geq f_x^y - s_i^j \quad \forall D^{s2s}_{\langle \tau_i^j, \tau_x^y \rangle} \in \boldsymbol{TC} \tag{5}$$

$$D^{\text{abs}}_{\tau_i^j} \geq f_i^j \quad \forall D^{\text{abs}}_{\tau_i^j} \in \boldsymbol{TC} \tag{6}$$

$$f_i^j - s_i^j \geq lb^a_{\tau_i^j} - M\left(1 - A^a_{\tau_i^j}\right) \quad \forall \tau_i^j \in \boldsymbol{\tau}, a \in \boldsymbol{A} \tag{7}$$

$$f_i^j - s_i^j \leq ub^a_{\tau_i^j} + M\left(1 - A^a_{\tau_i^j}\right) \quad \forall \tau_i^j \in \boldsymbol{\tau}, a \in \boldsymbol{A} \tag{8}$$

$$s_x^y - f_i^j \geq M\left(x_{\langle \tau_i^j, \tau_x^y \rangle} - 1\right) \quad \forall \left\langle \tau_x^y, \tau_i^j \right\rangle \in \boldsymbol{\tau_R} \tag{9}$$

$$s_i^j - f_x^y \geq -M x_{\langle \tau_i^j, \tau_x^y \rangle} \forall \left\langle \tau_i^j, \tau_x^y \right\rangle \in \boldsymbol{\tau_R} \tag{10}$$

$$s_x^y - f_i^j \geq M\left(1 + x_{\langle \tau_i^j, \tau_x^y \rangle} - A^a_{\tau_i^j} - A^a_{\tau_x^y}\right) \quad \forall \tau_i^j, \tau_x^y \in \boldsymbol{\tau} \tag{11}$$

$$s_i^j - f_x^y \geq M\left(2 - x_{\langle \tau_i^j, \tau_x^y \rangle} - A^a_{\tau_i^j} - A^a_{\tau_x^y}\right) \quad \forall \tau_i^j, \tau_x^y \in \boldsymbol{\tau}. \tag{12}$$

Employing branch-and-bound search to identify the optimal solution in this MILP-based formulation requires $O\left(2^{|A||\boldsymbol{\tau}|)^3}\right)$ just in terms of the integer variables for allocation ($A^a_{\tau_i^j}$) and sequencing ($x_{\langle \tau_i^j, \tau_x^y \rangle}$). Note that the number of possible sequencing permutations is $O(|\boldsymbol{\tau}|!)$, while the number of sequencing variables in this formulation is $O\left(|\boldsymbol{\tau}|^2\right)$. Within the manufacturing settings of interest, the number of tasks and subtasks is typically much larger than the number of agents, so the computational bottleneck when solving for a schedule occurs within the sequencing subproblem.

As noted in Section II, other related works have included formulation of similar scheduling problems as MILPs: For example, Korsah *et al.* [43] proposed a general formulation of the instantaneous assignment problem for the XD class. However, our formulation considers time-extended scheduling in which one must also determine how to sequence jobs. Pinto and Grossmann [61] proposed an MILP formulation that bears similarities to ours: both consider assignments of agents to tasks [see (2)], as well as the ordering of tasks for machines (or agents) as given by (11) and (12). However, our MILP formulation includes shared resource constraints [see (9) and (10)], as well as the unique capabilities or production rates of agents [see (7) and (8)].Other works have considered separate aspects of our problem, such as inclusion of heterogeneous agents (see [78]) or shared resources (see [41]). However, these works do not propose a mathematical formulation that addresses the XD [ST-SR-TA] problem variant with resource constraints.

Tercio approximately solves our problem by producing suboptimal makespan solutions. Section X demonstrates through empirical evaluation that the produced makespans are within 10% of optimal for the range of problems evaluated. To preserve temporal flexibility at execution, the solution is then reformulated as an STP that is flexibly dispatched [84] (see Section IV-A).

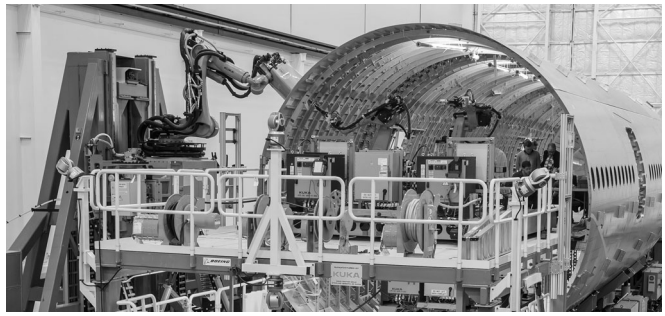

Fig. 2. This figure depicts the Boeing 777 Fuselage Upright Autonomous Build Process. Courtesy: The Boeing Company.

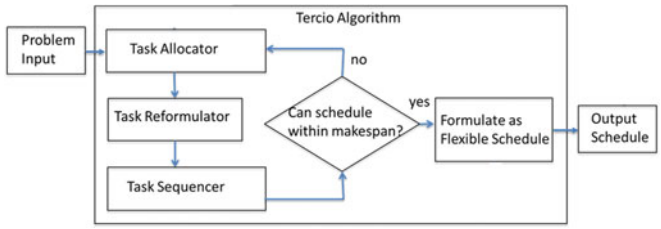

Fig. 3. This figure depicts the system architecture of Tercio.

A. Real-World Motivation Example

Our problem statement is motivated by real-world applications, such as the Boeing 777 Fully Autonomous Upright Build project (see Fig. 2). This type of application requires the coordination of six to eight robots for successful assembly of an aerospace structure. Precedence constraints among the work packages must be respected to ensure structural integrity during the build process. Task–task temporal constraints are imposed by process requirements for the timed application of sealant. The robots' work must be sequenced to ensure mutually exclusive access to tools, utilities, and floor space; efficient solutions to the problem involve an intricate choreography of robot movements. A fast dynamic scheduling method is necessary to efficiently and effectively respond to schedule disruptions such as those caused by process time variation, rework, and inspection.

IV. Tercio

In this section, we present Tercio, a centralized task assignment and scheduling algorithm that scales to multi-agent, factory-size problems and supports on-the-fly rescheduling with temporal and spatial proximity constraints. Fig. 3 depicts the system architecture, and the pseudocode for the Tercio algorithm is presented in Fig. 4. Tercio is made efficient through a fast, multi-agent task sequencer inspired by real-time processor scheduling techniques but adapted to leverage the hierarchical problem structure, wherein tasks are composed of precedence-related subtasks. Our approach decomposes the problem statement in Section III into subproblems of task allocation and sequencing. We use an MILP-based solution method (see Section V) to allocate agents to tasks and a polynomial-time task sequencer to efficiently solve the task sequencing problem (see Section VI). Although the task sequencer is satisficing and incomplete, it substantially improves worst-case time complexity.

TERCIO(τ, A, TC, $\tau_{\mathbf{R}}$, AC, z, cutoff/timeout)

1: makespan $\leftarrow \inf$
2: exclusions $\leftarrow \emptyset$
3: **while** makespan $>$ cutoff or runtime $<$ timeout **do**
4: A $\leftarrow$ ALLOCATION(z, $\mathbf{A}$, τ, TC, AC, $\tau_{\mathbf{R}}$, exclusions)
5: TC $\leftarrow$ update agent capabilities
6: makespan,seq $\leftarrow$ SEQUENCER($\mathbf{A}, \mathbf{TC}, \tau_{\mathbf{R}}$)
7: exclusions $\leftarrow$ exclusions $\cup$ $\mathbf{A}$
8: **end while**
9: TC $\leftarrow$ add ordering constraints to enforce seq
10: STP $\leftarrow$ DISPATCHABLE(TC)
11: **return** STP

Fig. 4. Pseudocode for the Tercio algorithm.

(We present time complexity analysis for each component of Tercio at the conclusion of their respective description below.)

A. Tercio Pseudocode

Tercio takes the inputs defined in Section III, along with a user-specified makespan *cutoff* intended to terminate the optimization process. This *cutoff* can often be derived from the temporal constraints of the manufacturing process: for example, a user may specify that a provided task set must be completed within an 8-hour shift. Tercio works by iterating through agent allocations until a schedule can be identified that satisfies the maximum allowable makespan for the problem. However, Tercio can also run as an anytime heuristic, terminating once the allotted time has expired.

As shown in Fig. 4, Tercio initializes the makespan (line 1) and previous solution (line 2), and then iterates lines 3–7 to compute a schedule that meets this makespan. A third-party optimizer (Gurobi) solves the agent-allocation MILP (line 4) and returns the agent allocation matrix A. Interval temporal (TC) constraints are updated according to this matrix by tightening task time intervals (line 5). For example, if a task is originally designated to take between 5 and 15 min, but the assigned robot can complete it in no fewer than 10 min, the interval tightens from [5, 15] to [10, 15].

The agent allocation, the capability-updated TCs, and the spatial map of tasks are then provided as input to the Tercio multi-agent task sequencer (line 6). The task sequencer (described further in Section VI) returns a tight upper bound on the optimal makespan for the given agent allocation, as well as a sequence of tasks for each agent. While this makespan is longer than *cutoff* (or while the algorithm's runtime has not exceeded the specified timeout), the algorithm iterates lines 3–7, each time adding a constraint (line 7) to exclude previously attempted agent allocations. Tercio terminates when either the returned makespan falls beneath *cutoff* or when no solution can be found after iterating through all feasible agent allocations. Note that, for each iteration, a search tree is generated for the agent allocation. We propose preserving the search tree across iterations in future work to reduce computation time, updating it with excluded allocations.

If the cutoff makespan is satisfied, agent and spatial resource sequencing constraints (interval form of $[0, \infty)$) are added to TC (line 9). The resulting STP, composed of the interval temporal constraints, is compiled into a dispatchable form (line 10) [57], [84], which guarantees that, for any consistent choice of a time point within a flexible window, there exists a solution that can be identified through one-step propagation of interval bounds. The dispatchable form maintains flexibility to increase robustness to disturbances and has been shown to decrease the amount of time spent recomputing solutions in response to disturbances for randomly generated structured problems by up to 75% [84].

V. Tercio: Agent Allocation

The Tercio agent allocator performs agent-task allocation by solving an MILP that includes (2), (7), and (8), ensuring that each task is assigned to exactly one agent and that the allocation does not violate the upper and lower bound temporal constraints. In this work, we investigate the objective of minimizing schedule makespan. This corresponds to solving the MILP defined in Section III according to the objective function depicted in (13), where we seek the optimal makespan (i.e., the overall process duration):

$$\min(z),\ z = g_{\text{MILP}}(A, \boldsymbol{\tau}) = \max_{\langle \tau_i^j, \tau_x^y \rangle} \left(f_i^j - s_x^y \right). \tag{13}$$

However, decomposition of task allocation and sequencing necessitates an objective function that guides the task allocation subroutine toward solutions that are likely to be favorable for the sequencing subroutine. As such, we developed the following objective function [see (14)], comprising three mixed-integer linear terms [see (15) and (16)]. Equation (15) minimizes the maximum work assigned to any one agent, which mitigates situations resulting in a single agent bottle-necking the schedule. Equation (16) minimizes the total amount of work time (i.e., "agent-hours") by selecting the most efficient agent for each subtask. As such, Tercio's agent-allocation subroutine maximizes (14) subject to (15) and (16). In our MILP-based task allocation subroutine, binary decision variable $A^a_{\tau_i^j}$ represents the assignment of agent a to subtask τ_i^j. The worst-case time complexity of assigning one of $a = |A|$ agents to each subtask in $\boldsymbol{\tau}$ via branch-and-bound search is given by $O\left(2^{|A||\boldsymbol{\tau}|}\right)$:

$$\min(z),\ z = \alpha_1 g_1(A, \boldsymbol{\tau}) + \alpha_2 g_2(A, \boldsymbol{\tau}) \tag{14}$$

$$g_1(A, \boldsymbol{\tau}) \geq \sum_{\tau_i^j \in \boldsymbol{\tau}} A^a_{\tau_i^j} \times C^a_{\tau_i^j} \quad \forall a \in \boldsymbol{A} \tag{15}$$

$$g_2(A, \boldsymbol{\tau}) \geq \sum_{a \in \boldsymbol{A}} \sum_{\tau_i^j \in \boldsymbol{\tau}} A^a_{\tau_i^j} \times C^a_{\tau_i^j}. \tag{16}$$

VI. Tercio: Multi-Agent Task Sequencer

The Tercio task sequencer takes the problem defined in Section III as input, along with a set of task assignments provided by the Tercio agent allocator described in Section V. The task sequencer is satisficing, meaning the produced schedule merely satisfies the constraints of the problem [see (2)–(12)] and does not take an objective function as input. The task sequencer returns a valid task sequence if the algorithm can identify one. Tercio schedules tasks in simulation using the dynamic

TERCIO-SEQUENCER($\boldsymbol{\tau}$)

1: reformulate $\boldsymbol{D^{s2s}}$ and $\boldsymbol{W}$
2: $t \leftarrow 0$ ▷ set simulation time to zero
3: ensure temporal feasibility of $\boldsymbol{D^{abs}}$
4: **while** true **do**
5: $availableTasks \leftarrow$ get and sort subtasks ready for execution at time t according to EDF
6: **for** all $\tau_i^j \in$availableTasks **do**
7: **if** executing τ_i^j at t will not violate temporospatial constraints **then**
8: $seq \leftarrow$ schedule τ_i^j
9: **end if**
10: **end for**
11: $t \leftarrow t + 1$ ▷ increment simulation time
12: **if** all tasks executed **then** break;
13: **end if**
14: **end while**
15: $makespan, seq \leftarrow$ extract multi-agent schedule
16: **return** $makespan, seq$

Fig. 5. Pseudocode for the Tercio multi-agent task sequencer.

priority policy earliest-deadline first (EDF), as well as an online schedulability test that guarantees satisfaction of temporospatial constraints for any opportunistic scheduling policy (i.e., a policy that executes a task if one is available to execute). We formulate this schedulability test as a CP problem and determine whether a full, feasible schedule can be developed if subtask τ_i^j is scheduled at time t.

The Tercio task sequencer is inspired by real-time processor scheduling techniques and operates on the structure of a real-time processor scheduling model, called the "self-suspending task model." The sequencer performs a rapid variant of "edge-checking" (similar to that performed in [45] and [82]), which we call the multiprocessor Russian dolls test. To our knowledge, the Tercio task sequencer is the first real-time scheduling method for multiprocessor systems that tests the schedulability of nonpreemptive, self-suspending tasks in scenarios where multiple tasks have more than one self-suspension and tasks are constrained by shared memory resources.

We now outline the steps taken by the Tercio task sequencer. In Section VII, we discuss the relationship of our problem to prior art in real-time processor scheduling and describe our problem as a real-time processor scheduling problem. We then present how the problem depicted in Section III is reformulated into a real-time processor scheduling problem. Finally, in Section VIII, we present the task sequencer's priority policy and multiprocessor Russian dolls test.

A. Multi-Agent Task Sequencer Walk-Through

Here, we address the mechanics of our task sequencing algorithm, as depicted in Fig. 5. In line 1, the task set is reformulated (if necessary) into a specific structure that makes the task sequencer operate more efficiently. The task sequencer requires that every subtask τ_i^j involved in an absolute deadline constraint $D^{\text{abs}}_{\tau_i^j}$ or subtask-to-subtask deadline constraint $D^{s2s}_{\langle \tau_a^b, \tau_i^j \rangle}$ have only one predecessor subtask τ_x^y. This means that either $\tau_x^y = \tau_i^{j-1}$, or else there exists a lower bound wait constraint $W_{\langle \tau_x^y, \tau_i^j \rangle}$. In turn, every τ_x^y must have exactly one such predecessor subtask. This recurses until reaching either the epoch subtask τ_0^0 in the case of an absolute deadline constraint, or τ_a^b in the case of a subtask-to-subtask deadline.

In Section VII-C, we prove that the reformulation process preserves correctness—any schedule that satisfies the constraints of the reformulated task set will also satisfy the constraints of the original set. The ability to reformulate depends upon the laxity of the deadlines within the task set and the structure of the constraints. The tighter the deadlines and the more connected the constraint graph, the less able the algorithm will be to reformulate the problem into the structure our schedulability test requires. However, this more restricted structure enables our schedulability test to compute an empirically tight schedulability test in polynomial time.

In line 2, simulation time is initialized to zero. In line 3, the algorithm determines whether the set of absolute deadlines $\boldsymbol{D}^{\text{abs}}$ in the reformulated task set is temporally consistent, meaning that the set of agents will be able to successfully schedule against those deadline constraints and their associated spatial constraints. This is determined using the multiprocessor Russian dolls test, as described in Section VIII. After ensuring schedule feasibility due to $\boldsymbol{D}^{\text{abs}}$, the algorithm begins to schedule all subtasks in $\boldsymbol{\tau}$ in simulation (lines 4–14). In line 5, the algorithm prioritizes the order in which it attempts to schedule available subtasks according to the EDF dynamic scheduling priority. EDF, commonly used in real-time systems [36], [86], [89] and multi-agent scheduling [11], attempts to execute the task τ_i^j with the earliest (smallest in magnitude) deadline d_i^j first.

Line 6 iterates over all available subtasks τ_i^j and applies the multiprocessor Russian dolls test (line 7) to determine whether a subtask can feasibly be scheduled at time t while satisfying the temporospatial constraints. If feasible, the subtask is scheduled at time t (line 8) and an all-pairs shortest path (APSP) computation [28], [40] is performed to update the temporal constraints. The simulation time is incremented (line 11) and the algorithm continues until all tasks have been scheduled (line 12). Finally, the algorithm returns the makespan and subtask sequence (line 15).

VII. Real-Time Processor Scheduling Analogy for the Task Sequencer

The design of our informative, polynomial-time task sequencer was inspired by a processor scheduling analogy in which each agent is a computer processor able to perform one task at a time, and a physical location in discretized space is modeled as a shared memory resource accessible by only one processor at a time. Wait constraints (lower bounds on interval temporal constraints) are modeled as "self-suspensions" [46], [69]—times during which a task is blocked while another piece of hardware completes a different, time-durative task.

Typically, assembly manufacturing tasks have more structure (e.g., parallel and sequential subcomponents) and more complex temporal constraints than real-time processor scheduling problems. AI scheduling methods address complex temporal

constraints and gain computational tractability by leveraging a hierarchical structure within the plan [71]. We bridged AI and real-time processor scheduling in order to develop a fast multi-agent task sequencer that satisfies tightly coupled upper and lower bound temporal deadlines and spatial proximity restrictions (shared resource constraints).

The Tercio task sequencer operates on an augmented *self-suspending task model* (defined in Section VII-B) and returns a valid task sequence if the algorithm can identify one. The task sequencer is satisficing and incomplete; however, we have empirically validated that it returns makespans within 10% of the optimal when integrated with the Tercio agent allocation algorithm (see Section X).

Processor scheduling of *self-suspending task systems* has been the focus of much prior work due to the integration of relatively recent hardware and supporting software systems (e.g., GPUs and PPUs) that trigger the external blocking of tasks [20], [46], [69]. Self-suspensions can be thought of as lower bound temporal constraints relating tasks: for example, a user might specify that a first coat of paint needs at least 30 min to dry before a second coat may be applied—this 30-minute wait time is a self-suspension of the painting task.

Prior work has computed the uniprocessor schedulability of a task set with single [51], [69] or multiple [32], [33] self-suspensions. In our work, we compute the multiprocessor schedulability of a task set in which multiple tasks have more than one suspension and each subtask has a resource constraint. Our approach incorporates a scheduling policy that partially restricts the behavior of the scheduler in order to reduce incidence of multiprocessor schedule anomalies due to self-suspensions. Our approach is similar in spirit to prior art that restricted behavior to reduce anomalies that inherently arise from application of uniprocessor scheduling methods to self-suspending task sets [32], [33], [47], [65], [69]. We first introduce our task model in this section. Second, we describe how our task sequencer satisfies temporospatial constraints in Section VIII.

A. Traditional Self-Suspending Task Model

The Tercio task sequencer relies upon a well-formed task model that captures hierarchical and precedence structure within the task network. The basis for our framework is the self-suspending task model, described as

$$\tau_i : \left(W_{\langle \tau_i^0, \tau_i^1 \rangle}, C_i^1, W_{\langle \tau_i^1, \tau_i^2 \rangle}, C_i^2, \ldots, C_i^m, T_i, D_i \right). \quad (17)$$

In this model, a set of tasks $\boldsymbol{\tau}$ must be processed by the computer. An instance of each task τ_i is *released* (eligible to execute) at every period T_i. The execution of the first subtask of τ_i may be delayed from the epoch start as specified by a lower bound wait constraint $W_{\langle \tau_i^0, \tau_i^1 \rangle}$, called the "phase offset." For each task, there are m_i subtasks, with $m_i - 1$ self-suspension intervals for each task $\tau_i \in \boldsymbol{\tau}$. We use τ_i^j to denote the jth subtask of τ_i. C_i^j is the expected duration (cost) of τ_i^j. A subtask τ_i^k is released once its predecessor subtask and preceding self-suspension have both finished executing. $W_{\langle \tau_i^{j-1}, \tau_i^j \rangle}$ is the lower bound wait constraint (or self-suspension) interval relating the end of τ_i^{j-1} and the start of τ_i^j. T_i and D_i are the period and deadline of τ_i, respectively.

B. Augmented Self-Suspending Task Model

The standard self-suspending task model provides a solid basis for describing many real-world processor scheduling problems of interest [20], [46], [69]. Scheduling problems within the manufacturing domain inherently have strong, hierarchical structures that are captured well by the traditional self-suspending task model. However, self-suspending task systems generally assume that processors are homogeneous and do not include more general temporal or resource constraints among tasks and subtasks [51]:

$$\tau_i : \Big(W_{\langle \tau_i^0, \tau_i^1 \rangle}, (C^a_{\tau_i^1}, R_i^1), W_{\langle \tau_i^1, \tau_i^2 \rangle}, (C^a_{\tau_i^2}, R_i^2), \ldots (C^a_{\tau_i^m}, R_i^m), T_i, D_i^{s2s}, D_i^{\text{abs}} \Big) ; \left\{ A^a_{\tau_i^j} \right\}, \left\{ W_{\langle \tau_i^j, \tau_x^y \rangle} \right\}. \quad (18)$$

We first augmented the model to encode the assignment of processors (i.e., agents) to subtasks, denoted by the set of binary decision variables $\{A^a_{\tau_i^j}\}$. The subtask cost $C^a_{\tau_i^j}$ is now dependent upon the capabilities of the agent a processing the subtask. The second augmentation enables a user to specify subtask-to-subtask deadlines $D^{s2s}_{\langle \tau_i^j, \tau_i^k \rangle}$ between the start of τ_i^j and end of τ_i^k [see (19)]; as well as absolute deadlines $D^{\text{abs}}_{\tau_i^j}$ for the absolute finish times of subtasks [see (20)], as shown in (19) and (20). Subtask-to-subtask deadlines are restricted to constrain subtasks within the same task, enabling fast edge-checking in our schedulability test (see Section VIII). In Section VII-C, we describe how general upper bound deadline constraints relating pairs of subtasks are reformulated into this augmented self-suspending task model:

$$D^{s2s}_{\langle \tau_i^j, \tau_i^k \rangle} \geq f_i^k - s_i^j \quad (19)$$

$$D^{\text{abs}}_{\tau_i^j} \geq f_i^j. \quad (20)$$

We define d_i^j as the implicit deadline on the finish time f_i^j of subtask τ_i^j, which is implied by the absolute and subtask deadlines $\boldsymbol{D}^{\text{abs}} \cup \boldsymbol{D}^{\boldsymbol{s2s}}$. These deadline constraints provide additional expressiveness to encode binary temporal constraints relating tasks. For instance, these constraints may be used to specify that a sequence of subtasks involving sealant application must be completed within 30 min of opening the sealant container. These types of constraints are commonly included in AI and operations research scheduling models and are vital for modeling many real-world problems [19], [57].

We augmented the model to express subtask-to-subtask wait constraints [see (21)] provided that the subtasks meet certain criteria. To describe this restriction, we first introduce two categories of subtasks: *free* and *embedded*.

Definition 1: A *free subtask* $\tau_i^j \in \boldsymbol{\tau}_{\mathbf{free}}$ is a subtask that does not share a deadline constraint with τ_i^{j-1}. In other words, a subtask τ_i^j is free *iff* for any deadline $D_{\langle \tau_i^a, \tau_i^b \rangle}$ associated with that

task $(j \leq a) \vee (b < j)$. We define τ_i^1 as free, since a preceding subtask does not exist.

Definition 2: An *embedded subtask* $\tau_i^{j+1} \in \boldsymbol{\tau}_{\mathbf{embedded}}$ is a subtask that shares a deadline constraint with τ_i^j (i.e., $\tau_i^{j+1} \notin \boldsymbol{\tau}_{\mathbf{free}}$). $\boldsymbol{\tau}_{\mathbf{free}} \cap \boldsymbol{\tau}_{\mathbf{embedded}} = \emptyset$:

$$W_{\langle \tau_i^j, \tau_x^y \rangle} \leq s_x^y - f_i^j. \tag{21}$$

For our augmented self-suspending task model, a wait constraint can be applied to subtasks across different tasks $\langle \tau_i^j, \tau_x^y \rangle$ if and only if τ_x^y is a *free* subtask. This restriction also enables fast edge-checking in our schedulability test, presented in Section VIII. We address how general lower bound wait constraints relating pairs of subtasks are reformulated into this model in Section VII-C.

We also extended the model to include shared memory resources. Each subtask τ_i^j requires that a set of shared memory resources $R_i^j = \{R_{i,1}^j, \ldots, R_{i,k_i}^j\}$ be utilized to perform that subtask (e.g., for memory shared among multiple processors). In a manufacturing setting, a shared memory resource corresponds to a region of space on the factory floor that must be physically unoccupied in order for an agent to execute a subtask there. Shared memory resources encode hard spatial constraints that prohibit agents from working in dangerously close physical proximity to one another.

We made four simplifying assumptions about this augmented self-suspending task model. First, we set the implicit deadlines of the tasks equal to the period of the task set. This modification accurately models many assembly line manufacturing processes wherein the set of tasks at each location are repeated once every "pulse" (i.e., period H) of the production line. In this scenario, the user allots a certain amount of time T for the set of tasks to be accomplished, and the set of tasks is repeated with a period of T.

We also assume that all subtasks are nonpreemptable, meaning that the interruption of a subtask significantly degrades its quality and that the cost of a subtask τ_i^j assigned to agent a is determined by the expected cost of agent a executing subtask τ_i^j. Finally, we assume that switching times between tasks (i.e., travel times on the factory floor) are small compared with subtask times and can be modeled as constant. We find this assumption suitable for many assembly manufacturing applications—for example, situations in which work is performed within a localized area and does not require lengthy traversals across the factory floor, or where calibration and set-up times at new work locations are long compared with travel times. This assumption would not be reasonable for applications involving the optimization of vehicle routes across a factory or satisfaction of more-complex geometric constraints. We leave the study of more general geometric and temporal logic constraints, such as those considered in [22] and [62], for future work.

C. Reformulating the Task Set

Tercio's task sequencer uses an online schedulability test to determine whether a given subtask τ_i^j can be scheduled at time t considering the deadline constraints within the task set. This schedulability test requires the specific structure of the augmented self-suspending task model in order for this inference to be efficient. In this section, we present our method for reformulating general upper bound deadlines $D^{s2s}_{\langle \tau_a^b, \tau_i^j \rangle}$ and lower bound wait constraints $W_{\langle \tau_a^b, \tau_x^y \rangle}$ into this more-structured form.

SEQUENCER-REFORMULATE($\boldsymbol{\tau}$)

while $\exists D^{s2s}_{\langle \tau_i^j, \tau_x^y \rangle}$ s.t. there is no line of precedence from τ_x^y to τ_i^j **do**
  $D^{abs}_{\tau_x^y} \leftarrow D^{s2s}_{\langle \tau_i^j, \tau_x^y \rangle} - APSP_{\langle \tau_x^y, \tau_0^0 \rangle}$
  Replace $D^{s2s}_{\langle \tau_i^j, \tau_x^y \rangle}$ with $D^{abs}_{\tau_x^y}$
end while
while $\exists \tau_i^j$ with constrained start time and two predecessors, **do**
  for counter $= 2$ to number of τ_i^j's predecessors **do**
    $\tau_a^b \leftarrow \tau_i^j$'s (counter$-1)^{th}$ predecessor
    $\tau_x^y \leftarrow \tau_i^j$'s (counter)th predecessor
    feasible1 $\leftarrow$ whether it is feasible for τ_a^b to precede τ_x^y
    feasible2 $\leftarrow$ whether it is feasible for τ_x^y to precede τ_a^b
    if feasible1 $\wedge$ **feasible2 then**
      if $|APSP_{\langle \tau_a^b, \tau_i^j \rangle}| \leq |APSP_{\langle \tau_x^y, \tau_i^j \rangle}|$ **then**
        Replace $W_{\langle \tau_a^b, \tau_i^j \rangle}$ with $W_{\langle \tau_a^b, \tau_x^y \rangle}$ (Eq 22)
      else
        Replace $W_{\langle \tau_x^y, \tau_i^j \rangle}$ with $W_{\langle \tau_x^y, \tau_a^b \rangle}$ (Eq 22)
      end if
    else if feasible1 then
      Replace $W_{\langle \tau_a^b, \tau_i^j \rangle}$ with $W_{\langle \tau_a^b, \tau_x^y \rangle}$ (Eq 22)
    else if feasible2 then
      Replace $W_{\langle \tau_x^y, \tau_i^j \rangle}$ with $W_{\langle \tau_x^y, \tau_a^b \rangle}$ (Eq 22)
    end if
    if feasible1 $\vee$ **feasible2 then**
      continue
    else if counter = # predecessor subtasks **then return null**
    end if
  end for
end while
return $\boldsymbol{\tau}$

Fig. 6. Pseudocode for the reformulation subroutine.

The reformulation ensures that for every absolute deadline constraint $D^{\text{abs}}_{\tau_i^j}$ or subtask-to-subtask deadline constraint $D^{s2s}_{\langle \tau_a^b, \tau_i^j \rangle}$, subtask τ_i^j has only one predecessor subtask τ_x^y. This means that either $\tau_x^y = \tau_i^{j-1}$, or that there exists a self-suspension $W_{\langle \tau_x^y, \tau_i^j \rangle}$. In turn, τ_x^y must have exactly one such predecessor subtask. This process recurses until reaching either the epoch subtask τ_0^0, in the case of an absolute deadline constraint, or τ_a^b, in the case of a subtask-to-subtask deadline.

The reformulation preserves correctness, meaning that any schedule that satisfies the constraints of the reformulated task set will also satisfy the constraints of the original task set. The reformulation rules are based on the *triangle rules* and proofs of correctness developed by Tsamardinos *et al.* for the reformulation of temporal networks [80]. However, note that the reformulation process for our algorithm is incomplete: it heuristically attempts to solve an NP-hard scheduling problem, does not explore the entire search space, and thus does not always return a sequence if one exists. The ability to reformulate depends upon the laxity of the deadlines in the task set and the structure of the constraints. The tighter the deadlines and the more connected the constraint graph, the less likely it is that the algorithm will be able to reformulate the problem into the structure our schedulability test requires. Fig. 6 depicts *SEQUENCER-REFORMULATE*, which

takes a task set as input and returns either a reformulated task set (if the algorithm can find a solution) or null (if no feasible reformulated task set can be identified).

D. Reformulation Pseudocode

Line 1 checks whether any subtask-to-subtask deadlines $D^{s2s}_{\langle \tau_i^j, \tau_x^y \rangle}$ exist such that there is neither a wait constraint $W_{\langle \tau_i^j, \tau_x^y \rangle}$ nor a set of wait constraints linking τ_i^j and τ_x^y via one or more intermediary subtasks (e.g., $W_{\langle \tau_i^j, \tau_a^b \rangle}$ and $W_{\langle \tau_a^b, \tau_x^y \rangle}$). If there exists such a deadline $D^{s2s}_{\langle \tau_i^j, \tau_x^y \rangle}$, it is replaced with a new absolute deadline $D^{\text{abs}}_{\tau_x^y}$ that implicitly enforces $D^{s2s}_{\langle \tau_i^j, \tau_x^y \rangle}$ (line 2). Note in line 2, $\text{APSP}_{\langle \tau_i^j, \tau_x^y \rangle}$ is the APSP temporal upper bound between τ_i^j and τ_x^y, and that the APSP temporal lower bound between ordered tasks is nonpositive.

For each subtask with its initiation constrained by a deadline and which involves more than one predecessor subtask, the algorithm iterates over the predecessor subtasks (lines 5 and 6). Lines 7 and 8 consider two predecessors at a time τ_a^b and τ_x^y and determine whether the lower bound constraints can be reformulated such that τ_a^b is the predecessor of τ_x^y, or vice-versa (lines 9 and 10). For a subtask τ_i^j with two predecessors τ_a^b and τ_x^y, the algorithm restructures the problem such that τ_a^b is the predecessor of τ_x^y by replacing $W_{\langle \tau_x^y, \tau_i^j \rangle}$ with $W_{\langle \tau_a^b, \tau_x^y \rangle}$, according to

$$W_{\langle \tau_a^b, \tau_x^y \rangle} = \max\left(W_{\langle \tau_a^b, \tau_i^j \rangle} - C_{\tau_x^y} - W_{\langle \tau_x^y, \tau_i^j \rangle}, W_{\langle \tau_a^b, \tau_x^y \rangle}\right). \quad (22)$$

In line 11, in the event that both methods of reformulating the problem are feasible, the algorithm employs a "tie-breaker": If the temporal distance path $\tau_i^j \rightarrow \tau_x^y \rightarrow \tau_a^b$ is shorter than that between $\tau_i^j \rightarrow \tau_a^b \rightarrow \tau_x^y$, the path from τ_i^j to τ_x^y is removed and a temporal wait constraint is added from τ_a^b to τ_x^y—as a result, τ_x^y is now the predecessor of τ_a^b. If only one method of reformulation is feasible (lines 17 and 19), the constraints are modified accordingly (lines 18 and 20). Each time the constraint network is modified, the reformulation process restarts (lines 22 and 23). The algorithm returns null if it is impossible to reorder τ_a^b and τ_x^y and if there are no more predecessor subtasks of τ_i^j to process (line 24); otherwise, the algorithm returns the reformulated task set (line 28).

E. Reformulation Example

In this section, we provide an example to illustrate the reformulation process. Figs. 7 and 8 depict a task set before and after reformulation, respectively. In these figures, subtask start and end times are denoted as nodes (black circles), and constraints are represented by edges. Blue edges indicate lower bound temporal constraints (phase offsets, self-suspensions, or wait constraints), and orange edges represent upper bound temporal constraints (absolute or subtask-to-subtask deadline constraints). Recall that a task set must satisfy two conditions in order to be correctly sequenced by Tercio: First, for every subtask-to-subtask deadline $D^{s2s}_{\langle \tau_i^j, \tau_x^y \rangle}$, there must be a path of precedence from τ_i^j to τ_x^y. Second, each subtask must have exactly one predecessor subtask.

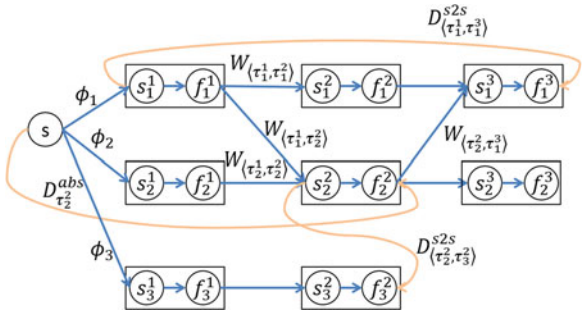

Fig. 7. This figure depicts a task set that must be reformulated to adhere to the augmented self-suspending task model.

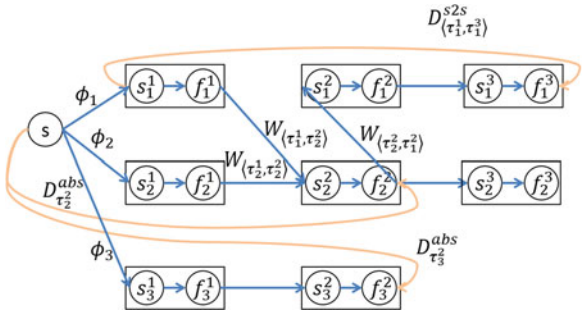

Fig. 8. This figure depicts the first two steps for the reformulation of the task set in Fig. 7.

Consider the task set depicted in Fig. 7. This example includes three temporal constraints that conflict with the above-mentioned requirements: $D^{s2s}_{\langle \tau_2^2, \tau_3^2 \rangle}$ does not satisfy the requirement of a directed path of precedence from τ_2^2 to τ_3^2. Also, embedded subtask τ_2^2 has more than one precedence constraint delaying its initiation: ($W_{\langle \tau_1^1, \tau_2^2 \rangle}$ and $W_{\langle \tau_2^1, \tau_2^2 \rangle}$).

The first step to addressing these temporal constraints is to reformulate $D^{s2s}_{\langle \tau_2^2, \tau_3^2 \rangle}$ as an absolute deadline $D^{\text{abs}}_{\tau_3^2}$, as shown in Fig. 8, such that $D^{\text{abs}}_{\tau_3^2}$ is sufficiently tight to guarantee that f_3^2 does not occur later than $s_2^2 + D^{s2s}_{\langle \tau_2^2, \tau_3^2 \rangle}$. To ensure consistency, the reformulation subroutine sets $D^{\text{abs}}_{\tau_3^2}$ equal to $-\text{APSP}_{\langle \tau_2^2, \tau_0^0 \rangle} + D^{s2s}_{\langle \tau_2^2, \tau_3^2 \rangle}$. The requirement of a path of precedence from τ_i^j to τ_x^y for every subtask-to-subtask deadline $D^{s2s}_{\langle \tau_i^j, \tau_x^y \rangle}$ is now satisfied.

Next, we address the wait constraints $W_{\langle \tau_1^1, \tau_2^2 \rangle}$ and $W_{\langle \tau_2^1, \tau_2^2 \rangle}$. In order to do so, we must first determine whether to require for τ_2^1 to precede τ_1^1, or vice-versa. If neither option is possible, the reformulation algorithm cannot reformulate the task set. If only one option is possible, the algorithm selects and enforces that precedence constraint. If both options are temporally feasible, the algorithm assigns the ordering to reduce the delay of τ_2^2. In this example, ordering τ_2^1 before τ_1^1 is preferred, so the algorithm replaces $W_{\langle \tau_2^1, \tau_2^2 \rangle}$ with $W_{\langle \tau_2^1, \tau_1^1 \rangle} = \max(W_{\langle \tau_2^1, \tau_2^2 \rangle} - (W_{\langle \tau_1^1, \tau_2^2 \rangle} C_1^2), 0)$ to ensure satisfaction of the original constraint. The task set can now be correctly sequenced by the Tercio task sequencer algorithm.

F. Proofs of Termination and Correct Reformulation

In this section, we show that the reformulation algorithm (see Fig. 6) will terminate for finite task sets. We also demonstrate that the algorithm preserves correctness—that any schedule that satisfies the constraints of the reformulated task set will also satisfy the constraints of the original task set.

Theorem VII.1: The reformulation subroutine preserves correctness, meaning that any schedule that satisfies the constraints of the reformulated task set will also satisfy the constraints of the original task set.

Proof by Deduction: Here, we show that each lower bound temporal constraint removed in lines 13, 15, 18, and 20 is made redundant by the newly added temporal constraint. Consider an embedded subtask τ_i^j, with predecessors τ_a^b and τ_x^y, ordered as

$$s_i^j \geq f_x^y + W_{\langle \tau_x^y, \tau_i^j \rangle} \tag{23}$$

and

$$s_i^j \geq f_a^b + W_{\langle \tau_a^b, \tau_i^j \rangle}. \tag{24}$$

The reformulation process either orders τ_x^y before τ_a^b, or vice-versa. If ordering τ_x^y before τ_a^b is temporally feasible [28], [40], the algorithm removes constraint $W_{\langle \tau_x^y, \tau_i^j \rangle}$ and adds constraint $W_{\langle \tau_x^y, \tau_a^b \rangle}$ according to (22). (Note that the proof for ordering τ_a^b before τ_x^y is symmetric.) When $W_{\langle \tau_a^b, \tau_i^j \rangle}$ in (22) is sufficiently large to contribute to the new wait constraint $W_{\langle \tau_x^y, \tau_a^b \rangle}$, (24) and (22) recover (23):

$$\begin{aligned} s_i^j &\geq W_{\langle \tau_a^b, \tau_i^j \rangle} + f_a^b \geq W_{\langle \tau_a^b, \tau_i^j \rangle} + C_{\tau_a^b} + s_a^b \\ &\geq W_{\langle \tau_a^b, \tau_i^j \rangle} + C_{\tau_a^b} + W_{\langle \tau_x^y, \tau_a^b \rangle} + f_x^y \\ &\geq W_{\langle \tau_a^b, \tau_i^j \rangle} + C_{\tau_a^b} + W_{\langle \tau_x^y, \tau_i^j \rangle} - \left(C_{\tau_a^b} + W_{\langle \tau_a^b, \tau_i^j \rangle} \right) + f_x^y \\ &\geq f_x^y + W_{\langle \tau_x^y, \tau_i^j \rangle}. \end{aligned} \tag{25}$$

If there already exists a constraint $W_{\langle \tau_x^y, \tau_a^b \rangle}$ and $W_{\langle \tau_x^y, \tau_i^j \rangle} - (C_{\tau_a^b} + W_{\langle \tau_a^b, \tau_i^j \rangle}) \leq W_{\langle \tau_x^y, \tau_a^b \rangle}$, then ordering τ_x^y before τ_a^b inherently satisfies (23). Likewise, if $W_{\langle \tau_x^y, \tau_a^b \rangle}$ does not exist and $W_{\langle \tau_x^y, \tau_i^j \rangle} - (C_{\tau_a^b} + W_{\langle \tau_a^b, \tau_i^j \rangle}) \leq 0$, then ordering τ_x^y before τ_a^b inherently satisfies (23). Equation (24) remains satisfied because $W_{\langle \tau_a^b, \tau_i^j \rangle}$ remains unaltered.

Theorem VII.2: The reformulation subroutine terminates.

Proof by Deduction: Lines 1–4 process a finite number of subtask-to-subtask deadlines, replacing each with an absolute deadline. Lines 5–27 add temporal precedence constraints to the network. Each lower bound temporal constraint removed in lines 13, 15, 18, and 20 is redundant given the newly added temporal constraint, meaning that any schedule that satisfies the new temporal constraint also satisfies the removed constraint (see Theorem VII.1). Therefore, the size of the set of feasible schedules only decreases with each reformulation step, and the algorithm is ultimately guaranteed to terminate.

G. SEQUENCER-REFORMULATE Complexity Analysis

Here, we present the computational complexity of *SEQUENCER-REFORMULATE*. Lines 1–4 of *SEQUENCER-REFORMULATE* convert a subset of the subtask-to-subtask deadlines into absolute deadlines. This involves a maximum of $|\boldsymbol{\tau}|^2$ operations. Lines 5–27 reformulate the task graph to ensure that all subtasks τ_i^j with constrained start times have no more than one predecessor. At most, $|\boldsymbol{\tau}|^2$ wait constraints are modified, and one wait constraint is removed at each iteration (lines 13, 15, 18, and 20). The feasibility of each modification is assessed using Johnson's algorithm [40], which has a complexity of $O(|\boldsymbol{\tau}|^2 \log|\boldsymbol{\tau}| + |\boldsymbol{\tau}||\mathrm{TC}|)$. Thus, the complexity of the reformulation algorithm is $O\left(|\boldsymbol{\tau}|^2 + |\boldsymbol{\tau}|^2 \left(|\boldsymbol{\tau}|^2 \log|\boldsymbol{\tau}| + |\boldsymbol{\tau}||\mathrm{TC}|\right)\right)$.

VIII. Multi-Agent Online Consistency Check

The Tercio task sequencer presented in Fig. 5 uses a schedulability test to ensure feasibility while scheduling tasks against deadline constraints and shared memory (or spatial) resource constraints. Our schedulability test, the multiprocessor Russian dolls test, works by determining whether the execution of one set of subtasks associated with a single deadline constraint can be nested within the laxity of a second set of subtasks associated with another deadline constraint. The test operates on the augmented self-suspending task model introduced in Section VII-B and makes the explicit assumption that any subtask with a start time tightened by a deadline constraint will be released upon completion of its single predecessor and any wait or self-suspension time. This assumption enables the algorithm to compute an empirically tight schedulability test in polynomial time.

Our test is a variant of a resource edge-finding algorithm [45], [82], the purpose of which is to determine whether an event must or may execute before or after a set of activities [2]. To our knowledge, our approach is the first to leverage the structure of the self-suspending task model to perform fast edge-checking.

We incorporate the test in two ways: First, we use it to ensure that the sequencing algorithm can identify a feasible schedule against $\boldsymbol{D}^{\mathrm{abs}}$ upon initiation of the scheduling simulation. Second, we use the test to ensure that scheduling a subtask τ_i^j at time t will not result in plan infeasibility due to $\boldsymbol{D}^{\mathrm{abs}} \cup \boldsymbol{D}^{\boldsymbol{s2s}}$.

To describe our test, we first define an *active deadline* (Definition 3) and an *active subtask* (Definition 4).

Definition 3: Active Deadline: A subtask-to-subtask deadline $D^{s2s}_{\langle \tau_i^j, \tau_x^y \rangle}$ is considered *active* for $t \in [s_i^j, f_x^y)$, and an absolute deadline $D^{abs}_{\tau_i^j}$ is considered *active* for $t \in [0, f_i^j)$.

Definition 4: Active Subtask: A subtask is *active* at time t if it is unexecuted at time t and is directly constrained by an active deadline.

A. Pseudocode Walk-Through

Fig. 9 depicts the pseudocode for the multiprocessor Russian dolls test. The test takes as input a subtask τ_i^j, the task set $\boldsymbol{\tau}$, the time t, and the *type* of test, and returns either the feasibility of the set of absolute deadlines (type $= 1$) or the feasibility of scheduling τ_i^j at time t (type $= 2$).

multiprocessorRussianDollsTest(τ_i^j,$\boldsymbol{\tau}$,t,type)

1: **if** type $= 1$ **then**
2: $\quad\boldsymbol{\tau_{group}} \leftarrow \boldsymbol{\tau_{active}}$
3: **else if** (type $= 2$) $\wedge \left(\exists D^{s2s}_{\langle \tau_i^j, \tau_i^z \rangle}\right)$ **then**
4: $\quad$**for** all $\tau_x^y \in \boldsymbol{\tau_{active}} \backslash \tau_i^j$ **do**
5: $\quad\quad$**if** τ_x^y and τ_i^j use the same agent or resource **then**
6: $\quad\quad\quad$**if** $t + C_i^j > d_x^y - C_x^y$ **then return** false
7: $\quad\quad\quad$**end if**
8: $\quad\quad$**end if**
9: $\quad$**end for**
10: $\quad\boldsymbol{\tau_{group}} \leftarrow \{\tau_i^k | j < k \leq z, \forall D^{s2s}_{\langle \tau_i^j, \tau_i^z \rangle}\}$
11: **else**
12: $\quad\boldsymbol{\tau_{group}} \leftarrow \emptyset$
13: **end if**
14: **for** all $\tau_i^k \in \boldsymbol{\tau_{group}}$ **do**
15: $\quad$**for** all $\tau_x^y \in \boldsymbol{\tau_{active}} \backslash \tau_i^k$ **do**
16: $\quad\quad$**if** τ_i^k and τ_x^y use the same agent or resource **then**
17: $\quad\quad\quad$**if** $\left(d_i^k > d_x^y - C_x^y\right) \wedge \left(d_i^k - C_i^k < d_x^y\right)$ **then return** false
18: $\quad\quad\quad$**end if**
19: $\quad\quad$**end if**
20: $\quad$**end for**
21: **end for**
22: **return** true

Fig. 9. Pseudocode describing the multiprocessor Russian dolls test.

Our test operates on a specific problem structure: For every absolute deadline constraint $D^{\text{abs}}_{\tau_i^j}$ or subtask-to-subtask deadline constraint $D^{s2s}_{\langle \tau_a^b, \tau_i^j \rangle}$, subtask τ_i^j must have only one predecessor subtask τ_x^y. This means that either $\tau_x^y = \tau_i^{j-1}$, or else there exists a self-suspension $W_{\langle \tau_x^y, \tau_i^j \rangle}$. In turn, each τ_x^y must have exactly one such predecessor subtask. This recurses until reaching either the epoch subtask τ_0^0, in the case of an absolute deadline constraint, or τ_a^b, in the case of a subtask-to-subtask deadline. This structure allows us to assume that a subtask constrained by a deadline will be released as soon as its predecessor subtask has executed and the wait constraint has expired. In Section VII-C, we describe the mechanism for reformulating a general task set into this structure.

Before sequencing the task, the algorithm tests the feasibility of the absolute deadlines by calling the Russian dolls test with $\tau_i^j = \tau_0^0$ (the epoch), the task set $\boldsymbol{\tau}$, time $t = 0$, and type $= 1$ as input. The test determines whether the set of subtasks constrained by each active deadline can feasibly be executed within the laxity of the other active deadlines. Recall that initially, when $t = 0$, only the absolute deadlines are active. The algorithm collects the set of active tasks (line 2) and iterates over all pairs of unique subtasks constrained by absolute deadlines (lines 14 and 15). If a pair of subtasks, τ_i^k and τ_x^y, is allocated to the same agent (i.e., processor) or requires the same resource, there is potential for contention between these subtasks. Line 17 checks whether the execution of one subtask τ_i^k, which must occur before d_i^k, can be nested within the laxity, $d_x^y - C_{\tau_i^k}$, of the deadline of subtask τ_x^y, or vice-versa. This nesting is possible if proposition $\left(d_i^k \leq d_x^y - C_x^y\right) \vee \left(d_i^k - C_i^k \geq d_x^y\right)$ holds; if no such nesting is possible (i.e., if $\left(d_i^k > d_x^y - C_x^y\right) \wedge \left(d_i^k - C_i^k < d_x^y\right)$), the system returns that it cannot successfully schedule against the set of absolute deadline constraints (line 17). If the system can perform this nesting for all such pairs of subtasks, it returns that it can successfully schedule τ_i^j at time t (line 22).

If the task sequencer calls the Russian dolls test to determine the feasibility of scheduling τ_i^j at time t (if type $= 2$), it must first consider the direct effect of executing τ_i^j. The algorithm iterates over all active subtasks (Definition 4) not including τ_i^j (line 4). If another active subtask τ_x^y is assigned to the same agent or requires the same resource as τ_i^j (line 5), the test determines whether it is possible to nest the execution of τ_i^j within the laxity of τ_x^y's deadline (line 6). If such nesting is impossible, the algorithm returns *false*, prohibiting the scheduling of τ_i^j at time t (line 13). Otherwise, if scheduling τ_i^j at time t would activate a subtask-to-subtask deadline $D^{s2s}_{\langle \tau_i^j, \tau_i^z \rangle}$ (Definition 3), the algorithm must consider the indirect effects of activation on the other subtasks influenced by this deadline constraint. If such activation would occur, the test stores the set of those subtasks explicitly constrained by $D^{s2s}_{\langle \tau_i^j, \tau_i^z \rangle}$, excluding τ_i^j (line 10). (Note that the algorithm has already tested the feasibility of τ_i^j.)

Next, the algorithm iterates over the subtasks in this group and over the set of active subtasks (lines 14-15). If an active subtask τ_x^y and a subtask τ_i^k constrained by $D^{s2s}_{\langle \tau_i^j, \tau_i^z \rangle}$ are assigned to the same agent or require the same resource (line 16), the test determines whether it is possible to nest the execution of τ_i^k within the laxity of τ_x^y's deadline, or vice-versa (line 17). The implicit deadline d_i^k for $\tau_i^k \in \boldsymbol{\tau}_{\mathbf{group}}$ is calculated assuming τ_i^j is executed at time t for the purposes of the test. If nesting is not feasible, the algorithm returns *false*, indicating there is no guarantee that a feasible schedule exists if τ_i^j is executed at time t (line 17). If nesting can be performed for all such pairs of subtasks, the algorithm returns *true*, indicating that τ_i^j can be scheduled at time t (line 22).

B. Proof of Correctness of Multiprocessor Russian Dolls Test

The multiprocessor Russian dolls test is polynomial in time complexity as it only performs pairwise comparisons between active subtasks and the tasks in τ_{group}, and is therefore not a complete schedulability test. However, we prove in this section that the algorithm is nonetheless correct, in that only pairwise comparisons of subtasks are necessary to ensure schedule feasibility.

The test leverages the problem structure inherent in the augmented self-suspending task model in order to perform efficient computation. Recall that the task model requires every subtask constrained by an absolute $D^{\text{abs}}_{\tau_i^j}$ or subtask-to-subtask $D^{s2s}_{\langle \tau_i^j, \tau_i^k \rangle}$ deadline to have exactly one predecessor. For absolute deadline $D^{\text{abs}}_{\tau_i^j}$, the precedence recurses from τ_i^j to τ_i^{j-1} and so on through τ_i^1. Likewise, for subtask-to-subtask deadline $D^{s2s}_{\langle \tau_i^j, \tau_i^k \rangle}$, the precedence recurses from τ_i^k to τ_i^{k-1} through τ_i^j. This problem structure restricts the interaction of deadlines in the task set.

Lemma VIII.1: Scheduling τ_i^j does not change the implicit deadlines or the laxity of any subtask τ_x^y unless $x = i$ and a subtask-to-subtask deadline $D^{s2s}_{\langle \tau_i^j, \tau_i^z \rangle}$ with $i \leq y \leq z$ exists.

Proof: Consider a set of intratask deadlines for τ_i. If the problem is correctly structured as an augmented self-suspending task model, then each embedded subtask in the set $\boldsymbol{\tau}_{\mathbf{group}} = \{\tau_i^{j+1}, \ldots, \tau_i^y, \tau_i^z\}$ has one predecessor that is either a member of $\boldsymbol{\tau}_{\mathbf{group}}$ or is τ_i^j. Thus, when τ_i^j is scheduled, implicit deadlines δ_x^y for subtasks $\tau_x^y \notin \{\tau_i^j, \tau_i^{j+1}, \ldots, \tau_i^y, \tau_i^z\}$ are unchanged.

Next, we use Lemma VIII.1 to demonstrate the correctness of the Russian dolls test.

Theorem VIII.2: The Russian dolls test is correct in that it requires only pairwise comparisons between active subtasks $\boldsymbol{\tau}_{\mathbf{active}}$ and subtasks in $\boldsymbol{\tau}_{\mathbf{group}}$.

Proof: When τ_i^j is scheduled, the implicit deadlines for active subtasks may be tightened. We show that this tightening will not result in temporal infeasibility. As in Lemma VIII.1, scheduling τ_i^j does not change the implicit deadlines or slack times of any subtask τ_x^y unless $x = i$ and a subtask-to-subtask deadline $D^{s2s}_{\langle \tau_i^j, \tau_i^z \rangle}$ with $i \leq y \leq z$ exists. In the case where $x \neq i$ or there is no $D^{s2s}_{\langle \tau_i^j, \tau_i^z \rangle}$, if it is feasible to nest an active subtask τ_x^y within the laxity of every other active subtask τ_p^q at time t_o, this schedule commitment will reduce the slack available for future scheduling commitments but will not tighten the deadlines of other active subtasks. As such, a pairwise comparison between subtasks in $\boldsymbol{\tau}_{\mathbf{group}}$ and subtasks in $\boldsymbol{\tau}_{\mathbf{active}}$ is sufficient to produce of a feasible schedule.

According to Theorem VIII.2, it is sufficient to consider all pairwise comparisons between subtasks in $\boldsymbol{\tau}_{\mathbf{active}}$ and $\boldsymbol{\tau}_{\mathbf{group}}$ to ensure correctness of the Russian dolls test. However, these comparisons are not necessary for correctness. While the computational complexity of the test is polynomial in time, as discussed in Section X, we seek to reduce the computational complexity further by leveraging the fact that these subtasks do not all utilize the same agents and resources.

Corollary VIII.3: The Russian dolls test is correct in that it requires only pairwise comparisons for each $\langle \tau_x^y, \tau_i^j \rangle$ where $\tau_x^y \in \boldsymbol{\tau}_{\mathbf{active}}$ and $\tau_i^j \in \boldsymbol{\tau}_{\mathbf{group}}$ (as in Theorem VIII.2), such that $A^a_{\tau_i^j} = A^a_{\tau_x^j}$ and $\langle \tau_x^y, \tau_i^j \rangle \in \boldsymbol{\tau}_{\boldsymbol{R}}$.

Proof: Scheduling any subtask in $\tau_i^j \in \boldsymbol{\tau}_{\mathbf{group}}$ can only result in infeasibility if the required agent or resource is not available to execute τ_i^j before d_i^j because the agent or resource is occupied by some $\tau_x^y \in \boldsymbol{\tau}_{\mathbf{active}}$. Thus, to ensure feasibility in the Russian dolls test, it is only necessary to consider pairwise comparisons between subtasks in $\boldsymbol{\tau}_{\mathbf{group}}$ and $\boldsymbol{\tau}_{\mathbf{active}}$ that share the same agent or resource.

C. Russian Dolls Test Complexity Analysis

Here, we analyze the computational complexity of the Russian dolls test. The test can be called under one of two conditions, type 1 or type 2, depending on how the test is being used. In either case, the complexity is the same. For type 1, at most $|\boldsymbol{\tau}|$ are added to $\boldsymbol{\tau}_{\mathbf{group}}$. Then, in lines 14–21, every member of $\boldsymbol{\tau}_{\mathbf{group}}$ is compared to each member of $\boldsymbol{\tau}_{\mathbf{active}}$ in order to determine whether each $\tau_i^k \in \boldsymbol{\tau}_{\mathbf{group}}$ can nest within the slack of $\tau_x^y \in \boldsymbol{\tau}_{\mathbf{active}}$, or vice-versa (line 17). As such, the complexity of the Russian dolls type 1 test is $O\left(|\boldsymbol{\tau}|^2\right)$. For type 2, the algorithm first checks the feasibility of scheduling τ_i^j at time t in lines 4–9. Here, the test considers pairings of τ_i^j to each subtask in $\boldsymbol{\tau}_{\mathbf{active}}$, resulting in a maximum of $|\boldsymbol{\tau}| - 1$ operations. Next, lines 14–21 consider pairings of each τ_i^k in $\boldsymbol{\tau}_{\mathbf{group}}$ to each subtask in $\boldsymbol{\tau}_{\mathbf{active}}$. There are at most $\boldsymbol{\tau} - 1$ in both $\boldsymbol{\tau}_{\mathbf{group}}$ and $\boldsymbol{\tau}_{\mathbf{active}}$, resulting in $|\boldsymbol{\tau}|(|\boldsymbol{\tau}| - 1)$ evaluations. Overall, the type 2 Russian dolls test is of complexity $O\left(|\boldsymbol{\tau}|(|\boldsymbol{\tau}| - 1) + (|\boldsymbol{\tau}| - 1)\right) = O\left(|\boldsymbol{\tau}|^2\right)$. Thus, the complexity of the Russian dolls test under type 1 or 2 is given by $O\left(|\boldsymbol{\tau}|^2\right)$.

IX. Complexity of Tercio

As presented in Section V, the computational complexity of Tercio's agent allocation subroutine is given by $O\left(2^{|A||\boldsymbol{\tau}|}\right)$. Here, we derive the complexity of Tercio's sequencing subroutine (see Fig. 5). Tercio's sequencing subroutine, *TERCIO-SEQUENCER*, begins in line 1 by calling the reformulation algorithm, *SEQUENCER-REFORMULATE* (see Fig. 6), to transform general task sets into the structured model required for the schedulability test. The complexity of this subroutine is given by $O\left(|\boldsymbol{\tau}|^2 + |\boldsymbol{\tau}|^2\left(|\boldsymbol{\tau}|^2 \log|\boldsymbol{\tau}| + |\boldsymbol{\tau}||TC|\right)\right)$. After reformulation, the sequencing subroutine uses the Russian dolls test (Fig. 9, type 1 test) to ensure the feasibility of $\boldsymbol{D}^{\text{abs}}$, which has a complexity of $O(|\boldsymbol{\tau}|)$ (see Section VIII-C).

Next, the sequencer iterates through time to sequence all tasks (line 1, Fig. 5). The sequencing algorithm iterates $f_{\max} = \max_{\tau_i^j} f_i^j$ time steps at most, assuming a unit time step, and $\frac{f_{\max}}{dt}$ otherwise. At each iteration, a maximum of n subtasks (lines 5 and 6) is available for scheduling with satisfied precedence constraints. The priority queue of available subtasks is sorted with $O(n\log(n))$. Line 7 of the sequencer (see Fig. 5) then evaluates the feasibility of scheduling each subtask using the Russian dolls type 2 test, the complexity of which is given by $O(|\boldsymbol{\tau}|)$ (see Section VIII-C). Finally, the sequencer performs an APSP computation (line 8, Fig. 5) for each subtask τ_i^j with a deadline constraint $D^{s2s}_{\langle \tau_i^j, \tau_x^y \rangle}$. This involves $O\left(|\boldsymbol{\tau}|^2 \log|\boldsymbol{\tau}| + |\boldsymbol{\tau}||\text{TC}|\right)$ operations. Thus, the sequencer's complexity is of $O(\text{Sequencer}) = O\left(|\boldsymbol{\tau}|^2 + |\tau^2| + (|\boldsymbol{\tau}|^2 + |\boldsymbol{\tau}|^2 n f_{\max} n\log(n)\right)(|\boldsymbol{\tau}|^2 \log|\boldsymbol{\tau}| + |\boldsymbol{\tau}||\text{TC}|)) = O((f_{\max} n^2 \log(n))(|\boldsymbol{\tau}|^4 \log|\boldsymbol{\tau}| + |\boldsymbol{\tau}|^3 |\text{TC}|))$.

Given the aforementioned, Tercio—including task allocation, the sequencer, and all sequencing subroutines—is of complexity $O(2^{|\boldsymbol{A}||\boldsymbol{\tau}|}(f_{\max} n^2 \log(n))(|\boldsymbol{\tau}|^4 \log|\boldsymbol{\tau}| + |\boldsymbol{\tau}|^3 |\text{TC}|))$.

X. Evaluation and Discussion

In this section, we empirically validate that Tercio quickly produces solutions within 10% of optimal for multi-agent scheduling problems with temporospatial constraints. Results were generated using an off-the-shelf laptop with an Intel Core i7-2820QM CPU 3.20 GHz (8 Cores) and 16 GB of RAM.

Tercio is composed of a task reformulator (see Section VII-C, Fig. 6), task allocator (see Section V), and task (see Section VI, Fig. 5). The reformulator is implemented in Java, the task allocation algorithm is solved using a Java interface to a third-party optimizer (Gurobi), and the task sequencer is in MATLAB.

Tercio is an iterative algorithm; for this evaluation, we limited the number of iterations to 25. We found that solutions returned by Tercio did not improve significantly with additional iterations for the problems we were able to benchmark against the optimal solution. For agent allocation, we set $\alpha_1 = 2$ and $\alpha_2 = 1$, as this generally achieves helpful allocations for the sequencer, and terminated Gurobi once the incumbent solution was found to be within 0.1% of optimal. We compared our approach against three benchmarks: 1) TeSSI [58], 2) object-coding genetic algorithm (OCGA) [88], and 3) an exact, MILP-based solution method. For all algorithms, a 60-minute timeout was applied.

We compared our approach to a technique based on TeSSI and its variants [54], [58]. At its core, TeSSI is an insertion heuristic. As discussed in [29], insertion heuristics function by deciding which subtask should next be inserted, where, and to which agent based on some prescribed criteria. TeSSI's criterion is the makespan of the agent to which the subtask is assigned. TeSSI operates on a more-restricted problem structure that involves neither upper and lower bound temporal constraints nor resource constraints. Thus, to compute each agent's makespan, TeSSI simply adds the duration of each task and the time spent traveling between tasks, which is linear in the number of subtasks assigned to the agent. In contrast, Tercio considers problems with upper and lower bound temporal constraints and resource constraints. In order to correctly schedule against upper bound temporal constraints, it is necessary to employ a temporal consistency check (e.g., Tercio's Russian dolls test) as commitments are made. A directed path consistency algorithm [19] may also serve this purpose as an alternative to the Russian dolls test. For our comparison, we used Snowball [63], a state-of-the-art algorithm that achieves complexity of $O(n^2)$ for certain cases, such as graphs of constant tree width. We refer to this APSP-variant of TeSSI as TeSSI*.

OCGA is a state-of-the-art GA developed for job shop scheduling problems [88]. We adopted the parameter prescribed by [88] for OCGA as follows: population size $N = 100$, pairs of parents replicating per iteration $r = 7$, probability for crossover $p_c = 1$, probability for shifting genes' loci $p_e = 1$, probability for re-allocating agent for a given subtask $p_m = 0.06$, degeneration ratio $R = 0.09$, and number of iterations $N = 2,500$. As with TeSSI*, it was necessary to add a directed path consistency algorithm to evaluate the quality of each schedule; we also used Snowball for this purpose [63]. In order to improve runtime, we used a hashing function, which stored the quality of previously computed schedules to reduce the number of calls to Snowball with successive iterations of OCGA. GAs such as OCGA are generally probabilistically guaranteed to find the optimal solution as the number of iterations approaches infinity. Thus, given enough time, OCGA will identify a solution better than that generated by Tercio. We find that a more-helpful benchmark than comparing solution quality between OCGA and Tercio is to measure how much time is required for OCGA to find a solution comparable to or better than that identified by Tercio. Thus, in our results, we report the computation time required for OCGA to find such a solution. For instances in which Tercio cannot identify a solution, we report the total computation time required for all 2500 iterations, as prescribed by [88].

The exact, MILP-based formulation, corresponding to solving the MILP defined in Section III and objective function in (13), was solved by calling Gurobi using its default settings, which sets the optimality threshold to 0.1%.

A. Generating Random Problems

We evaluated the performance of Tercio when applied to randomly generated problems. The lower bound agent-task times were of the form $lb^a_{\tau_i^j}$ and were drawn from a uniform distribution over the interval $[1, 10]$. Expected agent-task times $C^a_{\tau_i^j}$ were generated from $C^a_{\tau_i^j} \sim U[lb^a_{\tau_i^j}, 10]$. We did not explicitly constrain the maximum amount of time an agent spent performing a subtask. Approximately 25% of the generated precedence constraints $W_{\langle \tau_x^{y-1}, \tau_x^y \rangle}$ were wait constraints with nonzero lower bounds drawn from the interval $[1, 10]$. Approximately 25% of subtasks had a wait constraint $W_{\langle \tau_i^j, \tau_x^y \rangle}$, in addition to their self-suspension constraint $W_{\langle \tau_x^{y-1}, \tau_x^y \rangle}$.

In order to generate subtask-to-subtask deadlines, we randomly sampled two subtasks τ_i^j and τ_x^y, such that $y \geq j$, and added a deadline constraint $D^{s2s}_{\langle \tau_i^j, \tau_x^y \rangle}$. To add absolute deadlines, we randomly sampled a subtask τ_i^j and added a deadline constraint $D^{\text{abs}}_{\tau_i^j}$. We sought to generate problems of sufficient challenge for our validation. We established a metric $\hat{D} = \sum_{D^{s2s}_{\langle \tau_i^j, \tau_x^y \rangle}} (y - j + 1) + \sum_{D^{\text{abs}}_{\tau_i^j}} j$ and set it to $\frac{1}{4}$ for our empirical evaluation depicted by Fig. 10(a)–(l). The upper bound of each deadline constraint was drawn from a uniform distribution, with the lower bound set to the tightest feasible deadline and the upper bound set to the sum over subtask costs and wait constraint times. The one-dimensional physical locations of subtasks were drawn from a uniform distribution $[1, |\boldsymbol{\tau}|]$. The number of subtasks m_i within each task τ_i was drawn from a uniform distribution in the interval $[1, 2n]$, where n is the number of tasks τ_i in $\boldsymbol{\tau}$. While an agent performed subtask τ_i^j at resource location (x), no other agent could work on a subtask at location (u) if $x - 1 \leq u \leq x + 1$.

B. Computation Speed and Scalability

Fig. 10(a), (e), and (i) depict our evaluation of Tercio's scalability and computational speed. We present the median and upper/lower quartiles of the computation time for 50 randomly generated problems incorporating 5, 10, or 100 agents [see Fig. 10(a), (e), and (i), respectively] and between 4 and 1024 subtasks. Where possible, we provide the computation time for the MILP-based solution method, the insertion algorithm, and the time required for OCGA to find a solution as good as that identified by Tercio. Note that the median is typically reported for such optimization problems because the distributions are often skewed and the mean is a less informative measure [79].

Tercio can solve problems involving up to 100 agents and 1000 subtasks in $\sim$ 120 s, a substantial improvement over our benchmarks. We note that TeSSI* and OCGA compute more

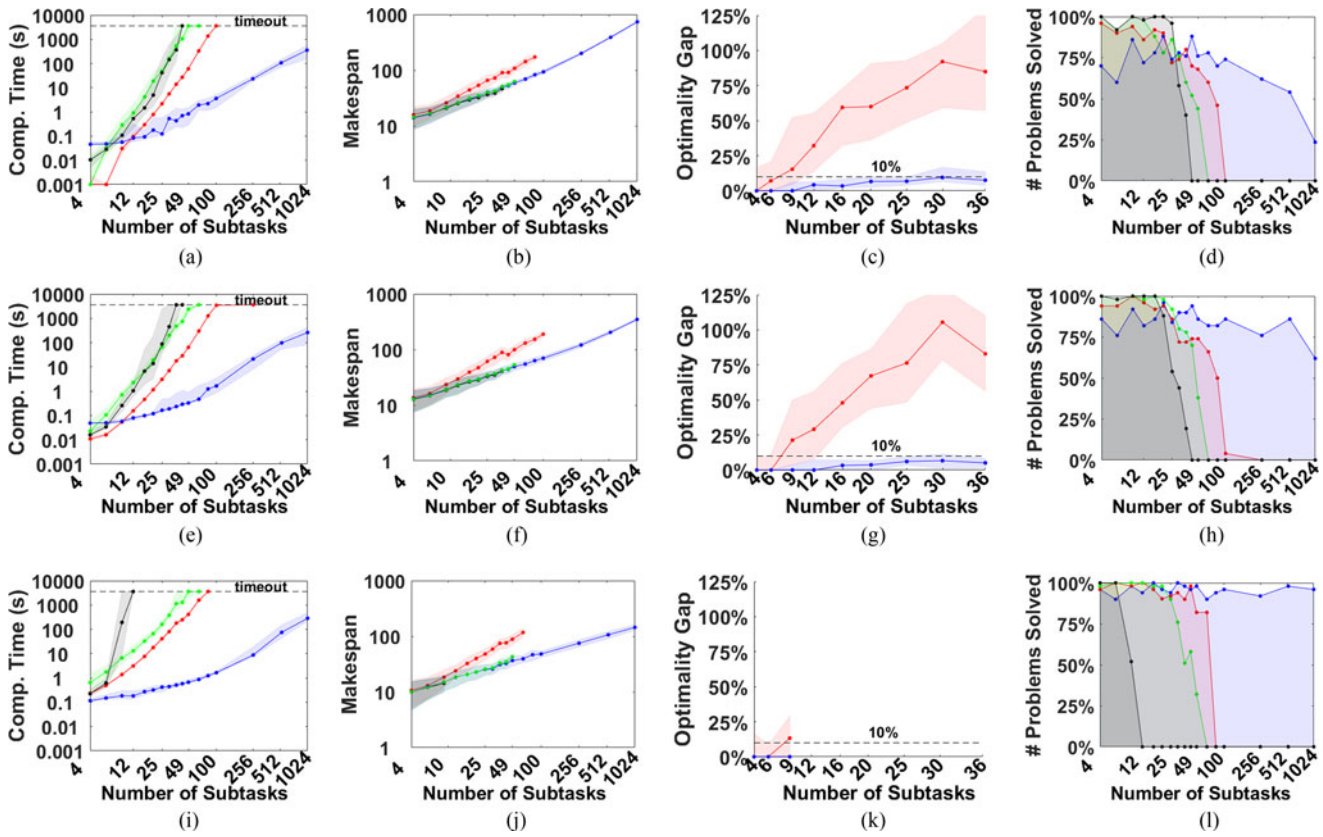

Fig. 10. This figure depicts the performance of Tercio relative to TeSSI*, OCGA, and the exact solution (where possible). (a)–(d), (e)–(h), and (i)–(l) present results for problems involving 5, 10, and 100 agents, respectively. When reporting computation time or solution quality (i.e., the makespan), the graphs depict the median and quartiles. Tercio is represented in blue, TeSSI* in red, OCGA in green, and the MILP-based solution technique in black. Note that OCGA is not depicted when reporting the optimality gap, as we terminated OCGA once it found a solution as good as that of Tercio.

slowly than previously published variants because of the need to include a temporospatial feasibility test for each assignment according to the upper and lower bound temporospatial constraints, requiring $O(n^2)$ step in the innermost loop of the algorithm. This feasibility test greatly increases computation time despite using the fastest technique available [63]. Our findings underscore the benefit to computation time provided by our schedulability test.

C. Optimality

We empirically validate that Tercio produces solutions within 10% of optimal. Fig. 10(c), (g), and (k) depict the median and upper/lower quartiles of the makespans produced by Tercio, along with the insertion algorithm for 50 randomly generated problems involving 5 and 10 agents and problem sizes spanning four to 42 tasks. The median deviation from optimal for Tercio was less than 10% for all testable problem sizes. However, the median deviation from optimal for the insertion algorithm increased up to 100% for larger problems.

D. Evaluating Completeness

Fig. 10(d), (h), and (l) depict the proportion of problems solved by Tercio with 5, 10, and 100 agents, respectively. While TeSSI* and OCGA were able to solve slightly more smaller problems than Tercio, Tercio's ability overtook that of both TeSSI* and OCGA as problem size increased. Furthermore, Tercio's completeness is proportional to the number of agents; conversely, OCGA and TeSSI* appear less able to identify satisfactory solutions as the number of agents increases.

E. Robustness

To test the robustness of our approach, we considered problems that are more- or less-constrained in Fig. 11(a)–(d), (e)–(h), and (i)–(l). We set $\hat{D} = \{\frac{0}{4}, \frac{1}{4}, \frac{2}{4}\}$. Furthermore, we varied the proportion of subtasks with wait constraints $W_{\langle \tau_i^j, \tau_x^y \rangle}$ such that $\tau_x^y \neq \tau_i^{j+1}$ in the set $\{\frac{0}{4}, \frac{1}{4}, \frac{2}{4}\}$. We found that, relative to OCGA and TeSSI*, Tercio's performance remained strong across a range of constraint settings. Tercio's completeness did degrade for the most-constrained problems [see Fig. 11(l)], but its scalability and solution quality remained strong relative to our benchmarks.

F. Robot Demonstration

Here, we demonstrate the use of Tercio in two hypothetical manufacturing scenarios. In both cases, a team (i.e., set) of robots worked to complete tasks on a simulated fuselage. The robots performed their tasks at specific locations on the factory floor, where there can be multiple subtasks at each location. In order to prevent collisions, each robot reserved the physical location for its subtasks, along with any immediately adjacent

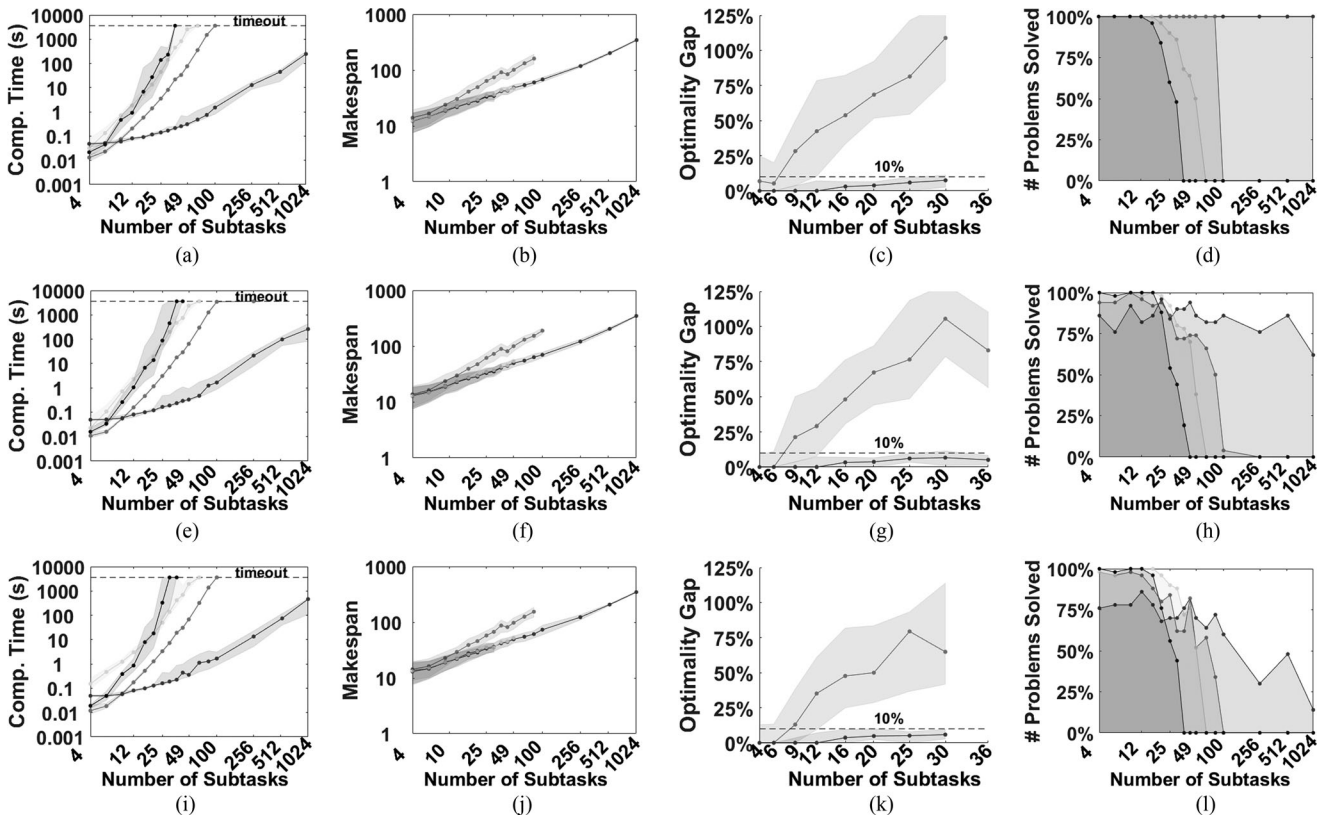

Fig. 11. This figure depicts the performance of Tercio relative to the exact solution, TeSSI*, and OCGA. (a)–(d), (e)–(h), and (i)–(l) present results for problems that involve ten agents and are lightly, moderately, and severely constrained, respectively. Tercio is depicted in blue, TeSSI* in red, OCGA in green, and the MILP-based solution technique in black.

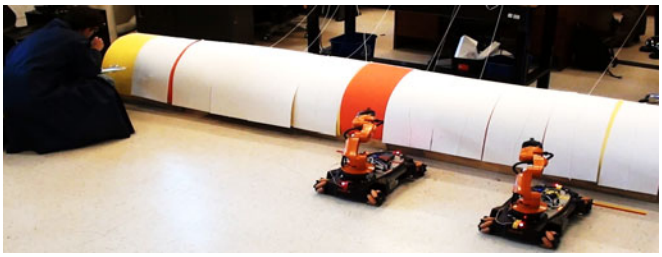

Fig. 12. Robotic team completes mock aerospace final-assembly tasks while maintaining a safe distance from human workers.

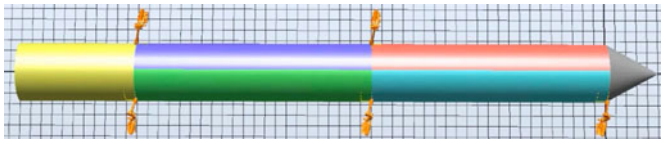

Fig. 13. Robotic team adapts performance of work related to a virtual fuselage in response to multiple disruptions.

task locations; thus, no two workers could be present at the same location (or in neighboring locations) at the same time. For simplicity, only absolute deadlines D^{abs} were considered, although other constraints could be easily incorporated.

In the first evaluation, two KUKA Youbots simulated completion of drilling tasks on an aerospace fuselage, as shown in Fig. 12 (video available at http://tiny.cc/t6wjxw). Initially, the robots planned to evenly split 12 identical tasks down the middle of the fuselage. After the robots completed their first subtasks, a worker then requested time to inspect the completed work along the left half of the fuselage; in the problem formulation, this corresponds to adding a resource reservation for the left half for a specified period of time. Tercio replanned in response to the addition of this new constraint and reallocated work in a reasonable manner to make productive use of both robots. For the purposes of this demonstration, the replanning process involved calling Tercio while forcing a fixed assignment and ordering of all subtasks completed prior to the replan request. More-efficient methods could also be considered, such as pruning completed subtasks and reformulating appropriate temporal constraints.

Second, we demonstrated Tercio on a larger simulated problem, as shown in Fig. 13 (video available at http://tiny.cc/jladjy). In this demonstration, which incorporated ABB's Robot Studio simulation environment, Tercio coordinated 5 robots to perform 110 identical tasks around an aerospace fuselage. Tercio was applied to replan in response to three disturbances: 1) a human worker's request to enter the space to perform a quality-assurance (QA) inspection, 2) a robot breakdown, and 3) changing task deadlines. The duration of the QA request and the robot breakdown were known at the time of the disturbance. Tercio modeled the QA request as a resource reservation for the section of the fuselage to be inspected (i.e., no robots could occupy that

space for a given duration). Tercio modeled the robot breakdown as an agent occupied by a repair subtask for a given duration.

These demonstrations indicate the applicability of the Tercio algorithm for solving real-world problems with application-specific features and complex objective functions. For example, localization errors among mobile robots must be accounted for during schedule generation. Intrarobot errors accumulate, which can result in gaps or overlaps in work. In order to reduce the number of opportunities for such errors to occur, we added a term to reduce the number of times when two different agents were used to perform adjacent work, as defined by $g_4(\boldsymbol{A}, \boldsymbol{\tau}) \geq \sum_{a \in \boldsymbol{A}} \sum_{\langle \tau_i^j, \tau_x^y \rangle \in \boldsymbol{\tau_R}} A^a_{\tau_i^j} \times (1 - A^a_{\tau_x^y})$. Also, schedule changes in response to disturbances must be interpretable by human workers. Small disturbances can potentially result in substantial changes to robot task assignments, and human workers may find it challenging to understand and anticipate the actions of the robots. In many cases, small schedule changes may be an acceptable tradeoff for a marginally suboptimal solution. As such, we included an additional term to minimize the number of assignment changes from one schedule's allocation $P^a_{\tau_i^j}$ to the next $A^a_{\tau_i^j}$, as defined by $g_5(\boldsymbol{A}, \boldsymbol{P}, \boldsymbol{\tau}) \geq \sum_{a \in \boldsymbol{A}} \sum_{\tau_i^j \in \boldsymbol{\tau}} A^a_{\tau_i^j} \times (1 - P^a_{\tau_i^j})$.

While practitioners might attempt to apply other scheduling technology (e.g., one of the benchmarks we employed), we believe Tercio provides a singular ability to rapidly and near-optimally reschedule robots in dynamic environment.

XI. Conclusion

We developed Tercio, a scheduling algorithm made efficient through a fast, multi-agent task sequencer inspired by real-time processor scheduling techniques. We used the task sequencer in conjunction with an MILP solver to compute an integrated multi-agent task sequence that satisfies precedence and temporal and spatial-proximity constraints. Although Tercio is incomplete, we empirically show that the algorithm produces schedules within 10% of the optimal for real-world, structured problems. We also show that Tercio is able to solve more problems and achieve better-quality solutions for such problems than the prior state-of-the-art technique. Finally, we provide physical and virtual demonstrations of Tercio coordinating the activities of a robotic team within a human work environment.

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