基于非稳态热传导模型的炉内元件温度拟合与分析

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摘要

本文解决了题目 A ——炉温曲线的问题一、问题二与问题三,并对问题四做了分析。基于傅里叶定律、牛顿冷却定律与能量守恒定律,建立非稳态热传导模型。本文考虑了温区之间的热导影响,解出了热传导偏微分方程的数值解,并较好地拟合了真实实验数据。在新的预设温度下,以固定步长遍历传送带的速度区间,在制程界限的限制下逐个检验每一个解,进行至多三次迭代,得到传送带运行速度的最大可行解。再将预设温度与传送带速度设为新的参数,利用微分法计算出目标函数的数值解,在约束条件下尝试参数的可能解,从而找到参数的最优解。

关键词

非稳态热传导模型,牛顿冷却定律,傅里叶定律,最小二乘法拟合,目标函数优化,迭代优化算法

1 问题重述

将安装电子元件的印刷电路板放置在回焊炉中,通过加热可与将电子元件自动焊接到电子板上。

焊炉内有 11 个小温区及炉前区域和炉后区域。小温区长 30.5 cm,相邻有 5 cm 间隙,炉前区域、炉后区域长 25 cm。回焊炉启动后,炉内空气温度视为瞬间达到稳定。炉前区域、炉后区域和小温区间隙温度与相邻温区的温度有关。生产车间温度保持在 25℃。设定温度分别为:小温区 1-5,175℃;小温区 6,195℃;小温区 7,235℃;小温区 8-9,255℃;小温区 10-11,25℃。传送带过炉速度为 70 cm/min;焊接区域厚度为 0.15 mm。

称焊接区域中心温度曲线为炉温曲线。焊接区域中心温度达到 30℃ 时开始,电路板进入回焊炉开始 计时。

可调节各温区的温度和传送带的过炉速度。在原始设定温度基础上,各小温区温度可进行 ±10℃ 范围内的调整。调整时要求小温区 1-5、小温区 8-9、小温区 10-11 中的温度保持 25℃。传送带的过炉速度调节范围为 65-100 cm/min。此外,炉温曲线应满足制程界限。

需解答下列问题:

- (1) 对焊接区域的温度变化规律建立数学模型。假设传送带过炉速度为 78 cm/min,各温区温度的设定值分别为:小温区 1-5,173℃;小温区 6,198℃;小温区 7,230℃;小温区 8-9,257℃。给出焊接区域中心的温度变化情况,列出小温区 3、6、7 中点及小温区 8 结束处焊接区域中心的温度,画出相应的炉温曲线,并记录每隔 0.5 s 焊接区域中心的温度。
- (2) 假设各温区温度的设定值分别为: 小温区 1-5: 182℃、小温区 6: 203℃、小温区 7: 237℃、小温区 8-9: 254℃,确定允许最大传送带过炉速度。
- (3) 确定在"使超过 217℃ 到峰值温度所覆盖的面积最小"要求下的最优炉温曲线,以及各温区的设定温度和传送带的过炉速度,并给出相应面积。
- (4) 结合问题 3, 进一步给出"以峰值温度为中心线的两侧超过 217℃ 的炉温曲线应尽量对称"最优炉温曲线,以及各温区设定的温度及传送带过炉速度,并给出相应指标值。

2 模型准备

2.1 背景知识

2.1.1 传热方式

热量在传递过程中主要有以下三种形式:

- **1. 热传导** 热传导指能量在物体内部由高温区到低温区的传递或在高温物体到与之接触的低温物体的传递。其限制发生传热的物质间没有宏观位移。由傅立叶定律描述该现象。
- 2. **热对流** 热对流指能量在流体的各部分间发生相对位移、冷热流体互相掺杂情况下的传递。其限制只发生在流体中。由牛顿冷却定律描述该现象。
- 3. 热辐射 热辐射指温度差产生的电磁波空间传递。由斯蒂芬-玻尔兹曼定理描述该物理现象。

2.1.2 傅里叶定律

傅里叶定律以微分方程的形式描述了热量在介质之间传导的规律。公式如下:

$$q = -\lambda \frac{\partial T}{\partial x}$$

其中, q 为热流密度, λ 为导热系数, 温度 T 是关于位置 x 的函数。我们使用其微分形式, 因为更加 关注其在局部的能量传导率,即元件进入加热器的前接触面和后接触面的热量传递过程。

2.1.3 牛顿冷却定律

牛顿冷却定律描述了高于周围环境温度的物体在冷却过程中向周围媒质传递热量的规律。公式如下:

$$\triangle t = \left| t_w - t_f \right|$$

$$q = h \triangle t$$

$$\Phi = qA = Ah \triangle t = \frac{\triangle t}{1/hA}$$

其中 q 为热流密度,h 为对流传热系数,和物体本身的性质和物体厚度有关。 ΔT 为物体和环境温度差。 Φ 为传热功率,即单位时间内的传热量,A 为接触面积。通过观察可以发现物体和周围环境的热交换速率和温度密切相关,且在累计时间上呈指数递减的趋势。

2.1.4 微分法

在一个二维坐标平面上,求解一个固定区间内两曲线所夹的面积时,可以考虑使用微分法进行计算。以x轴区间为例,具体做法为:将原始区间划分为若干极小区间。对于每个小区间,用常规多边形计算方式近似求出该区间的覆盖面积。一种计算方法为用矩形近似表示区间面积:求出小区间左右两侧边界的长度(即两曲线y轴坐标绝对值),用两个长度的平均值表示矩形的长,用区间的长度(即边界x轴坐标绝对值)表示矩形的宽,用矩形面积计算公式获得该矩形面积,并用其近似表示该小区间覆盖的面积(如图 1)。随着划分的小区间个数增加,算法的精度会不断增长,计算的结果会不断逼近真实情况。当小区间个数足够多时(即极小区间的区间宽足够小),误差是极小的。

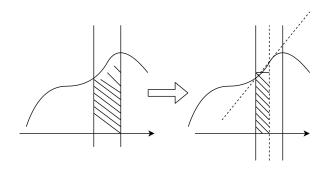


图 1: 微分法

2.2 模型抽象与假设

由于我们仅需要考虑物体在进入炉子之后的在运动方向上的运动变化,不需要考虑在垂直与运动方向上的热量扩散(物体的体积大小也允许我们忽略这部分的热量传导差异),同时由于元件高度较小,在竖直方向上也可以认为热量是均匀传导。因此,我们无需建立三维方向上的所有热量传导细节,只需考虑一维热传导的模型抽象。

除此之外,针对由于题目中提到各熔炉在启动后温度会快速达到稳定,所以也需要考虑熔炉之间的 热量传递:

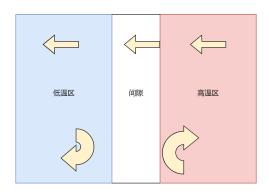


图 2: 温区之间传热的简易示意图

综上所述, 我们排除建模过程中无需考虑的元素, 将机理模型做适当简化, 做出如下假设:

- 假设 1: 间隙温度为相邻小温区之间的平均温度;
- 假设 2: 不考虑接触面之间的接触热阻, 并且认为接触面界面光滑连续;
- 假设 3: 简化问题为一维传热问题,不考虑其他不均匀热源和传热过程;

在物体运动方向上的一维简易模型示意图 3 所示。为了便于表示,这里展示的是二维的竖直截面的情况。此时,对水平方向上元件的空间移动,建立坐标系 $s-T_{fur}$,对元件内部,建立方向相反的坐标系(便于后续分析) $\alpha-T$ 。焊接中心厚度为 0.15mm,相对于估计的元件长度 $\alpha=3cm$ 很小,可以认为元件的温度即是电路板中点 $\alpha/2$ 位置的温度,而忽略元件内部的热量传递情况。

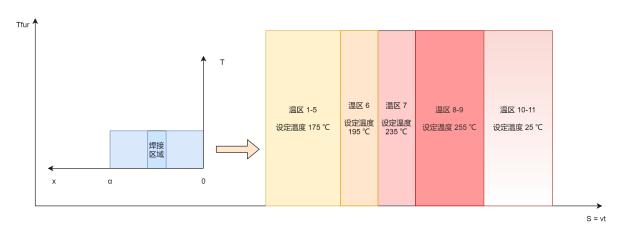


图 3: 焊接区域热传导的简易示意图

2.3 符号定义

本文构件的模型中出现的关键符号定义如表 1 所示。

符号	含义	值	单位
λ	电路板的导热系数	待拟合	$W/(m \cdot {}^{\circ}C)$
h_1, h_2	元件前、后侧与炉内空气间对流换热系数	待拟合	$W/(m^2 \cdot {}^{\circ} C)$
θ	炉间空气之间的换热系数	待拟合	$W/(m^2 \cdot {}^{\circ} C)$
${\cal H}$	热量控制方程	-	-
t	从炉前区域开始的运动时间	-	S
v	传送速度	65 - 100	cm/min
S	相对起始位置的运动距离	vt	S
\boldsymbol{x}	元件相对出发位置的横向坐标	-	ст
α	元件长度	3(估计)	cm
T_{env}	环境温度	25	°C
T(t,x)	电子元件的温度控制方程	-	°C
T_{fur_j}	第j个熔炉内部的空气温度	-	°C
T_{gap_j}	第j个熔炉间隙的空气温度	-	°C
L_{fur}	熔炉长度	30.5	ст
L_{gap}	熔炉间隙	5	ст
L_{edge}	炉前与炉后区域长度	25	ст
ho	电子元件密度	7.469(估计)	g/cm^3
С	电子元件比热容	480(估计)	J/g·° C
$\mathcal{F}(T_{fur_j},v)$	目标优化函数	-	-
$\mathcal{A}^{'}$	阴影区域面积	-	-

表 1: 符号定义

3 问题一

3.1 问题分析

3.1.1 模型构建-非稳态传热模型

如前文所述,本模型中主要存在的传热过程为热空气和电子元件间的热对流。传热的过程可以分为 稳态传热和非稳态传热。其中,稳态传热不受时间的影响,控制方程中不引入时间;而稳态传热受时间 的影响,控制方程中需要引入时间。在本模型中,由于焊接体的运动涉及到时间流动,在控制方程中需 要引入时间参数,所以我们建立非稳态传热模型。建立控制方程。焊接面在进入不同的炉中时,会和炉 内的高温气体相接触,形成对流传热。

传热控制方程 考虑到电子元件内部的温度随着时间的变化而变化,所以在本问题中,时间对温度的影响无法消去,应当建立一个非稳态的热传导模型。

再考虑电子元件前进时,热空气以垂直于电子原件表面的方向射入,与物体表面发生一个热对流,产生了热量交换。在电子元件的运动过程中,其他方向的热空气影响虽然存在但并不显著,所以暂时假设其影响几乎为零。因此我们可以把此问题抽象成一个一维的非稳态热传导模型。

由能量守恒定律,系统总能量的改变等于传入或者传出该系统的能量。在本模型中,考虑对电子元件这个小型系统进行分析,其热力学能的变化等于流入或者流出该系统的热流量的差值,因此得到一个偏微分控制方程:

$$\rho c \frac{\partial T}{\partial t} = \frac{\partial}{\partial x} (\lambda \frac{\partial T}{\partial x})$$

其中,等式左边表示电子元件热力学能的变化,等式右边表示电子元件流入或者流出的热流量的差值。对于具体的参数来讲, ρ 表示电子原件内部体积微元的密度,c 表示电子元件内部各个位置的比热容, λ 表示电子元件材料的导热系数,进一步整理方程得到温度控制方程 \mathcal{H} :

$$\frac{\partial T}{\partial t} = a^2 (\frac{\partial^2 T}{\partial x^2}), \ a^2 = \frac{\lambda}{\rho c}$$

边界条件以及初始条件 根据牛顿冷却定律,热量交换过程中热流量恒定,可以得到控制方程的边界条件:

$$\begin{cases} -\lambda \frac{\partial T}{\partial x} \bigg|_{x=0} = h \cdot (Tf(t) - T(0, t)) \\ -\lambda \frac{\partial T}{\partial x} \bigg|_{x=\alpha} = h \cdot (T(L, t) - Tf(t - \frac{\alpha}{v})) \end{cases}$$

其中,h 表示热对流系数, λ 表示热导系数, α 表示电子元件的长度,Tf(t) 表示炉内温度,T(0, t) 表示电子元件的头部位置,温度随时间变化的函数关系, $T(\alpha, t)$ 表示电子元件的尾部位置,温度随时间变化的函数关系。

根据已知的车间温度、用 Tenno 表示、于是控制方程的初始条件可以写成

$$T(x, 0) = T_{env} = 25^{\circ}C$$

于是总的初值条件与边界条件为:

$$\begin{cases} -\lambda \frac{\partial T}{\partial x} \Big|_{x=0} = h \cdot (Tf(t) - T(0, t)) \\ -\lambda \frac{\partial T}{\partial x} \Big|_{x=\alpha} = h \cdot (T(L, t) - Tf(t - \frac{\alpha}{v})) \\ T(x, 0) = T_{env} = 25^{\circ}C \end{cases}$$

参数估计 在本问题的非稳态传热模型中,电子元件两端的换热系数 h_1 , h_2 是未知量,同时电子元件内部的导热系数 λ 同样未知。题目中给定了特定温度下。利用已知的焊接区域温度随时间的变化情况,可以使用最小二乘法拟合出估计值 $\hat{h_1}$, $\hat{h_2}$ 。可以使用 MATLAB 中的 1sqcurvefit 来进行实现。

$$(\hat{h_1}, \hat{h_2}) = arg \min \sum_{i=1}^{N} [T(L, t_i; h_1, h_2) - T^*(t_i)]^2$$

对于其他参数,如元件的密度 ρ 、元件的比热容 c。由于题目中没有给出元件的材料信息,我们使用铁金属的数据作为代入。这里不可避免会和实际情况出现误差,但由于在偏微分方程的控制方程中,变量 $a=\frac{\lambda}{\rho c}$ 含有将会拟合的未知量 λ ,因此误差将会被拟合最小化,尽可能满足附件 1 中提供的实验数据,从拟合到的数据中填补模型的未知参数。

温区温度的变化 电子元件从低温区域进入高温区域,势必会造成温区的温度变化。温区的温度虽然随着时间的推移在基础温度上轻微地波动,但是当电子元件进入时,此温度会发生变化。在本模型中,环境温度认为恒定为 25 摄氏度,不受元件波动和炉的热辐射的影响。但在炉前具有的长为 25 cm 的区域部分,温度仍然会受到第一个炉的高温的影响,导致温度升高。同时,降温区和炉后区域的空缺也由于和相邻温区的温差过大,容易产生较强的热对流。根据牛顿冷却定律 $q = h \triangle T$,我们可以得知每个炉的温度都和其相邻的炉内的空气温度有关。接下来考虑该影响。

为了简化计算,我们首先假设在炉与炉之间的空隙的 5cm 部分的温度为相邻两个炉的设定温度的算术平均值。

$$T_{gapj}(t) = \frac{1}{2}(T_{furj-1} + T_{furj})$$

其次,令 $T_{furj}(t)$ 表示在 t 时间第 j 个炉的温度。从能量守恒定律我们知道每个炉从相邻炉获得的热流量将会转换成自己的热能,以温度的形式表现。同时,由于炉一直处在有外部能源加热状态,我我们可以认为所有炉的总能量在增加,但仍然存在流动。我们将牛顿冷却定律中的系数直接和温度正相关(这是因为在热流量和温度的转换中,温度和热量同样是线性的关系),将相关系数整合为炉间空气的换热系数 θ ,且有以下常微分方程:

$$\frac{\mathrm{d}T_{furj}(t)}{\mathrm{d}t} = T_{furj}(t) + \theta(T_{furj}(t) - T_{furj-1}(t)) + \theta(T_{furj}(t) - T_{furj+1}(t))$$

使用四阶龙格-库塔法(Runge-Kutta Method)求解该常微分方程。在 t 和 t+1 之间选取四个点求平均斜率。在微分方程 $\frac{\mathrm{d}T_{furj}(t)}{\mathrm{d}t}$ 中,令:

$$k_1 = T_{furj}(t)$$
 $k_2 = T_{furj}(t + 0.5 dt k_1)$
 $k_3 = T_{furj}(t + 0.5 dt k_2)$
 $k_4 = T_{furj}(t + dk_3)$
则有 $T_{furj}(t + 1) = T_{furj}(t) + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$.

在计算中取微分步长 dt = 0.5,对温度流动过程离散化,得到常微分方程的数值解。该过程使用 Matlab 可视化如图 4 所示:

3.1.2 参数拟合与模型检验

将炉内温度设定为题目所要求的初始问题,通过若干次迭代,得到未知参数的拟合结果如表2所示。

参数	猜测值	最小二乘法拟合值		
λ	$\lambda_{guess} = 50$	99.7946		
$h_1 = h_2$	$h_{guess} = 1000$	1813.6		
θ	$\theta_{guess} = 10$			

表 2: 参数拟合结果

将该拟合数据可视化,和原实验数据进行比对,得到如图 5 所示的结果。为了便于在二维图像中展

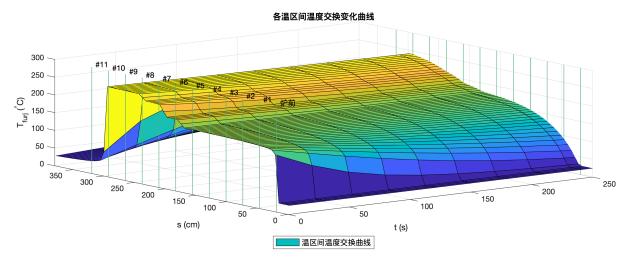


图 4: 温区间温度交换结果

示炉温的大致走向,引入炉温理想曲线的概念。即图中的红线为假设炉内温度不会受到旁边的温度的影响时的温度曲线,在实际计算中,使用的是上文提到的常微分方程计算。

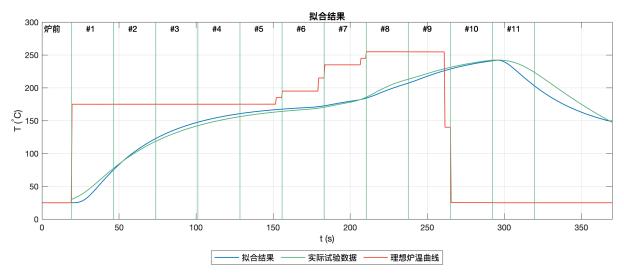


图 5: 拟合结果与实验结果

可以看到,该模型的拟合效果除了在最初升温阶段和后期的降温阶段以外,中间的拟合效果比较理想。考虑到我们在模型中极大地简化了炉间空气的交换流程,这样的结果已经可以接受。数值化地,我们根据参数拟合的结果计算了拟合数据与试验数据的均方差 $R^2=0.9212$,可以认为该模型具有较好的拟合效果。

3.1.3 方程求解

为了求解上述微分方程的数值解,可以使用 MATLAB 的 pdepe 函数求解。

考虑一般的情况,为了让 MATLAB 能够求解上述微分方程,首先需要将微分方程转换成以下标准形式:

$$c\left(x,t,u,\frac{\partial u}{\partial x}\right)\frac{\partial u}{\partial t} = x^{-m}\frac{\partial}{\partial x}\left(x^{m}f\left(x,t,u,\frac{\partial u}{\partial x}\right)\right) + s\left(x,t,u,\frac{\partial u}{\partial x}\right)$$

需要得到标准微分方程中的系数 m, c, f, s。改写热量控制方程 \mathcal{H} 为标准形式如下:

$$\frac{\rho c}{\lambda} \frac{\partial T}{\partial t} - x^0 \frac{\partial}{\partial x} (x^0 \frac{\partial T}{\partial x}) = 0$$

对应标准形式求得标准 PDE 系数为:

$$\begin{cases} m = 1 \\ c(x, t, u, \frac{\partial u}{\partial x}) = 1 \\ f(x, t, u, \frac{\partial u}{\partial x}) = \frac{\partial u}{\partial x} \\ s(x, t, u, \frac{\partial u}{\partial x}) = 0 \end{cases}$$

此外对于边界条件和初始条件,标准形式为:

$$p(x,t,u) + q(x,t)f\left(x,t,u,\frac{\partial u}{\partial x}\right) = 0$$

将 升 的边界条件和初始条件化为标准形式得:

$$\begin{cases} \lambda \frac{\partial T}{\partial x}(0,t) + h(T_{fur}(t) - T(0,t)) = 0\\ \lambda \frac{\partial T}{\partial x}(\alpha,t) + h(T(\alpha,t) - T_{fur}(t - \frac{\alpha}{t})) = 0\\ T(x,0) = T_{env} = 25^{\circ}C \end{cases}$$

对应得到各个参数的情况如下:

$$\begin{cases} m = 0 \\ c(x, t, u, \frac{\partial u}{\partial x}) = \frac{\rho c}{\lambda} \\ s(x, t, u, \frac{\partial u}{\partial x}) = 0 \\ f(x, t, u, \frac{\partial u}{\partial x}) = \frac{\partial T}{\partial x} \end{cases}$$

得到标准形式 PDE 系数后即可使用 MATLAB 得到偏微分方程的数值解,用于后续的拟合和预测。

3.2 解答结果

根据拟合所得到的偏微分方程参数,使用有限元差分法,以步长 dt 为 $0.5 \, s$ 进行计算,得到偏微分方程的数值解。题目中要求的几个特殊点的温度如表 3 所示。整次预测中每一时刻元件温度的详细结果文件见附件中的 result.csv。

位置	x / cm	t / s	T°C
小温区3中点	101.25	77.88	134.9095
小温区 6 中点	192.75	148.27	172.9624
小温区7中点	223.25	171.13	180.1546
小温区8结束处	269.00	206.92	203.7561

表 3: 部分位置的焊接点温度

预测的完整结果可视化地绘制成图像结果如图 6 所示。

图 6: 问题一: 焊接部分的温度曲线

4 问题二

4.1 问题分析

4.1.1 制程界限的数学表达

流程给出的四个制程的条件限制如表 4 所示。

界限名称	最低值	最高值	单位
温度上升斜率	0	3	ºC/s
温度下降斜率	-3	0	ºC/s
温度上升过程中在 150℃-190℃ 的时间	60	120	s
温度大于 217℃ 的时间	40	90	s
峰值温度	240	250	ºC

表 4: 制程界限

条件规定了元件在全过程中的温度变化情况,以及特定区域的持续时间。我们通过每次迭代后依次 计算 PDE 数值解的梯度和对应温区的持续时间,于是将约束条件形式化地改写如下:

$$\begin{cases}
-3 \le \frac{dT}{dt} \le 3 \\
60 \le t_{150^{\circ}C \le T \le 190^{\circ}C} \le 120 \\
40 \le t_{T \ge 217^{\circ}C} \le 90 \\
240 \le T_{max} \le 250
\end{cases}$$

4.1.2 拟合与迭代

由于传送带速度 v 的取值范围为 [65, 100],且需要找到同时满足以上条件的最大值,所以考虑从 100~cm/min 往 65 cm/min 按照 0.1 的步长进行迭代,依次判读新拟合出的函数 T(t) 是否满足上述约束条件。找到一个可行解之后,继续以 0.02 或者 0.01 的步长迭代,如此反复,最终得到一个较为精确的可行最大速度 v_{max} 。对于 cm/min 的速度单位和尺度而言,保留 2 位小数(即进行三次迭代)即能得到较高精度。

4.2 解答结果

先利用步长 0.5 cm/mim, 从 100 cm/min 的速度逐渐递减。该过程可以得到以下速度与各指标 (最大梯度、最小梯度、升温时间、高温时间、峰值温度) 趋势如图 7 所示。完整的结果参考附件中的 Q2_Trend.csv。其内包含有的传送速度和指定温度条件下,所有满足条件的解析。

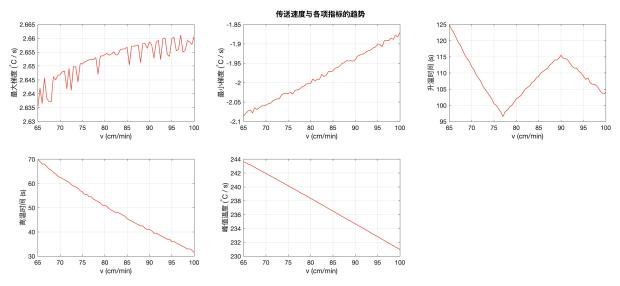


图 7: 传送速度与各项指标的趋势(粗粒度)

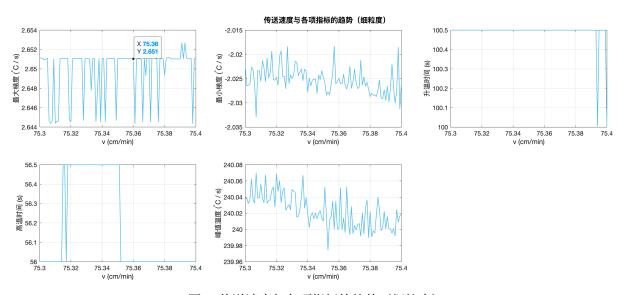


图 8: 传送速度与各项指标的趋势(细粒度)

于是可以发现在 65 cm/min 到 100 cm/min 之间的粗粒度计算,满足的最大速度在 76 cm/min 附近,于是我们在 76 cm/min 至 74 cm/min 以步长为 0.1 cm/min 逼近最大值,再以 75.4 cm/min 至 75.3 cm/min 以步长为 0.01 cm/min 逼近最大值。细粒度逼近的可视化结果如图 8 所示。

经过上述方法的计算, 我们得到能够满足以上所有制程限制条件的最大传送速度为 75.3520 cm/min。

5 问题三

5.1 问题分析

5.1.1 目标函数

目标优化函数 $A = \mathcal{F}(T_{fur_i}, v)$, 物理意义为目标区域面积, 其中:

$$T_{fur_j} = \begin{bmatrix} T_{fur_{1-5}} & T_{fur_6} & T_{fur_7} & T_{fur_{8-9}} \end{bmatrix}$$

冷却区域设定温度 $T_{fur_{10-11}}$ 温度固定为 25 摄氏度,不纳入优化。目标函数的优化可以形式化地写为

$$(T_{fur}, \hat{v}) = arg min A$$

5.1.2 迭代优化算法

使用 brute force 算法,枚举 4 个温区的温度和传送带速度,使用问题一中的模型,利用 T_{fur_j} 指定温度参数,计算出焊接区域的温度曲线,再依次审核制程界限,若不符合任意一条要求则不保留结果。对于满足制程界限的参数,计算积分面积 A。具体地讲,考虑从 165° C 到 185° C 枚举小温区 1.5 的温度;从 185° C 到 205° C 枚举小温区 6 的温度;从 225° C 到 245° C 枚举小温区 7 的温度;从 245° C 到 265° C 枚举小温区 1.5 的温度;从 1.50 的温度;从 1.51 的温度;从 1.52 的温度;从 1.53 的温度;从 1.54 的温度;从 1.55 的温度,从 1.55 的温度,

5.2 解答结果

记步长为 dv,参数矩阵为 $[v, T_{fur_{1-5}}, T_{fur_6}, T_{fur_7}, T_{fur_{8-9}}, T_{fur_{10-11}}, A]$ 在第一次粗粒度查找中,设置 dv = 3,遍历得到如下参数矩阵:

$$[v,\ T_{fur_{1-5}},\ T_{fur_6},\ T_{fur_7},\ T_{fur_{8-9}},\ T_{fur_{10-11}},\ \mathcal{A}]=[95,\ 183,\ 200,\ 240,\ 263,\ 25,\ 401.7657]$$

第一次粗粒度查找的结果如图 9 所示。为了便于展示,横坐标只绘制了 v 和 T_{fur_1} 参数和阴影面积 A 表面的情况。

我们在最优结果的参数值上下浮动 3, 即令在下一轮遍历中, 各参数的取值范围为

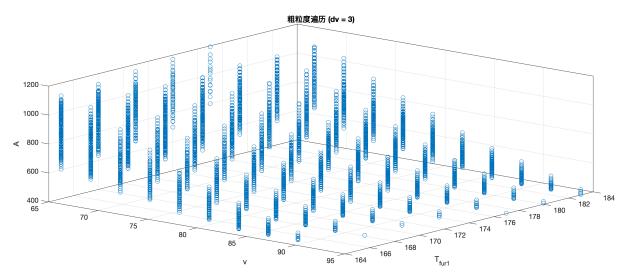


图 9: $T_{fur_1} - v - A$ 优化结果(粗粒度)

$$\begin{cases} v \in [92, 98] \\ T_{fur_{1-5}} \in [180, 186] \\ T_{fur_{6}} \in [197, 203] \\ T_{fur_{7}} \in [237, 243] \\ T_{fur_{8-9}} \in [260, 265] \end{cases}$$

接下来在 A 具备最小值的附近进行中粒度遍历。设置 dv=1,遍历得到具备最小 A 时的参数矩阵为:

$$[v,\ T_{fur_{1-5}},\ T_{fur_6},\ T_{fur_7},\ T_{fur_{8-9}},\ T_{fur_{10-11}},\ \mathcal{A}]=[98,\ 182,\ 203,\ 243,\ 264,\ 25,\ 393.5364]$$

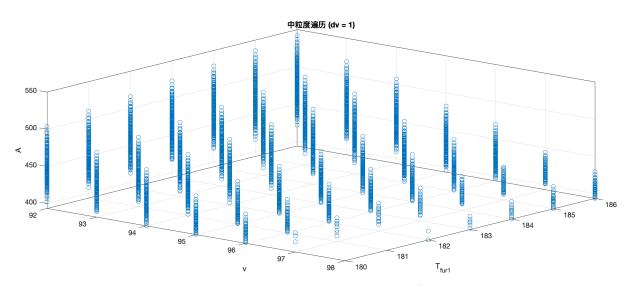


图 10: $T_{fur_1} - v - A$ 优化结果(中粒度)

此时可视化的结果如图 10 所示。进一步地进行细粒度查找,以得到更精确的参数价值。我们在拟合的结果上下浮动 0.6,并将步长缩小至 0.3,在具备最低值的各个参数附近值进行遍历。此时遍历范围为 2*dv=0.6。各参数的取值范围为:

$$\begin{cases} v \in [97.4, 98.6] \\ T_{fur_{1-5}} \in [181.4, 182.6] \\ T_{fur_{6}} \in [202.4, 203.6] \\ T_{fur_{7}} \in [242.4, 243.6] \\ T_{fur_{8-9}} \in [263.4, 264.6] \end{cases}$$

在第三次粗粒度查找中,设置更小的步长 dv = 0.3。经过遍历,此时的优化结果如图 11 所示。

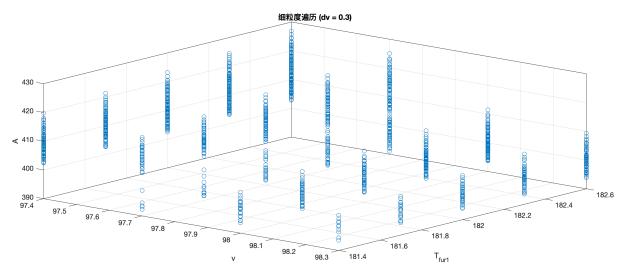


图 11: $T_{fur_1} - v - A$ 优化结果(细粒度)

得到最优参数矩阵如下:

$$[v,\ T_{fur_{1-5}},\ T_{fur_{6}},\ T_{fur_{7}},\ T_{fur_{8-9}},\ T_{fur_{10-11}},\ \mathcal{A}]=[97.7,\ 182.6,\ 203.0,\ 242.4,\ 263.7,\ 25,\ 391.0421]$$

最终,我们可以得到阴影区域的最小面积 \mathcal{A} 为 391.0421,这时传送带的速度为 97.7 cm/min,各个温度区的温度分别为: $T_{fur_{1-5}}=182.6^{\circ}C$, $T_{fur_{6}}=203.0^{\circ}C$, $T_{fur_{7}}=242.4^{\circ}C$, $T_{fur_{8-9}}=263.7^{\circ}C$, $T_{fur_{10-11}}=25^{\circ}C$ 。

6 *问题四

6.1 问题分析

6.1.1 目标函数

沿袭问题三的思路,有峰值左侧的区域面积为 $\mathcal{A}=\mathcal{F}(T_{fur_j},v)$,再设峰值右侧的区域面积为 $\mathcal{B}=\mathcal{F}(T_{fur_j},v)$,其中各参数仍然有:

$$T_{fur_j} = \begin{bmatrix} T_{fur_{1-5}} & T_{fur_6} & T_{fur_7} & T_{fur_{8-9}} \end{bmatrix}$$

冷却区域设定温度 $T_{fur_{10-11}}$ 温度固定为 25 摄氏度,不纳入优化。

考虑评价指标。要在问题 3 限制下使得 217 °C 到峰值温度左右两侧的温度尽可能对称,我们可以用右侧函数以 $t = t_{peak}$ 轴对称的函数 f_R 和峰值温度左侧的函数 f_L 做分析,计算其方差。为问题 3 再增加一个优化条件,目标为使得方差 R^2 尽可能的小。目标函数的优化可以形式化地写成:

$$(T_{fur}, \hat{v}) = arg \min(A - B)^2$$

通过不断调整参数来使 R² 尽可能小。

6.1.2 迭代优化算法

仍然考虑使用 brute-force 算法,枚举四个温区的温度和传送带速度,检验制程界限,舍去不满足条件的参数元组。对于符合制程界限的参数元组,维护差值平方的最小值,利用迭代优化算法,设定恰当的大小步长,逐步迭代逼近最终解。

6.2 解答结果

由于时间限制,仅对此问题做了分析,未得出答案与实现。

7 分析与结论

本文完成了「炉温曲线」问题中的问题一、问题二、问题三。

问题一 问题一基于非稳态传热模型,分析了两个维度的温度变换: 炉内空气和炉内的热对流; 以及基于牛顿冷却定律的炉子与炉子之间温度的互相影响。通过构件热力学偏微分方程并利用其数值解拟合实际试验数据,得到了元件导热系数 λ 和空气的对流换热系数 h,取得了较好的拟合效果。

问题二 在问题一构建的模型基础上,我们采取了两次迭代,从允许的最大速度开始依次向下预测元件的炉温曲线,并审核其制程界限。在已知的界限范围内部继续递归缩小步长,找到能够满足制程界限的最大温度,得到结果。

问题三 使用迭代优化法,使用微分法近似计算题目要求部分的阴影面积,将计算结果的最小化作为优化函数 $A = F(T_{fur_j}, v)$ 的目标。对优化函数的五个参数先进行粗粒度的遍历,找到全局的最优解的取值范围,再在全局最优范围附近缩小步长,递归遍历。经过三次迭代,找到了精确到一位小数的全局最优解。

本模型已经考虑了对流传热及炉间温度传导的情况,利用偏微分方程的数值解能够较好地拟合实际 试验的炉温曲线,在利用其它参数预测炉温曲线时也取得了较合理的结果。不过模型仍有可以改进之处,例如对于炉与炉缝隙处的温度传导过程,我们简化为了简单的左右瞬时温度的算术平方根,有不科学之处。如果能够将这部分的热量交换考虑进热量控制方程中,模型在炉后区域的温度下降阶段应当能够取得更好的结果。

此外,在模型构建中我们直接估计元件的长度为 3cm,进行所有的参数拟合。但实际这一参数题目并未给出,可以考虑将元件的长度也作为拟合参数加入最小二乘法的计算中,也有进一步提高拟合效果的可能性。

8 附录

8.1 源代码

本模型基于 MATLAB 实现,以下代码在 macOS 环境和 Windows 环境下的 MATLAB R2020a 版本成功运行。

TPDEFit.m 用于拟合偏微分方程。

```
%% 使用最小二乘方拟合 lambda 和 h
  global TfurCalced;
  TfurCalced = false;
6 lambdaguess = 50;
7 hguess = 1500;
8 thetaguess = 10;
  dt = 0.5;
10 % 优化参数
options=optimset('TolX',1e-5,'TolFun',1e-5,'MaxFunEvals',1200,'Display','iter');
expdata = xlsread('temp.xlsx');
13 t = 19:0.5:373; % 生成横向坐标
xdata = expdata(:,1);
15 xdata = (xdata - 19) * 2 + 1;
  ydata = expdata(:,2);
  % LSQ
   [coef,~, residual] = lsqcurvefit(@(para, data) PDF_for_fitting(para, data), ...
      [lambdaguess, hguess, thetaguess], xdata, ydata, [30 1000 0], [100 5000 100], options);
  lambdafit = coef(1)
23 hfit = coef(2)
thetafit = coef(3)
25 function r = PDF_for_fitting(params, x)
  %% 用于拟合的 PDF 函数
      res = TPDESolve(params(1), params(2), params(3), 70); % 70 = Velocity
      sol = res(:,15);
      sol = sol(19:728);
      r = sol(x);
```

TPDESolve.m 用于计算偏微分方程。

```
function [result] = TPDESolve(lambdaguess, hguess, thetaguess, velocity)
2 %% 使用 pdepe 求解元件温度的微分方程
3 % 参数配置
4 Tenv = 25; % 环境温度
5 V = velocity; % 传送速度
6 alpha = 3.2; % 物体长度 cm
8 % 物理参数
9 rho = 7.874; % 密度
10 C = 480; % 比热容
11 lambda = lambdaguess; % 导热系数
12  % h = lambda / (alpha / 100);
13 h = hguess; % 对流换热系数;
x = linspace(0, alpha, 30);
t = linspace(0, 400, 800);
m = 0;
17
sol = pdepe(m, @TPDE, @TPDE_IC, @TPDE_BC, x, t);
19 hold;
20 grid on;
21 plot(t, sol(:,15), 'LineWidth', 1)
  title('PDE 数值解');
24 result = sol;
      %% PDF
25
      % PDE 方程
      function [c, f, s] = TPDE(x, t, T, dudx)
         % 转换为标准 PDE
         c = rho * C / lambda;
         f = dudx;
         s = 0;
      end
      % PDE 初始条件
34
      function TO = TPDE_IC(x)
35
         % Initial Condition
         TO = Tenv;
37
      end
38
      % PDE 边界值
      function [pl, ql, pr, qr] = TPDE_BC(xl, ul, xr, ur, t)
41
         pl = h * Tfur2D(t, ul, thetaguess, V) - h * ul;
42
         ql = lambda;
         pr = h * ur - h * Tfur2D(t - alpha / V, ur, thetaguess, V);
         qr = lambda;
```

```
46 end47 end
```

Tfur2D.m 用于计算炉内温度。

```
function [tfur] = Tfur(t, m, theta, velocity)
   global Tfurdata;
      furnaceLength = 30.5; % 熔炉长度
      furnaceGap = 5; % 熔炉间隙
      edgeGap = 25; % 炉前与炉后
      if (nargin < 4)</pre>
         % 指定默认速度
         V = 70;
      else
         V = velocity; % 速度, cm/min
11
      end
      tempEnv = 25; % 环境温度
12
13
       % 附件 - 拟合实验温区温度配置
14
   %
          t1 = 175;
15
   %
          t2 = 195;
16
  %
          t3 = 235;
  %
          t4 = 255;
18
19
   %
          t5 = 25;
       % Q1 - 预测温区温度配置
  %
        t1 = 173;
22
   %
        t2 = 198;
23
   %
         t3 = 230;
  %
         t4 = 257;
         t5 = 25;
   %
      % Q2 - 制程界限温区温度配置
        t1 = 182;
        t2 = 203;
30
        t3 = 237;
31
        t4 = 254;
32
        t5 = 25;
33
34
      hf = 1e-4; % 炉间温度传播系数
      if t < 0.5
         t = 0.5;
      end
      tempMat = TfurNew([tempEnv, t1, t2, t3, t4, t5], hf);
      distance = V / 60 * t + (furnaceLength + furnaceGap - edgeGap);
41
```

```
furnaceTemp = tempMat(ceil(t * 2), 1:13);
42
       fCount = ceil(distance / (furnaceLength + furnaceGap));
43
       fCount(fCount < 1) = 1;</pre>
       dd = distance - (furnaceLength + furnaceGap) * (fCount - 1);
       if(fCount < 11)</pre>
          if dd <= furnaceLength</pre>
              tfur = furnaceTemp(fCount);
              tfur = (furnaceTemp(fCount) + furnaceTemp(fCount + 1)) / 2;
          end
       else
52
          tfur = furnaceTemp(11);
53
       end
54
       if(fCount > 10)
            tfur = theta * nthroot((m - tfur), 3) + tfur;
       end
       Tfurdata(ceil(t * 2))= tfur;
   end
```

Tfur.m 理想炉温曲线。

```
function [tfur] = Tfur(t)
      furnaceLength = 30.5; % 熔炉长度
      furnaceGap = 5; % 熔炉间隙
      edgeGap = 25; % 炉前与炉后
      V = 78; % 速度, cm/min
      tempEnv = 25; % 环境温度
       t1 = 175; % 附件 - 拟合实验温区温度配置
       t2 = 195;
       t3 = 235;
10
       t4 = 255;
       t5 = 25;
      hf = 0.00002; % 炉间温度传播系数
14
      if t < 0.5
         t = 0.5;
      end
17
      tempMat = TfurNew([tempEnv, t1, t2, t3, t4, t5], hf);
      distance = V / 60 * t - edgeGap;
      furnaceTemp = tempMat(ceil(t * 2), 2:12);
      if distance < 0</pre>
         tfur = tempEnv;
         return
24
      end
25
```

```
fCount = ceil(distance / (furnaceLength + furnaceGap));
       fCount(fCount < 1) = 1;</pre>
       dd = distance - (furnaceLength + furnaceGap) * (fCount - 1);
       if(fCount < 11)</pre>
          if dd <= furnaceLength</pre>
              tfur = furnaceTemp(fCount);
              tfur = (furnaceTemp(fCount) + furnaceTemp(fCount + 1)) / 2;
33
          end
34
       else
          tfur = furnaceTemp(11);
36
            tfur = 25;
   %
37
       end
   end
```

TfurNew.m 用于计算炉内空气温度的动态变化。

```
function [tfur] = TfurNew(mat, h)
   %% 考虑炉间热量传导的情况
  % if exist('dp.mat', 'file')
  %
        load 'dp.mat' 'dp';
  %
        tfur = dp;
6 %
        return;
7 % end
8 global dp;
  global TfurCalced;
  if TfurCalced
      tfur = dp;
13
      return;
14
  % 13 * 800 矩阵
  tfur = zeros(800, 13);
  dp = zeros(800, 13);
  for i = 1 : 13
      if i == 1
         dp(1, i) = mat(1);
      elseif i <= 6
21
         dp(1, i) = mat(2);
      elseif i == 7
         dp(1, i) = mat(3);
      elseif i == 8
         dp(1, i) = mat(4);
      elseif i <= 10
         dp(1, i) = mat(5);
      else
```

```
dp(1, i) = mat(6);
       end
31
   end
   for i = 2 : 800
       for j = 1 : 13
          if j == 1
              dp(i, j) = dp(i - 1, j) - h * nthroot((dp(i - 1, j) - dp(i - 1, j + 1)), 3);
          elseif j <= 12
              dp(i, j) = dp(i - 1, j) - h * nthroot((dp(i - 1, j) - dp(i - 1, j + 1)), 3) - h *
                  \hookrightarrow nthroot((dp(i - 1, j) - dp(i - 1, j - 1)), 3);
          else
              dp(i, j) = dp(i - 1, j) - h * nthroot((dp(i - 1, j) - dp(i - 1, j - 1)), 3);
          end
41
       end
42
   end
43
   tfur = dp;
   TfurCalced = true;
   end
```

Variables.m 用于存储模型常量。

```
%% 参数配置
2 global Tenv
  global V
4 global alpha
  global h
6 global rho
  global c
  global lambda
  global furnaceLength
  global furnaceGap
  global edgeGap
11
13 furnaceLength = 30.5; % 熔炉长度
14 furnaceGap = 5; % 熔炉间隙
  edgeGap = 25; % 炉前与炉后
  Tenv = 25; % 环境温度
  Ⅴ = 70; % 传送速度
  alpha = 3; % 物体长度
21 % 物理参数
h = 1500;
_{23} rho = 7.874;
c = 480;
```

```
25 lambda = 50;
```

Preview.m 用于预览拟合效果。

```
%% 设定温度曲线绘制
x = 0.5:0.5:400;
y = 1:800;
5 for i = 1:800
      y(i) = Tfur(x(i));
7 end
8 % figure;
9 box on; grid on; hold on;
10 width = 900;
11 height = 350;
set(gcf,'position',[0,0,width,height])
red = '#EB4537';
14 green = '#55AF7B';
15 blue = '#4286F3';
17 title('拟合结果')
19 ylabel('T (^{\circ}C)')
t = xlsread('temp.xlsx')
22 plot(t(:,1), t(:,2), 'Color', green, 'LineWidth', 1)
plot(x, y, 'Color',red, 'LineWidth', 1)
  for i=1:11
      lines = [lines, (25 + i * (furnaceLength + furnaceGap)) / (70 / 60)];
  for i=1:12
      line([lines(i) lines(i)], [0 300], 'linewidth', 0.7, 'color', green);
      if i ~= 1
31
         text(lines(i) - 18, 290,['#', num2str(i - 1)]);
32
33
         text(lines(i) - 18, 290,['炉前']);
34
      end
35
   end
38 lgd = {'拟合结果', '实际试验数据', '理想炉温曲线'};
  legend(lgd, 'location', 'southoutside', 'NumColumns', 3, 'FontSize', 10);
40 xlim([0 370])
exportgraphics(gcf,'pic/FitResult.png','Resolution',300)
42
```

```
saveas(gcf,['pic/','FurnacePreTemp','.png'])
```

Q1_Predict.m 问题一的主要求解代码。

```
clf; close all;
2 %% 特定条件下的预测
4 % 预测得到的参数
5 lambda = 99.7964;
6 h = 1813.6;
7 theta = 21.2916;
8 \quad V = 78;
9 sol = TPDESolve(lambda, h, theta, V);
10 res = sol(:,15);
lines = [25 / (V / 60)];
13 furnaceLength = 30.5; % 熔炉长度
14 furnaceGap = 5; % 熔炉间隙
15 edgeGap = 25; % 炉前与炉后
17 % 绘制炉
18 % figure;
width = 900; height = 350;
set(gcf,'position',[0,0,width,height])
21 red = '#EB4537';
green = '#55AF7B';
23 blue = '#4286F3';
24 plot(0.5:0.5:400, res, 'LineWidth', 2, 'Color', blue);
25 grid on;
26 hold on;
28 % 温度曲线
x = 0.5:0.5:400;
y = 1:800;
31 for i = 1:800
      y(i) = Tfur(x(i));
plot(x, y, 'Color', red, 'LineWidth', 1)
36 for i=1:11
      lines = [lines, (25 + i * (furnaceLength + furnaceGap)) / (V / 60)];
  end
      line([lines(i) lines(i)], [0 300], 'linewidth', 0.7, 'color', green);
    if i ~= 1
```

```
text(lines(i) - 18, 290,['#', num2str(i - 1)]);
43
      else
44
          text(lines(i) - 18, 290,['炉前']);
      end
   end
   title('问题一预测结果')
   xlabel('t (s)');
   ylabel('T (^{\circ}C)');
51
   % Mark maximum.
53
  amax = max(res);
54
   tmax = find(res==amax) / 2;
   plot(tmax,amax,'Color',red,'Marker','v','LineStyle','none','LineWidth',1);
   text(tmax-2,amax*0.9,['最大值: ', num2str(amax), '^{\circ}C']);
   lgd = {'预测结果', '炉内理想温度曲线'};
   legend(lgd, 'location', 'southoutside', 'NumColumns', 2, 'FontSize', 10);
   xlim([0, 350]);
   exportgraphics(gcf,'pic/Q1.png','Resolution',300)
63
64
   %% 3D Surface
   figure;
   t = 0.5:0.5:800;
  % surf(1:10:300,t(1:10:800),sol(1:10:800,:));
   surf(1:30:900,linspace(0,450,80),sol(1:10:800,:));
  set(gcf,'position',[0,0,width,height])
  title('各温区间温度交换变化曲线');
74 ylabel('s (cm)');
  xlabel('t (s)');
76 zlabel('T_{furj} (^{\circ}C)');
  xlim([0 250])
   ylim([0 380])
   for i=1:12
80
      line([0 0], [lines(i) lines(i)], [0 300], 'linewidth', 0.7, 'color', green);
81
      line([250 250], [lines(i) lines(i)], [0 300], 'linewidth', 0.7, 'color', green);
82
83
      if i ~= 1
84
          text(0, lines(i) - 6, 310,['#', num2str(i - 1)]);
      else
          text(0, lines(i) - 6, 310,['炉前']);
```

```
end
end
gl end
gl gd = {'温区间温度交换曲线'};
legend(lgd,'location','southoutside','NumColumns',2,'FontSize',10);
exportgraphics(gcf,'pic/TempExchange.png','Resolution',300)
```

Q2_Restrain.m 问题二的主要求解代码。

```
1 % 从最大速度开始筛选
2 detail = [];
3 red = '#EB4537';
green = '#55AF7B';
  blue = '#4286F3';
  for vv = 75.4:-0.001:75.3
  % for vv = 100:-0.5:65
      out = vv
      % 指定拟合的模型参数
      lambda = 99.7964;
      h = 1813.6;
11
      theta = 21.2916;
12
      sol = TPDESolve(lambda, h, theta, vv);
      sol = sol(:,15);
14
      restrains = {@RestrainDiff @RestrainRising @RestrainPeakTime @RestrainPeakTemp};
15
      flag = 0;
16
      [r1, dd(2), dd(3)] = RestrainDiff(sol);
      [r2, dd(4)] = RestrainRising(sol);
      [r3, dd(5)] = RestrainPeakTime(sol);
       [r4, dd(6)] = RestrainPeakTemp(sol);
21
      dd(1) = vv;
22
      if ~r1
          flag = 1;
24
      end
25
26
      if ~r2
27
         flag = 2;
      end
29
      if ~r3
          flag = 3;
      end
      if ~r4
          flag = 4;
      end
```

```
dd(7) = flag;
      detail = [detail; dd];
39
41
   end
   writematrix(detail, 'Q2.csv');
43
44
  %% 可视化
45
  figure;
47 hold on;
  grid on;
  box on;
  set(gcf,'position',[0,0,1000,450])
52 subplot(2,3,1);
plot(detail(:,1), detail(:,2), 'Color', red, 'LineWidth', 1);
54 xlabel('v (cm/min)')
55 ylabel('最大梯度 (^{\circ}C / s)');
56 subplot(2,3,2);
  plot(detail(:,1), detail(:,3), 'Color', red, 'LineWidth', 1);
58 xlabel('v (cm/min)')
59 ylabel('最小梯度 (^{\circ}C / s)');
60 subplot(2,3,3);
plot(detail(:,1), detail(:,4), 'Color', red, 'LineWidth', 1);
62 xlabel('v (cm/min)')
63 ylabel('升温时间(s)');
64 subplot(2,3,4);
plot(detail(:,1), detail(:,5), 'Color', red, 'LineWidth', 1);
66 xlabel('v (cm/min)')
67 ylabel('高温时间 (s)');
68 subplot(2,3,5);
  plot(detail(:,1), detail(:,6), 'Color', red, 'LineWidth', 1);
  xlabel('v (cm/min)')
   ylabel('峰值温度 (^{\circ}C / s)');
   exportgraphics(gcf,'pic/Trend.png','Resolution',300);
72
  function [res, valueMax, valueMin] = RestrainDiff(sol)
   %% 斜率是否超过 3?
      tdiff = 2 * diff(sol);
      rising = tdiff(tdiff > 0);
      descending = tdiff(tdiff < 0);</pre>
      [rc,~] = size(rising(rising > 3));
      [dc,~] = size(descending(descending < -3));</pre>
      if rc ~= 0
         sprintf("升温速度过大。")
         res = false;
```

```
return;
       end
       if dc ~= 0
          sprintf("降温速度过大。")
          res = false;
          return;
       valueMax = max(tdiff);
92
       valueMin = min(tdiff);
       res = true;
   end
95
   function [res, value] = RestrainRising(sol)
   %% 升温时间是否符合要求?
       % 只考虑升温过程
99
       sol = sol(1:600);
100
       count = sol(sol > 150 & sol < 190);</pre>
       [t, ~] = size(count);
       if t / 2 > 60 && t / 2 < 120</pre>
          res = true;
104
       else
105
          res = false;
106
          sprintf("升温时间不合要求。")
107
108
       value = t / 2;
109
   end
110
   function [res, value] = RestrainPeakTime(sol)
   %% 高温时间是否符合要求?
114
       count = sol(sol > 217);
       [t, ~] = size(count);
117
       if t / 2 > 40 && t / 2 < 90</pre>
118
          res = true;
119
       else
120
          res = false;
          sprintf("高温时间不合要求。")
       end
       value = t / 2;
124
   end
125
126
127
   function [res, value] = RestrainPeakTemp(sol)
   %% 峰值温度是否符合要求?
       mt = max(sol);
       if mt > 240 && mt < 250
131
```

```
      132
      res = true;

      133
      else

      134
      res = false;

      135
      sprintf("峰值温度不合要求。")

      136
      end

      137
      value = mt;

      138
      end
```

Q3_Optimize.m 问题三的主要求解代码。

```
1 Output = [];
0utputFail = [];
4 % 模型参数
5 lambda = 99.7964;
6 h = 1813.6;
7 theta = 21.2916;
9 % 优化参数
_{10} % V = 70;
11 % t1 = 175;
12 % t2 = 195;
13 % t3 = 240;
_{14} % t4 = 257;
15 % t5 = 25; % t5 恒定为 25
17 %% ROUND 1
18 % dt = 3;
19 % %
20 % Vb = [89 100];
21 % t1b = [165 185];
22 % t2b = [185 205];
23 % t3b = [225 245];
24 % t4b = [245 265];
26 %% ROUND 2
27 dt = 1;
Vb = [94 96];
29 t1b = [180 186];
30 t2b = [197 203];
31 t3b = [237 243];
32 t4b = [260 265];
34 % ROUND 3
35 dt = 0.15;
_{36} Vb = [98.3 98.6];
```

```
t1b = [181.4 182.6];
   t2b = [202.4 \ 203.6];
   t3b = [242.4 243.6];
   t4b = [263.4 264.6];
   for V=Vb(1):dt:Vb(2)
       for t1=t1b(1):dt:t1b(2)
          for t2=t2b(1):dt:t2b(2)
44
              for t3=t3b(1):dt:t3b(2)
                 for t4=t4b(1):dt:t4b(2)
                     [Aera, sol] = FF(lambda, h, theta, V, [t1 t2 t3 t4 t5]);
                     [success, flag] = Restrain(sol(:,15));
                     if success
                         Output = [Output; [V, t1, t2, t3, t4, t5, Aera]];
                     else
                         OutputFail = [OutputFail; [V, t1, t2, t3, t4, t5, flag]];
                     \quad \text{end} \quad
                     disp([V t1 t2 t3 t4 success flag round(Aera)]);
                  end
              end
          end
       end
   end
59
   % 参数上下界
61
   %% 优化目标函数
   function [aera, sol] = FF(lambda, h, theta, V, mat)
       sol = TPDESolveFO(lambda, h, theta, V, [mat(1) mat(2) mat(3) mat(4) mat(5)]);
64
       aera = getAera(sol(:,15));
       function res = getAera(sol)
          peakTemp = max(sol);
          peakTime = find(sol==peakTemp);
          sol = sol(1:peakTime);
          count = sol(sol > 217 & sol <= peakTemp);</pre>
          [1, \sim] = size(count);
          res = sum(count) / 2 - (1 / 2) * 217;
       end
73
   end
74
   %% 约束条件
   function [success, flag] = Restrain(sol)
       flag = 0;
       [r1] = RestrainDiff(sol);
       [r2] = RestrainRising(sol);
       [r3] = RestrainPeakTime(sol);
       [r4] = RestrainPeakTemp(sol);
       if ~r1
```

```
flag = 1;
84
       end
85
       if ~r2
           flag = 2;
       end
       if ~r3
           flag = 3;
91
       if ~r4
92
           flag = 4;
93
       end
94
       if flag == 0
95
           success = true;
       else
           success = false;
98
       end
       function [res, valueMax, valueMin] = RestrainDiff(sol)
       % 斜率是否超过 3?
           tdiff = 2 * diff(sol);
102
           rising = tdiff(tdiff > 0);
103
           descending = tdiff(tdiff < 0);</pre>
104
            [rc,~] = size(rising(rising > 3));
105
           [dc,~] = size(descending(descending < -3));</pre>
106
           if rc ~= 0
107
               res = false;
108
109
           end
110
           %if dc ~= 0
              % res = false;
113
           %end
115
           valueMax = max(tdiff);
           valueMin = min(tdiff);
116
           res = true;
117
118
119
       function [res, value] = RestrainRising(sol)
120
           % 升温时间是否符合要求?
           % 只考虑升温过程
           sol = sol(1:600);
           count = sol(sol > 150 & sol < 190);</pre>
124
           [t, ~] = size(count);
125
           if t / 2 > 60 && t / 2 < 120</pre>
               res = true;
           else
               res = false;
129
130
           \quad \text{end} \quad
```

```
value = t / 2;
131
       end
132
       function [res, value] = RestrainPeakTime(sol)
134
           % 高温时间是否符合要求?
           count = sol(sol > 217);
           [t, ~] = size(count);
138
           if t / 2 > 40 && t / 2 < 90
139
              res = true;
140
           else
141
              res = false;
142
           end
143
           value = t / 2;
       end
145
       function [res, value] = RestrainPeakTemp(sol)
           %峰值温度是否符合要求?
           mt = max(sol);
           if mt > 240 && mt < 250</pre>
150
              res = true;
151
           else
152
              res = false;
153
           end
154
           value = mt;
       end
156
   end
157
```

Plotter.m 用于绘制优化结果。

```
1 %% 寻找最小值
2
3 minRow = find(Output(:,7) == min(Output(:,7)));
4 Output(minRow,:)
5
6 %% 优化结果绘制
7 scatter3(Output(:,1), Output(:,2),Output(:,7));
8 view(40, 35);
9 set(gcf,'position',[0,0,1000,400]);
10 box on;
11 grid on;
12 % title("粗粒度遍历 (dv = 3)");
13 % title("中粒度遍历 (dv = 1)");
14 title("细粒度遍历 (dv = 0.3)");
15 xlabel('v');
16 ylabel('T_{fur1}');
```

```
zlabel('A');
exportgraphics(gcf,'OROUND3.png','Resolution',300);
```

8.2 结果文件

result.csv 问题一的解答:

```
34.500,49.9766820363965
                                                                                       70,124.395996345290
                                                                                                                                105.50,157.132110606820
                                                                                                                                                                       141,171.308622668490
      时间(s),温度(摄氏度)
                                               35,51,3636844129079
                                                                               143
                                                                                       70.500,125.077364087062
                                                                                                                        214
                                                                                                                                106,157.424110347357
                                                                                                                                                                285
                                                                                                                                                                       141.50,171.435916100469
      0,25
                                               35.500.52.7521159851482
                                                                               144
                                                                                       71.125.749482881087
                                                                                                                                106.50.157.712900549240
                                                                                                                                                                       142.171.561728563565
      0.5.25
                                                                                       71.500,126.412713645566
                                                                                                                                                                       142.50,171.686078032347
                                        74
                                               36.54.1392987586635
                                                                                                                                107.157.998542193704
                                                                                                                                                                287
                                                                               145
      1,25.0000000002737
                                               36.500,55.5225547389999
                                                                                                                                107.50,158.281096261984
                                                                                                                                                                       143,171.808982481385
                                                                                       72,127.067998321836
                                                                               146
      1.5000,25.0000000027697
                                               37,56.8997481020511
                                                                                147
                                                                                       72.500,127.715437773941
                                                                                                                                108,158.560623735318
                                                                                                                                                                        143.50,171.930459885248
      2.25.0000000147783
                                               37 500 58 2779064468243
                                                                               148
                                                                                       73 128 355132865927
                                                                                                                        219
                                                                                                                                108 50 158 837185594941
                                                                                                                                                                290
                                                                                                                                                                       144 172 050528218507
      2.5000.25.0000109376896
                                               38,59.6497898446136
                                                                               149
                                                                                       73.500,128.987184461837
                                                                                                                                109,159.110842822089
                                                                                                                                                                291
                                                                                                                                                                       144.50,172.169205455731
      3,25.0000218606008
                                               38.500,61.0136467266850
                                                                                                                                109.50,159.381656397998
                                                                                                                                                                       145,172.286509571490
                                                                               150
                                                                                       74,129.611693425716
                                                                                                                                                                292
      3,5000,25,0000327835121
                                                                                       74.500,130.228760621608
                                               39,62.3677255243048
                                                                                                                                110,159.649687303904
                                                                                                                                                                        145.50,172.402458540353
       4,25.0000437064234
                                               39.500.63.7102746687393
                                                                                       75,130.838486913558
                                                                                                                                110.50,159.914996521043
                                                                                                                                                                       146,172.517070336891
       4.5000.25.0000546293347
                                        82
                                               40.65.0486904290100
                                                                                       75.500.131.440973165611
                                                                                                                        224
                                                                                                                                111.160.177645030652
                                                                                                                                                                295
                                                                                                                                                                       146.50.172.630362935672
      5.25.0000655522459
                                               40.500,66.3770429205360
                                                                                                                                111.50,160.437693813966
                                                                                                                                                                       147,172.742354311267
                                        83
                                                                               154
                                                                                       76,132.036320241810
      5.5000,25.0000764751572
                                               41,67.6943715037984
                                                                                155
                                                                                       76.500,132.624629006200
                                                                                                                                112,160.693261937585
                                                                                                                                                                       147.50,172.853062438245
      6,25.0000873980685
                                        85
                                               41.500,68.9997397346458
                                                                               156
                                                                                       77,133.206000322826
                                                                                                                                112.50,160.943241219877
                                                                                                                                                                       148,172.962416606991
      6.5000.25.0000983209798
                                               42.70.2922111689269
                                                                                       77.500,133.780535055732
                                                                                                                                113,161.190275644736
                                                                                                                                                                       148.50.173.070402019597
                                                                                                                        228
16
       7.25.0001092438911
                                               42.500,71.5761084181703
                                        87
                                                                               158
                                                                                                                                113.50,161.434407863456
                                                                                                                                                                       149,173.177092089900
                                                                                       78,134.348334068963
                                                                                                                                                                300
17
      7.5000,25.0001465335839
                                               43.72.8486327387099
                                                                                159
                                                                                       78.500.134.909498226562
                                                                                                                                114.161.675680527331
                                                                                                                                                                        149.50.173.282497901028
      8,25.0001838232767
18
                                               43.500,74.1090430597860
                                                                                       79,135.464128392575
                                                                                                                                114.50,161.914136287655
                                                                                                                                                                        150,173.386630536105
       8.5000,25.0002211129696
19
                                        90
                                               44.75.3569163529188
                                                                               161
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397	197.50,195.701629321330	467	232.50,223.579810991936	537	267.50,239.677424818674	607	302.50,181.859285212842	677	337.50,149.545023898779
398	198,196.147552145974	468	233,223.936101927193	538	268,239.452437355801	608	303,181.189067858836	678	338,149.248392844472
399	198.50,196.595968692414	469	233.50,224.289120595498	539	268.50,239.156902457768	609	303.50,180.527044236890	679	338.50,148.955035659773
400	199,197.038056688476	470	234,224.638866996850	540	269,238.792273853934	610	304,179.873214347004	680	339,148.664894828469
401	199.50,197.477569728222	471	234.50,224.984150416015	541	269.50,238.360361553298	611	304.50,179.227578189178	681	339.50,148.377912834345
402	200,197.916230122954	472	235,225.324687492264	542	270,237.865732358561	612	305,178.590135763412	682	340,148.094032161189
403	200.50,198.354037872671	473	235.50,225.661501364493	543	270.50,237.313672624886	613	305.50,177.960033855077	683	340.50,147.813195292787
404	201,198.790992977375	474	236,225.994592032704	544	271,236.700867906306	614	306,177.331368914081	684	341,147.535344712925
405	201.50,199.226319103891	475	236.50,226.323959496897	545	271.50,236.040521434002	615	306.50,176.709938265361	685	341.50,147.262821246040
406	202,199.654713287553	476	237,226.649603757070	546	272,235.330127311447	616	307,176.095741908915	686	342,146.993996519281
407	202.50,200.080281371945	477	237.50,226.971524813224	547	272.50,234.572874217001	617	307.50,175.488779844743	687	342.50,146.728313233293
408	203,200.503023357067	478	238,227.289722665359	548	273,233.781203753630	618	308,174.889052072846	688	343,146.465725073647
409	203.50,200.922939242919	479	238.50,227.604197313476	549	273.50,232.948442377410	619	308.50,174.296558593224	689	343.50,146.206185725918
410	204,201.340029029501	480	239,227.914948757573	550	274,232.087970406422	620	309,173.711299405877	690	344,145.949648875679
411	204.50,201.753436905308	481	239.50,228.222316841883	551	274.50,231.199382298605	621	309.50,173.133274510804	691	344.50,145.696068208502
412	205,202.162184564500	482	240,228.526172334874	552	275,230.285506260074	622	310,172.562483908005	692	345,145.445397409962
413	205.50,202.566978343866	483	240.50,228.826394636166	553	275.50,229.364044269074	623	310.50,171.998927597481	693	345.50,145.197590165630
414	206,202.967628005885	484	241,229.122983745759	554	276,228.421718018414	624	311,171.442605579232	694	346,144.952600161082
415	206.50,203.363954198229	485	241.50,229.415939663653	555	276.50,227.458527508093	625	311.50,170.893517853258	695	346.50,144.710381081889
416	207,203.756062559616	486	242,229.705262389848	556	277,226.474472738112	626	312,170.351664419558	696	347,144.470886613625
417	207.50,204.143596964843	487	242.50,229.990951924345	557	277.50,225.494728491143	627	312.50,169.817045278132	697	347.50,144.234070441863
418	208,204.526829776326	488	243,230.273008267143	558	278,224.505719745238	628	313,169.285735071758	698	348,143.999886252177
419	208.50,204.905224289991	489	243.50,230.551431418242	559	278.50,223.506089606568	629	313.50,168.756930481207	699	348.50,143.768287730139
420	209,205.280249241609	490	244,230.826221377642	560 561	279,222.495838075134	630	314,168.234318731605	700	349,143.539228561324
421	209.50,205.652290210828	491	244.50,231.102102572260 245,231.375160386375	561 562	279.50,221.487387025584 280,220.482038345985	631	314.50,167.717899822952 315,167.207673755248	701 702	349.50,143.312662431303
422	210,206.022151250448	492	245,231.375160386375 245.50,231.645023966514	562 563	280,220.482038345985 280.50,219.474027723767	632		/02	350,143.088543025650
423 424	210.50,206.390150732612 211,206.756966179589	493 494	245.50,231.645023966514 246,231.911693312677	563 564	280.50,219.474027723767 281,218.463355158929	633 634	315.50,166.703640528493 316,166.205800142687		
424	411,400.730700177307	4.74	4-10,431.7110733140//	304	201,210.400000100727	0.04	510,100.205000142007		

Q2_trend.csv 问题二中速度在 65 - 100 期间的各项指标:

速度 v	最大梯度	最小梯度	升温时间	高温时间	峰值温度	满足状态(值为0时满足所有条件)
100	2.66075563748363	-1.87085565037546	104	31.5	230.964103878399	4
99.5	2.65776880341623	-1.88163670843562	103.5	32.5	231.169025428978	4
99	2.65869034628008	-1.87835952717262	104	33	231.319912826064	4
98.5	2.65919905538394	-1.8885499589926	105	33	231.531683144739	4
98	2.65562323539477	-1.88481239234665	106	33.5	231.710606381595	4
97.5	2.65503197393994	-1.89147742934426	106.5	34	231.865710600343	4

9.5. 2.66019/98/05/211 19/14/2074/67/35 108.5 35.5 222.687/10/37/65 4 9.6. 2.66585183442031 19/04/227746745 108.5 35.5 222.687/10/37/65 4 9.6. 2.66585183442031 19/04/227746745 108.5 35.5 222.687/10/37/65 4 9.6. 2.66595183442031 19/04/227746745 108.5 35.5 222.687/10/37/65 4 9.6. 2.66595183442031 19/04/227746745 108.5 35.5 222.687/10/37/25 4 9.6. 2.66595189969997 19/04/2074676548 110.5 37 233.096/202090 4 9.7. 2.66518798944 19/158303791138 111.5 38 233.56214998947 4 9.7. 2.66595789956 1-19.21589617404 112 38.5 233.5743007882 4 9.7. 2.6659571672061 1-19.25267917476 114 39.5 224.303331918 4 9.7. 2.6659571672061 1-19.25267914746 114 39.5 224.303381918 4 9.7. 2.6659571672061 1-19.25267914746 114 39.5 224.303381918 4 9.7. 2.6659571672061 1-19.25267914746 114 39.5 224.303381918 4 9.7. 2.6659571672061 1-19.25267914746 114 39.5 224.303831918 4 9.7. 2.665959769944 1-19.457004981314 114.5 40.5 224.69813015292 4 9.7. 2.665959769944 1-19.457004981314 114.5 40.5 224.69813015292 4 9.7. 2.665959769944 1-19.457004981314 114.5 41 224.6786761377 4 9.8. 2.66595980769944 1-19.4470318314 114.5 41 224.6786761377 4 9.8. 2.66595818697947 1-19.4470318314 114.5 42.5 235.1058793514 4 9.8. 2.665958186473 1-9441030618714 111.5 43.5 235.1058793514 4 9.8. 2.665958186473 1-9441030618714 111.5 43.5 235.1058793514 4 9.8. 2.665958186473 1-9441030618714 111.5 43.5 235.1058793514 4 9.8. 2.665958186473 1-9441030618714 111.5 43.5 235.1058793514 4 9.8. 2.665958186473 1-9441030618714 111.5 43.5 235.1058793514 4 9.8. 2.665958186473 1-9441030618714 111.5 44.5 225.25806787577 4 9.8. 2.665958186473 1-9441030618714 111.5 43.5 225.25806787575 4 9.8. 2.665958186473 1-944105866714 111.5 43.5 225.2580679777							
9.5. 2.65052183442631 -1.9074227746745 108.5 35.5 22.430584989425 4 9.5. 2.6598798098975 -1.908214502393 109 36 22.6597410912 4 9.5. 2.6598798098975 -1.908214502393 109 36 22.659814907355 4 9.5. 2.659879809897 -1.91914579665468 1105 37 23.000452052909 4 9.5. 2.661281798349 -1.91914579665468 1105 37 23.000452052909 4 9.5. 2.661281798349 -1.91914579665468 1105 37 23.5100272418033 4 9.5. 2.661281798349 -1.91914579665468 1105 37 23.5100272418033 4 9.5. 2.661281798949 -1.91914579665468 1105 37 23.5100272418033 4 9.5. 2.6623810579956 -1.930152861740 112 38.5 23.573430607882 4 9.5. 2.6598791672081 -1.9262309471426 114 39.5 23.5426157932 4 9.5. 2.6598791672081 -1.9262309471426 114 39.5 23.494813015292 4 9.5. 2.6598791672081 -1.9262309471426 114 39.5 24.303341918 4 9.5. 2.6598791672081 -1.9467200493318 1145 405 22.469813015292 4 9.5. 2.6598209079744 -1.94472004893318 1145 41 224.6786743177 4 9.5. 2.6598209079744 -1.944720638131 1145 41 224.6786743177 4 9.5. 2.6598209079744 -1.944720638131 1145 41 224.6786743177 4 9.5. 2.659930978944 -1.9447426265 9.5. 2.65973198571997 -1.968743592091 11 44 42.5 23.51368973514 4 9.5. 2.65973198571997 -1.968743592081 11 9.5. 2.6597319857994 -1.948747426265 9.5. 2.65973198571997 -1.968743592081 1 9.5. 2.65973198571997 -1.968743592081 1 9.5. 2.65973198571997 -1.968743592081 1 9.5. 2.65973198571997 -1.968743592081 1 9.5. 2.65973198571997 -1.99878435986 110 45 22.6597803847 9.5. 2.659741666731 -1.9437452655 9.5. 2.659746166731 -1.993751981084922 9.5. 2.659746166731 -1.993751981084922 9.5. 2.659746166731 -1.993751981084922 9.5. 2.659746166731 -1.99375198108492 9.5. 2.659746166731 -1.99375198108492 9.5. 2.659746166731 -1.99375198108492 9.5. 2.659746166731 -1.99375198108492 9.5. 2.659746166731 -1.99375198108492 9.5. 2.659746166731 -1.993751506103 9.5. 2.659746166731 -1.993751506103 9.5. 2.659746166731 -1.993751506103 9.5. 2.659746166731 -1.993705559996 9.5. 2.659746166731 -1.993705559996 9.5. 2.659746166731 -1.99370555	97	2.66117420266956	-1.89146653853464	106.5	34.5	232.053139819539	4
6.5.2 2.66898863757747 -1.90121.646795154 10.8 36 22.2688418973255 4 94.5 2.65899215785902 -1.90021313801122 110 37 233.000873025099 4 94.5 2.6614185149772 -1.91048795658488 110.5 37 233.0004972025099 4 95.5 2.6651918785449 -1.91048795658488 110.5 37 233.004967207727 4 95.2 2.665291816941675 -1.9155689713081 112 38.5 235.52460872947 4 91.5 2.6580791672081 -1.92527120813008 113 39 233.934604759902 4 91.5 2.6580791672081 -1.925203941746 114 39.5 2.241.937537640 4 91.5 2.658078164674 -1.9245700483184 114 39.5 2.241.94813015292 4 90.5 2.657575415466 -1.9135720451055 114.5 40.5 224.94813015292 4 93.5 2.6659209775744 -1.914474262652 114 41.5 225.507596291941 4	96.5	2.65600198795241	-1.89195556044842	107	35	232.268710443765	4
9.5. 2.6587915818977. 9.1. 2.6691281789812. 9.1. 9.1. 11. 11. 11. 11. 11. 11. 11. 11	96	2.65552183442631	-1.90742277465745	108.5	35.5	232.430589489425	4
9.4 2.6544.061.99772 - 1.098279.5784.0 110. 37 233.00952052090 4 9.4 2.6514.061.99773 - 1.0916879.6584.0 110.5 37.5 233.79461247172 4 9.3.5 2.66012817983440 - 1.9155068919369 111.5 37.5 233.79461247172 4 9.5.2 2.6592.816.09656 - 1.091528861740 112 38.5 233.57249052747 4 9.5.2 2.6592.816.09656 - 1.091528861740 112 38.5 233.572490507832 4 9.1 2.6592.77347247 - 1.92518701284626 114.5 39.5 234.1947839572 4 9.1 2.6592.77347247 - 1.92918970424636 114.5 39.5 234.19481015292 4 9.0 2.6592.873458466 - 1.09152.23462125 114.5 40.5 224.49481015292 4 9.0 2.6592.8883292488 - 1.09461147183663 115.5 41 234.67667613777 4 8.9 2.6592.0979.7974 - 1.094570.0189318 114.5 39.5 234.19481015292 4 8.8 2.6592.0907.99744 - 1.94474240652 114. 41.5 235.057604239147 4 8.8 2.6592.0979.7974 - 1.094870.0189318 114. 41.5 235.057604239147 4 8.8 2.6512.865799462 - 1.0941031122418 114. 42.5 235.21088993514 4 8.8 2.6512.865799462 - 1.094870.0189318 111.5 43.5 235.957604239147 4 8.8 2.6512.865799462 - 1.094870.0189318 111.5 43.5 235.957604239147 4 8.8 2.6512.865799462 - 1.094870.0189318 111.5 43.5 235.957604239147 4 8.8 2.6512.865799462 - 1.094870.0189318 111.5 43.5 235.9579608857 4 8.6 3 2.657618.251479 - 1.0948794527917 112.5 43 225.584960787567 4 8.6 3 2.657618.251479 - 1.0948748527917 112.5 43 225.584960787567 4 8.6 3 2.65763.053449 - 1.094798519197 110 45 226.384927167967 4 8.5 2 2.6567850384273 - 1.0947985191891 110.5 43.5 225.58496085925 4 8.6 2 2.657960.053449 - 1.094798519197 110 45 226.384927167967 4 8.8 2 2.656980.0533246 - 1.094798519197 110 45 226.284985851337 4 8.4 2.656.083327667 - 1.0947985191891 110 47 226.2840593522 4 8.5 2 2.6567960.05349 - 1.094798519197 110 45 226.284985781337 4 8.5 2 2.6567960.053349 - 1.094798519197 110 45 226.284985781337 4 8.6 2 2.657960.05349 - 1.094798519197 110 45 226.08495875177 4 8.7 2 2.65289801183 - 2.09479851981 110 5 48.5 226.084857857877 4 8.8 2 2.656980.053294 - 2.09479851981 110 5 48.5 226.084857857877 4 8.9 2 2.656980.05339 - 2.0018980.05418 1 100 5 5 2 228.08495191750	95.5	2.66038868757671	-1.90121646795154	108	36	232.653714410912	4
9.4	95	2.65987918908976	-1.90024459023903	109	36	232.838418973255	4
2.6502817987994034	94.5	2.65349215785902	-1.90821313804122	110	37	233.006952052909	4
9.2	94	2.65414636149772	-1.91048795655468	110.5	37	233.160272418033	4
9.2 2 2.685238160789565 -1.92015288617404 112 38.5 223.738430607882 4 91.5 2 2.6806791672081 -1.92625039417436 114 39.5 224.136778357602 4 91.5 2 2.6822877427247 -1.92628039417436 114.5 39.5 224.13078357602 4 90.5 2 2.6872875418466 -1.9418722462615 114.5 39.5 224.30383149184 4 90.5 2 2.68528912979744 -1.9448700498318 115.5 41 224.678676135777 4 88.5 2 2.685240026799744 -1.9448700498318 114.4 41.5 235.078604239147 4 88.5 2 2.68797613425181 -1.94401031132418 114 42.5 235.1978604239147 4 88.5 2 2.687198867937 -1.948770265714 113 42.5 235.2130887939144 4 8.7 2 2.6879718867937 -1.94871059667914 113 42.5 235.2130887937144 4 8.7 2 2.68797188672 -1.94871059667914 113 42.5 235.257975866184	93.5	2.66012817983449	-1.9155068919369	111.5	37.5	233.37961247172	4
92	93	2.65979975994034	-1.91755337501138	111.5	38	233.562140982947	4
92	92.5	2.65238150789556	-1.92015288617404	112	38.5	233.753430607882	4
91	92	2.65923816841675	-1.92547120843028	113			4
91	91.5	2.65867916172081	-1.92625039417436	114	39.5	234.136778357602	4
90. 2.65772774718466 -1.93152324621025 114.5 40.5 234.494813015022 4 4 989.5 2.6586883329468 -1.944601147183663 115.5 41 234.676876413777 4 4 98.5 2.65869329739744 -1.9445704983184 114.5 41 234.676876413777 4 4 4 4 4 4 4 5 235.1057601239147 4 4 8 8 2.65824902679344 -1.9445704983184 114.4 41.5 235.057601239147 4 4 8 8 2.65124687999462 -1.94401031132418 114 4 2.5 235.2310587935314 4 4 8 2 5 235.24687999462 -1.94401031132418 114 4 2.5 235.2310587935314 4 4 8 2 5 235.24687999462 -1.94874453727073 112.5 43 235.584960787567 4 4 8 2 5 2 5 2 5 2 5 2 5 2 5 2 5 2 5 2 5 2		2.65282773427427					
99							
89.5 2.6563920073744 -1.9445704863184 114.5 41 234.8892.65610596 4							
88 2.65824902679344 1.9434744262652 114 41.5 235.057604239147 4 88.5 2.65797613425181 1.94401031132418 114 42.5 235.22547966688 4 87.5 2.6570182514799 1.19454165727073 112.5 43 235.584960787567 4 86.7 2.657373158870737 112.5 43 235.5997138462 4 86.7 2.65715938621582 -1.950854339336308 111.5 43 235.7997138462 4 86.5 2.6571938621582 -1.96239393908088 110 445 226.132480799767 4 85.5 2.6508902871286 -1.96239393908288 110 45 236.349427167967 4 84.5 2.6561903877686 -1.9771891918891233 108.5 46.5 236.50307388778 4 84.5 2.6561033327467 -1.97129931885616 107.5 47 236.86104141258 4 83.5 2.6502451505164 -1.9981753480986 106 48 237.2701099932348 4 85.5							
88.5 2.65797613425181 -1.94401031132418 114 42.5 235.213058793514 4 88 2.65124687999462 -1.94341059607914 113 42.5 235.42257966588 4 87.5 2.6570182514799 -1.94854153727073 112.5 43 235.528957138642 4 86.5 2.6571938621582 -1.9571784290097 111 44 235.57897138642 4 86.5 2.6571938621582 -1.9571784290097 111 44 235.57897138677 4 85.5 2.650785808473 -1.960239359306088 110 45 265.349427167967 4 85.5 2.6567855038473 -1.960239359306088 110 45 266.540370387778 4 84.5 2.65619501877686 -1.9718991802516 10.75 47 236.6501937387778 4 84.5 2.65899997112281 107 47 256.6510373387377 4 83.5 2.65899999112251 107 47 256.60141412528 4 82.5 2.65314002338419							
88 2.651246879999402 1.94874453727073 112.5 43 235.84960787567 4 87.5 2.6576182514799 -1.94874453727073 112.5 43 235.84960787567 4 87.2 2.65737158877937 -1.99088433993608 111.15 43.5 235.75957188642 4 86.5 2.65715938621582 -1.95171842490097 111 44 235.95396906887 4 86.5 2.65038092884012 -1.96123497381888 110 45 236.349427167967 4 85.5 2.650980586473 -1.996498517919 45.5 265.05037388778 4 84.6 2.65598190187686 -1.97189189189223 108.5 46.5 236.50937385778 4 84.5 2.656190317766 -1.97189189189223 105.5 46.5 226.50937333374 4 84.5 2.656190317766 -1.978172840986 106 48 237.268814014125233 4 85.2 2.6579297112281 -1.99804749404747 105 48.5 2.256786232571177 4							
87.5 2.6576182514799 - 1.94874453727073							
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	65.5	2.6419810414464	-2.07892992586602	123.5	69	243.480000282951	2

表 5: 速度在 65 - 100 期间的各项指标

8.3 数据来源

- 热导率数据
 - https://en.wikipedia.org/wiki/Thermal_conductivity
- 密度数据
 - https://en.wikipedia.org/wiki/Density

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