CS 680 PSet 1: Linear Algebra Self-Assessment

Solutions

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Question 1:

(a) Given points $\mathbf{p}_1 = (1, 6, 5)$ and $\mathbf{p}_2 = (5, 3, -7)$, solve for $\mathbf{v}_2$ the vector from $\mathbf{p}_1$ to $\mathbf{p}_2$.
$\mathbf{v}_2 = (4, -3, -12)$

(b) Given a third point $\mathbf{p}_3 = (1, 6, 4)$, solve for $\mathbf{v}_3$ the vector from $\mathbf{p}_1$ to $\mathbf{p}_3$.
$\mathbf{v}_3 = (0, 0, -1)$

(c) Find the values for the magnitudes of $\mathbf{v}_2$ and $\mathbf{v}_3$.
$||\mathbf{v}_2|| = 13, ||\mathbf{v}_3|| = 1$

(d) Solve for the unit vectors in the directions of $\mathbf{v}_2$ and $\mathbf{v}_3$.
$\frac{1}{13}(4, -3, -12), (0, 0, -1)$

Question 2:

(a) Solve for the vector (cross) product $\mathbf{v}_2 \times \mathbf{v}_3$.
$(3, 4, 0)$

(b) Solve for $\mathbf{v}_3 \times \mathbf{v}_2$.
$(-3, -4, 0)$

(c) Solve for the scalar (dot) product $\mathbf{v}_3 \cdot \mathbf{v}_2$.
12

Question 3:

(a) If two vectors $\mathbf{u}, \mathbf{v} \in \Re^n$ are orthogonal, what is the value of their scalar (dot) product?
0

(b) If two vectors $\mathbf{u}, \mathbf{v} \in \Re^n$ are parallel, what is the value of their cross product?
0

Question 4:

Which of the following are unit vectors? (a) $(\frac{1}{2}, -\frac{1}{2}, 0)$ (b) $(0, -1, 0)$ (c) $\frac{1}{7}(-2, 3, 6)$
(b), (c)

Question 5:

We are given two non-zero vectors $\mathbf{u}, \mathbf{v} \in \mathbb{R}^3$. Assume the angle between $\mathbf{u}$ and $\mathbf{v}$ satisfies $0 < \theta < \frac{\pi}{2}$. Use dot products and/or cross products of $\mathbf{u}$ and $\mathbf{v}$ to give expressions for:

(a) $\cos\theta$ (b) $\sin\theta$ (c) A vector perpendicular to both $\mathbf{u}$ and $\mathbf{v}$.

Norm notation: $||\mathbf{w}|| = \sqrt{\mathbf{w} \cdot \mathbf{w}}$
$\frac{\mathbf{u} \cdot \mathbf{v}}{||\mathbf{u}||||\mathbf{v}||}$, $\frac{||\mathbf{u} \times \mathbf{v}||}{||\mathbf{u}||||\mathbf{v}||}$, $\mathbf{u} \times \mathbf{v}$

Question 6:

Given three square matrices $\mathbf{Q}, \mathbf{R}, \mathbf{S} \in \mathbb{R}^{n \times n}$, which statements are true **in general**? If the statement is false, please correct it.

(a) $(\mathbf{QRS})^{-1} = \mathbf{Q}^{-1}\mathbf{R}^{-1}\mathbf{S}^{-1}$, *False*, $(\mathbf{QRS})^{-1} = \mathbf{S}^{-1}\mathbf{R}^{-1}\mathbf{Q}^{-1}$

(b) $\mathbf{QR} = \mathbf{RQ}$, *False*, $\mathbf{QR} = \mathbf{QR}$, or $\mathbf{QR} = \mathbf{R}(\mathbf{R^{-1}QR}) = (\mathbf{QRQ^{-1}})\mathbf{Q}$ if $\mathbf{Q}, \mathbf{R}$ invertible

(c) $(\mathbf{QRS})^T = \mathbf{S}^T\mathbf{R}^T\mathbf{Q}^T$, *True*

(d) $(\mathbf{R} + \mathbf{S})\mathbf{Q} = \mathbf{SQ} + \mathbf{RQ}$, *True*

Question 7:

Given a square matrix $\mathbf{A} \in \mathbb{R}^{n \times n}$ whose columns form an orthonormal basis:

(a) What is the dot product of any pair of columns in $\mathbf{A}$?
0, if the columns are distinct; 1, if they are the same.

(b) What is the inverse of $\mathbf{A}$?
$\mathbf{A}^T$