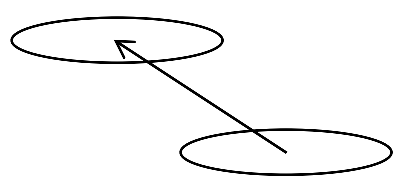
CS 480/CS680 PSet 3: Curves & Surfaces

Due: November 18, 2021 at 9:30 am

Submission guidelines:
Please prepare your answers neatly written or typed, on separate 8.5"x11" sheets. Submit **on Blackboard**. If your answers are hand-written please either upload a scan or photos of your answer sheets.

1. (20 points; 5 points extra credit)
   We are given the implicit function for a two-sheeted hyperboloid,
$$f(x, y, z) = (x - c_x)^2 + (y - c_y)^2 - z^2 + r^2.$$
   a) Give the 4×4 matrix $\boldsymbol{Q}$ for this cylindrical surface, such that $\boldsymbol{p}^T\boldsymbol{Q}\boldsymbol{p} = 0$ for any homogeneous point $p = (x, y, z, 1)$ that is on the cylindrical surface.
   b) Using the implicit function, derive a function that gives a unit normal vector at any point on the surface (x, y, z).
   c) Give an equivalent parametric equation for the sheet of the surface that is above the xy-plane ($z \geq 0$), in terms of θ and z. Assume that $-\pi \leq \theta \leq \pi$ and $z_{min} \leq z \leq z_{max}$. Give the parameters for z_{min}, z_{max}.
   d) Using the parametric equation, derive the function that gives the unit normal vector at any point on the sheet with parameter (θ, z).
   e) Extra Credit (5 points): Give a parametrization that utilizes hyperbolic trigonometric functions, and find the unit normal with this parametrization.

2. (30 points)
   We are given a 3D swept surface. A line segment with end points $\boldsymbol{p_0} = (2,0,0)$ and $\boldsymbol{p_1} = (0,0,6)$ is used as the sweep curve. The cross-section is an ellipse in the $z = 0$ plane, where $x = 2 + r_x \cos(2\pi v)$, $y = r_y \sin(2\pi v)$, $0 \leq v \leq 1$. The center of the ellipse is swept along the vector from $\boldsymbol{p_0}$ to $\boldsymbol{p_1}$, such that it remains parallel to the *x*-*y* plane.



   a) Derive the parametric equation in u for the directed line segment $\boldsymbol{p_0}\,\boldsymbol{p_1}$.
   b) Derive an equation for location of the point $\boldsymbol{P}(u, v)$ on the swept surface.
   c) Derive the equation for the swept surface normal $\boldsymbol{N}(u, v)$.

3. (20 points)
   We are given a 2D cubic Bezier curve segment, which has the following control points:
$$\boldsymbol{p_0} = (2, 3)$$
$$\boldsymbol{p_1} = (3, 0)$$

$\boldsymbol{p_2}$ = (4, 4)
$\boldsymbol{p_3}$ = (5, 1)

a) Draw the convex hull for this 2D Bezier curve segment.

b) Compute the value of $P'(0)$ for this 2D Bezier curve segment.

c) We are now given a second 2D Bezier curve segment, which has the control points:

$\boldsymbol{p_0}$ = (0, -1)
$\boldsymbol{p_1}$ = (-2, 4)
$\boldsymbol{p_2}$ = (0, 9)
$\boldsymbol{p_3}$ = (2, 3)

Does this segment join the previous segment with C1 continuity? Give a mathematical justification for your answer.

d) Which control point above (for the second curve in (c)) may we change to achieve C1 continuity? Write down the new position of the point that will achieve this.

4. (30 points)
We are given the following boundary conditions for a cubic spline section:

$$\boldsymbol{P}(0) = \boldsymbol{p}_k$$
$$\boldsymbol{P}(1) = \boldsymbol{p}_{k+1}$$
$$\boldsymbol{P}'(0) = \tfrac{1}{2}[(1+b)(\boldsymbol{p}_k - \boldsymbol{p}_{k-1}) + (1-b)(\boldsymbol{p}_{k+1} - \boldsymbol{p}_k)]$$
$$\boldsymbol{P}'(1) = \tfrac{1}{2}[(1+b)(\boldsymbol{p}_{k+1} - \boldsymbol{p}_k) + (1-b)(\boldsymbol{p}_{k+2} - \boldsymbol{p}_{k+1})]$$

In the textbook, we see this is a Cardinal Spline (Kochanek-Bartels spline with t=0 and c=0). In this case $M_{geom} = [p_{k-1}\ p_k\ p_{k+1}\ p_{k+2}]^T$ and the boundary conditions can be written:

$$\begin{bmatrix} \boldsymbol{P}(0) \\ \boldsymbol{P}(1) \\ \boldsymbol{P}'(0) \\ \boldsymbol{P}'(1) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\frac{1}{2}(1+b) & b & \frac{1}{2}(1-b) & 0 \\ 0 & -\frac{1}{2}(1+b) & b & \frac{1}{2}(1-b) \end{bmatrix} \begin{bmatrix} \boldsymbol{p}_{k-1} \\ \boldsymbol{p}_k \\ \boldsymbol{p}_{k+1} \\ \boldsymbol{p}_{k+2} \end{bmatrix}$$

a) Show how to compute the 4×4 coefficient matrix M_C given the boundary conditions written above. You do not need to compute a matrix inverse to find M_C (the relevant one is given in the textbook anyway). Just give the specific equations for M_C.

b) Given M_C write out the blending functions for this curve.

c) Do adjacent segments satisfy C1 continuity? Give a mathematical justification.

d) Does adjusting b change the tangent direction at the endpoints or only magnitude?