PS2

Question 1

(a)

```
/*
Input:  vertices[N]  //List of N polygon vertices
        point p      //A point within the interior
Output: True or False  //True if ccw, else False
*/
vertices.append(vertices[0])
find a point q outside the polygon by using MBR
pq = q - p             //pq vector
winding = 0
for i: 0 -> N-1
    if segsIntersect(vertices[i+1],verices[i],p,q):
        e = vertices[i+1] - verices[i]
        cross = e.x*pq.y - e.y*pq.x   //cross product of edge and pq
        winding += 1 if cross > 0 else -1
if winding > 0: return True
else: return False
```

(b)

Pick a point p near vertices within the MBR of the polygon and a point q outside the MBR. Calculate the winding number of point p. If winding number is 1 or -1, p is a point that in the interior of the polygon. If winding number is 0, find another point p until meet the requirement. (Mahir Patel and I discussed the approaches in this question.)

Question 2

(a)

The transform matrix should be:

$$\begin{matrix} \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} & a \\ 0 & 3 & 0 & b \\ -\frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} & c \\ 0 & 0 & 0 & 1 \end{matrix}$$

(b)

The order should be: $S(1,3,-1)R_y(-\frac{\pi}{4})T(a,b,c)$

which is:

$$\begin{matrix} 1 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{matrix} \times \begin{matrix} \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} & 0 \\ 0 & 1 & 0 & 0 \\ \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & 1 \end{matrix} \times \begin{matrix} 1 & 0 & 0 & a \\ 0 & 1 & 0 & b \\ 0 & 0 & 1 & c \\ 0 & 0 & 0 & 1 \end{matrix}$$

Question 3

Variables version: $T(a,b)Shear(\alpha,0)T(-a,-b)$

$$\begin{matrix} 1 & 0 & a \\ 0 & 1 & b \\ 0 & 0 & 1 \end{matrix} \times \begin{matrix} 1 & \alpha & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix} \times \begin{matrix} 1 & 0 & -a \\ 0 & 1 & -b \\ 0 & 0 & 1 \end{matrix}$$

Values version: $T(3,4)Shear(\frac{1}{2},0)T(-3,-4)$

$$\begin{matrix} 1 & 0 & 3 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{matrix} \times \begin{matrix} 1 & \frac{1}{2} & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix} \times \begin{matrix} 1 & 0 & -3 \\ 0 & 1 & -4 \\ 0 & 0 & 1 \end{matrix}$$

Question 4

(a)

We have $q_1 = (0, 0, \frac{1}{2}, \frac{\sqrt{3}}{2})$, Set the unit vector $\mu_1 = (x_1, y_1, z_1)$, we got:

$$\begin{matrix} \cos\left(\frac{\theta_1}{2}\right) = 0 \\ \sin\left(\frac{\theta_1}{2}\right) x_1 = 0 \\ \sin\left(\frac{\theta_1}{2}\right) y_1 = \frac{1}{2} \\ \sin\left(\frac{\theta_1}{2}\right) z_1 = \frac{\sqrt{3}}{2} \\ x_1^2 + y_1^2 + z_1^2 = 1 \end{matrix} \Rightarrow \text{one of the answers is:} \quad \begin{matrix} \theta_1 = \pi \\ \mu_1 = (0, \frac{1}{2}, \frac{\sqrt{3}}{2}) \end{matrix}$$

(b)

We have $q_2 = (\frac{\sqrt{3}}{2}, 0, -\frac{\sqrt{3}}{4}, \frac{1}{4})$, Set the unit vector $\mu_2 = (x_2, y_2, z_2)$, we got:

$$\begin{matrix} \cos\left(\frac{\theta_2}{2}\right) = \frac{\sqrt{3}}{2} \\ \sin\left(\frac{\theta_2}{2}\right) x_2 = 0 \\ \sin\left(\frac{\theta_2}{2}\right) y_2 = -\frac{\sqrt{3}}{4} \\ \sin\left(\frac{\theta_2}{2}\right) z_2 = \frac{1}{4} \\ x_2^2 + y_2^2 + z_2^2 = 1 \end{matrix} \Rightarrow \text{one of the answers is:} \quad \begin{matrix} \theta_2 = \frac{\pi}{3} \\ \mu_2 = (0, -\frac{\sqrt{3}}{2}, \frac{1}{2}) \end{matrix}$$

(c)

No. Because μ_1 and μ_2 are neither in the same direction nor opposite, and neither θ_1 nor θ_2 is zero.

(d)

Two rotations commute when at least one of θ_1 and θ_2 is zero, or, μ_1 and μ_2 are in the same direction or opposite. Otherwise they do not.

Question 5

In this case, the unit vector along the u-axis should be $u = (\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2})$. Similarly, we have $v = (-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2})$.

In order to scale the object with the requirement, we have operations:
$T_{out}(7,5,0)R_{out}(u,v)S(S_u,1,1)R_{in}(-u,-v)T_{in}(-7,-5,0)$, which in this case should be:

$$\begin{matrix} 1 & 0 & 0 & 7 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{matrix} \times \begin{matrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 & 0 \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{matrix} \times \begin{matrix} S_u & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{matrix} \times \begin{matrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 & 0 \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{matrix} \times \begin{matrix} 1 & 0 & 0 & -7 \\ 0 & 1 & 0 & -5 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{matrix}$$

Question 6

(I follow the 2D example in the note of lecture 6, but there's still something I couldn't figure out.)

We have a plane P with equation $ax + by + cz = d$, so there is a vector $N = (a, b, c)$ that is perpendicular to the plane P.

Set the vector $v = \frac{N}{\|N\|} = (v_x, v_y, v_z)$ is the unit vector of N. And two unit vectors $u = (u_x, u_y, u_z)$ and $w = (w_x, w_y, w_z)$ perpendicular to each other in plane P .

Set a point $p_0(p_x, p_y, p_z)$ in the plane P.

To reflect 3D points about plane P, we can: ($R_{out} = R^T{}_{in}$)

$$T_{out}(P_0)R_{out}(u, v, w)S(0, 0, 1)R_{in}(-u, -v, -w)T_{in}(-P_0)$$

which in matrices is:

$$\begin{matrix} 1 & 0 & 0 & p_x \\ 0 & 1 & 0 & p_y \\ 0 & 0 & 1 & p_z \\ 0 & 0 & 0 & 1 \end{matrix} \times \begin{matrix} u_x & v_x & w_x & 0 \\ u_y & v_y & w_y & 0 \\ u_z & vz & w_z & 0 \\ 0 & 0 & 0 & 1 \end{matrix} \times \begin{matrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{matrix} \times \begin{matrix} u_x & u_y & u_z & 0 \\ v_x & v_y & v_z & 0 \\ w_x & w_y & w_z & 0 \\ 0 & 0 & 0 & 1 \end{matrix} \times \begin{matrix} 1 & 0 & 0 & -p_x \\ 0 & 1 & 0 & -p_y \\ 0 & 0 & 1 & -p_z \\ 0 & 0 & 0 & 1 \end{matrix}$$