1.

(a)

Let $p^T Q p = [x\ y\ z\ 1] \begin{bmatrix} A & B & C & D \\ B & E & F & G \\ C & F & H & I \\ D & G & I & J \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = Ax^2 + 2Bxy + 2Cxz + 2Dx$

$+ Ey^2 + 2Fyz + 2Gy$

$+ Hz^2 + 2Iz + J$

$f(x,y,z) = (x-C_x)^2 + (y-C_y)^2 - z^2 + r^2$

$= x^2 - 2C_x x + C_x^2 + y^2 - 2C_y \cdot y + C_y^2 - z^2 + r^2$

$= x^2 + 0 \cdot xy + 0 \cdot x \cdot z + 2(-C_x) \cdot x$

$+ y^2 + 0 \cdot yz + 2(-C_y) \cdot y$

$+ (-1) z^2 + 0 \cdot z + r^2$

$\Rightarrow Q = \begin{bmatrix} 1 & 0 & 0 & -C_x \\ 0 & 1 & 0 & -C_y \\ 0 & 0 & -1 & 0 \\ -C_x & -C_y & 0 & r^2 \end{bmatrix}$

(b)

$$n(x,y,z) = \nabla f(x,y,z) = \begin{bmatrix} \frac{df}{dx} \\ \frac{df}{dy} \\ \frac{df}{dz} \end{bmatrix} = \begin{bmatrix} 2(x-C_x) \\ 2(y-C_y) \\ -2 \end{bmatrix}$$

(c)

$$P(\theta,z) = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \sqrt{z^2-r^2} \cdot \cos\theta + C_x \\ \sqrt{z^2-r^2} \cdot \sin\theta + C_y \\ z \end{bmatrix}, \quad r \leq z \leq +\infty$$

(d)

$$n(\theta,z) = \frac{dp}{d\theta} \times \frac{dp}{dz} = \begin{bmatrix} -\sqrt{z^2-r^2} \cdot \sin\theta \\ \sqrt{z^2-r^2} \cdot \cos\theta \\ 0 \end{bmatrix} \times \begin{bmatrix} \cos\theta \cdot z / \sqrt{z^2-r^2} \\ \sin\theta \cdot z / \sqrt{z^2-r^2} \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta \\ \sin\theta \\ \frac{-z}{\sqrt{z^2-r^2}} \end{bmatrix}$$

$$N(\theta,z) = \frac{n(\theta,z)}{\|n(\theta,z)\|}$$

2.

(a)

$$p(u)=\begin{bmatrix}x(u)\\y(u)\\z(u)\end{bmatrix}=(1-u)P_0+uP_1=(1-u)\begin{bmatrix}2\\0\\0\end{bmatrix}+u\begin{bmatrix}0\\0\\6\end{bmatrix}=\begin{bmatrix}2-2u\\0\\6u\end{bmatrix}$$

(b)

$$p(u,v)=\begin{bmatrix}x(u)\\y(u)\\z(u)\end{bmatrix}=\begin{bmatrix}2-2u+r_x\cos(2\pi v)\\r_y\sin(2\pi v)\\6u\end{bmatrix}$$

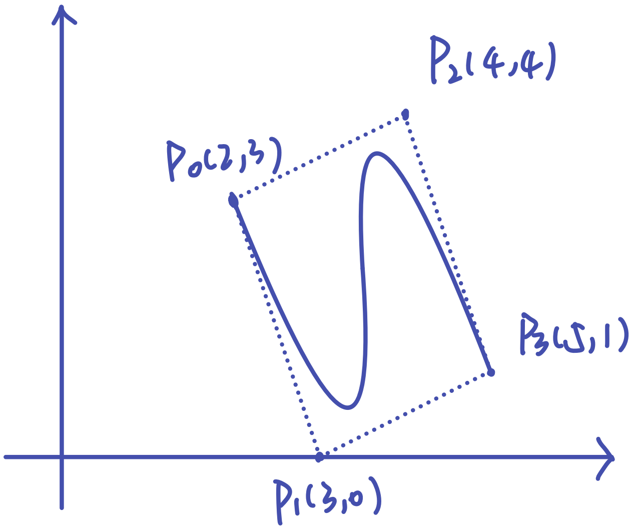
(c)

$$\nabla p(u,v)=\frac{dp}{du}\times\frac{dp}{dv}=\begin{bmatrix}-2\\0\\6\end{bmatrix}\times\begin{bmatrix}-2\pi\cdot r_x\cdot\sin(2\pi v)\\2\pi\cdot r_y\cdot\cos(2\pi v)\\0\end{bmatrix}=\begin{bmatrix}-12\pi\cdot r_y\cdot\cos(2\pi v)\\-12\pi\cdot r_x\cdot\sin(2\pi v)\\-4\pi\cdot r_y\cdot\cos(2\pi v)\end{bmatrix}$$

$$N(u,v)=\frac{\nabla p(u,v)}{\|\nabla p(u,v)\|}$$

3.

(a)



(b)

$$Curve_1(u) = [u^3 \; u^2 \; u \; 1] M_{Bez} \begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \end{bmatrix}, \quad M_{Bez} = \begin{bmatrix} 1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 3 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

$$Curve_1'(u) = [3u^2 \; 2u \; 1 \; 0] M_{Bez} \begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \end{bmatrix}$$

$$Curve_1'(0) = [0 \; 0 \; 1 \; 0] M_{Bez} \begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \end{bmatrix} = -3P_0 + 3P_1 = -3(2,3) + 3(3,0) = (3,-9)$$

(c)

NO.

① P_0 in $Curve_1$ = P_3 in $Curve_2$. C_0 ✓

② Similarly with (b), $Curve_2'(1) = -3P_3 + 3P_3 = (6,-18) \neq Curve_1'(0)$. C_1 ✗

(d)

$P_2(1,6)$

4.

(a)

$$A = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 3 & 2 & 1 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ -\frac{1}{2}(1+b) & b & \frac{1}{2}(1-b) & 0 \\ 0 & -\frac{1}{2}(1+b) & b & \frac{1}{2}(1-b) \end{bmatrix} \quad M_c = \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} \quad M_{geom} = \begin{bmatrix} P_{k-1} \\ P_k \\ P_{k+1} \\ P_{k+2} \end{bmatrix}$$

$$\begin{bmatrix} p(0) \\ p(1) \\ p'(0) \\ p'(1) \end{bmatrix} = A \cdot M_c = B \cdot M_{geom} \Rightarrow M_c = A^{-1} \cdot B \cdot M_{geom}$$