

# Hidden Markov Models

## Practice Problems

### 1. When to Use HMMs

For each of the following scenarios, is it appropriate to use a hidden markov model to model the dataset? Why or why not.

- (a) Stock market price data
- (b) Recommendations on a database of movie reviews (like the book reviews from the first practical)
- (c) Daily precipitation data in Boston
- (d) Optical character recognition

### 2. d-separation

Use the d-separation criterion to verify that the conditional independence properties (13.24)-(13.31) are satisfied by the joint distribution for the hidden Markov model defined by (13.6). (Bishop 13.9)

### 3. Alpha Message

Use the definition (8.64) of the messages passed from a factor node to a variable node in a factor graph, together with the expression (13.6) for the joint distribution in a hidden Markov model, to show that the definition (13.50) of the alpha message is the same as the definition (13.34). (Bishop 13.13)

### 4. Factor Graph

Show that the directed graph for the input-output hidden Markov model, given in Figure 13.18, can be expressed as a tree-structured factor graph of the form shown in Figure 13.15 and write down expressions for the initial factor  $h(z_1)$  and for the general factor  $f_n(z_{n-1}, z_n)$  where  $2 \leq n \leq N$ . (Bishop 13.17)

**5. E-M For HMM's, Bishop 13.6**

Show that if any elements of the parameters  $\pi$  (start probability) or  $A$  (transition probability) for a hidden Markov model are initialized to 0, then those elements will remain 0 in all subsequent updates of the EM algorithm.

**6. E-M For HMM's, Bishop 13.5**

Verify the M-step equation 13.18 (the update rule for  $\pi_k$ ) for the initial state probabilities of the hidden Markov model by maximization of the expected complete-data log likelihood function (given in eq. 13.17), using Lagrange multipliers to enforce the summation constraint on the components of  $\pi$ .