PhD Econometrics I (First Half) matias.guizzo.altube@nyu.edu
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Recitation 1

Exercise 1. An economy that lasts T periods is described by $(Y_t)_{t=1}^T$, where each $Y_t \in \mathbb{R}^K$ is a (random) state variable.
(a) Viewing the economy $(Y_t)_{t=1}^T$ as an outcome of an experiment, formulate an appropriate sample space and σ-algebra $(\Omega, \mathcal{F})$.
(b) An economist, despite being trained at NYU, does not observe Y_t, but instead only observes $h(Y_t)$ for some $\mathcal{F}/\mathcal{B}(\mathbb{R})$-measurable function $h : \mathbb{R}^K \to \mathbb{R}$. Formulate an appropriate sample space and σ-algebra from the economist's perspective.

Exercise 2. Let $\{\mathcal{F}_\gamma : \gamma \in G\}$ be a collection of σ-algebras on Ω, of which G is possibly uncountable. Show that $\mathcal{G} := \bigcap_{\gamma \in G} \mathcal{F}_\gamma$ is a σ-algebra. Using the definition of generated σ-algebra, explain why $\sigma(\mathcal{H})$ for any $\mathcal{H} \in 2^\Omega$ is non-empty and prove that $\sigma(\mathcal{H})$ is an σ-algebra.

Exercise 3. Let $(\Omega, \mathcal{F}, P)$ be a probability space.
(a) Suppose $A, B \in \mathcal{F}$. Show that if $A \subseteq B$ then $P(A) \leqslant P(B)$ and $P(B \backslash A) = P(B) - P(A)$.
(b) Suppose $A_1, A_2, \dots \in \mathcal{F}$. Show that P is *countably subadditivite:*

$$P\left(\bigcup_{i=1}^{\infty} A_i\right) \leqslant \sum_{i=1}^{\infty} P(A_i).$$

Explain why this implies P is *finitely subadditivit*e: $P(\bigcup_{i=1}^n A_i) \leqslant \sum_{i=1}^n P(A_i)$.

Exercise 4. Let $(\Omega, \mathcal{F})$ be a measurable space and suppose $\mu : \Omega \to [0, \infty]$ a *finitely additive* map, i.e.,

$$A_1, \dots, A_n \in \mathcal{F} \text{ disjoint } \implies \mu\left(\bigsqcup_{i=1}^{n} A_i\right) = \sum_{i=1}^{n} \mu(A_i).$$

Show that μ is *countably additive* if and only if the following condition holds: For any (A_i) non-decreasing sequence of sets in $\mathcal{F}$ (i.e., $A_1 \subseteq A_2 \subseteq \cdots$) it holds that $\mu(\bigsqcup_{i=1}^\infty A_i) = \lim_{i \to \infty} \mu(A_i)$.

Exercise 5. Let $\mathcal{G}$ be an algebra on Ω and $\mu : \mathcal{G} \to [0, \infty]$ be a countably additive map. Define the outer measure $\mu^* : 2^\Omega \to [0, \infty]$ with respect to μ as

$$\mu^*(A) = \inf\left\{\sum_{i \in \mathbb{N}} \mu(A_i) : A_1, A_2, \dots \in \mathcal{G}\, ;\, A \subseteq \bigcup_{i \in \mathbb{N}} A_i\right\}.$$

Show that $\mu^*(A) = \mu(A)$ for every $A \in \mathcal{G}$. (*Hint:* prove/use the fact that μ is countably subadditive.)