PhD Econometrics I (First Half) matias.guizzo.altube@nyu.edu
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Problem Set 1, due September 12

Problem 1. There are n urns, arranged from left to right, and k balls are thrown into these urns. Assume each ball land uniformly at random in each urn irrespective of other balls.
(a) Formulate an appropriate probability space for this experiment.
(b) For any generic probability space $(\Omega, \mathcal{F}, P)$, show the *law of total probability*, *i.e.*,

$$P(A) = \sum_{i=1}^{\infty} P(A \cap B_i) \qquad \text{for any } (B_i)_{i\in\mathbb{N}} \text{ in } \mathcal{F} \text{ that partitions } \Omega.$$

(c) Now let B_i denote the event that exactly i urns are empty. Write the probability of the event

$$A := \{\text{all empty urns are located to the left of all urns containing at least one ball}\}$$

in terms of $P(B_i)$'s.

Problem 2. Let $\mathcal{I} := \{(a,b) : a < b\ ;\ a,b \in \mathbb{R}\}$ and $\mathcal{F} := \sigma(\mathcal{I})$.
(a) Show that $\{(c,d] : c < d\ ;\ a,b,\in \mathbb{R}\} \subset \mathcal{F}$ and $\{(a,\infty) : a \in \mathbb{R}\} \subset \mathcal{F}$.
(b) Show that $\{[a,b] : a < b\ ;\ a,b,\in \mathbb{R}\}$ and $\{(-\infty,b] : b \in \mathbb{R}\}$ both generate $\mathcal{F}$.
(d) True or False (and explain.) $\mathcal{F}$ can be expressed as

$$\mathcal{F} = \mathcal{I} \cup \left\{ \bigcup_{i\in\mathbb{N}} S_i : S_i \in \mathcal{I} \text{ or } S_i^c \in \mathcal{I} \right\}.$$

Problem 3. Show that $\mathcal{L} := \{(x,y) \in \mathbb{R}^2 : x = y\ ;\ 0 \leqslant x \leqslant 1\}$ is a Borel set in $\mathbb{R}^2$.

Problem 4. Provide an example of a measure space $(\Omega, \mathcal{F}, \mu)$ and a non-increasing sequence of sets $A_1 \supseteq A_2 \supseteq \cdots$ in $\mathcal{F}$ such that $\mu(\bigcap_{i\in\mathbb{N}} A_i) \neq \lim_{i\to\infty} \mu(A_i)$.

Problem 5. Consider the measure space $(\mathbb{R}, \mathcal{B}(\mathbb{R}), \lambda)$ where λ denotes the Lebesgue measure.
(a) Show that $\lambda(\{a\}) = 0$ for every $a \in \mathbb{R}$.
(b) Show that $\lambda(A) = 0$ for every countable $A \in \mathcal{B}(\mathbb{R})$.
(c) Show that $\lambda([a,b]) = \lambda((a,b)) = \lambda((a,b]) = \lambda([a,b)) = b - a$ for every $a < b$.

Problem 6. Let $(\Omega, \mathcal{F}, P)$ be a probability space. The *symmetric difference* between two sets A and B is $A \triangle B = (A \cup B) \backslash (A \cap B)$, i.e., exactly one event occurs. Suppose $A, B \in \mathcal{F}$ and show the following upper bound on the difference in probabilities between A and B:

$$|P(A) - P(B)| \leqslant P(A \triangle B).$$

(*Hint*: it may be helpful to express $A \triangle B$ as a disjoint union.)