PhD Econometrics I (First Half)
Fall 2025

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Problem Set 2, due September 19

Problem 1.

1. Let $(\Omega, \mathcal{F})$ and $(\Omega', \mathcal{F}')$ be two measurable spaces and $f : \Omega \to \Omega'$. Show that

$$\mathcal{P} := \{A \subseteq \Omega' : f^{-1}(A) \in \mathcal{F}\}$$

is a σ-algebra.

2. Suppose $\mathcal{F}' = \sigma(\mathcal{A})$. Show that $f^{-1}(A) \in \mathcal{F}$ for each $A \in \mathcal{A}$ implies $f^{-1}(A) \in \mathcal{F}$ for each $A \in \mathcal{F}'$.

Problem 2.

1. Let $(X, \mathcal{F})$, $(Y, \mathcal{G})$, $(Z, \mathcal{H})$ be measurable spaces, $f : X \to Y$ be $\mathcal{F}/\mathcal{G}$ measurable, and $g : Y \to Z$ be $\mathcal{G}/\mathcal{H}$ measurable. Show that $g \circ f : X \to Z$ is $\mathcal{F}/\mathcal{H}$ measurable.

2. Let $(\Omega, \mathcal{F})$ be a measurable space and suppose $f : \Omega \to \mathbb{R}$ and $g : \Omega \to \mathbb{R}$ are both Borel-measurable. Show that $f + g$ and fg are Borel-measurable. (*Hint*: One way is using without proof the fact that $\omega \mapsto (f(\omega), g(\omega))$ is $\mathcal{F}/\mathcal{B}(\mathbb{R}^2)$ measurable.)

3. Show that if $X : \Omega \to \mathbb{R}$ is a random variable then $X^+(\omega) = \max\{X(\omega), 0\}$ and $X^-(\omega) = -\min\{X(\omega), 0\}$ are also random variables.

Problem 3. Let $(\Omega, \mathcal{F}, P)$ be a probability space and $X : \Omega \to \mathbb{R}$ a random variable.

1. Show that the distribution of X

$$P_X := P \circ X^{-1} : \mathcal{B}(\mathbb{R}) \to [0, 1]$$

is a Borel probability measure.

2. Show that $F_X : \mathbb{R} \to [0, 1]$ defined as $F_X(t) = P_X(-\infty, t]$ is non-decreasing, right-continuous, and $\lim_{t\to-\infty} F_X(t) = 0$, $\lim_{t\to+\infty} F_X(t) = 1$.

Problem 4. Let $(\Omega, \mathcal{F}, P)$ be a probability space and X, Y be $\mathbb{R}^{d_X}$ and $\mathbb{R}^{d_Y}$ random vectors on it respectively.

1. Show that if Y is constant, then X and Y are independent and also that Y and Y are independent.

2. Suppose X, Y are independent. Show $f(X)$ and $g(Y)$ are independent for every Borel-measurable $f : \mathbb{R}^{d_X} \to \mathbb{R}$ and $g : \mathbb{R}^{d_Y} \to \mathbb{R}$.