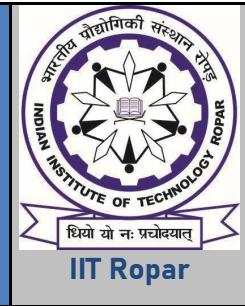


DEPARTMENT OF MECHANICAL ENGINEERING

INDIAN INSTITUTE OF TECHNOLOGY ROPAR

RUPNAGAR-140001, INDIA



## DESIGN LAB II

### PROJECT REPORT

For

**PROJECT TITLE:  
THRUST VECTOR CONTROL**

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Report Submitted On: 15-05-2025

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# 1 Abstract

In this project, we aim to perform thrust-vector control using a PID loop. On-board sensors measure the tilt and feed the error into a PID controller. This controller adaptively adjusts the gimbal angle as the mass and aerodynamic forces change, swivelling the nozzle to keep the vehicle vertical or maintain a desired orientation. We developed a Simulink model to simulate the rocket dynamics and control system, which successfully maintains the vehicle orientation throughout the powered flight.

# 2 Introduction

Thrust Vector Control (TVC) is a method used in rockets and missiles to control the attitude or angular velocity, or mass offset of the vehicle by directing the thrust. Conventional rockets use fixed nozzles, but TVC allows manoeuvring by gimbaling (tilting) the nozzle. This enables finer control, especially crucial during launch and atmospheric flight phases. In this project, we simulate a TVC mechanism using MATLAB Simulink to visualize and test our control system under various flight conditions.

# 3 Objectives

1. To design and simulate a thrust vector-controlled rocket in Simulink.
2. To model the dynamic behavior of a rocket under the influence of thrust gimbaling.
3. To apply PID control to achieve stable orientation.
4. To validate the model using Simulink simulations.

# 4 Theory and Approach :

+ ABOUT NASA    + NEWS & EVENTS    + MULTIMEDIA    + MISSIONS    + MY NASA    + WORK FOR NASA



## Rocket Thrust Summary

**Known:**

$p_t$  = Total Pressure       $\gamma$  = Specific Heat Ratio  
 $T_t$  = Total Temperature       $R$  = Gas Constant  
 $p_0$  = Free Stream Pressure       $A$  = Area

**Mass Flow Rate:**  $m = \frac{A^* p_t}{\sqrt{T_t}} \sqrt{\frac{\gamma}{R}} \left( \frac{\gamma+1}{2} \right)^{\frac{\gamma+1}{2(\gamma-1)}}$

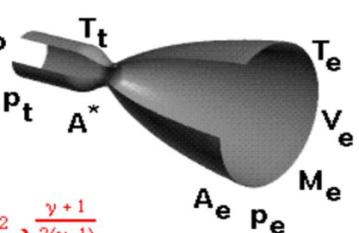
**Exit Mach:**  $\frac{A_e}{A^*} = \left( \frac{\gamma+1}{2} \right)^{\frac{\gamma+1}{2(\gamma-1)}} \frac{\left( 1 + \frac{\gamma-1}{2} M_e^2 \right)^{\frac{\gamma+1}{2(\gamma-1)}}}{M_e}$

**Exit Temperature:**  $\frac{T_e}{T_t} = \left( 1 + \frac{\gamma-1}{2} M_e^2 \right)^{-1}$

**Exit Pressure:**  $\frac{p_e}{p_t} = \left( 1 + \frac{\gamma-1}{2} M_e^2 \right)^{-\frac{\gamma}{\gamma-1}}$

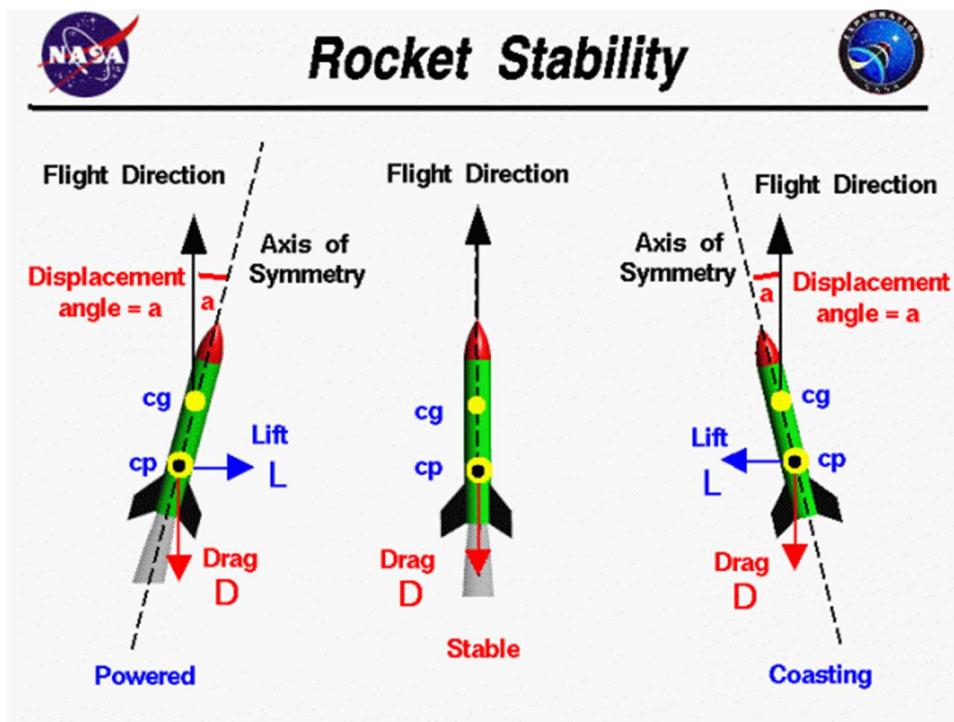
**Exit Velocity:**  $V_e = M_e \sqrt{\gamma R T_e}$

**Thrust:**  $F = \dot{m} V_e + (p_e - p_0) A_e$



**Rocket Stability:**

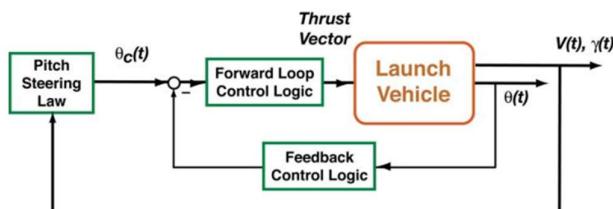
Stability of a rocket depends on the relative positions of the Center of Pressure (Cp) and Center of Gravity (Cg). A rocket is statically stable if Cp lies behind Cg. The distance between these points measured in calibers (rocket diameters) indicates the degree of stability.

**Nozzle Gimbaling:**

Gimbaling the nozzle provides a torque that corrects the rocket's orientation. This method of thrust vector control is effective even at low speeds or vacuum conditions where aerodynamic surfaces are ineffective.

## Feedback Control Law

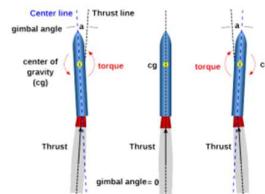
**Errors due to disturbances and modeling errors  
corrected by feedback control**



$$\text{Motor Gimbal Angle}(t) \triangleq \delta_G(t) = c_\theta [\theta_{des}(t) - \theta(t)] - c_q q(t)$$

$$\theta_{des} = \text{Desired pitch angle}; \quad q = \frac{d\theta}{dt} = \text{pitch rate}$$

$c_\theta, c_q$  : Feedback control law gains



## THRUST VECTOR CONTROL

## **Equations of Motion:**

The 3D motion of a rocket is governed by Newton's second law:

- Translation: 4 (2 for thrust and 2 for moment)
  - Rotation: 2 (each for thrust and moment)

For 2D simplification:

The thrust vector components are:

$$T_x = T \sin(\gamma + \theta)$$

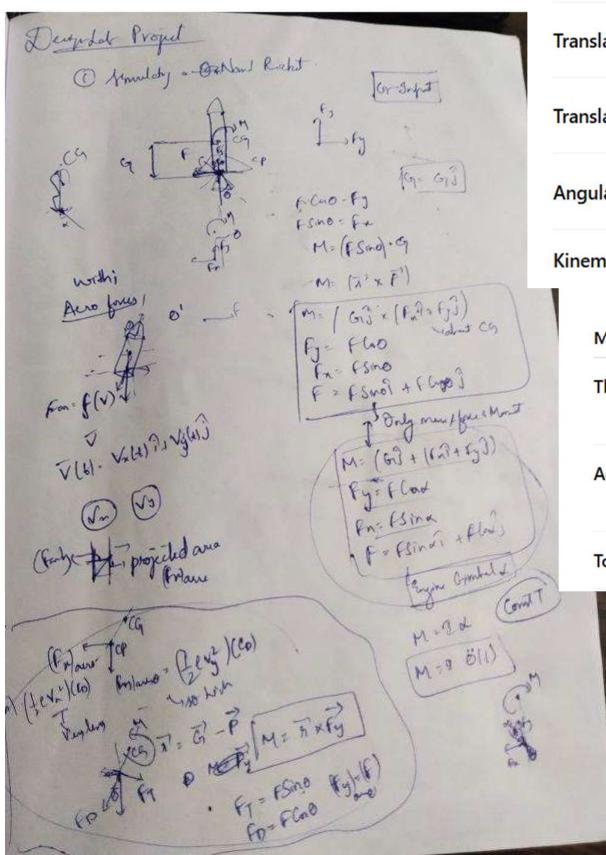
$$T_z = T \cos(\gamma + \theta)$$

### Translational motion equations:

$$m\ddot{x} = T_x = T \sin(\gamma + \theta)$$

$$m\ddot{z} = T_z - mg = T \cos(\gamma + \theta) - mg$$

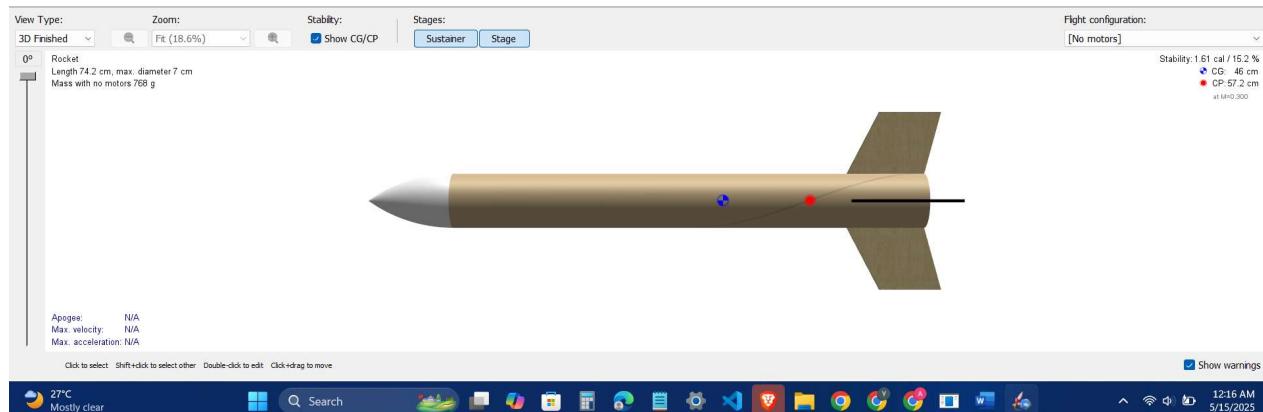
Quantity	Formula
Lateral thrust	$T_{\text{lat}} = T \sin \gamma$
Torque about CoM	$\tau = L T_{\text{lat}} = L T \sin \gamma$
Thrust-X component	$T_x = T \sin(\theta + \gamma)$
Thrust-Z component	$T_z = T \cos(\theta + \gamma)$
Translational accel. in $x$	$\ddot{x} = \frac{T_x}{m}$
Translational accel. in $z$	$\ddot{z} = \frac{T_z}{m} - g$
Angular acceleration	$\dot{\omega} = \frac{\tau}{I} = \frac{L T \sin \gamma}{I}$
Kinematic relation (pitch rate)	$\dot{\theta} = \omega$
Moment about COD	Expression
Thrust-vector torque	$\tau_{T,COD} = L_{COD} T \sin \gamma$ $L_{COD}$ = distance from COD to nozzle gimbal
Aerodynamic pitching moment	$M_{aero} = \frac{1}{2} \rho V^2 S l_{ref} C_m(\alpha, \delta)$ $l_{ref}$ = reference length
Total net moment about COD	$I \dot{\omega} = \tau_{T,COD} + M_{aero}$



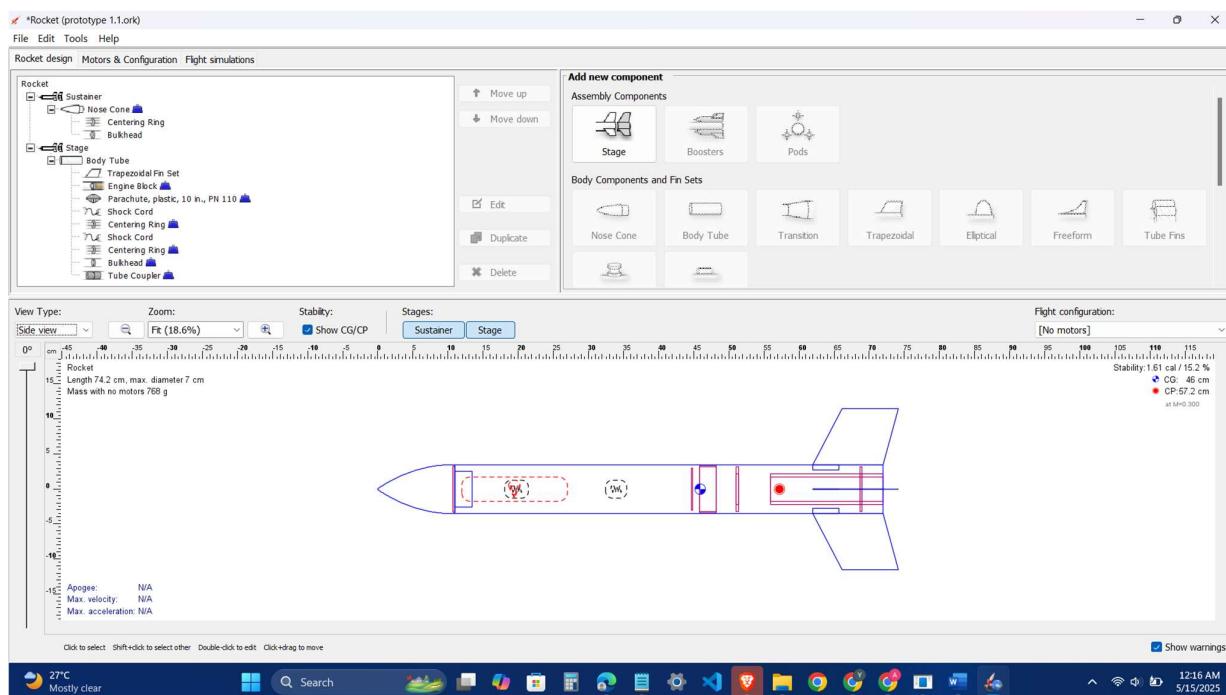
## THRUST VECTOR CONTROL

Quantity	Formula	Notes
Drag magnitude	$D = \frac{1}{2} \rho V^2 S C_d$	$\rho$ : air density, $V$ : speed, $S$ : ref. area
Drag X-component	$F_{dx} = -D \sin\theta$	Opposes motion along $x$
Drag Z-component	$F_{dz} = -D \cos\theta$	Opposes motion along $z$
Lift magnitude	$L = \frac{1}{2} \rho V^2 S C_l$	$C_l$ : lift coefficient
Lift X-component	$F_{\ell x} = -L \cos\theta$	Perpendicular to velocity
Lift Z-component	$F_{\ell z} = L \sin\theta$	Perpendicular to velocity
Total aero X-force	$F_{ax} = F_{dx} + F_{\ell x} = -D \sin\theta - L \cos\theta$	Sum of drag + lift
Total aero Z-force	$F_{az} = F_{dz} + F_{\ell z} = -D \cos\theta + L \sin\theta$	Sum of drag + lift

### STL FILE:

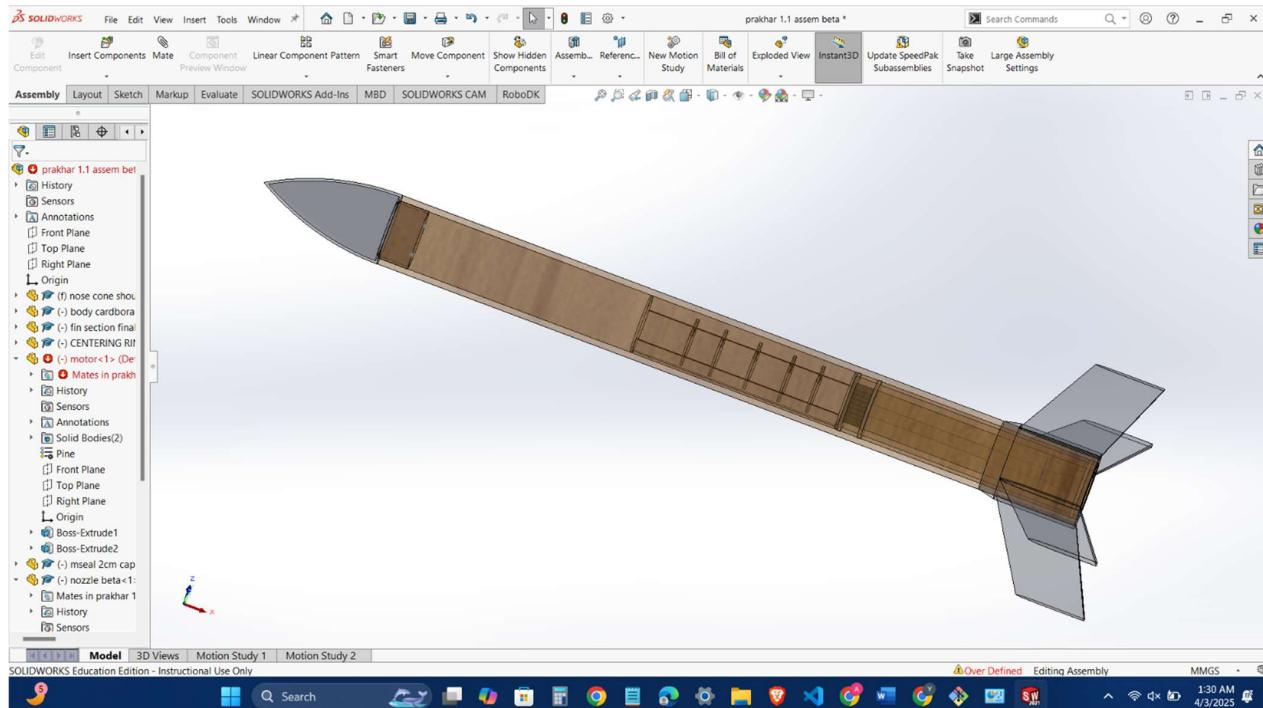


### OPEN ROCKET FILE:

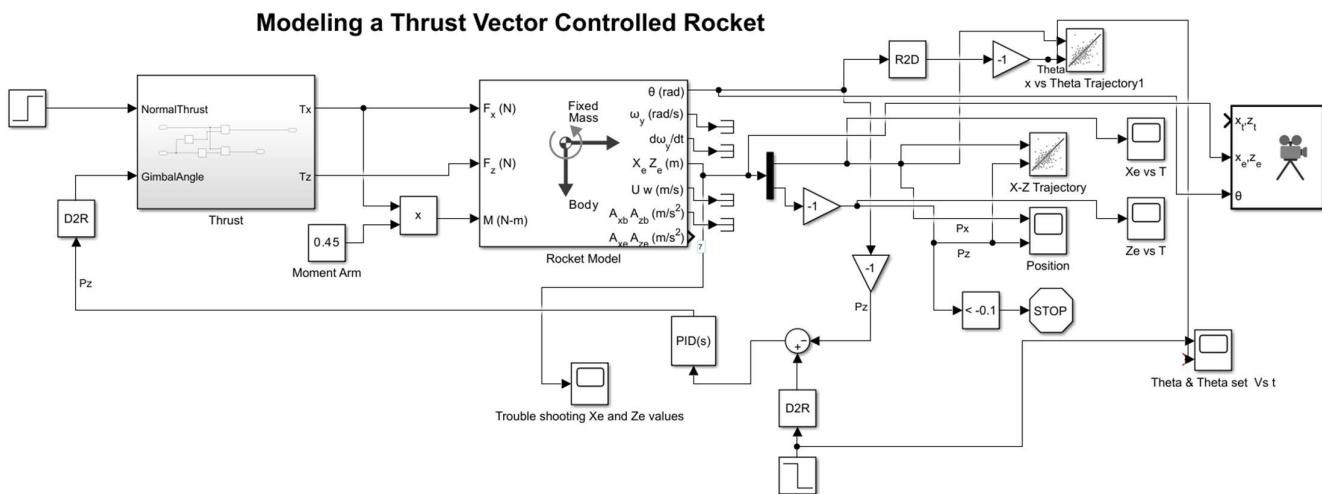


## THRUST VECTOR CONTROL

### CAD FILE



## 5 Simulation using Simulink:



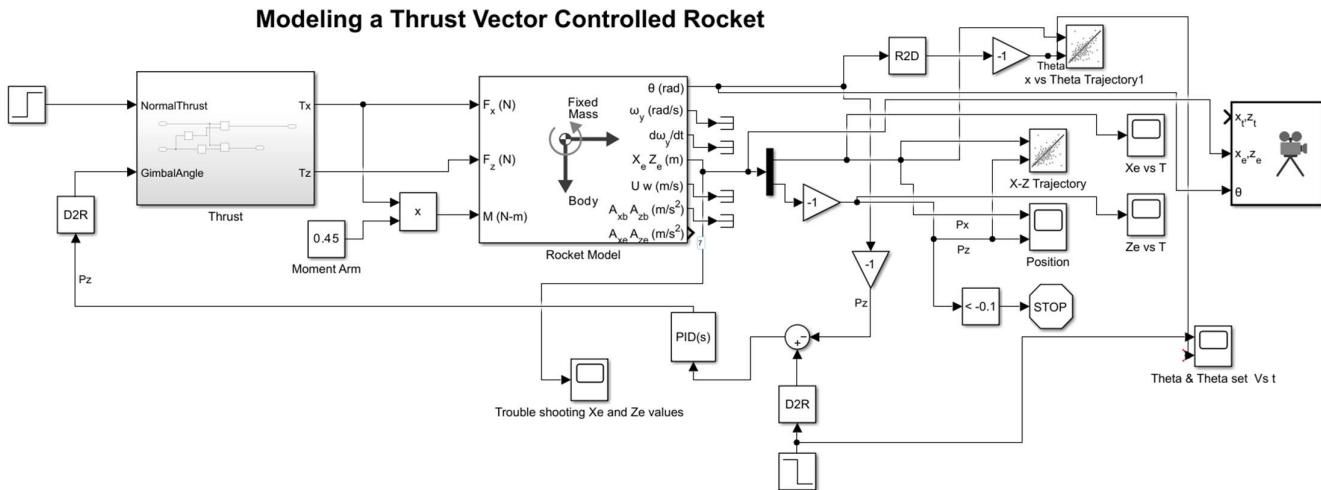
Simulation of Thrust Vector Control using Simulink.

To simulate this model, we developed a block diagram in Simulink consisting of:

- **3DOF Block:** Implements Newtonian dynamics based on input forces and moments.
- **Thrust Block:** Calculates lateral and vertical thrusts based on gimbal angle and total thrust.
- **PID Controller:** Receives pitch error () and outputs the required gimbal angle.

## THRUST VECTOR CONTROL

- **Moment Arm Multiplier:** Calculates torque as the product of lateral thrust and moment arm.
- **Feedback Loop:** Simulates measurement and feeds values back to controller.



The thrust is given as an input function, here we assumed to be a step function (constant thrust of 15N for 200s), and the thrust block will resolve the components with gimbal angle which is given by the PID Loop and the output Tx and Ty will act at the joint of Nozzle and the Rocket motor.

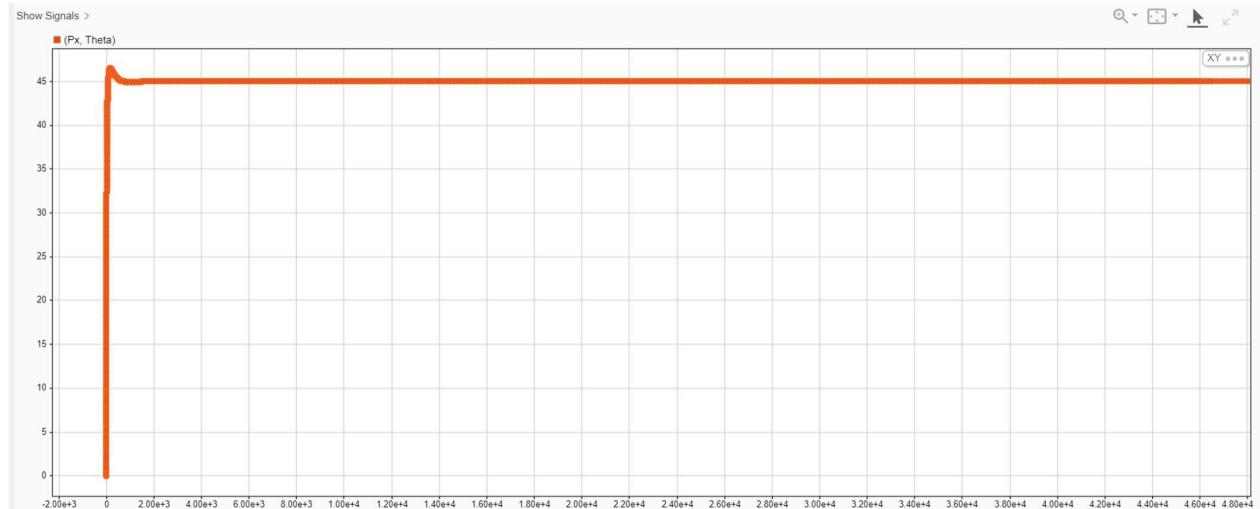
Therefore, a moment is also generated, the moment is calculated by multiplying it Tx with the distance of COM, which can be found from OR software, these are given as Input to 3 DOF block and the output theta, Xe, Ze are displayed on scopes which will give the trajectory of the rocket.

The theta (Angular orientation of the rocket is fed to the PID loop where it is compared against the desired, angular orientation and corrected according to the PID constants a gimbal angle alpha is the output which is given to the actuators to correct the nozzle orientation to achieve the set orientation.

## 6 Simulation Results

The system simulation produced various useful plots:

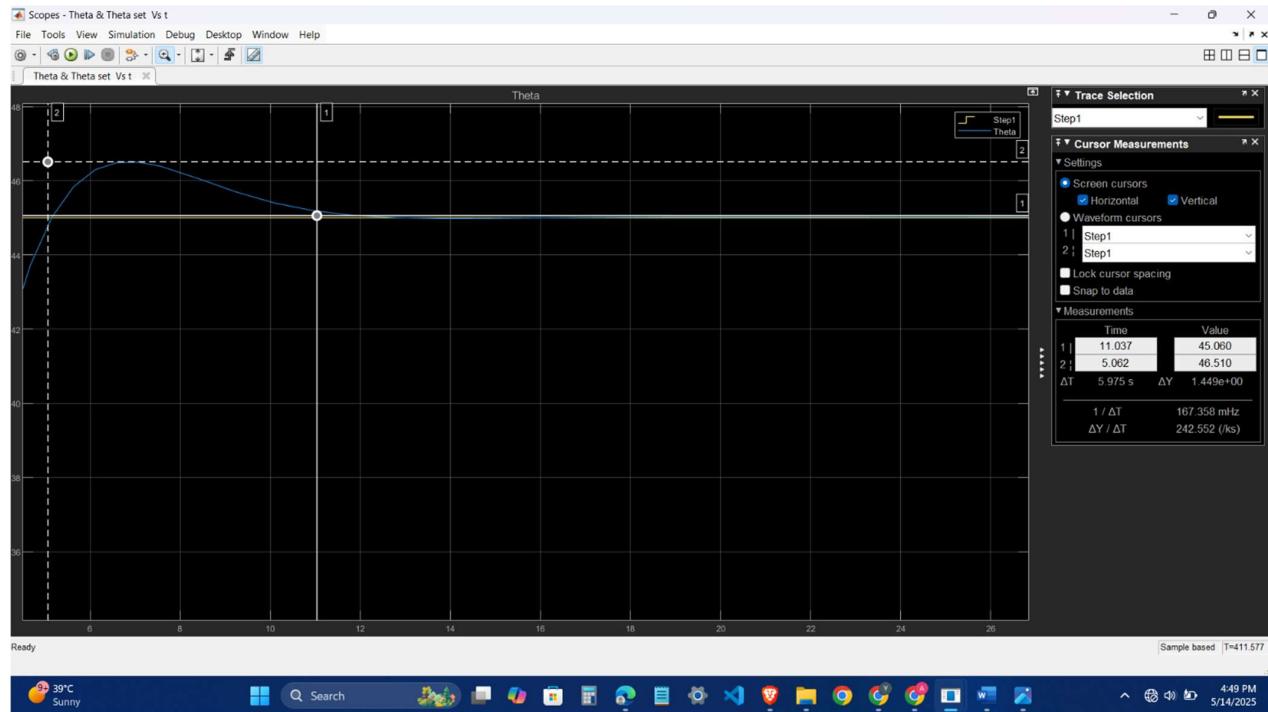
- **Px vs theta:** Visualizes orientation control responsiveness .



- The set point is 45degrees, this graph shows how the orientation changes with horizontal displacement.

## THRUST VECTOR CONTROL

- Set theta vs actual theta: Confirms PID controller's accuracy.



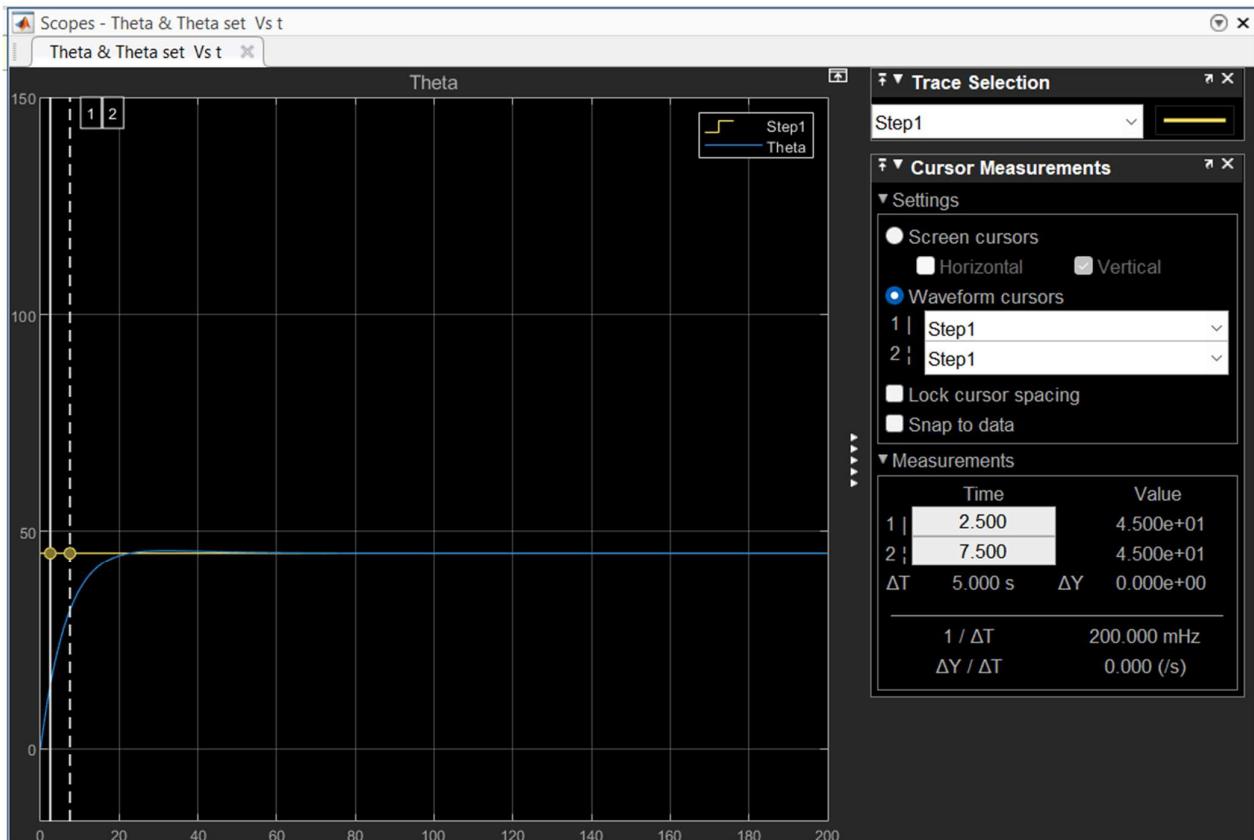
Constants have been tuned manually, and the set point is reached for the first time at 5.034 seconds.

With an over shoot of 1.6 degrees from the set point.



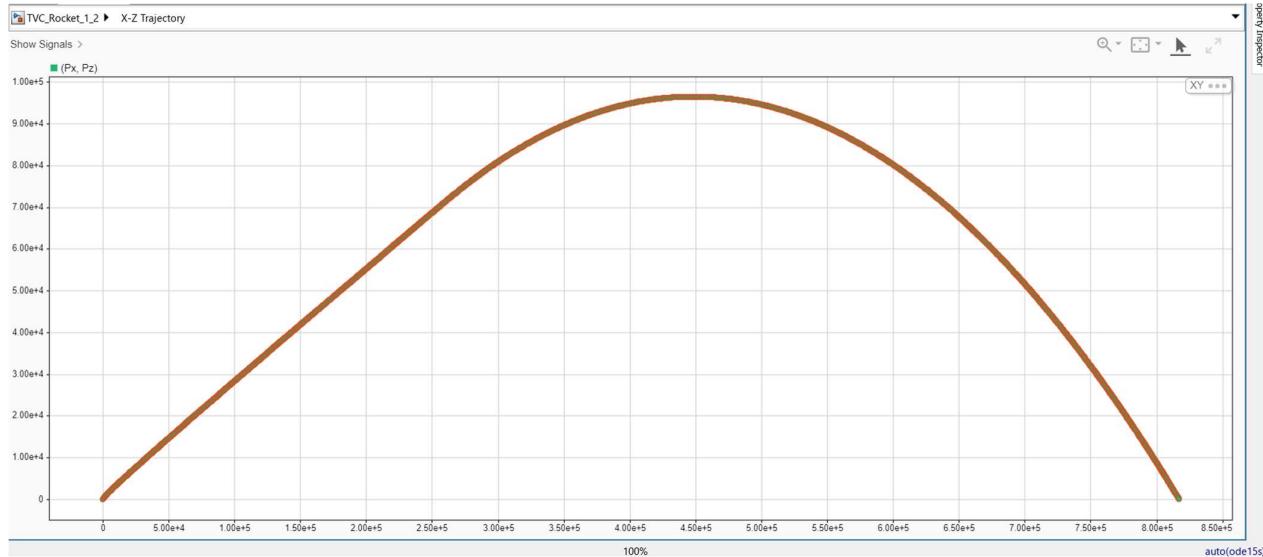
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## THRUST VECTOR CONTROL



Setpoint vs Actual orientation for different angle of attack and thrust., The parameters have to be tuned accordingly. The parameters have to be tuned differently for different angle of attack and thrust profiles.

- **X-Z Trajectory:** Plots actual flight path. The plot of Xe vs Ze (Horizontal vs Vertical Displacement of the Rocket)

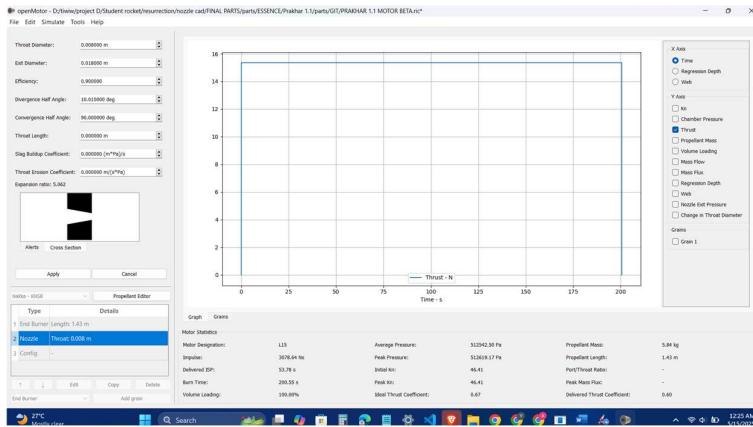


The Trajectory of the rocket, under desired set point is plotted in these graphs.

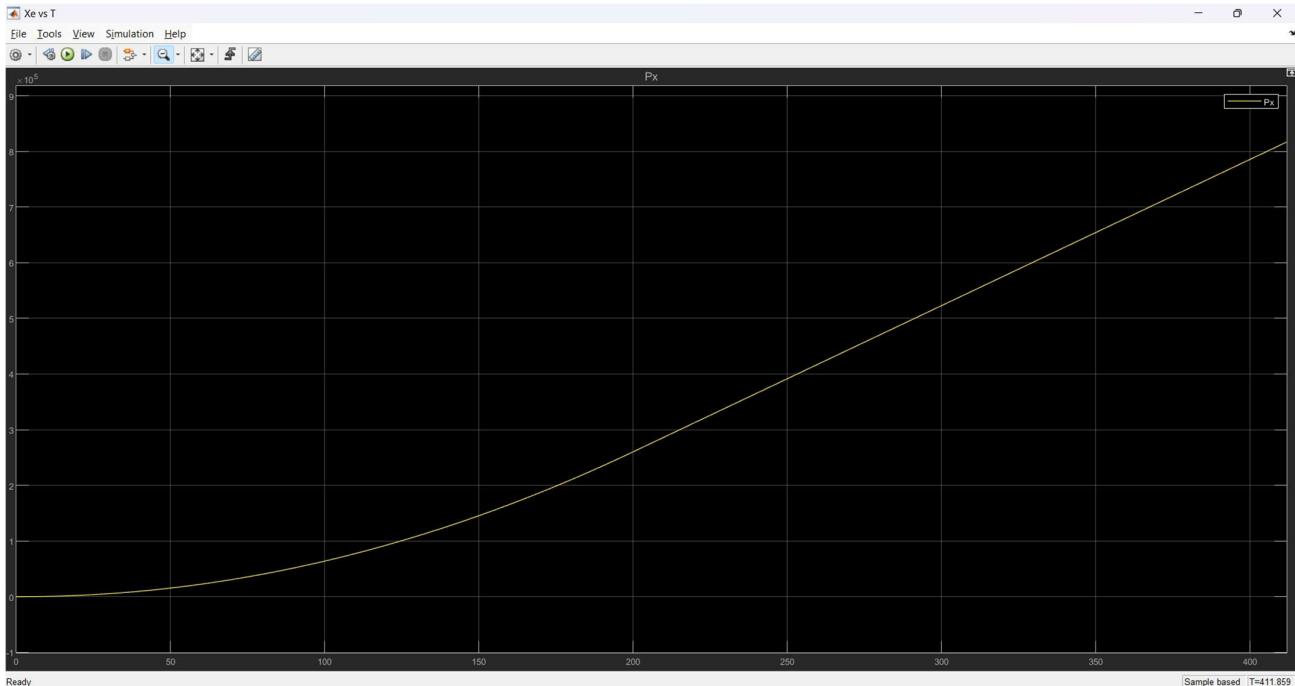
- Those enormous, displacements of magnitudes ( $10^5$ )m are not magic , but rocket science , this is what constant thrust of 15N does in 200s for an 800gm rocket .
- This can be verified with a real rocket simulation software (Openrocket).
-

## THRUST VECTOR CONTROL

SUCH THRUST PROFILES CAN WE ACHIEVED WITH END BURNER PROFILES:



- **Xe vs Time, Ze vs Time:** Shows time evolution of rocket's displacement along X and Z axis.

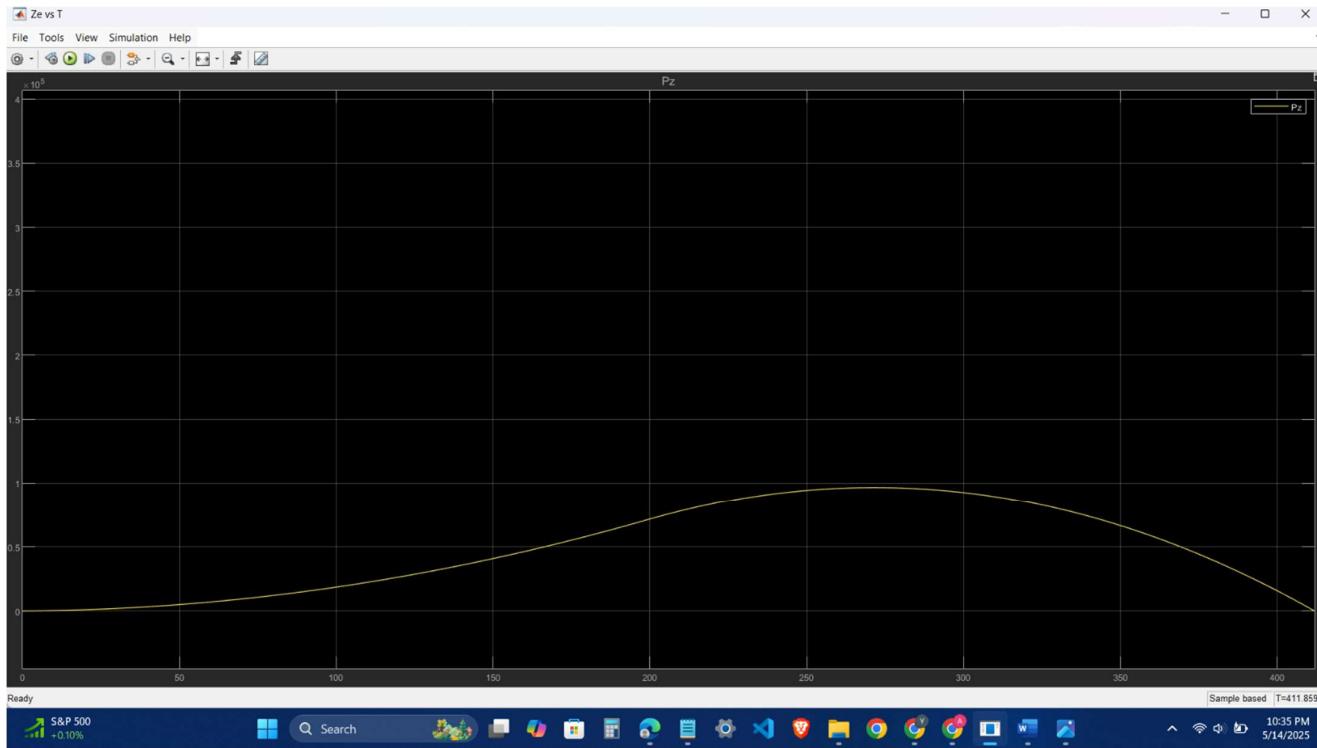


- The displacements can be justified with constant thrust travel of 15 N for 200 seconds.

Sampling time of 2000sec and time of flight of 411.89 seconds, this is the plot of displacement along x axis with t, we can see that the displacement just keeps on increasing this is because the thrust in horizontal direction is just constant.

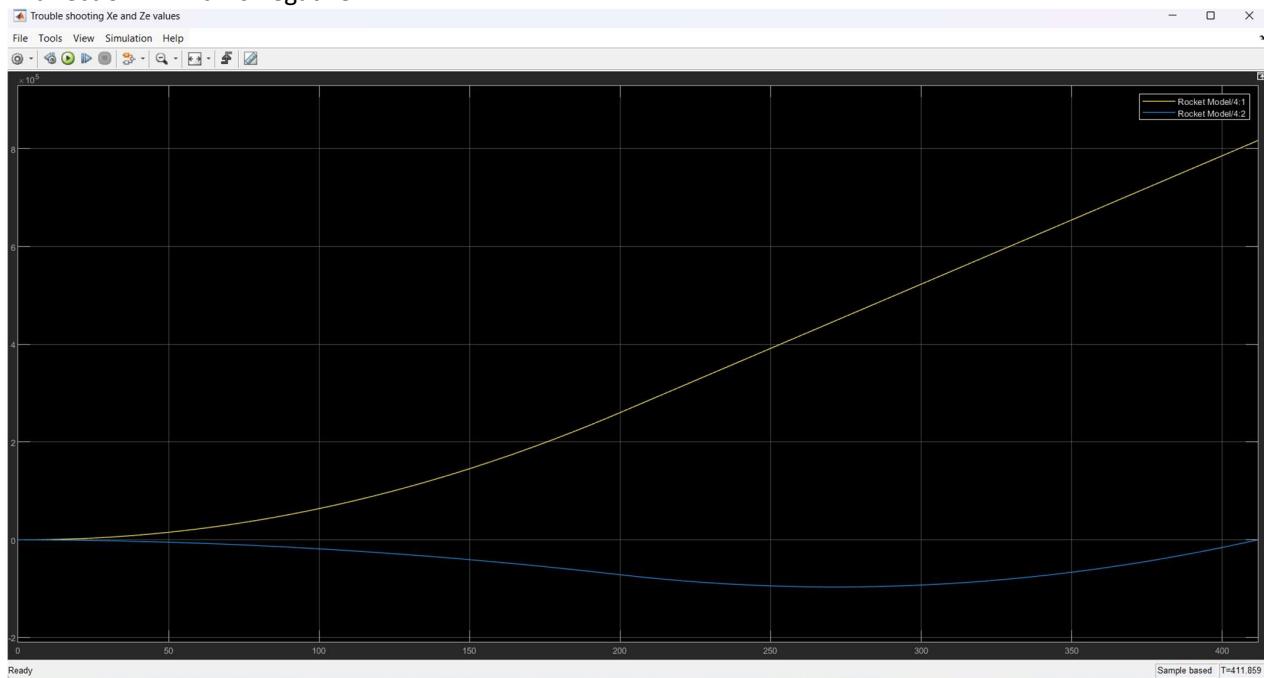
## THRUST VECTOR CONTROL

### Ze displacement vs Time:



These results validate that the system correctly adjusts orientation and maintains controlled flight.

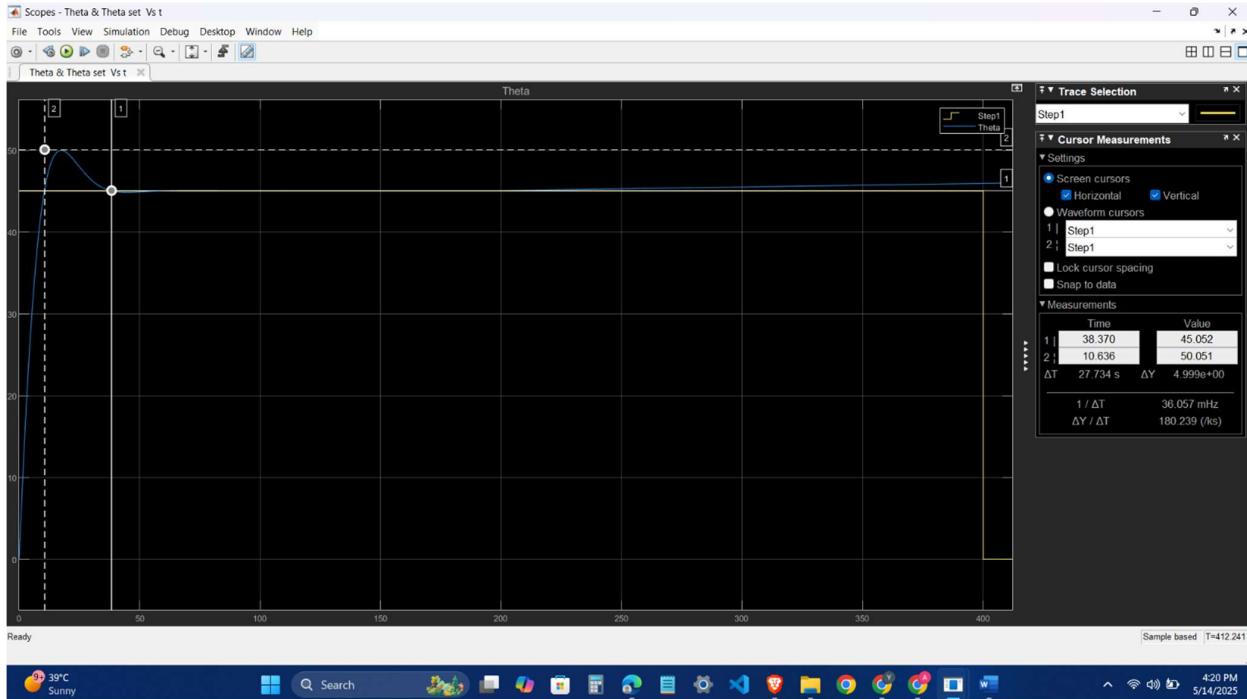
Plots of  $Xe$  and  $Ze$  coming inverted from the 3 DOF block , we used several gain blocks to modify the default direction Z which is negative .



## 7 Tuning of PID

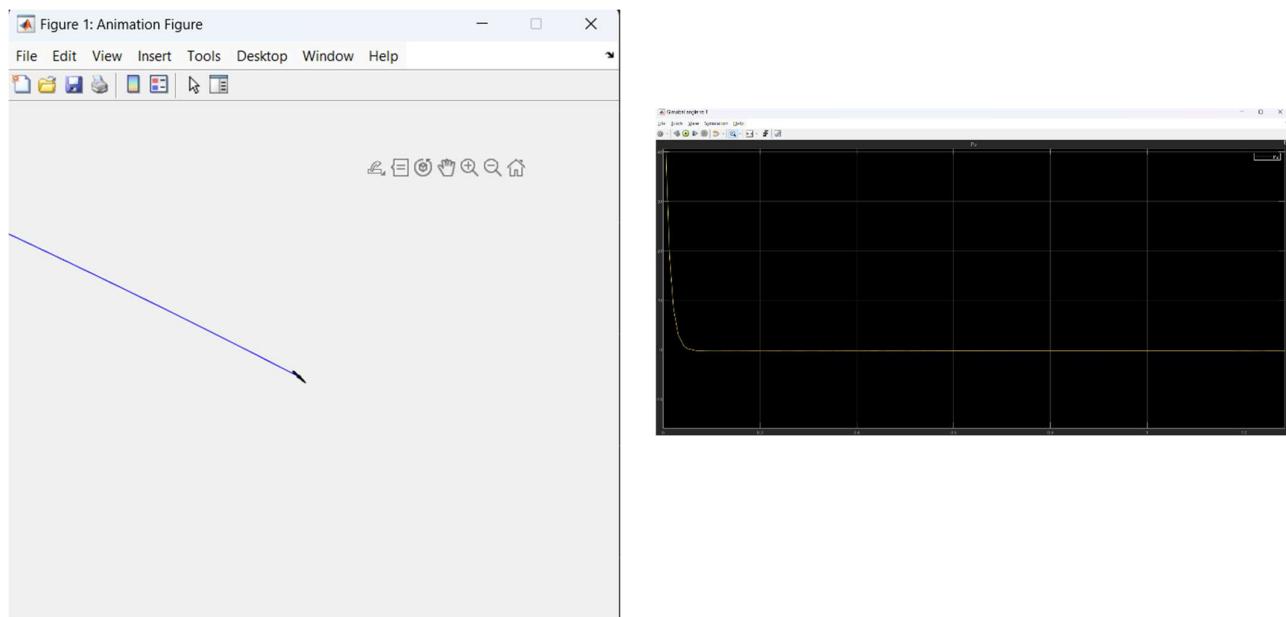
Tuning the PID controller was crucial for stable operation:

- **P (Proportional):** Controls rise time. High values reduce rise time but cause overshoot.
- **I (Integral):** Eliminates steady-state error but can introduce lag.



- **D (Derivative):** Reduces overshoot and damping oscillations.

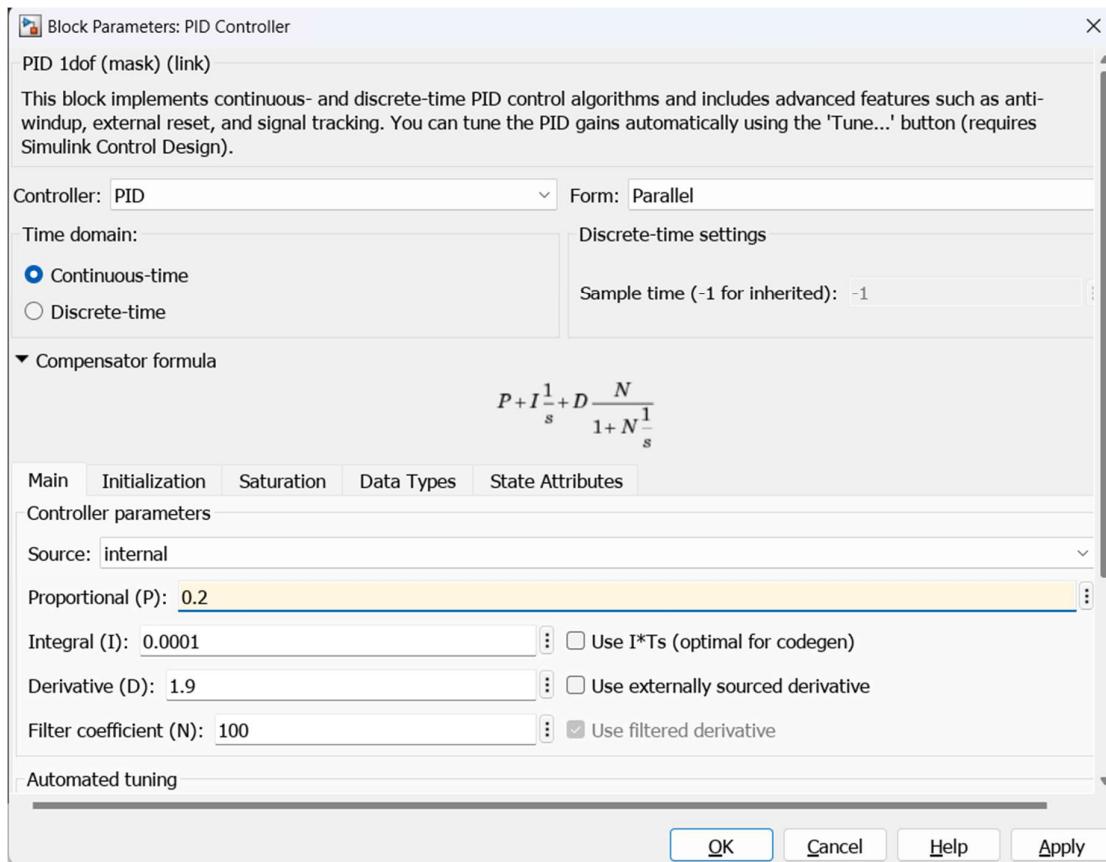
The parameters ( $K_p$ ,  $K_i$  &  $K_d$  have been tuned manually for achieving steady state in least amount of time)



Gimbal angle vs Time

## THRUST VECTOR CONTROL

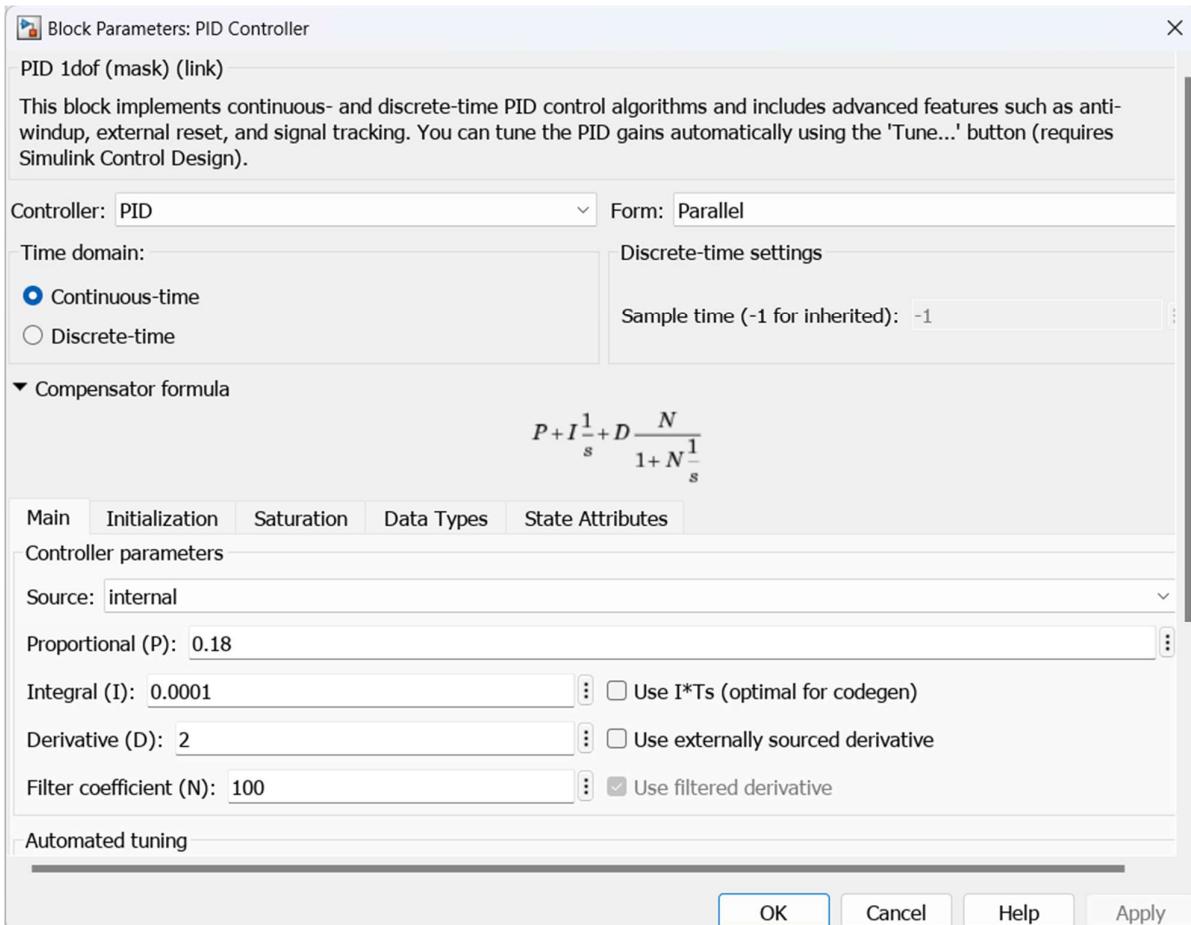
**These are few trials:**



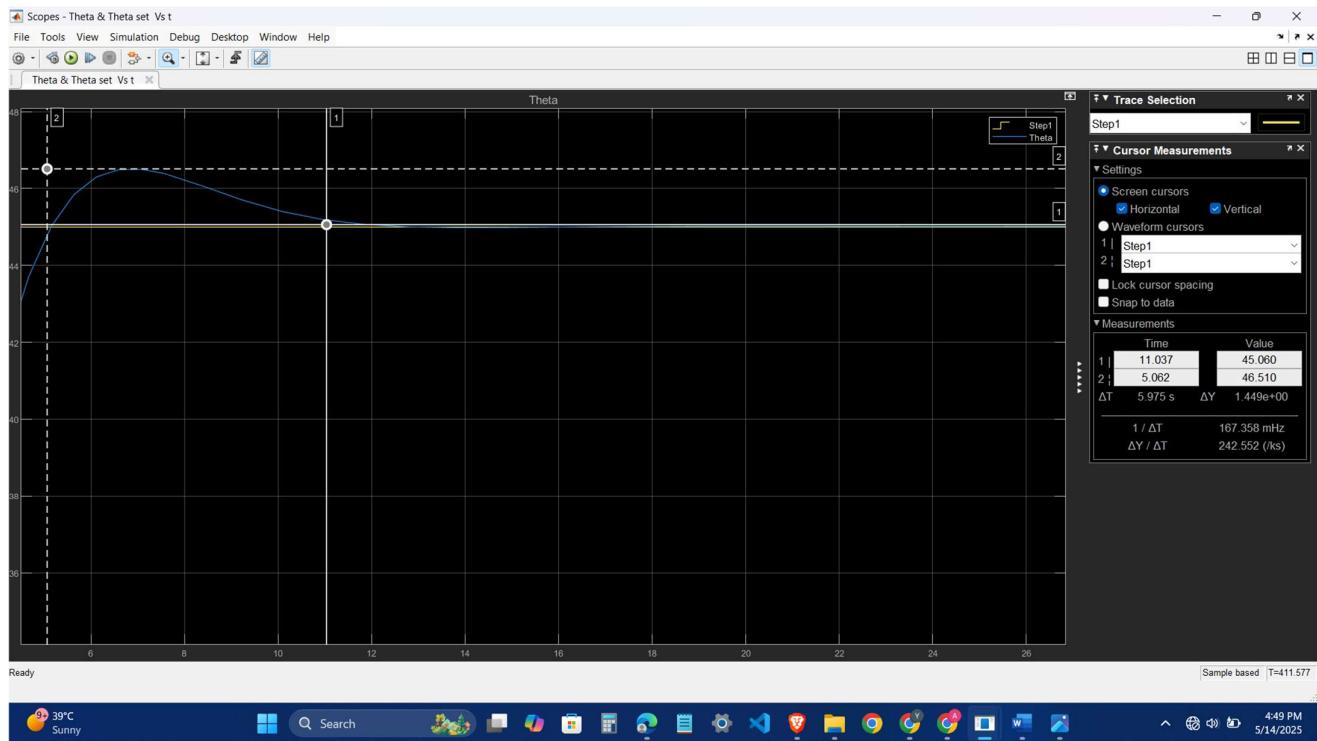
**TRAIL #2:**



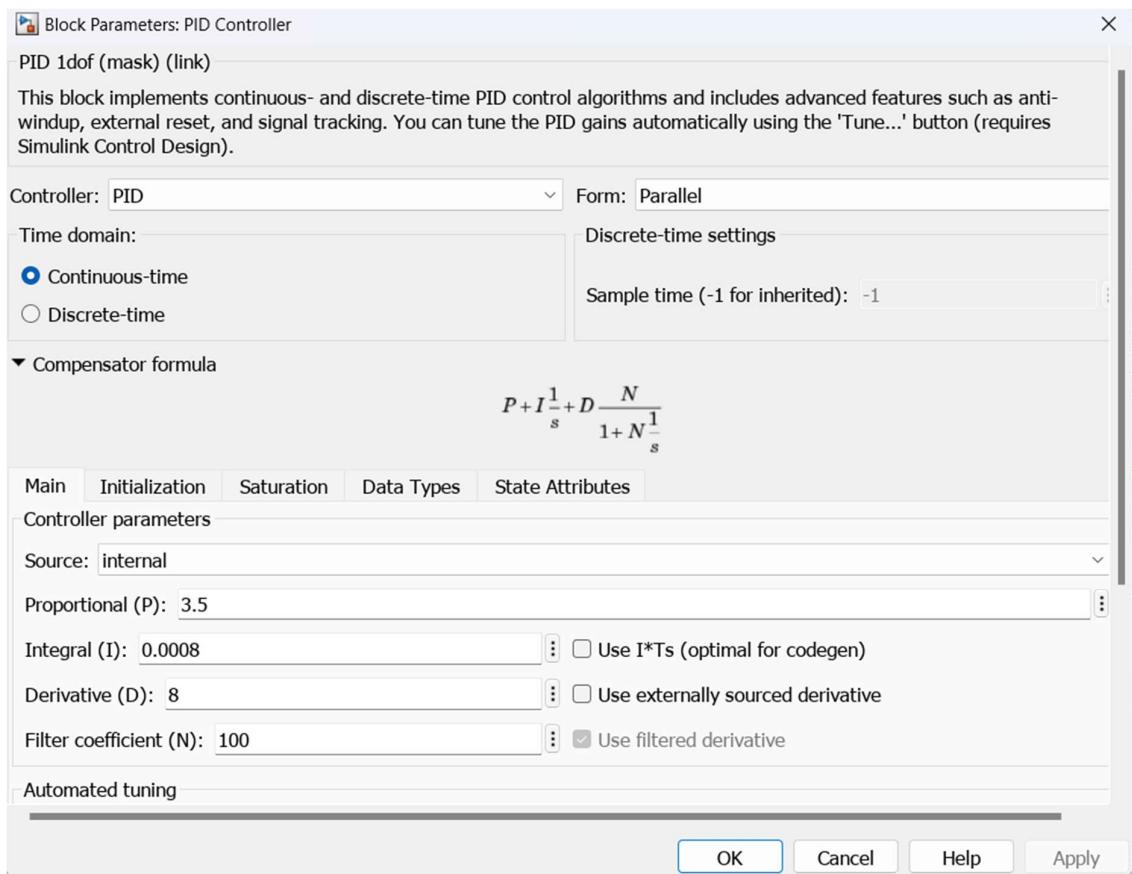
## THRUST VECTOR CONTROL



#### **FINAL TUNED BEST TUNED RESULTS:**



## THRUST VECTOR CONTROL

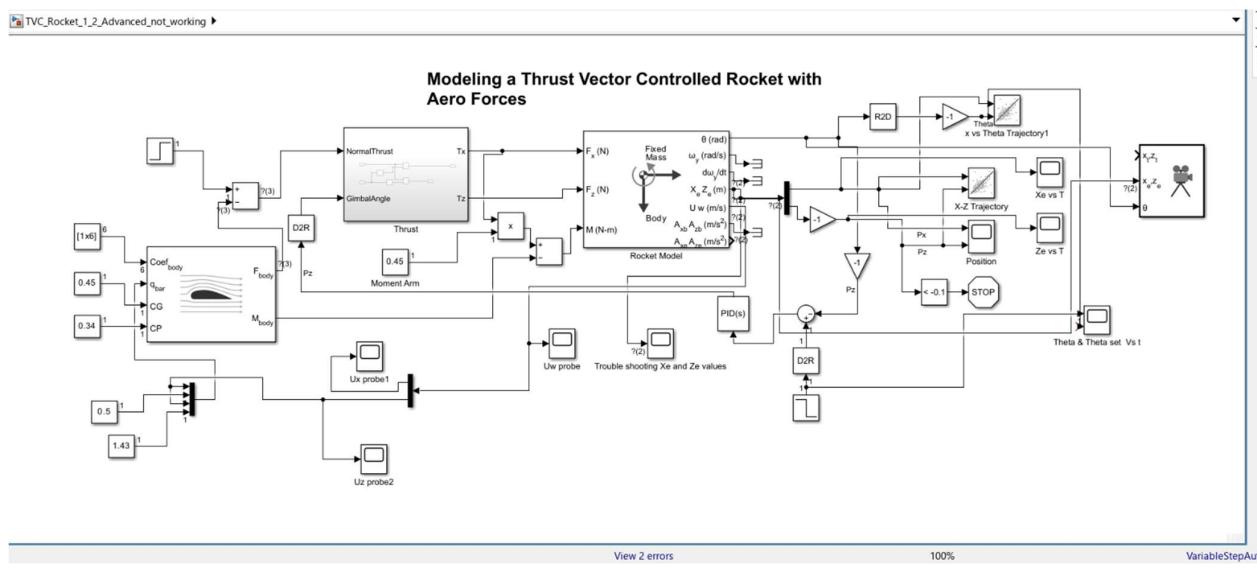


### Results:

- Underdamped systems showed large oscillations.
- Overdamped systems responded slowly.
- Proper tuning resulted in fast rise time, minimal overshoot, and steady convergence.

Finally, the least transient time can be achieved with Kp, Ki and Kd of (3.5,0.0008,8) for 15 N, and set angle of 45degrees.

### FUTILE ATTEMPTS TO SIMULATE AERO FORCES ACCURATELY



# GRAVITY TURN:

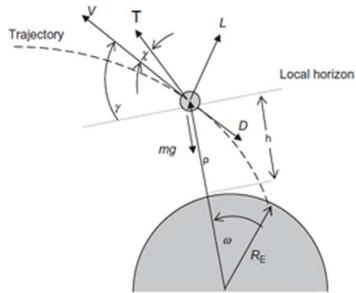
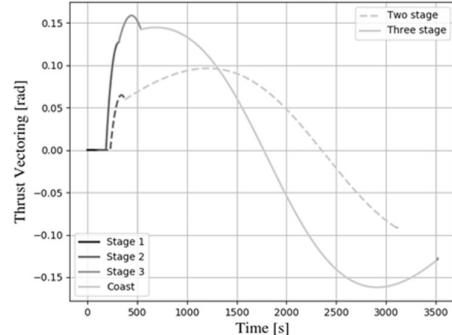
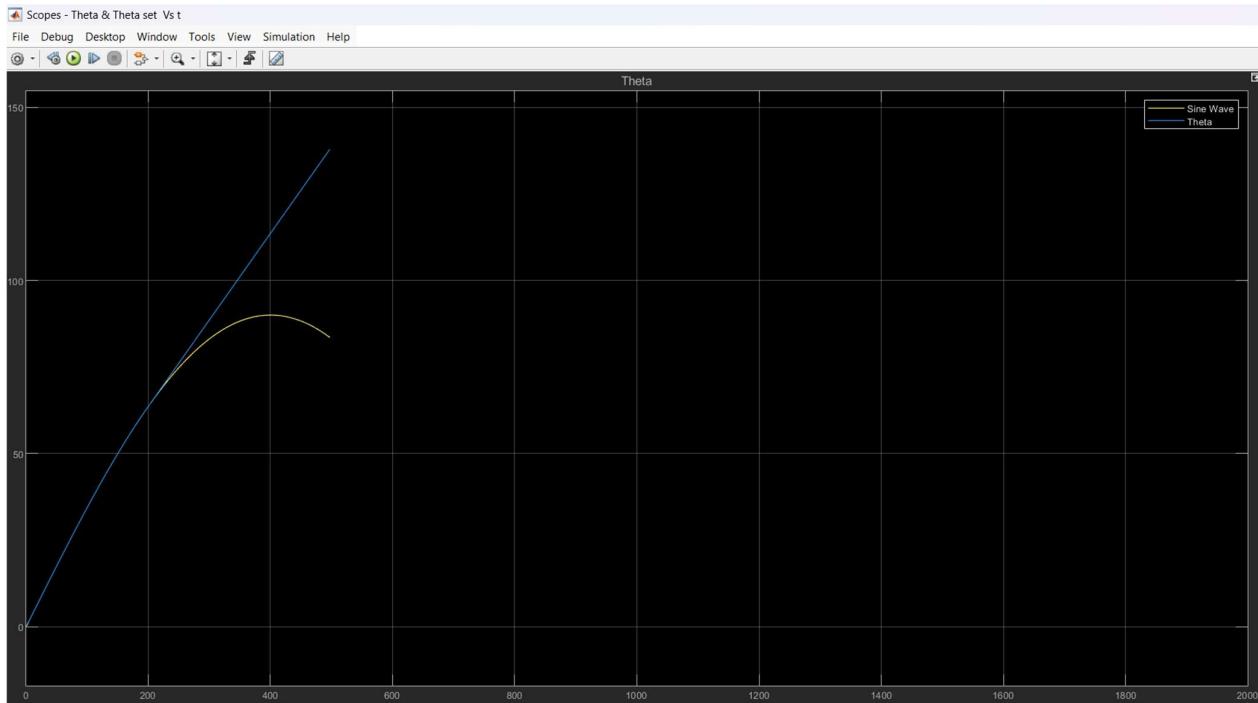


Figure 3: Rocket state variables and forces during flight [6].

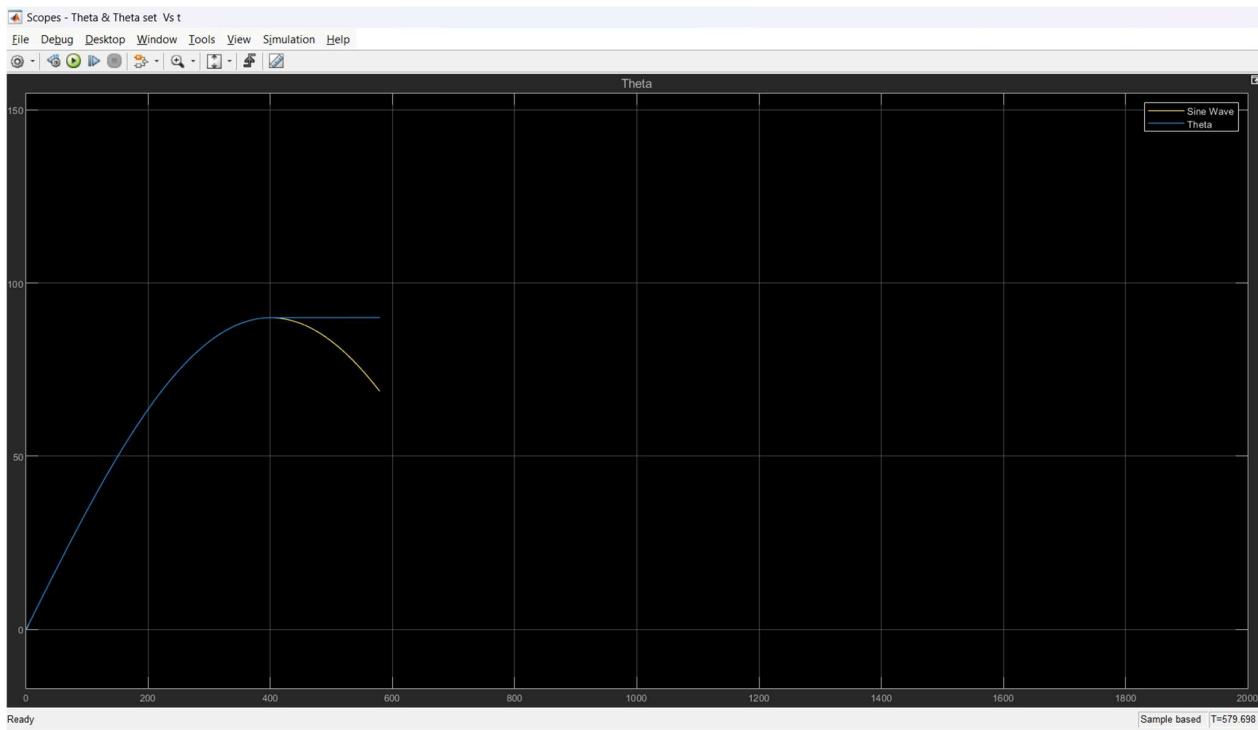


A gravity turn is a flight trajectory maneuver that uses the pull of gravity—rather than aerodynamic control surfaces—to gradually tip a rocket from its initial vertical ascent into a more horizontal path. After launch, the vehicle begins a small programmed pitch over; as it accelerates and its inertia carries it forward, gravity naturally bends its flight toward the desired downrange heading. This approach minimizes the control effort and aerodynamic loads on the airframe, conserves propellant by aligning thrust with the instantaneous velocity vector, and simplifies guidance by largely “handing off” attitude shaping to gravity. Gravity turns are widely used in orbital launches because they ensure efficient transition between vertical lift-off and horizontal orbital insertion with minimal structural stress and steering losses.



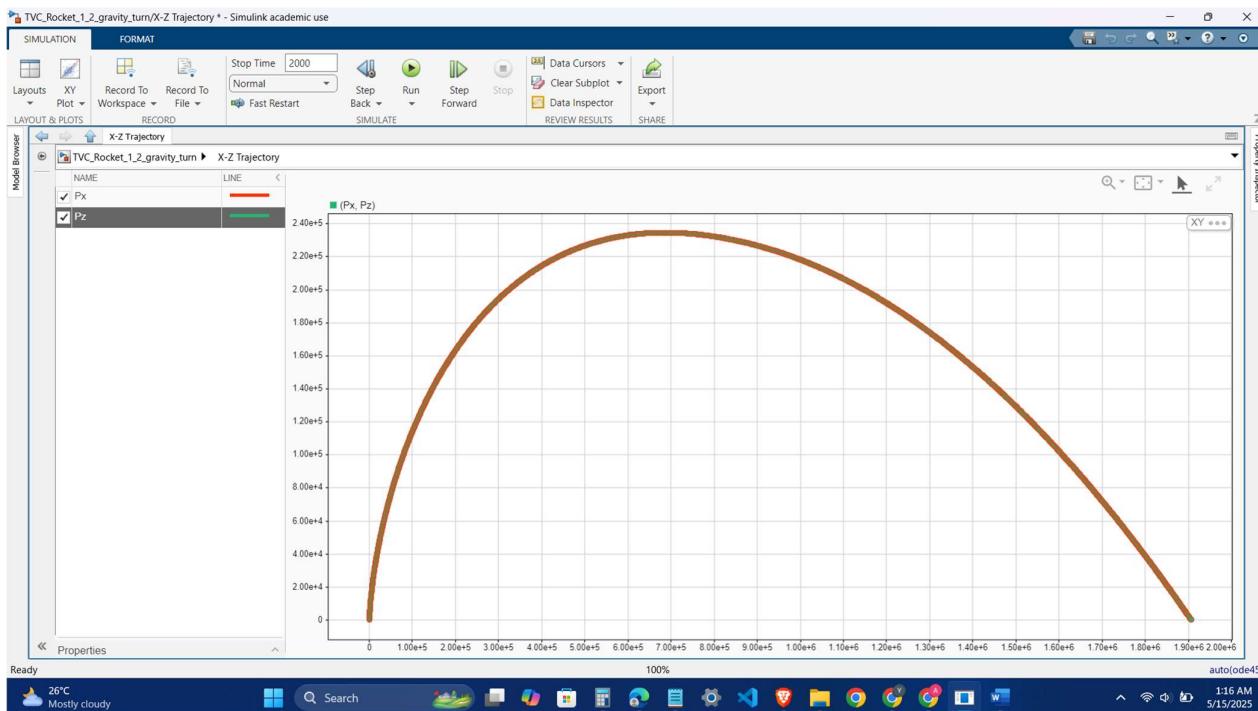
The Pid controller, could correct for changing desired angle , but as soon as the thrust goes out at  $T=200s$  , the rocket starts to deviate from planned angle , but this is planned at  $T=200$  , the angle is around 60 deg which still can give best range results .

## THRUST VECTOR CONTROL



**When the thrust is present, till T=400 sec, the desired path till 90 degrees which is nominal angle at orbital insertion is achieved successfully.**

### Maximum Range is achieved: (In presence of Atmospheric Drag)

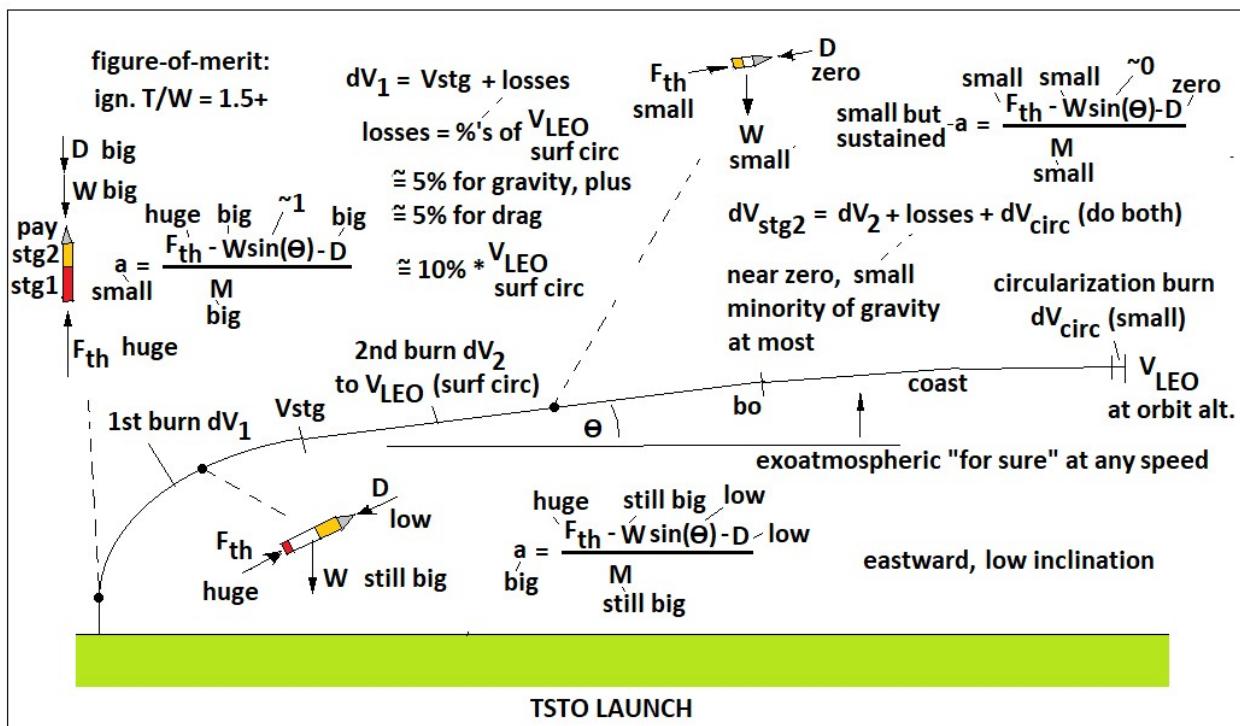


## 8 Conclusion:

In this work, we developed and validated a thrust-vector controlled rocket model using a PID-based gimbal system within MATLAB Simulink. We began by deriving the full set of translational and rotational equations of motion, then distilled these into concise force and moment relationships and packaged our aerodynamic coefficients in the stability-axes format. By feeding those coefficients, along with mass, CG/COD locations, and a gravity-turn angle profile, into the Aerospace Block set's 3-DOF Animation blocks, we demonstrated real-time trajectory control in the X-Z plane. Our PID tuning achieved rapid rise times and minimal overshoot across multiple step inputs and thrust profiles, while the gravity-turn manoeuvre illustrated efficient, gravity-shaped attitude transitions. Although the current model uses constant thrust and simplified aerodynamics, it provides a robust foundation for future extensions—such as variable-thrust burn profiles, drag coupling, full 3D control (roll/yaw), and hardware-in-the-loop validation—that will bring us closer to realistic launch vehicle guidance and control.

## ANSWER TO THE QUESTION POSED IN 5 MINS EVALUATION:

Sir the maximum range taught in Class XII for 45degrees is for single ballistic projectile, with no atmospheric drag conditions. But in real life for achieving maximum range in atmospheric drag, so the rocket starts at low inclinations and gradually increase the inclination angles at higher altitudes in thin atmospheric conditions , we generally use the gravity turn approach to achieve maximum range ,which was discussed above in the report , so with this thrust vector control we can continuously change the orientation of the rocket to match with the gravity turn profile , by changing the gimbal angle and thus achieving maximum range , We can also give our desired orientation angle for desired time intervals and the TVC will adjust the gimbal angle to attain those desired orientation , In lowest settling time possible dependent on the tuning the PID constants .

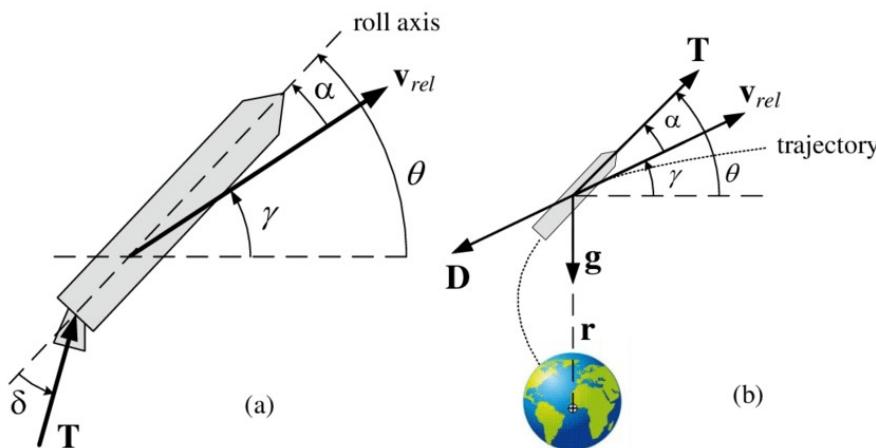
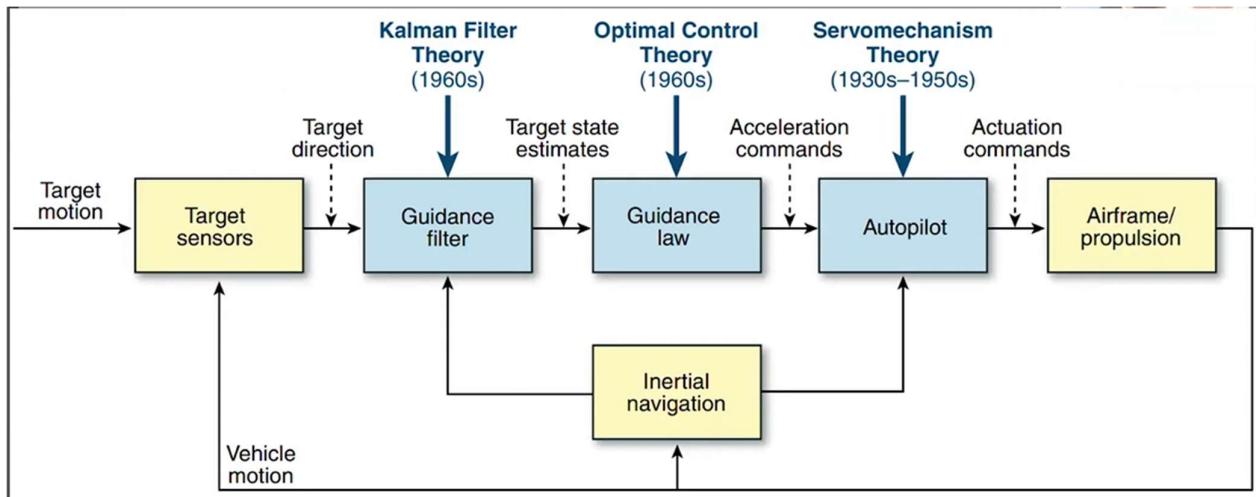


## EXTENSION, DERIVING FPA FOR GIVEN: TARGET CO-ORDINATES,

Right now the rocket uses the three-loop autopilot from the attached paper — it expects you to enter the Flight-Path Angle (FPA) and converts an acceleration command into nozzle gimbal motions. In the next phase I am adding an autonomous FPA generator that computes the desired angle from mission coordinates and the rocket's state — from a simple "point at the target" rule ( $\gamma = \text{atan}2(\Delta\text{alt}, \text{range})$ ) to a velocity-vector estimates or a full trajectory-optimization for best performance.

The autopilot then tracks the resulting acceleration command ( $A_{zc}$ ); this is a guidance (choose  $\gamma$ ) + control (track  $A_{zc}$ ) split and next step is to use a more optimal proportional navigation algorithm for generating FPA, it estimated the future position of the target and navigates to that estimated position rather than , following the current position,

To implement Proportional Navigation (PN) within the existing architecture, we will adopt a dual-mode guidance strategy that leverages our current FPA generator for the midcourse phase. We will continue to use our simple guidance algorithm after launch to fly an efficient trajectory toward the target's predicted location. Once our onboard seeker acquires the target and achieves a stable lock, a process we define as "handover," our guidance computer will transition its command logic. At this point, it will stop issuing acceleration commands based on the FPA rule and instead begin calculating commands using the Proportional Navigation law ( $a_c = NV_c \dot{\lambda}$ ). The existing three-loop autopilot is perfectly suited for this, as its fundamental design is to track an input acceleration command ( $A_{zc}$ ) regardless of the source. This allows us to integrate a high-performance terminal homing capability with minimal changes to our flight control system.



## THRUST VECTOR CONTROL

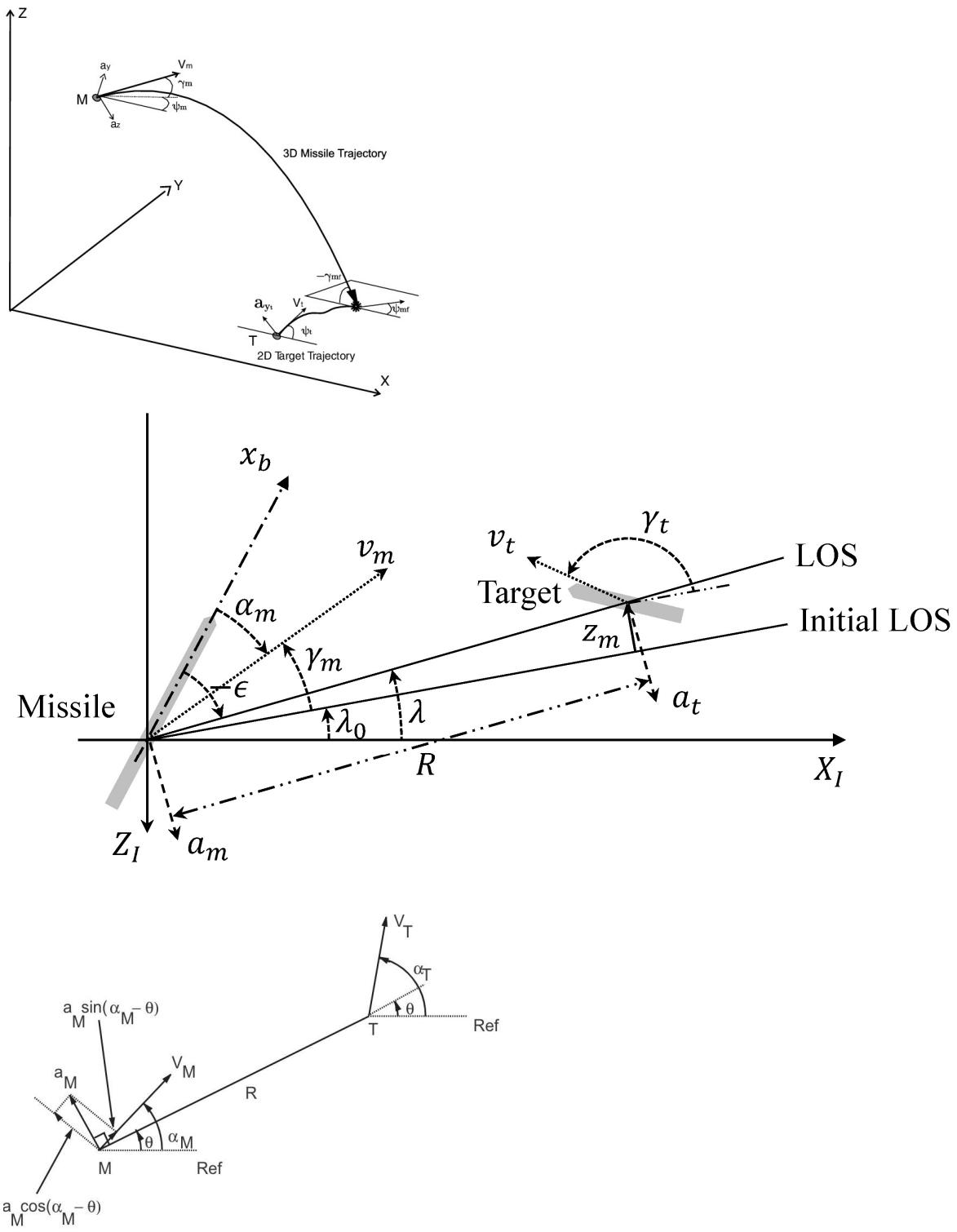


Figure 10.1: Missile-target engagement geometry for TPN

### 10.1 Original TPN with Non-Maneuvering Target

In this section we will consider the TPN law as it was first formulated in the non-linear setting. For our purposes we will call this the *original TPN* law. The missile latex is expressed as,

$$a_M = c\dot{\theta} \quad (10.1)$$

## REFERENCES

Here are the citations for the references you provided, formatted based on the available information:

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