GMM $\quad (\pi_1, \pi_2, \pi_3 \dots \pi_k)$

$$\sum_{i=1}^{k} \pi_i = 1$$

① E-step: To update the posterior $Pr(Z_n = k) \triangleq \gamma_{nk}$

responsibility

$$\gamma_{nk} \propto \underset{\pi_k}{\underset{\parallel}{\text{Prior}}} \times \text{likelihood}$$

$$\pi_k \times N(x_n | \mu_k, \Sigma_k) \Rightarrow \text{normalization} \quad \sum_{k=1}^{K} \gamma_{nk} = 1$$

for all n.

② M-step

①
$$\mathcal{L}_k^{\mu,\Sigma} = \sum_{n=1}^{N} \gamma_{nk} \log N(x_n | \mu_k, \Sigma_k)$$

$$\Downarrow \quad \partial_{\mu_k} \mathcal{L}_k^{\mu,\Sigma} = 0 \qquad \partial_{\Sigma_k} \mathcal{L}_k^{\mu,\Sigma} = 0$$

$$\mu_k = \frac{\sum_{n=1}^{N} \gamma_{nk} x_n}{N_k} \qquad N_k = \sum_{n=1}^{N} \gamma_{nk}$$

$$\Sigma_k = \frac{1}{N_k} \sum_{n=1}^{N} \gamma_{nk} (x_n - \mu_n) \cdot (x_n - \mu_n)^T$$

$$\mathcal{L}^{\pi} = \sum_{k=1}^{K} \sum_{n=1}^{N} \gamma_{nk} \log(\pi_k) + \lambda\Big(1 - \underbrace{\sum_{k=1}^{K} \pi_k}_{\text{why?}}\Big)$$

$$\partial_{\mu_k} \mathcal{L}^{\pi} = 0 \Rightarrow \pi_k = \frac{N_k}{N}$$

$$\sum_{k=1}^{K} \pi_k = 1$$

Repeat E-step & M-step until convergence. $\square$.

Doc1: $w_1 \; w_2 \cdots \; w_n$

Doc2: $w_1 \; w_2 \cdots \; w_n$

Doc3: $\vdots$. ——

1. K topics $(\alpha_1, \alpha_2 \cdots \alpha_K)$
2. Vocabulary $\{1, 2, \cdots V\}$
3. $$\beta_{|V| \times K} = \begin{pmatrix} \beta_{11} & \cdots & \beta_{1K} \\ \beta_{|V|1} & \cdots & \beta_{|V|K} \end{pmatrix}$$

$$\beta_{ij} = \Pr[w = i \mid z = j]$$

* ① Document $M \sim \text{Poisson}(\xi)$

② For each Doc, $\theta \sim \text{Dirichlet}(\alpha)$

③ $z_n \sim \text{Multinomial}(\theta)$

④ $w_n \sim \beta = \Pr(w_n = i \mid z_n = j)$

$i \in \{1, 2, \cdots V\}$

$j \in \{1, 2, \cdots K\}$,

VI. $p(\theta, z \mid w, \alpha, \beta) \Rightarrow$ Seth we want

approx.

$$q(\theta, z \mid \gamma, \phi_n) = q(\theta \mid \gamma) \cdot \prod_{i=1}^{n} q(z_n \mid \phi_n)$$

$$KL(q \| p) = -\mathbb{E}_q\left[\log\left(\frac{p}{q}\right)\right] \geq 0$$

$= 0$ iff $p = q$ a.s.

$$KL(q \| p) \neq KL(p \| q)$$

$$q(\theta \mid \gamma) \sim \text{Dirichlet}(\gamma)$$

$$q(z_n \mid \phi_n) \sim \text{Dirichlet}(\phi_n)$$