$\alpha \longrightarrow \theta$ Dist. of topics for each Doc. $\longrightarrow z_n$: Topic of each word

$\alpha \in [0,1]^k$

$\theta \in Simplex(k-1)$

$\sum_{i=1}^{k} \theta_i = 1$

$0 \le \theta_i \le 1$

$\theta \sim Dirichlet(\alpha)$

$z_n \in \{1, 2, \cdots k\}$

$Multinomial(\theta)$

β

w_n

$Pr[w_n = i \mid z_n = j] = \beta_{ij}$

$\beta = \begin{pmatrix} \beta_{11} & \beta_{12} \cdots & \beta_{1k} \\ \vdots & & \\ \beta_{|V|1} & \beta_{|V|2} \cdots & \beta_{|V|k} \end{pmatrix}$

$Pr(\theta, z \mid \alpha, \beta, w)$

parameters Data

$q(\theta \mid \gamma) \cdot \prod_{i=1}^{n} q(z_i \mid \phi_i)$

$Dirichlet(\gamma)$ $Multinomial(\phi_i)$

$KL[q \| p] = -E_q\left[\log\left(\frac{p}{q}\right)\right] \ge 0$

$$KL[q||p] = -\int \{\log[p(\theta, z|\alpha, \beta, w)] - \log(q(\theta|\gamma)) - \sum_{n=1}^{N} \log(z_n|\phi_n)\} q(\theta|\gamma) \prod_{n=1}^{N} q(z_n|\phi_n) \cdot d\theta\, dz_1\, dz_2 \cdots dz_N$$

EM-Algo.

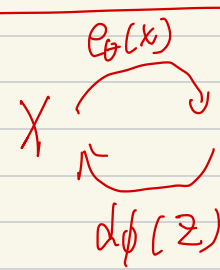
① E-step: $(\alpha, \beta) \Rightarrow (\gamma^*, \phi^*)$

② M-step: $(\gamma, \phi) \Rightarrow (\alpha^*, \beta^*)$

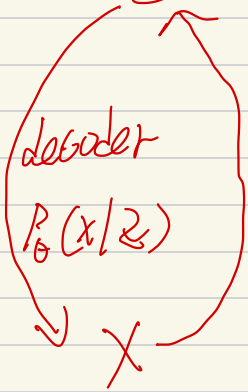
③ Iterate Until Convergence.

VAE

Autoencoder.



$$\text{loss} = \|X - \hat{X}\|_2 = \|X - d_\phi(e_\theta(x))\|_2$$



① Reconstruction loss

$\max \mathbb{E}_{q_\phi(z|x)} [\log p_\theta(x|z)]$

② $\min KL(q_\phi(z|x) \| p(z))$

Prior Matching

$ELBO = \mathbb{E}_{q_\phi(z|x)}[\log p_\theta(x|z)] - KL(q_\phi(z|x) \| p(z))$

Evidence

Lower bound

(Want to maximize)

$\log p_\theta(x) \geq ELBO$