1. (5 points) How many $\sigma \in S_5$ are there such that $\sigma(1) = 2$, $\sigma(2) = 1$ and σ is conjugate to $(1\ 5)(2\ 3)$?

2. (5 points) G and G' are groups. $f : G \longrightarrow G'$ is a map. Define the set $H = \{(g, g') \in G \times G' | g' = f(g)\}$. Prove H is a subgroup of $G \times G'$ if and only if f is a homomorphism.

3. (5 points) How many subgroups are there in $\mathbb{Z}/14\mathbb{Z}$?

4. (5 points) What is the order of the group $\mathrm{Aut}(\mathrm{Aut}(\mathbb{Z}/10\mathbb{Z}))$?

5. (5 points) G is a group. If $|g| = 2$ for any non-identity $g \in G$, prove that G is abeian.

6. (5 points) $\mathbb{R}$ is the group of real numbers with addition. $\mathbb{Q}$ is the subgroup of rational numbers. r is a nonzero real number, denote $r\mathbb{Q} = \{rq \in \mathbb{R} | q \in \mathbb{Q}\}$. Prove $\mathbb{R}/r\mathbb{Q} \cong \mathbb{R}/\mathbb{Q}$.

7. (5 points) $f : G \longrightarrow G'$ is a homomorphism. $K = \ker(f)$. H is a subgroup of G, and $H' = \{f(h) \in G' | h \in H\}$, $M = \{g \in G | f(g) \in H'\}$. Prove that $M = HK$.