Math 140 - Fall 2017 - Midterm Exam - Version B

You have 105 minutes to complete this midterm exam. Books, notes and electronic devices are not permitted. Read and follow directions carefully. Show and check all work. Label graphs and include units where appropriate. If a problem is not clear, please ask for clarification.

Multiple Choice 1-10	/30
Free Response 11	/10
Free Response 12	/10
Free Response 13	/10
Free Response 14	/10
Free Response 15	/10
Total	/80

I pledge that I have completed this midterm exam in compliance with the NYU CAS Honor Code. In particular, I have neither given nor received unauthorized assistance during this exam.

Name ____

N Number ____

Signature ____

Date ____

1 Multiple Choice

(30 points) For problems 1-10, circle the best answer choice.

1. Let $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 2 \\ 2 & 2 \\ 2 & 2 \end{bmatrix}$. Then $AB =$

   (a) $\begin{bmatrix} 0 & 12 \\ 0 & 12 \end{bmatrix}$

   (b) $\begin{bmatrix} 0 & 0 \\ 12 & 12 \end{bmatrix}$

   (c) $\begin{bmatrix} 6 & 4 & 2 \\ 6 & 4 & 2 \\ 6 & 4 & 2 \end{bmatrix}$

   (d) $\begin{bmatrix} 6 & 6 & 6 \\ 4 & 4 & 4 \\ 2 & 2 & 2 \end{bmatrix}$

   (e) AB is not defined

2. Let $\boldsymbol{u}$ and $\boldsymbol{v}$ be two nonzero vectors in $\mathbb{R}^n$. Consider the angles $\theta_1, \theta_2, \theta_3$ and θ_4 defined as follows:

   θ_1 is the angle between $\boldsymbol{u}$ and $\boldsymbol{v}$

   θ_2 is the angle between $-\boldsymbol{u}$ and $\boldsymbol{v}$

   θ_3 is the angle between $\boldsymbol{u}$ and $-\boldsymbol{v}$

   θ_4 is the angle between $-\boldsymbol{u}$ and $-\boldsymbol{v}$

   What is the relationship between $\theta_1, \theta_2, \theta_3$ and θ_4?

   (a) $\theta_1 = \pi + \theta_2 = \pi + \theta_3 = \theta_4$

   (b) $\theta_1 = \theta_2 = \theta_3 = \theta_4$

   (c) $\theta_1 = \pi - \theta_2 = \pi - \theta_3 = \theta_4$

   (d) $\theta_1 = \pi + \theta_2 = \theta_3 = \pi + \theta_4$

   (e) $\theta_1 = \theta_4$ and $\theta_2 = \theta_3$, but there is no relationship between θ_1 and θ_2

3. For what values of t is $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & t \\ 1 & 4 & t^2 \end{bmatrix}$ NOT invertible?

(a) $t \in \mathbb{R}$

(b) $t > 0$

(c) $t < 0$

(d) $t = \pm 1$

(e) $t = 1$ and $t = 2$

4. For which value(s) h is the linear system associated with the following augmented matrix solvable?

$$\left[\begin{array}{rrr|r} 0 & 6 & 1 & -3 \\ -2 & 6 & 6 & h \\ -2 & -12 & 3 & 5 \end{array}\right]$$

(a) 0

(b) 4

(c) -4

(d) 14

(e) None of the above

5. Which of the following matrices is in reduced row-echelon form?

(a) $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix}$

(d) $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$

(e) $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

6. Let $A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & c & 2 \\ c & 8 & c \end{bmatrix}$. For which value of c does A have rank 1?

(a) 0

(b) -1

(c) 1

(d) 8

(e) 4

7. A basis for $\mathbb{P}_2$, the space of all polynomial functions with real coefficients of degree at most 2, is $\{1, x, x^2\}$. Which of the following give another basis for $\mathbb{P}_2$?

(a) $\{(x-1)^2, x-1, 1\}$

(b) $\{(x-1)^2, x-1\}$

(c) $\{(x-1)^2\}$

(d) None of the above; the basis of $\mathbb{P}_2$ is unique.

(e) None of the above; a basis of $\mathbb{P}_2$ is not unique, but the bases above are not of $\mathbb{P}_2$.

8. The dimension of $\mathbb{P}_n$, the space of all polynomial functions with real coefficients of degree at most n, is

(a) 1

(b) $n-1$

(c) n

(d) $n+1$

(e) n^2

9. Span$\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ -1 \\ 0 \end{bmatrix} \right\} =$

(a) $\mathbb{R}^2$

(b) $\mathbb{R}^3$

(c) a line in $\mathbb{R}^2$

(d) a line in $\mathbb{R}^3$

(e) a plane in $\mathbb{R}^3$

10. Suppose A is a 53×32 matrix whose rank is 17. Which of the following is a true statement?

(a) Col A is a 17-dimensional subspace of $\mathbb{R}^{53}$ and Nul A^T is a 17-dimensional subspace of $\mathbb{R}^{32}$.

(b) Col A is a 36-dimensional subspace of $\mathbb{R}^{53}$ and Nul A^T is a 17-dimensional subspace of $\mathbb{R}^{53}$.

(c) Col A is a 17-dimensional subspace of $\mathbb{R}^{53}$ and Nul A^T is a 36-dimensional subspace of $\mathbb{R}^{53}$.

(d) Col A is a 17-dimensional subspace of $\mathbb{R}^{32}$ and Nul A^T is a 15-dimensional subspace of $\mathbb{R}^{32}$.

(e) Col A is a 15-dimensional subspace of $\mathbb{R}^{32}$ and Nul A^T is a 17-dimensional subspace of $\mathbb{R}^{32}$.

2 Free Response

For problems 11-15, show all work and justify each step to receive full credit. Draw a box around your answers.

11. (10 points) Consider the matrix $A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix}$

(a) Compute A^{-1}.

(b) Compute $K = A^T A$.

(c) Compute K^{-1}. (*Hint: Use* A^{-1}.)

(d) Solve $K\boldsymbol{x} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$. (*Hint: Use* K^{-1}.)

12. (10 points) Find the LDU factorization for $A = \begin{bmatrix} 2 & 1 & 1 \\ -8 & -7 & -3 \\ 2 & -14 & 7 \end{bmatrix}$.

13. (10 points) Recall that $\mathbb{M}_{2\times 2} = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} \middle| \, a, b, c, d \in \mathbb{R} \right\}$ is a vector space.

Determine which of the following subsets $W \subset \mathbb{M}_{2\times 2}$ are subspaces of $\mathbb{M}_{2\times 2}$. Support your answer.

(a) $W = \{A \,|\, a + d = 0\}$

(b) $W = \{A \,|\, b = c\}$

(c) $W = \{A \,|\, ad - bc \neq 0\}$

14. (10 points) Find the complete solution $\boldsymbol{x} = \boldsymbol{x}_p + \boldsymbol{x}_n$ to the following linear system.

$$\begin{bmatrix} 1 & 2 & 2 & 2 \\ 2 & 4 & 6 & 8 \\ 3 & 6 & 8 & 10 \end{bmatrix} \boldsymbol{x} = \begin{bmatrix} 1 \\ 5 \\ 6 \end{bmatrix}$$

15. (10 points) Find a basis and dimension for each of the four vector spaces associated with the matrix.

$$R = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$