Q2.10. $P(\text{recover}) = 0.8$

$P(\text{exactly 2 of the next 3 recover})$

$= 0.8 \cdot 0.8 \cdot 0.2 + 0.8 \cdot 0.2 \cdot 0.8 + 0.2 \cdot 0.8 \cdot 0.8$

Discrete r.v.

p.m.f. $f(x) = P(X = x)$

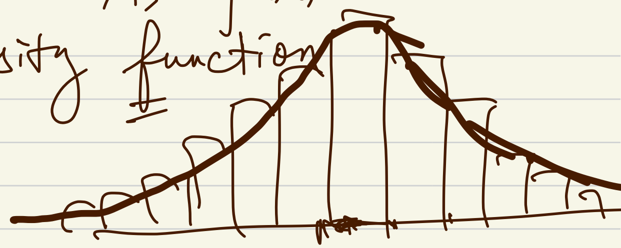
c.d.f. $F(x) = P(X \leq x)$

Continuous r.v.

for any x, $P(X = x) = 0$

$P(X = 5'3'') = 0$

Def: For a continuous r.v. X, $f(x)$ is said to be a probability density function (p.d.f.) if:



① $f(x) \geq 0$ for any x.

② $\int_{-\infty}^{\infty} f(x)\,dx = 1$

③ $P(a \leq X \leq b) = \int_a^b f(x)\,dx$

$a < X < b$

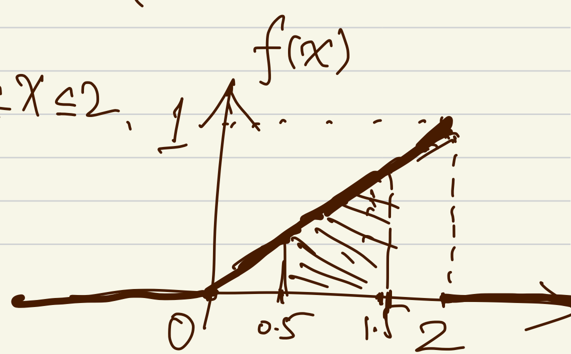
$a \leq X < b$

$a < X \leq b$

Ex: ① X is a random number in $[0,1]$

$$f(x) = \begin{cases} 1 & 0 \leq x \leq 1 \\ 0 & \text{elsewhere} \end{cases}$$

② $f(x) = \frac{x}{2}$ $0 \leq x \leq 2$.



a $P(X \in [0.5, 1.5])$

$= \frac{1}{2}$

③ X is a cont r.v. with pdf $f(x) = c \cdot x^2 \quad -1 \le x \le 2$.

ⓐ $c = ?$ $\frac{1}{3}$

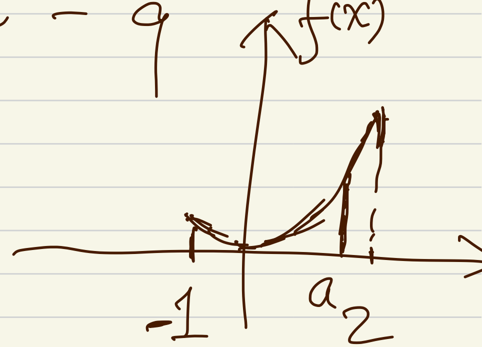
ⓑ $P(X \le 0)$

ⓒ what's the median of ~~this~~ dist ?

$$\int_{0.5}^{1.5} \frac{x}{2}\,dx = \frac{x^2}{4}\Big|_{0.5}^{1.5} = \frac{1}{2}$$

$$1 = \int_{-1}^{2} c x^2\,dx = \frac{c}{3} x^3 \Big|_{-1}^{2} = 3c$$

$$P(X \le 0) = \int_{-1}^{0} \frac{x^2}{3}\,dx = \frac{1}{9}$$



$$a = \sqrt[3]{\frac{7}{2}}$$

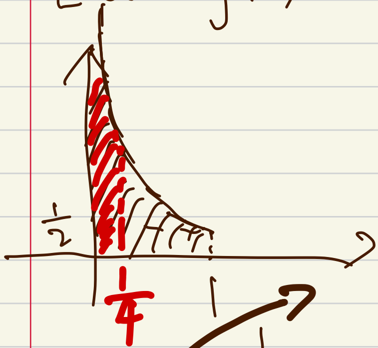
$$\frac{7}{18} = \int_{0}^{a?} \frac{x^2}{3}\,dx$$

$\frac{1}{2}$ (-1)

$$P(X \le a) = \frac{1}{2}$$

Def: a is the median of the dist of X: $P(X \le a) = P(X \ge a) = \frac{1}{2}$

Ex: $f(x) = \dfrac{1}{2\sqrt{x}} \qquad 0 < x < 1.$



$$\int_0^1 \frac{1}{2\sqrt{x}}\,dx = 1$$

median = ?

$$\int_0^{\frac14} \frac{1}{2\sqrt{x}} = \sqrt{x}\,\Big|_0^{\frac14} = \frac12 \qquad \checkmark$$

$$\int_0^{a} \frac{1}{2\sqrt{x}}\,dx = \frac12$$

$a = ?$

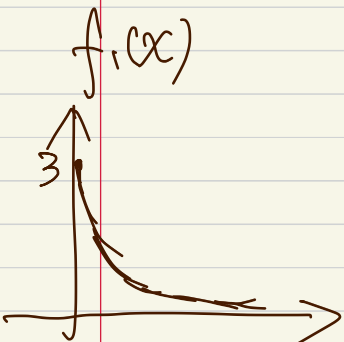
$f_1(x) = c_1 e^{-3x} \qquad x \geq 0$

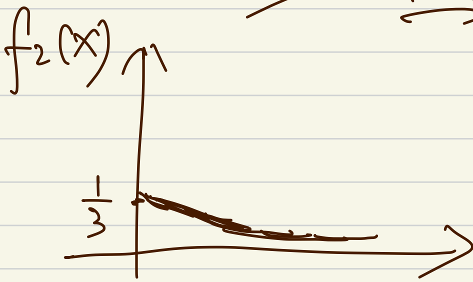
$f_2(x) = c_2 e^{-x/3} \qquad x \geq 0.$

$f_1(x) = 3e^{-3x} \qquad x \geq 0$

$f_2(x) = \frac13 e^{-\frac{x}{3}} \qquad x \geq 0$

a. $$1 = \int_0^\infty c e^{-3x}\,dx = c\left(-\frac{1}{3}\right) e^{-3x} \Big|_0^\infty = \frac{c}{3} \Rightarrow c = 3.$$

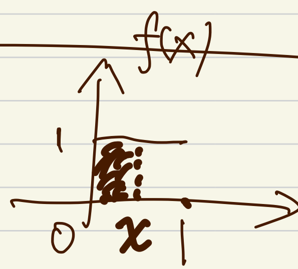




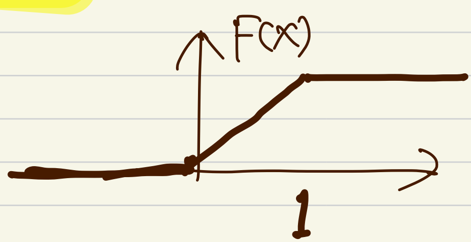
c.d.f. $$F(x) = P(X \le x) = \int_{-\infty}^{x} f(t)\,dt.$$

$$F'(x) = f(x)$$

$$f(x) = 1 \quad 0 < x < 1$$

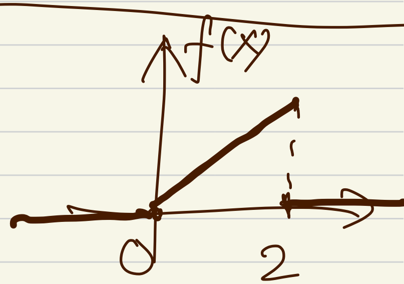


$$F(x) = \begin{cases} 0 & x < 0 \\ x & 0 \le x \le 1 \\ 1 & x > 1 \end{cases}$$



$$F(x) = \int_0^x 1 \, dt = x$$

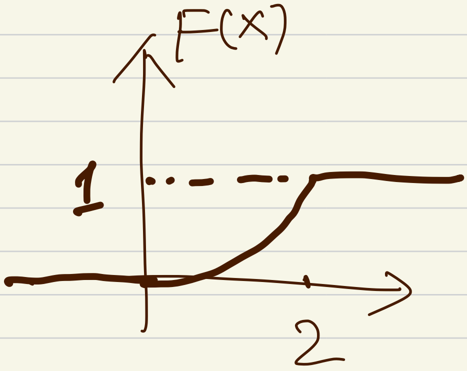
$$f(x) = \frac{x}{2} \quad 0 \le x \le 2$$



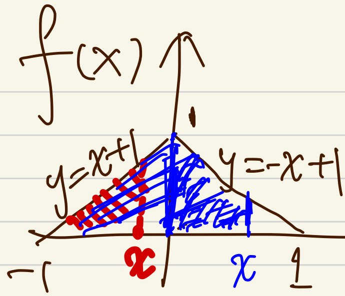
$$F(x) = \begin{cases} 0 & x < 0 \\ \frac{x^2}{4} & 0 \le x \le 2 \\ 1 & x > 2 \end{cases}$$

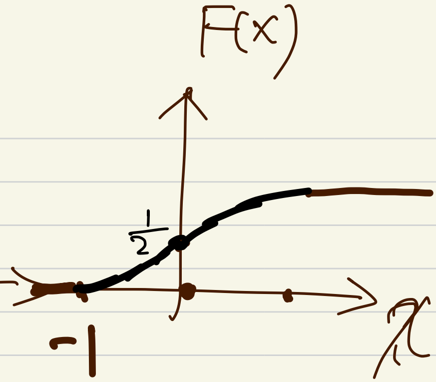
$x \in [0, 2)$,

$$F(x) = \int_0^x \frac{t}{2} \, dt = \frac{x^2}{4}$$



Ex.





$$f(x)=\begin{cases} x+1 & -1\le x\le 0 \\ -x+1 & 0\le x\le 1 \end{cases}$$

$-1 \le x \le 0$

$$F(x)=\int_{-1}^{x} t+1\,dt = \left(\frac{t^2}{2}+t\right)\Big|_{-1}^{x} = \frac{x^2}{2}+x+\frac{1}{2}$$

$0 \le x \le 1$

$$F(x)=\frac{1}{2}+\int_{0}^{x} -t+1\,dt = \frac{1}{2}+\left(-\frac{t^2}{2}+t\right)\Big|_{0}^{x}$$

$$= -\frac{x^2}{2}+x+\frac{1}{2}$$

$$F(x)=\begin{cases} 0 & x\le -1 \\ \frac{x^2}{2}+x+\frac{1}{2} & -1\le x\le 0 \\ -\frac{x^2}{2}+x+\frac{1}{2} & 0\le x\le 1 \\ 1 & x\ge 1 \end{cases}$$