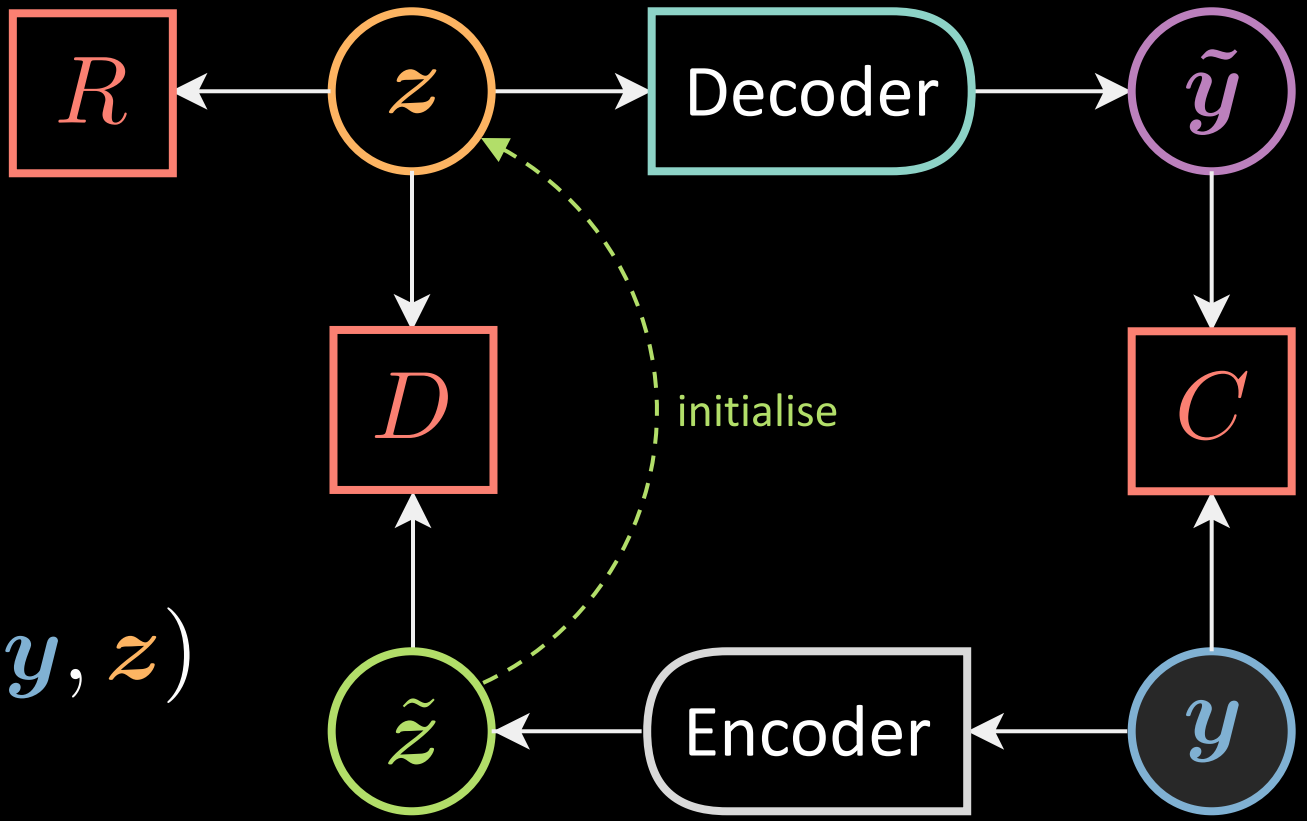
Target prop(agation)



Given an observation y,

- Compute $\tilde{z} = \mathrm{Enc}(y)$
- Compute $\check{z} = \arg\min_{z} E(y, z)$
- Minimise $\mathcal{L}(F_\infty(\cdot), Y)$:

$$\boldsymbol{w}_{\mathrm{Dec}} \leftarrow \boldsymbol{w}_{\mathrm{Dec}} - \eta \nabla_{\boldsymbol{w}_{\mathrm{Dec}}} C(y, \tilde{y})$$

$$\boldsymbol{w}_{\mathrm{Enc}} \leftarrow \boldsymbol{w}_{\mathrm{Enc}} - \eta \nabla_{\boldsymbol{w}_{\mathrm{Enc}}} D(\check{z}, \tilde{z})$$

$$E(y, z) = C(y, \tilde{y}) + R(z) + D(z, \tilde{z})$$

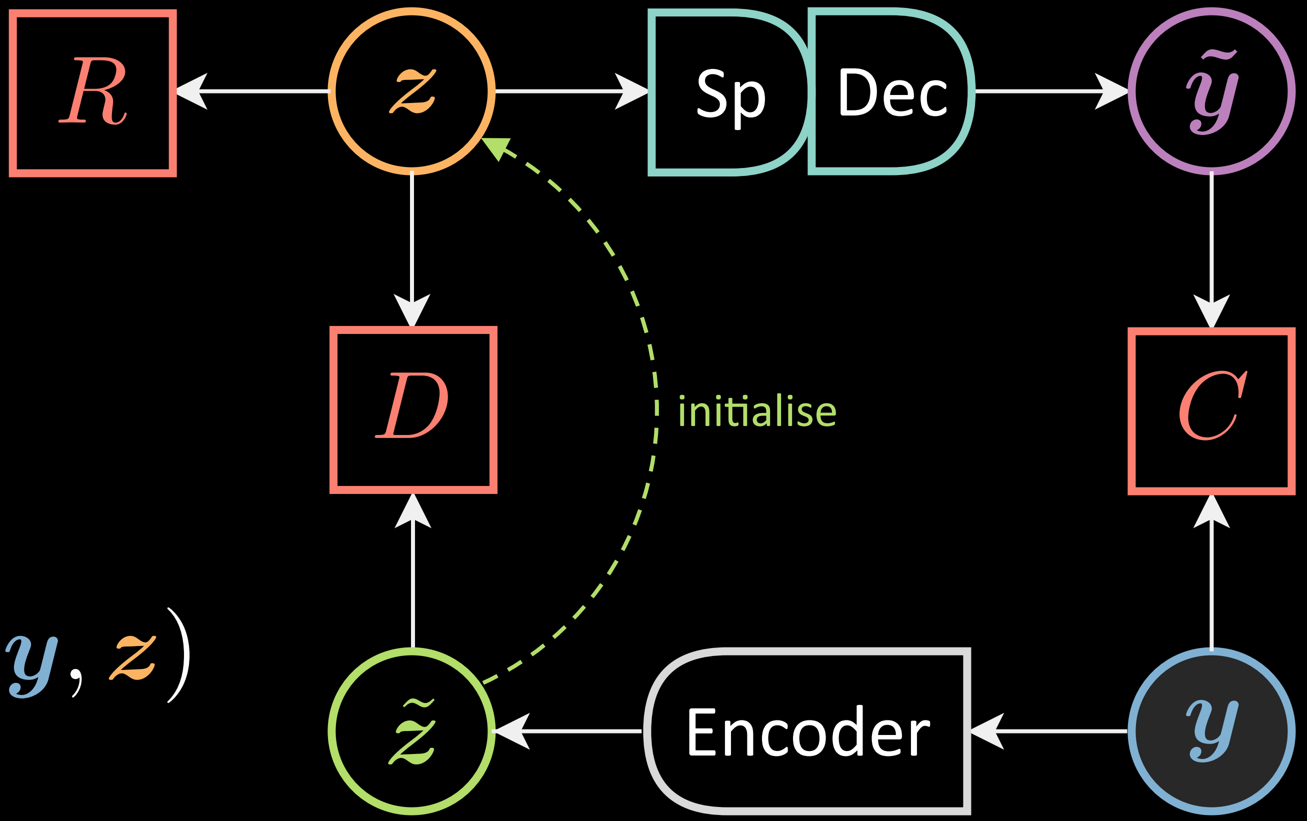
Non-linear actvtn

Given an observation $\boldsymbol{y}$,

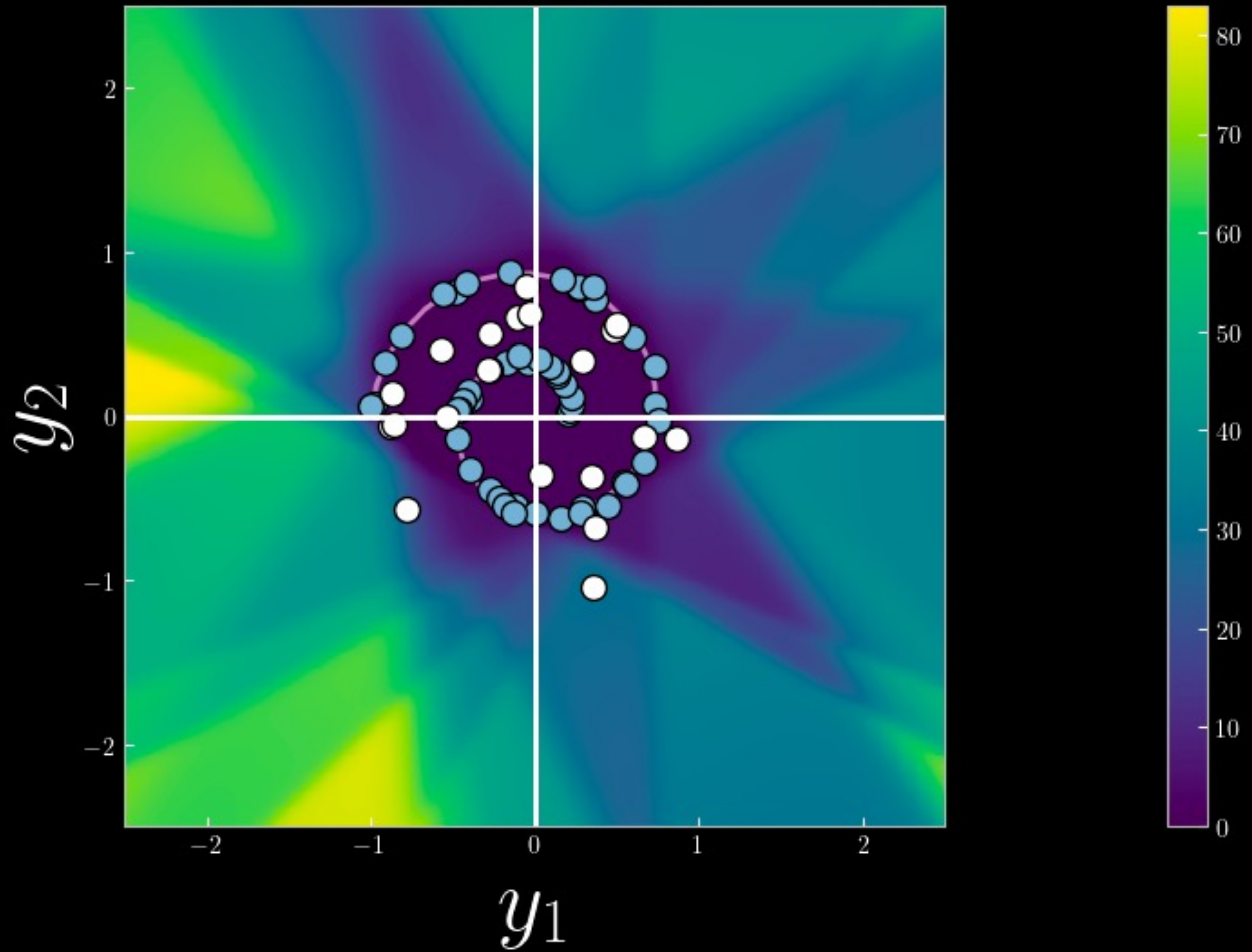
- Compute $\tilde{\boldsymbol{z}} = \mathrm{Enc}(\boldsymbol{y})$
- Compute $\check{\boldsymbol{z}} = \arg\min_{\boldsymbol{z}} E(\boldsymbol{y}, \boldsymbol{z})$
- Minimise $\mathcal{L}(F_\infty(\,\cdot\,), \boldsymbol{Y})$:

$$\boldsymbol{w}_{\mathrm{Dec}} \leftarrow \boldsymbol{w}_{\mathrm{Dec}} - \eta \nabla_{\boldsymbol{w}_{\mathrm{Dec}}} C(\boldsymbol{y}, \tilde{\boldsymbol{y}})$$

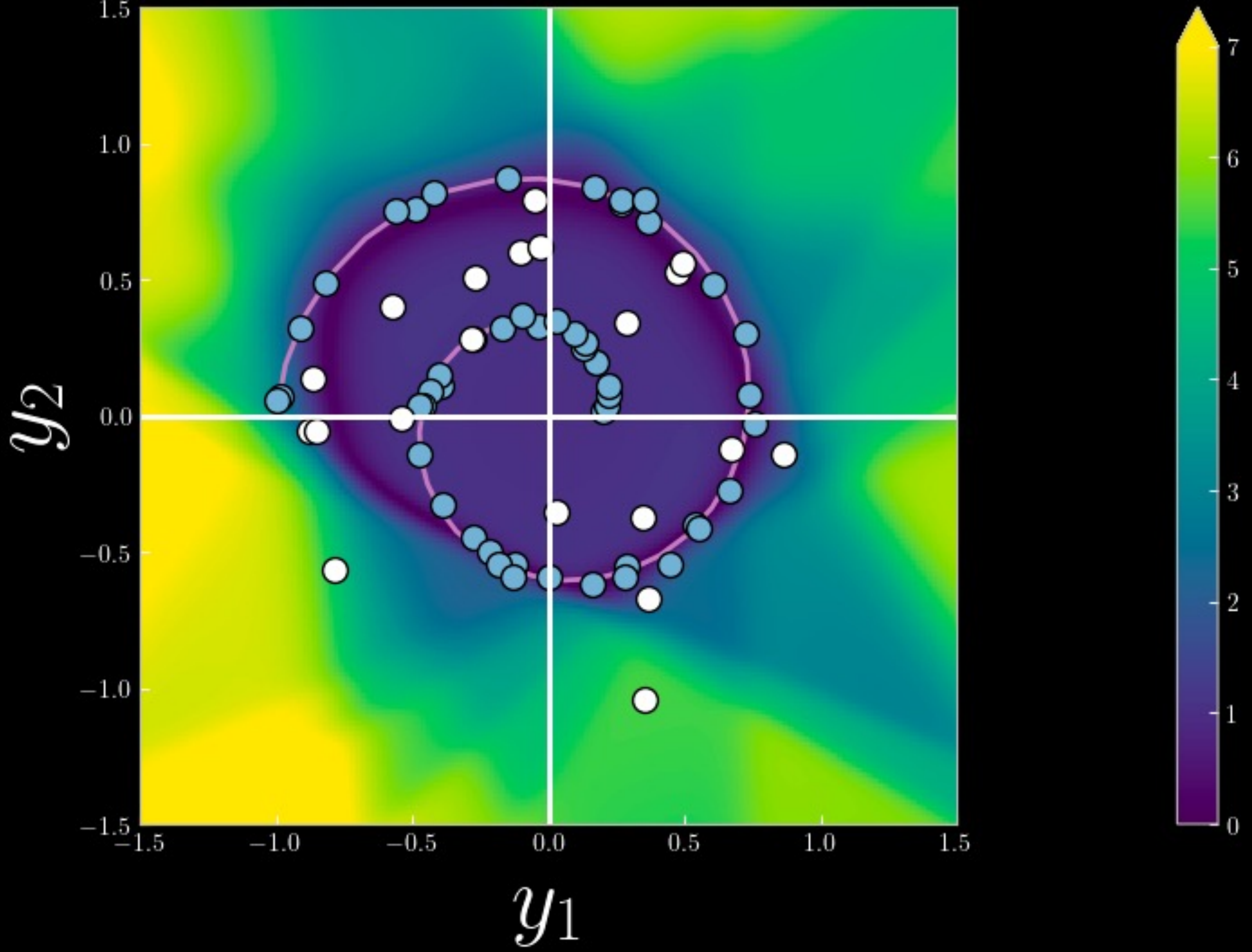
$$\boldsymbol{w}_{\mathrm{Enc}} \leftarrow \boldsymbol{w}_{\mathrm{Enc}} - \eta \nabla_{\boldsymbol{w}_{\mathrm{Enc}}} D(\check{\boldsymbol{z}}, \tilde{\boldsymbol{z}})$$



$$E(\boldsymbol{y}, \boldsymbol{z}) = C(\boldsymbol{y}, \tilde{\boldsymbol{y}}) + R(\boldsymbol{z}) + D(\boldsymbol{z}, \tilde{\boldsymbol{z}})$$

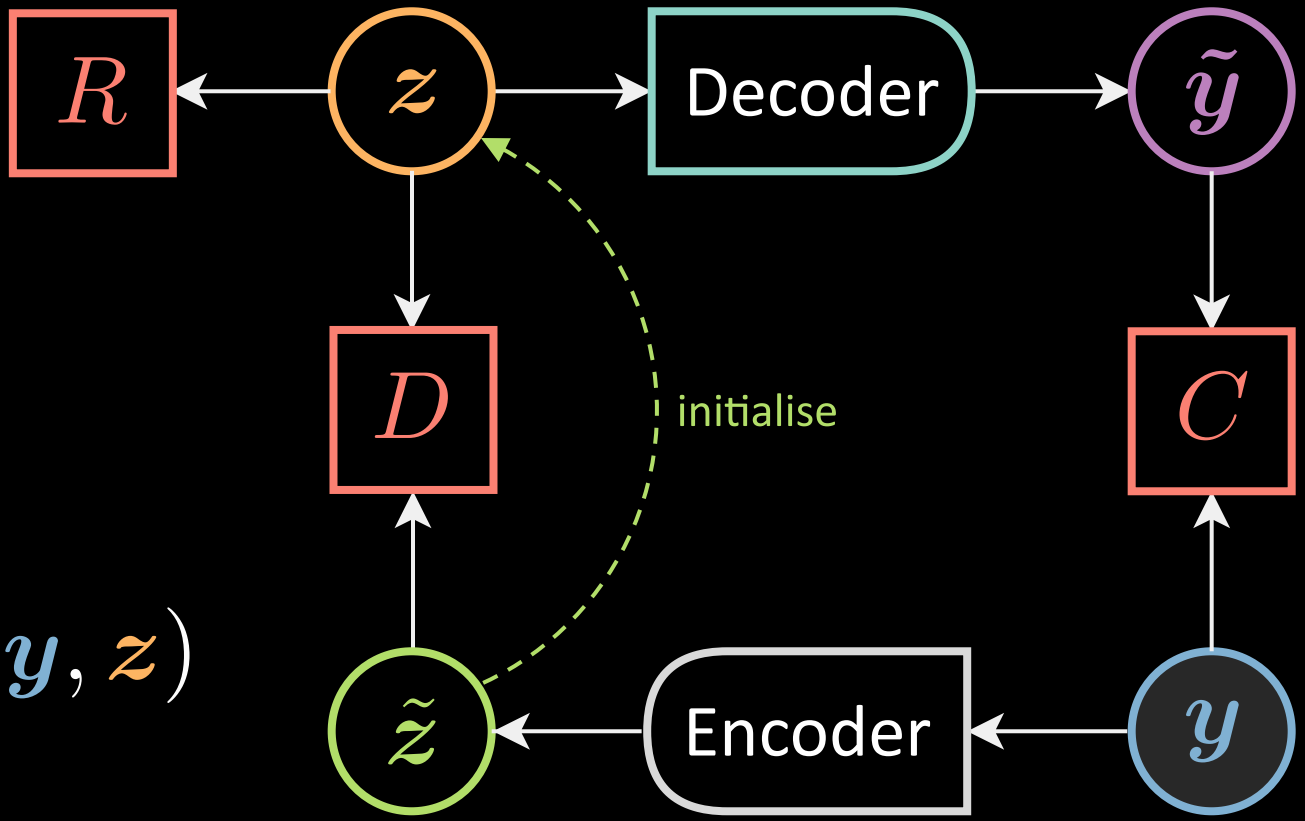


y_2
y_1
2
1
0
−1
−2
−2
−1
0
1
2
80
70
60
50
40
30
20
10
0



y_1
y_2
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5
0
1
2
3
4
5
6
7

Target prop(agation)



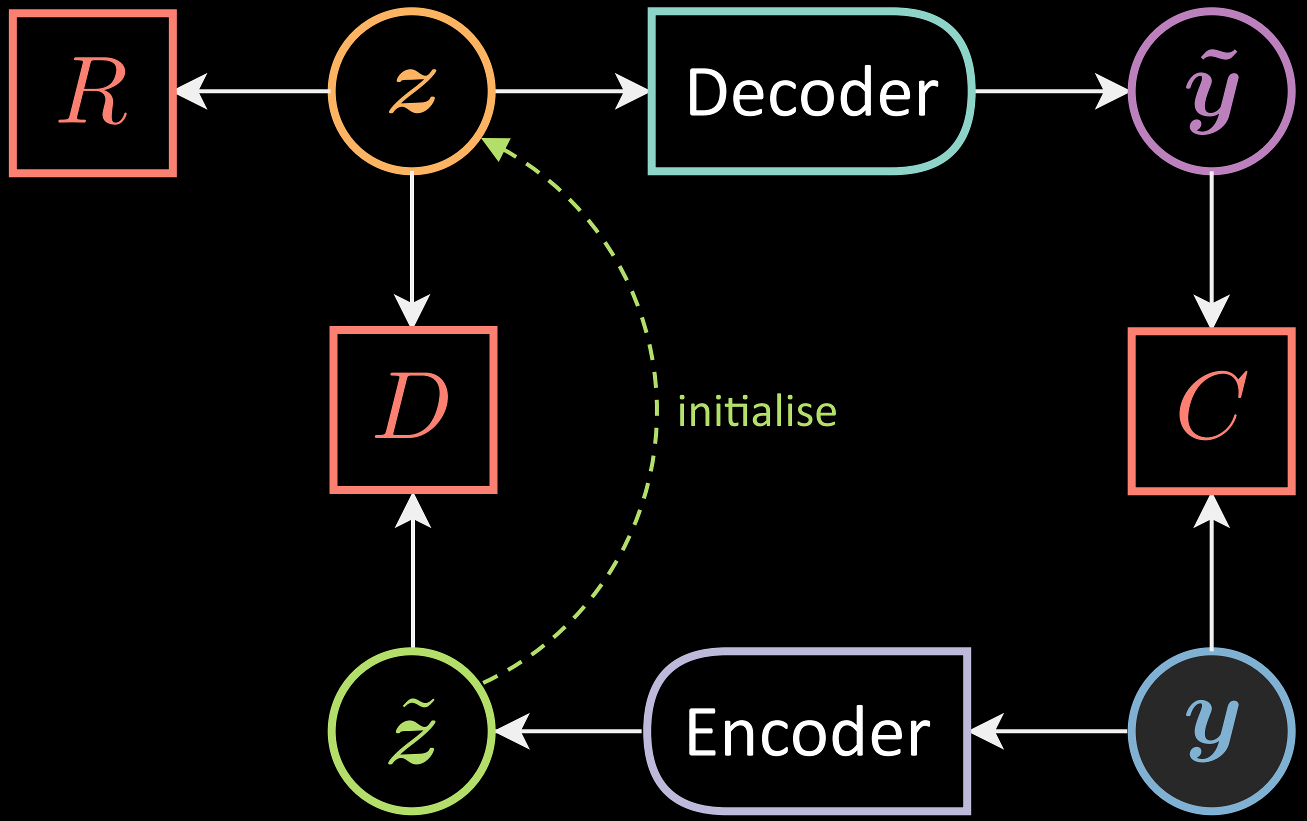
Given an observation y,

- Compute $\tilde{z} = \mathrm{Enc}(y)$
- Compute $\check{z} = \arg\min_{z} E(y, z)$
- Minimise $\mathcal{L}(F_\infty(\overset{z}{\cdot}), Y)$:

$$w_{\mathrm{Dec}} \leftarrow w_{\mathrm{Dec}} - \eta \nabla_{w_{\mathrm{Dec}}} C(y, \tilde{y})$$

$$w_{\mathrm{Enc}} \leftarrow w_{\mathrm{Enc}} - \eta \nabla_{w_{\mathrm{Enc}}} D(\check{z}, \tilde{z})$$

$$E(y, z) = C(y, \tilde{y}) + R(z) + D(z, \tilde{z})$$



R
z
Decoder
$\tilde{y}$
D
initialise
C
$\tilde{z}$
Encoder
y

Autoencoder

$$\boldsymbol{h} = f(\boldsymbol{W_h y} + \boldsymbol{b_h})$$

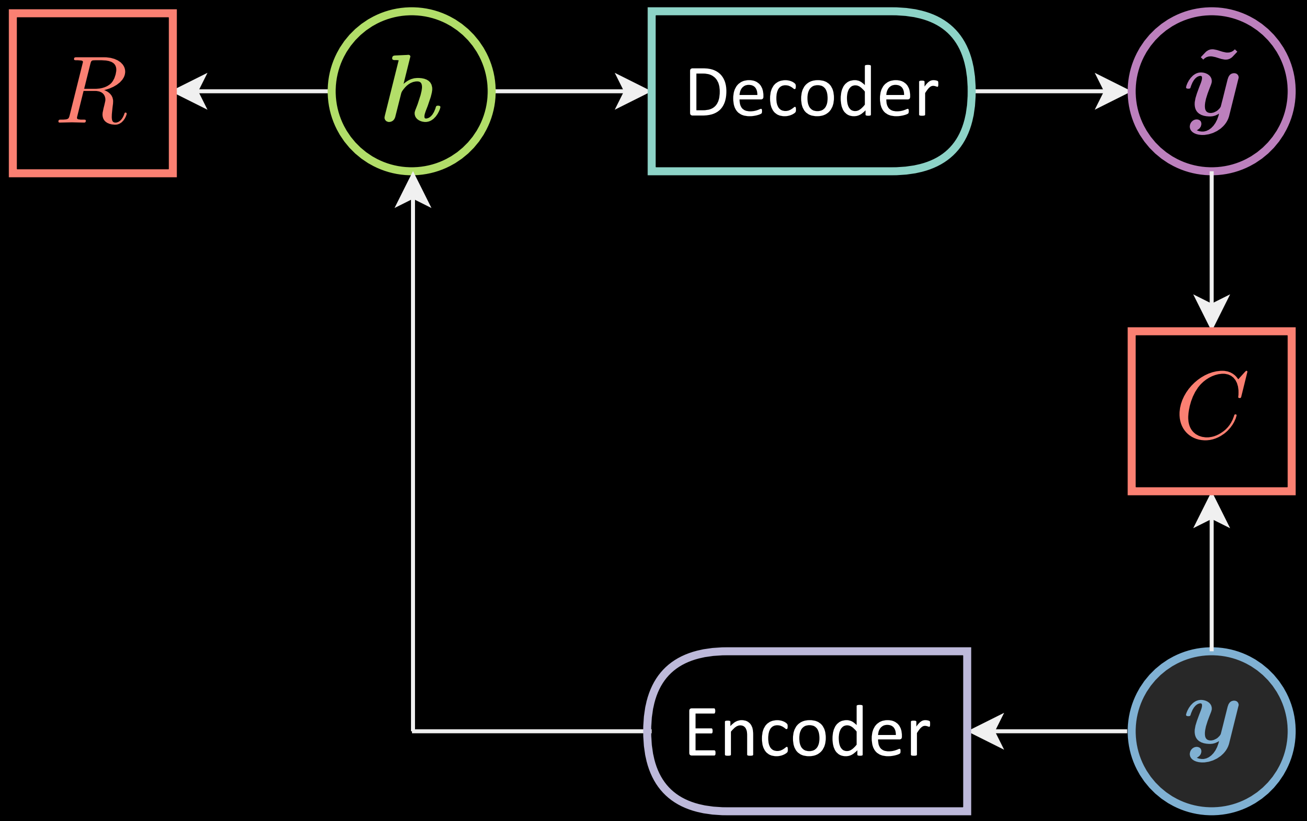
$$\tilde{\boldsymbol{y}} = g(\boldsymbol{W_{\tilde{y}} h} + \boldsymbol{b_{\tilde{y}}})$$

$$\boldsymbol{y}, \tilde{\boldsymbol{y}} \in \mathbb{R}^K$$

$$\boldsymbol{h} \in \mathbb{R}^d$$

$$\boldsymbol{W_h} \in \mathbb{R}^{d \times K}$$

$$\boldsymbol{W_{\tilde{y}}} \in \mathbb{R}^{K \times d}$$



$$\boldsymbol{h} = \mathrm{Enc}(\boldsymbol{y}) \quad \tilde{\boldsymbol{y}} = \mathrm{Dec}(\boldsymbol{h})$$

$$F(\boldsymbol{y}) = C(\boldsymbol{y}, \tilde{\boldsymbol{y}}) + R(\boldsymbol{h})$$

Reconstruction costs

real valued input

$$C(\boldsymbol{y}, \tilde{\boldsymbol{y}}) = \|\boldsymbol{y} - \tilde{\boldsymbol{y}}\|^2 = \|\boldsymbol{y} - \mathrm{Dec}\left[\mathrm{Enc}(\boldsymbol{y})\right]\|^2$$

binary input

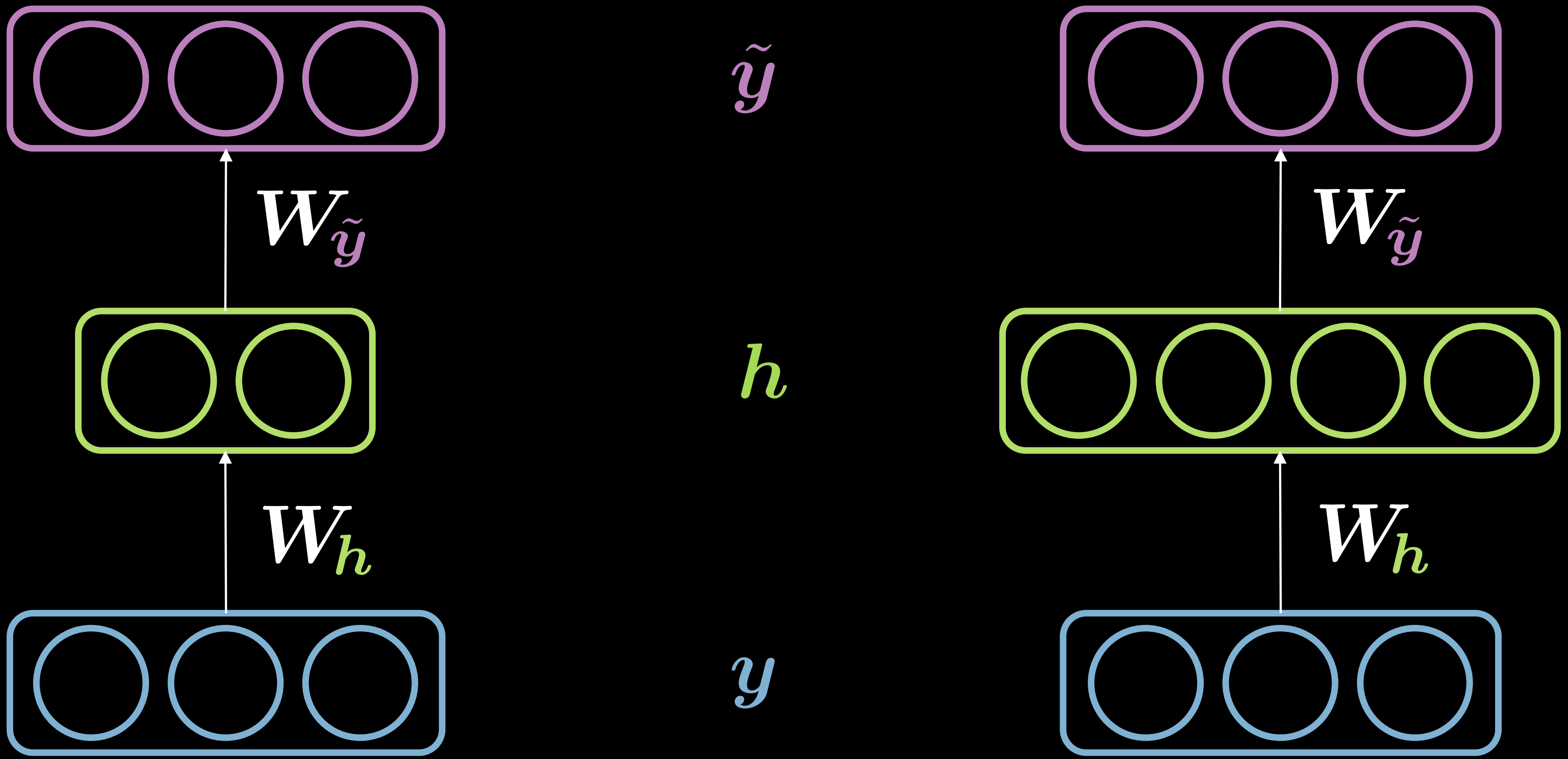
$$C(\boldsymbol{y}, \tilde{\boldsymbol{y}}) = -\sum_{k=1}^{K}\left[y_k \log(\tilde{y}_k) + (1 - y_k)\log(1 - \tilde{y}_k)\right]$$

Loss functional

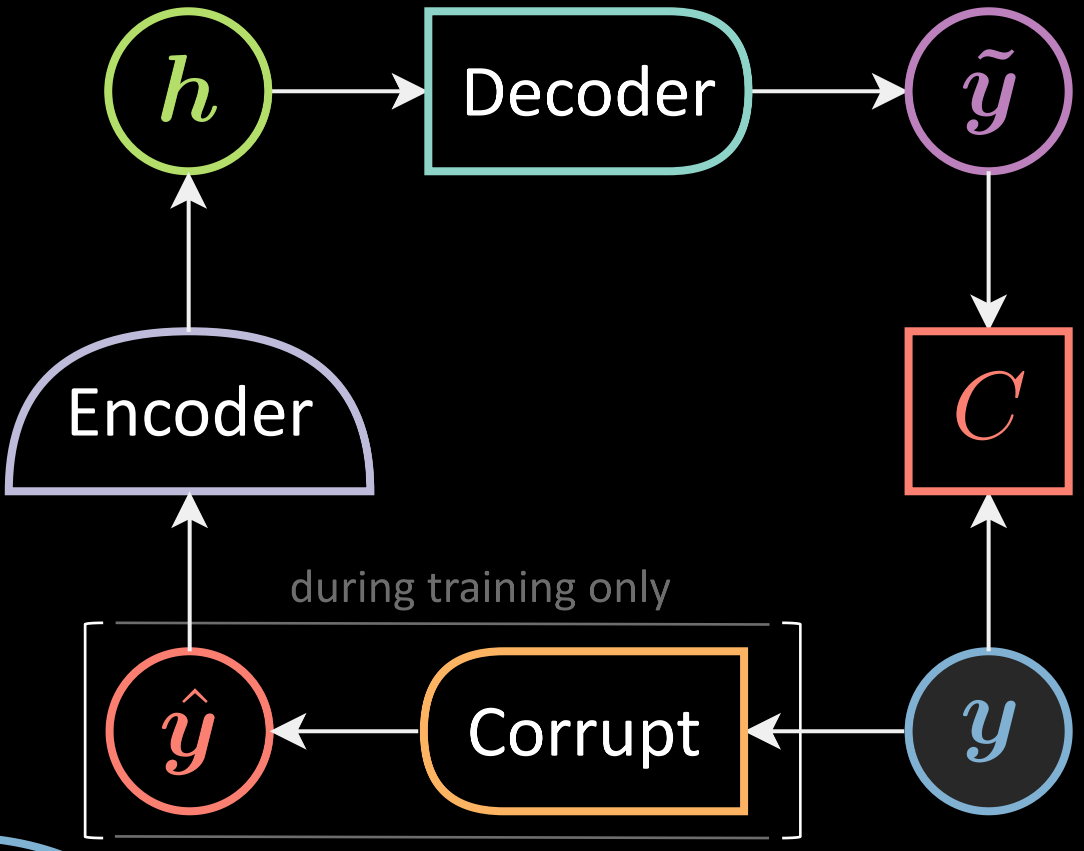
$$\mathcal{L}[F(\mathcal{Y}), \mathcal{S}] \doteq \frac{1}{P} \sum_{p=1}^{P} L[F(\mathcal{Y}), \boldsymbol{y}^{(p)}]$$

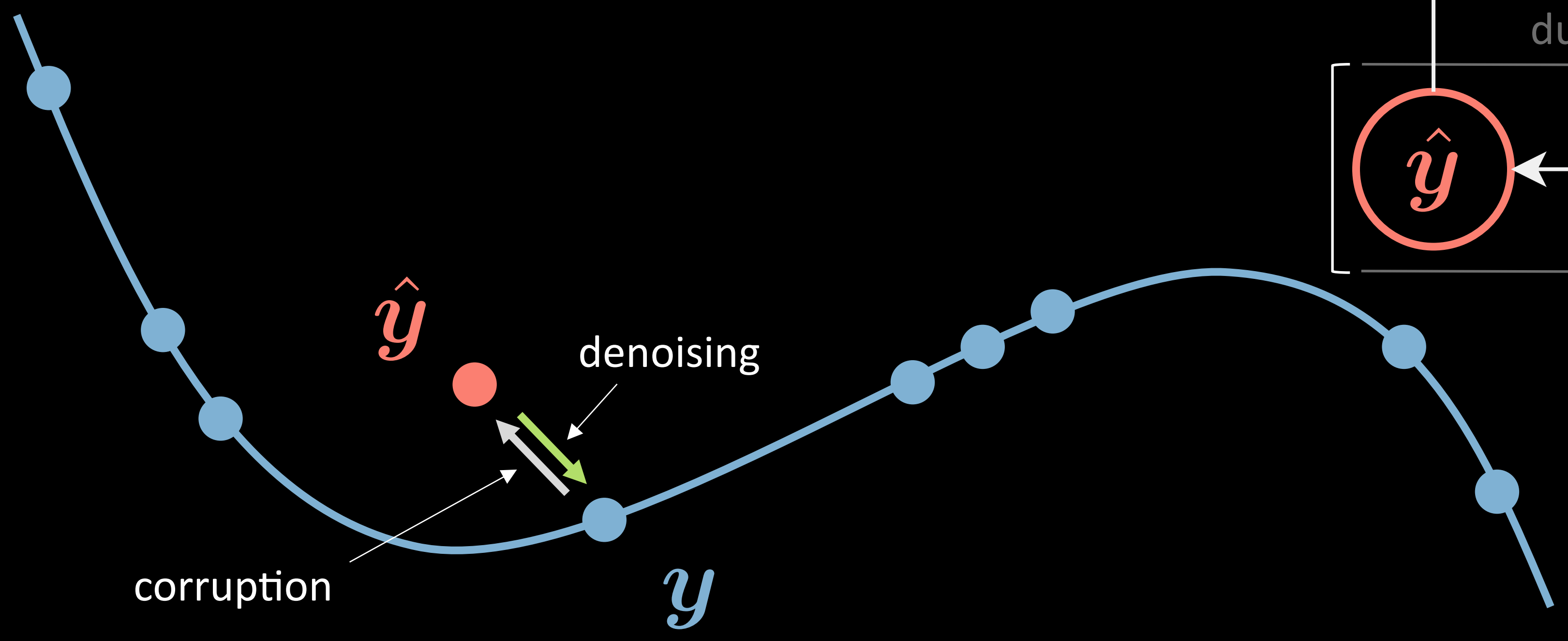
$$L_{\text{energy}}[F(\mathcal{Y}), \boldsymbol{y}] = F(\boldsymbol{y})$$

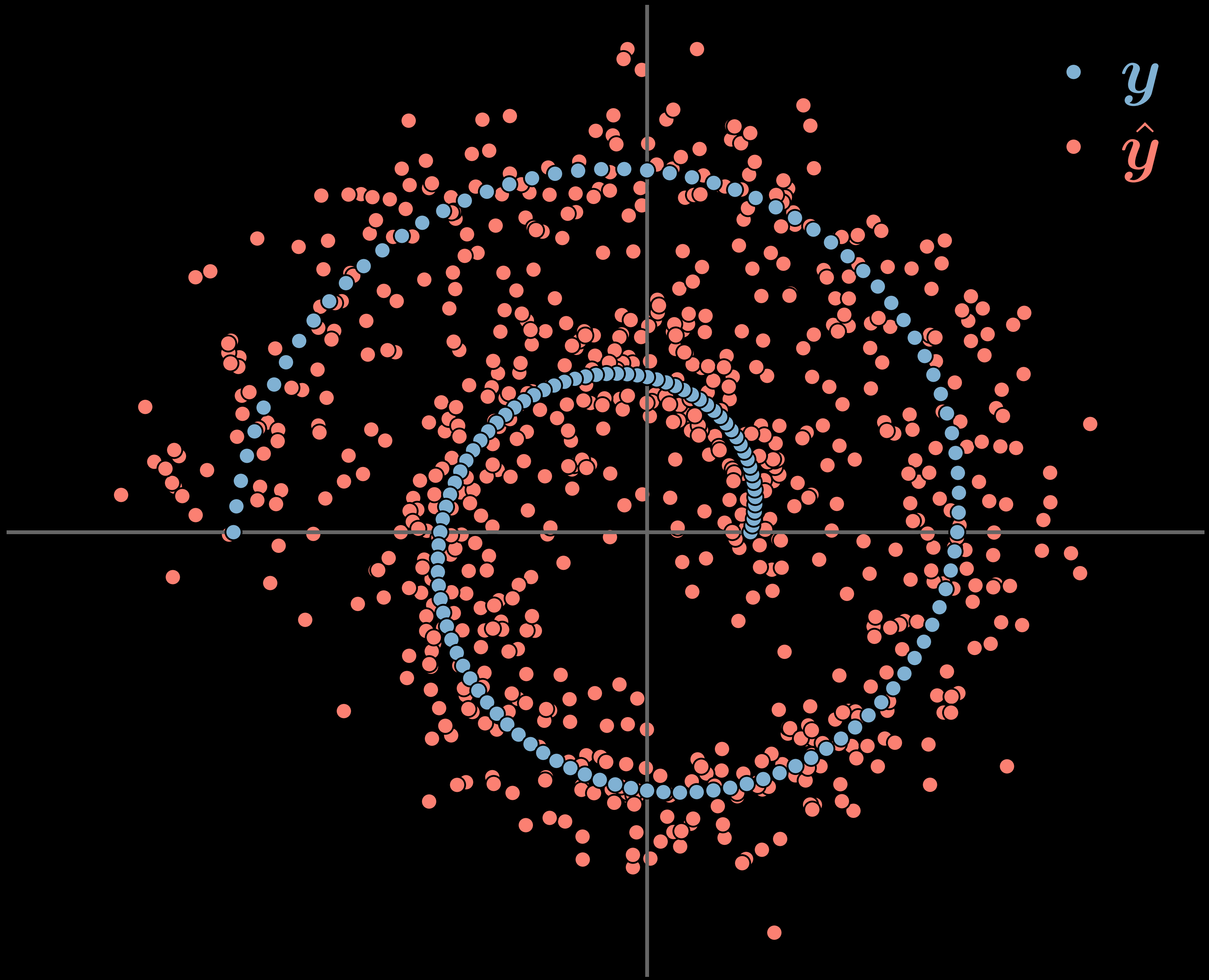
Under-/over-complete hidden layer

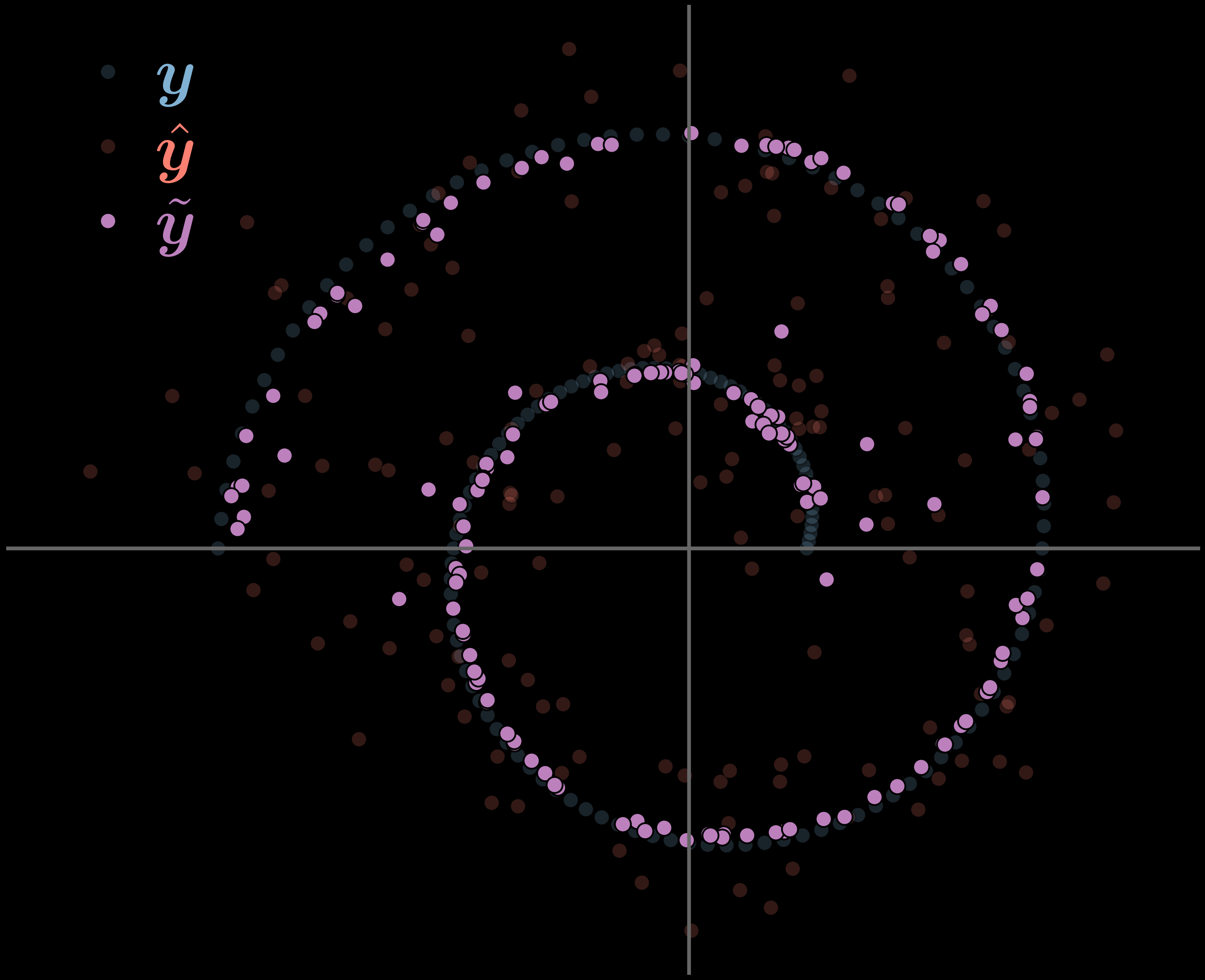


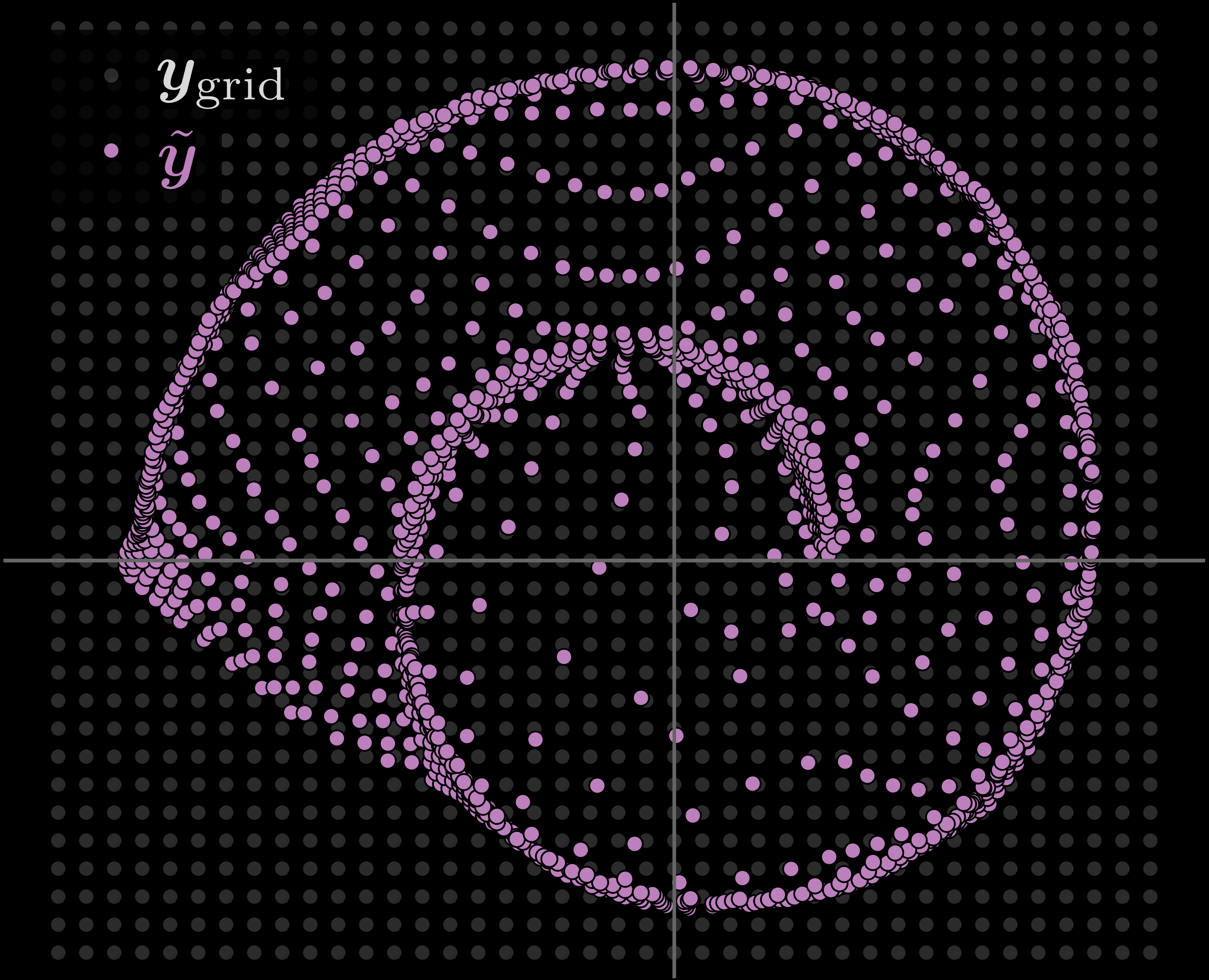
Denoising autoencoder





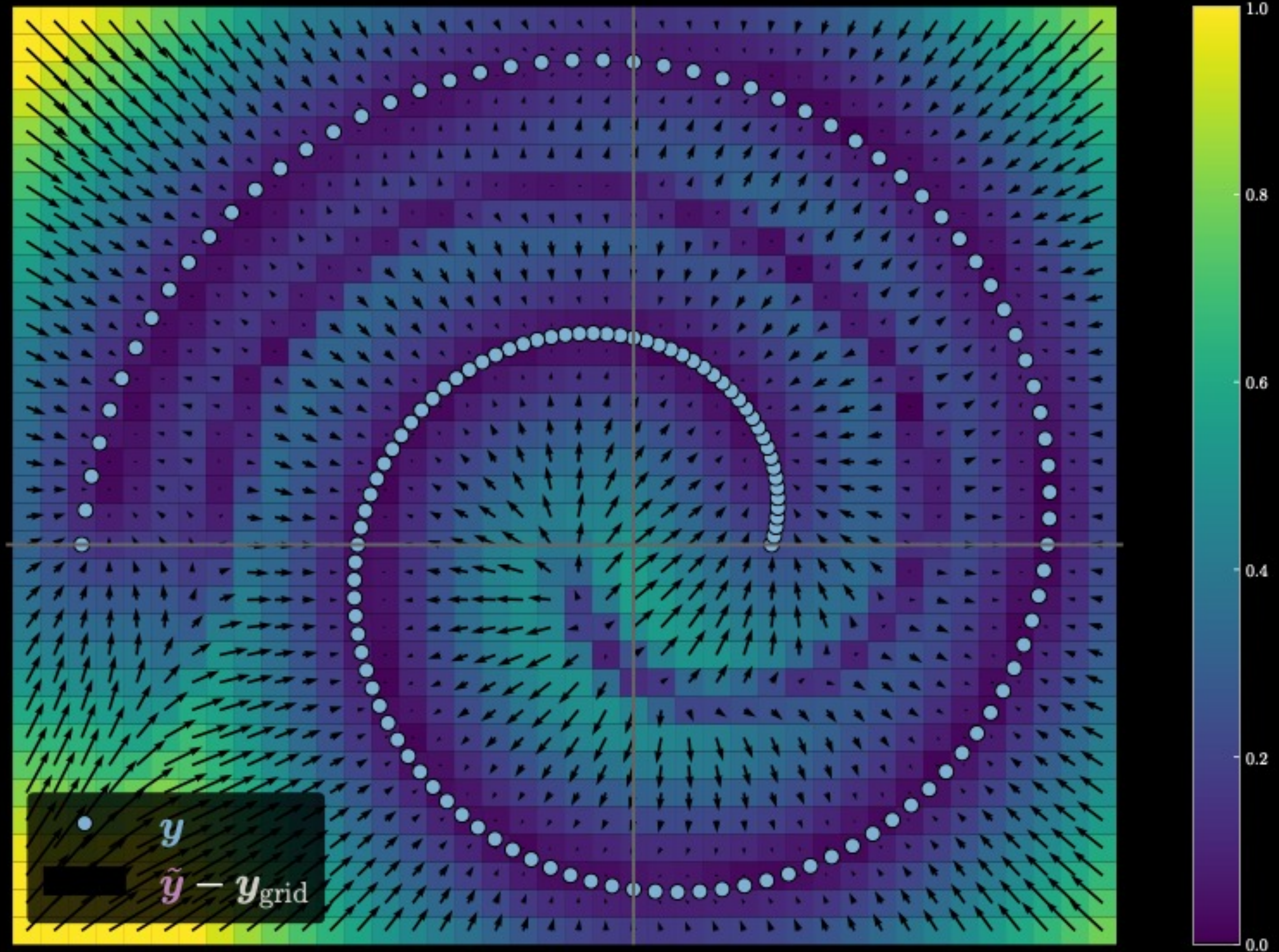

y
$\hat{y}$


y
$\hat{y}$
$\tilde{y}$



$\boldsymbol{y}_{\text{grid}}$
$\tilde{\boldsymbol{y}}$

Denoising AE



Denoising AE, near. neigh.