Lecture 2 : Basics of Graphs Continued.

Review : def of IS / def of EC.

Prop I is indep $\iff$ $V \setminus I$ is an edge cover.

Proof :

Assume I indep but $V \setminus I$ is not an EC.

$\Rightarrow \exists \, e \in E \mid \partial e \cap V \setminus I = \varnothing$

$\Rightarrow \exists \, e \in E \mid \partial e \subseteq I \Rightarrow I$ NOT independent.

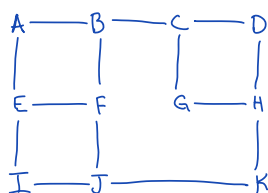Assume $V \setminus I$ an edge cover & I NOT indep.

$\Rightarrow \exists \, e \in E \mid \partial e \subseteq I \Rightarrow \exists e \in E \mid \partial e \not\subseteq V \setminus I$
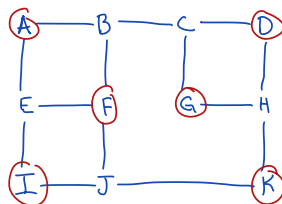
$\Rightarrow V \setminus I$ not an edge cover.

Cor The largest independent set corresp to the smallest edge cover.

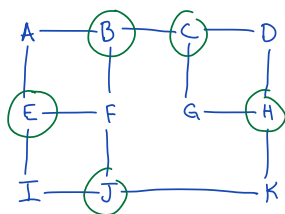Eg Find the smallest edge cover of :

Find largest IS.

Notice IS can be built from balls of size 2.

$\rightsquigarrow$ EC of size $\underline{\underline{5}}$

def : A directed graph every edge has an arrow, two fixed vertices are connected by at most one arrow each way.

def : a (directed) path is a sequence of distinct verts w/ each consecutive pair joined by an edge.
↳ All edges flow in the direction of the sequence.

def : a circuit is a path whose final vertex is adj to its initial vertex.

def G is cctd if there's a path between any two vertices.

def vertex basis is $B \subseteq V$ s.t. there's a directed path to any vertex from one in B.

Eg:
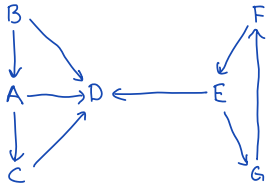
$\{B, F\}$ form a v.b.
↳ b/c source
↳ Then remove its image.

Notation for a path ?
$P = V_1 - V_2 - V_3 - \cdots - V_k.$
$p : V_1 \mapsto V_k.$

Note Loops act on the vertex bases.

End Lecture 2