Ex. regulated $\sim N(200, 15^2)$

$n=9$, $\bar{x}$ falls in $(191, 209)$ ✓

otherwise, we conclude $\mu \neq 200$. adjust.

$H_0: \mu = 200$ $\qquad$ $H_1: \mu \neq 200$.

① When $\mu = 200$, find type I error error.
(the significance level)

$$\alpha = P(\bar{X} < 191 \text{ or } \bar{X} > 209)_{\mu=200} \qquad \bar{X} \sim N\left(200, \frac{15^2}{9}\right)$$

$$= 2 * P\left(Z > \frac{209-200}{15/3}\right) = 2P(Z > 1.8)$$

$$= 2 \times 0.0359 = 0.0718$$

② Find β when $\mu = 215$.

$$\beta = P(191 < \bar{X} < 209, \ \mu = 215)$$

$$= P\left(\frac{191-215}{15/3} < Z < \frac{209-215}{15/3}\right)$$

$$= P(-4.8 < Z < -1.2)$$

$$= 0.1151$$

Ex: --- $\sim N(\mu, 15^2)$

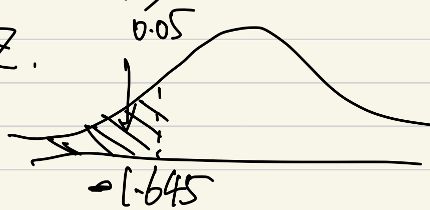
$H_0: \mu = 200$ $\quad H_1: \mu < 200$ $\quad \alpha = 0.05$

$n = 9$.

① Find C. (decision rule)

under H_0, $\bar{X} \sim N\left(200, \frac{15^2}{9}\right)$

$$\frac{\bar{X} - 200}{15/3} \sim Z.$$



$$C = \left\{ \frac{\bar{X} - 200}{15/3} < -1.645 \right\}$$

$$= \left\{ \bar{X} < 200 - \frac{15}{3} * 1.645 \right\}$$

$$= \left\{ \bar{X} < 191.775 \right\}$$

② Find β when $\mu = 190$.

$$\beta = P(\bar{X} \geq 191.775, \mu = 190)$$

$$= P\left(Z \geq \frac{191.775 - 190}{15/3}\right) = P(Z \geq 0.355)$$

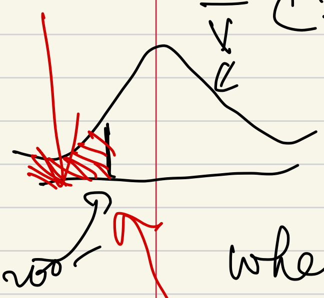
$$= 0.3594$$

③ If we wish to keep $\alpha = 0.05$ and $\beta = 0.10$ when $\mu = 190$, what's the n need to be?

C.

for $\alpha = 0.05$.

$$C = \left\{ \frac{\bar{X} - 200}{15/\sqrt{n}} < -1.645 \right\}$$



$$= \left\{ \bar{X} < 200 - 1.645 \times \frac{15}{\sqrt{n}} \right\}$$

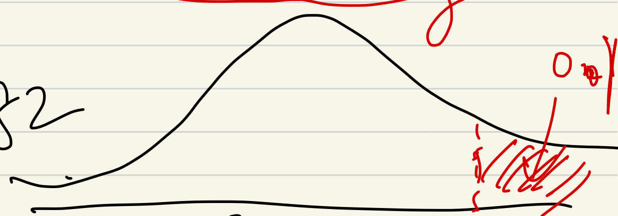
when $\mu = 190$.

$$0.1 = \beta = P\left(\bar{X} \geq 200 - 1.645 \cdot \frac{15}{\sqrt{n}} \right)$$

$\bar{X} \sim N\left(190, \frac{15^2}{n}\right)$

$$= P\left(Z \geq \frac{200 - 1.645 \cdot \frac{15}{\sqrt{n}} - 190}{15/\sqrt{n}} \right)$$

$$= P\left(Z \geq \frac{10\sqrt{n}}{15} - 1.645 \right)$$



$$\frac{10\sqrt{n}}{15} - 1.645 = 1.282$$

$1.282 = z_{0.1}$

$$n = \frac{(1.282 + 1.645)^2 \times 15^2}{10^2}$$

≈ 20

In the case of a 1-sided test, to achieve type I error α, and type II β when a specific μ_1 in H_1 is the mean,

$$n \approx \frac{(z_\alpha + z_\beta)^2 \sigma^2}{\delta^2}, \quad \text{where } \delta = |\mu_1 - \mu_0|$$

If it's a 2-sided test.

$$n = \frac{(z_{\frac{\alpha}{2}} + z_\beta)^2 \sigma^2}{\delta^2}.$$

§10.8. One sample, test on a single proportion

H_0: $p = p_0$ survey n people.

H_1: $p < p_0$

under H_0, test stat. $\hat{p} \sim N\left(p_0, \frac{p_0 q_0}{n}\right)$

$$\frac{\hat{p} - p_0}{\sqrt{p_0 q_0 / n}} \sim Z.$$

Ex: Claims: $p = 70\%$. Would you agree if a survey of 15 houses showed only 8 had heat pumps?

$\alpha = 0.10$.

$H_0: \quad p = 0.7$

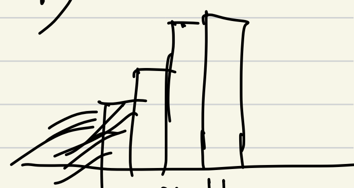
$H_1: \quad p \neq 0.7$

Y = # of houses with heat pump.

under H_0, $Y \sim Bin(15, 0.7)$

$$p\text{-value} = 2 * P(Y \le 8)$$

$$= 2 * 0.11\ldots \quad > \alpha$$



$\Rightarrow$ we cannot reject H_0.

$$Y \sim B(n, p_0) \approx N(np_0, np_0 q_0)$$

$$\hat{p} = \frac{Y}{n} \approx N\left(p_0, \frac{p_0 q_0}{n}\right)$$

If use normal to find p-value.

$Y \sim Bin(15, 0.7)$

$\approx N(10.5, 3.15)$

$$p\text{-value} = 2 * P(Y \le 8)$$

$$= 2P\left(Z \le \frac{8.5 - 10.5}{\sqrt{3.15}}\right) = P(Z < -1.13)$$

$$= 0.1292$$

§10.9. 2 sample p_1 & p_2

$H_0: p_1 = p_2 = p_0$ $\quad n_1 \quad Y_1 \quad \hat{p}_1$

$H_1: p_1 \neq p_2$ $\quad n_2 \quad Y_2 \quad \hat{p}_2$

$\boxed{\alpha}$

under H_0:

$$\hat{p}_1 - \hat{p}_2 \sim N\left(0, \frac{\hat{p}_0 \hat{q}_0}{n_1} + \frac{\hat{p}_0 \hat{q}_0}{n_2}\right)$$

$$\hat{p}_0 = \frac{Y_1 + Y_2}{n_1 + n_2}$$

$$\hat{p}_1 - \hat{p}_2 \sim N\left(0, \hat{p}_0 \hat{q}_0 \left(\frac{1}{n_1} + \frac{1}{n_2}\right)\right)$$

Ex: Town voter: 120 of 200 support

County voters: 240 of 500 support

$\hat{p}_1 = 0.60$. $\hat{p}_2 = .48$.

$H_0: p_1 = p_2$

$H_1: p_1 > p_2$

$\alpha = 0.05$

$$\hat{p}_0 = \frac{360}{700} = 0.514$$

under H_0, $\hat{p}_1 - \hat{p}_2 \sim N\left(0, \hat{p}_0 \hat{q}_0 \left(\frac{1}{200} + \frac{1}{500}\right)\right)$

$$\text{p-value} = P(\hat{p}_1 - \hat{p}_2 > 0.12)$$

$$= P\left(Z > \frac{0.12 - 0}{\sqrt{0.514 \cdot 0.486 \cdot \left(\frac{1}{200} + \frac{1}{500}\right)}}\right)$$

$$= P(Z > 2.9) = 0.0019 < \alpha$$

$\Rightarrow$ Reject H_0.

Ex. claim: battery life $\sim N(\mu, 0.9^2)$

sample: $n = 10$. $S = 1.2$

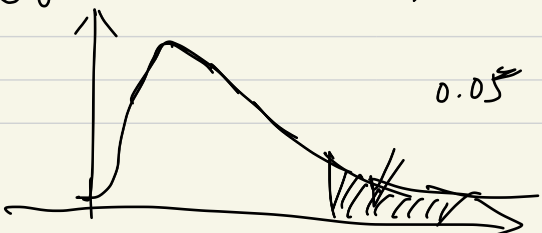
Do you think this indicates $\sigma > 0.9$

$\alpha = 0.05$

$H_0: \sigma = 0.9$

$H_1: \sigma > 0.9$

under H_0, $\dfrac{(n-1)S^2}{\sigma_0^2} \sim \chi^2(n-1)$



a) as $C = \{ \chi^2 > 16.919 \}$ $\quad \chi^2_{0.05}(9) = 16.919$.

$$\chi^2_{obs} = \frac{9 * 1.2^2}{0.9^2} = 16 \notin C$$

$\Rightarrow$ can not reject H_0

b $C = \left\{ \frac{(n-1) S^2}{\sigma_0^2} = \frac{9 S^2}{0.9^2} > 16.919 \right\}$

$= \{ S > 1.234 \}$

$S_{obs} = 1.2 \notin C$.

can not rej H_0.