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Office Hours:
Tuesday 9–11 am
Wed. 2:30–3:30 pm

2-Metro 8-

Chapter 2 Probability

- Random experiment: an experiment whose outcome can't be predetermined.
- Outcome space: the total collection of all possible outcomes of a random exper.

Ex: ① flip a coin. $S = \{H, T\}$

② throw a die $S = \{1, 2, 3, 4, 5, 6\}$

$E_0 = \{1, 3, 5\}$

$E_1 = \{6\}$

$E_2 = \{2, 4, 6\}$

$E_3 = \{5, 6\}$

$E_2 \cup E_3 = \{2, 4, 5, 6\}$

$E_2 \cap E_3 = \{6\}$

$E_0 \cap E_2 = \phi$

$E_0' = E_2$

③ throw 2 dice (red, blue)

$$S = \left\{ \begin{array}{cccc} (1,1), & (1,2), & \cdots & (1,6) \\ (2,1) & (2,2) & \cdots & (2,6) \\ \vdots & & & \\ (6,1) & (6,2) & \cdots & (6,6) \end{array} \right\}$$

④ throw 2 dice. Observe the sum on the 2.

$S = \{2, 3, 4, \cdots 11, 12\}$

$E = \{10, 11, 12\}$

$P(E) = \frac{1}{6}$

⑤ Throw a pair of dice. Stop when (5,5) shows. Observe the times you had to throw.

$$S = \{1, 2, 3, 4, \cdots\}$$

⑥ $[0,1]$ Randomly pin a point in $[0,1]$

$$S = \{x : 0 \le x \le 1\}$$
$$= \{x : x \in [0,1]\}$$

Examples
1–5 : Discrete S
6 : Continuous outcome space

Equally likely outcome space

$$P(A) = \frac{\#\text{ of outcomes in } A}{\#\text{ of outcomes in } S}$$

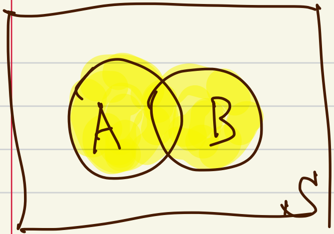
$\in$: an element of

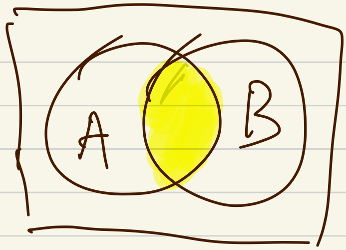
Event: A subset of S

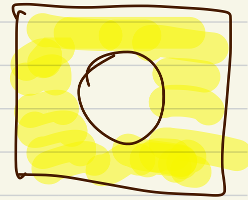
union $A \cup B = \{x : x \in A \text{ or } x \in B\}$

intersection $A \cap B = \{x : x \in A \text{ and } x \in B\}$

complement $A' = \{x : x \notin A\}$







$A \cup B$ $\qquad$ $A \cap B$ $\qquad$ A'

S. whole set

ϕ empty set

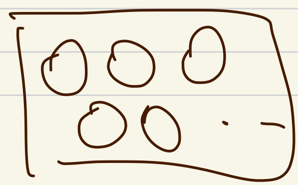
Generals of Prob.

1. $P(S) = 1$
2. For any A, $0 \le P(A) \le 1$
3. If $A_1, A_2, \cdots A_k \cdots$ are mutually exclusive then

$$P(A_1 \cup A_2 \cup \cdots \cup A_k \cup \cdots) = P(A_1) + P(A_2) + \cdots + P(A_k) + \cdots$$

A and B are mutually exclusive

$\iff A \cap B = \phi$



④ $P(A') = 1 - P(A)$

40 people.

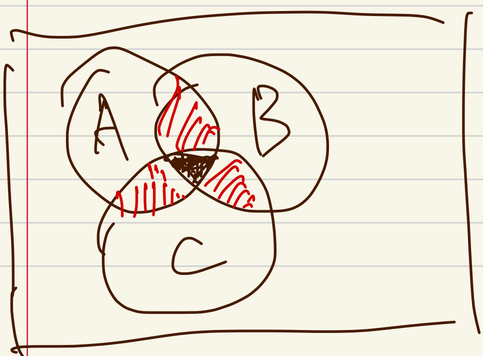
P(some people have the same birthday)

= 1 − P(everyone is born on a diff day)

$$= 1 - \frac{365}{365} * \frac{364}{365} * \frac{363}{365} \cdots \frac{326}{365}$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$$



Ex: A box of chocolate, 5 dark choc. 3 milk choc. 2 white.

Randomly pick 1,

$P(\text{dark choc.}) = \frac{1}{2}$ $\quad \frac{\binom{5}{1}\binom{3}{1}}{\binom{10}{2}}$

Randomly pick 2,

$P(1 \text{ dark } \& \text{ } 1 \text{ milk})$

$= \frac{3}{20}$ X $\qquad \cancel{\frac{1}{2} * \frac{3}{10}}$

$\frac{1}{6}$ X

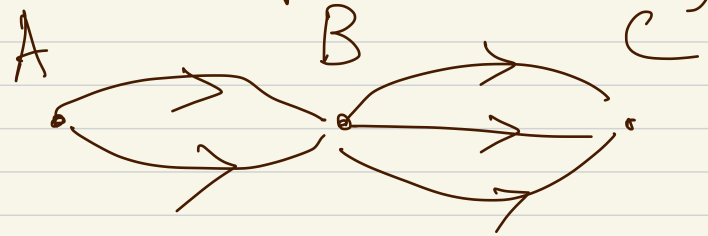
$= \frac{5}{10} * \frac{3}{9} + \frac{3}{10} * \frac{5}{9} = \frac{30}{90} = \frac{1}{3}$

Counting Technique.

① Multiplication Rule (Principle).

consecutive steps $E_1 E_2 \cdots E_k$.

For each step, there are n_i ways to do it.



Total # of ways $= n_1 \cdot n_2 \cdots n_k$.

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3 letters A, B, C. $\underline{3} \cdot \underline{2} \cdot \underline{1}$

10 people, stand in a straight line:

$10 \cdot 9 \cdot 8 \cdots 3 \cdot 2 \cdot 1 = 10!$

A permutation of n objects is an ordered arrangement of these objects.

There are $n!$ diff permutations of n objects.

Ex: 10 people in a competition.

1st 2nd 3rd prize. (no tie)

$\underline{10} \cdot \underline{9} \cdot \underline{8} = 720$

ABC ACB
BAC BCA
CAB CBA

Ex: 20 problems. choose 4 to make a quiz. How many ways to make the quiz if we care about the order?

$\underline{20}\ \underline{19}\ \underline{18}\ \underline{17}$

Ex: A permutation of n obj choosing r at a time is an ordered arrangement of r objects from the n objects $(r \le n)$

How many such permution?

$$P(n,r) = n(n-1)(n-2)\cdots(n-r+1)$$
$$= \frac{n!}{(n-r)!}$$

Ex. 10 people in a competition, 3 can advance to the next round. How many ways?

x

$3! \cdot x = 720$

$\parallel$

$\frac{720}{3!}$

$\boxed{ABC}$

ABD

:

FGI

Ex: 10 problems, pick 4 to assignment as a homework set. (Assume order does not matter.)

$$\frac{10 \cdot 9 \cdot 8 \cdot 7}{4!}$$

$C(n,r)$ = # of ways to pick r objects out of n obj. order does not matter

$$\binom{n}{r} = \frac{P(n,r)}{r!} = \frac{n!}{(n-r)!\,r!}$$

$\uparrow$

n choose r.

$$\binom{5}{2} = \frac{5 \cdot 4}{2!} = 10$$

12 23 34 45
13 24 35
14 25
15

$$\binom{7}{3} = \frac{7 \cdot 6 \cdot 5}{3!} = 35 \qquad \frac{7!}{4!\,3!}$$

$$\binom{30}{28} = \binom{30}{2} = \frac{30 \cdot 29}{2} = 435$$

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40 students, 10 Freshmen, 30 not

Randomly pick 10,

$$P(\text{exactly 2 freshmen}) = \frac{\binom{10}{2}\binom{30}{8}}{\binom{40}{10}}$$