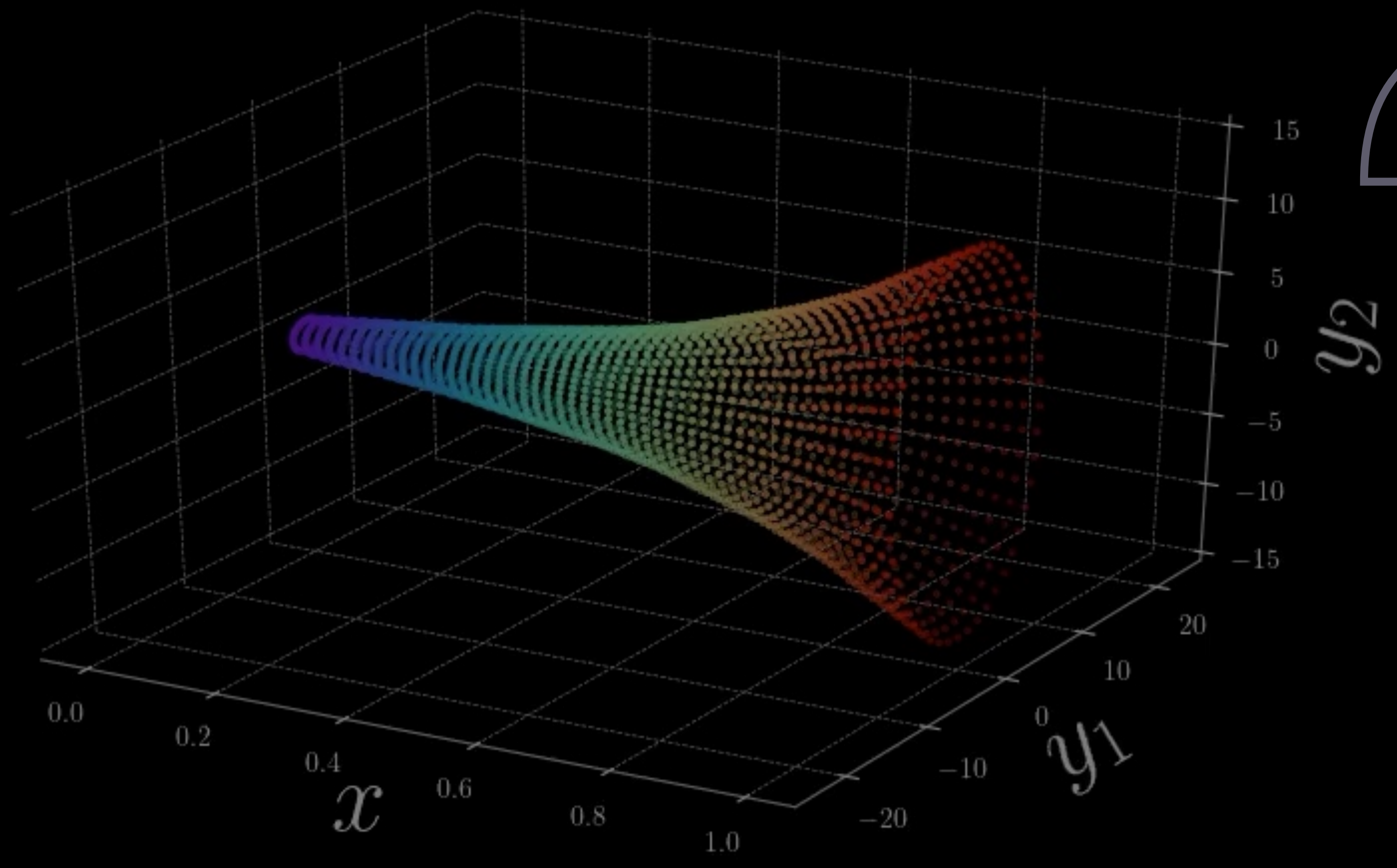
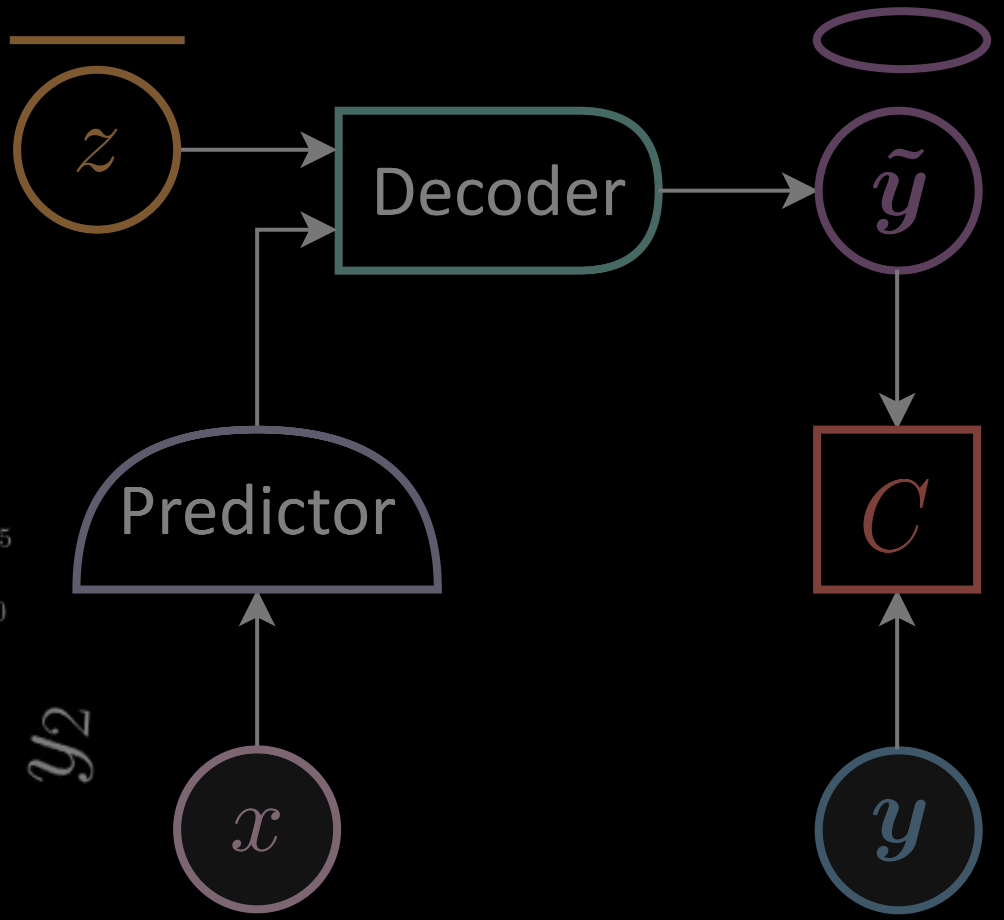
from the *EBM* lecture

Trained model manifold



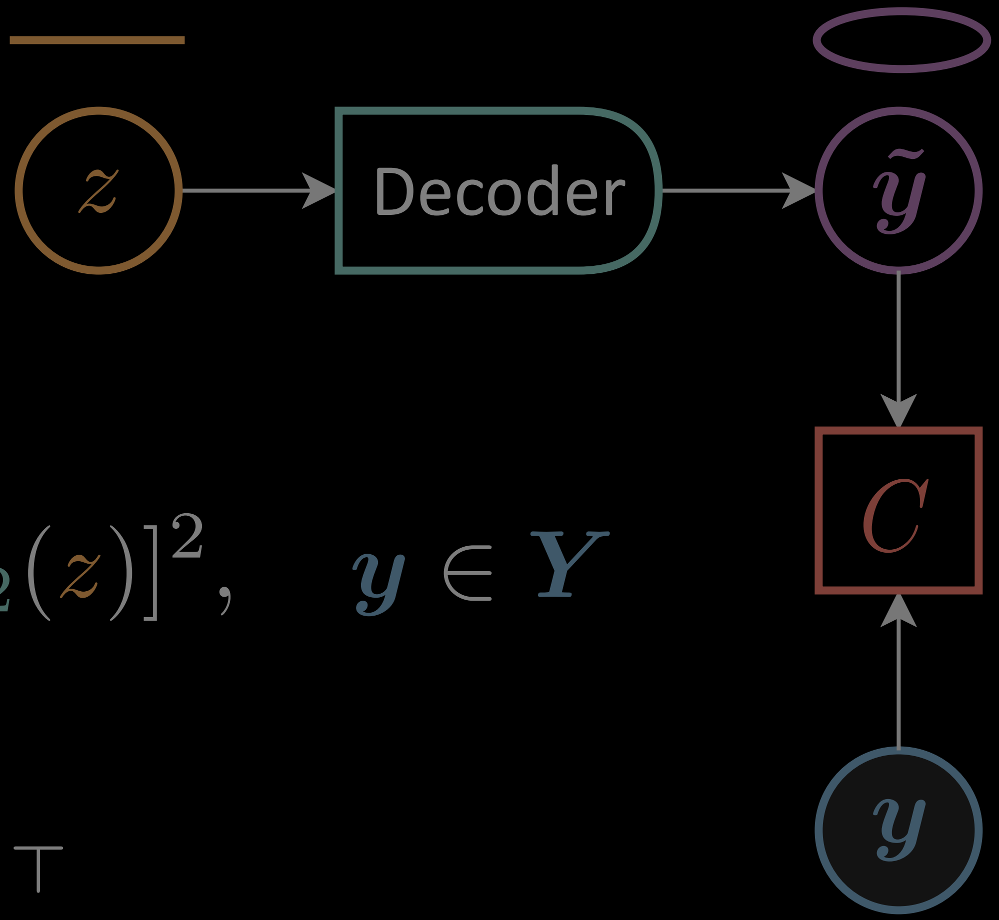


$$z = \left[0 : \frac{\pi}{24} : 2\pi\right[$$

$$x = \left[0 : \frac{1}{50} : 1\right]$$

from the *EBM* lecture

Energy function $E(\boldsymbol{y}, z)$

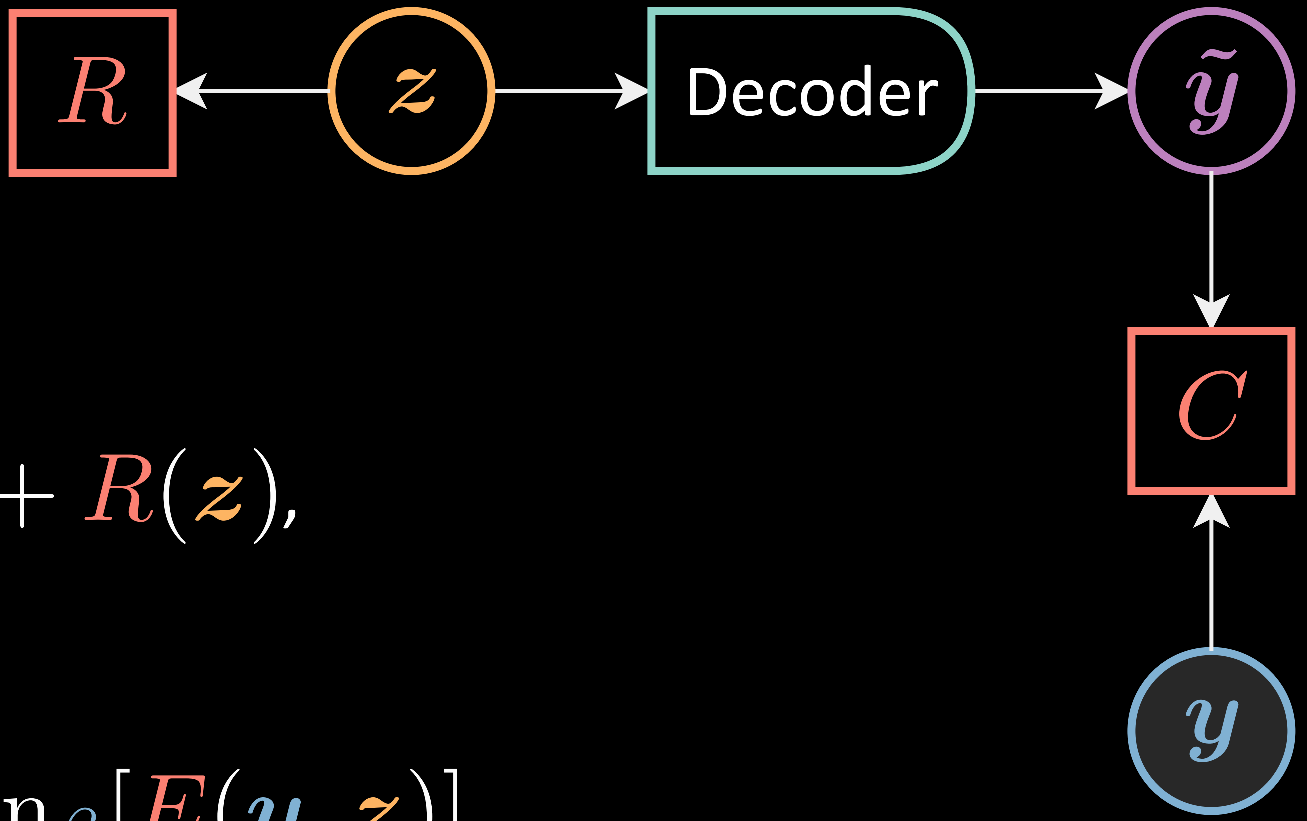


$$E(\boldsymbol{y}, z) = [y_1 - g_1(z)]^2 + [y_2 - g_2(z)]^2, \quad \boldsymbol{y} \in \boldsymbol{Y}$$

$$g = \begin{bmatrix} g_1 & g_2 \end{bmatrix}^\top : \mathbb{R} \to \mathbb{R}^2$$

$$z \mapsto \begin{bmatrix} w_1 \cos(z) & w_2 \sin(z) \end{bmatrix}^\top$$

Training recap



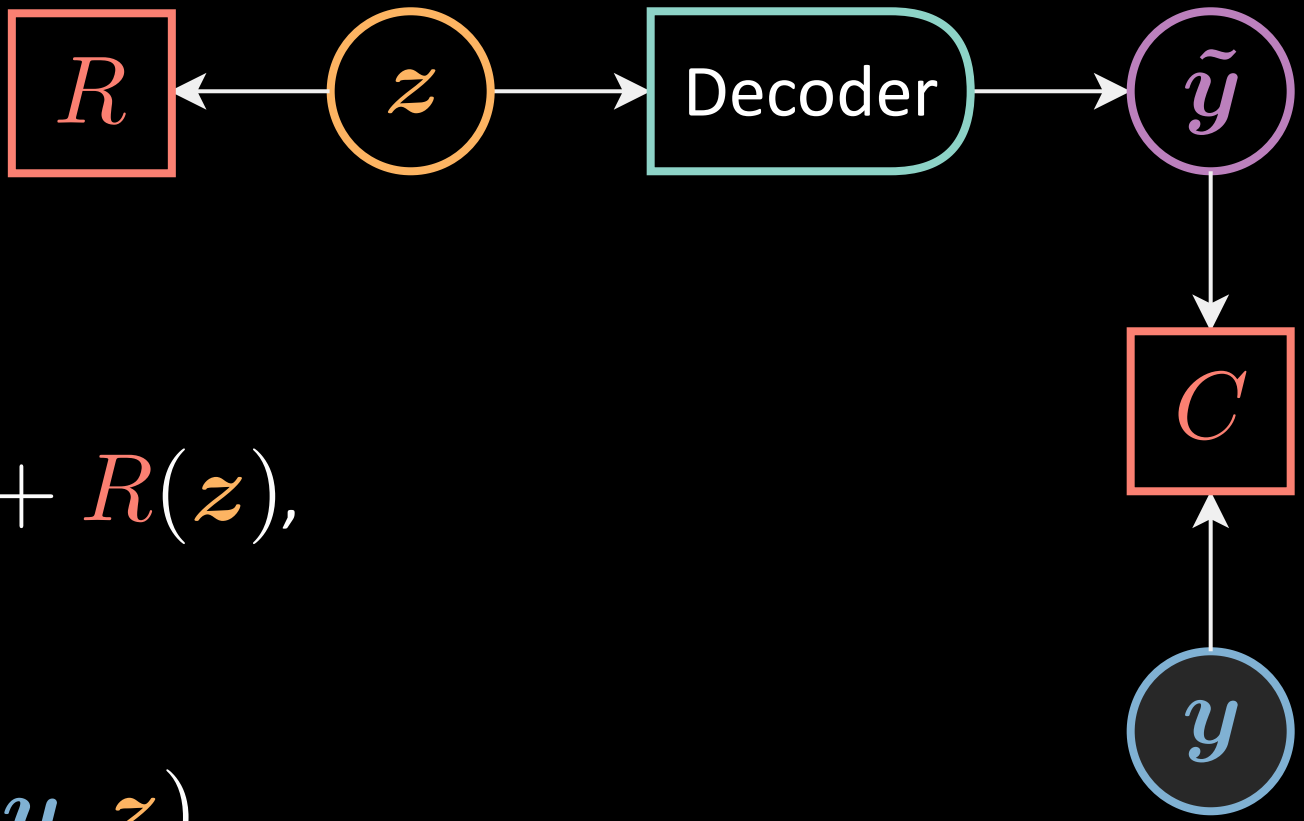
Given an observation $\boldsymbol{y}$,

given $E(\boldsymbol{y}, \boldsymbol{z}) = C(\boldsymbol{y}, \tilde{\boldsymbol{y}}) + R(\boldsymbol{z})$,

where $\tilde{\boldsymbol{y}} = \mathrm{Dec}(\boldsymbol{z})$

- Compute $F_\beta(\boldsymbol{y}) = \underset{\boldsymbol{z}}{\mathrm{softmin}_\beta}[E(\boldsymbol{y}, \boldsymbol{z})]$
- Minimise $\mathcal{L}[F_\beta(\mathcal{Y}), \boldsymbol{Y}]$

Zero temp. limit



Given an observation y,

given $E(y, z) = C(y, \tilde{y}) + R(z)$,

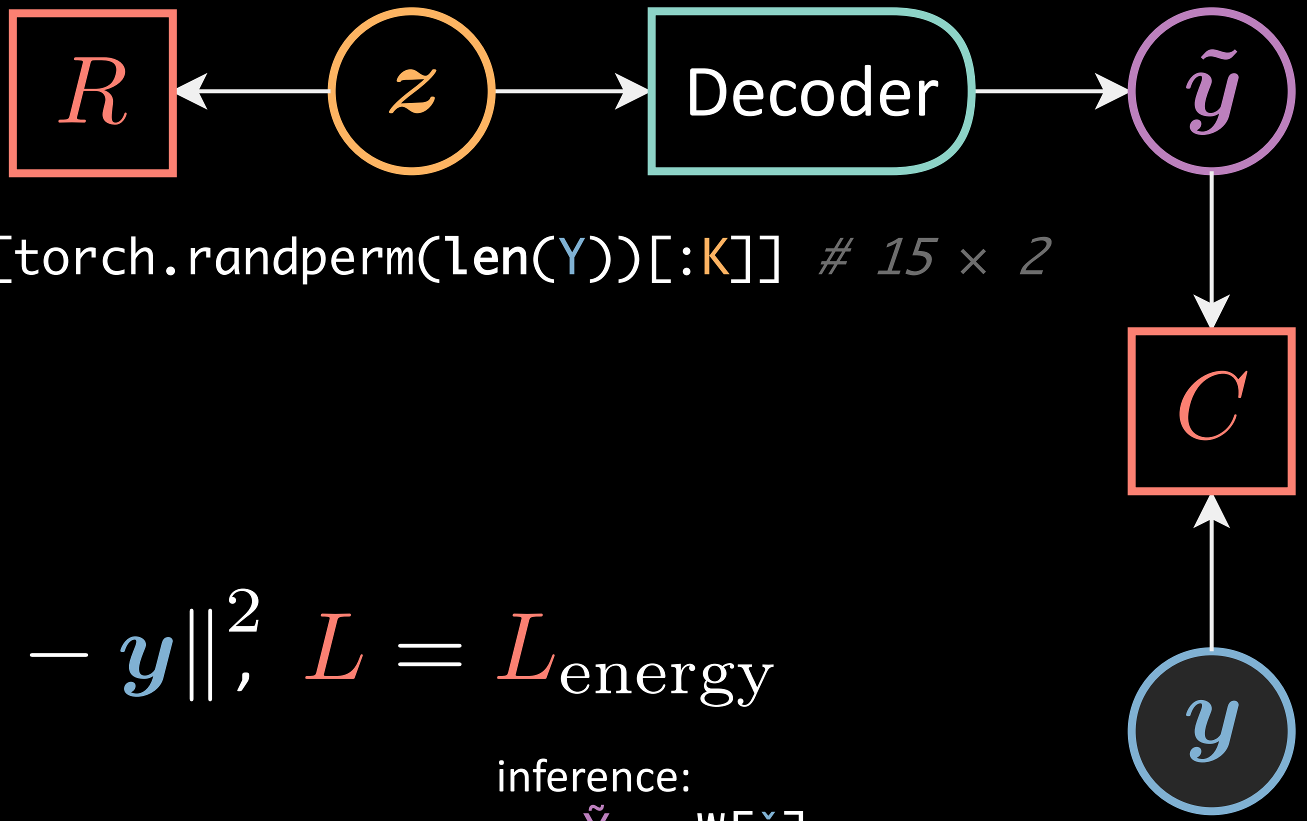
where $\tilde{y} = \mathrm{Dec}(z)$

- Compute $\check{z} = \underset{z}{\arg\min}\, E(y, z)$

$$F_\infty(y) = \min_{z} E(y, z) = E(y, \check{z})$$

- Minimise $\mathcal{L}[F_\infty(\mathcal{Y}), Y]$

K-means




```
W = Y[torch.randperm(len(Y))[:K]] # 15 × 2
```

Let $z \in \mathbb{I}_K$ (one-hot),

$\tilde{y} = \mathrm{Dec}(z) = \boldsymbol{W} z$, and

$$E(y, z) = C(y, \tilde{y}) = \|\tilde{y} - y\|^2, \; L = L_{\mathrm{energy}}$$

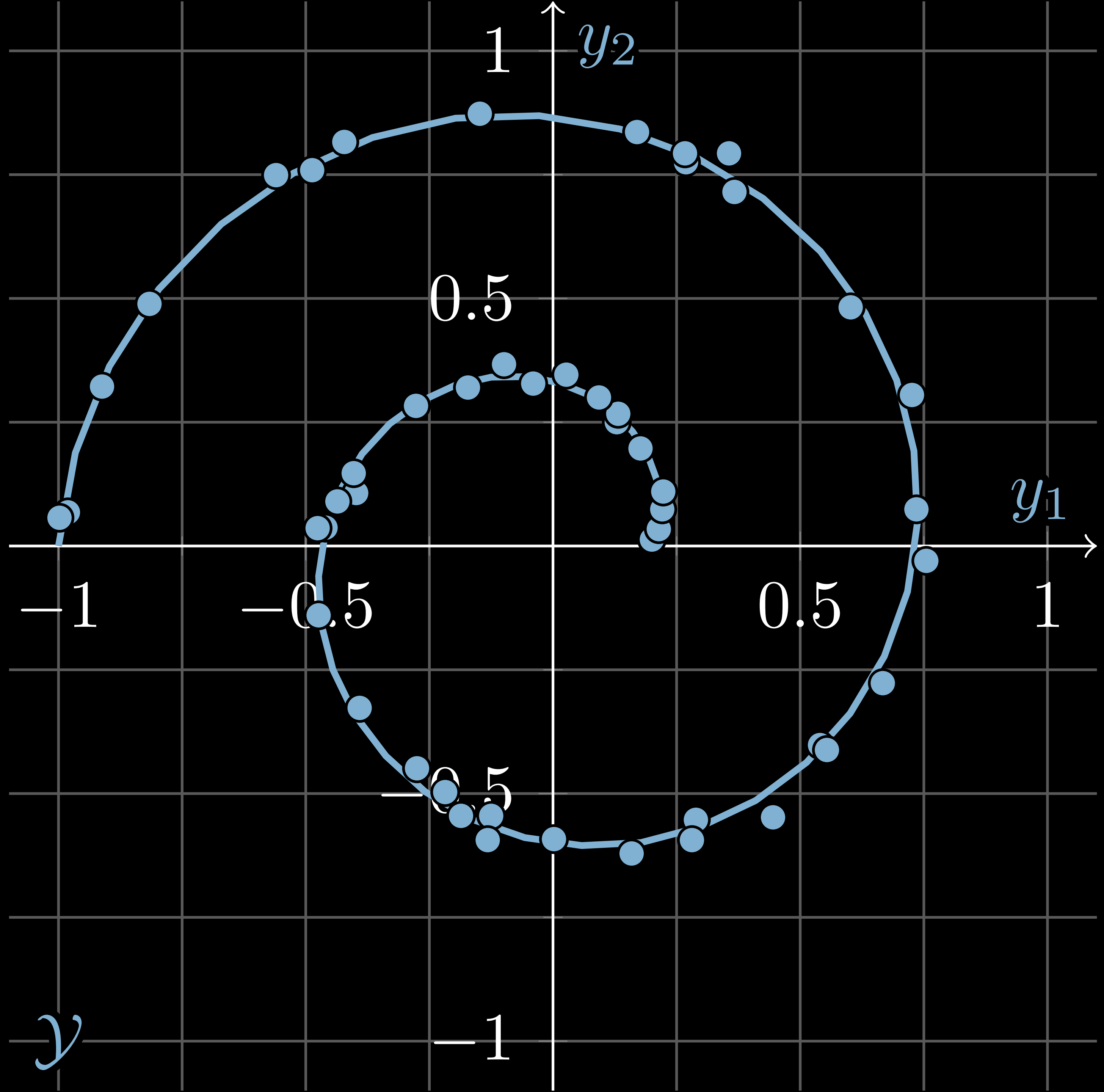
training:

```
E = torch.cdist(Y, W).pow(2) # 50 × 15
F, ž = E.min(dim=-1)
for k in range(K):
    W[k] = Y[ž==k].mean(dim=0)
```

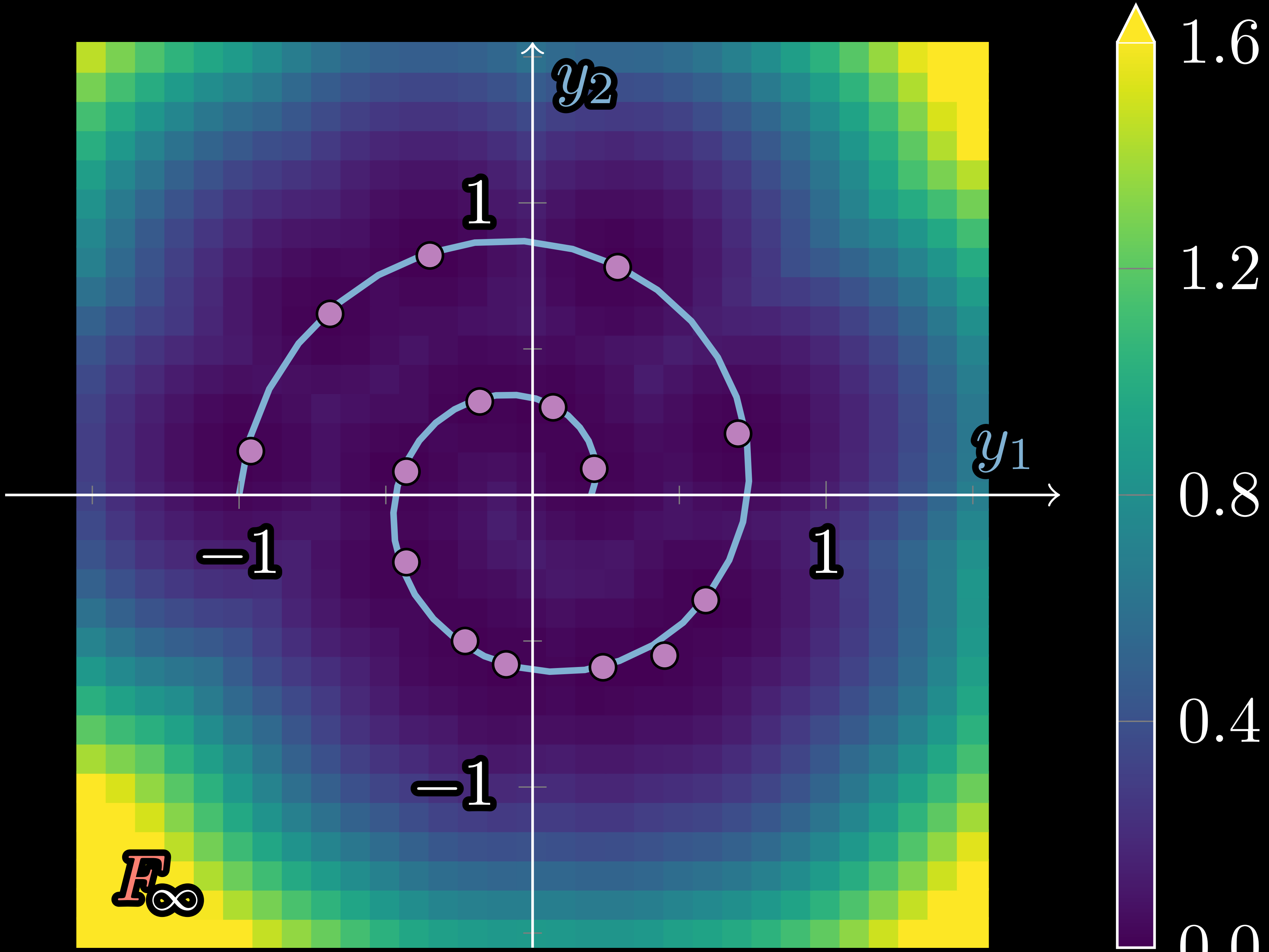
inference:

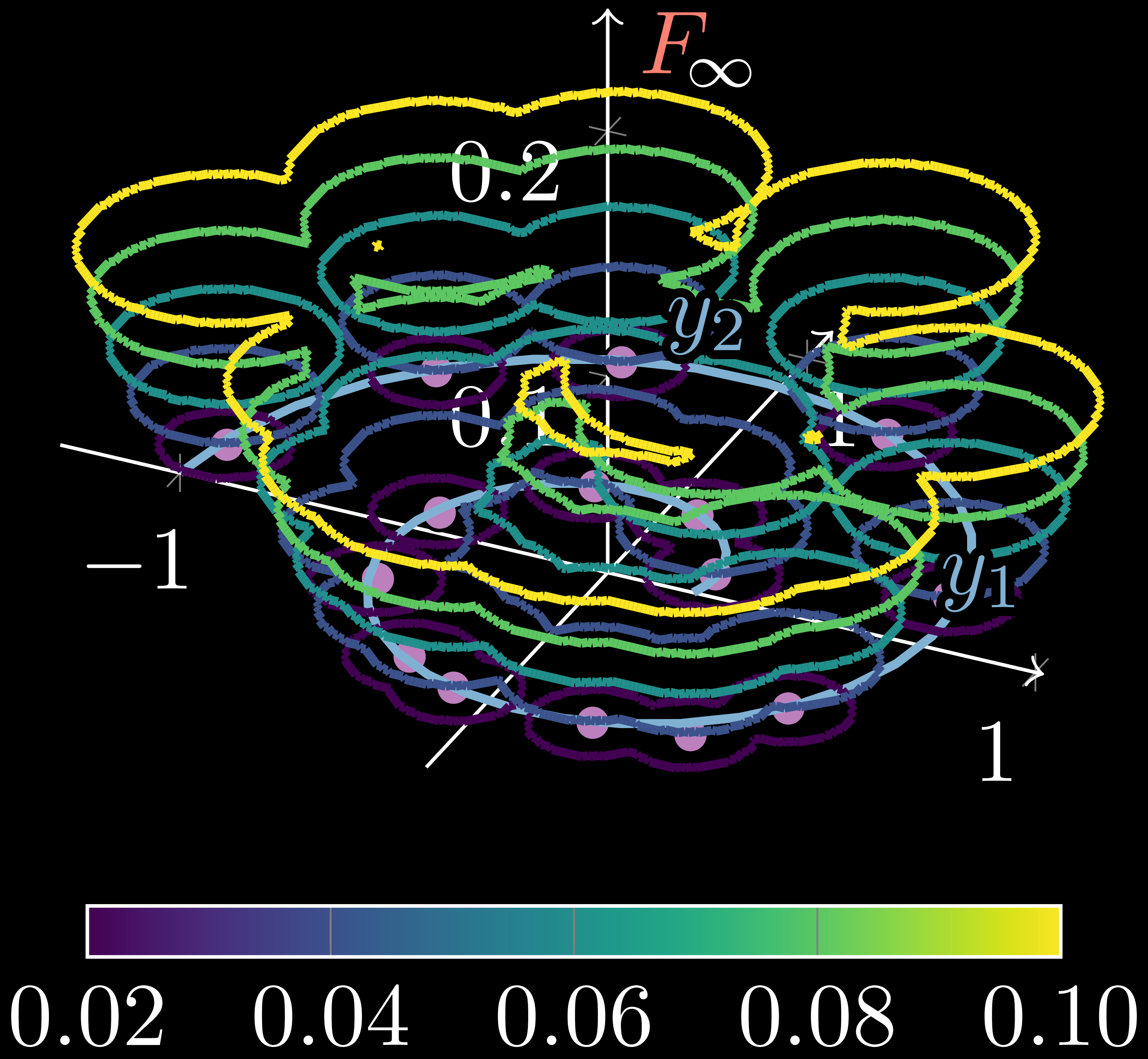
```
Ỹ = W[ž]
```

```
Y # 50 × 2
```

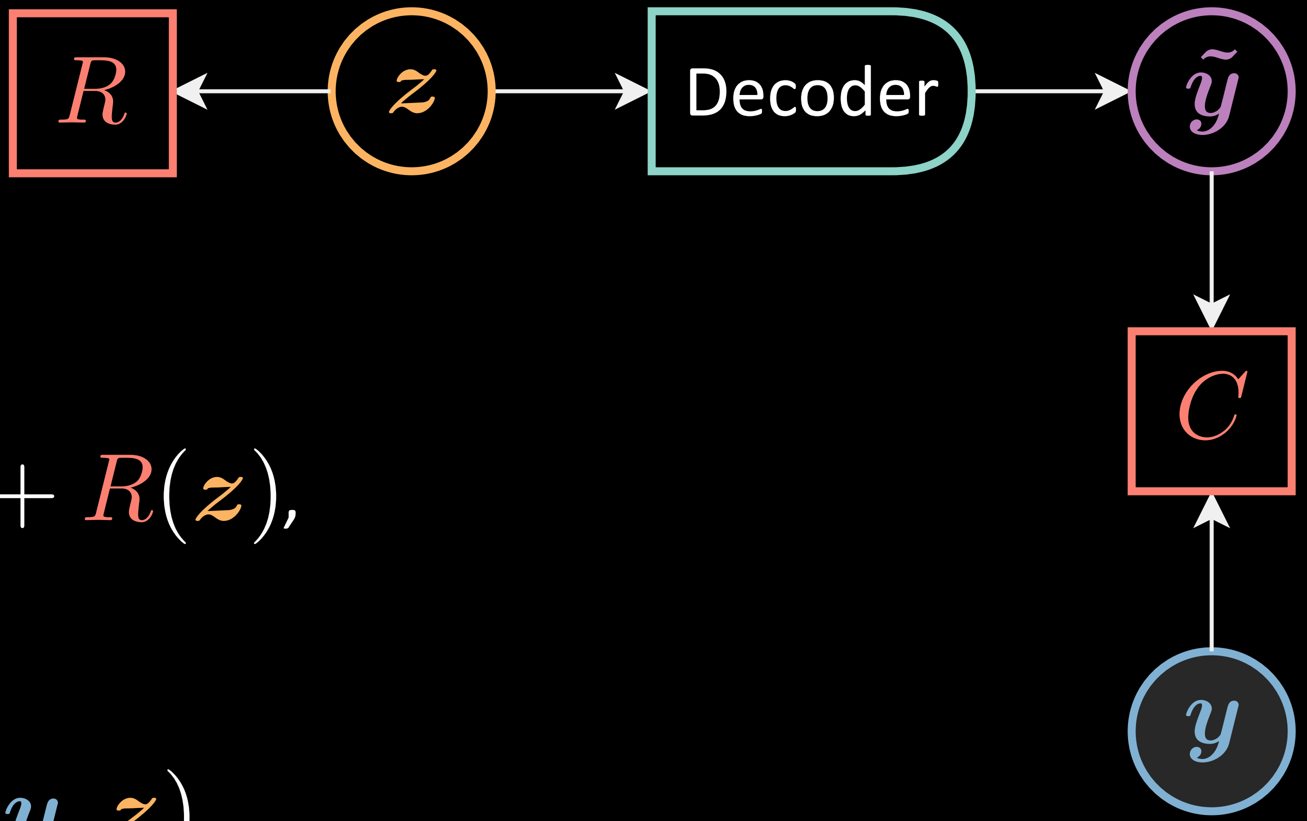


y_2
1
0.5
y_1
−1
−0.5
0.5
1
−0.5
−1
$\mathcal{Y}$


y_2
1
y_1
-1
1
-1
F_∞
1.6
1.2
0.8
0.4
0.0


F_∞
0.2
y_2
-1
y_1
1
0.02
0.04
0.06
0.08
0.10

Zero temp. limit



Given an observation y,

given $E(y, z) = C(y, \tilde{y}) + R(z)$,

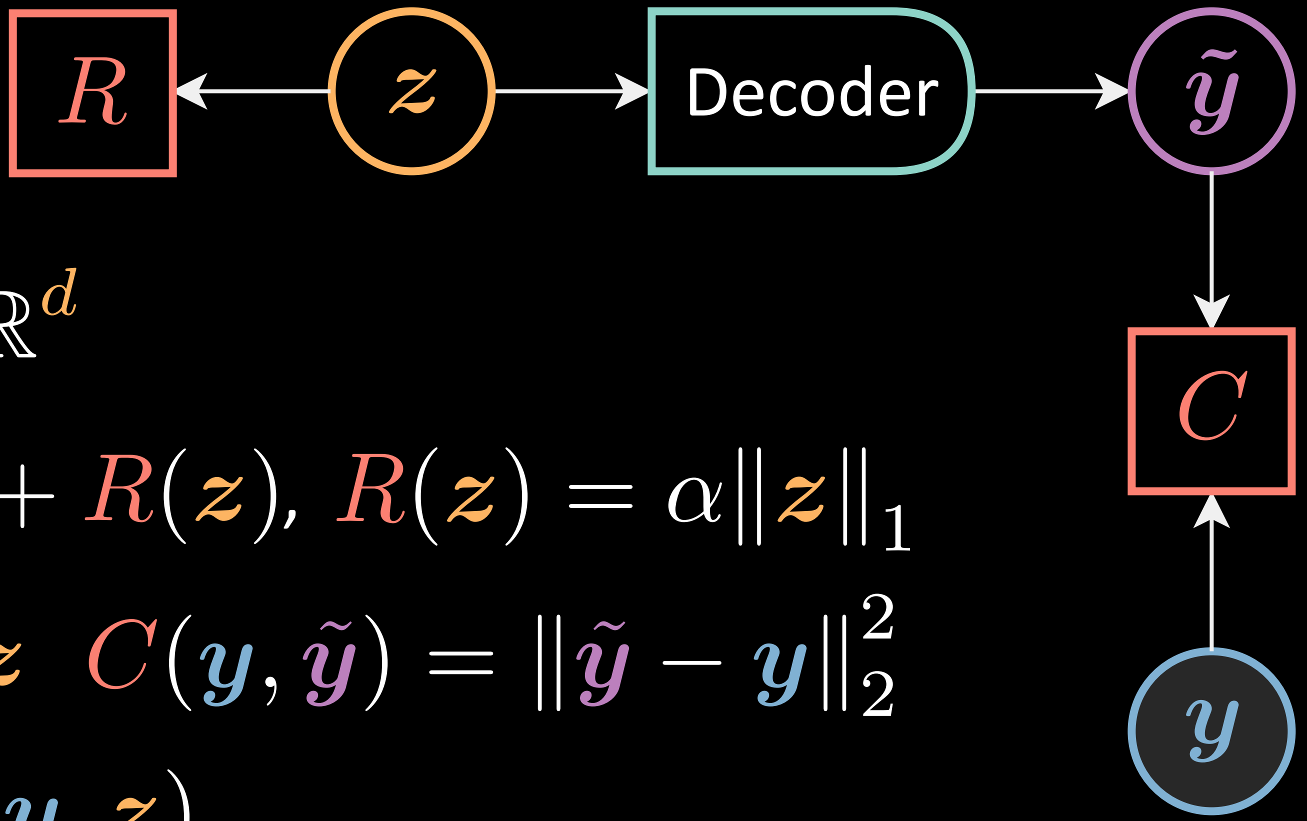
where $\tilde{y} = \mathrm{Dec}(z)$

- Compute $\check{z} = \arg\min_{z} E(y, z)$

$$F_\infty(y) = \min_{z} E(y, z) = E(y, \check{z})$$

- Minimise $\mathcal{L}[F_\infty(\mathcal{Y}), Y]$

Sparse coding



Given an observation y, $z \in \mathbb{R}^d$

given $E(y, z) = C(y, \tilde{y}) + R(z)$, $R(z) = \alpha \|z\|_1$

where $\tilde{y} = \mathrm{Dec}(z) = \boldsymbol{W} z$ $\; C(y, \tilde{y}) = \|\tilde{y} - y\|_2^2$

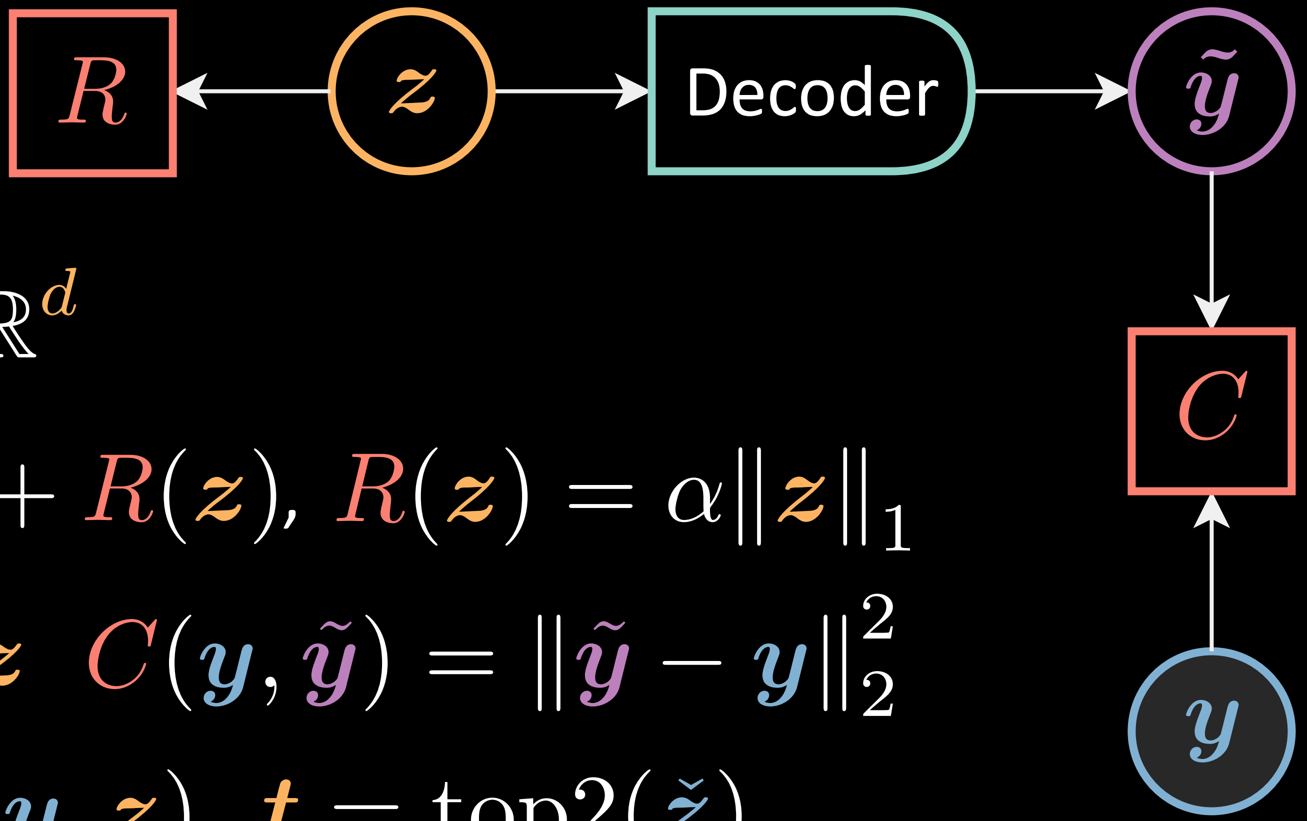
- Compute $\check{z} = \arg\min_{z} E(y, z)$

$$F_\infty(y) = \min_{z} E(y, z) = E(y, \check{z})$$

- Minimise $\mathcal{L}[F_\infty(\mathcal{Y}), Y]$

$$\|\boldsymbol{w}_j\|_2 \overset{\downarrow}{=} 1$$

Sparse coding



Given an observation $\boldsymbol{y}$, $\boldsymbol{z} \in \mathbb{R}^{d}$

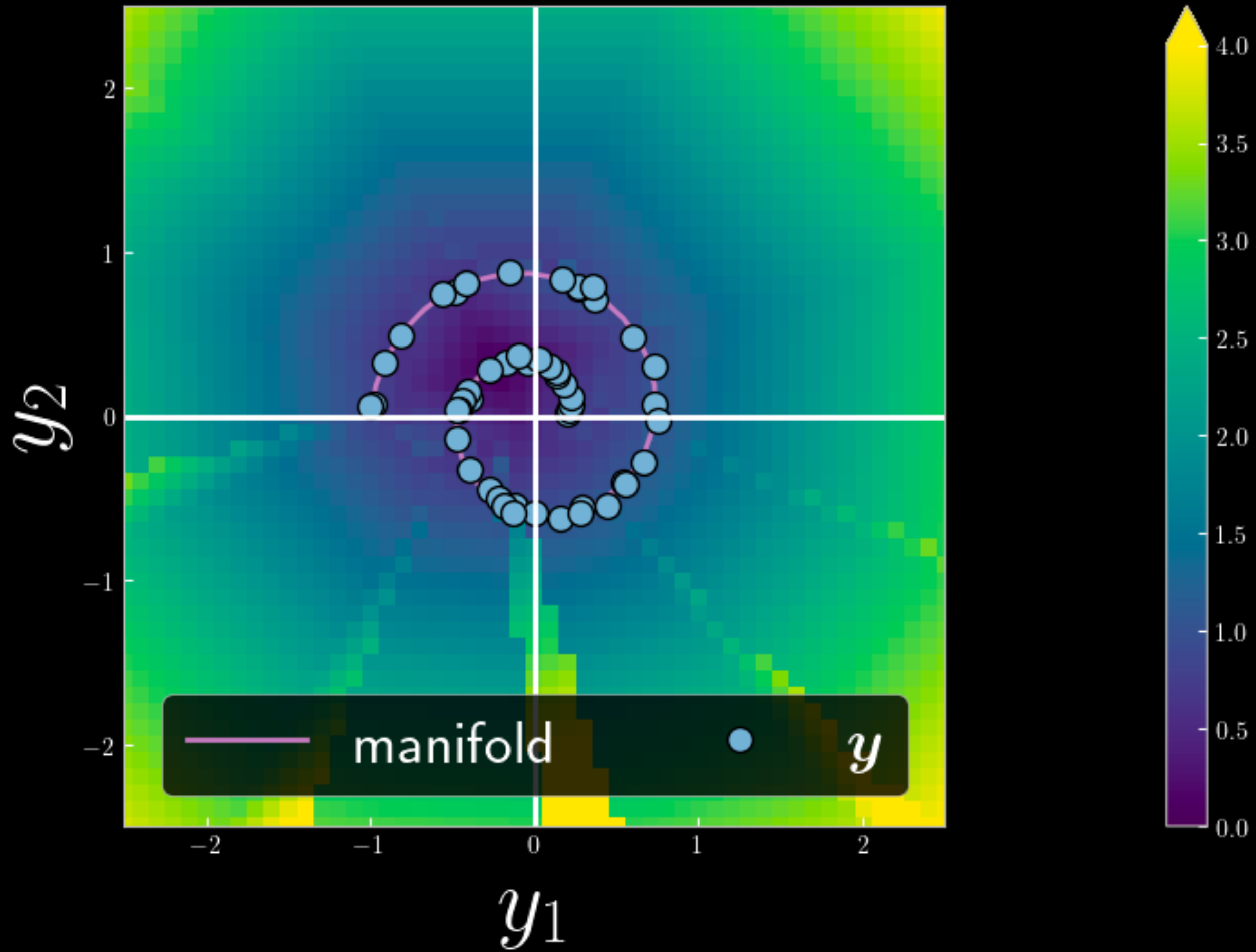
given $E(\boldsymbol{y}, \boldsymbol{z}) = C(\boldsymbol{y}, \tilde{\boldsymbol{y}}) + R(\boldsymbol{z})$, $R(\boldsymbol{z}) = \alpha \|\boldsymbol{z}\|_1$

where $\tilde{\boldsymbol{y}} = \mathrm{Dec}(\boldsymbol{z}) = \boldsymbol{W}\boldsymbol{z}$ $C(\boldsymbol{y}, \tilde{\boldsymbol{y}}) = \|\tilde{\boldsymbol{y}} - \boldsymbol{y}\|_2^2$

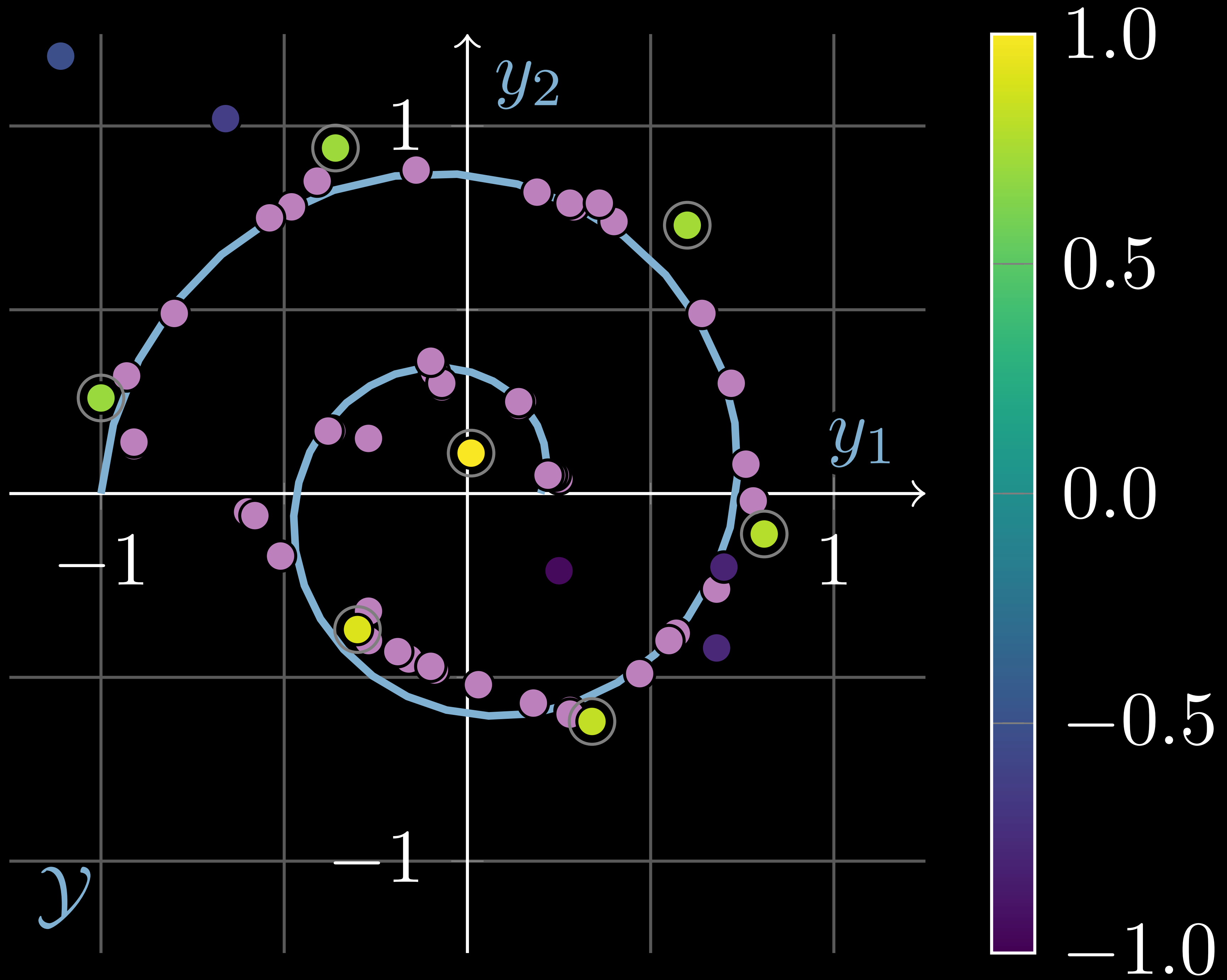
- Compute $\check{\boldsymbol{z}} = \underset{\boldsymbol{z}}{\arg\min}\, E(\boldsymbol{y}, \boldsymbol{z})$ $\boldsymbol{t} = \mathrm{top2}(\check{\boldsymbol{z}})$
- compute $\check{\boldsymbol{z}}_2 = \underset{\boldsymbol{t}}{\arg\min}\, C[\boldsymbol{y}, \mathrm{Dec}(\boldsymbol{t})]$
- choose $L[F_\infty(\mathcal{Y}), \boldsymbol{y}] = F_\infty(\boldsymbol{y}) = E(\boldsymbol{y}, \check{\boldsymbol{z}}_2)$

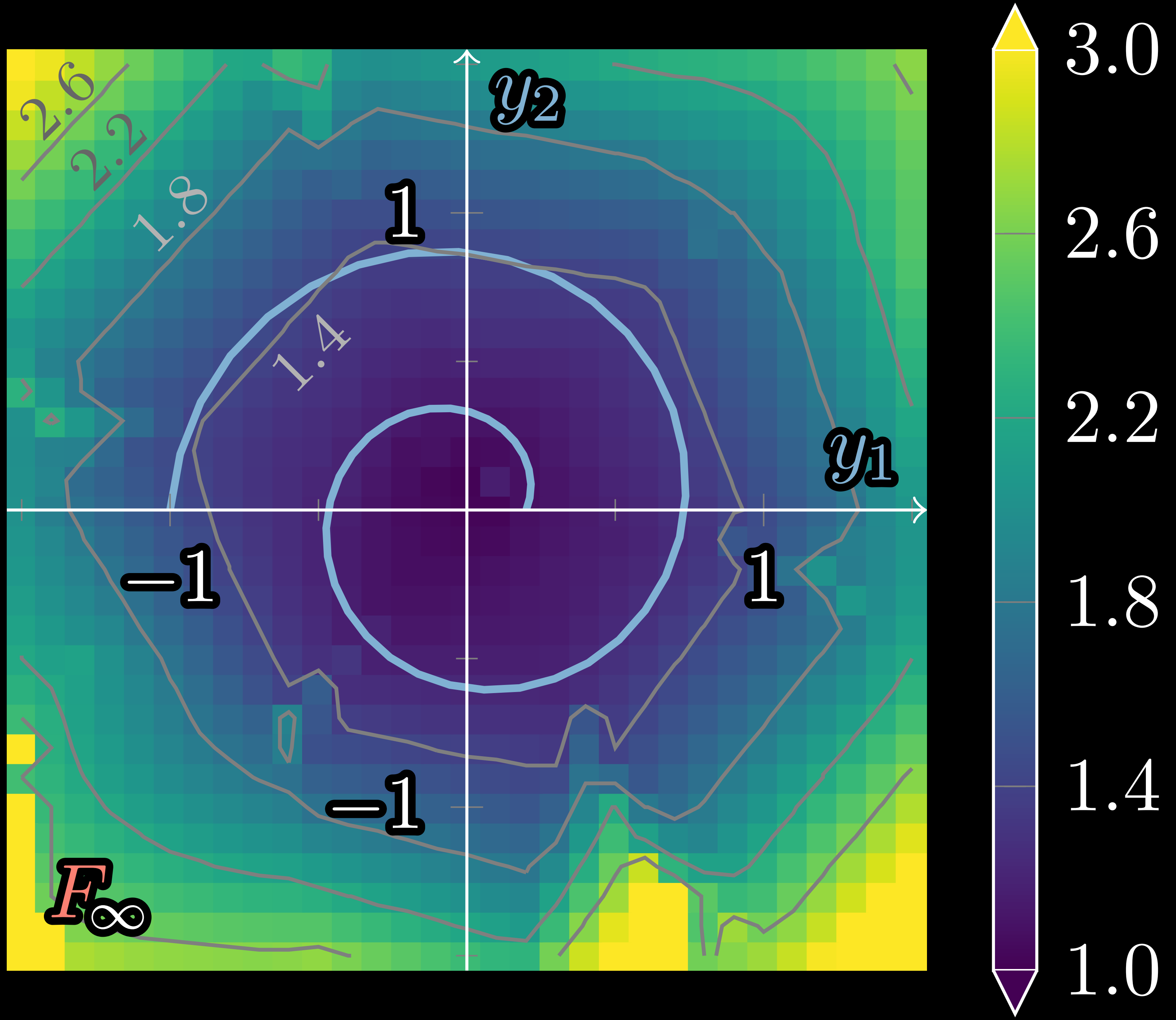
```
Y # 50 × 3
```

$\dot{\boldsymbol{y}} \doteq \begin{bmatrix} 1 \\ \boldsymbol{y} \end{bmatrix}$

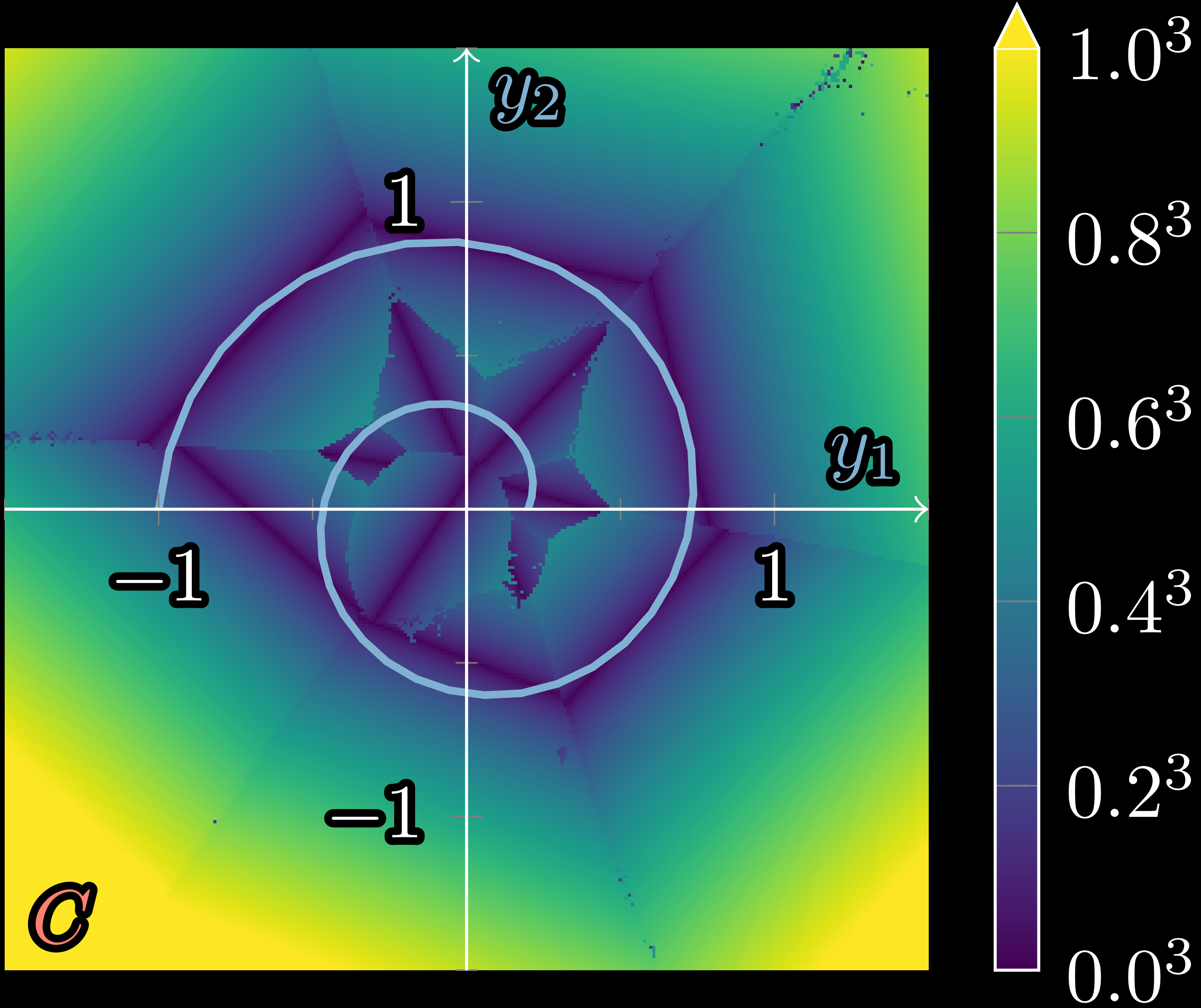


manifold
y
y1
y2
−2
−1
0
1
2
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0


y_2
1
y_1
-1
1
-1
$\mathcal{Y}$
1.0
0.5
0.0
-0.5
-1.0



2.6
2.2
1.8
1.4
1
−1
1
−1
y_2
y_1
F_∞
3.0
2.6
2.2
1.8
1.4
1.0


y_2
y_1
1
−1
1
−1
$\mathcal{C}$
1.0^3
0.8^3
0.6^3
0.4^3
0.2^3
0.0^3