14.4

3. $z = e^{x-y}$ $(2,2,1)$

find tangent plane.

need $z_x = \frac{\partial}{\partial x}(e^{x-y}) = e^{x-y}$

$\frac{\partial}{\partial x} e^{x-y} = \frac{\partial}{\partial x} e^{x} e^{-y}$

$= e^{-y} \frac{\partial}{\partial x} e^{x}$

$= e^{-y} e^{x}$

$= e^{x-y}$

$z_y = -e^{x-y}$

need $z_x$ at $(2,2)$ : $e^{2-2} = 1$

$z_y$ at $(2,2)$ : $-e^{2-2} = -1$

$z_x = f_x = \frac{\partial f}{\partial x} = \frac{\partial z}{\partial x}$

plane: $z - z_0 = f_x(a,b)(x-a) + f_y(a,b)(y-b)$

$a = x_0$, $b = y_0$

use $(2,2,1)$ and $z_x(2,2) = 1$ $z_y(2,2) = -1$

$z - 1 = (1)(x-2) + (-1)(y-2)$

$z = x - 2 - y + 2 + 1$

$z = x - y + 1$

$f_x(a,b)(x-a) + f_y(a,b)(y-b) - (z - z_0) = 0$

normal vector: $\langle f_x(a,b), f_y(a,b), -1 \rangle$

⑤ $z = x\sin(x+y)$ at $(-1,1,0)$ find tangent plane.

$z_x = \sin(x+y) + x\cos(x+y)$ $\quad$ $z_y = x\cos(x+y)$

evaluate at $x=-1$, $y=1$

$z_x(-1,1) = \sin(0) + (-1)\cos(0)$ $\quad$ $z_y(-1,1) = (-1)\cos(0) = -1$

$= -1$

plane: $\quad z - 0 = (-1)(x-(-1)) + (-1)(y-1)$

$z = -(x+1) - (y-1)$

⑫ $f(x,y) = \sqrt{xy}$ at $(1,4)$ why is $f$ diff'able at the point.

$f(x,y) = x^{\frac{1}{2}} y^{\frac{1}{2}}$

$z = f(x,y)$ is diff'able at $(a,b)$

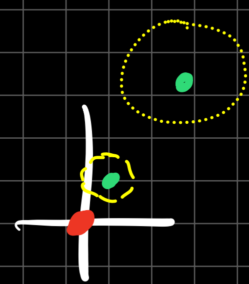
if $\Delta z = f_x(a,b)\Delta x + f_y(a,b)\Delta y + \varepsilon_1 \Delta x + \varepsilon_2 \Delta y$

where $\varepsilon_1, \varepsilon_2 \to 0$ as $\Delta x, \Delta y \to 0$

**OR** if $f_x, f_y$ exist near $(a,b)$ and are continuous at $(a,b)$

then $f$ diff'able at $(a,b)$.

$f_x = \frac{1}{2} x^{-\frac{1}{2}} y^{\frac{1}{2}}$ $\quad$ $f_y = \frac{1}{2} x^{\frac{1}{2}} y^{-\frac{1}{2}}$



now find local linearization at the given point.
(find tangent plane).

$f(x,y)=\sqrt{xy}$ at $(1,4)$

$f_x(1,4)=1$ $\qquad$ $f_y(1,4)=\frac{1}{4}$

$f(1,4)=\sqrt{(1)(4)}=2.$ ~ z-value.

$$L(x,y)=f(a,b)+f_x(a,b)(x-a)+f_y(a,b)(y-b)$$

near $(1,4)$ $\qquad$ $f(x,y)\approx L(x,y)$

$$L(x,y)=2+1(x-1)+\frac{1}{4}(y-4)$$

$$=2+x-1+\frac{1}{4}y-1$$

$$=x+\frac{1}{4}y$$

(19) $f$ is differentiable

$f(2,5)=6$ $\qquad$ $f_x(2,5)=1$ $\qquad$ $f_y(2,5)=-1$

estimate $f(2.2,4.9)$

$$L(x,y)=6+1(x-2)+(-1)(y-5)$$

$$L(2.2,4.9)=6+1(2.2-2)+(-1)(4.9-5)$$

$$=6+0.2+0.1$$

$$=6.3$$

(26) $u=\sqrt{x^2+3y^2}$ $\qquad$ find the differential

u depends on $x, y$

$$du = \frac{\partial u}{\partial x}dx + \frac{\partial u}{\partial y}dy = u_x dx + u_y dy$$

$$\frac{\partial u}{\partial x} = \frac{1}{2}(x^2+3y^2)^{-\frac{1}{2}}(2x) = \frac{x}{\sqrt{x^2+3y^2}}$$

$$\frac{\partial u}{\partial y} = \frac{1}{2}(x^2+3y^2)^{-\frac{1}{2}}(6y) = \frac{3y}{\sqrt{x^2+3y^2}}$$

$$\underbrace{du = \frac{x}{\sqrt{x^2+3y^2}}dx + \frac{3y}{\sqrt{x^2+3y^2}}dy}$$

use to see how changes in $x$ & $y$ affect changes in $u$.

32) $z = x^2 - xy + 3y^2$ $\quad (x, y)$ changes from $(3, -1)$ to $(2.96, -0.95)$

find $\Delta z = f(2.96, -0.95) - f(3, -1) = -0.71$

$$dz = (2x - y)dx + (-x + 6y)dy$$

$dx = -0.04 \quad x = 3$

$dy = .05 \quad y = -1$

$dz$ at

$$dz = 7dx + (-9)dy = 7(-0.04) - 9(0.05)$$

$$= -.28 - .45$$

$= -.73$

also use $dz = z_x\,dx + z_y\,dy$

to find upper bound for change.

$|a+b| \leq |a| + |b|$

$$|dz| \leq |z_x||dx| + |z_y||dy|$$

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Plane given 3 points.

$A(0,0,0)$, $B(1,2,3)$, $C(4,5,6)$

$\vec{u}$ connects A to B

$\vec{v}$ connects A to C

$\vec{u} = \langle 1,2,3 \rangle$ $\quad$ $\vec{v} = \langle 4,5,6 \rangle$

find $\vec{u} \times \vec{v}$ = normal. $\quad$ use normal + point.

$a\hat{i} + b\hat{j} + c\hat{k}$ $\quad$ $a(x-x_0) + b(y-y_0) + c(z-z_0) = 0$

. line through point $\parallel$ to a line.

$(-6,2,3)$ $\quad \parallel$ to $\quad \frac{1}{2}x = \frac{1}{3}y = z+1$

$\frac{1}{2}x = t \quad \frac{1}{3}y = t \quad z+1 = t$

now use

$x = 2t \quad y = 3t \quad z = t-1$

$\boxed{\langle -6,2,3 \rangle} + t\langle 2,3,1 \rangle$

$\vec{r}(t) = \langle 2t, 3t, t-1 \rangle$

$= \langle 0, 0, -1 \rangle + t\langle 2, 3, 1 \rangle$

$\vec{r}_0 + t\vec{v}$

direction vector.

## acceleration

$\vec{r}''(t) = \vec{a} = a_T\vec{T} + a_N\vec{N}$

$= v'\vec{T} + \kappa v^2\vec{N}$

$\vec{T} = \frac{\vec{r}'}{\|\vec{r}'\|}$    $\vec{N} = \frac{\vec{T}'}{\|\vec{T}'\|}$

$a_T = v'$

$v = \|\vec{r}'\|$

then find $\frac{d}{dt}(v)$

$a_N = \kappa v^2$

speed.

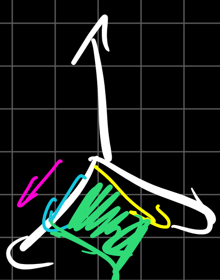
curvature.

v = speed.

**V** = velocity.

$\vec{v}$

Right hand Rule.



Zoom link $6^{30} - 7$