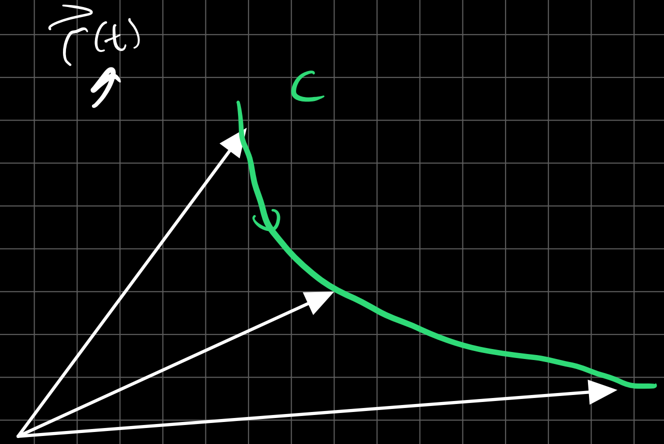


$\vec{r}(u,v)$, $\vec{r}(s,t)$, $\vec{r}(x,y)$, $\vec{r}(r,\theta)$, ...

① $\vec{r}(u,v) = \langle u+v,\ u-2v,\ 3+u-v \rangle$

$P(4,-5,1)$ $\quad Q(0,4,6)$ are the points on the surface?

P

(I) $u+v = 4$

(II) $u-2v = -5$

(III) $3+u-v = 1$

(I)+(III): $3+2u = 5 \Rightarrow \boxed{u=1}$

then by (I) $\boxed{v=3}$

make sure all 3 equations satisfied

(II): $1-2(3) = -5$ ✓

P is on surface.

Q

(I) $u+v = 0$

(II) $u-2v = 4$

(III) $3+u-v = 6$

(I)+(III): $3+2u=6 \Rightarrow 2u=3 \Rightarrow u=\frac{3}{2}$

from (I) $v = -\frac{3}{2}$

(II): $\frac{3}{2} - 2\left(-\frac{3}{2}\right) \stackrel{?}{=} 4$

$\frac{3}{2} + 3 \neq 4$ ✗

Q not on surface.

Can we look at traces? Want to find the surface.

$\vec{r}(u,v) = \langle u+v,\ u-2v,\ 3+u-v \rangle$

let $u=0$ $\quad \vec{r}(0,v) = \langle v, -2v, 3-v \rangle$

$u=1$ $\quad \vec{r}(1,v) = \langle 1+v, 1-2v, 4-v \rangle$

$u=2$ $\quad \vec{r}(2,v) = \langle 2+v, 2-2v, 5-5 \rangle$

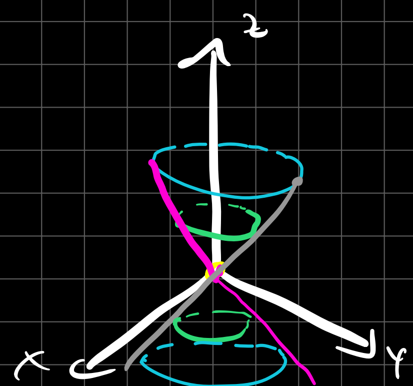
---

(5) $\vec{r}(s,t) = \langle s\cos t, s\sin t, s \rangle$ Identify surface.

$s=0$ $\quad \vec{r}(0,t) = \langle 0,0,0 \rangle$

$s=1$ $\quad \vec{r}(1,t) = \langle \cos t, \sin t, 1 \rangle$

$s=2$ $\quad \vec{r}(2,t) = \langle 2\cos t, 2\sin t, 2 \rangle$



$x=s \quad y=0 \quad z=s$

$t=0$ $\quad \vec{r}(s,0) = \langle s, 0, s \rangle$

$t=\frac{\pi}{2}$ $\quad \vec{r}(s,\frac{\pi}{2}) = \langle 0, s, s \rangle$

$t=\pi$ $\quad \vec{r}(s,\pi) = \langle -s, 0, s \rangle$

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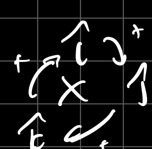
(19) Find the plane containing $\hat{i}-\hat{j}$ & $\hat{j}-\hat{k}$ that passes through origin.

then find param. equation.



$$(\hat{i}-\hat{j})\times(\hat{j}-\hat{k}) = \hat{k}-(-\hat{j})+\hat{i} = \hat{i}+\hat{j}+\hat{k}$$

$\hat{n} = \langle 1,1,1 \rangle$ $\quad$ point $(0,0,0)$

$$1(x-0)+1(y-0)+1(z-0)=0$$

$x+y+z=0.$ $\rightarrow$ $z=-x-y=f(x,y)$

Let $x=x$, $y=y$, $z=f(x,y)$.

$$\boxed{\vec{r}(x,y) = \langle x, y, -x-y \rangle}$$

(x) could also let $x=u,\ y=v \qquad z=-u-v$

or $\quad x=u \quad z=v \qquad y=-u-v$

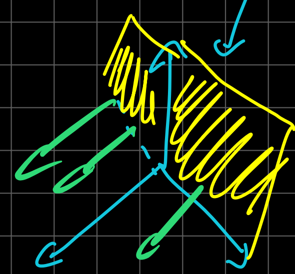
or $\quad x=s,\ y=t$

(21) find param for part of $4x^2-4y^2-z^2=4$ that lies in front of yz plane.

$$z^2 = 4x^2-4y^2-4$$

$$z = \pm\sqrt{4x^2-4y^2-4}$$

stinks!!



$x>0$

instead, solve for x

$$x^2 = y^2+\tfrac{1}{4}z^2+1$$

$$x = \sqrt{y^2+\tfrac{1}{4}z^2+1}$$

let $y=y \quad , z=z \quad , x=\sqrt{y^2+\tfrac{1}{4}z^2+1}$

$$\vec{r}(x,y) = \left\langle \sqrt{y^2+\tfrac{1}{4}z^2+1}, y, z \right\rangle$$

(32)

$$x = 2\cos\theta + r\cos(\theta/2) \qquad -\tfrac{1}{2} \le r \le \tfrac{1}{2}$$

$$y = 2\sin\theta + r\cos(\theta/2) \qquad 0 \le \theta \le 2\pi$$

$$z = r\sin(\theta/2)$$

See zoom recording

~ @ 1:00

(34) $x = u^2+1$

$y = v^3+1$

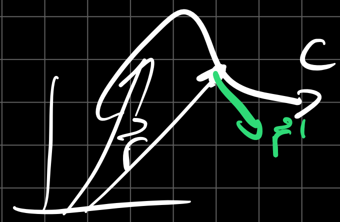
$z = u+v$

$(5,2,3)$ find tan plane to surface at the point.

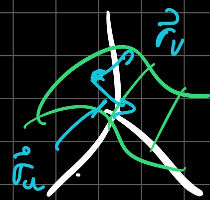
$\vec{r}(u,v) = \langle u^2+1,\ v^3+1,\ u+v \rangle$

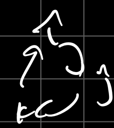
$\vec{r}_u(u,v) = \langle 2u,\ 0,\ 1 \rangle$

$\vec{r}_v(u,v) = \langle 0,\ 3v^2,\ 1 \rangle$



$$\vec{r}_u \times \vec{r}_v = (2u\hat{\imath} + \hat{k}) \times (3v^2\hat{\jmath} + \hat{k})$$
$$= 6uv^2\hat{k} + 2u(-\hat{\jmath}) + 3v^2(-\hat{\imath})$$
$$= -3v^2\hat{\imath} - 2u\hat{\jmath} + 6uv^2\hat{k}$$





find u,v when we are at $\boxed{(5,2,3)}$

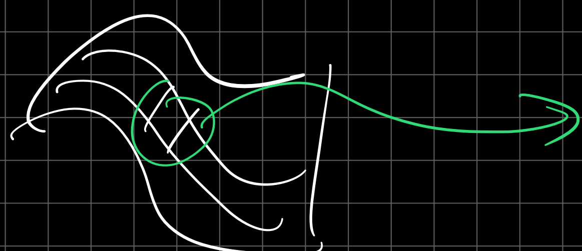
$u^2+1=5$ | $v^3+1=2$ | $u+v=3$

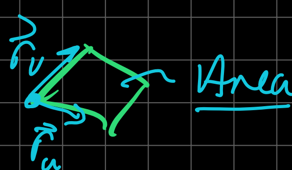
$u=\pm 2$ | $v=1$ | $u=2,\ v=1$

$$(\vec{r}_u \times \vec{r}_v)(2,1) = -3\hat{\imath} - 4\hat{\jmath} + 12\hat{k}$$

$$-3(x-5) - 4(y-2) + 12(z-3) = 0.$$

(47) Surface area.





$\|\vec{r}_u \times \vec{r}_v\|$

$$SA = \iint_D \|\vec{r}_u \times \vec{r}_v\|\, dA$$

looks like arc length

Special case: $x=x,\ y=y,\ z=f(x,y)$

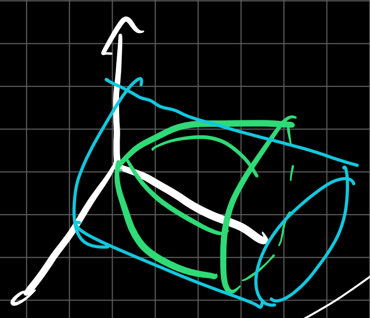
$\vec{r}(x,y) = \langle x,\ y,\ f(x,y) \rangle$

$$\int_a^b \|\vec{r}\,'(t)\|\,dt$$

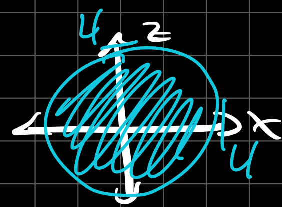
$\vec{r}_x = \langle 1, 0, f_x \rangle \quad \vec{r}_y = \langle 0, 1, f_y \rangle$

$\|\vec{r}_x \times \vec{r}_y\| = \sqrt{f_x^2 + f_y^2 + 1}$

find Surface area of part of paraboloid $y = x^2 + z^2$ that lies inside $x^2 + z^2 = 16$.



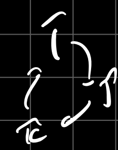
restriction is on x, z



$y = f(x, z)$

two possible approaches:

① $\vec{r}(x, z) = \langle x, x^2 + z^2, z \rangle$



$\vec{r}_x = \langle 1, 2x, 0 \rangle \qquad \vec{r}_x \times \vec{r}_z = (\hat{\imath} + 2x\hat{\jmath}) \times (2z\hat{\jmath} + \hat{k})$

$\vec{r}_z = \langle 0, 2z, 1 \rangle \qquad = 2z\hat{k} + (-\hat{\jmath}) + 2x(\hat{\imath})$

$= 2x\hat{\imath} - \hat{\jmath} + 2z\hat{k}$

$\|\vec{r}_x \times \vec{r}_z\| = \sqrt{4x^2 + 4z^2 + 1}$

$$SA = \iint_D \sqrt{4x^2 + 4z^2 + 1}\, dA$$

or do this:

② $y = f(x, z)$ and $SA = \iint_D \sqrt{f_x^2 + f_z^2 + 1}\, dA$

$= x^2 + z^2$

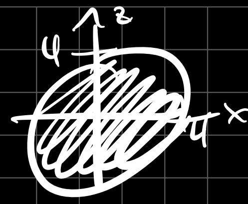
$f_x = 2x \quad f_z = 2z \qquad = \iint_D \sqrt{4x^2 + 4z^2 + 1}\, dA.$

Now integrate:

$$\iint_D \sqrt{4x^2+4z^2+1}\,dA$$

$$= \int_0^{2\pi}\int_0^{4} \sqrt{4r^2+1}\; r\,dr\,d\theta$$

simple



$0 \le r \le 4$

$0 \le \theta \le 2\pi$

$x^2+z^2=r^2$