MA-UY 2314: Discrete Mathematics

Exam 1

- Print Name: : ____________________________________
- NetID: ____________________________________
- You have 80 minutes for the exam. No notes or calculators are permitted.
- Remember the famous mathematical formula: No work = No credit (unless stated otherwise)

I pledge that I have observed the NYU honor code, and that I have neither given nor received unauthorized assistance during this examination.

Signature:

Multiple Choice

This is the multiple choice section of the exam. Each problem is worth 3 points. No partial credit is given and there is no need to justify your answer. All multiple choice answers must be recorded here. Only this page will be graded.

1. (A) (B) (C) (D) (E) (F) (G) (H) (I) (J)

2. (A) (B) (C) (D) (E) (F) (G) (H) (I) (J)

3. (A) (B) (C) (D) (E) (F) (G) (H) (I) (J)

1. The statement

No good grade is easy to obtain.

is equivalent to which of the following statements

(a) For any grade, if the grade was not good then it was not easy to obtain.

(b) For any grade, if the grade was not good then it was easy to obtain.

(c) For any grade, if the grade is good then it was not easy to obtain.

(d) For any grade, if the grade is good then it was easy to obtain.

(e) None of the above.

2. The negation of the statement

$$\forall \varepsilon > 0, \forall \delta > 0, \exists \nu < 0, \text{ if } f(x) > \nu \text{ then } \delta < x < \varepsilon$$

is

(a) $\exists \varepsilon > 0, \exists \delta > 0, \forall \nu < 0,\ (f(x) \leq \nu)$ and $(\delta < x < \varepsilon)$.

(b) $\exists \varepsilon \leq 0, \exists \delta \leq 0, \forall \nu \geq 0,\ (f(x) > \nu)$ and $(x \leq \delta$ or $x \geq \varepsilon)$.

(c) $\exists \varepsilon > 0, \exists \delta > 0, \forall \nu < 0,\ (f(x) > \nu)$ and $(x \leq \delta$ or $x \geq \varepsilon)$.

(d) $\exists \varepsilon \leq 0, \exists \delta \leq 0, \forall \nu \geq 0,\ (f(x) \leq \nu)$ and $(\delta < x < \varepsilon)$.

(e) None of the above.

3. The statement

The disjunction of any two tautologies is logically equivalent to a contradiction

written formally reads

(a) $\forall$ tautologies $\mathbf{t}, \forall$ tautologies $\mathbf{s}, \mathbf{t} \wedge \mathbf{s} \equiv \mathbf{c}$

(b) $\forall$ tautologies $\mathbf{t}, \forall$ tautologies $\mathbf{s}, \mathbf{t} \vee \mathbf{s} \equiv \mathbf{c}$

(c) $\exists$ tautologies $\mathbf{t}, \exists$ tautologies $\mathbf{s}, \mathbf{t} \vee \mathbf{s} \equiv \mathbf{c}$

(d) $\exists$ tautologies $\mathbf{t}, \exists$ tautologies $\mathbf{s}, \mathbf{t} \wedge \mathbf{s} \equiv \mathbf{c}$

(e) None of the above.

Open Response

This is the open response section of the exam. All answers must be justified, unless otherwise stated.

4. Determine whether the given statement is true or false. If the statement is true, then no need to expand further. Simply write true and move on to the next part. If the statement is false, then write false and justify your answer with a counter-example or a proof by exhaustion.

   Remark. *The notations* $\mathbb{Z}$ *and* $\mathbb{Z}^+$ *indicate the integers and the postive integers respectively. The domain* $D = \{0, 1, 2, 3\}$.

   (a) (3 points) $\forall x \in \mathbb{Z}, x < 0 \vee x > 0$.

   False: $x = 0$ is an integer and $x < 0 \wedge x > 0$ is false.

   Remark. *You should say a few words about your counterexample. Note that this is the only counterexample possible.*

   (b) (3 points) $\exists x \in D,\ x^2 + x + 1 = 11$.

   False

   When $x = 0$, $0^2 + 0 + 1 = 1 \neq 1$

   When $x = 1$, $1^2 + 1 + 1 = 3 \neq 1$

   When $x = 2$, $2^2 + 2 + 1 = 7 \neq 1$

   When $x = 3$, $3^2 + 3 + 1 = 13 \neq 1$

   Remark. *You have to show the predicate is false for each element in* D.

   (c) (3 points) $\forall x \in D,\ x^2 \neq x$.

   False For $x = 0$, $0^2 < 0$ is false.

   Remark. *This is not the only counterexample possible.*

5. (20 points) Prove the logical equivalence

$$\sim (p \vee (\sim p \wedge q)) \equiv \sim p \wedge \sim q$$

using Theorem 2.1.1. Annotate.

Remark. *Failure to use Theorem 2.1.1. will result in zero points. Failure to annotate will result in points deducted (lots of em).*

Proof:

$$\begin{aligned}
\sim (p \vee (\sim p \wedge q)) &\equiv \sim p \wedge \sim (\sim p \wedge q) \;\; (DeMorgan's\ Law) \\
&\equiv \sim p \wedge (\sim (\sim p) \vee \sim q) \;\; (DeMorgan's\ Law) \\
&\equiv \sim p \wedge (p \vee \sim q) \;\; (Double\ Negation\ Law) \\
&\equiv (\sim p \wedge p) \vee (\sim p \wedge \sim q) \;\; (Distribution\ Law) \\
&\equiv \mathbf{c} \vee (\sim p \wedge \sim q) \;\; (Negation\ Law) \\
&\equiv \sim p \wedge \sim q \;\; (Identity\ Law)
\end{aligned}$$

6. Let $C(x)$ be the predicate *x has a cat*, let $D(x)$ be the predicate *x has a dog*, and let $F(x)$ be the predicate *x has a ferret*. Express each of these statements in terms of $C(x)$, $D(x)$, $F(x)$, quantifiers, and logical connectives. Let the domain consist of all students in your class.

   (a) (4 points) A student in your class has a cat, a dog, and a ferret.

   $\exists x \in D, C(x) \wedge D(x) \wedge F(x)$

   Remark. *I've used D to denote the domain all students, but any letter will do. Some students may chose to write something along the lines*

   $\exists$ *student* $x, C(x) \wedge D(x) \wedge F(x)$

   This is OK! Lastly, the order in which you write the predicates does not matter.

   (b) (4 points) All students in your class have a cat, a dog, or a ferret.

   $\forall x \in D, C(x) \vee D(x) \vee F(x)$

   (c) (4 points) Some student in your class has a cat and a ferret, but not a dog.

   $\exists x \in D, (C(x) \wedge F(x)) \wedge \sim D(x)$

   Remark. *You will need the parenthesis around the predicates* $C(x)$ *and* $D(x)$*.*

7. (20 points) Given the set of premises and conclusion below, deduce the conclusion from the premises using the valid argument forms listed in **Table 2.3.1**. Annotate.

   (a) $(p \wedge \sim q) \to r$.

   (b) $(s \vee t) \to p$.

   (c) $q \to v$.

   (d) $s \to \sim v$

   (e) s

   (f) $\therefore\ r$

Modus Ponens
$s \to \sim v$
s
$\therefore \sim v$

Modus Tollens
$q \to v$
$\sim v$
$\therefore \sim q$

Generalization
s
$\therefore\ s \vee t$

Modus Ponens
$s \vee t \to p$
$s \vee t$
$\therefore\ p$

Conjunction
p
$\sim q$
$\therefore\ p \wedge \sim q$

Modus Ponens
$p \wedge \sim q \to r$
$p \wedge \sim q$
$\therefore\ r$