worksheet 3 due after Exam 2.

↳ on Gradescope 2 hrs.

open 10am close at 6pm.

- open book

Project Part II due on 12th

- Record video today at 2pm

Cylindrical & Spherical Coordinates

↓

polar + z

$x = r\cos\theta$

$y = r\sin\theta$

$z = z$

$dV = r\,dr\,d\theta\,dz$

we can change to x or y.

$x = \rho\sin\phi\cos\theta$

$y = \rho\sin\phi\sin\theta$

$z = \rho\cos\phi$

$\rho^2 = x^2+y^2+z^2$

$dV = \rho^2\sin\phi\,d\rho\,d\phi\,d\theta$

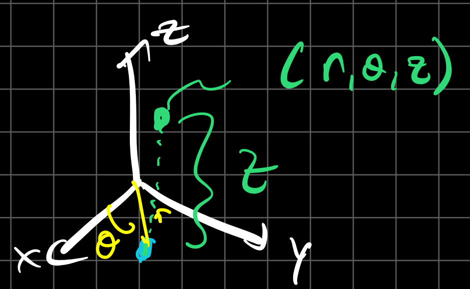
15.7

① $(4, \frac{\pi}{3}, -2)$ cylindrical find point in (x, y, z)

$r \quad \theta \quad z$

$$\left.\begin{aligned} x &= 4\cos\tfrac{\pi}{3} = 4\left(\tfrac{1}{2}\right) = 2 \\ y &= 4\sin\tfrac{\pi}{3} = 4\left(\tfrac{\sqrt{3}}{2}\right) = 2\sqrt{3} \\ z &= -2 \end{aligned}\right\} \quad (2, 2\sqrt{3}, -2)$$

$(x, y, z) = (r\cos\theta, r\sin\theta, z)$



③ $(-1,1,1)$ change to cylindrical $\qquad \theta = \arctan\left(\frac{y}{x}\right)$

$x=-1$
$y=1$ } need r,θ

$z=1 \longrightarrow z=1$

$\frac{1 = r\sin\theta}{-1 = r\cos\theta} \Rightarrow -1 = \tan\theta$

$\theta = \arctan(-1)$

$\boxed{\theta = -\frac{\pi}{4}}$

$r^2 = x^2+y^2$

$r^2 = (-1)^2+(1)^2$

$r = \sqrt{2}$

$(-1,1,1) \to (\sqrt{2}, -\frac{\pi}{4}, 1)$

⑨ $x^2 - x + y^2 + z^2 = 1$ write using cylindrical.

we know $x^2+y^2 = r^2$

$r^2 - r\cos\theta + z^2 = 1$

$\cos 2\theta = \cos^2\theta - \sin^2\theta.$

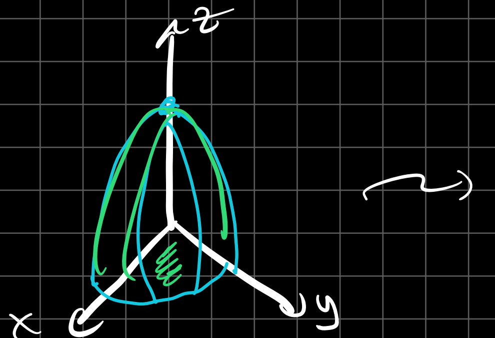
$z = x^2 - y^2 \longrightarrow z = r^2\cos^2\theta - r^2\sin^2\theta = r^2\cos(2\theta)$

$z = r^2\cos(2\theta)$

⑭ $\iiint_E (x+y+z)\, dV$ where E solid in 1^{st} octant under $z = 4-x^2-y^2$

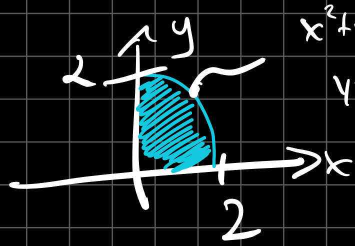
$x \geq 0$
$y \geq 0$
$z \geq 0$

Set up using cartesian & cylindrical.



$4-x^2-y^2 = 4-(x^2+y^2)$
$= 4-r^2$

$\rightsquigarrow$ in xy plane



$x^2+y^2=4$

$y = \sqrt{4-x^2}$

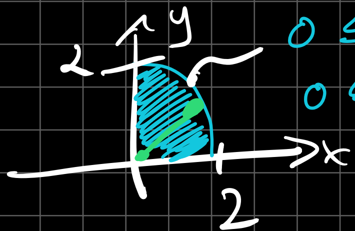
$0 \leq x \leq 2$

$0 \leq y \leq \sqrt{4-x^2}$

$$0 \le z \le 4-x^2-y^2$$

$$\iiint_E (x+y+z)\,dV = \int_0^2 \int_0^{\sqrt{4-x^2}} \int_0^{4-x^2-y^2} (x+y+z)\,dz\,dy\,dx$$

try cylindrical



$0 \le r \le 2$

$0 \le \theta \le \frac{\pi}{2}$

now z

$0 \le z \le 4-x^2-y^2$ needs to be in terms of r, θ

$0 \le z \le 4-r^2$

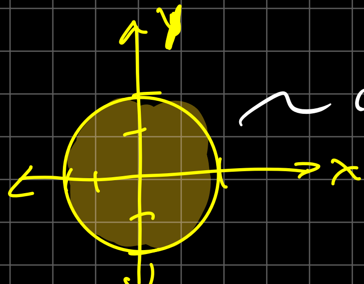
$$\iiint_E (x+y+z)\,dV = \int_0^{\frac{\pi}{2}} \int_0^{2} \int_0^{4-r^2} (r\cos\theta + r\sin\theta + z)\, r\,dz\,dr\,d\theta$$

29 $$\int_{-2}^{2} \int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}} \int_{\sqrt{x^2+y^2}}^{2} xz\,dz\,dx\,dy$$

change to cylindrical.

Draw xy plane part.

$-2 \le y \le 2$ $\quad -\sqrt{4-y^2} \le x \le \sqrt{4-y^2}$



convert to r, θ

$0 \le r \le 2$

$0 \le \theta \le 2\pi$

take cone of z. $\sqrt{x^2+y^2} \le z \le 2$

turn into r, θ, $r \le z \le 2$

$$\int_{-2}^{2} \int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}} \int_{\sqrt{x^2+y^2}}^{2} xz \, dz\,dx\,dy = \int_{0}^{2\pi} \int_{0}^{2} \int_{r}^{2} r\cos\theta \, z \, r \, dz\,dr\,d\theta$$

15.8

① $(6, \frac{\pi}{3}, \frac{\pi}{6})$ Convert from spherical to cartesian. (rectangular)

$x = \rho \sin\phi \cos\theta$

$y = \rho \sin\phi \sin\theta$

$z = \rho \cos\phi$

$\rho^2 = x^2+y^2+z^2$

$\rho = 6$ - rho

$\theta = \frac{\pi}{3}$

$\phi = \frac{\pi}{6}$

$x = 6 \sin(\frac{\pi}{6}) \cos(\frac{\pi}{3}) = 6(\frac{1}{2})(\frac{1}{2}) = \frac{6}{4} = \frac{3}{2}$

$y = 6 \sin(\frac{\pi}{6}) \sin(\frac{\pi}{3}) = 6(\frac{1}{2})(\frac{\sqrt{3}}{2}) = \frac{3\sqrt{3}}{2}$

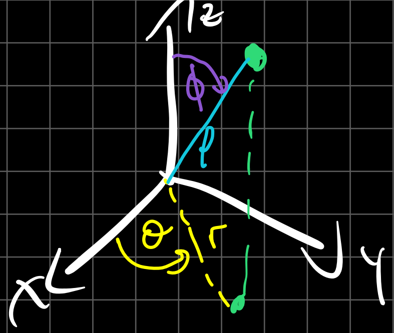
$z = 6 \cos(\frac{\pi}{6}) = 6(\frac{\sqrt{3}}{2}) = 3\sqrt{3}$

$(6, \frac{\pi}{3}, \frac{\pi}{6}) \rightarrow (\frac{3}{2}, \frac{3\sqrt{3}}{2}, 3\sqrt{3})$

comment: θ same concept as cylindrical.

$\rho^2 = r^2+z^2 = x^2+y^2+z^2$

ϕ angle coming down from positive z-axis.



(10) $z = x^2 - y^2$ write in spherical.

$x = \rho \sin\phi \cos\theta$

$y = \rho \sin\phi \sin\theta$

$z = \rho \cos\phi$

$$\rho\cos\phi = \rho^2 \sin^2\phi \cos^2\theta - \rho^2 \sin^2\phi \sin^2\theta$$

$$\rho\cos\phi = \rho^2 \sin^2\phi \cos 2\theta$$ if $\rho \neq 0$ then $\cos\phi = \rho \sin^2\phi \cos 2\theta$

(9) $\boxed{x^2+y^2+z^2=9}$ write in cylindrical & spherical.

$\rho = 3$

$r^2 + z^2 = 9$

$\rho^2 = 9$

$\rho = 3$

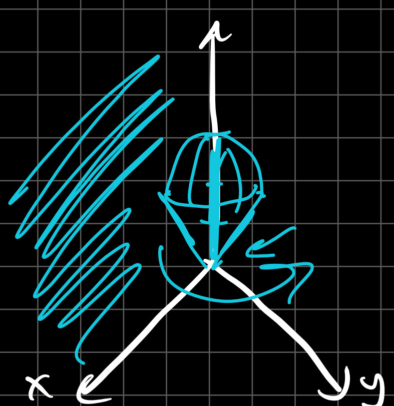
(17) $$\int_0^{\pi/6} \int_0^{\pi/2} \int_0^3 \rho^2 \sin\phi \, d\rho \, d\theta \, d\phi = \iiint_E dV = \text{Volume of } E$$

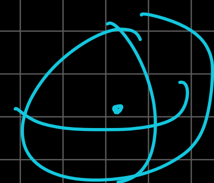
What is E?

$0 \le \rho \le 3$

$0 \le \theta \le \frac{\pi}{2}$

$0 \le \phi \le \pi/6$





(23) $\iiint_E (x^2+y^2)\,dV$ E lies in between $x^2+y^2+z^2=4$ & $x^2+y^2+z^2=9$

write in spherical cylindrical and cartesian.

$2\le\rho\le 3$

$0\le\theta\le 2\pi$

$0\le\phi\le\pi$

$$\iiint_E (x^2+y^2)\,dV$$

$$=\int_0^{2\pi}\int_0^{\pi}\int_2^{3}\Big(\underbrace{\rho^2\sin^2\phi\cos^2\theta+\rho^2\sin^2\phi\sin^2\theta}_{\rho^2\sin^2\phi\left(\underbrace{\cos^2\theta+\sin^2\theta}_{1}\right)}\Big)\rho^2\sin\phi\,d\rho\,d\phi\,d\theta$$

$$=\int_0^{2\pi}\int_0^{\pi}\int_2^{3}\left(\rho^2\sin^2\phi\right)\rho^2\sin\phi\,d\rho\,d\phi\,d\theta=\underbrace{\int_0^{2\pi}d\theta}_{2\pi}\,\underbrace{\int_0^{\pi}\sin^3\phi\,d\phi}\,\underbrace{\int_2^{3}\rho^4\,d\rho}_{\frac{\rho^5}{5}\Big|_{\rho=2}^{\rho=3}}$$

$$\int_0^{\pi}\sin^2\phi\,\sin\phi\,d\phi=\int_0^{\pi}(1-\cos^2\phi)\sin\phi\,d\phi$$

$u=\cos\phi$

$du=-\sin\phi\,d\phi$

$$=-\int_1^{-1}(1-u^2)\,du$$

$$= \int_{-1}^{1} (1-u^2)\,du = \left(u - \frac{u^3}{3}\right)\Big|_{u=-1}^{u=1} = \left(1-\frac{1}{3}\right) - \left(-1+\frac{1}{3}\right)$$

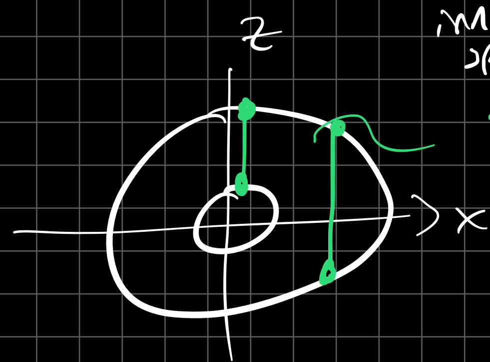
$$= 1 - \frac{1}{3} + 1 - \frac{1}{3} = 2 - \frac{2}{3} = \frac{4}{3}.$$

$$\text{Integral} = (2\pi)\left(\frac{4}{3}\right)\left(\frac{3^5}{5} - \frac{2^5}{5}\right)$$

Think about changing to cylindrical : $2 \le r \le 3$

$0 \le \theta \le 2\pi$

inner sphere $\le z \le$ Outer sphere.



becomes an issue.

but x^2+y^2 becomes r^2

dV becomes $r\,dr\,d\theta\,dz$

inequality for z stinks but other parts are "nice"

Check out Q43