13.4

(3) $\vec{r}(t) = \langle -\frac{1}{2}t^2, t \rangle$

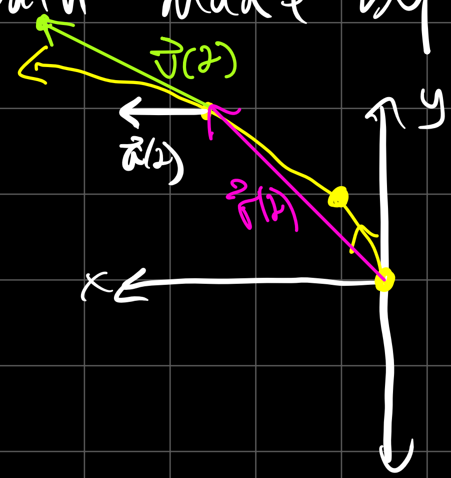
find $\vec{v}(t) = \vec{r}\,'(t) = \langle -t, 1 \rangle$

$\vec{a}(t) = \vec{v}\,'(t) = \vec{r}\,''(t) = \langle -1, 0 \rangle$

and speed. $\|\vec{v}\| = v = \sqrt{t^2+1}$

Sketch curve/path made by $\vec{r}(t)$.

t	$\vec{r}$
0	$\langle 0,0 \rangle$
1	$\langle -\frac{1}{2}, 1 \rangle$
2	$\langle -2, 2 \rangle$



draw $\vec{v}(2) = \langle -2, 1 \rangle$

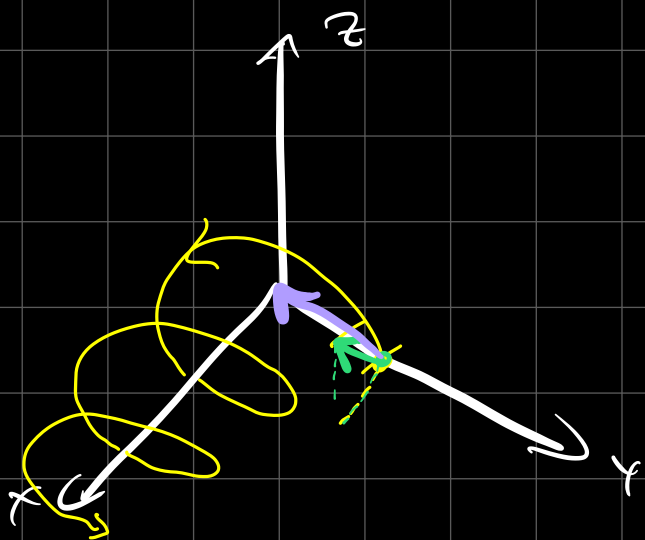
$\vec{a}(2) = \langle -1, 0 \rangle$

(8) $\vec{r}(t) = t\hat{\imath} + 2\cos t\,\hat{\jmath} + \sin t\,\hat{k}$ $\quad \vec{r}(0) = 2\hat{\jmath}$

Sketch path, find $\vec{a}(0)$, $\vec{v}(0)$

$\vec{v}(t) = \hat{\imath} - 2\sin t\,\hat{\jmath} + \cos t\,\hat{k}$, $\vec{v}(0) = \hat{\imath} + \hat{k}$

$\vec{a}(t) = -2\cos t\,\hat{j} - \sin t\,\hat{k}$, $\vec{a}(0) = -2\hat{j}$



⑯ $\vec{a}(t) = \sin t\,\hat{i} + 2\cos t\,\hat{j} + 6t\,\hat{k}$

$\vec{v}(0) = -\hat{k}$ $\quad$ $\vec{r}(0) = \hat{j} - 4\hat{k}$

find $\vec{r}(t)$.

$$\vec{v}(t) = \int \vec{a}(t)\,dt = \underbrace{-\cos t\,\hat{i} + 2\sin t\,\hat{j} + 3t^2\hat{k} + \vec{C}}_{\text{put } t=0 \text{ in and set } = \vec{v}(0)}$$

$$-\hat{k} = -\hat{i} + \vec{C} \Rightarrow \vec{C} = \hat{i} - \hat{k}$$

$$\vec{v}(t) = (1-\cos t)\hat{i} + 2\sin t\,\hat{j} + (3t^2 - 1)\hat{k}$$

$$\vec{r}(t) = \int \vec{v}(t)\,dt$$

$$= (t-\sin t)\hat{\imath} - 2\cos t\,\hat{\jmath} + (t^3 - t)\hat{k} + \vec{D}$$

find $\vec{D}$ using $\vec{r}(0)$.

$$\underbrace{\hat{\jmath} - 4\hat{k}}_{\vec{r}(0)} = -2\hat{\jmath} + \vec{D} \Rightarrow \vec{D} = 3\hat{\jmath} - 4\hat{k}$$

$$\boxed{\vec{r}(t) = (t-\sin t)\hat{\imath} + (3-2\cos t)\hat{\jmath} + (t^3 - t - 4)\hat{k}}$$

19 $\vec{r}(t) = \langle t^2, 5t, t^2 - 16t \rangle$

find when speed is a minimum.

$$\vec{v}(t) = \vec{r}\,'(t) = \langle 2t, 5, 2t-16 \rangle$$

$$v(t) = \|\vec{v}(t)\| = \sqrt{(2t)^2 + (5)^2 + (2t-16)^2}$$

$$= \sqrt{8t^2 - 64t + 281}$$

$b^2 - 4ac < 0 \Rightarrow \underline{8t^2 - 64t + 281 > 0} \quad \forall t$

for all.

to find min

option ① complete the square

② use axis of symmetry

③ use calculus

$$\frac{d}{dt}\left(8t^2-64t+281\right)=0$$

$$\boxed{16t-64}=0 \Rightarrow \underline{t=4.}$$

$\ominus$ $\oplus$

$t=4$ — min at $t=4$.

$$\min = 8(4)^2-64(4)+281=153$$

$\Rightarrow$ min speed $\sqrt{153}$

35

$\vec{r}\,'(t)=\vec{c}\times\vec{r}(t)$, $\vec{c}$ is constant vector.

describe path given by $\vec{r}(t)$.

$\vec{r}\,' \perp \vec{c}$

$\vec{r}\,' \perp \vec{r}$

$\vec{r}\,' \cdot \vec{r} = 0$

$\vec{v} \perp \vec{r}$

$\|\vec{r}\| = \text{constant}$

$\forall t$

circular motion.

$$\|\vec{r}\|^2 = \vec{r}(t) \cdot \vec{r}(t)$$

$$\frac{d}{dt}\left(\vec{r}(t) \cdot r(t)\right) = \vec{r}\,'(t) \cdot \vec{r}(t) + \vec{r}(t) \cdot \vec{r}\,'(t)$$

$$= 2\vec{r}\,'(t) \cdot \vec{r}(t) = 0$$

b/c $\vec{r}\,' \perp \vec{r}$

$$\Rightarrow \quad \frac{d}{dt}\|\vec{r}\|^2 = 0$$

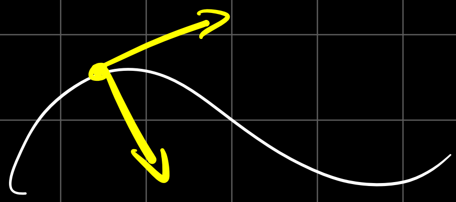
$$\Rightarrow \quad \|\vec{r}\|^2 = \text{Constant.}$$

$$\|\vec{r}\| = \sqrt{\text{constant}}$$

path given by $\vec{r}$ is a circle.

(38) $\vec{r}(t) = 2t^2\hat{\imath} + \left(\frac{2}{3}t^3 - 2t\right)\hat{\jmath}$

find tangential and normal components of acceleration.



find $\vec{a}$, $\quad \vec{v}(t) = 4t\hat{\imath} + (2t^2-2)\hat{\jmath}$

$$\vec{a}(t) = 4\hat{\imath} + 4t\hat{\jmath}$$

$$= \underline{a_T}\vec{T} + \underline{a_N}\vec{N}$$

Recall $\vec{T} = \frac{\vec{r}'}{\|\vec{r}'\|} = \frac{\vec{v}}{v}$

$$\vec{N} = \frac{\vec{T}'}{\|\vec{T}'\|}$$

$\vec{v}(t) = 4t\hat{\imath} + (2t^2-2)\hat{\jmath}$, $\quad \|\vec{v}(t)\| = \sqrt{(4t)^2 + (2t^2-2)^2}$

$$= \sqrt{16t^2 + 4t^4 - 8t^2 + 4}$$

$$\vec{T} = \frac{4t}{2t^2+2}\hat{\imath} + \frac{2t^2-2}{2t^2+2}\hat{\jmath}$$

$$= \sqrt{4t^4+8t^2+4}$$

$$= \sqrt{(2t^2+2)^2}$$

$$= 2t^2+2.$$

To find $\vec{N}$ need $\vec{T}'$ & $\|\vec{T}'\|$

(exercise)

Need $a_T = v'$ $\quad a_N = \underset{\substack{| \\ \text{Curvature.}}}{\kappa} v^2$

$$\vec{a} = a_T\vec{T} + a_N\vec{N}$$

$$v(t) = 2t^2+2, \quad v' = 4t$$

$$\kappa = \frac{\|\vec{T}'\|}{\|\vec{r}'\|} = \frac{\|\vec{T}'\|}{v} = \frac{\|\vec{T}'\|}{2t^2+2}$$

need to find $\vec{T}'$ to get κ, N.

$$\vec{a} = 4t\left(\frac{4t}{2t^2+2}\hat{\imath} + \frac{2t^2-2}{2t^2+2}\hat{\jmath}\right) + \kappa\underbrace{(2t^2+2)^2}_{v^2}\vec{N}$$

amount in Tangential direction $\quad (a_T)(\vec{T})$

amount in normal direction

$$(\vec{T})' = \left(\frac{4t}{2t^2+2}\hat{\imath} + \frac{2t^2-2}{2t^2+2}\hat{\jmath}\right)' = \text{little messy.}$$

Try to find a_N