

Theorem 2.1.1 Logical Equivalences

Ken

① commutativity

$$p \vee q \equiv q \vee p$$

$$p \wedge q \equiv q \wedge p$$

② associativity

$$(p \vee q) \vee r \equiv p \vee (q \vee r)$$

$$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$$

③ distributivity

$$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$$

$$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$$

④ identity

$$p \wedge T \equiv p$$

$$p \vee \perp \equiv p$$

⑤ negation laws

$$p \vee \neg p \equiv T \quad (\text{tautology})$$

$$p \wedge \neg p \equiv \perp \quad (\text{contradiction})$$

⑥ negation elimination
(double negative law)

$$\neg(\neg p) \equiv p$$

⑦ idempotence

$$p \vee p \equiv p$$

$$p \wedge p \equiv p$$

⑧ universal bound

$$p \vee T \equiv T$$

$$p \wedge \perp \equiv \perp$$

⑨ DeMorgan's laws

$$\neg(p \vee q) \equiv \neg p \wedge \neg q$$

$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$

⑩ absorption

$$p \vee (p \wedge q) \equiv p$$

$$p \wedge (p \vee q) \equiv p$$

⑪ tautology & contradiction
negation

$$\neg T \equiv \perp$$

$$\neg \perp \equiv T$$