Quiz 3.

Q1. $\sim N(4.25,\ 0.004)$

a $P(X<4.1) = P\left(Z < \frac{4.1-4.25}{\sqrt{0.004}}\right) = P(Z<-2.37)$

$= 0.0089.$

b $n=15$

$\sum_{i=1}^{15} X_i \sim N(15*4.25,\ 15*0.004) = N(63.75, 0.06)$

c $P\left(\sum_{i=1}^{15} X_i > 64\right) = P\left(Z > \frac{64-63.75}{\sqrt{0.06}}\right) = P(Z>1.02)$

$= 0.1534$

Q2. $X \sim Exp(\mu = 12)$ $f(x) = \frac{1}{12} e^{-x/12}$ $x > 0$.

a $P(X > 15 \mid X > 6) = P(X > 9) = \int_9^\infty \frac{1}{12} e^{-x/12} dx$

$\frac{\int_{15}^\infty \frac{1}{12} e^{-x/12} dx}{\int_6^\infty \frac{1}{12} e^{-x/12} dx}$ $= e^{-9/12} = 0.472$

b $p = P(X < 3) = \int_0^3 \frac{1}{12} e^{-x/12} dx = 1 - e^{-1/4}$

$= 0.221$

$Y \sim Geo(0.221)$ $E(Y) = \frac{1}{p} \approx \frac{1}{0.221}$

Quiz 4.

Q1. (a) $\bar{X}$ ✓ S^2 ✓

(b). $\mu = 1.6$ mg $\sigma^2 = 0.15$

$n = 100$, $P(\bar{X} > 1.55) = P\left(Z > \frac{1.55 - 1.6}{\sqrt{0.15/100}}\right)$

$N\left(1.6, \frac{0.15}{100}\right)$ $= P(Z > -1.29)$

$= 0.9015$

(c) $\$1 = 8/00$

$$P\left(\sum_{i=1}^{8100} X_i > 12,900\right) = P\left(Z > \frac{12,900 - 12960}{\sqrt{8100 * 0.15}}\right)$$

$$N(8100 * 1.6,\ 8100 * 0.15) \qquad = P(Z > -1.72)$$

$$= 0.9573$$

Q2 $n = 80$, $X \sim Bin(80, 0.47) \approx N(37.6,\ 37.6 * 0.53)$

$$P(X \geq 35) \approx P\left(Z > \frac{34.5 - 37.6}{\sqrt{37.6 * 0.53}}\right) = \text{——}$$

Sample Ex 2. $X \sim N(\mu, \sigma^2)$

Q2. $n = 20$, $\bar{X} = 180$, $S = 30$.

(a) 95% C.I. for μ.

$$\bar{X} \pm t^{(19)}_{0.025} \frac{S}{\sqrt{n}} = 180 \pm 2.093 \frac{30}{\sqrt{20}}$$

$$= (\quad ,\quad)$$

(b) $X_0 - \bar{X} \sim N\left(0,\ \sigma^2\left(1 + \frac{1}{n}\right)\right)$

$$\frac{X_0 - \bar{X}}{\sigma\sqrt{1 + \frac{1}{n}}} \sim Z$$

$$\frac{\bar{X}_0 - \bar{X}}{S\sqrt{1+\frac{1}{n}}} \sim t(n-1)$$

$$\bar{X} \pm t_{\frac{\alpha}{2}}(n-1)\, S \cdot \sqrt{1+\frac{1}{n}}$$

Q4.

diff: 7, 13, 2, 5, -2, 6, 6, 5, 2, 6

from a $N(\mu, \sigma^2)$

Q1. $X_1, X_2, \cdots X_{16} \sim N(90, 10^2)$

$$P(S^2 < 166.64)$$

$$= P\left(\frac{15S^2}{100} < \frac{15 * 166.64}{100}\right)$$

$$= P(\chi^2(15) < 25)$$

$$\approx 0.95$$

$$\frac{(n-1)S^2}{\sigma^2} \sim \chi^2(n-1)$$

166.66
8.33

26

Q5. $X_1, X_2, \cdots X_{80} \sim f(x) = 3x^2 \quad 0 < x < 1.$

$$P\left(58 \le \sum_{i=1}^{80} X_i \le 64\right)$$

$$\sum_{i=1}^{80} X_i \sim N(n\mu, n\sigma^2) \sim N(60, 3)$$

$\mu = ?\ 3/4$

$\sigma^2 = ?\ 0.0375$

$3/80$

$$\approx P\left(\frac{58-60}{\sqrt{3}} \le Z \le \frac{64-60}{\sqrt{3}}\right) = \text{---}$$

Or: $P\left(\frac{58}{80} \le \bar{X} \le \frac{64}{80}\right)$

$$= P\left(\frac{\frac{58}{80} - \frac{3}{4}}{\sqrt{3/80}/\sqrt{80}} < Z < \frac{\frac{64}{80} - \frac{3}{4}}{\sqrt{3/80}/\sqrt{80}}\right)$$

$$\bar{X} \sim N(\mu, \sigma^2/n)$$

$$\sum_{i=1}^{n} X_i \sim N(n\mu, n\sigma^2)$$

WSS. Q7.

$n = 200$, $\bar{Y} = 114$. $\hat{p} = 0.57$.

96% CI for p.

$$\frac{\hat{p} - p}{\sqrt{p\bar{q}/n}} \sim Z.$$

CI for p: $\hat{p} \pm z_{\frac{\alpha}{2}} \sqrt{\frac{\hat{p}\hat{q}}{n}}$

$= 0.57 \pm 2.055 \sqrt{\frac{0.57 \cdot 0.43}{200}}$

$= (\_\_, \_\_)$

W3. Q6

a) $N(\mu = 7.36, \sigma = 0.29)$

$P(X > 8) = \_\_\_$

b) $p = P(X > 7.5) = P\left(Z > \frac{7.5 - 7.36}{0.29}\right)$

$= P(Z > 0.48)$

$= 0.3156.$

$Y \sim Bin(n = 12, p = 0.3156)$

$P(Y \geq 10) = \binom{12}{10} 0.3156^{10} \cdot 0.6844^{2}$

$+ \binom{12}{11} 0.3156^{11} 0.6844$

$+ 0.3156^{12}$

$= 0.0083.$

(c). $n=15$ ✗

$n=16$ ✓