Black-Scholes PDE

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Example. [Black-Scholes PDE] We assume that the underlying (stock, for instance) follows a geometric Brownian motion. That is in the risk-neutral measure it satisfies SDE

$$dS_t = rS_t dt + \sigma S_t dB_t, \tag{10}$$

where r is the risk-free rate which we assume to be constant. The payoff of a European option at maturity T is known and is equal to $V(S_T)$. Then to find the value of the option at some earlier time $t < T$ we have to compute

$$\mathbb{E}_{S_t=x}\left(e^{-rt}V(S_T)\right). \tag{11}$$

From the Feynman-Kac formula we conclude that $u(t,x)$ solves partial differential equation

$$u_t + rxu_x + \frac{1}{2}\sigma^2 x^2 u_{xx} - ru = 0 \tag{12}$$

with $u(T,x) = V(x)$. Equation (12) is the famous known Black-Scholes PDE.