Exp $(\beta = 5)$ $\quad E(X) = \beta$

$$f(x) = \frac{1}{5} e^{-x/5}, \quad x > 0$$

Exp $(\lambda = 5)$ $\quad f(x) = 5 e^{-5x} \quad x > 0$ $\quad$ 2 diff dist.

$\lambda = \frac{1}{5}$ $\quad f(x) = \frac{1}{5} e^{-x/5} \quad x > 0$

$E(X) = \frac{1}{\lambda}$

$\lambda = \frac{1}{\beta}$

these 2 are the same

Review:

① $X_1, X_2, \dots X_n$ indep with μ, σ^2.

then $E(\bar{X}) = \mu$

$Var(\bar{X}) = \sigma^2/n$

② $X_1, X_2, \dots X_n$ indep $N(\mu, \sigma^2)$, then

$\bar{X} \sim N(\mu, \sigma^2/n)$

③ C.L.T. $X_1, X_2, \dots X_n$ indep $\mu, \sigma^2 < \infty$.

$\bar{X} \underset{approx}{\approx} N(\mu, \sigma^2/n)$ $(n \geq 30)$ but can be smaller

$\Updownarrow$

$\sum_{i=1}^{n} X_i \approx N(n\mu, n\sigma^2)$

Ex: A bus, weight ≤ 5000 lbs.

passengers: $\mu = 180$, $\sigma = 20$.

27 people on the bus.

P(weight exceeds the limit of 5000)

ⓐ $= P(\sum_{i=1}^{27} X_i > 5000) \approx P(Z > \frac{5000 - 4860}{20 \cdot \sqrt{27}})$

By CLT, $\sum_{i=1}^{27} X_i \sim N(4860, 27 * 20^2)$

$$= P(Z > 1.35) = 0.0885$$

(b) $= P\left(\bar{X} > \frac{5000}{27}\right) \approx P\left(Z > \frac{\frac{5000}{27} - 180}{20/\sqrt{27}}\right)$

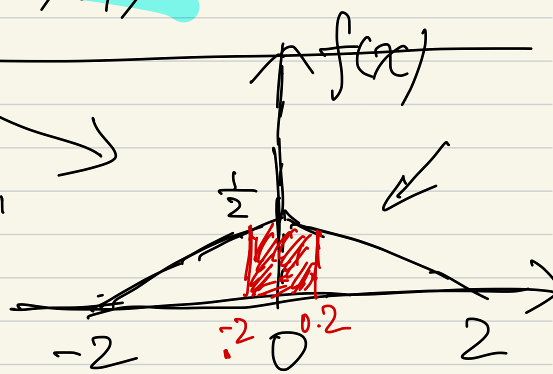
$= P(Z > 1.35)$

$= 0.0885$

By CLT, $\bar{X} \sim N(\mu, \sigma^2/n)$

$\sim N(180, 20^2/27)$

Ex: $X \sim f(x)$ →



① Random pick an X,

$P(-0.2 < X < 0.2) = 0.19$

② Randomly pick $X_1, X_2, - - X_{50}$. $\bar{X}$.

$P(-0.2 < \bar{X} < 0.2) =$

By C.L.T., $\bar{X} \approx N(\mu, \sigma^2/n)$

$\approx N\left(0, \frac{2}{3 * 50}\right)$

$$f(x) = \begin{cases} \frac{x}{4} + \frac{1}{2} & -2 \leq x \leq 0 \\ -\frac{x}{4} + \frac{1}{2} & 0 \leq x \leq 2 \end{cases}$$

$$\sigma^2 = E(X^2) - \mu^2 = E(X^2)$$

$$= \int_{-2}^{0} x^2 \cdot \left(\frac{x}{4} + \frac{1}{2}\right) dx + \int_{0}^{2} x^2 \left(-\frac{x}{4} + \frac{1}{2}\right) dx$$

$$= \cdots = 2/3$$

$$P\left(-0.2 < \bar{X} < 0.2\right)$$

$$\approx P\left(\frac{-0.2 - 0}{\sqrt{2/150}} < Z < \frac{0.2 - 0}{\sqrt{2/3}/\sqrt{50}}\right)$$

$$= P(-1.73 < Z < 1.73)$$

$$= 0.9582 - 0.0418 = 0.9164$$

§6.5 Normal approx to Binomial r.v.

Ex: Toss a fair coin 10 times, what's the prob of getting 4, 5, or 6 heads?

$$\sum_{x=4}^{6} \binom{10}{x} \frac{1}{2^{10}} = 0.6562$$

Ex 2. 1000 times, 480 — 520 heads? (inclusive)

$X \sim Bin(1000, 0.5)$ $\mu = p$ $\sigma^2 = pq$

$X_1, X_2, \cdots X_n \sim Bernoulli(p)$

$Y = X_1 + X_2 + \cdots + X_n \sim B(n, p)$

$\overset{CLT}{\approx} N(np, npq)$ if np & $nq \geq 5$.

$B(1000, 0.5) \approx N(500, 250)$

$$P(480 \leq X \leq 520) \approx P\left(\frac{\cancel{480}\ 479.5 - 500}{\sqrt{250}} < Z \leq \frac{\cancel{520}\ 520.5 - 500}{\sqrt{250}}\right)$$

$= P(-1.26 < Z < 1.26) = 0.8962 - 0.1038$

$= 0.7924$

$X \sim Bin(10, .5) \approx N(5, 2.5)$

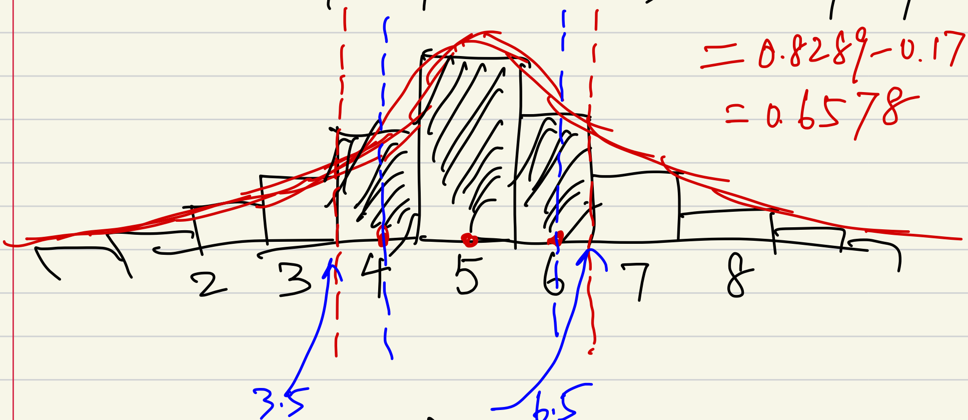
$P(4 \le X \le 6) = P\left(\frac{\cancel{4}^{\,3.5}-5}{\sqrt{2.5}} \le Z \le \frac{\cancel{6}^{\,6.5}-5}{\sqrt{2.5}}\right)$

$= P(\cancel{-0.63}^{\,-0.95} < Z < \cancel{0.63}^{\,0.95})$

~~$= 0.7357 - 0.2643 = 0.4714$~~

$= 0.8289 - 0.1711$

$= 0.6578$



$P(4 < X \le 7) \approx P\left(\frac{4.5-5}{\sqrt{2.5}} < Z < \frac{7.5-5}{\sqrt{2.5}}\right)$

$= P(-0.32 < Z < 1.58)$

$= 0.9429 - 0.3745$

$= 0.5684$