13.1

② $\vec{r}(t) = \cos t\,\hat{\imath} + \ln t\,\hat{\jmath} + \frac{1}{t-2}\,\hat{k}$

find domain:

$t > 0$ $\quad$ $t \neq 2$

$(0,2) \cup (2,\infty)$.

Q: what does a vector $\vec{r}(t) = \langle x(t), y(t), z(t) \rangle$ really do?

Traces out curve in $\mathbb{R}^3$

④ $\lim_{t \to 1} \left(\frac{t^2 - t}{t-1}\,\hat{\imath} + \sqrt{t+8}\,\hat{\jmath} + \frac{\sin \pi t}{\ln t}\,\hat{k} \right)$

$= \underbrace{\lim_{t \to 1} \left(\frac{t^2 - t}{t-1} \right)}_{\cancel{t(t-1)}}\hat{\imath} + \underbrace{\left(\lim_{t \to 1} \sqrt{t+8} \right)}\hat{\jmath} + \underbrace{\left(\lim_{t \to 1} \frac{\sin \pi t}{\ln(t)} \right)}_{\frac{0}{0}}\hat{k}$

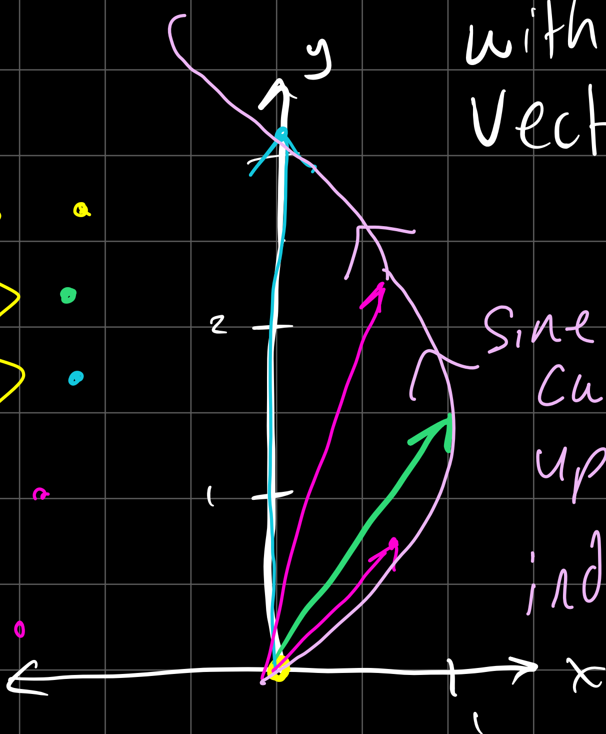
$\cancel{t-1}$

$$= (1)\hat{\imath} + 3\hat{\jmath} + \Big(\underbrace{\lim_{t\to 1} \frac{\pi\cos\pi t}{1}}_{-\pi}\Big)\hat{k}$$

$$= \hat{\imath} + 3\hat{\jmath} - \pi\hat{k}$$

(7) $\vec{r}(t) = \langle \sin t, t \rangle$ Sketch curve with the given Vector equation.

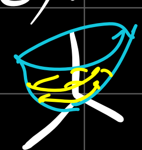
t	$\vec{r}(t)$
0	$\langle 0, 0 \rangle$
$\frac{\pi}{2}$	$\langle 1, \pi/2 \rangle$
π	$\langle 0, \pi \rangle$
$\frac{\pi}{4}$	$\langle \frac{1}{\sqrt{2}}, \frac{\pi}{4} \rangle$
$\frac{3\pi}{4}$	$\langle \frac{1}{\sqrt{2}}, \frac{3\pi}{4} \rangle$



Sine curve up y-axis.

indicate direction

(31) $\vec{r}(t) = t\hat{\imath} + (2t - t^2)\hat{k}$



Paraboloid.

$z = x^2 + y^2$

where does path intersect surface.

$x = t \qquad y = 0 \qquad z = 2t - t^2$

put into $z = x^2 + y^2$

$$2t - t^2 = t^2 + 0^2$$

$$\cancel{2}t = \cancel{2}t^2 \Rightarrow t^2 - t = 0$$

$$t(t-1) = 0$$

$$t = 0, \; t = 1$$

$\vec{r}(t) = t\hat{i} + (2t - t^2)\hat{k}$

$$\vec{r}(0) = \vec{0}$$

$$\vec{r}(1) = \hat{i} + \hat{k}$$

Intersection occurs at $(0,0,0)$ & $(1,0,1)$.

(43) $z = \sqrt{x^2 + y^2}$ - cone

$z = 1 + y$ - plane

find $\vec{r}(t)$ that describes intersection

$$1 + y = \sqrt{x^2 + y^2}$$

let $x=t$ find y in terms of t.

$$1+y=\sqrt{t^2+y^2}$$

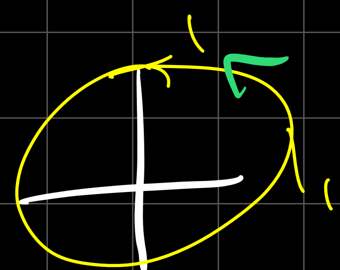
$$1+2y+\cancel{y^2}=t^2+\cancel{y^2} \Rightarrow y=\frac{t^2-1}{2}$$

$$z=1+y=1+\frac{t^2-1}{2}=\frac{t^2+1}{2}$$

$$\vec r(t)=\left\langle t, \frac{t^2-1}{2}, \frac{t^2+1}{2}\right\rangle$$

Could have let $y=t$ or $z=t$

note:



$\vec r(t)=\langle \cos t, \sin t\rangle$

$\vec r(t)=\langle \cos(2t), \sin(2t)\rangle$

$\vec r(t)=\langle \cos(t^2), \sin(t^2)\rangle$

(49) $\vec r_1(t)=\langle t^2, 7t-12, t^2\rangle$

$\vec r_2(t)=\langle 4t-3, t^2, 5t-6\rangle$

Do the particles ever collide?

$t^2 = 4t - 3 \Rightarrow t^2 - 4t + 3 = 0$ $\quad t = 1, 3$

$(t-3)(t-1)$

$7t - 12 = t^2$ $\quad t^2 - 7t + 12 = 0$ $\quad t = 3, 4$

$(t-4)(t-3)$

$t^2 = 5t - 6$ $\quad t^2 - 5t + 6 = 0$ $\quad t = 2, 3$

$(t-2)(t-3)$

collide when $t = 3$

at $\vec{r}(3) = \langle 9, 9, 9 \rangle$

which gives the point $(9, 9, 9)$.

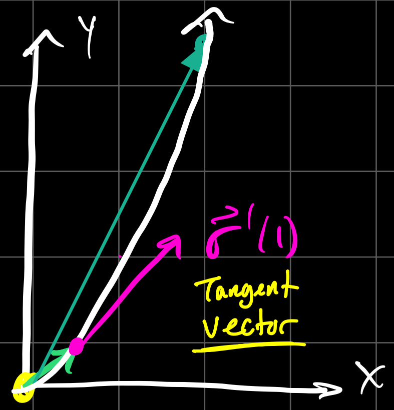
13.2

(4) $\vec{r}(t) = \langle t^2, t^3 \rangle$

find $\vec{r}'(t) = \langle 2t, 3t^2 \rangle$

$\vec{r}'(1) = \langle 2, 3 \rangle$

Sketch $\vec{r}(t), \vec{r}'(1)$.



t	$\vec{r}(t)$
0	$\langle 0,0 \rangle$
1	$\langle 1,1 \rangle$
2	$\langle 4,8 \rangle$

To find unit tangent vector at $t=1$,

$$\frac{\vec{r}\,'(1)}{\|\vec{r}\,'(1)\|} = \frac{\langle 2,3 \rangle}{\sqrt{2^2+3^2}} = \frac{\langle 2,3 \rangle}{\sqrt{13}}$$

$$= \left\langle \frac{2}{\sqrt{13}}, \frac{3}{\sqrt{13}} \right\rangle$$

(40) $\displaystyle\int \left(te^{2t}\,\hat{\imath} + \frac{t}{1-t}\,\hat{\jmath} + \frac{1}{\sqrt{1-t^2}}\,\hat{k} \right) dt$

$$= \underbrace{\int te^{2t}\,dt}_{\text{IBP}}\,\hat{\imath} + \Big(\underbrace{\int \frac{t}{1-t}\,dt}_{u=1-t}\Big)\hat{\jmath} + \Big(\underbrace{\int \frac{1}{\sqrt{1-t^2}}\,dt}_{\text{trig sub or use } \frac{d}{dt}\arcsin t = \frac{1}{\sqrt{1-t^2}}}\Big)\hat{k}$$

$\hat{\imath}$: $u = t \qquad dv = e^{2t}\,dt$

$du = dt \qquad v = e^{2t}/2$

$$\int te^{2t}\,dt = \frac{te^{2t}}{2} - \underbrace{\frac{1}{2}\int e^{2t}\,dt}_{\frac{e^{2t}}{4}} = \frac{te^{2t}}{2} - \frac{e^{2t}}{4} + C_1$$

$$\hat{j}: \int \frac{t}{1-t}\,dt = \int \frac{1-u}{u}(-du) = \int \frac{u-1}{u}\,du = \int 1 - \frac{1}{u}\,du$$

$$= u - \ln|u| + C_2$$

$u = 1-t \rightarrow t = 1-u$

$du = -dt$

$$= (1-t) - \ln|1-t| + C_2$$

$$\hat{k}: \int \frac{1}{\sqrt{1-t^2}}\,dt = \arcsin t + C_3$$

$$\int \left(te^{2t}\hat{i} + \frac{t}{1-t}\hat{j} + \frac{1}{\sqrt{1-t^2}}\hat{k} \right) dt$$

$$= \left(\frac{te^{2t}}{2} - \frac{e^{2t}}{4} + C_1 \right)\hat{i} + \left((1-t) - \ln|1-t| + C_2 \right)\hat{j} + (\sin^{-1} t + C_3)\hat{k}$$

$$= \left(\frac{te^{2t}}{2} - \frac{e^{2t}}{4} \right)\hat{i} + (1-t-\ln|1-t|)\hat{j} + \sin^{-1} t\,\hat{k} + \vec{C}$$

$\langle C_1, C_2, C_3 \rangle$