Girsanov 练习题

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1. 考虑 $\mathbb{1}_{z_1>10}\cdot\mathbb{1}_{z_2>10}\cdots+\mathbb{1}_{N\in 2^n}$

取 $N(\xi)\to P(z>10)$

考虑 $P(z>10)=\int_{10}^{+\infty}\frac{1}{\sqrt{2\pi}}e^{-\frac{z^2}{2}}dz=\frac{1}{10\sqrt{2\pi}}e^{-50}$

$$\le\int\frac{z}{10}\frac{1}{\sqrt{2\pi}}e^{-\frac{z^2}{2}}dz$$

$$\le\frac{1}{\sqrt{2\pi}\cdot 10}\cdot e^{-50}.$$

Girsanov to rescue!

$$\tilde{P}(Z\in dz)=e^{az-\frac{a^2}{2}}P(Z\in dz)$$

$N(a,1)\leftarrow N(0,1)$

取 $a=10$

有 $\tilde{P}(Z>10)=1/2$

且 $P(Z\in dz)=e^{-az+\frac{a^2}{2}}\tilde{P}(Z\in dz)$

则 $E(\mathbb{1}_{Z>a})=\tilde{E}(e^{-aZ+\frac{a^2}{2}}\cdot\mathbb{1}_{Z>a})$

考虑 $Z\sim N(a,1)=Y+a$, $Y\sim N(0,1)$ under $\tilde{P}$

$\therefore E(\mathbb{1}_{Z>a})=\tilde{E}(e^{-a(Y+a)+a^2/2}\mathbb{1}_{Y+a>a})$

$$=e^{-\frac{a^2}{2}}\tilde{E}(e^{-aY}\mathbb{1}_{Y>0}).$$

$$= e^{-\frac{a^2}{2}} \tilde{\mathbb{E}}\left(e^{-aY} \mathbb{1}_{Y>0}\right).$$

$$= e^{-\frac{a^2}{2}} \cdot \left[e^{-aY_1} \mathbb{1}_{Y_1>0} + e^{-aY_2} \mathbb{1}_{Y_2>0} \right]$$