Compound Poisson Process

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11.3.1 Construction of a Compound Poisson Process

$N_t \sim PP(\lambda)$. iid $Y_1, Y_2, \cdots$

Let $N(t)$ be a Poisson process with intensity λ, and let $Y_1, Y_2, \ldots$ be a sequence of identically distributed random variables with mean $\beta = \mathbb{E}Y_i$. We assume the random variables $Y_1, Y_2, \ldots$ are independent of one another and also independent of the Poisson process $N(t)$. We define the *compound Poisson process*

$$Q(t) = \sum_{i=1}^{N(t)} Y_i, \quad t \geq 0. \tag{11.3.1}$$

jump 与 jump sizes 可能有关！

* $\mathbb{P}(Q_t < a)$ 不可解.

则考虑 char. function. $\varphi_{Q_t}(u) = \mathbb{E}e^{iuQ_t} = \mathbb{E}\, e^{iu\sum_{k=1}^{N_t} Y_k}$

$$= \sum_{m=0}^{\infty} \mathbb{E}\left(e^{iu\sum_{k=1}^{N_t} Y_k}\right) \cdot \mathbb{P}(N_t = m)$$

考虑 $\mathbb{E}\left(e^{iu\sum_{k=1}^{N_t} Y_k}\right) = \prod_{k=1}^{n} \underbrace{\mathbb{E}\, e^{iuY_k}}_{\varphi_Y(u)} = \left[\varphi_Y(u)\right]^n$

$$\therefore \varphi_{Q_t}(u) = e^{\varphi_Y(u)\lambda t - \lambda t} = e^{(\varphi_Y(u) - 1)\lambda t}$$