Stochastic & PDE

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Monte Carlo 收敛速度 $\frac{1}{\sqrt{N}}$ $\Rightarrow$ 为1% 误差需 $N > 10000$

JP Morgan engine for 3-Dimensional PDEs

$\mathbb{E}[\text{Payoff}] \longleftrightarrow$ PDE

Lemma 1: R.V. X on $(\Omega, \mathcal{F}, \mathbb{P})$. filtration $(\mathcal{F}_t)_{t\ge0}$

则有 $Z_t = \mathbb{E}(X \mid \mathcal{F}_t)$ is a martingale w.r.t. $\mathcal{F}_t$

证明: Tower Property

Cor. 考虑 X_t 为 Markov Process, $\mathcal{F}_t = \sigma(X_s, s \le t)$

Fix T, function V (pay off).

则 $\mathbb{E}[V(X_T) \mid \mathcal{F}_t]$ 也为 martingale.

则 $\mathbb{E}[V(X_T) \mid X_t]$ 也 martingale.

则 令其为 $f(t, X_t)$ 有 $df = \sigma\, dX_t$ $\boxed{\text{Driftless!}}$

例 $dX_t = a(t, X_t)dt + \sigma(t, X_t)dB_t$

2维定价 u. 费曼卡茨 $u(t,x) = \mathbb{E}_{X_t = x}\left[V(X_T)\right]$ 亦可写作 $\mathbb{E}\left[V(X_T) \mid X_t = x\right]$

PDE $\begin{cases} u_t + a(t,x)u_x + \frac{1}{2}\sigma^2(t,x)u_{xx} = 0 \\ u(T,x) = V(x). \quad \text{B.C.} \end{cases}$

证明: 考虑 $du(t, X_t) = u_t dt + u_x dX_t + \frac{1}{2}u_{xx}(dX_t)^2$

$= u_t dt + u_x(a dt + \sigma dB_t) + \frac{1}{2}u_{xx}\sigma^2 dt.$

drift $= \left[u_t + a u_x + \frac{1}{2}u_{xx}\sigma^2\right] dt.$

考虑 $u(T,x) = \mathbb{E}_{X_T = x} V(X_T) = V(x)$