Introduction to Robot Intelligence
[Spring 2023]

Optimization and Math 101

January 26, 2023

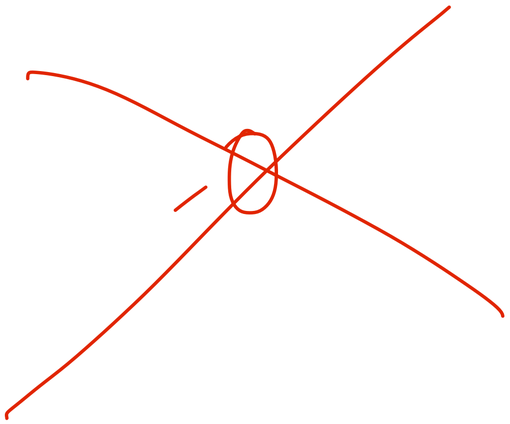
Lerrel Pinto

Some class logistics

- Office hours:
    - Lerrel (Tues 5-6pm @ 60 Fifth Ave, Room 505)
    - Ulyana (Thurs 1-2pm @ 60 Fifth Ave, Room 540)
- Late day policy for HWs:
    - No late days / No extensions granted (unless approved by Dean)

Goal for today: A soft introduction to optimization + Essential math for this class

Solve this:



- Problem 1:
  - Given: $x_1 + x_2 = 7; x_1 + 2x_2 = 11$. What is x_1 and x_2?

$x_1 = 7 - x_2$

Solve this:

- Problem 1:
  - Given: $x_1 + x_2 = 7; x_1 + 2x_2 = 11$. What is x_1 and x_2?
- Problem 2:
  - Given m linear equations with n variables.What is the value of each variable?

$x_1, x_2, \ldots x_n$

$f_{g_1} \quad \sum_{j=1}^{n} a_j x_j = b_i$

Solve this:

- Problem 3:
  - Given: $y = x_1^2 + 2x_1 + 4$. What is the smallest possible value of y?

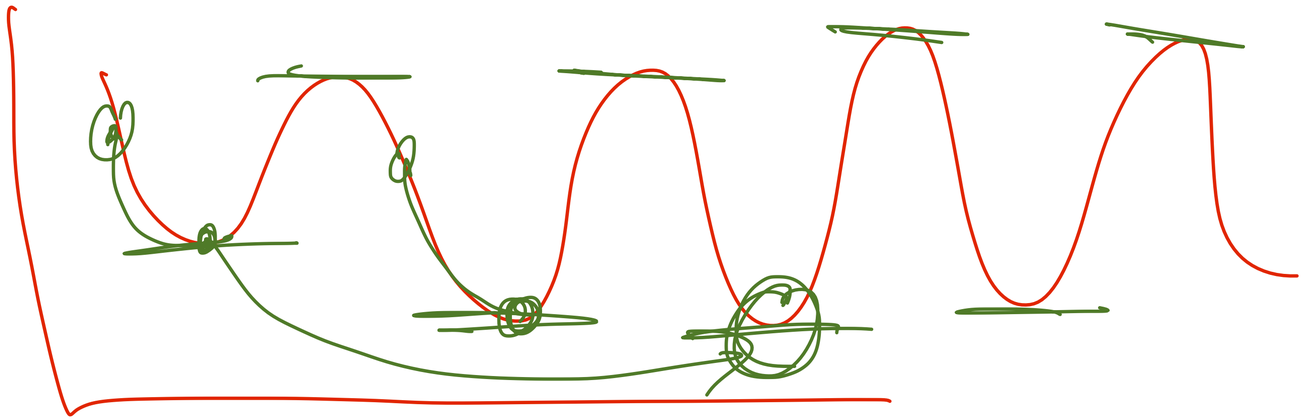
$\frac{dy}{dx_1} = 2x_1 + 2 = 0 \rightarrow x_1 = -1$ $\quad y = 3$

Solve this:

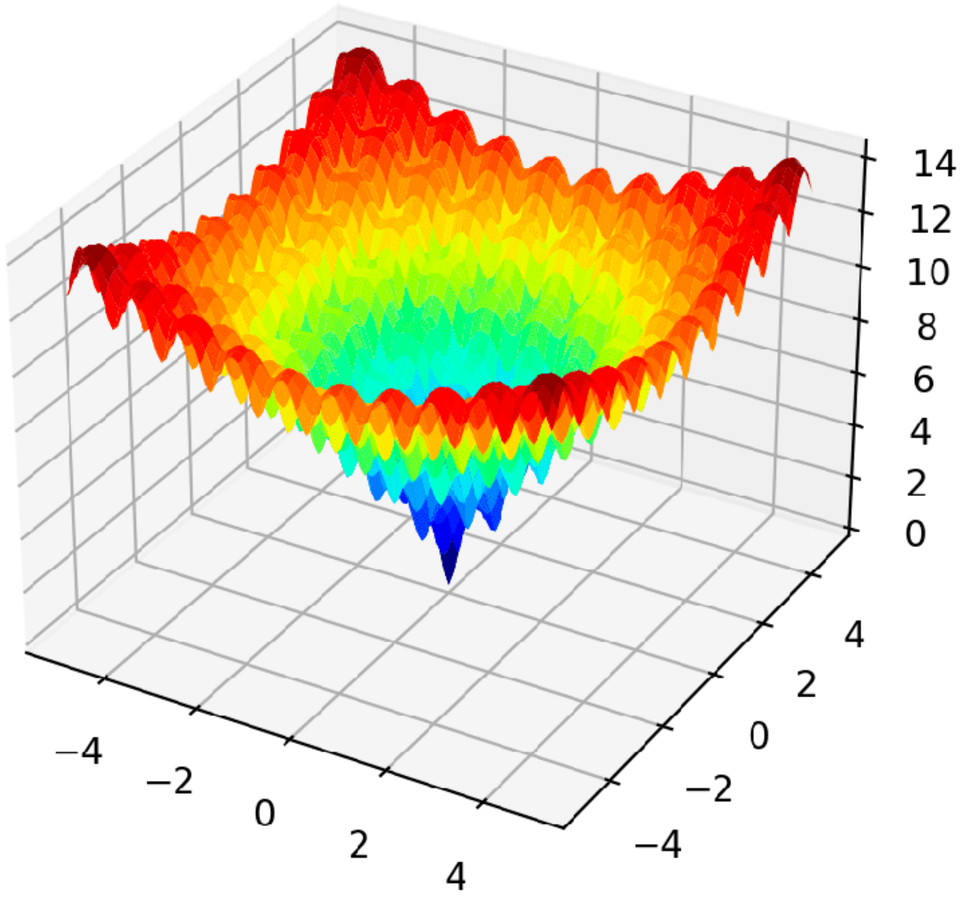
- Problem 3:
  - Given: $y = x_1^2 + 2x_1 + 4$. What is the smallest possible value of y?
- Problem 4:
  - Given: $y = x_1^2 + x_2^2 + 4x_1x_2 + 2x_1 + 2x_2 + 4$. What is the smallest possible value of y?

Solve this:

- Problem 5:
  - Given: $y = -20e^{-0.2\sqrt{0.5(x_1^2 + x_2^2)}} - e^{0.5(cos(2\pi x_1)+cos(2\pi x_2))} + e + 20$. Find smallest value of y?



Solve this:

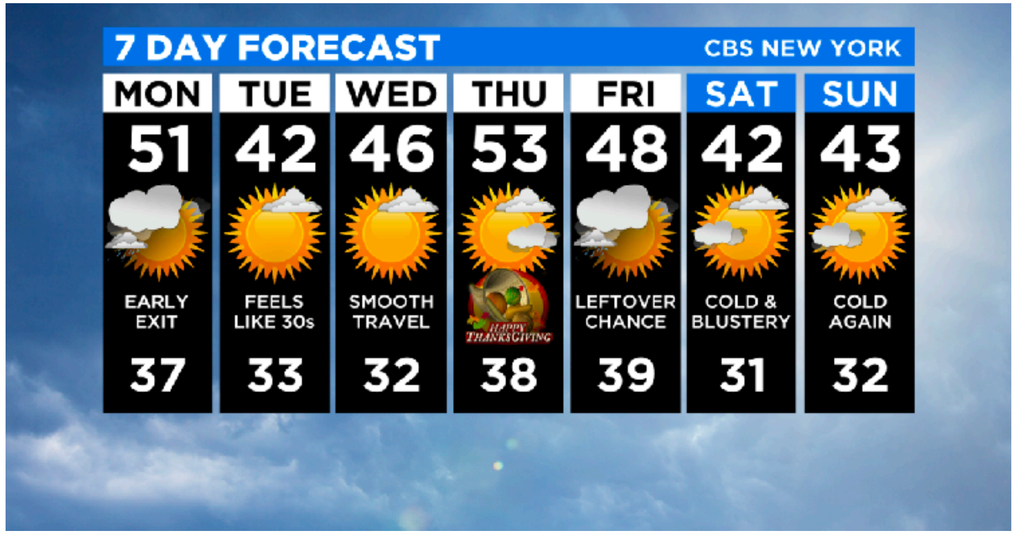


Some takeaways

- For some class of problems, we have methods to solve exactly
    - System of linear equations, quadratic objectives, etc.
- In general, these exact solutions are hard to compute exactly.

What if you do not have the analytic equation?

Financial markets



Weather models

Image recognition

$y \rightarrow$ 'catness'

$x \rightarrow$ 

How much water to give your plant?

x mL of water / day

$x \geq 0$

$x \leq M$

How long plant will live (days)

y

maximize y

$y = f(x)$

maximize / minize Objective

s.t

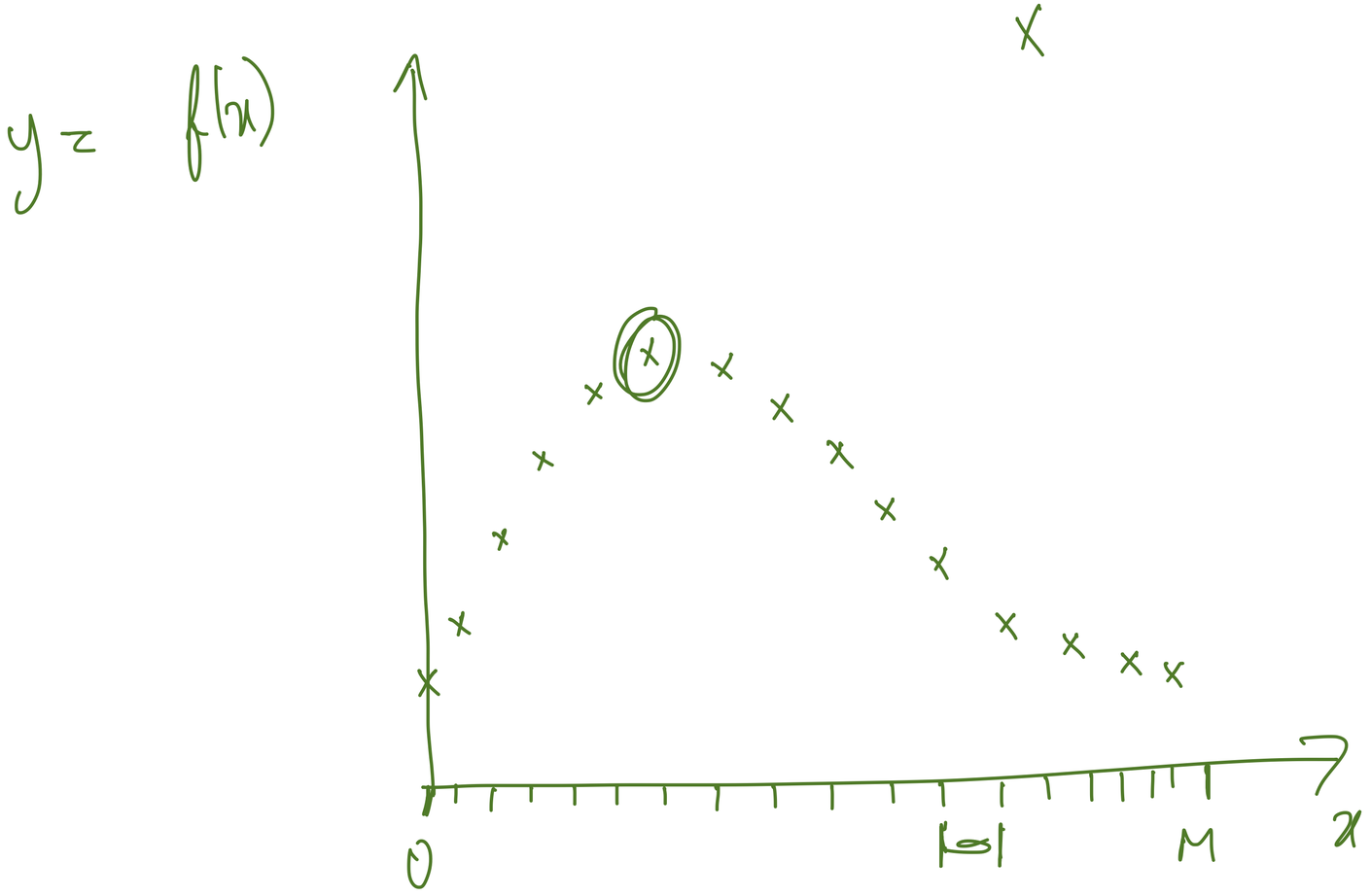
$x \geq 0$

$x \leq M$

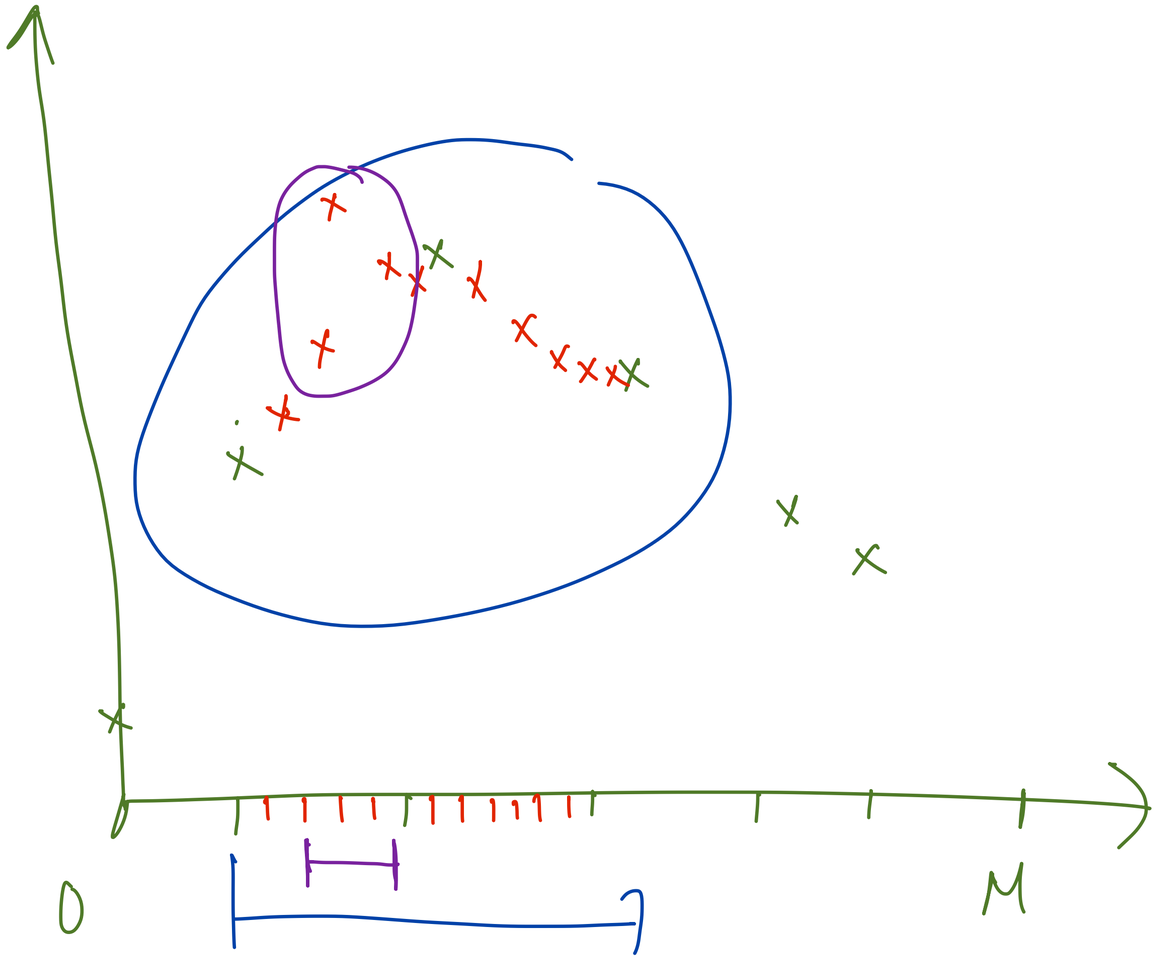
~~$x + e^{x} + \cos(x) \geq 5$~~

Plant watering as an optimization problem

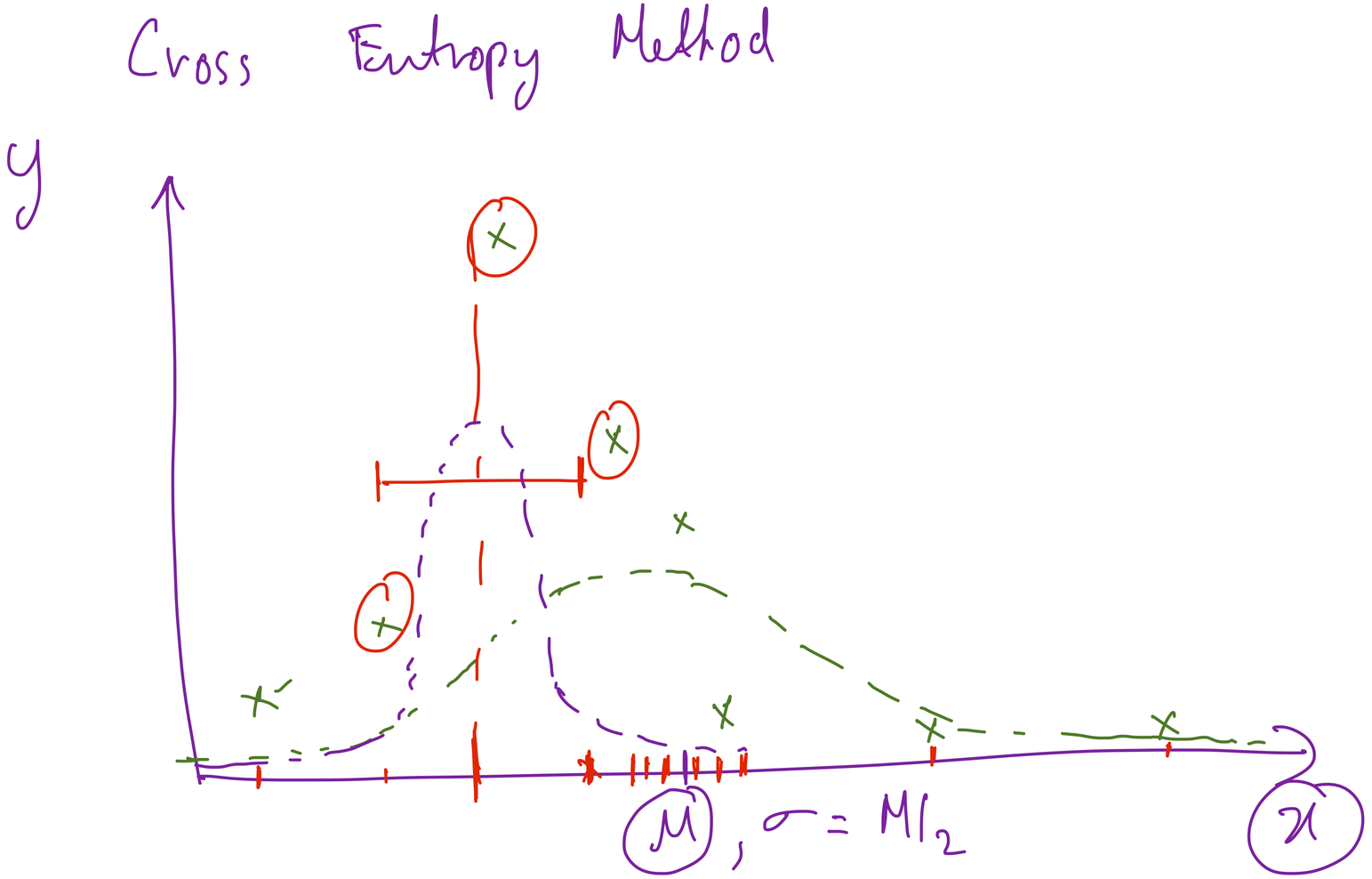
Case 0: Brute-force search



Case 1: No analytic model of the plant



Black-box / Derivative free optimization.



Cross Entropy Method

Initialize $\mu \in \mathbb{R}^d, \sigma \in \mathbb{R}^d$

for iteration $= 1, 2, \ldots$ **do**

Collect n samples of $\theta_i \sim N(\mu, \text{diag}(\sigma))$

Perform a noisy evaluation $R_i \sim \theta_i$

Select the top $p\%$ of samples (e.g. $p = 20$), which we'll call the **elite set**

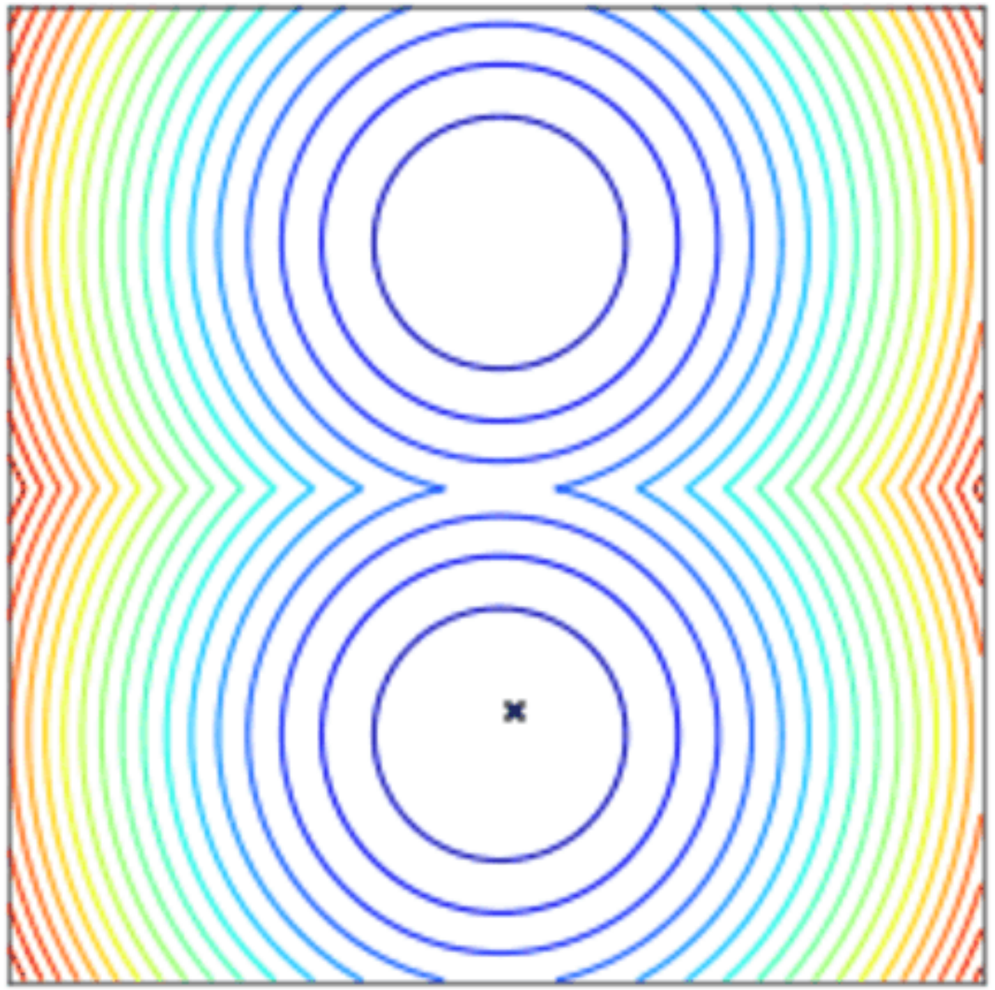
Fit a Gaussian distribution, with diagonal covariance, to the elite set, obtaining a new μ, σ.

end for

Return the final μ.

Slide credits: MLSS 2016 on Deep Reinforcement Learning by John Schulman

Visualizing DFO with two variables



Essential Math

Space, Points, and Vectors

Properties of Vectors

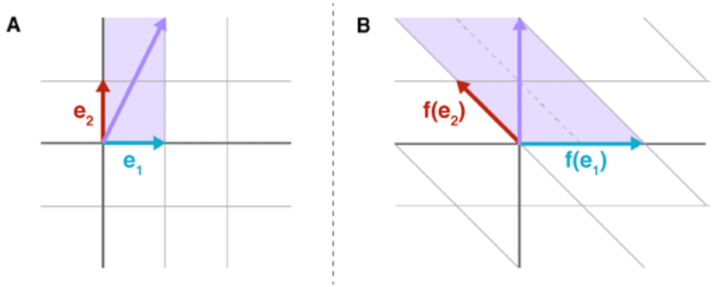
Multiple Dimensions

Functions

Linear Transformations

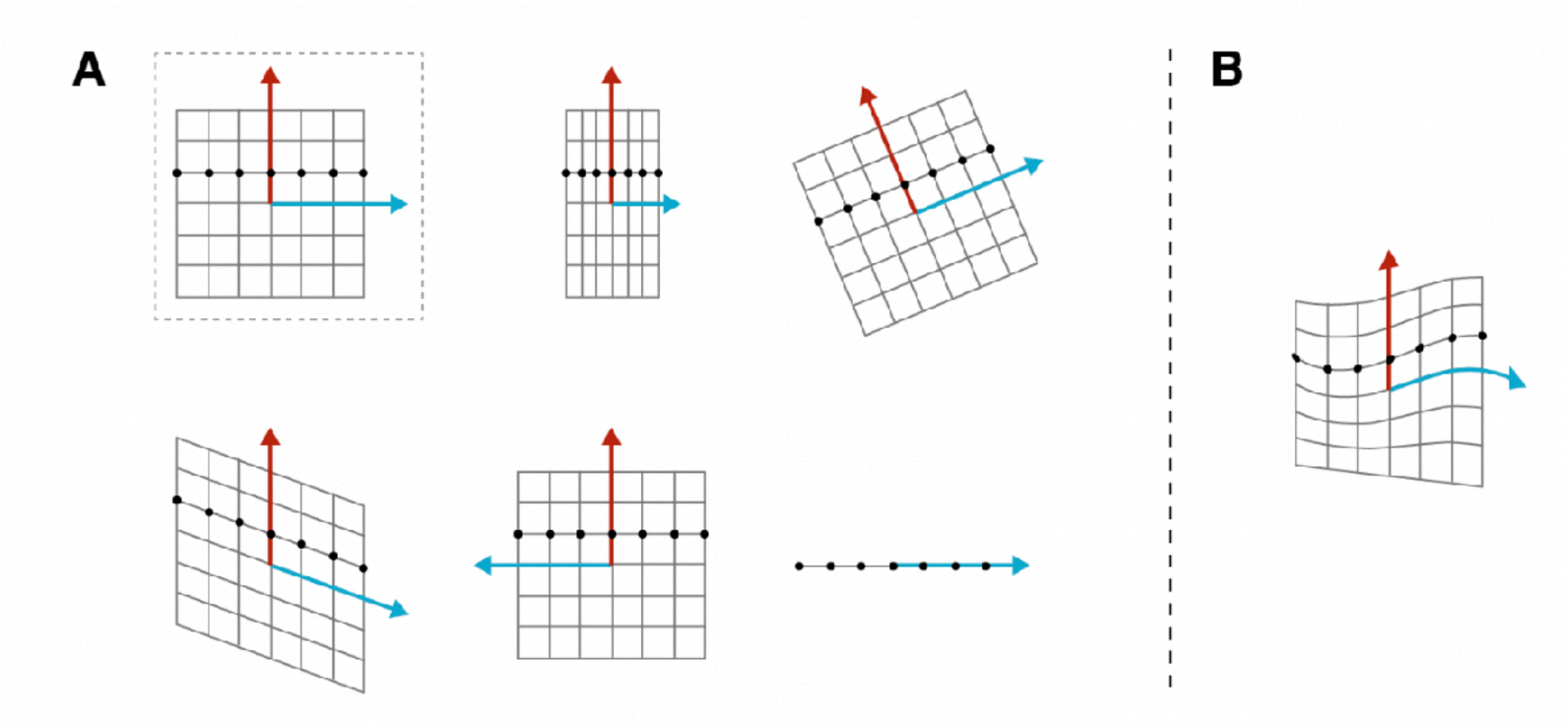
Operations on Matrices

$$\overbrace{\begin{bmatrix} 2 & -1 \\ 0 & 1 \end{bmatrix}}^{A} \overbrace{\begin{bmatrix} 1 \\ 2 \end{bmatrix}}^{\mathbf{x}} = \overbrace{\begin{bmatrix} 0 \\ 2 \end{bmatrix}}^{f(\mathbf{x})}$$



Credits: https://gregorygundersen.com/blog/2018/10/24/matrices/

Operations on Matrices



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Operations on Matrices

Operations on Matrices

Additional Reading

- Cross entropy method: https://people.smp.uq.edu.au/DirkKroese/ps/aortut.pdf
- Intro to Optimization: http://www.lewissoft.com/pdf/INTRO_OPT.pdf
- Linear Algebra Blog from Gregory Gundersen (https://gregorygundersen.com/blog/tags/la/)
- Mathematics for Machine Learning e-book by Deisenroth, Faisal, Ong (https://mml-book.github.io/book/mml-book.pdf)

Questions?