$$\binom{n}{k} = \frac{n\cdot(n-1)\cdot\cdots\cdot(n-k+1)}{k!}$$

$$= \frac{n!}{(n-k)!\,k!}$$

$$\binom{50}{3} = \frac{50\cdot 49\cdot 48}{3!}$$

$$\binom{50}{47} = \binom{50}{3}$$

Ex: 35 people. 7 people in front row.
Randomly choose 5 from 35. What's the prob ① all 5 are in the front row?

$$\frac{\binom{7}{5}\binom{28}{0}}{\binom{35}{5}} = \frac{7\cdot 6\cdot 5\cdot 4\cdot 3/5!}{35\cdot 34\cdot 33\cdot 32\cdot 31/5!}$$

② 2 are from front row, other 3 are not?

$$\frac{\binom{7}{2}\binom{28}{3}}{\binom{35}{5}}$$

Ex: A hand of 5 out of a standard deck of cards.

① $P(3\ 5\text{'s} \ \&\ 2\ \text{Aces}) = \dfrac{\binom{4}{3}\binom{4}{2}}{\binom{52}{5}}$

② $P(\text{a triple} \ \&\ \text{a pair}) = \dfrac{13\binom{4}{3}\,12\cdot\binom{4}{2}}{\binom{52}{5}}$

③ $P(\text{a triple} \ \&\ 2\ \text{differet cards})$

5 5 5 7 10

$$= \frac{\binom{13}{1}\binom{4}{3}\ \binom{12}{1}\binom{11}{1}\binom{4}{1}\binom{4}{1}/2!}{\binom{52}{5}}$$

$$= \frac{\binom{13}{1}\binom{4}{3}\binom{12}{2}\binom{4}{1}\binom{4}{1}}{\binom{52}{5}}$$

④ $P(2\ \text{pairs}) = \dfrac{\binom{13}{2}\binom{4}{2}\binom{4}{2}\binom{11}{1}\binom{4}{1}}{\binom{52}{5}}$

5 5 8 8 Q

$$= \frac{\binom{13}{1}\binom{4}{1}\binom{12}{2}\binom{4}{2}\binom{4}{2}}{\binom{52}{5}}$$

⑤ $P(1\ \text{pair}) = \dfrac{\binom{13}{1}\binom{4}{2}\binom{12}{3}\binom{4}{1}^3}{\binom{52}{5}}$

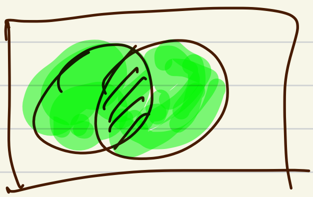
AA 4 6 7

$$\frac{\binom{13}{1}\binom{4}{2}\binom{12}{1}\binom{4}{1}\binom{11}{1}\binom{4}{1}\binom{10}{1}\binom{4}{1}/3!}{\binom{52}{5}}$$

Additive Rule & Product Rule.

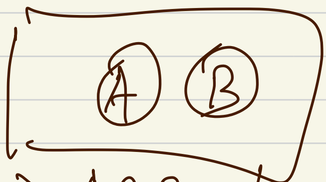
- Additive Rule (union)

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$



when A & B are mutually exclusive,

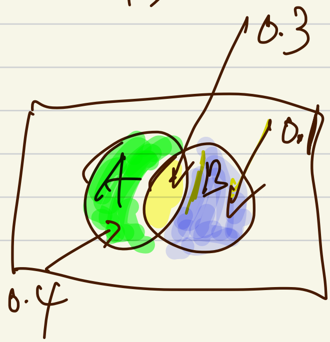
$$P(A \cup B) = P(A) + P(B)$$



mutually exclusive $\Longleftrightarrow A \cap B = \emptyset$

Ex: $P(A) = 0.7$ $P(B) = 0.4$, $P(A \cap B) = 0.3$

$P(A \cup B) = ?$ 0.8



Product Rule (intersection)

Ex: Draw 2 cards consequtively from a deck. what's the prob the 1st is a spades, the 2nd is a heart?

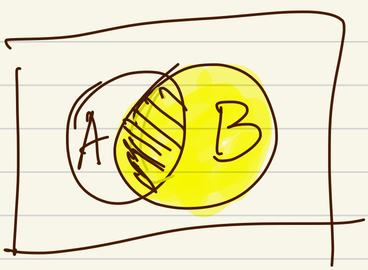
$$\frac{13}{52} \cdot \frac{13}{51}$$

A: Drawing a spades. B: heart.

$\underline{P(A \cap B)}$

Conditional Prob of A given B.

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$



$$P(A\cap B) = P(B)\cdot P(A|B)$$

$$P(A\cap B) = P(A)\cdot P(B|A)$$

A & B are independent means knowing A happen or not doesn't affect the prob of B.

A & B are independent $\Longleftrightarrow$ $P(A|B) = P(A)$

$P(B|A) = P(B)$

$\Updownarrow$

$$P(A\cap B) = P(A)\cdot P(B)$$

Ex. A & B are independent.

$P(A) = 0.8 \quad P(B) = 0.4$

$P(A\cup B) = ?\ 0.8 + 0.4 - 0.32$

$= 0.88$

$P(B)$

or: $P(A\cap B) = P(A)\cdot P(B|A) = 0.8\cdot 0.4$

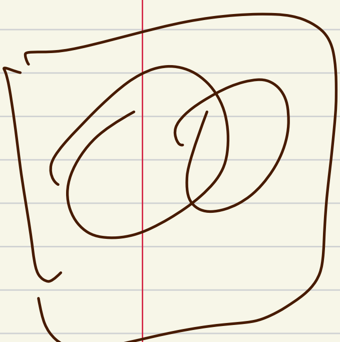
$= 0.32$

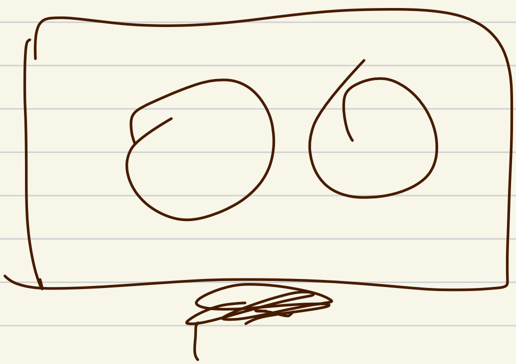
$$P(B|A) = \frac{P(A\cap B)}{P(A)}$$

Ex: $P(A) = 0.8$. $P(B) = 0.5$

$P(A|B) = 0.6$.

$$P(A\cup B) = P(A) + P(B) - P(A\cap B)$$
$$= 0.8 + 0.5 - P(B)\cdot P(A|B)$$
$$= 1.3 - 0.5 \cdot 0.6 = 1.$$





Ex: Consecutively pick 2 cards, what's the prob the 1st is an ace, & the 2nd is a spades?

$$\frac{1}{52}\cdot\frac{12}{51} + \frac{3}{52}\cdot\frac{13}{51}$$

Ex:

A: 5 red, 3 blue

1st step randomly transfer 1 → 2

B: 10 red, 3 blue

2nd step pick a ball. 2

P(the ball picked from B is blue)

1 blue / 1 red

$$\frac{5}{8}\cdot\frac{3}{14} + \frac{3}{8}\cdot\frac{4}{14} = \frac{27}{112}$$

Ex: $\boxed{\begin{array}{c}8 \text{ blue} \\ 5 \text{ red}\end{array}}$

① Randomly pick 5, what's the prob more red than blue.

$$= P(3 \text{ red}, 2 \text{ blue}) + P(4 \text{ red}, 1 \text{ blue}) + P(5 \text{ red})$$

$$= \frac{\binom{5}{3}\binom{8}{2} + \binom{5}{4}\binom{8}{1} + \binom{5}{5}\binom{8}{0}}{\binom{13}{5}}$$

② What's the prob the 5th ball picked out is the 1st red ball?

$$\frac{8}{13} \cdot \frac{7}{12} \cdot \frac{6}{11} \cdot \frac{5}{10} \cdot \frac{5}{9} = \frac{\binom{8}{4}}{\binom{13}{4}} \cdot \frac{5}{9}$$

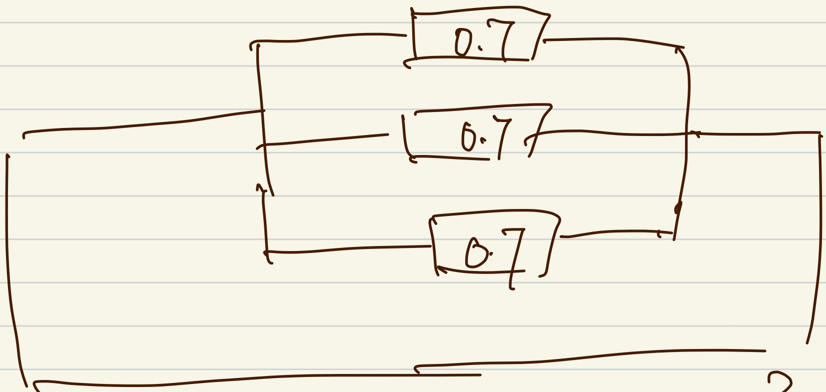
③ What's the prob the 5th ball pick out is the 3rd red?

$$\frac{\binom{8}{2}\binom{5}{2}}{\binom{13}{4}} \cdot \frac{3}{9}$$

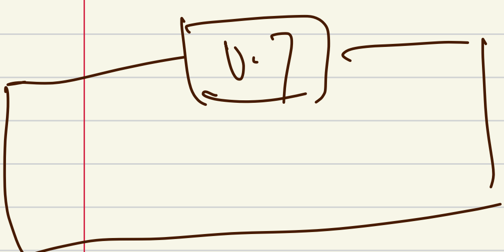
Independent events.

$$P(A \cap B) = P(A) \cdot P(B)$$

Ex:



$$P(\text{system functions}) = 1 - 0.3^3$$
$$= 1 - 0.027$$
$$= 0.973$$

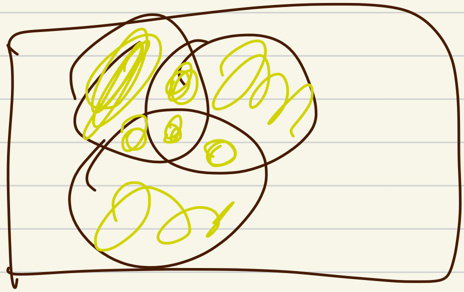


Ex: A puzzle to A, B, C. independently

$P(A) = 0.4$
$P(B) = 0.3$
$P(C) = 0.2$

① P(puzzle will be solved) =

$1 - 0.6 \cdot 0.7 \cdot 0.8 = 0.664$



② P(exactly 1 person will solve the puzzle)

$= 0.4 \cdot 0.7 \cdot 0.8 + 0.3 \cdot 0.6 \cdot 0.8$
$+ 0.2 \cdot 0.6 \cdot 0.7$
$= 0.224 + 0.144 + 0.084$

$= 0.452$

(3) P(exactly 2 people solve the puzzle)

$= 0.4 \cdot 0.3 \cdot 0.8 + 0.4 \cdot 0.2 \cdot 0.7$

$+ 0.3 \cdot 0.2 \cdot 0.6$

$= 0.096 + 0.056 + 0.036$

$= 0.188$

(4) P(all 3 solve) $= 0.4 \cdot 0.3 \cdot 0.2$

$= 0.024$