Homework 1 Solutions

Due: Friday Sept. 17, by 11:59pm,
via Gradescope

- Failure to submit homework correctly will result in a zero on the homework.
- Homework must be in LaTeX. Submit the pdf file to Gradescope.
- Problems assigned from the textbook come from the 5th edition.
- No late homework accepted. Lateness due to technical issues will not be excused.

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1. (6 points) Show the logical equivalences using truth tables and say a few words explaining why your truth table shows *equiv*

(a) $\sim (p \vee q) \equiv \sim p \wedge \sim q$ (DeMorgan's law)

Solution : Let's set up the truth table. Note that I will number the columns and rows so that I can refer to them if need be. I encourage you to do the same.

	1	2	3	4	5	6	7
1	p	q	$p \vee q$	$\sim (p \vee q)$	$\sim p$	$\sim q$	$\sim p \wedge \sim q$
2	T	T	T	F	F	F	F
3	T	F	T	F	F	T	F
4	F	T	T	F	T	F	F
5	F	F	F	T	T	T	T

DeMorgan's law holds since column 4 = column 7.

Remark. *Everyone should have 7 columns. Your first two columns should be the statement variables. The order in which my columns 3-7 are written may not necessarily be order in which your columns are written. That is OK.*

(b) $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$ (Distributive law). Note that your truth table will have 8 rows

Solution :

	1	2	3	4	5	6	7	8
1	p	q	r	$q \wedge r$	$p \vee (q \wedge r)$	$p \vee q$	$p \vee r$	$(p \vee q) \wedge (p \vee r)$
2	T	T	T	T	T	T	T	T
3	T	T	F	F	T	T	T	T
4	T	F	T	F	T	T	T	T
5	F	T	T	T	T	T	T	T
6	F	F	T	F	F	F	T	F
7	F	T	F	F	F	T	F	F
8	T	F	F	F	T	T	T	T
9	F	F	F	F	F	F	F	F

The distributive law holds since column 5 = column 8.

Remark. *The order in which you have written columns 4-8 may be different than mine. That is OK.*

2. (3 points) Prove that $(p \vee q) \to r \equiv (p \to r) \wedge (q \to r)$ using Theorem 2.1.1. Annotate your proof. For reference, see example 2.1.14

Solution :

$$\begin{aligned}
(p \vee q) \to r &\equiv \sim (p \vee q) \vee r && (1)\\
&\equiv (\sim p \wedge \sim q) \vee r \;\; (DeMorgan's) && (2)\\
&\equiv r \vee (\sim p \wedge \sim q) \;\; (Commutative) && (3)\\
&\equiv (r \vee \sim p) \wedge (r \vee \sim q) \;\; (Distributive) && (4)\\
&\equiv (\sim p \vee r) \wedge (\sim q \vee r) \;\; (Commutative) && (5)\\
&\equiv (p \to r) \wedge (q \to r) && (6)
\end{aligned}$$

Remark.

- *There is nothing to annotate when using the logical equivalence $p \to q \equiv \sim p \vee q$.*
- *I know some of you did not use either of the commutative properties. That is OK. With that said, everyone should have lines (1), (2), (4), (6).*

3. (3 points) Find all values of p and q for which $p \to q$ is not equal to $q \to p$. For which values of p, q are the statement forms equal?

Solution : Let's look at the truth table.

	1	2	3	4
1	p	q	$p \to q$	$q \to p$
2	T	T	T	T
3	T	F	F	T
4	F	T	T	F
5	F	F	T	T

It's clear that the statement forms $p \to q$ is equal to $q \to p$ if and only if statement variables p, q have the same truth values.

Remark. *The results of problem 3 tell us that the conditional is not logically equivalent to its converse.*

4. (3 points) Show that $[(p \to q) \wedge (p \to \sim q)] \to \sim p$ is a tautology using Theorem 2.1.1. Annotate your proof. For reference, see example 2.1.14

Solution : Let's set $P = p \to q$ and $Q = p \to \sim q$. Let's start.

$$\begin{aligned}
[P \wedge Q] \to \sim p &\equiv \sim [P \wedge Q] \vee \sim p && (7)\\
&\equiv \sim P \vee \sim Q \vee \sim p \;\; (DeMorgan's) && (8)\\
&\equiv \sim P \vee (\sim Q \vee \sim p) \;\; (Associative\ Law) && (9)\\
&\equiv \sim P \vee P && (10)\\
&\equiv \mathbf{t} \;\; (NegationLaws) && (11)
\end{aligned}$$

Of course, I bet you're thinking. How in the world did we get line (11)? Let's now show that $P \equiv \sim Q \vee \sim p$. We have

$$\begin{aligned}
\sim Q \vee \sim p &\equiv \sim (p \to \sim q) \vee \sim p && (12)\\
&\equiv \sim (\sim p \vee \sim q) \vee \sim p && (13)\\
&\equiv\ [\sim (\sim p) \wedge \sim (\sim q)] \vee \sim p\ \ (DeMogan's) && (14)\\
&\equiv\ (p \wedge q) \vee \sim p\ \ (Double\ Negation\ Laws) && (15)\\
&\equiv \sim p \vee (p \wedge q)\ \ (Commutative\ Law) && (16)\\
&\equiv\ (\sim p \vee p) \wedge (\sim p \vee q)\ \ (Distribution) && (17)\\
&\equiv \mathbf{t} \wedge (\sim p \vee q\ \ (NegationLaws) && (18)\\
&\equiv (\sim p \vee p)\ \ (Identity\ Laws) && (19)\\
&\equiv (p \to q) && (20)\\
&\equiv P && (21)
\end{aligned}$$

Remark. *The argument above is not the only argument. Here is a very clever argument that a student presented in office hour!*

$$\begin{aligned}
(p \to q) \wedge (p \to \sim q) &\equiv\ (\sim p \vee q) \wedge (\sim p \vee \sim (\sim q) && (22)\\
&\equiv\ (\sim p \vee q) \wedge (\sim p \vee q)\ \ (Double\ Negative\ Law) && (23)\\
&\equiv \sim p \vee (q \wedge \sim q)\ \ (DistributiveLaw) && (24)\\
&\equiv \sim p \vee \mathbf{c}\ \ (Negation\ Law) && (25)\\
&\equiv \sim p\ \ (IdentityLaw) && (26)
\end{aligned}$$

Therefore, we have

$$\begin{aligned}
[(p \to q) \wedge (p \to \sim q)] \to \sim p &\equiv \sim p \to \sim p && (27)\\
&\equiv \sim (\sim p) \vee \sim p && (28)\\
&\equiv\ p \vee \sim p\ \ (DoubleNegativeLaw) && (29)\\
&\equiv \mathbf{t}\ \ (NegationLaw) && (30)
\end{aligned}$$

5. (9 points) Section 2.1 #31 (see page 52).

   Solution :

   (a) This is the set of all strings s of length two where the first entry is a 0 or a 1 and the second must be a 1 or a 2.

   Remark. *BTW. I am happy with the description given above. If we agree to denote a string of length two as (x, y), then then set of such strings looks like*

   $$\{(0,1),(0,2),(1,1),(1,2)\}$$

   (b) The first entry must be a 2 and the second entry must be a 1 or a 2.

(c) The first entry must be a 1 or a 2. The second entry must be a 1 or a 0.

6. (3 points) Section 2.1 #46(a) (see page 53)

   Solution : I will give a solution that will differ from the text and use Theorem 2.1.1.

$$\begin{aligned} p \oplus p &\equiv (p \vee p) \wedge \sim (p \wedge p) \\ &\equiv p \wedge \sim p \;\; (Idempotent\ Laws) \\ &\equiv \mathbf{c} \;\; (Negation\ Laws) \end{aligned}$$

   Similarly,

$$\begin{aligned} (p \oplus p) \oplus p &\equiv \mathbf{c} \oplus p && (31) \\ &\equiv (\mathbf{c} \vee p) \wedge \sim (\mathbf{c} \wedge p) && (32) \\ &\equiv (p \vee \mathbf{c}) \wedge \sim (p \wedge \mathbf{c}) \;\; (Commutative\ Law) && (33) \\ &\equiv p \wedge \mathbf{c} \;\; (Identity\ Laws\ andUniversal\ Bound\ Laws) && (34) \\ &\equiv \mathbf{c} \;\; (Universald\ Bound\ Laws) && (35) \end{aligned}$$