

# Unification of 3 types & Extension

2022年7月27日 19:10

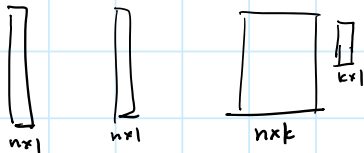
$$\mathcal{L}u = a u_x + \frac{1}{2} \sigma^2 u_{xx} \quad \mathcal{L} - \text{operator.}$$

$$\Rightarrow \text{Type 1: } \begin{cases} u_t + \mathcal{L}u = 0 \\ u(T, x) = V(x) \end{cases}$$

$$\text{Type 2: } \begin{cases} u_t + \mathcal{L}u - bu = 0 \\ u(T, x) = V(x) \end{cases}$$

$$\text{Type 3: } \begin{cases} u_t + \mathcal{L}u + b = 0 \\ u(T, x) = 0 \end{cases}$$

若  $\vec{F}$  为  $d\vec{x}_t = \vec{a} \cdot dt + \sum d\vec{B}_t$



则  $\mathcal{L}u = \sum_{i=1}^n a^i u_{x_i} + \frac{1}{2} \sum_{i,j=1}^n a^{ij} u_{x_i x_j}$  (trivial:  $n=1$ )

$$a^{ij} = \sum_{k=1}^n \sigma^{ik} \sigma^{jk}$$

(Pf.)  $df(t, \vec{x}) = f_t dt + \sum_i f_{x_i} d\vec{x}_t^i + \frac{1}{2} \sum_{i,j} f_{x_i x_j} d\vec{x}_t^i d\vec{x}_t^j$

$\xrightarrow{\text{替换为}} a^i dt + \sum \sigma^{ij} dB_t^j$

$d\vec{x}_t^i d\vec{x}_t^j = (a^i dt + \sum \sigma^{ik} dB_t^k)(a^j dt + \sum \sigma^{jl} dB_t^l)$

$(\text{Result}) = \sum_k \sigma^{ik} \sigma^{jk} dt$