Today: $\boxed{\text{Ken}}$

$\boxed{2.3}$ Valid & Invalid Arguments

Last time:

$\boxed{2.1}$ Logical Form & Equivalence

$\boxed{2.2}$ Conditional Statements

$\boxed{2.3}$ Valid & Invalid Arguments

(b) C & D approach but only C speaks.

C says: Both of us are knaves.

What are C & D?

subproof

1. Suppose C is telling the truth. assumption
2. C & D are knaves. what C said
3. C is a knave. specialization
4. C is lying. by def. of knave
5. C is telling the truth but lying. conjunction
6. C is lying. contradiction rule
7. C is a knave. by def. of knave
8. Not both C & D are knaves. negation of what C said
9. C is a knight or D is a knight. DeMorgan's laws
10. Since C is a knave, D is a knight. elimination

$p \lor q$

$\neg p$

$\therefore q$

$\neg(p \lor \neg p)$

p

$p \lor \neg p$

$\bot$

$\neg p$

$p \lor \neg p$

$\bot$

$\neg\neg(p \lor \neg p)$

$p \lor \neg p$

Summary of Rules of Inference

Table 2.3.1 Valid Argument Forms

Modus Ponens $p \to q$ p $\therefore q$	**Elimination** a) $p \vee q$ $\sim q$ $\therefore p$ b) $p \vee q$ $\sim p$ $\therefore q$
Modus Tollens $p \to q$ $\sim q$ $\therefore \sim p$	**Transitivity** $p \to q$ $q \to r$ $\therefore p \to r$
Generalization a) p $\therefore p \vee q$ b) q $\therefore p \vee q$	**Proof by Cases** $p \vee q$ $p \to r$ $q \to r$ $\therefore r$
Specialization a) $p \wedge q$ $\therefore p$ b) $p \wedge q$ $\therefore q$	**Proof by Contradiction** $\sim p \to \bot$ $\therefore p$
Conjunction p q $\therefore p \wedge q$	

A set of premises and a conclusion are given. Use the valid argument forms listed in Table 2.3.1 to deduce the conclusion from the premises, giving a reason for each step as in Example 2.3.8. Assume all variables are statement variables.

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a. $\neg p \vee q \to r$ 3

b. $s \vee \neg q$ 5

c. $\neg t$ 2

d. $p \to t$ 1

e. $\neg p \wedge r \to \neg s$ 4

f. $\therefore \neg q$ 6

$p \to t$ d

$\neg t$ c

$\therefore \neg p$ i modus tollens

$\neg p$ i

$\therefore \neg p \vee q$ ii generalization

$\neg p \vee q \rightarrow r$ (a)

$\neg p \vee q$ (ii)

$\therefore r$ (iii) modus ponens

$\neg p$ (i)

r (iii)

$\therefore \neg p \wedge r$ (iv) conjunction

$\neg p \wedge r \rightarrow \neg s$ (e)

$\neg p \wedge r$ (iv)

$\therefore \neg s$ (v) modus ponens

$s \vee \neg q$ (b)

$\neg s$ (v)

$\therefore \neg q$ (vi) elimination

#43 (a) $\neg p \to r \wedge \neg s$ (4)
(b) $t \to s$ (5)
(c) $u \to \neg p$ (3)
(d) $\neg w$ (2)
(e) $u \vee w$ (1)
(f) $\therefore \neg t$ (6)

$u \vee w$ (e)
$\neg w$ (d)
$\therefore u$ (i) elimination

$u \to \neg p$ (c)
u (i)
$\therefore \neg p$ (ii) modus ponens

$\neg p \to r \wedge \neg s$ (a)
$\neg p$ (ii)
$\therefore r \wedge \neg s$ (iii) modus ponens

$r \wedge \neg s$ (iii)
$\therefore \neg s$ (iv) specialization

$t \to s$ (b)

$\neg s$ (iv)

$\therefore \neg t$ (v) modus tollens

$\boxed{3.1}$ Predicates & Quantifiers I

<u>Some important sets:</u>

$\emptyset$ or $\{\}$	empty set	
$\mathbb{N}$	natural numbers	$\{1,2,3,\ldots\}$ OR $\{0,1,2,\ldots\}$
$\mathbb{Z}$	integers	$\{0,1,-1,2,-2,3,-3,\ldots\}$
$\mathbb{Z}^-$	negative integers	$\{-1,-2,-3,\ldots\}$
$\mathbb{Z}^+$	positive integers	$\{1,2,3,\ldots\}$
$\mathbb{Z}^{nonneg}$	nonnegative integers	$\{0,1,2,\ldots\}$
$\mathbb{Q}$	rational numbers	$\{m/n : m,n \in \mathbb{Z}; n \neq 0\}$
$\mathbb{Q}^-$	negative rational numbers	$\{r \in \mathbb{Q} : r < 0\}$
$\mathbb{Q}^+$	positive rational numbers	$\{r \in \mathbb{Q} : r > 0\}$
$\mathbb{Q}^{nonneg}$	nonnegative rational numbers	$\{r \in \mathbb{Q} : r \geq 0\}$
$\mathbb{R}$	real numbers	
$\mathbb{R}^-$	negative real numbers	
$\mathbb{R}^+$	positive real numbers	
$\mathbb{R}^{nonneg}$	nonnegative real numbers	
$\mathbb{C}$	complex numbers	$\{a+bi : a,b \in \mathbb{R};\ i=\sqrt{-1}\}$
$\mathbb{H}$	quaternions	

Some set theory and operations

Suppose A, B are sets.

$a \in A$	a is an element of A
$a \notin A$	a is not an element of A
$\{a_1, \dots, a_n\}$	set with elements $a_1, \dots, a_n$
A^c	A complement
$A \subset B$ or $A \subseteq B$	A is a subset of B
$A \not\subset B$ or $A \not\subseteq B$	A is not a subset B
$A = B$	A equals B
$A \cup B$	A union B
$A \cap B$	A intersect B
$A - B$	set difference A minus B
(x, y)	ordered pair
$(x_1, x_2, \dots, x_n)$	ordered n-tuple
$A \times B$	Cartesian product of A & B
$A_1 \times \cdots \times A_n$	Cartesian product of sets $A_1, \dots, A_n$
$\mathcal{P}(A)$	power set of A

$A = \{1, 2\}$ $\quad 2^2 = 4$

$\mathcal{P}(A) = \{\emptyset, \{1\}, \{2\}, \{1, 2\}\}$

$A \times B = \{(a,b) : a \in A \text{ and } b \in B\}$

$p \rightarrow q \equiv \neg p \vee q$

Theorem *

(formerly "Awesome law" in Spring 2021)