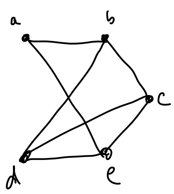
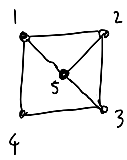
Lecture 3:

Topics: Isom, adj matrix, subgraph, configuration, complement, Removing vertices/edges, degree sequence, Handshake Lemma, Spanning Trees. (P1)

(P2): χ "chromatic #", χ-$\geq$ & K_p, part to indep sets, Greedy bnd $\chi \leq d_{max} + 1$.
$\chi \leq d_{max}$ except K_n or odd cycle, chrom polyn, $\lceil \frac{n}{\alpha} \rceil \leq \chi$

def $G \cong G' \iff \exists \phi : V \to V' \mid adj(v_i, v_j) = adj'(\phi(v_i), \phi(v_j))$.





$$\begin{pmatrix} a & b & c & d & e \end{pmatrix} \xrightarrow{\phi} \begin{pmatrix} 4 & 1 & 5 & 2 & 3 \end{pmatrix}$$

Eg $v(c) = \{b, d, e\} \xrightarrow{\phi} \{1, 2, 3\} = v(5)$

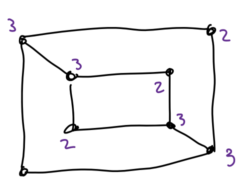
Lemma: $G \cong G' \iff A_G \equiv A_{G'}$ by permuting rows/cols.

Q When are graphs $\cong$? When not?

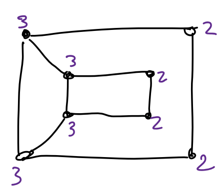
Ideas

1. List $\deg(v_i)$ in dec. order for G. (If $(d_1, \ldots, d_n) \neq (d_1' \ldots d_n')$, Then $G \not\cong G'$)
2. Remove all verts of deg = fixed. If $G \cong G'$ then $G \setminus d^{-1}(k) \cong G' \setminus d^{-1}(k)$.
   when you lose vert, also take cctd edges.
3. Circuits are dtd by length.

Eg.



vs



Remove all verts of deg 3

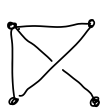
How many edges?

$d_n = (3,3,3,3,2,2,2,2) \quad d_n' = (3,3,3,3,2,2,2,2)$

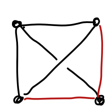
def: The complement of G is the graph obtained by

$V^c = V$ & $adj^c = 1 - adj^G$.

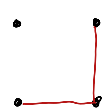
Eg:



G $\xrightarrow{\text{add edges}}$



$\xrightarrow{\text{forget orig}}$



def: A subgraph $H \subseteq G$ is a graph $H = (V_H, E_H)$ w/ $V_H \subseteq V_G$ & $E_H \subseteq E_G$.

def: A subdivision of G is a graph obtained by adding vertices to edges of G.

Obs $\cong$ gives bijections between {subgraphs} and {subdivs} of G, G'.

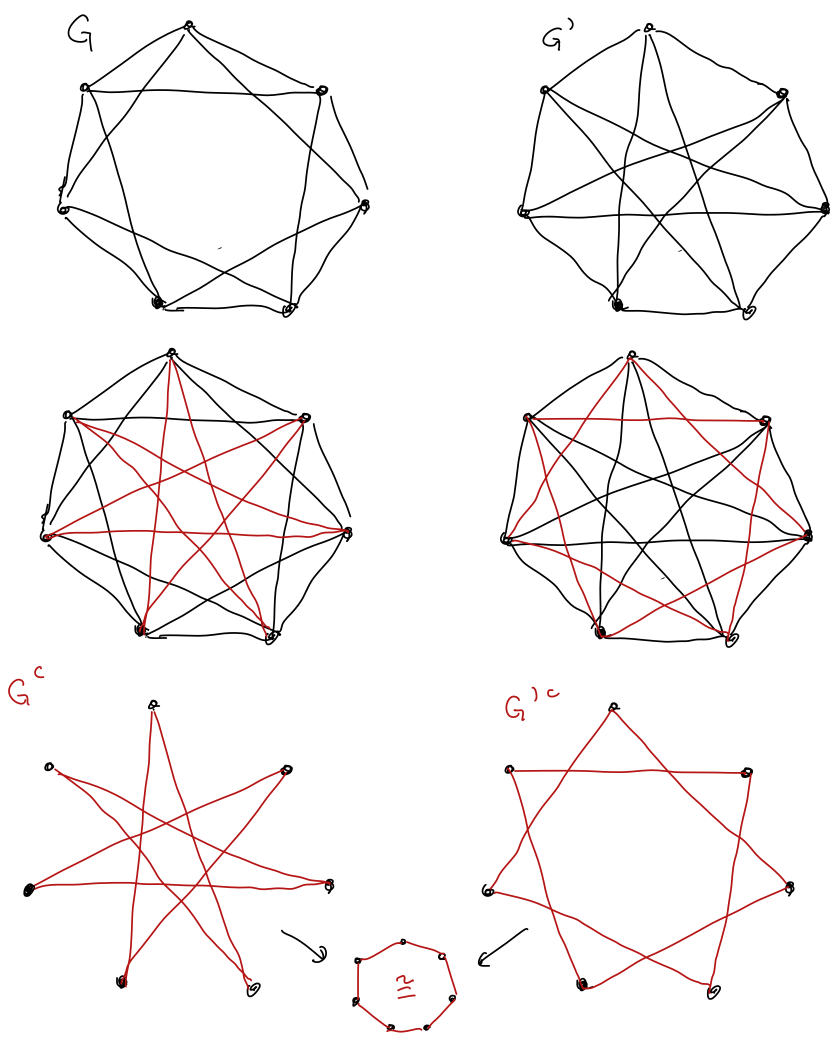
Prop $G \cong G' \iff G^c \cong G'^c$

Pf G is det'd by Adj but Adj dets $1 - \text{Adj}$ etc. so G^c dets G.

If $\phi : G \xrightarrow{\cong} G'$ then $\phi : G^c \to G'^c$

b/c $\text{Adj}_{G^c}(v_i, v_j) = 1 - \text{Adj}_G(v_i, v_j) = 1 - \text{Adj}'(\phi(v_i), \phi(v_j)) = \text{Adj}_{G'^c}(\phi(v_i), \phi(v_j))$ ∎

Eg:



Isom ?

<u>Lemma</u> [Handshake Lemma] Let $V = V_{even} \cup V_{odd}$
$d \in 2\mathbb{Z}$ ↲ ↳ $d \in 2\mathbb{Z}-1$

(a) $\sum_{v \in V} d(v) = 2|E|$

(b) $|V_{odd}| \in 2\mathbb{Z}$.

<u>Pf</u>: (a) Every edge touches two verts. so it gets counted exactly 2x.

(b) $\sum d(v) = \sum_{V_{odd}} d(v) + \sum_{V_{even}} d(x) = 2|E|$

$\equiv \sum_{V_{odd}} 1 + \sum_{V_{even}} 0 \equiv 0 \Rightarrow |V_{odd}| \equiv 0 \pmod 2$.

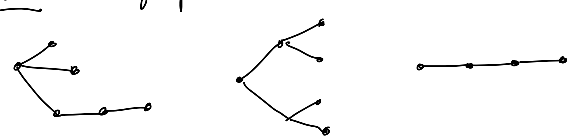
$\square$.

Use this to rule out deg sequences ?

<u>Q</u> Is there a graph w/ deg seq $(5,4,4,2,1,1)$?

No, 3 are odd.

<u>def</u> a <u>Tree</u> is a graph which is cctd & disconnects when you lose any edge.

Eg.  <u>etc</u>.

<u>Equiv</u>: G is a Tree $\Leftrightarrow$ (a) $|E| = |V| - 1$

(b) No circuits

(c) $\exists!\ p : v_1 \mapsto v_2 \quad (\forall v_1 \neq v_2 \in V)$.

<u>def</u>: A spanning tree for G is $T \subseteq G$ s.t. $V_T = V_G$. (More Later).