MA-UY 2314: Discrete Mathematics

Exam 2

- Print Name: : ______________________________
- NetID: ______________________________
- You have 80 minutes for the exam. No notes or calculators are permitted.
- Remember the famous mathematical formula: No work = No credit (unless stated otherwise)

I pledge that I have observed the NYU honor code, and that I have neither given nor received unauthorized assistance during this examination.

Signature:

Multiple Choice

This is the multiple choice section of the exam. Each problem is worth 3 points. No partial credit is given and there is no need to justify your answer. All multiple choice answers must be recorded here. Only this page will be graded.

1. (A) (B) (C) (D) (E) (F) (G) (H) (I) (J)

2. (A) (B) (C) (D) (E) (F) (G) (H) (I) (J)

3. (A) (B) (C) (D) (E) (F) (G) (H) (I) (J)

1. Student is asked to prove

$$P(n), \quad \forall n \geq 2$$

using Strong Mathematical Induction. Student shows $P(2) \wedge P(3) \wedge P(4)$ is true in the base case. What is strong induction hypothesis?

(a) Assume $k \geq 2$ such that $P(i)$ is true for $i = 2, \ldots k$.

(b) Assume $k \geq 2$ such that $P(i)$ is true for $i = 4, \ldots k$.

(c) Assume $k \geq 4$ such that $P(i)$ is true for $i = 2, \ldots k$.

(d) Assume $k \geq 4$ such that $P(i)$ is true for $i = 4, \ldots k$.

(e) None of the above.

Answer is (c)?

2. Select all statements that are true.

Remark: There are at least two correct options. You must select fill in all correct options on the scantron. There is no partial credit.

(a) 1 mod 3 = 3 mod 1

(b) 2 mod 3 $\neq$ 3 mod 2

(c) 3 div 2 = 10 div 8

(d) -46 div 3 = 15

(e) 5 mod (6 mod 11) = 1

3. The contrapositive of the statement

$$\forall x \in \mathbb{Z}, \text{ if } x \text{ is any non-zero integer, then } x^2 \text{ is positive.}$$

(a) $\forall x \in \mathbb{Z}$, if x is zero, then $x^2 < 0$.

(b) $\forall x \in \mathbb{Z}$, if x is zero, then $x^2 \leq 0$.

(c) $\forall x \in \mathbb{Z}$, if $x^2 < 0$, then x must be zero.

(d) $\forall x \in \mathbb{Z}$, if $x^2 \leq 0$,then x must be zero.

(e) $\forall x \in \mathbb{Z}$, if $x^2 > 0$, then x is any non-zero integer.

Open Response

This is the open response section of the exam. All answers must be justified, unless otherwise stated.

4. (20 points) Prove

$$21 \text{ divides } 4^{n+1} + 5^{2n-1}$$

for all integers $n \geq 1$ using the Principle of Mathematical Induction.

Remark: You must use the Principle of Mathematical Induction in order to obtain any sort of credit.

Proof:

Base Case: When $n = 1$, $4^{n+1} + 5^{2n-1} = 16 + 5 = 21$ and $21|21$.

Induction Step: Assume k is any integer ≥ 1 such that 21 divides $4^{k+1} + 5^{2k-1}$. We need to show that 21 divides $4^{k+2} + 5^{2k+1}$. BY the induction hypothesis we know that

$$4^{k+1} + 5^{2k-1} = 21m \quad (*)$$

for some integer m. Let's multiply $(*)$ by 4. We obtain

$$4^{k+2} + 4 \cdot 5^{2k-1} = 21 \cdot 4m$$

Now $4 \cdot 5^{2k-1} = (25 - 21)5^{2k-1} = 5^{2k+1} - 21 \cdot 5^{2k-1}$ Therefore

$$4^{k+2} + 5^{2k+1} = 21 \cdot 4m + 21 \cdot 5^{2k-1} = 21s$$

where $s = 4m + 5^{2k-1}$ $\square$

5. (20 points) **Prove by contradiction:** For any primes p_1 and p_2, if $p_1 + p_2$ is prime, then $p_1 = 2$ or $p_2 = 2$.

   Remark: You must use proof by contradiction in order to obtain any sort of credit.

   Proof: Assume p_1 and p_2 are any two primes such that $p_1 + p_2$ is prime and $p_1 \neq 2$ and $p_2 \neq 2$. Then p_1 and p_2 are both odd. Therefore $p_1 = 2k + 1$ and $p_2 = 2m + 1$ for some integers k and m. It follows that $p_1 + p_2 = 2(k + m + 1)$, hence 2 is a factor of $p_1 + p_2 \Rightarrow\Leftarrow \square$

6. (20 points) Prove

$$1 + 4 + 7 + \ldots (3n - 2) = \frac{n(3n-1)}{2}$$

for all integers $n \geq 1$ using the Principle of Mathematical Induction.

Remark: You must use the Principle of Mathematical Induction in order to obtain any sort of credit.

Proof:

Base Case: When $n = 1$, $3n - 2 = 1$ so the sum is equal to 1. While $n(3n-1)/2 = 2/2 = 1$.

Induction Step: Assume k is any integer $k \geq 1$ such that

$$1 + 4 + 7 + \ldots (3k - 2) = \frac{k(3k-1)}{2} \quad (*)$$

Let's add $3(k+1) - 2$ to $(*)$. We obtain

$$\begin{aligned}
1 + 4 + 7 + \ldots (3(k+1) - 2) &= \frac{k(3k-1)}{2} + 3k + 1 \\
&= \frac{k(3k-1) + (3k+1)2}{2} \\
&= \frac{3k^2 + 5k + 2}{2} \\
&= \frac{(k+1)(3k+2)}{2}
\end{aligned}$$

$\square$

7. (20 points) **Prove directly:** Let a and b be any integers and let m be any positive integer. If a mod $m = b$ mod m, then m divides $b - a$.

   Remark: You must provide a direct proof in order to obtain any sort of credit.

   Proof: Assume a and b are any integers and m is any positive integer such that a mod $m = m$ mod a. Let $r = a$ mod m. Using the Quotient-Remainder Theorem, we know that $a = qm + r$ and $b = q'm + r$. It follows that $b - a = m(q' - q)$. Therefore, m divides $b - a$ $\square$