Poisson Process

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Poisson Process Construction.

Define. $\tau \sim$ Exp dist. w/ $\lambda > 0$.

为. $f(t) = \begin{cases} 0 & \text{if } t<0 \\ \lambda e^{-\lambda t} & \text{if } t \geq 0 \end{cases}$

且有 $E\tau = \int_0^{+\infty} t\lambda e^{-\lambda t}\,dt = t\cdot(-e^{-\lambda t})\Big|_0^{\infty} - \int_0^{+\infty} -e^{-\lambda t}\,dt = \frac{1}{\lambda}$

Memoriless Property: $P(x>t+s \mid x>t) = \frac{P(x>t+s \cap x>t)}{P(x>t)} = \frac{e^{-\lambda(t+s)}}{e^{-\lambda t}} = e^{-\lambda s} = P(x>s)$.

Construction: 为 $\tau_1, \tau_2, \cdots \tau_n$ 为 iid Exp(λ) RV.

有 $N_t \begin{cases} 0 & \text{if } t<\tau_1 \\ 1 & \text{if } \tau_1 \leq t < \tau_2 \\ \vdots & \\ k & \text{if } \tau_k \leq t < \tau_{k+1} \end{cases}$