Worksheet 4 posted - Due 12/8

Extend last part of Project.

Final Exam - 12/17 $3^{30}-5^{30}$ pm.

16.7 Surface Integrals

replace x,y,z with u,v.

$$\iint_{\substack{S \\ \vec{r}(u,v)}} f(x,y,z)\,dS = \iint_D f(\vec{r}(u,v))\,\|\vec{r}_u \times \vec{r}_v\|\,dA$$

domain for u,v.

$$\iint_{\substack{S \\ z=g(x,y)}} f(x,y,z)\,dS = \iint_D f(x,y,g(x,y))\sqrt{g_x^2+g_y^2+1}\,dA.$$

domain for x,y.

unit normal.

Flux through surface $\iint_S \vec{F}\cdot d\vec{S} = \iint_S \vec{F}\cdot\hat{n}\,dS$

Positively oriented

upwards or outwards.

S is given by $\vec{r}(u,v)$ $\quad$ Flux $= \iint_D \underline{\vec{F}(\vec{r}(u,v))}\cdot\underline{(\vec{r}_u\times\vec{r}_v)}\,dA$

S is given by $z=g(x,y)$ $\quad$ Flux $= \iint_D (-Pg_x - Qg_y + R)\,dA$

$\vec{F} = \langle P, Q, R\rangle$

$\vec{F}\cdot(-g_x\hat{i} - g_y\hat{j} + \hat{k})$

⑤ $\iint_S (x+y+z)\,dS$ S is given by

$x = u+v$
$y = u-v$
$z = 1+2u+v$ $\quad 0 \le u \le 2 \quad 0 \le v \le 1$

$$\iint_S f(x,y,z)\,dS = \iint_D f(\vec{r}(u,v))\,\|\vec{r}_u \times \vec{r}_v\|\,dA$$

$\vec{r}(u,v) = \langle u+v,\ u-v,\ 1+2u+v \rangle$

$\vec{r}_u = \langle 1, 1, 2 \rangle \qquad \vec{r}_v = \langle 1, -1, 1 \rangle$

$\vec{r}_u \times \vec{r}_v = (\hat{i} + \hat{j} + 2\hat{k}) \times (\hat{i} - \hat{j} + \hat{k}) = -\hat{k} + (-\hat{j}) + (-\hat{k}) + \hat{i} + 2\hat{j} + 2\hat{i}$

$= 3\hat{i} + \hat{j} - 2\hat{k}$

$\|\vec{r}_u \times \vec{r}_v\| = \sqrt{3^2 + (1)^2 + (-2)^2} = \sqrt{14}$

$f(\vec{r}(u,v)) = u+v + u-v + 1+2u+v = 4u + v + 1$

$$\iint_S (x+y+z)\,dS = \int_0^2 \int_0^1 (4u+v+1)\sqrt{14}\,dv\,du = \ldots$$

⑩ $\iint_S xz\,dS$ S is part of $2x+2y+z=4$ that lies in first octant.

$x, y, z \ge 0$.

$$\iint_S f(x,y,z)\,dS = \iint_D f(x,y,g(x,y))\sqrt{g_x^2+g_y^2+1}\,dA.$$

note: $g(z,y) \rightarrow \sqrt{g_z^2+g_y^2+1}$

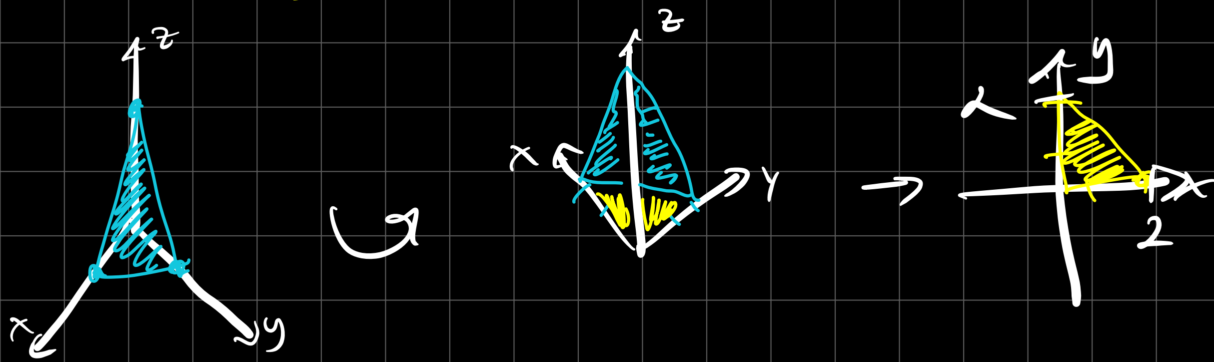
Try $g(x,z)$ or $g(y,z)$ later.

$g(x,y) = 4-2x-2y$

$g_x = -2 \qquad g_y = -2$

$f(x,y,g(x,y)) = x(4-2x-2y)$

$$\iint_S xz\,dS = \iint_D x(4-2x-2y)\sqrt{(-2)^2+(-2)^2+1}\,dA$$



$0 \le x \le 2$

$0 \le y \le 2-x$

$$\iint_S xz\,dS = \int_0^2\int_0^{2-x}(4x-2x^2-2xy)\,3\,dy\,dx = \ldots$$

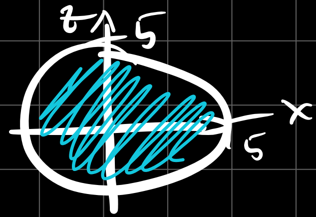
(14) $\iint_S y^2z^2\,dS$ $\quad S$ is part of cone $y=\sqrt{x^2+z^2}$, $0\le y\le 5$

option 1:

use $y = g(x,z)$

$g_x = \dfrac{x}{\sqrt{x^2+z^2}} \qquad g_z = \dfrac{z}{\sqrt{x^2+z^2}}, \qquad \sqrt{g_x^2+g_z^2+1} = \sqrt{2}$

need domain $\sqrt{x^2+z^2} \le 5 \Rightarrow x^2+z^2 \le 25$



$-5 \le x \le 5$

$-\sqrt{25-x^2} \le z \le \sqrt{25-x^2}$

$$\iint_S y^2 z^2 \, dS = \int_{-5}^{5}\int_{-\sqrt{25-x^2}}^{\sqrt{25-x^2}} (x^2+z^2)\, z^2 \sqrt{2}\, dz\, dx$$

convert to polar!!

$x = r\cos\theta$

$z = r\sin\theta$

$0 \le r \le 5$

$0 \le \theta \le 2\pi.$

$$= \int_0^{2\pi}\int_0^5 (r^2)\, r^2\sin^2\theta \sqrt{2}\, r\, dr\, d\theta$$

$= \ldots$

option 2: $y = \sqrt{x^2+z^2}$ $\quad 0 \le y \le 5$

$y = g(r,\theta) = r$

$y = r$ $\quad 0 \le r \le 5$

$g_r = 1$

$g_\theta = 0$

$x = r\cos\theta$, $z = r\sin\theta$ $\quad 0 \le \theta \le 2\pi.$

$$\int_0^{2\pi}\int_0^5 r^2 (r\sin\theta)^2 \underbrace{\sqrt{g_r^2 + g_\theta^2 + 1}}_{\sqrt{2}}\, r\, dr\, d\theta = \text{Same as option 1.}$$

(23) $\vec{F}(x,y,z) = \underbrace{xy}_{P}\,\hat{i} + \underbrace{yz}_{Q}\,\hat{j} + \underbrace{zx}_{R}\,\hat{k}$

S is part of parab. $z = \underbrace{4-x^2-y^2}_{g(x,y)}$ above $0 \le x \le 1$, $0 \le y \le 1$

oriented upwards.

Find flux of $\vec{F}$ through S.

$$\text{Flux} = \iint_D (-Pg_x - Qg_y + R)\,dA \rightsquigarrow \vec{n} = \langle -g_x, -g_y, 1\rangle$$

upwards.

$g_x = -2x \qquad g_y = -2y$

downward would be $-\vec{n}$

$\langle g_x, g_y, -1\rangle$

$$\text{Flux} = \int_0^1\int_0^1 \left(-xy(-2x) - y(4-x^2-y^2)(-2y) + (4-x^2-y^2)x\right) dx\,dy$$

$$= \int_0^1\int_0^1 \left(2x^2y + 8y^2 - 2x^2y^2 - 2y^4 + 4x - x^3 - xy^2\right) dx\,dy$$

$= \ldots$

(25) $\vec{F}(x,y,z) = x\hat{i} + y\hat{j} + z^2\hat{k}$

S unit sphere oriented outwards.

Find flux of $\vec{F}$ through S.

$\rho = 1$

$$\vec{r}(u,v) = \langle \underbrace{\sin u\cos v}_{x}, \underbrace{\sin u \sin v}_{y}, \underbrace{\cos u}_{z}\rangle$$

$0 \le u \le \pi$

$0 \le v \le 2\pi$

$\vec{r}_u = \langle \cos u\cos v, \cos u\sin v, -\sin u\rangle$

$\hat{k}\times\hat{i}$ $\quad$ $\hat{k}\times\hat{j}$

$\vec{r}_v = \langle -\sin u\sin v, \sin u\cos v, 0\rangle$

$\hat{i} \to \hat{j} \to \hat{k}$

$$\vec{r}_u \times \vec{r}_v = \sin u\cos u\cos^2 v\,\hat{k} - \cos u\sin u\sin^2 v(-\hat{k}) + \sin^2 u\sin v\,\hat{j} - \sin^2 u\cos v(-\hat{i})$$

$$= \sin^2 u\cos v\,\hat{i} + \sin^2 u\sin v\,\hat{j} + \sin u\cos u\,\hat{k}$$

$\uparrow \quad + \quad \uparrow \quad + \quad \uparrow$

$\vec{F}(\vec{r}(u,v)) = \sin u \cos v\,\hat{i} + \sin u \sin v\,\hat{j} + \cos^2 u\,\hat{k}$

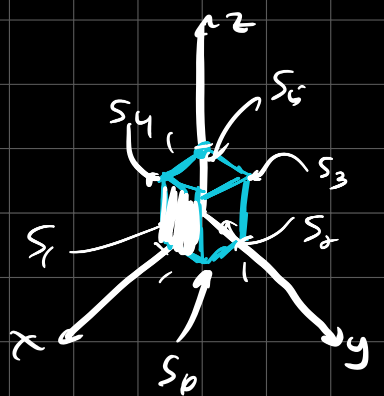
$\vec{F}\cdot(\vec{r}_u \times \vec{r}_v) = \sin^3 u \cos^2 v + \sin^3 u \sin^2 v + \sin u \cos^3 u$
$= \sin^3 u + \sin u \cos^3 v.$

Flux through surface $= \int_0^{2\pi}\int_0^{\pi} \sin^3 u + \sin u \cos^3 u \, du\, dv$

factor $\sin u$ out

$\sin u(\underbrace{\sin^2 u}_{1-\cos^2 u} + \cos^3 u)$ let $w = \cos u$

Comment if $S =$ surface of a cube. oriented out.



6 - sides.

Surface	$\hat{n}$
S_1	$\hat{i}$
S_2	$\hat{j}$
S_3	$-\hat{i}$
S_4	$-\hat{j}$
S_5	$\hat{k}$
S_6	$-\hat{k}$

$$\iint_S \vec{F}\cdot d\vec{S} = \iint_S \vec{F}\cdot\hat{n}\, dS$$

$$= \iint_{S_1} \vec{F}\cdot\hat{i}\, dS + \iint_{S_2} \vec{F}\cdot\hat{j}\, dS + \iint_{S_3} \vec{F}\cdot(-\hat{i})\, dS + \iint_{S_4} \vec{F}\cdot(-\hat{j})\, dS$$

$$+ \iint_{S_5} \vec{F}\cdot\hat{k}\, dS + \iint_{S_6} \vec{F}\cdot(-\hat{k})\, dS.$$

if $\vec{F} = \langle x, y, z \rangle$

on S_1 : $x = 1$ $0 \le y, z \le 1$ $\vec{F} = \langle 1, y, z \rangle$ $\hat{n} = \hat{\imath}$

$\vec{F} \cdot \hat{n} = 1$

flux: $\iint_{S_1} \vec{F} \cdot \hat{n} \, dS = \iint_{S_1} 1 \, dS = 1 \, A(S_1) = 1$

repeat for $S_2, \ldots, S_6$ and add.

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