Homework 3 Solutions

Due: Friday Oct. 1, by 11:59pm,
via Gradescope

- Failure to submit homework correctly will result in a zero on the homework.
- Homework must be in LaTeX. Submit the pdf file to Gradescope.
- Problems assigned from the textbook come from the 5th edition.
- No late homework accepted. Lateness due to technical issues will not be excused.

---

1. (21 points) Section 3.1 # 4, 20, 32(b)(d).

   Solution: Regarding problem # 4

   (a) Notice that $x < y$ is true while $x^2 < y^2$ is false.

   (b) $x = -1, y = 0$ would work. Infinitely many correct values of x and y.

   (c) This is because $x < y$ and $x^2 < y^2$ are both true.

   (d) $x = 8$, $y = 3$ will work. Do you see why?

   Here is problem 20

   Every positive real number has a positive square root.

   Every real number that has a non-positive square root must be non-positive.

   Remark. *Non-positive is* ≤ 0.

   Both parts (b) and (d) are T. They did not ask for a justification. However, you can always see this from the graph of $y = x^2$. Also, about part (d). Note that $|x| > 2 \leftrightarrow x < -2 \vee x > 2$.

2. (9 points) Section 3.2 # 15(b)(d)(e).

   Solution: (b) is T. $x = -8, -14 - 48$ are the only negative values in D and are all even.

   (d) is T. $x = 32$ is the only value in D where the ones digit is 2. Note that the tens digit is 3.

   (e) is F. $x = 36$ has a ones digit equal to 6 and the tens digit is a three.

3. (9 points) Section 3.2 # 12, 40, 46.

   Solution: Problem 12. The proposed negation is incorrect. I find it useful to write the given statement formally. Then negate. Then write the informal negation. The given statement formally reads

$$\forall \text{ irrational } x, \forall \text{ rational } y, xy \text{ is irrational}$$

The formal negation reads

$$\exists \text{ irrational } x, \exists \text{ rational } y, xy \text{ is rational}$$

The informal negation reads:

There exists a rational number and an irrational number such that the product is rational.

Problem 40. $\forall x \in \mathbb{R}$ if x is divisible by 8, then x is divisible by 4.

Problem 46. The *not necessary* is the negation of a universal necessary. That is,

$$\sim [\forall x, p(x) \text{ is necessary for } q(x)]$$

This simplies to

$$\exists x, \sim p(x) \wedge q(x)$$

Here we used the fact that

$$\sim (\sim p \to \sim q) \equiv \sim (\sim (\sim p) \vee \sim q) \equiv \sim (p \vee \sim q) \equiv \sim p \wedge \sim (\sim q) \equiv \sim p \wedge q$$

The final answer reads

Someone does not have a large income and is happy

4. (9 points) Section 3.3 #43, 44, 45

   Solution: Problem 43.

   $$\exists \varepsilon > 0, \forall \delta > 0, \exists x \in \mathbb{R}, (|x-a| < \delta) \wedge (x \neq a) \wedge (|f(x) - L| \geq \varepsilon)$$

   Remark. *I used the red parenthesis simply because I thought it would be easier to read.*

   Problem 44. Part (a) is T. The only value for which $xy = y$ for all real numbers y is $x = 1$. Part (b) is F. The ratio $\frac{1}{x}$ is an integer when set $x = 1$ and $x = -1$. Part (c) is F. When $x = 1$, then $y = -1$, and when $x = 2$, we must have y = -2.

   Problem 45. There is one and only one value in D for which P is T.

   Remark. *Problem 4 should be worth 15 points (problem 44 should be worth 9 points). Why didn't anyone send me a friendly email ☹*

5. (6 points) Section 3.3 # 56, 57.

   Solution: Problem 56. This is false. Let $D = \{1, 2\}$ and

   $P(x) : x$ is even. $Q(x) : x$ is odd.

   Then $\exists x \in D, P(x) \wedge Q(x)$ is F while $\exists x \in D, P(x)$ is T and $\exists x \in D, Q(x)$ is T. Therefore

   $$(\exists x \in D, P(x)) \wedge (\exists x \in D, Q(x)) \text{ is T}$$

   Problem 57. This is false. Using the same example in problem 56

   Then $\forall x \in D, P(x) \vee Q(x)$ is T while $\forall x \in D, P(x)$ is F and $\forall x \in D, Q(x)$ is F. Therefore

   $$(\forall x \in D, P(x)) \vee (\forall x \in D, Q(x)) \text{ is F}$$