Math 140 - Fall 2017 - Midterm Exam 2 - Version B

You have 110 minutes to complete this midterm exam. Books, notes and electronic devices are not permitted. Read and follow directions carefully. Show and check all work. Label graphs and include units where appropriate. If a problem is not clear, please ask for clarification.

Multiple Choice 1-10	/30
Free Response 11	/5
Free Response 12	/10
Free Response 13	/10
Free Response 14	/10
Free Response 15	/15
Total	/80

I pledge that I have completed this midterm exam in compliance with the NYU CAS Honor Code. In particular, I have neither given nor received unauthorized assistance during this exam.

Name ____

N Number ____

Signature ____

Date ____

1 Multiple Choice

(30 points) For problems 1-10, circle the best answer choice.

1. Which of the following matrices is orthogonal?

(a) $\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$

(b) $\begin{bmatrix} 1/2 & -1/2 \\ 1/2 & 1/2 \end{bmatrix}$

(c) $\begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}$

(d) $\begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 1 \\ 1/\sqrt{2} & 1/\sqrt{2} & 1 \end{bmatrix}$

(e) $\begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}$

2. Let $W = \text{Span}\left\{ \begin{bmatrix} 1 \\ 0 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 6 \\ 0 \end{bmatrix} \right\}$. Which of the following vectors belong to $W^{\perp}$?

(a) $\begin{bmatrix} 1 \\ 12 \\ 0 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 2 \\ 17 \\ 0 \\ -1 \end{bmatrix}$

(b) $\begin{bmatrix} 2 \\ 4 \\ 11 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 2 \\ 2 \\ 2 \\ -8 \end{bmatrix}$

(c) $\begin{bmatrix} 0 \\ 8 \\ 0 \\ 9 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 2 \\ 0 \\ 17 \end{bmatrix}$

(d) $\begin{bmatrix} 1 \\ 0 \\ 12 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 12 \\ 0 \\ -6 \\ 0 \end{bmatrix}$

(e) cannot be determined from given information

3. Let B be an $n \times n$ matrix such that $B^2 = B$. Which of the following statements MUST be true?

   (a) $\det B = 0$.

   (b) $\det B^3 = \det B$.

   (c) B is invertible.

   (d) The only possible eigenvalues of B are 1 and -1.

   (e) none of the above

4. What is the volume of a parallelpiped determined by vectors $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ -1 \end{bmatrix}$?

   (a) 0

   (b) 1

   (c) 2

   (d) 3

   (e) none of the above

5. Suppose A and B are 5×5 matrices with $\det A = -4$ and $\det B = 8$, then $\det A^{-1}B^T =$

   (a) -32

   (b) $\frac{1}{2}$

   (c) -2

   (d) 2

   (e) $-\frac{1}{2}$

6. What is the sum of the eigenvalues of the matrix $\begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$?

(a) 0

(b) 1

(c) 2

(d) 3

(e) 5

7. Which of the following matrices has eigenvectors $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ and eigenvalues -2 and 2, respectively?

(a) $\begin{bmatrix} -2 & 0 \\ -8 & 2 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 2 & 0 \\ 0 & -2 \end{bmatrix}$

(d) $\begin{bmatrix} 2 & 0 \\ 8 & -2 \end{bmatrix}$

(e) not enough information given to determine A

8. If an $n \times n$ matrix A is symmetric, then which of the following statements is may NOT be true?

   (a) A is diagonalizable.

   (b) A has orthogonal eigenvectors.

   (c) A has real-valued eigenvalues.

   (d) A has n distinct eigenvalues.

   (e) A has n linearly independent eigenvectors.

9. If A and B have the same eigenvalues, then

   (a) A and B must be similar matrices.

   (b) A and B must have the same eigenvectors.

   (c) det $A =$ det B.

   (d) (a) and (b)

   (e) all of the above

10. If A is a positive definite matrix, then which of the following must also be positive definite?

    (a) A^T

    (b) A^2

    (c) A^{-1}

    (d) (a) and (c)

    (e) all of the above

2 Free Response

For problems 11-15, show all work and justify each step to receive full credit. Draw a box around your answers.

11. (5 points) Let $W = \text{Col}\begin{bmatrix} 3 & 0 & 6 \\ 0 & 1 & -1 \\ 4 & 1 & 7 \end{bmatrix}$. Find a basis for $W^{\perp}$, the orthogonal complement of W.

12. (10 points) Find two numbers whose sum is 14, whose average is 2, and whose difference is 4. If that's not possible, get as close as you can with the least-squares solution.

13. (10 points) Let $A = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$

(a) Compute $\det(A)$.

(b) Solve for a and b in

$$A^{-1} = \begin{bmatrix} -a & -a & 2a & a \\ a & a & b & -a \\ -a & a & b & a \\ a & -a & b & a \end{bmatrix}.$$

(c) Solve $A\boldsymbol{x} = \begin{bmatrix} 4 \\ 3 \\ 2 \\ 1 \end{bmatrix}$.

14. (10 points) Let A be a matrix whose only eigenvalues are 1 and -1.

(a) Prove that if A is 2×2, then $A = A^{-1}$.

(b) Prove that this is not true if A is 3×3.

15. (15 points) Let $A = \begin{bmatrix} 4 & 1 & 1 \\ 1 & 4 & 1 \\ 1 & 1 & 4 \end{bmatrix}$ with eigenvalues 6 and 3.

(a) Compute the eigenvectors of A.

(b) Find an orthogonal set of three eigenvectors of A.

(c) Write A^{2017} as the product of three matrices. (*Hint: Diagonalize* A.)