13.4

(3) $\vec{r}(t) = \langle -\frac{1}{2}t^2, t \rangle$, $t=2$.

$\vec{v}(t) = \vec{r}\,'(t) = \langle -t, 1 \rangle$ - velocity

$\vec{a}(t) = \vec{v}\,'(t) = \vec{r}\,''(t) = \langle -1, 0 \rangle$ - acceleration

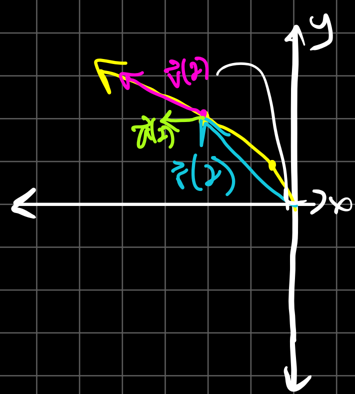
speed $= \|\vec{v}(t)\| = \sqrt{(-t)^2 + (1)^2} = \sqrt{t^2+1}$

find all at $t=2$.

$\vec{r}(2) = \langle -2, 2 \rangle$, $\vec{v}(2) = \langle -2, 1 \rangle$, $\vec{a}(2) = \langle -1, 0 \rangle$

$v(2) = \sqrt{5}$

plot $\vec{r}(t) = \langle -\frac{1}{2}t^2, t \rangle$



(8) $\vec{r}(t) = t\hat{i} + 2\cos t\,\hat{j} + \sin t\,\hat{k}$ at $t=0$.

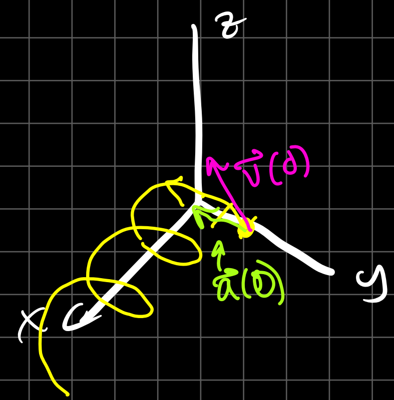
$\vec{v}(t) = \hat{i} - 2\sin t\,\hat{j} + \cos t\,\hat{k}$

$\vec{a}(t) = -2\cos t\,\hat{j} - \sin t\,\hat{k}$

$v(t) = \sqrt{(1)^2 + 4\sin^2 t + \underbrace{\cos^2 t}_{(1-\sin^2 t)}} = \sqrt{2 + 3\sin^2 t}$

$\vec{r}(0) = 2\hat{j}$

$\vec{v}(0) = \hat{i} + \hat{k}$ $\quad \vec{a}(0) = -2\hat{j}$ $\quad v(0) = \sqrt{2}$



When is speed largest? $v(t) = \sqrt{2 + 3\sin^2 t}$

biggest when $\sin^2 t = 1$

$v = \sqrt{5}$

find these values.

$\sin t = \pm 1$

$t = \frac{\pi}{2} + \pi k, k \in \mathbb{Z}$

Could also find

$v'(t) = 0$

Do standard optimization from calc I.

(16) $\vec{a}(t) = \sin t\,\hat{\imath} + 2\cos t\,\hat{\jmath} + 6t\,\hat{k}$

$\vec{v}(0) = -\hat{k}$ $\quad \vec{r}(0) = \hat{\jmath} - 4\hat{k}$

find $\vec{r}(t)$.

$$\vec{v}(t) = \int \vec{a}(t)\,dt = \underbrace{-\cos t\,\hat{\imath} + 2\sin t\,\hat{\jmath} + 3t^2\hat{k}}_{t=0} + \vec{C}$$

find $\vec{C}$

$$\underbrace{-\hat{k}}_{\vec{v}(0)} = -\hat{\imath} + \vec{C} \Rightarrow \vec{C} = \hat{\imath} - \hat{k}$$

$$\vec{v}(t) = -\cos t\,\hat{i} + 2\sin t\,\hat{j} + 3t^2\hat{k} + \hat{i} - \hat{k}$$

$$= (1-\cos t)\hat{i} + 2\sin t\,\hat{j} + (3t^2-1)\hat{k}$$

$$\vec{r}(t) = \int \vec{v}(t)\,dt = (t - \sin t)\hat{i} - 2\cos t\,\hat{j} + (t^3 - t)\hat{k} + \vec{D}$$

$$\vec{r}(0) = \hat{j} - 4\hat{k} = -2\hat{j} + \vec{D} \quad \Rightarrow \quad \vec{D} = 3\hat{j} - 4\hat{k}$$

$$\boxed{\vec{r}(t) = (t - \sin t)\hat{i} + (3 - 2\cos t)\hat{j} + (t^3 - t - 4)\hat{k}}$$

(19) $\vec{r}(t) = \langle t^2, 5t, t^2 - 16t\rangle$

when is speed a minimum?

$\vec{v}(t) = \vec{r}\,'(t) = \langle 2t, 5, 2t-16\rangle$

speed $v(t) = \sqrt{(2t)^2 + 25 + (2t-16)^2}$

$$= \sqrt{4t^2 + 25 + 4t^2 - 64t + 256}$$

$$= \sqrt{\boxed{8t^2 - 64t + 281}}$$

look at quadratic separately.

$$8\left(t^2 - 8t + \frac{281}{8}\right) \qquad \left(\frac{-8}{2}\right)^2 = (-4)^2$$

$$= 8\left(t^2 - 8t + (-4)^2 - (-4)^2 + \frac{281}{8}\right)$$

$$= 8\left((t-4)^2 - 16 + \frac{281}{8}\right)$$

$$= 8\left((t-4)^2 + \frac{281-128}{8}\right)$$

where min occurs.

$$= 8(t-4)^2 + 153 \qquad \text{vertex: } (4, 153)$$

min of quad.

min speed occurs when $t=4$

min speed $= \sqrt{153}$

(35) $\vec{r}(t)$ is position vector of a particle.

$$\vec{r}\,'(t) = \vec{c} \times \vec{r}(t)$$

$\vec{c}$ constant vector.

$\vec{a} \times \vec{b} = \vec{c}$

$\perp$

describe the path of the particle.

$\vec{r}\,' \perp \vec{r} \qquad \vec{r}\,' \perp \vec{c}$

$\Rightarrow$ velocity vector $\perp$ position vector. for all t.

$\Rightarrow$ we have circular motion.

path is a circle.

$\frac{d}{dt} \|\vec{r}(t)\|^2 = \frac{d}{dt} c$

$2\|\vec{r}(t)\| \underline{\frac{d}{dt}\|\vec{r}\|} = 0$

or

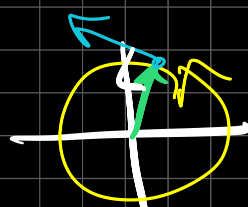
$\frac{d}{dt}(\underbrace{\vec{r} \cdot \vec{r}}_{\|\vec{r}\|^2}) = \vec{r}\,' \cdot \vec{r} + \vec{r} \cdot \vec{r}\,' = 0$

$2\vec{r} \cdot \vec{r}\,' = 0$

$\vec{r} \cdot \vec{r}\,' = 0$

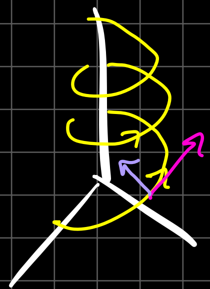
$\vec{r} \cdot \vec{v} = 0$

$\underline{\vec{r} \perp \vec{v}} \Leftrightarrow \underline{\|\vec{r}\| = c}$



39 $\vec{r}(t) = \cos t\,\hat{\imath} + \sin t\,\hat{\jmath} + t\hat{k}$

find tangential & normal components of acceleration.



$\vec{v}(t) = -\sin t\,\hat{\imath} + \cos t\,\hat{\jmath} + \hat{k}$

$\vec{a}(t) = -\cos t\,\hat{\imath} - \sin t\,\hat{\jmath}$

too hard to decompose.

$$\boxed{\vec{a} = a_T\vec{T} + a_N\vec{N}}$$

$a_T = v'$

$a_N = \kappa v^2$

$\vec{T} = \dfrac{\vec{r}\,'}{\|\vec{r}\,'\|} = \dfrac{\vec{v}(t)}{v(t)}$

$\vec{N} = \dfrac{\vec{T}\,'}{\|\vec{T}\,'\|} = ?$

$\|\vec{T}\,'\|\vec{N} = \vec{T}\,'$

use $\kappa = \dfrac{\|\vec{T}\,'\|}{\|\vec{r}\,'\|}$

$v\kappa = \|\vec{T}\,'\|$

speed.

then $\vec{N} = \dfrac{\vec{T}\,'}{v\kappa}$

$\vec{v}(t) = -\sin t\,\hat{\imath} + \cos t\,\hat{\jmath} + \hat{k}$

$v(t) = \sqrt{2}$ $\quad v'(t) = 0$

$\vec{T}(t) = -\frac{1}{\sqrt{2}}\sin t\,\hat{\imath} + \frac{1}{\sqrt{2}}\cos t\,\hat{\jmath} + \frac{1}{\sqrt{2}}\hat{k}$

$\vec{T}\,'(t) = -\frac{1}{\sqrt{2}}\cos t\,\hat{\imath} - \frac{1}{\sqrt{2}}\sin t\,\hat{\jmath}$

$\|\vec{T}\,'(t)\| = \frac{1}{\sqrt{2}}$

$\vec{N} = -\cos t\,\hat{\imath} - \sin t\,\hat{\jmath}$

$\kappa = \dfrac{\frac{1}{\sqrt{2}}}{\sqrt{2}} = \frac{1}{2}$

$\vec{a} = v'\vec{T} + \kappa v^2\vec{N}$

$= (0)\vec{T} + \left(\frac{1}{2}\right)(\sqrt{2})^2\vec{N}$

$= \vec{N} = -\cos t\,\hat{\imath} - \sin t\,\hat{\jmath}$