HW 8

x:	27	45	72	58	31	60	34	74
y:	250	285	320	295	265	298	267	321.

$\bar{x} = 50.125$ $\quad$ $\bar{y} = 287.625$

$S_x = 18.497$ $\quad$ $S_y = 25.867$

$\Sigma X^2 = 22495$ $\quad$ $\Sigma Y^2 = 666509$ $\quad$ $\Sigma XY = 118652$

$\sum_{i=1}^{8} X_i^2$ $\quad$ $\sum_{i=1}^{8} y_i^2$ $\quad$ $\sum_{i=1}^{8} X_i y_i$

$\boxed{\text{Thursday } 9:30 - 10:30 \text{ pm}}$

$S_{XX} = \Sigma X^2 - n\bar{X}^2 = \underline{2394.875}$

$= (n-1) \cdot S_x^2 = 7 * 18.497^2 = \underline{2394.97}$

$\sum_{i=1}^{n} (x_i - \bar{x})^2 = (n-1) \cdot S_x^2$

$S_{yy} = (n-1) \cdot S_y^2 = 7 * 25.867^2 = 4683.7$

$S_{xy} = \Sigma xy - n\bar{x}\bar{y}$

$= 118652 - 8 * 50.125 * 287.625$

$= 3314.375$

$b_1 = \frac{S_{xy}}{S_{xx}} = \frac{3314.375}{2394.875} = 1.384$

$b_0 = \bar{y} - b_1\bar{x} = 287.625 - 1.384 * 50.125$

$= 218.25$

① $\hat{y} = 218.25 + 1.384x.$

② $\hat{y} = 218.25 + 1.384 * 65 =$

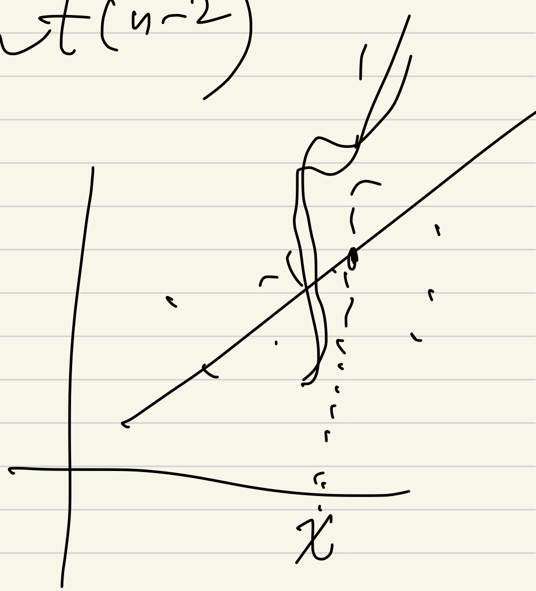
③ $s^2 = \frac{SSE}{n-2} = \frac{S_{yy} - b_1 S_{xy}}{n-2}$

$$= \frac{4683.7 - 1.384 * 3314.375}{6}$$

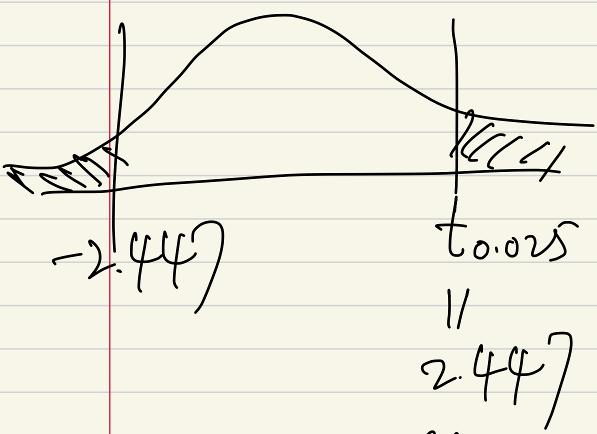
$$= \frac{96.6}{6} = 16.1 \qquad s = 4.01$$

(4) $H_0: \beta_1 = 0 \qquad H_1: \beta_1 \neq 0. \qquad \alpha = 0.05$

under H_0: $\dfrac{b_1 - 0}{s/\sqrt{S_{xx}}} \sim t(n-2)$



$$C = \{ |t(6)| > 2.447 \}$$



$$t_{obs} = \frac{1.384}{4.01 / \sqrt{2394.875}} = 16.89 \in C$$

reject H_0.

$$p\text{-value} = 2 * P(t(6) > t_{obs}) \approx 0$$

$t_{obs} = 16.89$

Reject H_0.

5) $$R^2 = 1 - \frac{SSE}{SST} = 1 - \frac{SSE}{S_{yy}}$$

$$= 1 - \frac{96.6}{4683.7}$$

$$= 0.98$$

6) 95% CI for β_1

WS. 7.
Q7.

①

	Men	Women	total
chosen	X 8	10-X 2	10
not chosen	1	4	5
Total	9	6	15

②

$x = 9,$	8	7	6	5	4
0	1	2	3	4	5

③ $$P(X=x) = \frac{\binom{9}{x}\binom{6}{10-x}}{\binom{15}{10}}$$

$x = 4, 5, 6, 7, 8, 9$

$$E(X) = 4*P(4) + 5*P(5) + \cdots + 9*P(9)$$

$\stackrel{?}{=} \text{---} \; 6$

④ under H_0, $E(X) = 6$. $X_{obs} = 8$

$$p\text{-value} = P(X \geq 8) = P(8) + P(9)$$

$$= \frac{\binom{9}{8}\cdot\binom{6}{1}}{\binom{15}{10}} + \frac{\binom{9}{9}\binom{6}{0}}{\binom{15}{10}} = \_\_\_$$

5 If p-value < 0.05, yes ✓

$\not<$ no.

Q6. $H_0: \mu_1 = \mu_2$

$H_1: \mu_1 > \mu_2$.

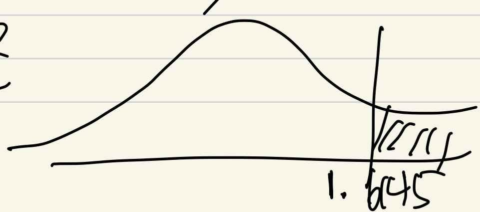
normal, $\sigma_1^2 = \sigma_2^2 = 32$.

$n_1 = n_2 = 25$, $\alpha = 0.05$, what's C?

1 under H_0, $\bar{X} - \bar{Y} \sim N\left(0, \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}\right)$

$$= N\left(0, \frac{64}{25}\right)$$

$$\frac{\bar{X} - \bar{Y} - 0}{8/5} \sim Z$$



$$C = \left\{ \frac{\bar{X} - \bar{Y}}{8/5} > 1.645 \right\} = \left\{ \bar{X} - \bar{Y} > 2.632 \right\}$$

② If $\mu_1 = \mu_2 + 3$ what's the power $= ?$

$$\text{power} = 1 - \beta = P\left(\text{rej } H_0 \text{ when } \mu_1 = \mu_2 + 3\right)$$

$$= P\left(\bar{X} - \bar{Y} > 2.632,\ \mu_1 - \mu_2 = 3\right)$$

$$= P\left(Z > \frac{2.632 - 3}{8/5}\right)$$

$$= P(Z > -0.23) = 0.5910$$

or: If $\mu_1 = \mu_2 + 3$, what's β?

$$\beta = P\left(\text{not rej } H_0,\ \mu_1 = \mu_2 + 3\right)$$

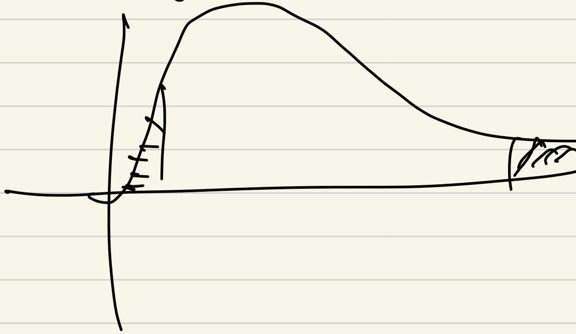
$$= P\left(\bar{x} - \bar{y} < 2.632,\ \mu_1 - \mu_2 = 3\right)$$

Decision \ reality	H_0 true	H_0 false
Reject H_0	Type I α	✓ power
Not rej H_0	✓	Type II β

WS 7. Q1,

① H_0. $\sigma_1^2 = \sigma_2^2$. H_1: $\sigma_1^2 \neq \sigma_2^2$.

use t_0: $\frac{S_1^2}{S_2^2} \sim f(24, 24)$



② H_0: $\mu_1 = \mu_2$

H_1: $\mu_1 < \mu_2$

under H_0, $\dfrac{\bar{X} - \bar{Y}}{S_p \cdot \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \sim t(48)$

Quiz. 6.

Q1. $H_0: p = 0.75$ $\quad H_1: p \neq 0.75$

Better: $H_0: p \geq 0.75$ $\quad H_1: p < 0.75$

under H_0, $Y \sim Bin(25, 0.75) \sim N(18.75, 4.6875)$

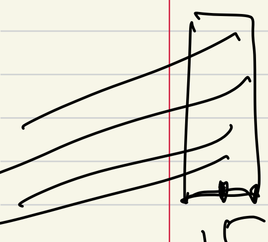
$Y_{obs} = 15$.

$np \geq 5$

$nq \geq 5$

p-value $= P($tail prob of obsered value of the test stat$)$

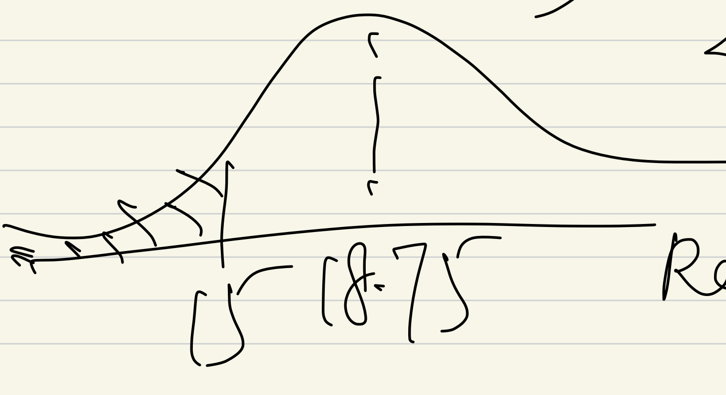
$= P(Y \leq 15) = P\left(Z \leq \dfrac{15.5 - 18.75}{\sqrt{4.6875}}\right)$



15

$= P(Z < -1.5) = 0.0668$

$< \alpha$



15 18.75

Rej H_0

why not use $\hat{p}$? use $\hat{p}$ when n is large.

$$Y \sim Bin(n, p) \sim N(np, npq)$$

$$\hat{p} = \frac{Y}{n} \sim N\left(p, \frac{pq}{n}\right)$$

WS 7.

Q3.

	smoker	non-smoker	
TB	42	218	260
w/o TB	15	245	260

$P(TB \mid Smoker) = ?$

1. $H_0: p_1 = p_2$ $\quad \hat{p}_1 = \frac{42}{260}$

$H_1: p_1 \neq p_2$. $\quad \hat{p}_2 = 15/260$

$$\hat{p}_0 = \frac{57}{520} = \text{—}$$

under H_0:

$$\frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}_0 \hat{q}_0 \left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} \sim Z.$$

$$z_{obs} = \frac{\frac{42}{260} - \frac{15}{260}}{\sqrt{\cdots}} \approx 3.79.$$

p-value $< 2 * 0.0002 < \alpha$.