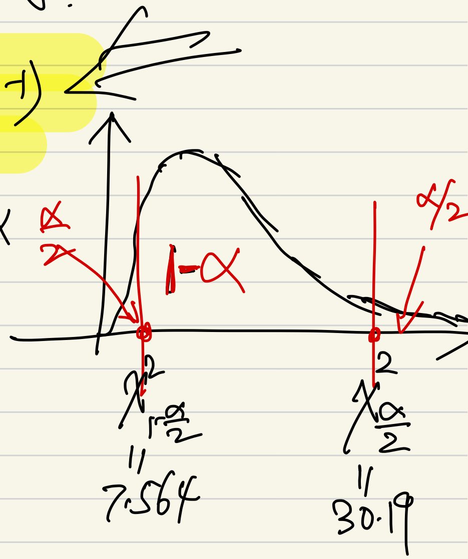
Quiz 5. Q1.

$\sim N(\mu, \sigma^2)$ $\quad n=18, \quad \bar{x}=82, \quad s=3.$

Find a 95% CI for σ.

$$\frac{(n-1)S^2}{\sigma^2} \sim \chi^2(n-1)$$



$$P\left(\boxed{7.564 < \frac{(n-1)S^2}{\sigma^2} < 30.19}\right) = 1-\alpha$$

Solve for σ^2.

$$\frac{(n-1)s^2}{30.19} < \frac{\sigma^2}{\cancel{(n-1)s^2}} < \frac{(n-1)s^2}{7.564}$$

$$\left(\sqrt{\frac{17*9}{30.19}},\ \sqrt{\frac{17\times 9}{7.564}}\right) = (2.25, 4.50)$$

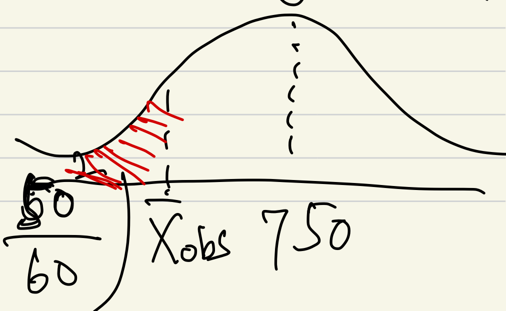
Q2. $\quad n=60, \quad \bar{x}=730, \quad s=50$

Do you have evidence to rej. $\mu \geq 750$?

$H_0: \mu \geq 750$

$H_1: \mu < 750$

under H_0, $\bar{X} \sim N\left(750, \frac{50}{60}\right)$



(a) p-value.

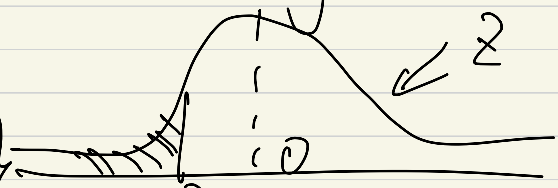
$$P(\bar{X}<730) = P\left(Z < \frac{730-750}{50/\sqrt{60}}\right)$$

$$= P(Z < -3.10) \approx 0.001 < \alpha$$

$\Rightarrow$ Rej H_0.

(b) C-Region.

$$C = \left\{ \frac{\bar{X}-750}{50/\sqrt{60}} < -1.75 \right\}$$



$$-z_{0.04} = -1.75$$

$$= \left\{ \bar{X} < 750 - 1.75 \cdot \frac{50}{\sqrt{60}} \right\}$$

$$= \{ \bar{X} < 738.7 \}$$

$\bar{X}_{obs} = 730 \in C$

$\Rightarrow$ Rej H_0.

(c)

One more Quiz on Monday, Dec 6.

§ 10.5 – 10.13

Last Wednesday,

① find size n such that the test satisfies specific requirement on α & β.

$$\boxed{n = \frac{(z_\alpha + z_\beta)^2 \sigma^2}{\delta^2}} \quad \text{1-sided}$$

or

$$n = \frac{(z_{\frac{\alpha}{2}} + z_\beta)^2 \sigma^2}{\delta^2}$$

2-sided

Before: μ, $\mu_1 - \mu_2$

② tests on p ✓

③ - - - - $p_1 - p_2$ } often Z test. can be Binomial

④ - - - - σ^2 — χ^2-test

$$\hat{p}_0 = \frac{y_1 + y_2}{n_1 + n_2}$$

Today: Test on $\frac{\sigma_1^2}{\sigma_2^2}$.

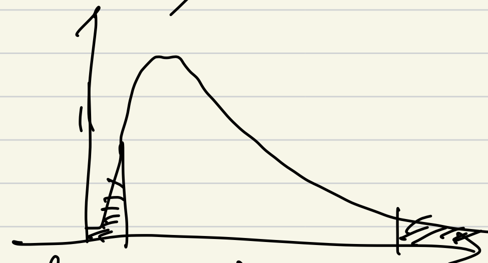
$$\frac{S_1^2/\sigma_1^2}{S_2^2/\sigma_2^2} \sim f(n_1-1, n_2-1)$$

$H_0: \sigma_1^2 = \sigma_2^2$

$H_1: \sigma_1^2 \neq \sigma_2^2$ (2-sided)

$\left.\begin{array}{l} \sigma_1^2 > \sigma_2^2 \\ \sigma_1^2 < \sigma_2^2 \end{array}\right\}$ 1-sided.

under H_0: $\dfrac{S_1^2}{S_2^2} \sim f(n_1-1, n_2-1)$



$f_{1-\frac{\alpha}{2}}(n_1-1, n_2-1) = \dfrac{1}{f_{\frac{\alpha}{2}}(n_2-1, n_1-1)}$

$f_{\frac{\alpha}{2}}(n_1-1, n_2-1)$

Ex: 2 normal populations

from popul. $n_1 = 10$ $\bar{x}_1 = 80$, $s_1 = 4$

from popu 2. $n = 9$. $\bar{x}_2 = 78$, $s_2 = 5$

2-samples are indep.

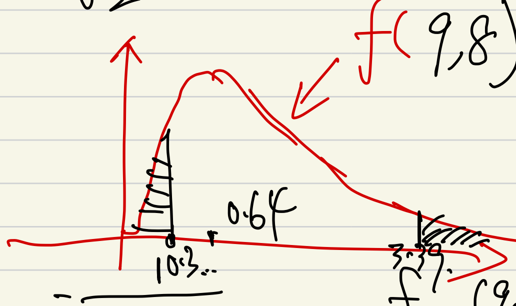
Test $H_0: \sigma_1^2 = \sigma_2^2$

$H_1: \sigma_1^2 \neq \sigma_2^2$

$\alpha = 0.10$

$f_{obs} = \dfrac{s_1^2}{s_2^2} = \dfrac{16}{25} = 0.64$

$\notin C$



$0.3 = \dfrac{1}{3.23} = \dfrac{1}{f_{0.05}(8,9)}$

$f_{0.05}(9,8) = 3.39$

$\Rightarrow$ Can not rej $H_0: \sigma_1^2 = \sigma_2^2$

Chi-square

§10.11 Goodness-of-Fit Test

Ex: Toss a die 180 times and you get the following data

	1	2	3	4	5	6	
observed freq	32	35	26	30	31	26	$\Leftarrow O_i$
expected	30	30	30	30	30	30	$\Leftarrow e_i$

Test at $\alpha = 0.05$ that this is a fair die

H_0: $p_1 = p_2 = p_3 = p_4 = p_5 = p_6 = \frac{1}{6}$

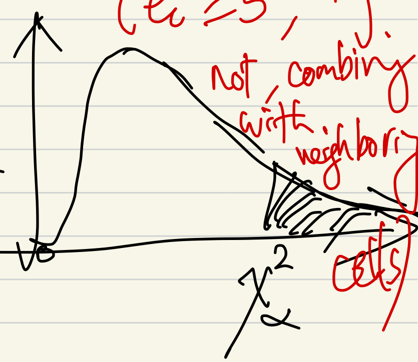
H_1: Not H_0.

under H_0: $\chi^2 = \sum_{i=1}^{n} \frac{(e_i - O_i)^2}{e_i} \sim \chi^2(n-1)$

($e_i \geq 5$, if not, combining with neighboring cells)

$$\chi^2_{obs} = \frac{4^2}{30} + \frac{1^2}{30} + \frac{0^2}{30} + \frac{2^2}{30} + \frac{5^2}{30} + \frac{4^2}{30}$$

$$= \frac{62}{30} = 2.07$$



$$C = \{ \chi^2 > \chi^2_{0.05}(5) = 11.07 \}$$

$$\chi^2_{obs} = 2.07 \notin C \Rightarrow$$ can not rej H_0.

Ex: Test a random data of 40 can be fitted by a $N(3.5, 0.7^2)$

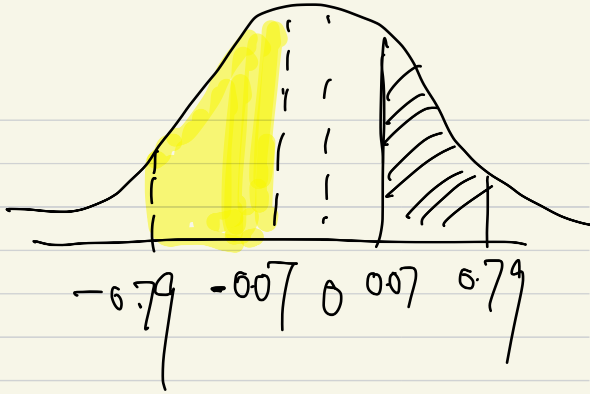
	observed		expected		
1.45-1.95	2	7	0.5	8.5	$\leftarrow \frac{1.5^2}{0.5} = 4.5$
1.95-2.45	1		2.1		
2.45-2.95	4		5.9		
2.95-3.45	15		10.3		$\frac{4.7^2}{10.3} = 2.1$
3.45-3.95	10		10.7		
3.95-4.45	5	8	7.0	10.5	
4.45-4.95	3		3.5		

$$P(2.95 < X < 3.45)$$
$$= P\left(\frac{2.95-3.5}{0.7} < Z < \frac{3.45-3.5}{0.7}\right)$$
$$= P(-0.79 < Z < -0.07)$$

$= 0.7852 - 0.5279$

$= 0.2573$

$40 \times 0.2573 = 10.3$



§10.12. Test for independence (categorical data)

last Wed.	Yes	No	
< 250 mile	14 (14.5)	28 (27.5)	42
1000 > > 250 miles	5 (4.83)	9 (9.17)	14
> 1000	10 (9.67)	18 (18.37)	28
	29	55	84

H_0: Attend class or not last wed. was indep of how far the student's home is.

H_1: Not independent.

$$P(<250 \cap \text{Yes}) = P(<250) \cdot P(\text{Yes})$$

$$= \frac{42}{84} \cdot \frac{29}{84}$$

$$e_i(<250, \cap \text{Yes}) = 84 \cdot \frac{42}{84} \cdot \frac{29}{84} = \frac{42 \cdot 29}{84}$$

$$e_{ij} = \frac{i^{th}\text{ row sum} \times j^{th}\text{ colume sum}}{\text{total sum}}$$

under H_0, $$\chi^2 = \sum_{i=1}^{r} \sum_{j=1}^{c} \frac{(e_{ij} - O_{ij})^2}{e_{ij}}$$

$$\sim \chi^2((r-1)(c-1))$$

10.13. Test for homogeneity.

$$\boxed{\boxed{\text{Real } 10.3}}$$