Cross Product

$\mathbb{R}^3$ or V_3

$\vec{a} = \langle a_1, a_2, a_3 \rangle$

$\vec{b} = \langle b_1, b_2, b_3 \rangle$

$$\vec{a} \times \vec{b} = \langle a_2 b_3 - b_2 a_3,\ a_3 b_1 - b_3 a_1,\ a_1 b_2 - b_1 a_2 \rangle$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

<u>12.4</u>

① $\vec{a} = \langle 2, 3, 0 \rangle \qquad \vec{b} = \langle 1, 0, 5 \rangle$

$$\vec{a} \times \vec{b} = \langle (3)(5) - (0)(0),\ (0)(1) - (5)(2),\ (2)(0) - (1)(3) \rangle$$
$$= \langle 15, -10, -3 \rangle$$

$\vec{a} \cdot \vec{b} = 2 \quad \vec{a} \not\perp \vec{b}$

is $\vec{a} \times \vec{b} \perp$ to $\vec{a}$ or $\vec{b}$?

$$\vec{a} \cdot (\vec{a} \times \vec{b}) = 0$$
$$\vec{b} \cdot (\vec{a} \times \vec{b}) = 0$$

in general $\quad \vec{u} \times \vec{v} \perp \vec{u}$ AND $\vec{v}$

what if in problem ① we found $\vec{b} \times \vec{a} = \langle -15, 10, 3 \rangle$

$$= -\vec{a} \times \vec{b}$$
$$= (-\vec{a}) \times \vec{b}$$
$$= \vec{a} \times (-\vec{b})$$

Comment

$$(\alpha\vec{a})\times(\beta\vec{b})$$

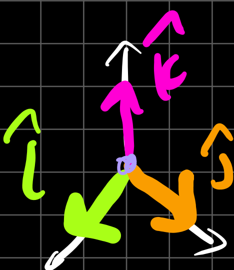
$\vec{a} = \langle 2,3,0\rangle = 2\hat{i}+3\hat{j}$

$\vec{b} = \langle 1,0,5\rangle = \hat{i}+5\hat{k}$

$$\vec{a}\times\vec{b} = (2\hat{i}+3\hat{j})\times(\hat{i}+5\hat{k})$$

$$=(2\hat{i})\times\hat{i} + (2\hat{i})\times(5\hat{k}) + (3\hat{j})\times(\hat{i}) + (3\hat{j})\times(5\hat{k})$$

$$= 2\underbrace{(\hat{i}\times\hat{i})}_{\vec{0}} + 10\underbrace{(\hat{i}\times\hat{k})}_{-\hat{j}} + 3\underbrace{(\hat{j}\times\hat{i})}_{-\hat{k}} + 15\underbrace{(\hat{j}\times\hat{k})}_{\hat{i}}$$



$$= -10\hat{j} - 3\hat{k} + 15\hat{i}$$

$$= 15\hat{i} - 10\hat{j} - 3\hat{k}$$

Right Hand Rule.

$\vec{a}\times\vec{a} = \vec{0}$

$\vec{a} \parallel \vec{b}$

$\vec{a}\times\vec{b} = \vec{0}$

⑥ $\vec{a} = t\hat{i} + \cos t\,\hat{j} + \sin t\,\hat{k}$

$\vec{b} = \hat{i} - \sin t\,\hat{j} + \cos t\,\hat{k}$

find $\vec{a}\times\vec{b}$

$$(t\hat{i} + \cos t\,\hat{j} + \sin t\,\hat{k})\times(\hat{i} - \sin t\,\hat{j} + \cos t\,\hat{k})$$

$$= -t\sin t\,\hat{k} + t\cos t\,(-\hat{j}) + \cos t\,(-\hat{k}) + \cos^2 t\,(\hat{i}) + \sin t\,(\hat{j}) - \sin^2 t\,(-\hat{i})$$

$$= \hat{i} + (\sin t - t\cos t)\,\hat{j} - (\cos t + t\sin t)\,\hat{k}$$

(11) $(\hat{j}-\hat{k})\times(\hat{k}-\hat{i}) = \hat{i} - (-\hat{k}) + \hat{j} = \hat{i}+\hat{j}+\hat{k}$

(13) $\vec{a}\cdot(\vec{b}\times\vec{c})$ ✓ are any meaningful?

vector • vector = scalar.

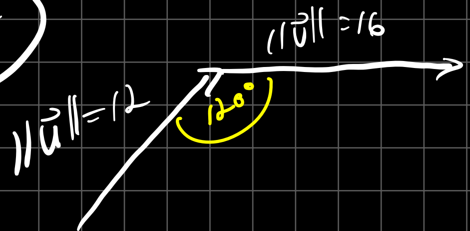
$\vec{a}\times(\vec{b}\times\vec{c})$ ✓

vector x vector = vector

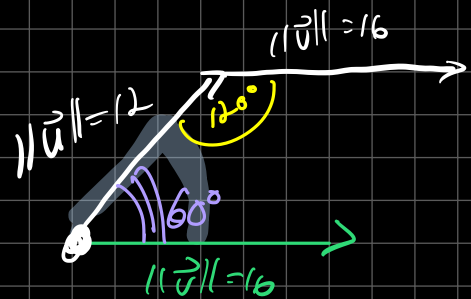
$(\vec{a}\cdot\vec{b})\times(\vec{c}\cdot\vec{d})$ X scalar X scalar

Not defined

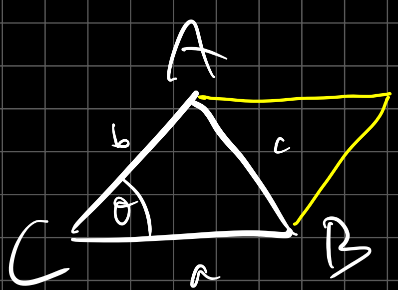
(15)



find $\|\vec{u}\times\vec{v}\| = \|\vec{u}\|\,\|\vec{v}\|\sin\theta$



$$\|\vec{u}\times\vec{v}\| = 12(16)\sin(60^\circ)$$
$$= (12)(16)(\sqrt{3}/2)$$
$$= 96\sqrt{3}$$



What is the significance

Area of 

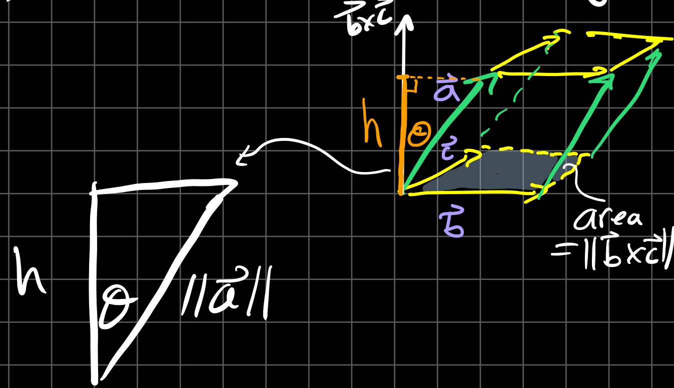
$A = \frac{1}{2}ab\sin C$

what direction does $\vec{u}\times\vec{v}$

point in? into page.

(33) $\vec{a} = \langle 1, 2, 3 \rangle$
$\vec{b} = \langle -1, 1, 2 \rangle$
$\vec{c} = \langle 2, 1, 4 \rangle$

$\Big\}$ good enough for us to find volume of parallelepiped



$$\cos\theta = \frac{h}{\|\vec{a}\|} \Rightarrow h = \|\vec{a}\|\cos\theta$$

$$\text{Volume} = \left| \|\vec{b}\times\vec{c}\| \|a\| \cos\theta \right|$$

$$= \left| \vec{a}\cdot(\vec{b}\times c) \right|$$

$\hookrightarrow$ scalar triple product.

Now find volume of parallelepiped formed by:

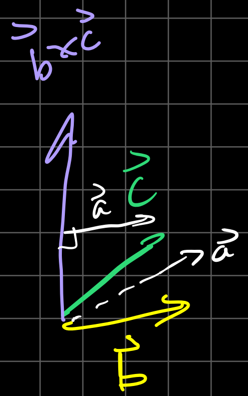
$\vec{a} = \langle 1, 2, 3 \rangle$
$\vec{b} = \langle -1, 1, 2 \rangle$
$\vec{c} = \langle 2, 1, 4 \rangle$

$$\vec{b}\times\vec{c} = \langle 2, 8, -3 \rangle$$

$$|\vec{a}\cdot(\vec{b}\times\vec{c})| = 2 + 16 + (-9) = 9$$

follow up: when is $\vec{a}\cdot(\vec{b}\times\vec{c}) = 0$?

$\vec{a}, \vec{b}, \vec{c}$ are all in same plane.



$\vec b \parallel \vec c$

$\vec b$ or $\vec c = \vec 0$

or $\vec a = \vec 0$

$\vec a \perp \vec b \times \vec c$

coplanar

Quick comments:

$\vec a \times \vec b = -\vec b \times \vec a$

$(\alpha \vec a) \times (\beta \vec b) = (\alpha\beta)(\vec a \times \vec b)$

$\vec a \times (\vec b + \vec c) = \vec a \times \vec b + \vec a \times \vec c$

may be interesting to look at #'s 47, 48.

Almost done w/ worksheet

Quiz Th-F open 12 am Th timed due F 11:59 pm.
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