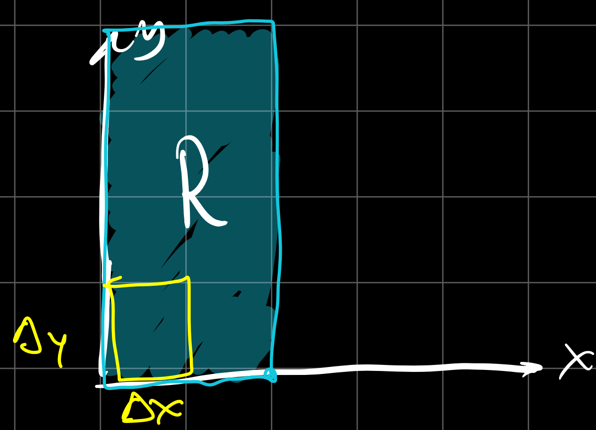
Double Integrals 15.1, 15.2

15.1

(10) $\iint_R (2x+1)\,dA \quad R=\{(x,y) \mid 0 \le x \le 2,\ 0 \le y \le 4\}$

$z = 2x+1$



$\Delta A = \Delta x \Delta y$

$dA = dx\,dy.$

$$\iint_R (2x+1)\,dA$$

$$= \int_0^4 \int_0^2 (2x+1)\,dx\,dy$$

we can switch!!

$$= \int_0^2 \underbrace{\int_0^4 (2x+1)\,dy}_{\text{int. w.r.t } y.}\,dx$$

or

find $\frac{\partial}{\partial y}(\ ?\ ) = 2x+1$

$\downarrow$

$2xy+y$

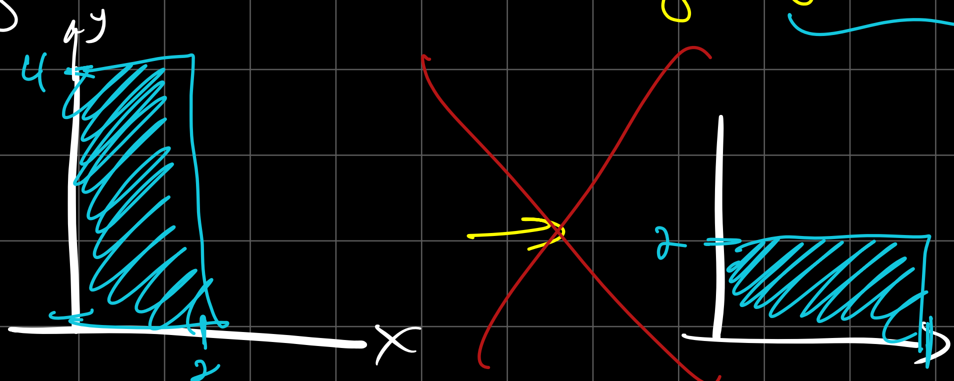
$$= \int_0^2 (2xy+y)\Big|_{y=0}^{y=4}\, dx = \int_0^2 (8x+4)\, dx$$

$$= (4x^2+4x)\Big|_{x=0}^{x=2}$$

$$= 16+8=24.$$

Q: if you switch dx, dy but don't switch integrals.

$$\int_0^2\int_0^4 f(x,y)\,dy\,dx \longmapsto \int_0^2\int_0^4 f(x,y)\,dx\,dy$$



change domain.

(13) $f(x,y)= x+3x^2y^2$ find $\int_0^2 f(x,y)\,dx$

$$\int_0^2 (x+3x^2y^2)\,dx = \left(\frac{x^2}{2} + x^3y^2\right)\Big|_{x=0}^{x=2}$$

$$= 2+8y^2$$

Two things. ① $\iint f(x,y)\,dA$

need a domain. No indefinite double integrals here.

② look at: $\int (x+3x^2y^2)\,dx = \underbrace{\frac{x^2}{2}+x^3y^2}+g(y)$

indef: $\int f(x,y)\,dx$

$\frac{\partial}{\partial x}\left(\frac{x^2}{2}+x^3y^2+g(y)\right)$

$= x+3x^2y^2$

Comment: Some authors write: $\iint_R f(x,y)\,dA$

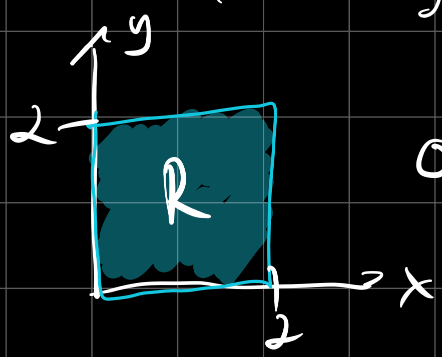
$= \int_R f(x,y)\,dA$

⑰ $\int_0^1 \underbrace{\int_1^2 (x+e^{-y})\,dx}_{\text{treat } y \text{ constant.}}\,dy$

$= \int_0^1 \left(\frac{x^2}{2}+xe^{-y}\right)\Big|_{x=1}^{x=2} dy = \int_0^1 2+2e^{-y}-\left(\frac{1}{2}+e^{-y}\right)dy$

$= \int_0^1 \left(\frac{3}{2}+e^{-y}\right)dy = \left(\frac{3}{2}y-e^{-y}\right)\Big|_{y=0}^{y=1}$

(33) $\iint\limits_R y e^{-xy}\, dA \qquad R = \underbrace{[0,2]}_{x} \times \underbrace{[0,2]}_{y}$



$0 \le x \le 2$

$0 \le y \le 2$

$$= \int_0^2 \int_0^2 y e^{-xy}\, dx\, dy$$

$$= \int_0^2 \left(-e^{-xy}\right)\Big|_{x=0}^{x=2} dy$$

Think: $\int y e^{-xy}\, dx = y \int e^{-xy}\, dx$

$y\left(\frac{e^{-xy}}{-y}\right)$

$-e^{-xy}.$

$$= \int_0^2 \left(-e^{-2y} + 1\right) dy$$

Missing limits

$$= \left(\frac{e^{-2y}}{2} + y\right)\Big|_{y=0}^{y=2}$$

$$= \dots$$

(*) If we tried: $\int_0^2 \int_0^2 y e^{-xy}\, dy\, dx$ need IBP.

Comment: if $f(x,y) = g(x)h(y)$ over $R = [a,b]\times[c,d]$
Rectangle.

$$\iint\limits_R f(x,y)\, dA = \int_a^b g(x)\,dx \int_c^d h(y)\,dy$$

Ex) $\int_0^1 \int_0^2 x y^2\, dx\, dy = \int_0^1 y^2\, dy \int_0^2 x\, dx$

(47) $f(x,y) = x^2 y$ find avg value of f over Rectangle with vertices. $(-1,0)$, $(-1,5)$, $(1,5)$, $(1,0)$

$$f_{avg} = \frac{\iint_R f(x,y)\,dA}{A(R)} = \frac{\iint_R f(x,y)\,dA}{\iint_R 1\,dA}$$

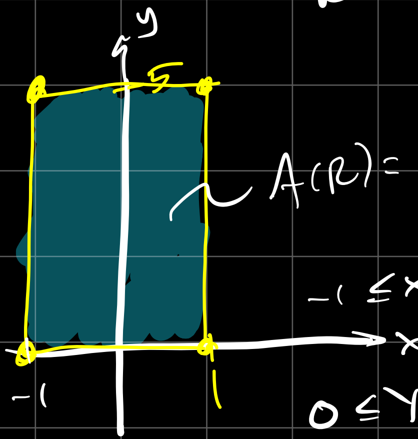
Calc II

$f(x)$ on $[a,b]$

$$f_{avg} = \frac{\int_a^b f\,dx}{b-a}$$

$$\int_a^b 1\,dx$$

find $A(R)$



$A(R) = (2)(5) = 10.$

$-1 \le x \le 1$

$0 \le y \le 5$

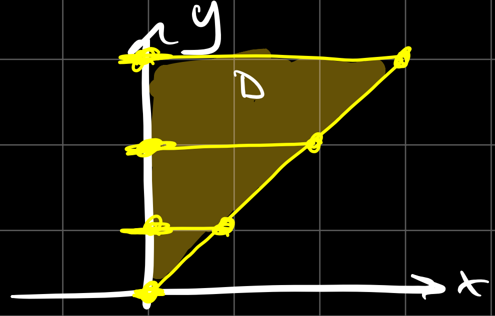
$$\iint_R 1\,dA = A(R).$$

$$f_{avg} = \frac{\int_0^5 \int_{-1}^1 x^2 y\,dx\,dy}{10} = \frac{\int_0^5 y\,dy \int_{-1}^1 x^2\,dx}{10} = \frac{\left(\frac{25}{2}\right)\left(\frac{2}{3}\right)}{10} = \frac{25}{30} = \frac{5}{6}.$$

15.2

(9) $\iint_D e^{-y^2}\,dA$ $D = \{(x,y) \mid 0 \le y \le 3,\ 0 \le x \le y\}.$

$$= \int_0^3 \int_0^y e^{-y^2}\,dx\,dy$$



$$= \int_0^3 \left(x e^{-y^2} \right) \Big|_{x=0}^{x=y} dy = \int_0^3 y e^{-y^2} dy$$

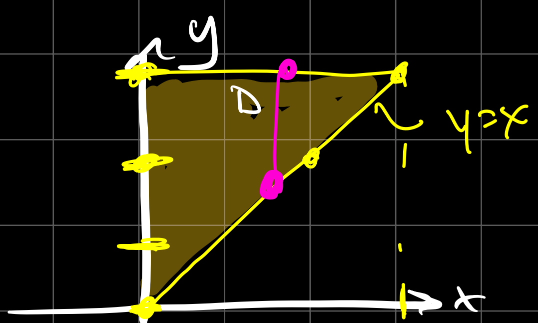
$u = -y^2$

$du = -2y\,dy$

$$= -\frac{1}{2} \int_0^{-9} e^u \, du$$

$$= -\frac{1}{2} \left(e^u \right) \Big|_{u=0}^{u=-9}$$

easy!!



Can we switch order?

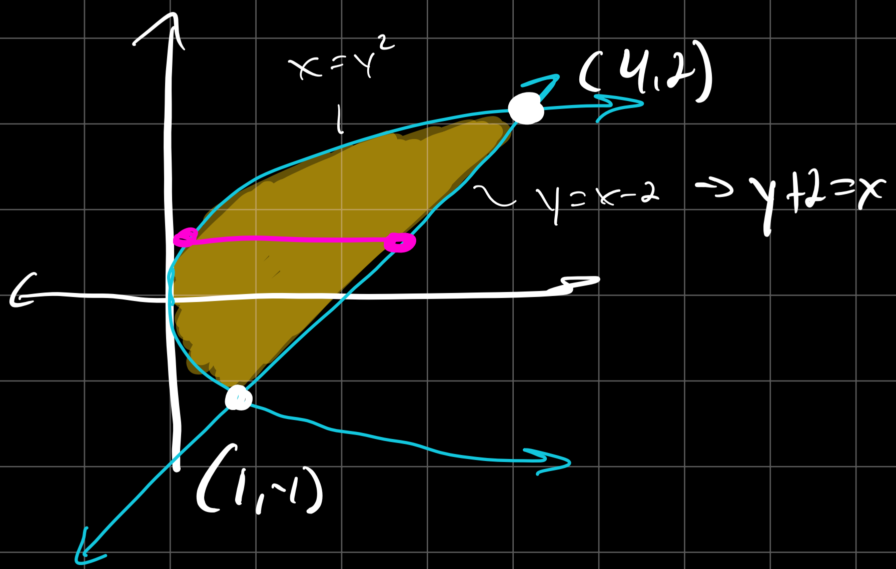
given $0 \le y \le 3$ $\quad$ $0 \le x \le 3$

$0 \le x \le y$ $\quad$ $x \le y \le 3$

then: $\displaystyle \iint_D f(x,y)\, dA = \underbrace{\int_0^3 \int_x^3 e^{-y^2} dy\, dx}_{\text{not possible for us.}}$

(15) $\iint_D y\,dA$ D is bounded by $y = x-2$

$x = y^2$



find intersections

$y = y^2 - 2$

$y^2 - y - 2 = 0$

$(y-2)(y+1) = 0$

$y=2$, $y=-1$

$x=4$ $\quad$ $x=1$

$-1 \le y \le 2$

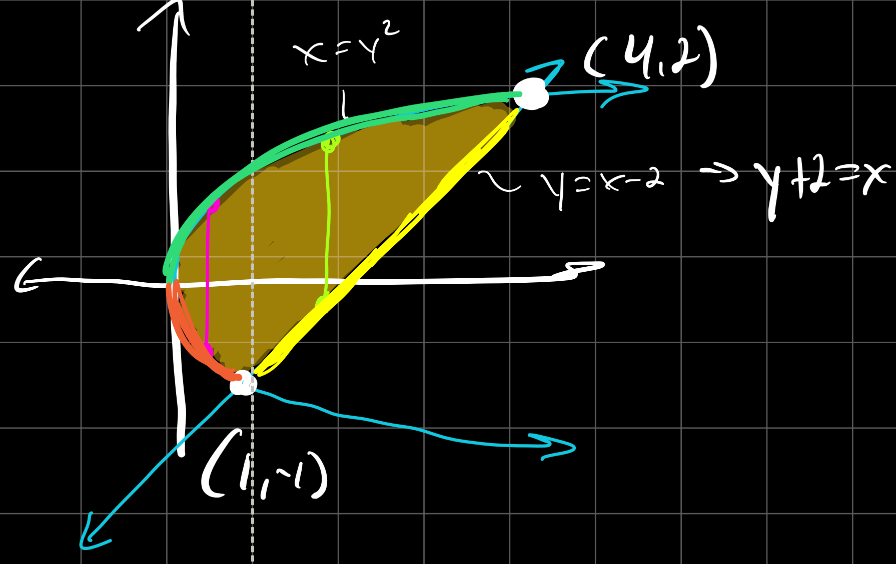
parab. $\le x \le$ line

$y^2 \le x \le y+2$

$$\iint_D y\,dA = \int_{-1}^{2}\int_{y^2}^{y+2} (y)\,dx\,dy = \int_{-1}^{2} (xy)\Big|_{x=y^2}^{x=y+2} dy$$

$$= \int_{-1}^{2} \left[(y+2)y - y^2(y)\right] dy = \underline{\text{continue.}}$$

What if we switch limits?



$0 \le x \le 1$ $\quad$ $1 \le x \le 4$

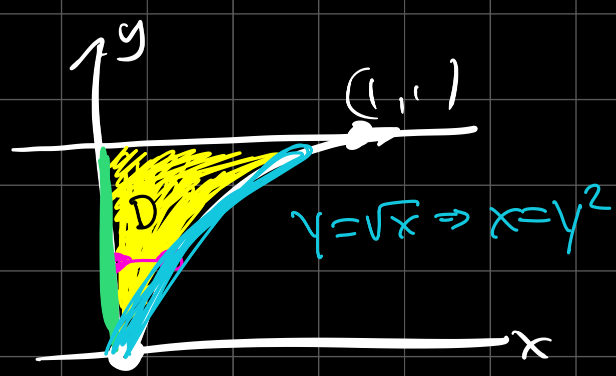
$-\sqrt{x} \le y \le \sqrt{x}$ $\quad$ $x-2 \le y \le \sqrt{x}$

$$\iint_D y\,dA = \int_0^1 \int_{-\sqrt{x}}^{\sqrt{x}} y\,dy\,dx + \int_1^4 \int_{x-2}^{\sqrt{x}} y\,dy\,dx$$

53 $\displaystyle\int_0^1 \int_{\sqrt{x}}^1 \sqrt{y^3+1}\,dy\,dx$

$\sqrt{x} \le y \le 1$

$0 \le x \le 1$



$0 \le y \le 1$

$0 \le x \le y^2$

$$\int_0^1 \int_{\sqrt{x}}^1 \sqrt{y^3+1}\, dy\, dx = \int_0^1 \int_0^{y^2} \sqrt{y^3+1}\, dx\, dy$$

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$$= \int_0^1 x\sqrt{y^3+1} \Big|_{x=0}^{x=y^2} dy$$

$$= \int_0^1 y^2 \sqrt{y^3+1}\, dy$$

basic substitution!!