Math 140 - Spring 2017 - Midterm Exam 2 - Version B

You have 105 minutes to complete this midterm exam. Books, notes and electronic devices are not permitted. Read and follow directions carefully. Show and check all work. Label graphs and include units where appropriate. If a problem is not clear, please ask for clarification.

Multiple Choice 1-10	/30
Free Response 11	/10
Free Response 12	/10
Free Response 13	/10
Free Response 14	/10
Free Response 15	/10
Total	/80

I pledge that I have completed this midterm exam in compliance with the NYU CAS Honor Code. In particular, I have neither given nor received unauthorized assistance during this exam.

Name ______________________________

N Number ______________________________

Signature ______________________________

Date ______________________________

1 Multiple Choice

(30 points) For problems 1-10, circle the best answer choice.

1. Which of the following is an orthonormal basis of $\mathbb{R}^3$?

   (a) $\left\{ \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}$

   (b) $\left\{ \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ 0 \end{bmatrix}, \begin{bmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \\ 0 \end{bmatrix} \right\}$

   (c) $\left\{ \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}$

   (d) $\left\{ \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ 0 \end{bmatrix}, \begin{bmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}$

   (e) $\left\{ \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix}, \begin{bmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\}$

2. If W is a subspace of $\mathbb{R}^n$ with dimension m, then the dimension of $W^\perp$ is

   (a) n

   (b) m

   (c) $n - m$

   (d) any number between n and $n - m$

   (e) cannot be determined from given information

3. An $n \times n$ matrix A is called *idempotent* if $A^2 = A$. Which of the following are idempotent?

   (a) any permutation matrix

   (b) any projection matrix

   (c) any orthonormal matrix

   (d) (a) and (b)

   (e) all of the above

4. What is the volume of a parallelpiped determined by vectors $\begin{bmatrix} 2 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ -2 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ -5 \\ 1 \end{bmatrix}$?

   (a) 0

   (b) 1

   (c) 2

   (d) 4

   (e) none of the above

5. Suppose A and B are 6×6 matrices with det $A = 10$ and det $B = -5$, then det $A^T B^{-1} =$

   (a) -50

   (b) 50

   (c) -2

   (d) 2

   (e) $-\frac{1}{2}$

6. Which of the following is the larger of the eigenvalues of the matrix $\begin{bmatrix} 5 & 1 \\ 1 & 5 \end{bmatrix}$?

(a) 4

(b) 5

(c) 6

(d) 10

(e) 12

7. For which value(s) of t is $\boldsymbol{x} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ an eigenvector of $A = \begin{bmatrix} 2 & 3 \\ t & -1 \end{bmatrix}$?

(a) 1

(b) $\frac{9}{4}$

(c) $t > 0$

(d) $t \in \mathbb{R}$

(e) -1

8. If an $n \times n$ matrix is symmetric, then it

(a) has n distinct eigenvalues

(b) has n linearly independent eigenvectors

(c) is diagonalizable

(d) (b) and (c)

(e) all of the above

9. If A and B are similar matrices, then they may have different

   (a) eigenvalues

   (b) eigenvectors

   (c) traces

   (d) ranks

   (e) determinants

10. If A is a positive definite matrix, then which of the following must also be positive definite?

    (a) A^T

    (b) cA where $c \in \mathbb{R}$

    (c) A^{-1}

    (d) (a) and (c)

    (e) all of the above

2 Free Response

For problems 11-15, show all work and justify each step to receive full credit. Draw a box around your answers.

11. (10 points) Let $W = \text{Nul}\begin{bmatrix} 1 & 2 & 2 & 1 \\ 3 & 4 & 2 & 3 \end{bmatrix}$.

    (a) Find a basis for $W^{\perp}$.

    (b) Apply the Gram-Schmidt process to orthogonalize the basis you found in part (a).

12. (10 points) The taxi fare associated with the distance traveled was recorded and shown in the table below.

Distance	1 mile	2 miles	3 miles
Fare	\$5	\$8	\$10

(a) Find the least-squares linear model to fit the data.

(b) Use your model from part (a) to predict the taxi fare for a 4 mile ride.

13. (10 points) Let $A = \begin{bmatrix} 1 & 2 & -1 & 0 \\ 1 & 0 & 2 & 0 \\ 1 & 2 & 0 & -1 \\ 1 & 0 & 3 & 0 \end{bmatrix}$

(a) Compute $\det(A)$.

(b) Solve for a and b in

$$A^{-1} = \begin{bmatrix} a & 3b & a & -2b \\ b/2 & -2b & a & 3b/2 \\ a & -b & a & b \\ b & -b & -b & b \end{bmatrix}.$$

(c) Solve $A\boldsymbol{x} = \begin{bmatrix} 1 \\ 2 \\ -1 \\ 2 \end{bmatrix}$.

14. (10 points) Prove the statement in general or disprove the statement by providing a counterexample:

(a) If A has eigenvalue λ, then A^2 has eigenvalue λ^2.

(b) If A has eigenvalue λ and B has eigenvalue μ, then AB has eigenvalue $\lambda\mu$.

15. (10 points) Given $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & 1 \\ 3 & 1 & 1 \end{bmatrix}$ has eigenvalues $5, 2, -2$ and eigenvectors $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, $\begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$, $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$, respectively.

(a) Compute the eigenvector corresponding to eigenvalue -2.

(b) Find the diagonalization of A.

(c) Use your answer from part (b) to write an expression for A^{10}.