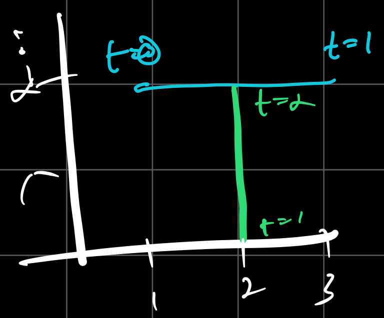
Project comment:

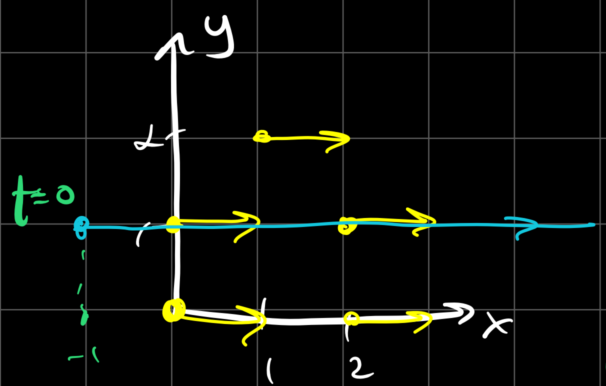


$$\vec{r}(t) = \begin{cases} \vec{r}_1(t), & 0 \le t < 1 \\ \vec{r}_2(t), & 1 \le t \le 2 \end{cases}$$

$$\vec{r}\,'(t) = \begin{cases} \vec{r}_1\,', & 0 < t < 1 \\ r_2\,', & 1 < t < 2 \end{cases}$$

16.1 Vector Fields

7. $\vec{F}(x,y) = \hat{i}$



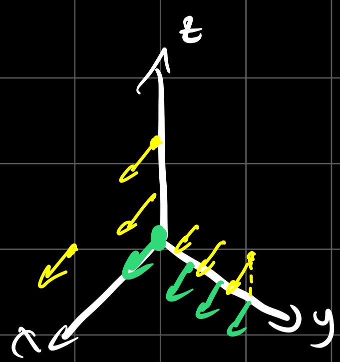
flow line

stream line.

$y=1$, $x=-1$ to ∞

$$\vec{r}(t) = \langle t-1, 1 \rangle, \; t \ge 0.$$

$F(x,y,z) = \hat{i}$



25. $f(x,y) = \frac{1}{2}(x-y)^2$ find $\vec{\nabla} f = \langle x-y, y-x \rangle$

$= (x-y)\hat{i} + (y-x)\hat{j}$

Vector field!!

See video for graph

16.2 Line Integrals

C is given by $\vec{r}(t) = \langle x(t), y(t) \rangle$, $a \le t \le b$

$$\int_C f(x,y)\,ds$$

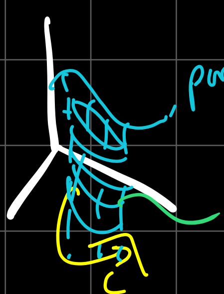
$$= \int_a^b \underbrace{f(x(t), y(t))}_{f(\vec{r}(t))} \underbrace{\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}}_{\|\vec{r}\,'(t)\|}\,dt$$

$$= \int_a^b f(\vec{r}(t))\,\|\vec{r}\,'(t)\|\,dt$$

$$\int_C \vec{F} \cdot d\vec{r}$$

$$= \int_a^b \vec{F}(\vec{r}(t)) \cdot \vec{r}\,'(t)\,dt$$

Measuring how much $\vec{F}$, $\vec{r}\,'$ point in same direction.



part of f above C.

area b/w f and C

We can also write $x = x(t)$ $dx = x'(t)\,dt$

$$\int_C f(x,y)\,dx = \int_a^b f(x(t), y(t))\,x'(t)\,dt$$

same for y

common practice to write: $\int_C f(x,y)\,dx + g(x,y)\,dy$

$$= \int_C f(x,y)\,dx + \int_C g(x,y)\,dy$$

② $\int_C \left(\frac{x}{y}\right) ds$ $\quad C: x = t^3,\ y = t^4 \quad 0 \le t \le 2$

$\frac{dx}{dt} = 3t^2 \qquad \frac{dy}{dt} = 4t^3$

$$\int_C \left(\frac{x}{y}\right) ds = \int_0^2 \left(\frac{t^3}{t^4}\right) \sqrt{(3t^2)^2 + (4t^3)^2}\, dt$$

$$= \int_0^2 \frac{1}{t} \sqrt{9t^4 + 16t^6}\, dt$$

factor out t^4

$$= \int_0^2 \frac{t^2}{t} \sqrt{9 + 16t^2}\, dt$$

$$= \int_0^2 t \sqrt{9 + 16t^2}\, dt$$

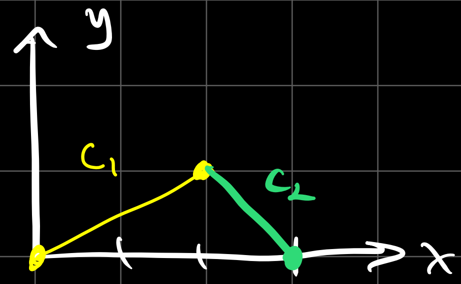
$u = 9 + 16t^2$
$du = 32t\,dt$

easy to evaluate

⑦ $\int_C (x+2y)\,dx + x^2\,dy$ $\quad C$ is line segments from $(0,0)$ to $(2,1)$ and $(2,1)$ to $(3,0)$.

$$= \int_{C_1} (x+2y)\,dx + x^2\,dy + \int_{C_2} (x+2y)\,dx + x^2\,dy$$

Need to parametrize C_1 & C_2



line segment

$\vec{r}(t) = (1-t)\,\vec{r}_0 + t\,\vec{r}_1$, $0 \le t \le 1$

C_1: $\vec{r}_0 = \langle 0,0 \rangle$ $\vec{r}_1 = \langle 2,1 \rangle$

$\vec{r}(t) = \langle 2t, t \rangle$ $x = 2t$ $y = t$ $dx = 2dt$ $dy = dt$

$0 \le t \le 1$

$$\int_{C_1} (x+2y)\,dx + x^2\,dy = \int_0^1 [(2t) + 2(t)]\,\underline{2\,dt} + (2t)^2\,\underline{dt}$$

$$= \int_0^1 (8t + 4t^2)\,dt = 4t^2 + \frac{4}{3}t^3 \Big|_{t=0}^{t=1} = \frac{16}{3}$$

C_2: $\vec{r}_0 = \langle 2,1 \rangle$ $\vec{r}_1 = \langle 3,0 \rangle$ $\vec{r}(t) = (1-t)\langle 2,1 \rangle + t\langle 3,0 \rangle$

$= \langle 2+t, 1-t \rangle$

$x = 2+t$ $y = 1-t$ $0 \le t \le 1$

$dx = dt$ $dy = -dt$

$$\int_{C_2} (x+2y)\,dx + x^2\,dy = \int_0^1 (\underbrace{(2+t) + 2(1-t)}_{2+t+2-2t})\,dt + (2+t)^2(-dt)$$

$$= \int_0^1 (4-t) - (2+t)^2\,dt = \boxed{}$$

$$\int_C (x+2y)\,dx + x^2\,dy = \frac{16}{3} + \boxed{}$$

(21) $\vec{F}(x,y,z) = \sin x\,\hat{i} + \cos y\,\hat{j} + xz\,\hat{k}$

$\vec{r}(t) = \underbrace{t^3}_{x}\hat{i} - \underbrace{t^2}_{y}\hat{j} + \underbrace{t}_{z}\hat{k} \qquad 0 \le t \le 2$

$$\int_C \vec{F}\cdot d\vec{r} = \int_a^b \vec{F}(\vec{r})\cdot\vec{r}\,'(t)\,dt$$

$\vec{r}\,'(t) = 3t^2\hat{i} - 2t\hat{j} + \hat{k}$

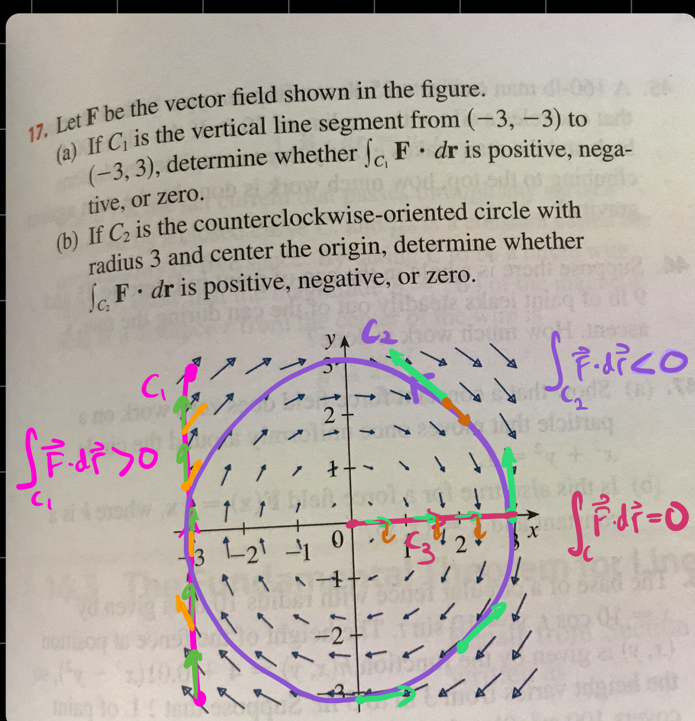
$\vec{F}(\vec{r}(t)) = \sin(t^3)\hat{i} + \cos(-t^2)\hat{j} + \underbrace{(t^3)(t)}_{t^4}\hat{k}$

$\vec{F}(\vec{r}(t))\cdot\vec{r}\,'(t) = 3t^2\sin(t^3) - 2t\cos(-t^2) + t^4$

$$\int_C \vec{F}\cdot d\vec{r} = \int_0^2 \left(3t^2\sin(t^3) - 2t\cos(-t^2) + t^4\right)dt$$

$$\left(-\cos(t^3) + \sin(-t^2) + \frac{t^5}{5}\right)\Bigg|_{t=0}^{t=2}$$

(17)



17. Let $\mathbf{F}$ be the vector field shown in the figure.

(a) If C_1 is the vertical line segment from $(-3, -3)$ to $(-3, 3)$, determine whether $\int_{C_1} \mathbf{F}\cdot d\mathbf{r}$ is positive, negative, or zero.

(b) If C_2 is the counterclockwise-oriented circle with radius 3 and center the origin, determine whether $\int_{C_2} \mathbf{F}\cdot d\mathbf{r}$ is positive, negative, or zero.