$f(t)$ = vector as a function of t.

yields vector at diff. points in space

13.1

② $\vec{r}(t) = \cos t\,\hat{\imath} + \ln t\,\hat{\jmath} + \frac{1}{t-2}\hat{k}$

what is the domain of $\vec{r}$?

$t > 0$

$t \neq 2$

$(0, 2) \cup (2, \infty)$

$(0, \infty) \setminus \{2\}$

④ $\lim_{t\to 1} \left(\frac{t^2 - t}{t-1}\hat{\imath} + \sqrt{t+8}\,\hat{\jmath} + \frac{\sin \pi t}{\ln t}\hat{k} \right)$

$$= \underbrace{\lim_{t\to 1}\left(\frac{t^2-t}{t-1}\right)\hat{\imath}} + \underbrace{\lim_{t\to 1}\sqrt{t+8}\,\hat{\jmath}}_{3\hat{\jmath}} + \lim_{t\to 1}\frac{\sin \pi t}{\ln t}\hat{k}$$

$$= \underbrace{\lim_{t\to 1}\frac{t\cancel{(t-1)}}{\cancel{t-1}}\hat{\imath}}_{\hat{\imath}} + 3\hat{\jmath} + \frac{0}{0}$$ use L'Hôpital's Rule.

$$\lim_{t\to 1}\frac{\sin \pi t}{\ln t}\hat{k} = \lim_{t\to 1}\frac{\pi \cos \pi t}{\frac{1}{t}} = -\pi$$

$$\lim_{t\to 1} \left(\frac{t^2 - t}{t-1}\hat{\imath} + \sqrt{t+8}\,\hat{\jmath} + \frac{\sin \pi t}{\ln t}\hat{k} \right) = \hat{\imath} + 3\hat{\jmath} - \pi\hat{k}$$

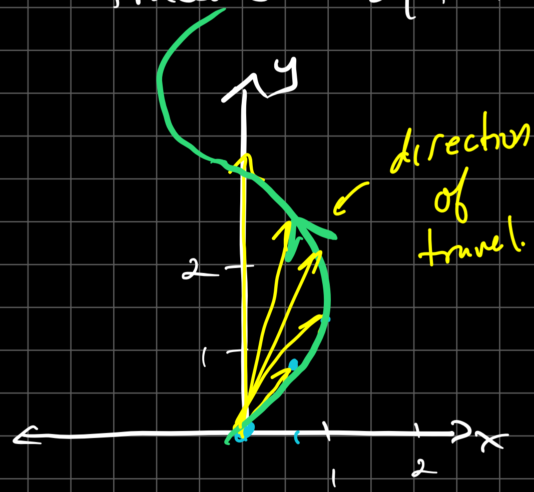
⑦ $\vec{r}(t) = \langle \sin t, t \rangle$ Sketch curve traced out by this.

$t=0$ $\vec{r}(t) = \langle 0, 0 \rangle$

$t=\frac{\pi}{4}$ $\vec{r}(\frac{\pi}{4}) = \langle \frac{\sqrt{2}}{2}, \frac{\pi}{4} \rangle$

$t=\frac{\pi}{2}$ $\vec{r}(\frac{\pi}{2}) = \langle 1, \frac{\pi}{2} \rangle$

$t=\pi$ $\vec{r}(\pi) = \langle 0, \pi \rangle$



(31) $\vec{r}(t) = t\hat{i} + (2t - t^2)\hat{k}$ when does curve intersect the surface?

$z = x^2 + y^2$

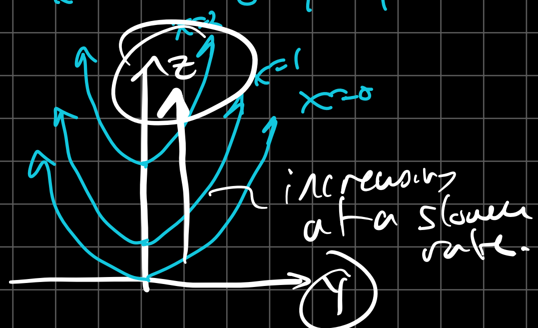
draw traces as a review.

fix x

$x=0$ $z = y^2$

$x=1$ $z = y^2+1$

$x=2$ $z = y^2+4$



fix y

$y=0$ $z = x^2$

$y=1$ $z = x^2+1$

$y=2$ $z = x^2+4$

looks same.

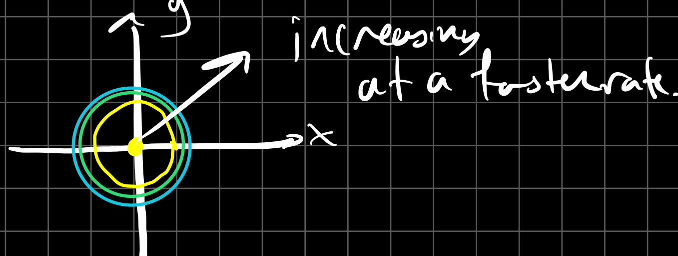
fix z

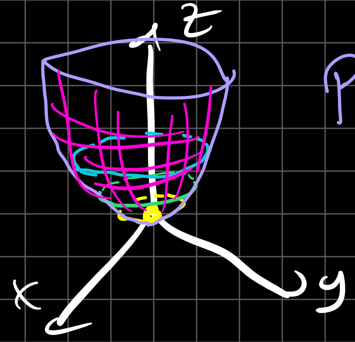
$z=0$ $x^2+y^2=0$

$z=1$ $x^2+y^2=1$

$z=2$ $x^2+y^2=2$

$z=3$ $x^2+y^2=3$





$\vec{r}(t) = t\hat{\imath} + (2t - t^2)\hat{k}$ find where these touch.

$z = x^2 + y^2$

$\vec{r}(t) = x(t)\hat{\imath} + y(t)\hat{\jmath} + z(t)\hat{k}$

$x = t$

$y = 0$

$z = 2t - t^2$

$2t - t^2 = t^2 + 0^2 \rightarrow \cancel{2}t = \cancel{2}t^2 \Rightarrow t^2 - t = 0$

$t(t-1) = 0$

$\boxed{t=0}$, $t - 1 = 0$

$\boxed{t=1}$

when they intersect.

$\vec{r}(0) = \vec{0}$

$\vec{r}(1) = \hat{\imath} + \hat{k}$

Intersection: $(0,0,0)$ & $(1,0,1)$.

(43) $z = \sqrt{x^2 + y^2}$

$z = 1 + y$

find $\vec{r}(t)$ that traces the intersection.

$1 + y = \sqrt{x^2 + y^2}$

let $y = t$ find x & z in terms of t.

$1+t=\sqrt{x^2+t^2}$ solve for x.

$(1+t)^2 = x^2+t^2$

$1+2t+\cancel{t^2} = x^2+\cancel{t^2}$

$x=\pm\sqrt{1+2t}$, $t \geq -\frac{1}{2}$.

we might improve later.

$z=1+t$

two

$\vec{r}_1(t) = \langle \sqrt{1+2t}, t, 1+t \rangle$

$\vec{r}_2(t) = \langle -\sqrt{1+2t}, t, 1+t \rangle$

could also have let $x=t$

or

let $z=t$

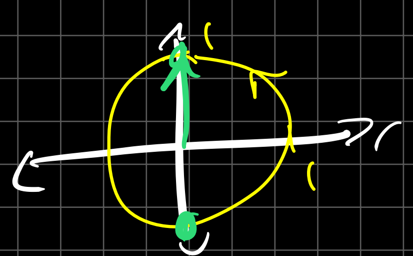
Param. of a curve is **NOT** unique.

could we have chosen $x=t^2$ or t^3

or $x=e^t$

depend on restrictions on x.

Ex)



$\vec{r}(t) = \langle \cos t, \sin t \rangle$, $t \geq 0$.

$\vec{r}(t) = \langle \cos 10t, \sin 10t \rangle$

$\vec{r}(t) = \langle -\sin t, \cos t \rangle \qquad t \geq 0$

$t=0 \quad \langle 0, 1 \rangle$

(49) $\vec{r}_1(t) = \langle t^2, 7t-12, t^2 \rangle$

$\vec{r}_2(t) = \langle 4t-3, t^2, 5t-6 \rangle$

paths of two particles.

Do they collide? Do the paths cross at same time?

$t^2 = 4t-3 \rightarrow t^2-4t+3=0, \quad t=1, 3$

$7t-12 = t^2 \rightarrow t^2-7t+12=0, \quad t=3, 4$

$t^2 = 5t-6 \rightarrow t^2-5t+6=0, \quad t=2, 3$

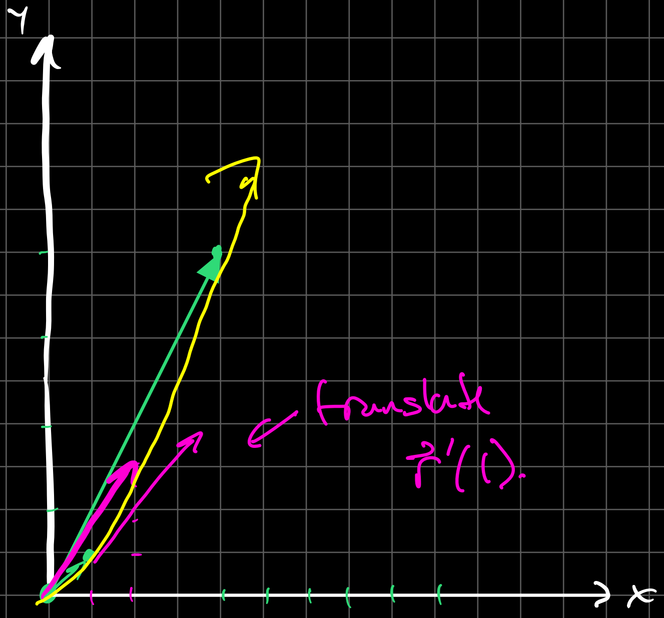
collide at $t=3$.

13.2

(4) $\vec{r}(t) = \langle t^2, t^3 \rangle, \quad t \geq 0.$

(a) sketch

t	$\vec{r}(t)$
0	$\langle 0, 0 \rangle$
1	$\langle 1, 1 \rangle$
2	$\langle 4, 8 \rangle$
3	$\langle 9, 27 \rangle$



$x = t^2$

$y = t^3 \Rightarrow y = (t^2)^{3/2} = x^{3/2}$

$\vec{r} = \langle t^2, t^3 \rangle$

(b) find $\vec{r}\,'(t) = \langle 2t, 3t^2 \rangle$

(c) find $\vec{r}\,'(1) = \langle 2, 3 \rangle$ — sketch.

tangent vector to path at $t = 1$.

(18) $\vec{r}(t) = \langle \tan^{-1}(t), 2e^{2t}, 8te^t \rangle$ find $\vec{T}(0) = \dfrac{\vec{r}\,'(0)}{\|\vec{r}\,'(0)\|}$

$\vec{r}\,'(t) = \left\langle \dfrac{1}{1+t^2}, 4e^{2t}, 8e^t + 8te^t \right\rangle$

$\vec{r}\,'(0) = \langle 1, 4, 8 \rangle$

$\|\vec{r}\,'(0)\| = \sqrt{1^2 + 4^2 + 8^2} = \sqrt{1 + 16 + 64} = 9.$

$\vec{T}(0) = \left\langle \frac{1}{9}, \frac{4}{9}, \frac{8}{9} \right\rangle.$

(40) $\displaystyle\int \left(te^{2t}\,\hat{\imath} + \frac{t}{1-t}\,\hat{\jmath} + \frac{1}{\sqrt{1-t^2}}\,\hat{k} \right) dt$

$\displaystyle = \left(\int te^{2t}\,dt \right)\hat{\imath} + \left(\int \frac{t}{1-t}\,dt \right)\hat{\jmath} + \left(\int \frac{1}{\sqrt{1-t^2}}\,dt \right)\hat{k}$

by parts $\quad$ $u = 1-t$ $\quad$ trig sub or

$u = t$ $\quad$ $dv = e^{2t}dt$

$du = dt$ $\quad$ $v = \frac{1}{2}e^{2t}$

$\frac{d}{dt}(\arcsin t) = \frac{1}{\sqrt{1-t^2}}$

$\hat{i}$: $\left(\frac{t}{2}e^{2t} - \int \frac{1}{2} e^{2t}dt\right) = \underline{\frac{te^{2t}}{2} - \frac{1}{4}e^{2t} + C_1}$

$\hat{j}$: $u = 1-t$

$du = -dt$

$t = 1-u$

$\int \frac{t}{1-t}dt = \int \frac{1-u}{u}(-du) = \int \frac{u-1}{u}du = \int 1 - \frac{1}{u}du$

$\underline{u - \ln|u| + C_2}$

$1-t-\ln|1-t|$

$\hat{k}$: $\underline{\arcsin t + C_3}$

$$\int \left(te^{2t}\hat{i} + \frac{t}{1-t}\hat{j} + \frac{1}{\sqrt{1-t^2}}\hat{k}\right)dt$$

$u = 1-t$

$$= \left(\frac{te^{2t}}{2} - \frac{1}{4}e^{2t} + C_1\right)\hat{i} + \left(u - \ln|u| + C_2\right)\hat{j} + \left(\arcsin t + C_3\right)\hat{k}$$

$$= \left(\frac{te^{2t}}{2} - \frac{1}{4}e^{2t}\right)\hat{i} + \left(1-t-\ln|1-t|\right)\hat{j} + \arcsin t\,\hat{k} + \vec{C}$$

Constant vector

$\vec{C} = \langle C_1, C_2, C_3 \rangle$