Homework 0 - Written Answer Key

Question 1:

(Question)

If $f(x) = 7x^2 + 8x + 10$, then $f(10) = 790$.

The best straight line approximation to $f(x)$ at any point x is the line $14x + 8$.

Without evaluating the function at $f(10+0.01)$ and $f(10-0.01)$, can you determine if $f(10+0.01) < f(10)$ or is $f(10-0.01) < f(10)$?

Why or why not?

(Answer(s))

- $$f(10+0.01) = f(10.01) = 7(10.01^2) + 8(10.01) + 10 = 791.4807$$
  $$791.4807 > 790$$
  $$f(10+0.01) > f(10)$$
  $$f(10-0.01) = f(9.99) = 7(9.99^2) + 8(9.99) + 10 = 788.5207$$
  $$788.5207 < 790$$
  $$f(9.99) < f(10)$$

  However, given the best straight line approximation to the function at the point x, the above conclusions can be reached without evaluating the function explicitly at x-values 10.01 and 9.99.

- This is because if one knows ahead of time that the best straight line approximation is $14x + 8$, then one would also know that the slope at any positive x value will always be positive. So, when x is positive, as x increases, so does y. That means that choosing a base positive x-value (10), any larger x-value (10.01) would always be greater, and any smaller positive x-value (9.99) would be lower.

Question 2:

(Question)

Suppose you decided to spend the day mountain climbing.

Having spent a fun day climbing, you notice it is time to get back to your car which is in the valley at the base of the mountain (since the sun has gone down and you can no longer see the beautiful views, ... or anything else). Unfortunately, you have lost yourself in the woods. Describe an algorithm that allows you to get back as quickly as possible, where you assume that the side of the mountain you are on has only one valley.

I am just looking for a simple 1 or 2 sentence intuitive answer for this question. (Don't overthink this question.)

(Answer(s))

If the mountain only has one valley, then it is convex, so there is no need to worry about going to the "wrong" base of the mountain. I would take one step forward, regardless of the direction I am currently facing. One of three possibilities can occur.

- Gravity can tell me that I took one step up the mountain. In this case, I am going 180° in the wrong direction. I turn around 180° and continue walking down the mountain until I detect that I am no longer going "down". When this happens, I am at the base.

- Gravity could also tell me that I just went one step down the mountain. In this case, I am going the correct direction and continue walking until I detect that I am no longer going "down".

- Lastly, if I feel like I am walking on even ground, this means that I just took a step left or right instead of up or down. I turn 90° in either direction and encounter either case 1 or case 2.

<u>Question 3</u>:

(Question)

If you knew the part of the mountain you are walking on doesn't have only one valley - would this make a difference.

What could go wrong? (Don't overthink this question.)

(Answer(s))

Yes, if the mountain I am on has more than one valley, the aforementioned algorithm would not always work. This is because the mountain would then be non-convex. When I reach the end of the original algorithm (I just stopped descending the mountain and am about to start ascending), then I am surely at a valley. However, there is no way to know whether or not this is the valley where my car is actually located. All valleys would share the property of descending and ascending that the algorithm relies on, so there is no way to distinguish the "correct" valley.

Question 4:

(Question)

Suppose you traveled to Los Vegas and you were playing a coin game on the street. You are pretty sure the coin was not fair. You would like to go back and win your \$50 back. If in 50 coin flips, there were 30 heads (and 20 tails). What is the most

likely probability of the coin having heads? Let p_h be the probability of the coin being heads.

What p_h would maximize $(p_h)^{30}(1-p_h)^{20}$?

Since we know that $p_h \neq 1$ and $p_h \neq 0$ since both heads and tails occurred, would finding the p_h that maximizes $(p_h)^{30}(1-p_h)^{20}$ be the same as finding the p_h that maximizes $\log\left((p_h)^{30}(1-p_h)^{20}\right) = 30\log(p_h) + 20\log(1-p_h)$?

Why or why not?

(Answer(s))

- The most likely probability of the coin landing heads is $30/(30+20) = 0.6$

- $p_h = 0.6$

- When $p_h = 0.6, (p_h)^{30}(1-p_h)^{20}$ is maximized. This is because this is the p_h value where the slope of $(p_h)^{30}(1-p_h)^{20}$ is equal to 0, which means that $p_h = 0.6$ is the "turning point" where the function stops increasing and starts decreasing.

- If the base of the log is greater than 1, then $\log(x)$ is monotonic. As x increases, the function does at well, even if at a slower pace. The highest possible value of x would thus also yield the highest possible value of $log(x)$. Additionally, in $log((p_h)^{30}(1-p_h)^{20})$, the maximum of $(p_h)^{30}(1-p_h)^{20}$ would be the same as the maximum of x in $\log(x)$. Thus, $p_h = 0.6$ would also maximize $log((p_h)^{30}(1-p_h)^{20})$.