Variance Swap

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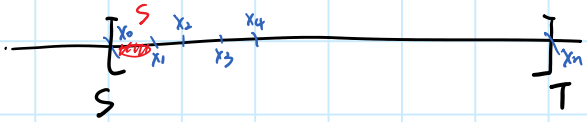

Observation { Period. / Frequency

$(X_1-X_0)^2+(X_2-X_1)^2+\cdots+(X_n-X_{n-1})^2 \approx QV \quad \langle X_t, X_t\rangle_{[S,T]}$

如何推 $\langle X_t, X_T\rangle$?

考虑 $f(t,X)=(X-X_0)^2$

$f_t=0$

$f_x=2(X-X_0)$

$f_{xx}=2$

$\therefore (X_T-X_0)^2=0+\int_0^T 0\,dt+\int_0^T 2(X_t-X_0)\,dX_t+\frac{1}{2}\int_0^T 2(dX_t)^2$

$\therefore (X_T-X_S)^2=2\int_S^T (X_t-X_S)\,dX_t+\langle X_t,X_t\rangle_{[S,T]}$

即 $E\left[(X_T-X_S)^2\right]-2E\left[\int_S^T (X_t-X_S)\,dX_t\right]=E\left[\langle X_t,X_t\rangle_{[S,T]}\right]$

① ② ③

LHS = ① − 2② = RHS = ③

① 只需已知 dist of X_T (通常已知 X_S).

↳ 通常由期权 (tail options) 决定. vanilla puts & calls

② 若 X_t martingale $\Rightarrow$ ② = 0

若 X_t 有 drift $\Rightarrow$ Gaussian approxi. 或用 Var 计算 Expectation

③ = ① − 2×②