Today: $\boxed{\text{Ken}}$

Syllabus review

$\boxed{2.1}$ Logical Form & Equivalence

$\boxed{2.2}$ Conditional Statements

Ken Cereste

cereste@nyu.edu

kc1946@nyu.edu

Office: 2 MetroTech Center Room 857

Office Hours: MW 10:00am - 11:00am

TuTh 10:30am - 12:30pm

see syllabus / Brightspace for Zoom links

LaTeX

$\boxed{2.1}$ Definition

$\boxed{2.2}$ A statement (or proposition) is a sentence that is either true or false (not both).

examples

① ⓐ true statement $e < \pi$

ⓑ false statement $\pi < e$

Logical Operators, Compound Statements, Truth Tables

① negation $\sim, \neg$

$\left.\begin{matrix} \sim p \\ \neg p \end{matrix}\right\}$ not p

p	$\neg p$
T	F
F	T

$e < \pi$

$e \geq \pi$

② disjunction $\vee$

$p \vee q$ p or q

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

$2^2 = 4$

③ conjunction $\wedge$

$p \wedge q$ p and q

$2^2 = 4$

p	q	$p\wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

④ conditional $\rightarrow$

$p \rightarrow q$

if p then q

p implies q

p only if q

p	q	$p\rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

if $e \geq \pi$ then $0 > 1$

hypothesis or antecedent

(in this case p)

conclusion or consequent
(in this case q)

⑤ biconditional $\longleftrightarrow$

$p \leftrightarrow q$

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

p if and only if q

e.g. $f(x) = y \iff x = f^{-1}(y)$

⑥ order of operations
negation first, followed by disjunction or conjunction,
then conditional and biconditional

$p \vee q \rightarrow r$ interpreted $(p \vee q) \rightarrow r$

$\neg p \vee q \rightarrow r$ interpreted $((\neg p) \vee q) \rightarrow r$

ambiguous cases

$p \vee q \wedge r$ don't

$(p \vee q) \wedge r$

$p \vee (q \wedge r)$ ok

$(p \vee q) \to \neg r$ $p \vee q \to \neg r$

$(p \vee \neg q) \to (p \wedge r)$ $p \vee \neg q \to p \wedge r$

$p \wedge q \to r \not\equiv p \wedge (q \to r)$

Definition

A statement form (or propositional form) is an expression made up of statement variables (like $p, q, r, \ldots$) and logical connectives (like $\neg, \vee, \wedge, \to, \leftrightarrow$) that becomes a statement when actual statements are substituted for the compound statement variables.

examples

I ⓐ "but"

It's 62F outside but the sun's out.

p: it is 62F outside

q: the sun is out

$p \wedge q$

ⓑ "neither-nor," "either-or"

it's neither raining nor snowing

p: it's raining

q: it's snowing

$\neg p \wedge \neg q \equiv \neg(p \vee q)$

"equals"

$A = B$ A, B sets

$2 = 2$ 2 is a integer

ⓒ "implies," "only if"

p only if q   means "if not q then not p"

$\neg q \rightarrow \neg p \equiv p \rightarrow q$

f is differentiable only if f is continuous.

diff $\longrightarrow$ cont

ⓓ "iff" if and only if

"p iff q" $p \longleftrightarrow q$

Ⅲ ⓐ If x and y are nonzero real numbers, then xy is a nonzero real number.

$$x \neq 0 \wedge y \neq 0 \implies xy \neq 0$$

$$xy = 0 \implies x = 0 \vee y = 0$$

"zero product property"

ⓑ (contrapositive)

"if not q, then not p"

$$\neg q \rightarrow \neg p$$

p	q	$\neg p$	$\neg q$	$p \to q$	$\neg q \to \neg p$
T	T	F	F	T	T
T	F	F	T	F	F
F	T	T	F	T	T
F	F	T	T	T	T

Goal: $p \to q \equiv \neg q \to \neg p$

p	$\neg p$	$p \wedge \neg p$
T	F	F
F	T	F

$p \wedge \neg p$
"contradiction"

f is differentiable only if
f is continuous.

f is not continuous implies
f is not differentiable.

sve.math@nyu.edu