14.4

(3) $z = e^{x-y}$ $\quad (2,2,1)$

$f(x,y) =$

find tangent plane to function at

$z - z_0 = f_x(x_0,y_0)(x-x_0) + f_y(x_0,y_0)(y-y_0)$

$z_x = e^{x-y}$ $\qquad z_y = -e^{x-y}$

z_y at $(2,2)$ $= -1.$

$$\left.\begin{array}{l} z_x \text{ at } (2,2) \\ f_x(2,2) \\ z_x\big|_{(x,y)=(2,2)} \end{array}\right\} = e^{2-2} = 1$$

$z - 1 = (1)(x-2) + (-1)(y-2)$

(5) $z = x\sin(x+y)$ $\quad$ find tangent plane at $(-1,1,0)$

$z_x = \sin(x+y) + x\cos(x+y)$

at $(-1,1)$ $\quad z_x = -1$

$z_y = x\cos(x+y)$

at $(-1,1) = -1$

$$z - 0 = (-1)[x - (-1)) + (-1)(y-1)$$

$$= -x - 1 - y + 1$$

$$z = -x - y \iff x + y + z = 0$$

(12) $f(x,y) = \sqrt{xy}$ at $(1,4)$

$= x^{\frac{1}{2}} y^{\frac{1}{2}}$

$\underbrace{\text{find local Linearization of}}_{\text{find tangent plane.}}$ $f(x,y)$.

$$L(x,y) = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$$

$$f_x = \frac{1}{2}x^{-\frac{1}{2}}y^{\frac{1}{2}} = \frac{\sqrt{y}}{2\sqrt{x}} = \frac{1}{2}\sqrt{\frac{y}{x}}$$

$$f_y = \frac{1}{2}y^{-\frac{1}{2}}x^{\frac{1}{2}} = \frac{\sqrt{x}}{2\sqrt{y}} = \frac{1}{2}\sqrt{\frac{x}{y}}$$

$$L(x,y) = f(1,4) + f_x(1,4)(x-1) + f_y(1,4)(y-4)$$

$$= 2 + (1)(x-1) + \left(\frac{1}{4}\right)(y-4)$$

Simplify

(19) $f(2,5)=6$ $\quad f_x(2,5)=1$ $\quad f_y(2,5)=-1$

estimate $f(2.2, 4.9)$

$L(x,y) = 6 + (1)(x-2) - 1(y-5)$

$$\begin{aligned} L(2.2, 4.9) &= 6 + (2.2-2) - 1(4.9-5) \\ &= 6 + 0.2 + 0.1 \\ &= 6.3 \end{aligned}$$

(26) $u = \sqrt{x^2+3y^2}$
$= (x^2+3y^2)^{1/2}$

find du.

if $z = f(x,y)$
$\underset{\text{differential}}{dz} = f_x\,dx + f_y\,dy$

$dz = z_x\,dx + z_y\,dy$

$u_x = \frac{1}{2}(x^2+3y^2)^{-1/2}(2x)$

$= \dfrac{x}{\sqrt{x^2+3y^2}}$

$u_y = \dfrac{3y}{\sqrt{x^2+3y^2}}$

$$du = \frac{x}{\sqrt{x^2+3y^2}}\,dx + \frac{3y}{\sqrt{x^2+3y^2}}\,dy$$

32 $z = x^2 - xy + 3y^2$

if (x,y) changes from $(3,-1)$ to $(2.96, -0.95)$

find Δz & dz

$$\Delta z = f(2.96, -0.95) - f(3,-1) = -.7189$$

$$dz = z_x dx + z_y dy = (2x - y)\underbrace{dx}_{-.04} + (-x + 6y)\underbrace{dy}_{.05}$$

$x=3, \quad y=-1$

$$dz = (2(3)+1)(-.04) + (-3-6)(.05)$$
$$= 7(-.04) + (-9)(.05)$$
$$= -.28 - .45 = -.73$$

note:

$|a+b| \le |a| + |b|$

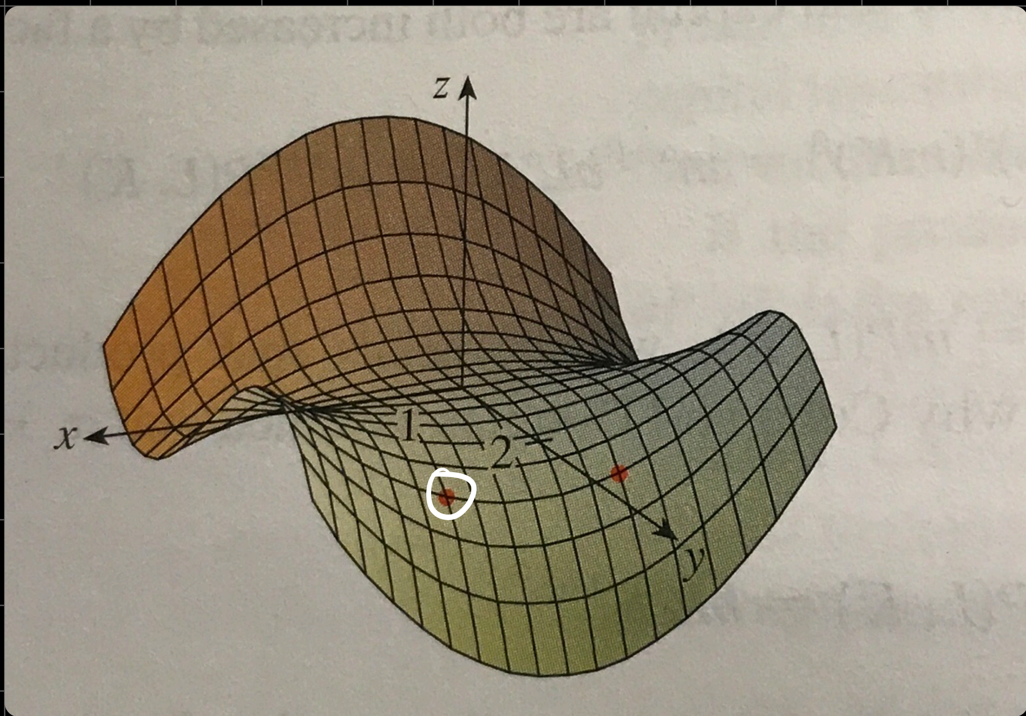
$|ab| = |a||b|$

$$|dz| = |z_x dx + z_y dy|$$

$$|dz| \le |z_x||dx| + |z_y||dy|$$

Exam

finding if 2^{nd} p.d. is >0 or <0 on a graph.



$f_{xx} > 0$

$f_{yy} < 0$

$f_{xy} > 0$

Intersection of Surfaces

$z = f(x,y)$ $\quad z = g(x,y)$

$z = x^2 + y^2$ $\quad z + x + y = 1$

$z = 1 - x - y$

$1 - x - y = x^2 + y^2$

$x^2 + x + y + y^2 - 1 = 0$

Solve for x or y $\quad$ let x or y be t find z in terms of t.

or given surface & plane containing points

Points A, B, C

$\vec{u}$ connects A to B

$\vec{v}$ connects A to C

Create 2 vectors

find cross product.

use

$a(x-x_0)+b(y-y_0)+c(z-z_0)=0$

Vector $\perp$ to plane.

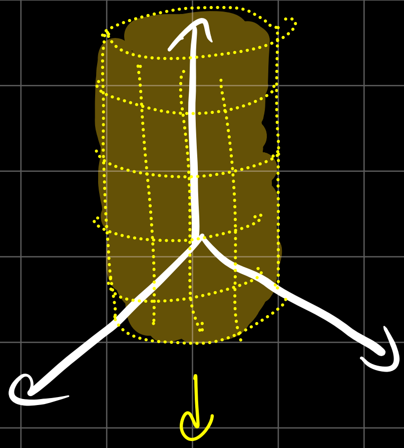
$ax+by+cz=d \qquad \vec{n}=\langle a,b,c\rangle$

Domain & Range

$$f(x,y,z) = \ln(9-x^2-y^2)+z$$

need $9-x^2-y^2>0$, $z \in \mathbb{R}$

$$x^2+y^2<9$$



$\|\vec{r}(t)\| = 1$ then $\|\vec{r}'\| =$ constant.

for all t.

Circular motion.

$\mathbb{R}^2$ $\quad \vec{r}(t) = x(t)\hat{\imath} + y(t)\hat{\jmath}$

$$\|\vec{r}\| = \sqrt{x^2+y^2} = 1$$

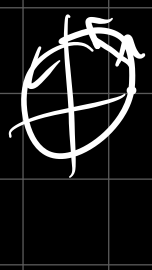
$$x^2+y^2=1$$

$\cancel{x = \cos t}$
$\cancel{y = \sin t}$

$x = \cos(f(t))$
$y = \sin(f(t))$
$x^2+y^2=1$

$\|\vec{r}'\|^2 = \vec{r}' \cdot \vec{r}'$

$$\|\vec{r}\|^2 = 1$$
$$\vec{r}\cdot\vec{r} = 1$$
$$2\vec{r}\cdot\vec{r}' = 0$$
$$\vec{r}\cdot\vec{r}' = 0$$
$$\boxed{\vec{r} \perp \vec{r}'}$$



tells nothing about $\|\vec{r}'\|$

$\vec{r} = \langle \cos f(t), \sin f(t) \rangle$

$\vec{r}\,' = \langle -\sin(f(t))\, f'(t),\ \cos(f(t))\, f'(t) \rangle$

$\|\vec{r}\,'\| = \sqrt{(f'(t))^2} = |f'(t)|$ does not have to be constant.

particle collision / path intersection.

$\vec{r}_1(t)$ $\vec{r}_2(t)$.

collide $\Rightarrow$ same place at same time.

$\left.\begin{array}{c} x_1(t) = x_2(t) \\ \vdots \end{array}\right\}$ setting components equal.

path intersect same place diff time