NYU - Mathematics at Tandon

MA-UY 2224 EXAM 1 Sample

Print Name:	Signature:
Section:	Instructor:
ID #:	

Directions: You have **110 minutes** to answer the following questions. ***You must show all your work*** as neatly and clearly as possible and indicate the final answer clearly. You may use only a TI-30 calculator.

If you are feeling ill you should inform the proctor. The proctor will note your name, student ID and accept any written statement(s) that you may wish to make regarding your illness. Cell phones and other electronic devices may **NOT** be used during the exam.

Problem	Possible	Points
1	15	
2	12	
3	20	
4	16	
5	17	
6	20	
Total	100	

Mathematics Department After-Test Survey:
"We care what you think!"

1. This exam was

   (a) too hard (b) fair (c) easy

2. This exam reflected the assigned homework/worksheet problems

   (a) a lot (b) somewhat (c) not much

3. The time I was given to complete this exam was

   (a) too much (b) just right (c) too little

(1) Show your work.

(a) (6 points) Let E and F be two events such that $P(E) = .75$, and $P(F|E) = .4$.

(i) Could E and F be independent events? If yes, find $P(F)$ and $P(E \cap F)$. If no, explain why.

(ii) Could E and F be mutually exclusive events? If yes, find $P(F)$ and $P(E \cap F)$. If no, explain why.

(b) (9 points) Consider two independent random variables X and Y where $E(X) = 5$ and $Var(X) = 7$; Y is a Binomial random variable with $n = 12$ and $p = 0.4$. Let T be the random variable given by $T = 2X - 5Y + 20$. Fill in each of the following blanks.

(i) $E(T) =$____________.

(ii) $Var(T) =$____________.

(iii) $E(X^2) =$____________.

(2) Two basketball teams, Team A and Team B, are playing in the NBA Eastern Conference Finals; the first team to win 4 games is declared the overall winner of the series (Best-of-7 series. No tied game allowed).

Assume that Team A is stronger and wins each game with probability 0.55. Assume the outcome of each game is independent of other games.

Given the information that the series went to game 7, what is the conditional probability that Team A won the series?

(3) A bag contains 20 red marbles and 4 blue marbles. Randomly pick 3 marbles from the bag, without replacement.

(a) Let X represent the number of the blue marbles in the chosen 3. Find the probability mass function (p.m.f.) of X. Show your work.

(b) Suppose we play the following game: Out of the 3 marbles that you picked out,

If no blue marble was picked, you lose \$2.
If 1 blue marble is picked, you come out even.
If 2 blue marbles are picked, you win \$5.
If 3 blue marbles are picked, you win \$80.
Would this be a fair game to you? Answer this by calculating your expected gain or loss from each play. Show your work.

(4) The "fill" problem is important in many industries, like those making toothpaste, cereal, beer and so on. If such an industry claims it is selling 12 ounces of its product in a container, it must have a mean greater than 12 ounces or else the FDA will crack down, although FDA will allow a very small percentage of the containers to have less than 12 ounces.

(a) If the contents of a container, X, has a normal distribution with $\mu = 12.2$ and $\sigma = 0.12$, find $P(X < 12)$.

(b) If the contents of a container, X, has a normal distribution with $\sigma = 0.1$, find the value of μ, such that $P(X < 12) = 0.01$.

(5) Consider three basketball players: Anna, Beth, and Carolyn. The probability that Anna will make a free throw is 0.9, Beth 0.6, and Carolyn 0.3. Assume that all performances are independent.

(a) If each of them attempts a free throw, what is the probability that nobody makes the shot?

(b) If Anna attempts 10 free throws in a row, that is the probability that she makes at most 8 of the 10 shots?

(c) If Beth decides that she won't take a break until she makes 10 free throws, what is the probability that she makes her 10th free throw on the 15th attempt?

(d) What is the expected number of shots that Carolyn needs to attempt until she makes one?

(6) The life in years of a certain type of electronic switch, X, has the following probability density function:

$$f(x) = \frac{1}{c}\,e^{-x/2}, \qquad x > 0.$$

(a) Find the value of the constant c.

(b) Find the probability that a randomly selected switch will last more than 3 years. In other words, find $P(X > 3)$.

(c) Given that a switch has been working for 1 year already, find the probability that the switch will last more than 3 additional years.

(d) If 5 of the new switches are installed in different independent systems, what is the probability that there will be 3 of these 5 switches fail during the first three years after the installation?