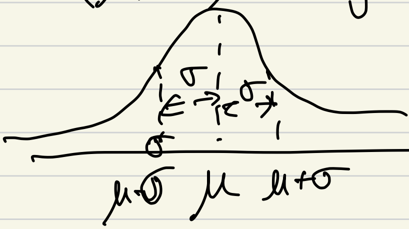
$X \sim N(\mu, \sigma^2)$ $\qquad f(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$ $\qquad -\infty < x < \infty$



$Z \sim N(0,1)$

$f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$ $\qquad -\infty < x < \infty$

numerical methods

Table

Thm: If $X \sim N(\mu, \sigma^2)$, then $Y = \frac{X-\mu}{\sigma} \sim Z$.

Pf: $F_Y(y) = P(Y \le y) = P\left(\frac{X-\mu}{\sigma} \le y\right)$

$$= P(X \le \mu + \sigma y) = \int_{-\infty}^{\mu+\sigma y} \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx$$

$t = \frac{x-\mu}{\sigma}$

$dt = \frac{1}{\sigma} dx$

$dx = \sigma \cdot dt$

$$\int_{-\infty}^{y} \frac{1}{\sqrt{2\pi}\,\cancel{\sigma}} e^{-\frac{t^2}{2}} \cancel{\sigma}\, dt$$

$$= \int_{-\infty}^{y} \frac{1}{\sqrt{2\pi}} e^{-t^2/2} dt$$

$\Rightarrow Y \stackrel{d}{=} Z$ $\qquad \square$

Ex: $X \sim N(10, 4)$ Find:

~~the~~ z-score of 12

① $P(8 < X < 12) = P\left(\frac{8-10}{2} < \frac{X-10}{2} < \frac{12-10}{2}\right) = P(-1 < Z < 1)$

$= 0.6826$

z-score of 8

② $P(X > 7)$

③ Find c, such that $P(X < c) = 0.95$

$z_{0.05} = 1.645$

$c = 10 + 1.645 \cdot 2$

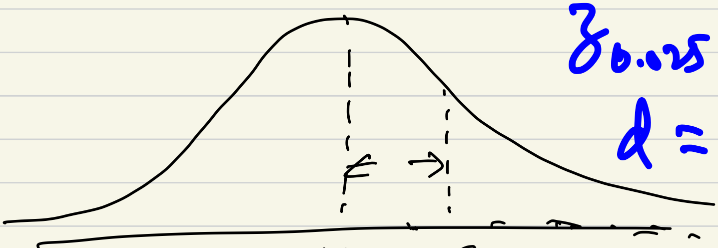
$= 13.29$

④ Find d, such that $P(|X-10| < d) = 0.95$

$z_{0.025} = 1.96$

$d = 1.96\sigma$

$= 3.92$



10   12

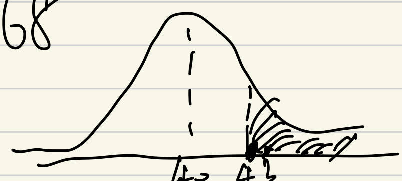
1 std above 10

Ex. electrical resistor, $\mu = 40$, $\sigma = 2$.
Normal.

① Assume the resistance can be measured to any degree of accuracy, what percentage of resistors have resistence exceeding 43 ohms?

$$P(X > 43) = P\left(Z > \frac{43-40}{2}\right) = P(Z > 1.5)$$

$$= 0.0668$$



② If resistence can only be measured to the nearest ohm, what percentage of resistors have resistence exceeding 43?

$$P(X > 43.5) = P\left(Z > \frac{43.5-40}{2}\right)$$

$$= P(Z > 1.75) \approx 0.04$$

§6.2-6.4. ✓

§6.5. Normal approx. to Binomial $\to$ later

§6.6. Gamma and Exponential Distribution.

$E(X)=\frac{1}{2} \Rightarrow f(x) = \underset{\substack{\shortparallel \\ 2.}}{c} \cdot e^{-2x} \qquad x>0. \qquad c=?$

$$f(x) = \frac{1}{\beta} e^{-x/\beta} \qquad x>0, \qquad \beta \text{ is a positive const.}$$

$Exp(\beta)$

$$E(X) = \int_0^\infty \underline{\underline{x}} \cdot \underbrace{\frac{1}{\beta} e^{-x/\beta}}_{dv = \frac{1}{\beta}e^{-x/\beta}} dx$$

$u = x \qquad dv = \frac{1}{\beta} e^{-x/\beta}$

$du = dx \qquad v = -e^{-x/\beta}$

$$= x\cdot(-e^{-x/\beta})\Big|_0^\infty - \int_0^\infty (-e^{-x/\beta})\, dx$$

$$= \beta \int_0^\infty \underline{\frac{1}{\beta} e^{-x/\beta}}\, dx = \beta$$

$$E(X^2) = \int_0^\infty \underline{\underline{x^2}}\, \underbrace{\frac{1}{\beta} e^{-x/\beta} dx}_{dv}$$

$u = x^2 \qquad dv$

$du = 2x\,dx \qquad v = -e^{-x/\beta}$

$$= \cancel{-x^2 e^{-x/\beta}\Big|_0^\infty}^{\;\to 0} + \beta 2 \int_0^\infty x \frac{1}{\beta} e^{-x/\beta}\, dx$$

$$= 2\beta \cdot \underbrace{\int_0^\infty x f(x)\, dx}_{E(X)=\beta} = 2\beta^2$$

$$Var(X) = E(X^2) - (E(X))^2 = 2\beta^2 - \beta^2 = \beta^2$$

Ex: $T \sim$ time to failure $\sim \exp(\beta = 5)$

① What is the prob that a component will last for more than 8 years?

$$P(T > 8) = \int_8^\infty \frac{1}{5} e^{-x/5}\, dx = -e^{-x/5}\Big|_8^\infty$$

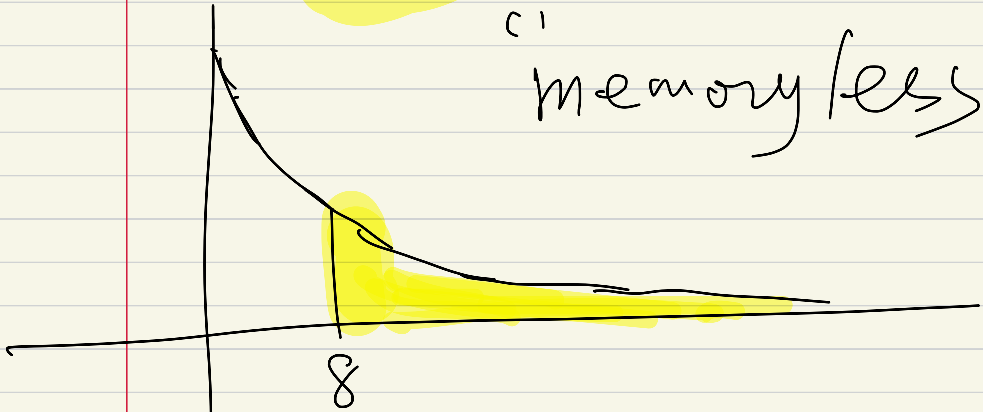
$$= e^{-1.6}$$

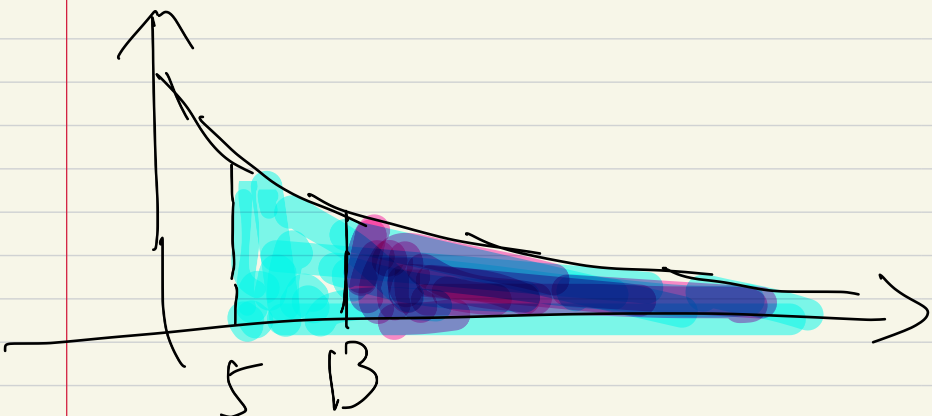
② A component has been functioning for 5 years, what's the prob that it will last for at least another 8?

$$P(X > 13 \mid X > 5) = \frac{P(X > 13 \cap X > 5)}{P(X > 5)} = \frac{e^{-13/5}}{e^{-5/5}}$$

$= e^{-8/5}$

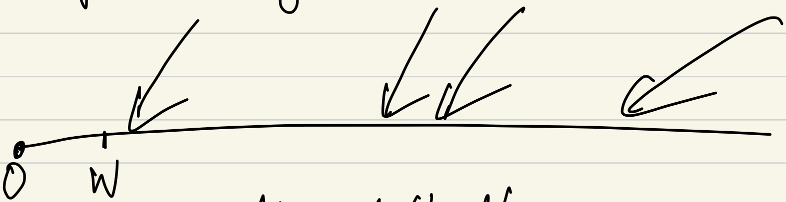
"memoryless"





Connection between Exp(β) and Poi(λ)

Ex: # of earthquakes occur $\sim$ Poi(λ)



W = waiting time till the next earthquake.

$$F_W(w) = P(W \le w) = 1 - P(W > w)$$

$$= 1 - P(\text{There is 0 earthq in } [0, w])$$

$$= 1 - e^{-w\lambda} \frac{(w\lambda)^0}{0!} = 1 - e^{-w\lambda}$$

$$f_W(w) = \left(F_W(w)\right)' = \underbrace{\lambda e^{-\lambda w}}_{\exp\left(\frac{1}{\lambda}\right)} \qquad w > 0.$$