Ex1.

Ex: $X_1, X_2, \dots X_{16} \sim N(\mu, 100)$

$\bar{X} = 24$

① $H_0: \mu = 20$ $\qquad \alpha = 0.05$

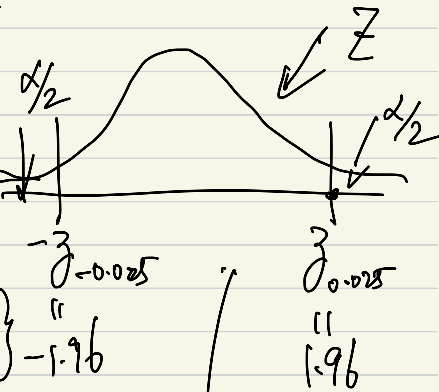
$H_1: \mu \neq 20$

② under H_0, $\bar{X} \sim N\left(\mu_0, \frac{\sigma^2}{n}\right)$

$\sim N\left(20, \frac{100}{16}\right)$

$\Leftrightarrow \frac{\bar{X} - 20}{10/\sqrt{16}} \sim Z.$

③ based on α:

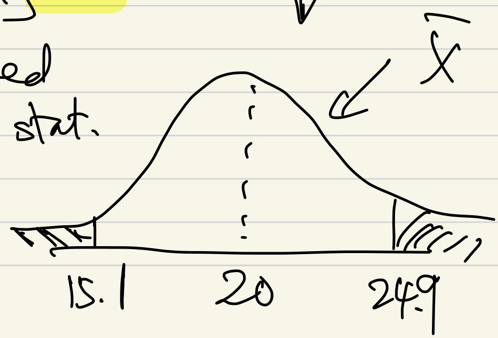


$C = \{ Z > 1.96 \text{ or } Z < -1.96 \}$

$= \left\{ \frac{\bar{X} - 20}{10/4} > 1.96 \text{ or } \frac{\bar{X} - 20}{10/4} < -1.96 \right\}$

$= \{ \bar{X} > 24.9 \text{ or } \bar{X} < 15.1 \}$

④ look at the observed value of the test stat.



$\bar{X}_{obs} = 24 \notin C$

$\Rightarrow$ cannot rej H_0!

Ex 2.

(1)

Ex: $H_0: \mu = 100$

$H_1: \mu > 100$.

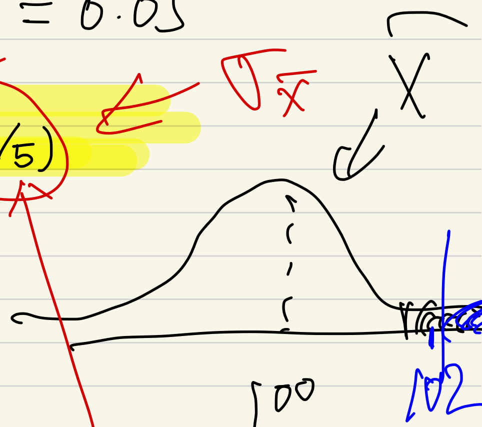
$X_1, X_2, \dots X_{25} \sim N(\mu, 25)$

$\bar{X} = 102$ $\qquad \alpha = 0.05$

Under H_0, $\bar{X} \sim N(100, 5/5)$ $\leftarrow \sigma_{\bar{X}}$

$\sim N(100, 1)$

$Z \sim \frac{\bar{X} - 100}{1}$



$C = \{\bar{X} - 100 > 1.645\}$

$= \{\bar{X} > 101.645\}$

$\frac{25}{25}$ — $\frac{\sigma^2}{n}$

$\bar{X}_{obs} = 102$

$\hookrightarrow$ in $C \rightarrow$ reject H_0 ✓

(add). p-value $= P(\bar{X} > 102) = P\left(Z > \frac{102-100}{1}\right)$

$= P(Z > 2) = 0.0228 < \alpha$

$\Rightarrow$ reject H_0

Ex3:

100 tires $\bar{X} = 54,500$, $S = 2,000$

Do you have enough evidence to reject the claim the average life is at least 55,000? $\alpha = 0.05$.

① $\cancel{H_0: \mu = 55000}$ $\quad H_0\ \mu \geq 55,000$

$\cancel{H_1: \mu > 55000}$ $\quad H_1: \mu < 55,000$ ✓

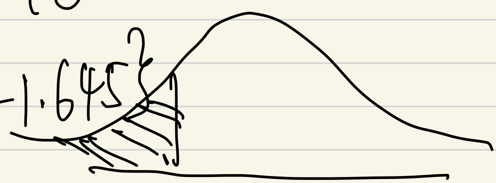
$\cancel{H_0: \mu < 55000}$ $\quad H_0: \mu = 55000$

$\cancel{H_1\ \mu \geq 55000}$ $\quad H_1: \mu < 55,000$ ✓

② under H_0, $\bar{X} \approx N\left(\mu_0, \frac{S^2}{n}\right)$

$$\Longleftrightarrow \frac{\bar{X} - 55,000}{2000/10} \sim Z$$

③ $C = \left\{ \frac{\bar{X} - 55000}{200} < -1.645 \right\}$



$= \{ \bar{X} < 55,000 - 329 \}$ $\quad -1.645 = -z_{0.05}$

$= \{ \bar{X} < 54{,}671 \}$

④ $\bar{X}_{obs} = 54{,}500 \in C$

$\Rightarrow$ Yes, rej H_0 :

Ex 4.

Ex: fix machine. average time 93 min.

new model: easier to fix.

new model: $n=18$. $\bar{x} = 88.8$ $s = 16.6$

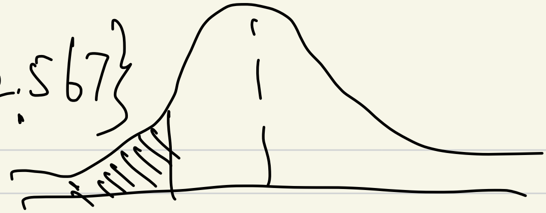
$\alpha = 0.01$. Assume ~~the~~ fix time $\sim$ Normal

① H_0: $\mu = 93$

H_1: $\mu < 93$

② under H_0, $\dfrac{\bar{X} - 93}{s/\sqrt{n}} \sim t(n-1)$

③ $C = \left\{ \dfrac{\bar{X} - 93}{16.6/\sqrt{18}} < -2.567 \right\}$



$= \{ \bar{X} < 83.0 \}$

$-t_{0.01} = -2.567$

④ $\bar{X}_{obs} = 88.8 \notin C$

$\Rightarrow$ cannot accept H_1.

reject H_0, can not

Critical Region Approach

Next:

p-value approach.

Ex 1:

Ex: $X_1, X_2, \cdots X_{16} \sim N(\mu, 100)$

$\bar{X} = 24$

① $H_0: \mu = 20$

$H_1: \mu \neq 20$

$\alpha = 0.05$

② under H_0, $\bar{X} \sim N(\mu_0, \frac{\sigma^2}{n})$

$\sim N(20, \frac{100}{16})$

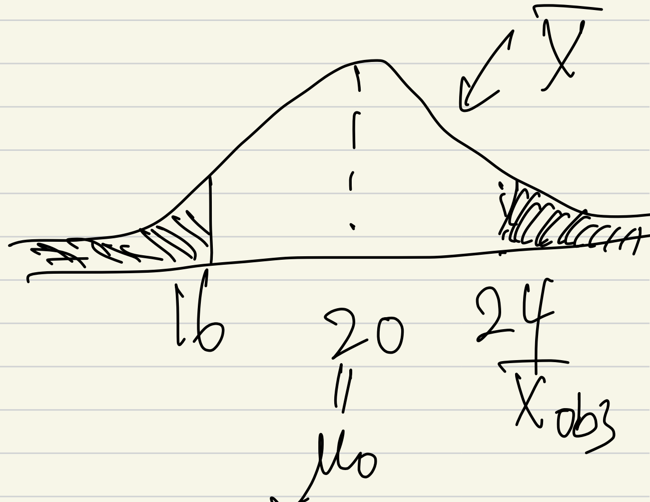
$$\Leftrightarrow \frac{\bar{X} - 20}{2.5} \sim Z$$

③ look at the observed value of the test stat,

p-value = under H_0, the prob of observing the observed value or outcomes even more extreme than the observed value.

= the tail prob of the obser stat.

(if it's a 2-sided test, the tail × 2.)

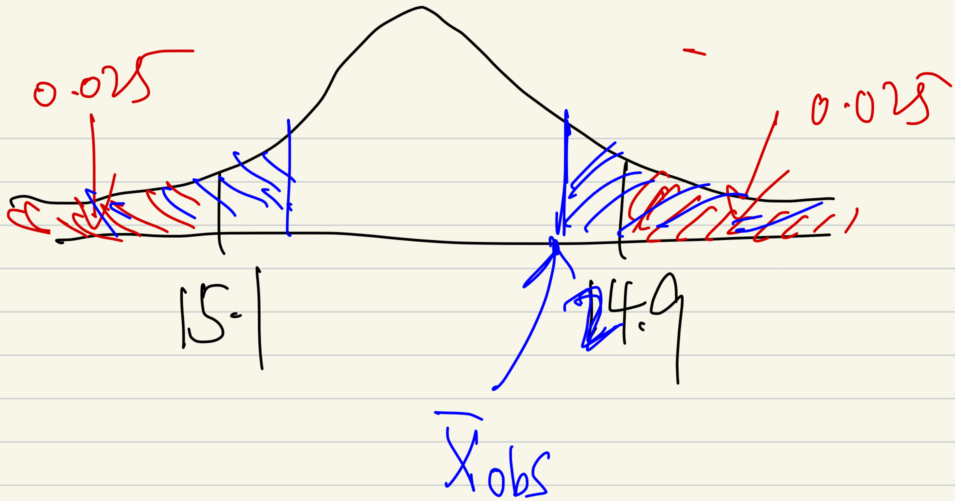


p-value $= 2 \cdot P(\bar{X} > 24)$

$= 2P\left(Z > \frac{24-20}{10/4}\right) = 2P(Z > 1.6)$

$= 2 \times 0.0548 = 0.1096$

(4) p-value $> \alpha \Rightarrow$ cannot reject H_0.



p-value : blue.

α : red.

Ex: $\sim N(\mu, \sigma^2 = 20^2)$

① $H_0: \mu = 75$

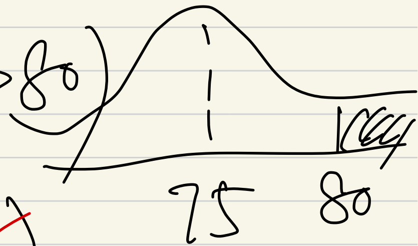
$H_1: \mu > 75.$

Based on a sample of size 16 (100), $\bar{x} = 80$. Can you support H_1?

② under H_0, $\bar{X} \sim N\left(75, \frac{20^2}{\cancel{16}\ 100}\right)$

$$\frac{\bar{X} - 75}{\cancel{5}\ 2} \sim Z.$$

③ p-value $= P(\bar{X} > 80)$



$= P\left(Z > \frac{80-75}{\cancel{5}\ 2} = \cancel{1}\ 2.5\right)$

$\approx \cancel{0.16}$ 0.0062

④ p-value pretty big.

Can not rej H_0. Can not support H_1 at this evid..