1. Element Set Theory

1.1 Functions

A function $f : X \to Y$ is injective if $f(x_1) = f(x_2)$ in Y implies $x_1 = x_2$ in X.

A function $f : X \to Y$ is surjective if for any $y \in Y$, there exists $x \in X$ such that $y = f(x)$.

$g : Y \to X$ is the inverse function of $f : X \to Y$ if $f \circ g = id_Y$ and $g \circ f = id_X$. If f has inverse function, we say f is invertible.

Prop. f is invertible $\iff$ f is bijective.

1.3 Equivalence Relations

A relation on a set S is a subset $R \subseteq S \times S$, that is, a subset of the ordered pairs of elements in S.

A relation R on S is called an equivalence relation if it satisfies:

1. Reflexive: $x \in S \to x \sim x$
2. Symmetric: $x \sim y \to y \sim x$
3. Transitive: $x \sim y, y \sim z \to x \sim z$

Given an equivalence relation on a set S, define the equivalence class of $a \in S$ to be the subset $[a] = \{b \in S | a \sim b\}$.

Prop. The equivalence classes of an equivalence relation on S give a partition of S, and conversely, a partition of S defines an equivalence relation on S.

Important equivalent relations on a group:

- $x \sim y$ if $y^{-1}x \in H$, this leads to cosets
- $x \sim y$ if $y = gxg^{-1}$ for some g, this leads to conjugacy classes