Midterm Review

Secant Method: $x_{k+1} = x_k - f(x_k)\frac{x_k - x_{k-1}}{f(x_k) - f(x_{k-1})}$

Newton's method: $x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}$

rate of convergence: $|\frac{f''(\eta)}{2f'(x_k)}| \rightarrow |\frac{f''(\xi)}{2f'(\xi)}|$ convergence $\leftrightarrow \frac{|f''(x)|}{|f'(y)|} \leq A$

$0 = f(\xi) = f(x_k) + f'(x_k)(\xi - x_k) + \frac{f''(\eta)}{2}(\xi - x_k)^2$

$\xi - x_k = -\frac{(\xi - x_k)^2 f''(\eta)}{2f'(x_k)} - \frac{f(x_k)}{f'(x_k)}$ $\xi - x_{k+1} = -\frac{(\xi - x_k)^2 f''(\eta)}{2f'(x_k)}$

$\lim_{k\to\infty} \frac{|x_{k+1} - \xi|}{|x_k - \xi|^2} = \lim_{k\to\infty} |\frac{f''(\eta)}{2f'(x_k)}| = |\frac{f''(\xi)}{2f'(\xi)}|$

Bisection method: $c_k = \frac{1}{2}(a_k + b_k)$

$(a_{k+1}, b_{k+1}) = (a_k, c_k)$ if $f(c_k)f(b_k) > 0$

$= (c_k, b_k)$ if $f(c_k)f(b_k) > 0$

A vector norm on X is a function $p : X \rightarrow R$

1. $p(x + y) \leq p(x) + p(y)$
2. $p(sx) = |s|p(x)$
3. for all $x \in X, p(x) = 0 \rightarrow x = 0$ or $p(x) = 0 \leftrightarrow x = 0$

$||v||_1 = |v1| + ... + |v_n|$

$||v||_2 = \sqrt{v_1^2 + ... + v_n^2}$

$||v||_\infty = max|v_i|$

$||v||_p = (|v_1|^p + ... + |v_n|^p)^{\frac{1}{p}}$

A matrix norm is a vector norm in a vector space whose elements are matrices, and $||AB|| \leq ||A||||B||$

$||A||_p = \max \frac{||Ax||_p}{||x||_p}$

$||A||_1$ is the maximum absolute column sum of the matrix

$||A||_\infty$ is the maximum absolute row sum of the matrix

$||A||_2 = \sqrt{\lambda_{max}(A^T A)} = \sigma_{max}(A^T A))$ is the largest singular value (squareroot of eigenvalue)

$Ax = b$

$LUx = b \quad Ly = b \quad ForwardSovle \rightarrow y \quad Ux = y \quad BackwardSolve \rightarrow x$

$QRx = b \quad x = Q^T R^{-1} b$ Gram Schmidt $\rightarrow Q \quad R = Q^T A$