Review. Sampling dist of $\bar{X}$:

① $X_1, X_2, \cdots X_n \sim N(\mu, \sigma^2)$, and σ^2 known.

$$\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right) \Leftrightarrow \frac{\bar{X}-\mu}{\sigma/\sqrt{n}} \sim Z$$

$$\Updownarrow$$

$n\bar{X}$

$$\sum_{i=1}^{n} X_i \sim N(n\mu, n\sigma^2) \Leftrightarrow \frac{\sum_{i=1}^{n} X_i - n\mu}{\sqrt{n}\cdot\sigma} \sim Z.$$

② $X_1, X_2, \cdots X_n \sim \mu, \sigma^2 < \infty$, then

$$\left.\begin{aligned} \bar{X} &\overset{\text{approx}}{\approx} N\left(\mu, \frac{\sigma^2}{n}\right) \\ \sum_{i=1}^{n} X_i &\overset{\text{approx}}{\approx} N(n\mu, n\sigma^2) \end{aligned}\right\} \text{C.L.T.}$$

- sampling dist of $S^2 = \frac{\sum (X_i - \bar{X})^2}{n-1}$

Thm: Let $X_1, X_2, \cdots X_n$ be a random sample from $N(\mu, \sigma^2)$, then

$$\frac{(n-1)S^2}{\sigma^2} \sim \chi^2(n-1)$$

$$\frac{\sum (X_i - \bar{X})^2}{\sigma^2} = \sum_{i=1}^{n} \left(\frac{X_i - \bar{X}}{\sigma}\right)^2 \sim \chi^2(n-1)$$

t-dist. similar to Z.

family of dist

$t(n)$ degree of freedom n.

bell-shaped

centered at 0.

more tail prob.

$t(n) \xrightarrow[n\to\infty]{} Z$

Thm: Let Z be a $N(0,1)$, and $\chi^2(r)$ be an χ^2-dist with d.o.f. r, and Z & $\chi^2(r)$ are indep. then:

$$\frac{Z}{\sqrt{\chi^2(r)/r}} \sim t(n).$$

$\frac{(n-1)S^2}{\sigma^2} \sim \chi^2(n-1)$

Ex: $X_1, X_2, --X_{20} \sim N(80, 10)$

① $P(\bar{X} > 81) = P\left(Z > \frac{81-80}{\sqrt{10}/\sqrt{20}}\right) = P(Z > 1.41)$
$= 0.0793$

$\bar{X} \sim N\left(80, \frac{10}{20}\right)$

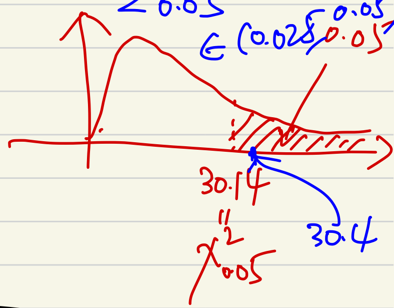
② $P(S > \cancel{5}\ 4) = P(S^2 > \cancel{25}\ 16) = P\left(\frac{19 \cdot S^2}{10} > \frac{19 \times \cancel{25}\ 16}{10}\right)$

$= P(\chi^2(19) > \cancel{47.5}\ 30.4) < \cancel{0.01}$

≈ 0.05

< 0.05

$\in (0.025, 0.05)$ 0.05



t - Table !!

Thm: $X_1, X_2, \cdots X_n \sim N(\mu, \sigma^2)$, but σ^2 unknow

Then $\frac{\bar{X} - \mu}{S/\sqrt{n}} \sim t(n-1)$

Only use $t(n-1)$ when underlying is normal, σ is unknown and n is small.

pf: $\dfrac{\bar{X}-\mu}{s/\sqrt{n}} = \dfrac{\bar{X}-\mu \big/ \sigma/\sqrt{n}}{s/\cancel{\sqrt{n}} \big/ \sigma/\cancel{\sqrt{n}}} = \dfrac{Z}{s/\sigma}$

$= \dfrac{Z}{\sqrt{(n-1)S^2/\sigma^2 (n-1)}} = \dfrac{Z}{\sqrt{\chi^2(n-1)/n-1}}$

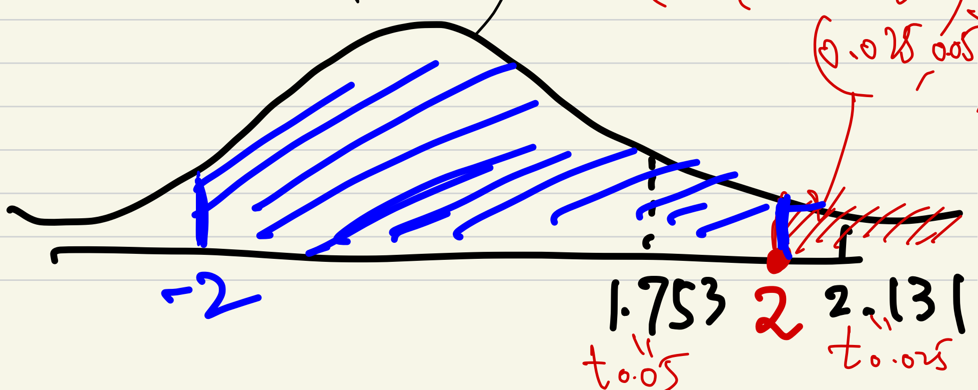
$\sim t(n-1)$ ∎

Ex: $X_1, X_2, \cdots X_{16} \sim N(\mu, \sigma^2)$

$\bar{X} = 40, \quad S = 2.$

$P(|\bar{X}-\mu| < 1) = P\left(\dfrac{|\bar{X}-\mu|}{S/\sqrt{n}} < \dfrac{1}{2/\sqrt{16}}\right)$

$= P(|t(15)| < 2) \in (0.90, 0.95)$



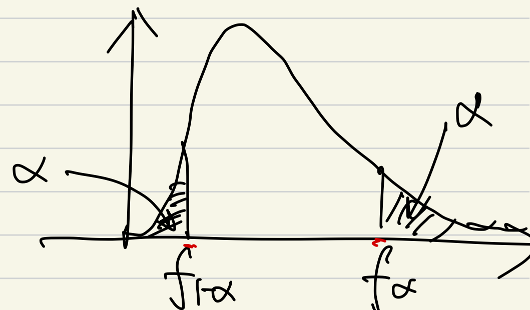
F - dist. (n_1, n_2)

numerator d.o.f.

denominator d.o.f.

Thm: Let W_1 & W_2 be indep. chi-square dist with d.o.f. n_1 & n_2 respectively. then

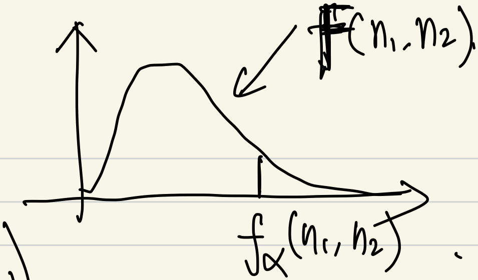
$$\frac{W_1/n_1}{W_2/n_2} \sim F(n_1, n_2)$$



F-table !!

Thm: $$F_{1-\alpha}(n_1, n_2) = \frac{1}{f_\alpha(n_2, n_1)}$$

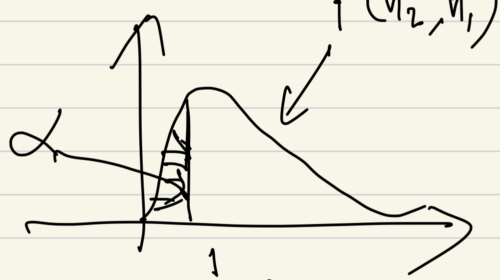
Pf:



$$P\left(F(n_1, n_2) > f_\alpha(n_1, n_2)\right) = \alpha$$

$$P\left(\frac{1}{F(n_1, n_2)} < \frac{1}{f_\alpha(n_1, n_2)}\right) = \alpha$$

$$P\left(F(n_2, n_1) < \frac{1}{f_\alpha(n_1, n_2)}\right) = \alpha$$



$$\Rightarrow f_{1-\alpha}(n_2, n_1) = \frac{1}{f_\alpha(n_1, n_2)}$$

$$\frac{1}{f_\alpha(n_1, n_2)} = f_{1-\alpha}(n_2, n_1)$$

$\blacksquare$

Ex. $f_{0.05}(8,10)$ & $f_{0.95}(8,10) = \dfrac{1}{f_{0.05}(10,8)} = \dfrac{1}{3.35} = 0.299$

$f_{0.05}(8,10) = 3.07$

$$P(0.299 < F(8,10) < 3.07) = 0.90$$