

Insertion Sort $O(n^2)$

Input: array $A[1 \dots n]$

Goal: sort in increasing order

Algo in words: Scan the array from left to right. For each position j , insert $A[j]$ to its correct position in $A[1 \dots j]$ by moving all elements larger than it one position to the right.

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for j = 2 to n:
    key <- A[j]
    i = j - 1
    while i > 0 and A[i] > key:
        A[i+1] <- A[i]
        i <- i - 1
    A[i+1] = key
```

Proof of correctness:

Loop invariant: at beginning of the j th iteration (or end of $j-1$ st iteration), $A[1 \dots j-1]$ contains the same elements it initially contained, but in sorted order.

If true, then at the end of the n th iteration, A is sorted, as required.

Claim: Loop invariant holds for $j = 2, 3, \dots, n+1$.

Proof: By induction

When $j = 2$ (base of induction), $A[1]$ is just one number and is therefore sorted.

Next, assume holds for j , and let us show that holds for $j+1$. By assumption, $A[1 \dots j-1]$ is sorted. During the j th iteration, we insert $A[j]$ into its correct position. Therefore, the invariant holds at the end of the j th iteration, as required.

Running time: $O(n^2)$

$$\sum_{j=2}^n (c + c'c_j) = c^2/2n^2 + n - () = O(n^2)$$