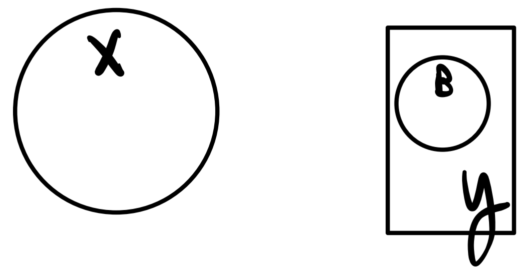
$(A-B)\cup(C-B)=(A\cup C)-B$ $\boxed{K}$

$F: X \twoheadrightarrow Y$



$\forall B\subset Y\,(F(F^{-1}(B))=B)$

$F^{-1}(B)=\{x\in X : F(x)\in B\}$

$F(A)=\{y\in Y : \exists x\in A\,(F(x)=y)\}$

Let $b\in F(F^{-1}(B))$. By definition of range, there exists a $x\in F^{-1}(B)$ such that $F(x)=b$.

Since $x\in F^{-1}(B)$, $F(x)\in B$ but $b=F(x)$ so $b\in B$.

$F(F^{-1}(B))\subset B$

Let $b\in B$. Since F is onto, there exists $x\in X$ such that $F(x)=b$. Since $F(x)=b\in B$, $x\in F^{-1}(B)$. So there exists $x\in F^{-1}(B)$ such that $b=F(x)$ and $b\in F(F^{-1}(B))$.

$B\subset F(F^{-1}(B))$.

$F: X \hookrightarrow Y$; $\forall A\subset X\,(F^{-1}(F(A))=A)$

e.g. $f : \mathbb{R} \to (0,\infty)$

$f(x) := e^x$

$f(1) = e$

$f(\mathbb{R}) = (0,\infty)$

$f([-1,0) \cup (1,2]) = [1/e, 1) \cup (e, e^2]$

$f : \mathbb{N} \times \mathbb{N} \times \mathbb{N} \longrightarrow \mathbb{N}$

① $f(x,y,0) = y+1$

② $f(x,0,1) = x$

③ $f(x,0,2) = 0$

④ $f(x,0,z+3) = 1$

⑤ $f(x,y,z) = f(x,w,z-1)$ where $w = f(x,y-1,z)$

$y \geq 1$ and $z \geq 1$

$f(2,2,2) = f(2,w_1,1) = f(2,2,1) = f(2,w_3,0)$

$= w_3 + 1 = 3 + 1 = 4$ but $w_3 = f(2,2-1,1)$

$w_3 = f(2,1,1) = f(2,w_4,0) = 2+1 = 3$ where $w_4 = 2$

$w_4 = f(2,0,1) = 2$

$w_1 = f(2,1,2) = f(2, w_2, 1) = f(2, 0, 1) = 2$

$w_2 = f(2,0,2) = 0$

---

① element method

$x \in A - B$.
then $x \in A$ and $x \notin B$.
by specialization,
$x \notin B$. so $x \in B^c$.
then by specialization
$x \in A$. So $x \in A$
and $x \in B^c$ by
conjunction.
Then $x \in A \cap B^c$ by
def of intersection

② Theorem 6.2.2

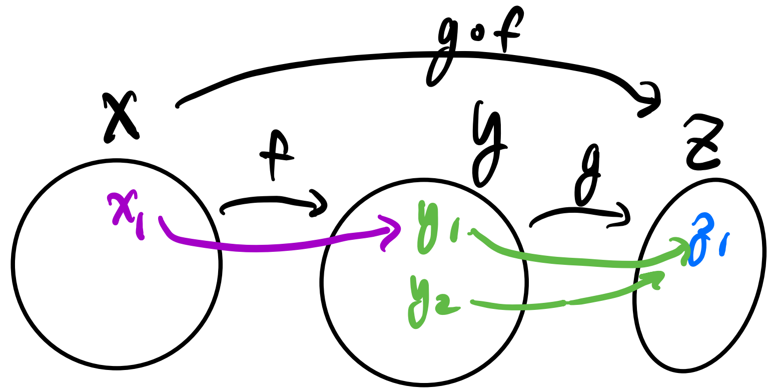
$A - B = A \cap B^c$

$$\left.\begin{array}{l} ①\ A \subset B \\ ②\ B \subset A \end{array}\right\} A = B$$

Show set A is empty.
Assume $A \neq \emptyset$ so
there exists some $x \in A$.

$f: X \to Y$

$g: Y \to Z$



$g \circ f$ is one-to-one

must g be one-to-one? NO

Let $X = \{x_1\}$, $Y = \{y_1, y_2\}$, $Z = \{z_1\}$.

Define $f(x_1) = y_1$, $g(y_1) = z_1$, $g(y_2) = z_1$, and

so $(g \circ f)(x_1) = z_1$.

one-to-one	injective
onto	surjective
one-to-one correspondence (Epp textbook terminology)	bijective

---

e.g. For any set X, define $id_X : X \to X$ such that $\forall x \in X \, (id_X(x) = x)$. Then id_X is called the identity function on X.

e.g. Suppose $X = \mathbb{Z}$.

(a) Is it true that

$\forall x, y \in \mathbb{Z} \, (id_{\mathbb{Z}}(x+y) = id_{\mathbb{Z}}(x) + id_{\mathbb{Z}}(y))$?

Let $x, y \in \mathbb{Z}$. $id_{\mathbb{Z}}(x+y) = x+y = id_{\mathbb{Z}}(x) + id_{\mathbb{Z}}(y)$

(b) Is it true that

$\forall x, y \in \mathbb{Z} \, (id_{\mathbb{Z}}(xy) = id_{\mathbb{Z}}(x) \, id_{\mathbb{Z}}(y))$?

Let $x, y \in \mathbb{Z}$. $id_{\mathbb{Z}}(xy) = xy = id_{\mathbb{Z}}(x) \, id_{\mathbb{Z}}(y)$

(c) Suppose $f: \mathbb{Z} \to \mathbb{Z}$ is any function with the property $\forall x, y \in \mathbb{Z} \, (f(x+y) = f(x) + f(y))$.

Why is $f(0) = 0$?

Let $x, y \in \mathbb{Z}$. Suppose $f(x+y) = f(x) + f(y)$.

$f(0) = f(0+0) = f(0) + f(0)$

$-f(0) \qquad\qquad -f(0)$

$0 = f(0)$

$f : X \to Y$

$id_X : X \to X$

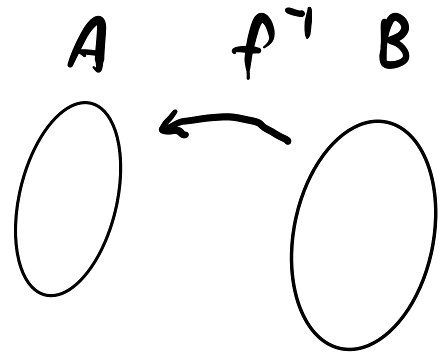
$id_Y : Y \to Y$

$f \circ id_X = f$

$id_Y \circ f = f$

$f : A \to B$

$T \subset B$



$f^{-1}(T) = \{x \in A : \exists t \in T (f^{-1}(t) = x)\}$

(range) image of $T \subset B$ under <u>inverse function</u> f^{-1}

$f^{-1}(T) = \{t \in A : f(t) \in T\}$

inverse image of $f : A \to B$

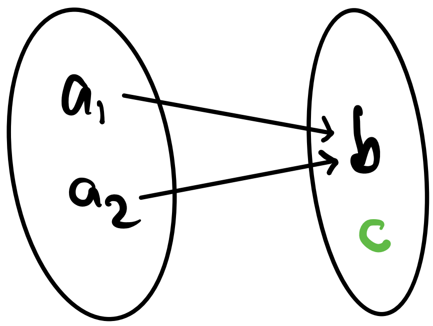
$$f^{-1}(b) = a$$

$$f(a_1) = f(a_2) = b$$

$$\cancel{f^{-1}(b) = a_1 ?\quad = a_2 ?}$$

f^{-1} not a function

$$\underline{f^{-1}(\{b\}) = \{a_1, a_2\}}$$



$$f^{-1}(B) \subset A$$

$$f^{-1}(\{c\}) = \emptyset$$