Math 140 - Spring 2017 - Midterm Exam 1 - Version A

You have 105 minutes to complete this midterm exam. Books, notes and electronic devices are not permitted. Read and follow directions carefully. Show and check all work. Label graphs and include units where appropriate. If a problem is not clear, please ask for clarification.

Multiple Choice 1-10	/30
Free Response 11	/10
Free Response 12	/10
Free Response 13	/10
Free Response 14	/10
Free Response 15	/10
Total	/80

I pledge that I have completed this midterm exam in compliance with the NYU CAS Honor Code. In particular, I have neither given nor received unauthorized assistance during this exam.

Name ____

N Number ____

Signature ____

Date ____

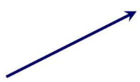
1 Multiple Choice

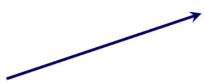
(30 points) For problems 1-10, circle the best answer choice.

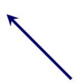
1. Let $\boldsymbol{u}$ and $\boldsymbol{v}$ be the vectors graphed below.

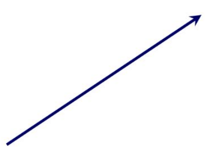
Which of the following is equal to $\boldsymbol{u} - 2\boldsymbol{v}$?

(a) 

(b) 

(c) 

(d) 

(e) 

2. Let $\boldsymbol{u} = \{1, 3, 1\}$ and $\boldsymbol{v} = \{2, 1, -3\}$. The angle formed by $\boldsymbol{u}$ and $\boldsymbol{v}$ is

(a) acute

(b) obtuse

(c) right

(d) straight

(e) positive

3. The vector $(1, 2, m, 5)$ is a linear combination of the vectors $(0, 1, 1, 1), (0, 0, 0, 1)$ and $(1, 1, 2, 0)$ if only if $m =$

(a) 0

(b) 1

(c) 2

(d) 3

(e) 4

4. Let A be a 4×6 matrix. Which elimination matrix corresponds with the row operations "switch rows 1 and 4, and scale row 3 by 7"?

(a) $\begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 7 & 7 & 7 & 7 \\ 1 & 0 & 0 & 0 \end{bmatrix}$

(b) $\begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 7 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$

(c) $\begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 7 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

(d) $\begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 7 & 7 & 7 & 7 & 7 & 7 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

(e) $\begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 7 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$

5. Let $A = \begin{bmatrix} 1 & 3 & -1 \\ 0 & 6 & -2 \\ 0 & 4 & 5 \end{bmatrix}$. Which of these entries is a pivot of A?

(a) -2

(b) 0

(c) 2

(d) 4

(e) 6

6. Suppose A and B are invertible 3×3 matrices. Which of the following is **not** necessarily invertible?

(a) $A + B$

(b) $A^T B$

(c) BA

(d) ABA^{-1}

(e) A^2

7. Let $A = \begin{bmatrix} 2 & d & d \\ 2 & d & d \\ 3 & 3 & 3 \end{bmatrix}$. For which value of d does A have rank 1?

(a) 0

(b) $\frac{2}{3}$

(c) 2

(d) 3

(e) 6

8. Span$\left\{\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}\right\} =$

(a) $\mathbb{R}^2$

(b) $\mathbb{R}^3$

(c) a line in $\mathbb{R}^2$

(d) a line in $\mathbb{R}^3$

(e) a plane in $\mathbb{R}^3$

9. A basis for the space $\mathbb{P}_2(x)$, the set of all polynomial functions with real coefficients of degree at most 2 is

(a) $\{1, x, x^2\}$

(b) $\{1 + x + x^2\}$

(c) $\{x^2\}$

(d) $\left\{\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ x \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ x^2 \end{bmatrix}\right\}$

(e) $\left\{\begin{bmatrix} 1 & x \\ 0 & x^2 \end{bmatrix}\right\}$

10. Suppose A is a 53×32 matrix whose rank is 17. Which of the following is a true statement?

(a) Col A is a 36-dimensional subspace of $\mathbb{R}^{53}$ and Nul A^T is a 17-dimensional subspace of $\mathbb{R}^{53}$.

(b) Col A is a 17-dimensional subspace of $\mathbb{R}^{53}$ and Nul A^T is a 36-dimensional subspace of $\mathbb{R}^{53}$.

(c) Col A is a 17-dimensional subspace of $\mathbb{R}^{32}$ and Nul A^T is a 15-dimensional subspace of $\mathbb{R}^{32}$.

(d) Col A is a 15-dimensional subspace of $\mathbb{R}^{32}$ and Nul A^T is a 17-dimensional subspace of $\mathbb{R}^{32}$.

(e) Col A is a 17-dimensional subspace of $\mathbb{R}^{53}$ and Nul A^T is a 17-dimensional subspace of $\mathbb{R}^{32}$.

2 Free Response

For problems 11-15, show all work and justify each step to receive full credit. Draw a box around your answers.

11. (10 points) Consider the matrix $A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -2 & 1 & 0 & 0 \\ 0 & -2 & 1 & 0 \\ 0 & 0 & -2 & 1 \end{bmatrix}$

(a) Compute A^{-1}.

(b) Compute $K = A^T A$.

(c) Compute K^{-1}. (*Hint: Use* A^{-1}.)

(d) Solve $K\boldsymbol{x} = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}$. (*Hint: Use* K^{-1}.)

12. (10 points) Find an LU factorization for $A = \begin{bmatrix} 1 & -2 & -2 & -3 \\ 3 & -9 & 0 & -9 \\ -1 & 2 & 4 & 7 \\ -3 & -6 & 26 & 2 \end{bmatrix}$ and use it to solve $A\boldsymbol{x} = \begin{bmatrix} 1 \\ 6 \\ 0 \\ 3 \end{bmatrix}$.

13. (10 points) Determine whether each set W below is a subspace of $\mathbb{R}^3$. Support your answer.

(a) $W = \{(x, y, z) | 3x + 2y^2 + z = 0, x, y, z \in \mathbb{R}\}$

(b) $W = \{(x, y, z) | 2x + 3y - z = 0, x, y, z \in \mathbb{R}\}$

14. (10 points) Find the complete solution $\boldsymbol{x} = \boldsymbol{x}_p + \boldsymbol{x}_n$ to the following linear system.

$$\begin{bmatrix} 2 & 4 & 0 & 8 \\ 1 & 2 & 1 & 10 \end{bmatrix} \boldsymbol{x} = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$$

15. (10 points) Find a basis and dimension for each of the four vector spaces associated with the matrix.

$$A = \begin{bmatrix} 1 & 0 & 0 & -5 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$