$X_1, X_2, \ldots X_{100} \sim Exp(20)$ $\mu = \sigma = 20$

① $P(18 < \bar{X} < 23) = \cancel{\int_{18}^{23} \frac{1}{20} e^{-x/20}\, dx}$

By CLT, $\bar{X} \sim N(\mu, \sigma^2/n) = N(20, \frac{20^2}{100})$

$P(18 < \bar{X} < 23) = P\left(\frac{18-20}{20/10} < Z < \frac{23-20}{20/10}\right)$

$= P(-1 < Z < 1.5) = 0.7745$?

② what's the prob at least 35 of these 100 will be < 10 ?

$p = P(X < 10) = \int_0^{10} \frac{1}{20} e^{-x/20}\, dx = -e^{-x/20}\Big|_0^{10}$

$= 1 - e^{-1/2} \approx 0.393$

$Y \sim Bin(100, 0.393) \approx N(np, npq)$

$$P(Y \geq 35) = P\left(Z \geq \frac{34.5 - 39.3}{\sqrt{npq}}\right)$$

$23. --$

$$\cancel{p = P(X < 10) \approx P\left(Z < \frac{10-20}{20}\right)}$$

Confidence Intervals for

μ, $\mu_1 - \mu_2$, p, $p_1 - p_2$

Now: σ^2, σ_1^2/σ_2^2

§9.12. One sample: Estimate the variance

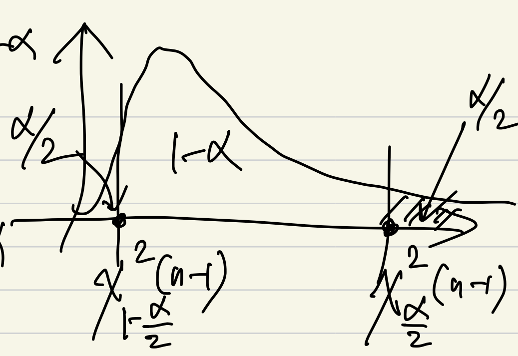
$X_1, X_2, \ldots X_n \sim N(\mu, \sigma^2)$

Goal: To est. σ^2.

Point estimator: S^2

sampling dist of S^2: $\dfrac{(n-1)S^2}{\sigma^2} \sim \chi^2(n-1)$

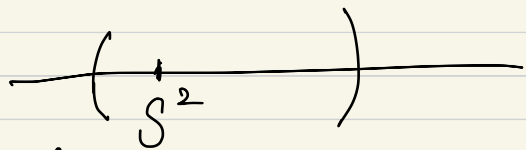
$$P\left(\chi^2_{1-\frac{\alpha}{2}}(n-1) \le \frac{(n-1)S^2}{\sigma^2} \le \chi^2_{\frac{\alpha}{2}}(n-1)\right) = 1-\alpha$$



$$P\left(\frac{(n-1)S^2}{\chi^2_{\frac{\alpha}{2}}(n-1)} \le \sigma^2 \le \frac{(n-1)S^2}{\chi^2_{1-\frac{\alpha}{2}}(n-1)}\right) = 1-\alpha$$

$100(1-\alpha)\%$ C.I. for σ^2 is

$$\left(\frac{(n-1)S^2}{\chi^2_{\frac{\alpha}{2}}}, \frac{(n-1)S^2}{\chi^2_{1-\frac{\alpha}{2}}}\right),$$ where d.o.f $= n-1$.

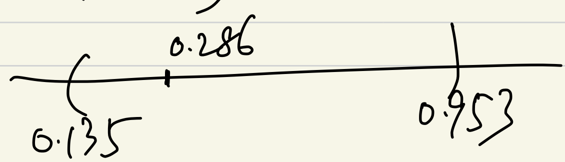


Ex: $\sim$ normal distribution.

$n=10$: 46.4, 46.1, 45.8, 47.0, 46.1
45.9, 45.8, 46.9, 45.2, 46.0

Find a 95% CI for σ^2. $\boxed{S^2 = 0.286}$

$$\left(\frac{\overset{9\times 0.286}{(n-1)S^2}}{19.023}, \frac{\overset{9\times 0.286}{(n-1)S^2}}{2.700}\right) = (0.135, 0.953)$$



9.13. 2-samples: estimating the ratio of 2 variances.

indep $\begin{cases} X_1, X_2, \dots X_{n_1} \sim N(\mu_1, \sigma_1^2) \\ Y_1, Y_2, \dots Y_{n_2} \sim N(\mu_2, \sigma_2^2) \end{cases}$

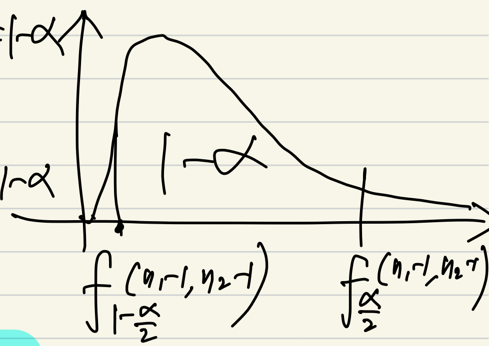
Goal: To est $\dfrac{\sigma_1^2}{\sigma_2^2}$

point est: $\dfrac{S_1^2}{S_2^2}$

sampling dist of $\dfrac{S_1^2}{S_2^2}$: $\dfrac{S_1^2/\sigma_1^2}{S_2^2/\sigma_2^2} \sim F(n_1-1, n_2-1)$

$$P\left(f_{1-\frac{\alpha}{2}}(n_1-1, n_2-1) \le \frac{S_1^2\,\sigma_2^2}{S_2^2\,\sigma_1^2} \le f_{\frac{\alpha}{2}}(n_1-1, n_2-1)\right) = 1-\alpha$$

$$P\left(\frac{S_1^2}{S_2^2}\cdot\frac{1}{f_{\frac{\alpha}{2}}(n_1-1,n_2-1)} \le \frac{\sigma_1^2}{\sigma_2^2} \le \frac{S_1^2}{S_2^2}\,\frac{1}{f_{1-\frac{\alpha}{2}}(n_1-1,n_2-1)}\right) = 1-\alpha$$



$$f_{1-\frac{\alpha}{2}}(n_1-1, n_2-1) = \frac{1}{f_{\frac{\alpha}{2}}(n_2-1, n_1-1)}$$

$100(1-\alpha)\%$ C.I. for $\dfrac{\sigma_1^2}{\sigma_2^2}$ is:

$$\left(\frac{S_1^2}{S_2^2}\cdot\frac{1}{f_{\frac{\alpha}{2}}(n_1-1,n_2-1)}\ ,\ \frac{S_1^2}{S_2^2}\cdot f_{\frac{\alpha}{2}}(n_2-1, n_1-1)\right)$$

Ex: 2 normal popu.

$n_1=10$, $s_1=3.07$ Find a 95% CI for σ_1^2/σ_2^2.

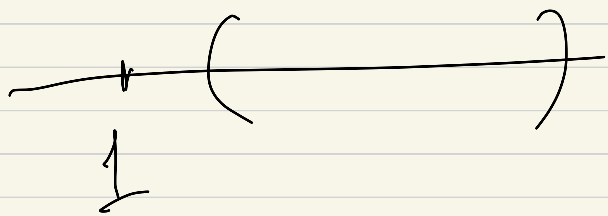
$n_2=8$, $s_2=0.80$

$$\left(\frac{3.07^2}{0.80^2}\,\frac{1}{4.82},\ \frac{3.07^2}{0.8^2}\cdot 4.2\right)$$

$$=\left(3.06,\ 61.9\right)$$

95% CI for $\frac{\sigma_1}{\sigma_2}$: $\left(\sqrt{3.06},\ \sqrt{61.9}\right)$

$$=\left(1.75,\ 7.8\right)$$



Chapter 10. Tests of Hypotheses.

Hypothesis: a statement, or conjecture, concerning (some parameter(s) of) a population or populations

(not about sample or statistic)

H_0: Null Hypothesis ← ($\mu = 70$)

H_1: Alternative Hypothesis ← ($\mu \neq 70$)

test statistic

H_0: $p = 0.5$ Toss the coin 10 times

H_1: $p \neq 0.5$

Decision \ reality	H_0 is true	H_0 is false, H_1 is true
Not reject H_0	✓	Type II error
Reject H_0	Type I error	✓

H_0: The defendant is innocent.
H_1: — — — — guilty

under H_0: $X \sim Bin(10, 0.5)$

x	0	1	2	3	4	5	6	7	8	9	10
$P(X=x)$	0.001	0.01	0.044	0.117	0.205	0.246	0.205	0.117	0.044	0.01	0.001

0.246 (5–5)

0.656 (4–6)

2, 8

0.89 (3–7)

0.978 (2–8)

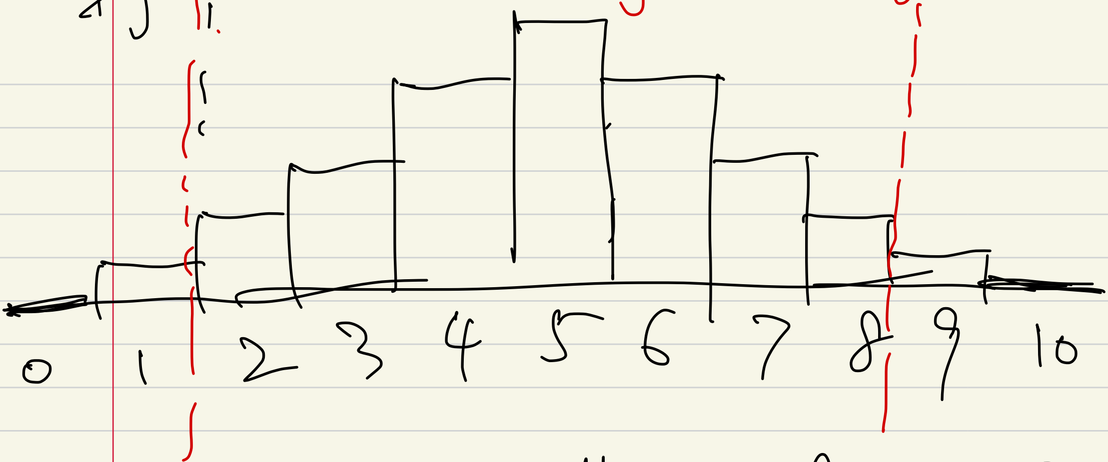
0.998 (1–9)

1

α: The significance level of the test.

If, $\alpha = 0.05$. Reject H_0 if $x = 0, 1, 9, 10$

If $\alpha = 0.01$. reject H_0 if $x = 0, 10$.



α: the max. allance of type I error when H_0 is true.

① State H_0 & H_1

② under H_0, __ __ ~ ___

test statistic   dist.

③ decision   rejection region (critical region)

p-value.

④ state the conclusion.