Homework 2

Due: Friday Sept. 17, by 11:59pm,
via Gradescope

- Failure to submit homework correctly will result in a zero on the homework.
- Homework must be in LaTeX. Submit the pdf file to Gradescope.
- Problems assigned from the textbook come from the 5th edition.
- No late homework accepted. Lateness due to technical issues will not be excused.

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1. (24 points) Section 2.2 # 22, 23.

   Remark. *For both problems, parts (b), (c), (e), (g) only.*

   Only a solution for problem # 22 will be provided.

   (b) If tomorrow is not January then today is not New Years Eve.
   (c) If r is irrational then the decimal expansion is non-terminating.
   (e) If x is not positive and x is not zero, then x is negative.
   (g) If n is not divisible by 2 or n is not divisible by 3, then n is not divisible by 6.

2. (9 points) Section 2.2 # 14, 38, 43.

   Remark. *When doing #14, do not use truth tables. Use Theorem 2.1.1. and*

   **Theorem ***_Let p and q be statement variables. $p \to q \equiv \sim p \lor q$._

   # 14. Only part (a) will be done.

$$\begin{aligned}
p \to q \lor r &\equiv \sim p \lor (q \lor r)\ (Theorem*)\\
&\equiv (\sim p \lor q) \lor r\ (Associative)\\
&\equiv \sim (p \land \sim q) \lor r\ (DeMorgan's)\\
&\equiv p \land \sim q \to r\ (Theorem*)\\
&\equiv \sim (p \land \sim q) \lor r\ (Theorem*)\\
&\equiv (\sim p \lor q) \lor r\ (DeMorgan's)\\
&\equiv \sim p \lor (q \lor r)\ (Associative)\\
&\equiv \sim p \lor (r \lor q)\ (Commutative)\\
&\equiv (\sim p \lor r) \lor q\ (Associative)\\
&\equiv \sim (p \land \sim r) \lor q\ (DeMorgan's)\\
&\equiv p \land \sim r \to q\ (Theorem*)
\end{aligned}$$

   # 38. If it does not rain, then Ann will go.

   # 43.

(a) If Jim passes the course, then Jim does homework regularly.

(b) If Jim does not do homework regularly, then Jim does not pass the course.

3. (9 points) Section 2.3 # 9, 12 (b), 23.

Solution : Only a solution to problems # 9 and # 23 will be provided.

	1	2	3	4	5	6	7	8	9	10
	p	q	r	$p \wedge q$	$\sim r$	$\sim q$	$p \wedge q \to \sim r$	$p \vee \sim q$	$\sim q \to p$	$\sim r$
1	T	T	T	T	F	F	F	T	T	F
2	T	T	F	T	T	F	T	T	T	T
3	T	F	T	F	F	T	T	T	T	F
4	F	T	T	F	F	F	T	F	T	F
5	F	F	T	F	F	T	T	T	F	F
6	F	T	F	F	T	F	T	F	T	T
7	T	F	F	F	T	T	T	T	T	T
8	F	F	F	F	T	T	T	T	F	T

The premises are columns 7, 8, and 9. The conclusion is column 10. The critical rows are rows 2, 3, 7. The argument form is invalid due to the conclusion being F in critical row 3.

Remark. *In addition to the instructions associated to these problems. Complete all truth tables even if argument is invalid. Identify all critical rows even if the argument is invalid.*

23. Here is the associated argument form.

$$p \vee q$$
$$p \to r$$
$$\therefore q \vee \sim r$$

While I won't construct the truth table here, notice how the premises will be true for $p = T = r$ and $q = F$. Meanwhile the conclusion will be false.

4. (6 points) Section 2.3 # 29, 38(d).

Solution :

#29. The associated argument form is

$$p \to q \tag{1}$$
$$\sim p \tag{2}$$
$$\therefore \sim q \tag{3}$$

The argument form is invalid (inverse error). Therefore, the argument is invalid.

#38(d) Here you are given a series of statements that you assume to be true. Then use the table on page 76 to obtain a conclusion. Let's write the associated statement form for each statement

$$(a)\ p \to q$$
$$(b)\ r \to \sim q$$
$$(c)\ p$$
$$(d)\ r \vee s$$
$$(e)\ t \to u$$

We have

$$p \to q$$
$$p$$
$$\therefore q \ \ (Modus\ Tollens)$$

$$r \to \sim q$$
$$q$$
$$\sim r \ \ (Modus\ Tollens)$$

$$r \vee s$$
$$\sim r$$
$$\therefore s \ \ (Elimination)$$

Therefore the treasure is buried underneath the flagpole. Note that statement (e) was never used.

Solution:

Let's assume that U is a knight.

$\therefore$ No one is a knight.

$\therefore$ U is not a knight (contradiction. U is a knight.).

Let's assume that V is a knight.

$\therefore$ There are at least three knights.

$\therefore$ $Z \wedge Y$ are knaves.

Since U is also a knive $\to V \wedge X \wedge W$ are the only knights.

$\therefore$ X tells the truth (contradiction. There are exactly three knights).

Let's assume that X is a knight.

$\therefore$ There are exactly 5 knights.

Since $U \wedge V$ are knaves we have at most 4 knights ($W \wedge X \wedge Y \wedge Z$) (contradiction).

Let's assume Z is a knight.

$\therefore$ Therefore there is exactly one knight.

$\therefore$ Z is a knight.

$\therefore$ $W \wedge Y$ are knaves.

$\therefore$ W lies (contradiction. W is telling the truth).

Notice that both W and Y tell the truth. They are the knights!

5. (6 points) Section 2.3 # 42, 44

   Solution : Here we are trying to show an argument form is valid using the the chart on page 76.

$$\begin{array}{l} q \to r \\ \sim r \\ \therefore \sim q \ \ (Modus\ Tollens) \end{array}$$

$$\begin{array}{l} p \vee q \\ \sim q \\ \therefore p \ \ (Elmination) \end{array}$$

$$\begin{array}{l} \sim q \to u \wedge s \\ \sim q \\ \therefore u \wedge s \ \ (ModusPonens) \end{array}$$

$$\begin{array}{l} u \wedge s \\ \therefore s \ \ (Specialization) \end{array}$$

$$\begin{array}{l} p \\ s \\ \therefore p \wedge s \ \ (Conjuction) \end{array}$$

$$\begin{array}{l} p \wedge s \to t \\ p \wedge s \\ \therefore t \ \ (Modus\ Ponens) \end{array}$$

$$\begin{array}{l} \sim q \vee s \\ \sim s \\ \therefore q \ \ (Elimination) \end{array}$$

Remark. *Your justification may not be in the same order as the solution provided.*

$r \vee s$
$\sim s$
$\therefore r \ \ (Elimination)$

$\sim s \rightarrow \sim t$
$\sim s$
$\therefore \sim t \ \ (ModusPonens)$

$w \vee t$
$\sim t$
$\therefore w \ \ (Elimination)$

$\sim q \vee s$
$\sim s$
$\therefore \sim q \ \ (Elimination)$

$p \rightarrow q$
$\sim q$
$\therefore \sim p \ \ (ModusPonens)$

$\sim p$
r
$\therefore \sim p \wedge s \ \ (Conjunction)$

$\sim p \wedge s \rightarrow u$
$\sim p \wedge s$
$\therefore u \ \ (Modus\ Ponens)$

u
w
$\therefore u \wedge w \ \ (Conjunction)$

Remark. *Your justification may not be in the same order as the solution provided.*