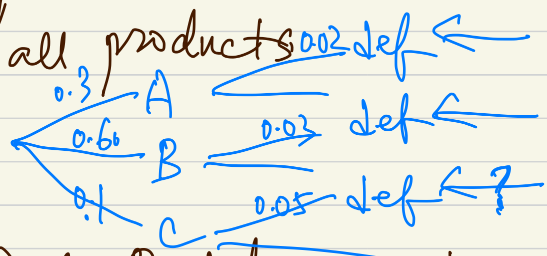
Ex: A factory has 3 production lines. A, B, C.

A makes 30% of all products

B 60%

C 10%



A, B, C has 2%, 3% & 5% defective rate respectively.

a. Randomly pick a finished product, P(defective) = ?

$= 0.3 * 0.02 + 0.60 * 0.03 + 0.10 * 0.05$

$= 0.006 + 0.018 + 0.005 = 0.029$

b. Given that the item is defective, what's the prob it was made by C

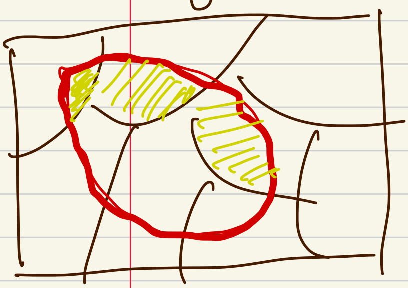
$$P(C \mid \text{defective}) = \frac{P(C \cap \text{def})}{P(\text{def})} ?$$

$$= \frac{0.005}{0.029} = 0.172$$

$P(A \mid \text{defective}) = 0.207$

$P(B \mid \text{def}) = 0.621$

Bayes' Theorem.



$B_1, B_2 \cdots B_k$ constitutes a partition of S.

$\left(B_i \cap B_j = \phi \ i \neq j, \ \bigcup_{i=1}^{k} B_i = S \right)$

$P(B_i)$ $i = 1, 2, \cdots k$ are known.

prior probabilities.

$P(A \mid B_i)$ are known for $i = 1, 2 \cdots k$

Then
$$P(B_r \mid A) = \frac{P(A \cap B_r)}{\sum_{i=1}^{k} P(A \cap B_i)}$$

$r = 1, 2, \cdots k$.

$$= \frac{P(B_r) \cdot P(A \mid B_r)}{\sum_{i=1}^{k} P(B_i) \cdot P(A \mid B_i)}$$

posterior prob.

Ex:

A	step 1	B
20 white 5 red balls	1 ball $\longrightarrow$ randomly	5 white 4 red 6 blue

step 2: pick 2 randomly

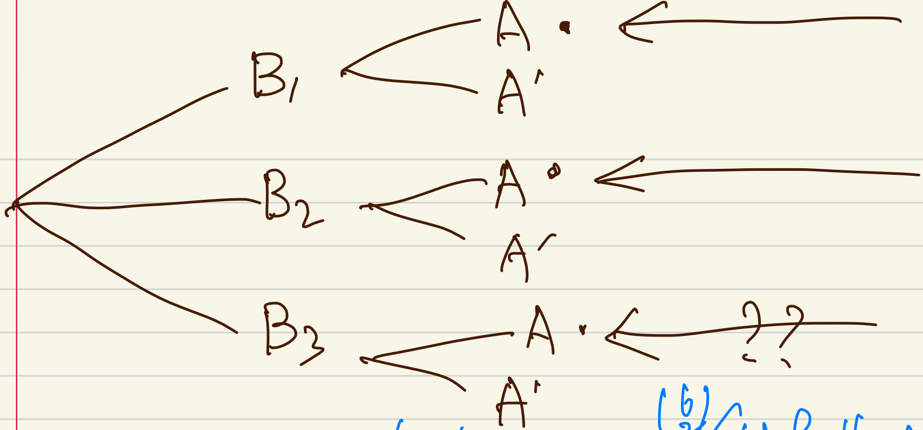
Given the balls in step 2 were both white, what's the prob the ball in step 1 was white also?

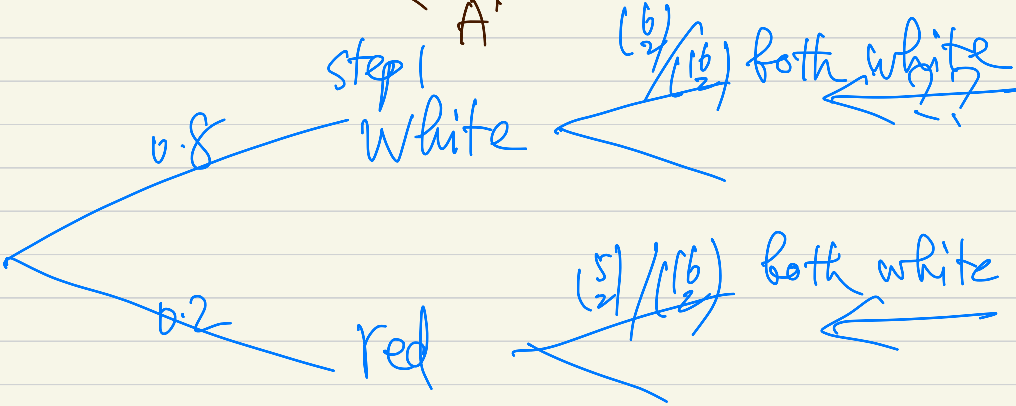
$$\frac{4}{5}$$

$$P\left(\begin{matrix}\text{1st is}\\ \text{white}\end{matrix} \,\middle|\, 2\text{ whites}\right) = \frac{P\left(\text{1st is white}, \begin{matrix}\text{2nd both}\\ \text{white}\end{matrix}\right)}{P\left(\begin{matrix}\text{2nd both}\\ \text{white}\end{matrix}\right)}$$

$$= \frac{\frac{20}{25}\frac{\binom{6}{2}}{\binom{16}{2}}}{\frac{20}{25}\frac{\binom{6}{2}}{\binom{16}{2}} + \frac{5}{25}\frac{\binom{5}{2}}{\binom{16}{2}}}$$

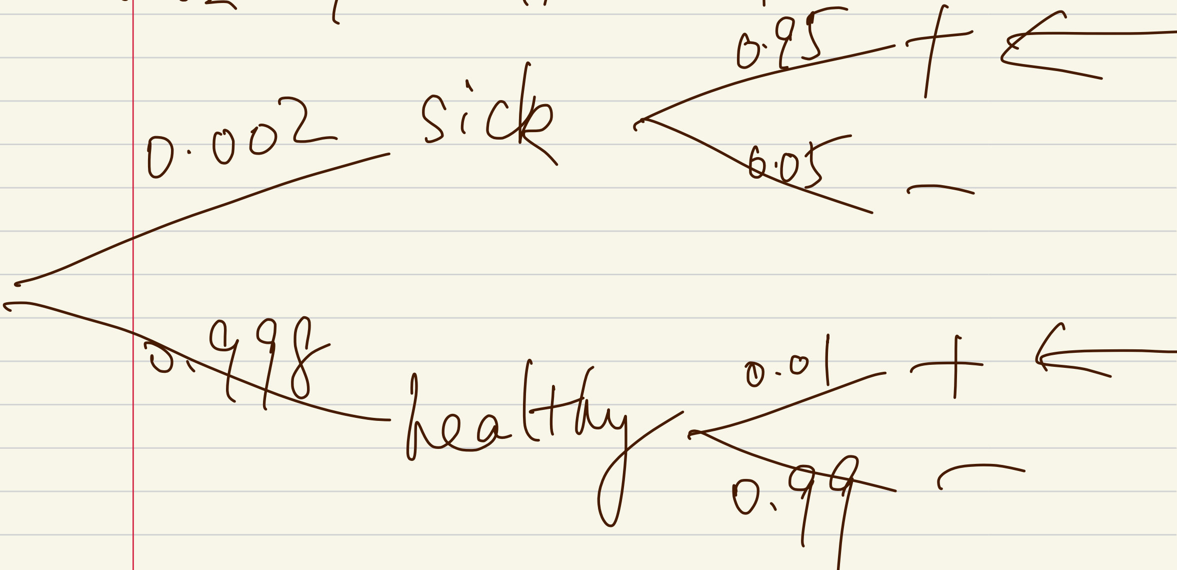
$$= \frac{20 \cdot 15}{20 \cdot 15 + 5 * 10} = \frac{300}{350} = \frac{6}{7}$$





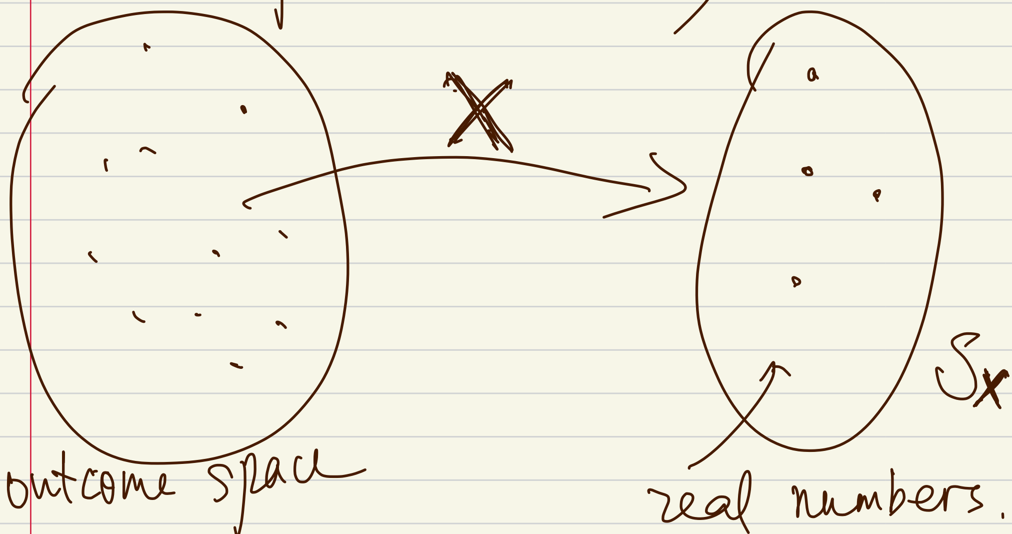
Ex: A rare disease, it exists in about 0.2% of the population.
It will give a correct positive result 95% of the time.
It also give 1% of false positive.
Given that a randomly selected person tested positive, what's the prob he actually has the disease?

$$\frac{0.002 * 0.95}{0.002 * 0.95 + 0.998 * 0.01} = 0.16$$



Chapter 3. Random Variables and Distributions.

Random Variable: (r.v.) A real valued function defined on the outcome space of a random experiment. (It associates a real value to each possible outcome.)



How to describe a r.v. X?

① what are the possible values X can be? the space of X

② what's the prob of X taking on each value in S_X?

Ex: Toss 2 fair dice. X = the larger of the 2.
Describe X.

$$S_X = \{1, 2, 3, 4, 5, 6\}$$

$$f(1) = P(X=1) = 1/36$$
$$f(2) = P(X=2) = 3/36$$
$$f(3) = P(X=3) = 5/36$$
$$f(4) = P(X=4) = 7/36$$
$$f(5) = P(X=5) = 9/36$$
$$f(6) = P(X=6) = 11/36$$

$\Leftrightarrow$

$$f(x) = \frac{2x-1}{36}$$
$x = 1, 2, 3, 4, 5, 6.$

1.

Ex:

8 students
3 m. majors
5 not

Randomly pick 2.
X = the math major being picked.

Describe X.

$$S_X = \{0, 1, 2\}.$$

$$f(0)=P(X=0)=\frac{\binom{5}{2}}{\binom{8}{2}}=\frac{10}{28}$$

$$f(1)=P(X=1)=\frac{\binom{3}{1}\binom{5}{1}}{\binom{8}{2}}=\frac{15}{28}$$

$$f(2)=P(X=2)=\frac{\binom{3}{2}}{\binom{8}{2}}=\frac{3}{28}$$

1.

Discrete r.v.:

If S_X is a finite set or an infinite set that is equivalent to the integers the X is called a discrete r.v.

Ex: Toss a coin, step only when the 1st Heads show up. $X = \#$ of tosses. Describe X.

$$S_X=\{1,2,3,\cdots\}$$

$$f(x)=P(X=x)=\frac{1}{2^x} \qquad x=1,2,3,\cdots$$

$$\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\cdots$$

Def: For a discrete r.v. X, $f(x)$ is called the Probability mass function if: (p.m.f)

① $f(x) \geq 0$

② $\sum_x f(x) = 1$

③ $f(x) = P(X=x)$

Ex: $f(x)$ is pmf of a r.v. X.

$$f(x) = c \cdot \binom{3}{x}\binom{3}{3-x} \qquad x = 0, 1, 2.$$

Determine the value of c.

$$f(0) = c\binom{3}{0}\binom{3}{3} = c.$$

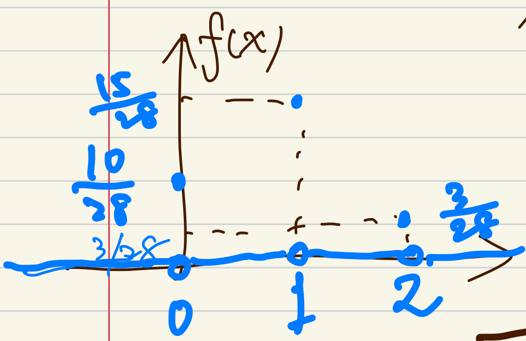
$$f(1) = c \cdot \binom{3}{1}\binom{3}{2} = 9c$$

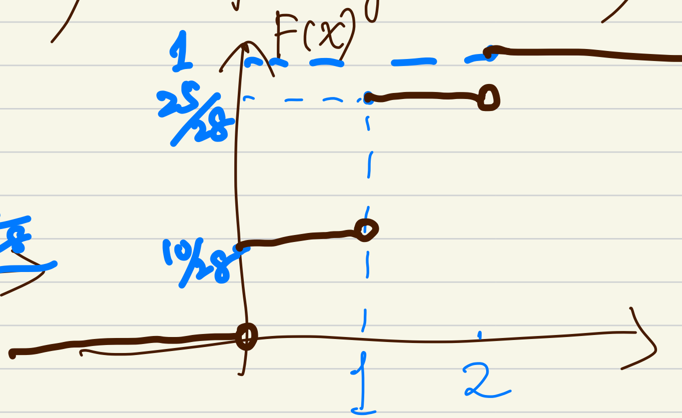
$$f(2) = c \cdot \binom{3}{2}\binom{3}{1} = 9c. \quad / 19c$$

$$c = \frac{1}{19}.$$

Cumulative Dist. Function (c.d.f.) $F(x)$

$F(x) = P(X \le x)$ for any $x \in (-\infty, \infty)$





$$F(1) = P(X \le 1) = P(X=0) + P(X=1) = \frac{10}{28} + \frac{15}{28} = \frac{25}{28}$$

Ex: $F(x) = \begin{cases} 0 & x < 1 \\ \frac{1}{5} & 1 \le x < 3 \\ \frac{1}{3} & 3 \le x < 5 \\ \frac{3}{4} & 5 \le x < 10 \\ 1 & x \ge 10 \end{cases}$

$f(x) = \begin{cases} \frac{1}{5} & x = 1 \\ \frac{2}{15} & x = 3 \\ \frac{5}{12} & x = 5 \\ \frac{1}{4} & x = 10 \end{cases}$

1

① $P(X=3) = \frac{2}{15}$ $\quad \left(\frac{1}{3} - \frac{1}{5}\right)$

② $P(X=4) = 0$

③ $P(X=5) = \frac{5}{12}$ $\quad \frac{3}{4} - \frac{1}{3}$

④ $f(x)$?