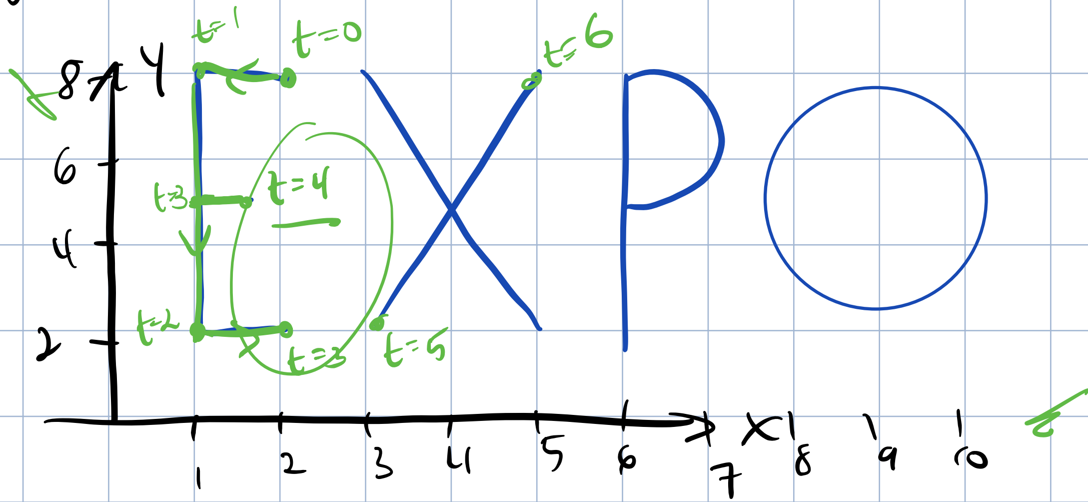
- Worksheet 2 is out.

$$\frac{d}{dx}\left(\int_0^{x} \cos(\theta^2)\,d\theta\right) = \cos(x^2) \qquad \text{FTC.}$$

- Project is out.

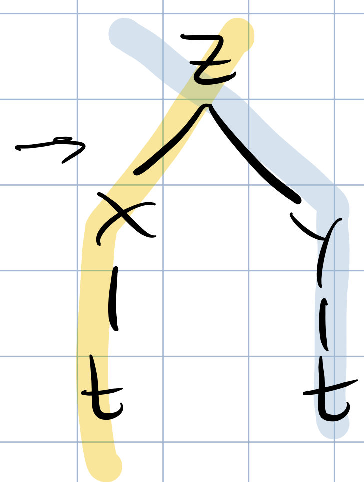


Chain Rule

$z = f(x,y) \qquad x(t),\ y(t)$

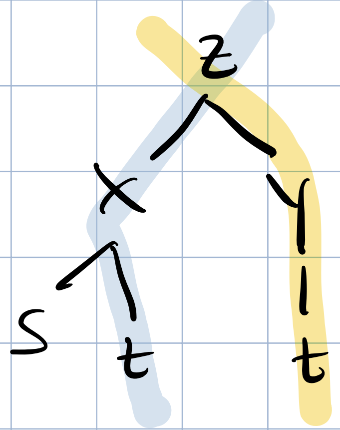
2 ⇒ ∂

1 ⇒ d



$$\frac{dz}{dt} = \frac{\partial z}{\partial x}\frac{dx}{dt} + \frac{\partial z}{\partial y}\frac{dy}{dt}$$

$z = f(x,y) \qquad x(s,t) \qquad y(t)$

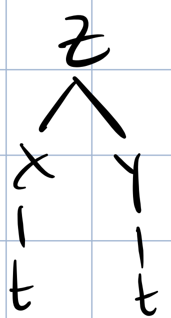


find $\dfrac{\partial z}{\partial s} = \dfrac{\partial z}{\partial x}\dfrac{\partial x}{\partial s}$

find $\dfrac{\partial z}{\partial t} = \dfrac{\partial z}{\partial x}\dfrac{\partial x}{\partial t} + \dfrac{\partial z}{\partial y}\dfrac{dy}{dt}$

14.5

(2) $z = \dfrac{x-y}{x+2y}$, $x = e^{\pi t}$ $y = e^{-\pi t}$ find $\dfrac{dz}{dt}$



$$\frac{dz}{dt} = \frac{\partial z}{\partial x}\frac{dx}{dt} + \frac{\partial z}{\partial y}\frac{dy}{dt}$$

$$= \frac{(x+2y)(1)-(x-y)(1)}{(x+2y)^2}\,\pi \underbrace{e^{\pi t}}_{x} + \frac{(x+2y)(-1)-(x-y)(2)}{(x+2y)^2}\left(-\pi \underbrace{e^{-\pi t}}_{y}\right)$$

$$= \frac{3\pi xy}{(x+2y)^2} + \frac{3\pi xy}{(x+2y)^2}$$

$$= \frac{6\pi xy}{(x+2y)^2}$$

(13) $p(t) = f(x,y)$ $x = g(t)$ $y = h(t)$

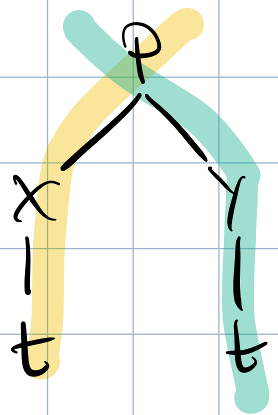
$g(2)=4$ $h(2)=5$

$g'(2)=-3$ $h'(2)=6$

$f_x(4,5) = 2$

$f_y(4,5) = 8$

find $p'(2) = \left.\frac{dp}{dt}\right|_{t=2}$.



$$\frac{dp}{dt} = \frac{\partial p}{\partial x}\frac{dx}{dt} + \frac{\partial p}{\partial y}\frac{dy}{dt}$$

now evaluate at $t=2$.

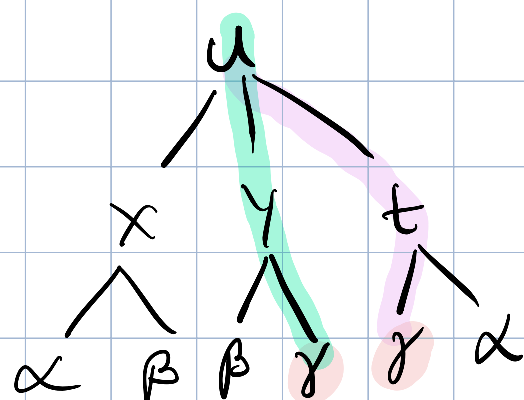
when $t=2$, $x=4$, $y=5$

$\left.\frac{dx}{dt}\right|_{t=2} = -3$ $\quad \left.\frac{dy}{dt}\right|_{t=2} = 6$

$$p'(2) = (2)(-3) + (8)(6)$$

(26) $u = xe^{ty}$ $\quad x = \alpha^2\beta,\ y = \beta^2\gamma,\ t = \gamma^2\alpha.$

find, $\frac{\partial u}{\partial \gamma}$



$$\frac{\partial u}{\partial \gamma} = \frac{\partial u}{\partial y}\frac{\partial y}{\partial \gamma} + \frac{\partial u}{\partial t}\frac{\partial t}{\partial \gamma}$$

$$= xte^{ty}\beta^2 + xye^{ty}(2\gamma\alpha)$$

(29) $\tan^{-1}(x^2y) = x + xy^2$ find $\frac{dy}{dx}$.

Use: $F(x,y) = 0$

$$F_x \cancel{\frac{dx}{dx}} + F_y \frac{dy}{dx} = 0 \rightarrow \boxed{\frac{dy}{dx} = -\frac{F_x}{F_y}}$$

$$\underbrace{\tan^{-1}(x^2y) - x - xy^2}_{F} = 0$$

$$\boxed{\frac{dy}{dx} = -\frac{\frac{1}{1+(x^2y)^2}(2xy) - 1 - y^2}{\frac{1}{1+(x^2y)^2}(x^2) - 2xy}}$$

(33) $e^z = xyz$ find $\frac{\partial z}{\partial x}$

use $F(x,y,z) = 0$

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z}$$

$$\underbrace{e^z - xyz}_{F} = 0$$

$$\frac{\partial z}{\partial x} = \boxed{-\frac{-yz}{e^z - xy}}$$

14.6 Directional deriv & Gradient vector.

$f(x,y)$, grad $f = \nabla f = \langle f_x, f_y \rangle$

$f(x,y,z)$ $\quad \vec{\nabla} f = \langle f_x, f_y, f_z \rangle$ $\quad$ direction of max rate of change

Max rate $= \|\vec{\nabla} f\|$

$\vec{u}$ = unit vector, $\quad D_{\vec{u}} f = \vec{\nabla} f \cdot \vec{u}$

14.6

15 $\quad f(x,y,z) = x^2 y + y^2 z \qquad \vec{v} = \langle 2, -1, 2 \rangle$

find $D_{\vec{u}} f(x,y,z)$

$$\vec{u} = \frac{\vec{v}}{\|\vec{v}\|} = \frac{\langle 2,-1,2 \rangle}{\sqrt{2^2+(-1)^2+2^2}}$$

$$= \langle \tfrac{2}{3}, \tfrac{-1}{3}, \tfrac{2}{3} \rangle$$

$$\vec{\nabla} f = \langle 2xy, x^2+2yz, y^2 \rangle$$

$$D_{\vec{u}} f = \langle 2xy, x^2+2yz, y^2 \rangle \cdot \langle \tfrac{2}{3}, \tfrac{-1}{3}, \tfrac{2}{3} \rangle$$

$$= \frac{4}{3}xy - \frac{x^2}{3} - \frac{2yz}{3} + \frac{2y^2}{3}$$

Now find $D_{\vec{u}} f(1,2,3) = \frac{4}{3}(1)(2) - \frac{1}{3} - \frac{2(2)(3)}{3} + \frac{2(2)^2}{3}$

$$= \frac{8}{3} - \frac{1}{3} - 4 + \frac{8}{3}$$

$$= 5 - 4 = \boxed{1.}$$

(22) $f(s,t) = te^{st}$ find max rate of change and direction of max rate of ch. at $(0,2)$

$$\text{grad} f = \vec{\nabla} f = \langle t^2 e^{st}, e^{st} + ste^{st} \rangle$$

$$\text{grad} f(0,2) = \langle 4, 1 \rangle$$ direction

max rate $\|\langle 4,1\rangle\| = \sqrt{17}$.

(29) $f(x,y) = x^2 + y^2 - 2x - 4y$

find where max rate of change is in direction of $\hat{\imath} + \hat{\jmath}$.

Set $\vec{\nabla} f = \hat{\imath} + \hat{\jmath}$

$$\langle 2x-2, 2y-4 \rangle = \langle 1, 1 \rangle$$

$2x-2=1$ and $2y-4=1$

$x = \frac{3}{2}$ $y = 5/2$.

at $\left(\frac{3}{2}, \frac{5}{2}\right)$

(37) $u(x,y), v(x,y)$ $a,b \in \mathbb{R}$.

$$\nabla(au+bv) = ?$$

$$\frac{\partial}{\partial x}(au+bv) = au_x + bv_x$$

$$\frac{\partial}{\partial y}(au+bv) = au_y + bv_y$$

$$\nabla(au+bv) = \langle au_x + bv_x, au_y + bv_y \rangle$$

$$= \langle au_x, au_y \rangle + \langle bv_x, bv_y \rangle$$

$$= a\langle u_x, u_y \rangle + b\langle v_x, v_y \rangle$$

$$= a\nabla u + b\nabla v$$

$$\nabla(uv) = (\nabla u)v + u\nabla v$$

$$\nabla\left(\frac{u}{v}\right) = \frac{v\nabla u - u\nabla v}{v^2}$$

$$\nabla(u^n) = nu^{n-1}\nabla u$$

39 $D^2_{\vec{u}} f = D_{\vec{u}}(D_{\vec{u}} f)$ Second directional derivative.

$f(x,y) = x^3 + 5x^2y + y^3 \qquad \vec{u} = \langle \frac{3}{5}, \frac{4}{5} \rangle$

find $D_{\vec{u}}^2 f(2,1)$

$$D_{\vec{u}} f = \nabla f \cdot \vec{u} = \langle 3x^2 + 10xy,\ 5x^2 + 3y^2 \rangle \cdot \langle \frac{3}{5}, \frac{4}{5} \rangle$$

$$= \frac{9}{5}x^2 + 6xy + 4x^2 + \frac{12}{5}y^2$$

$$D_{\vec{u}}^2 f = D_{\vec{u}}\left(\qquad \right)$$

$$= \langle \frac{58}{5}x + 6y,\ 6x + \frac{24}{5}y \rangle \cdot \langle \frac{3}{5}, \frac{4}{5} \rangle$$

$$= \frac{58}{5}\left(\frac{3}{5}\right)x + \frac{18}{5}y + \frac{24}{5}x + \frac{24}{5}\left(\frac{4}{5}\right)y$$

now find $D_{\vec{u}}^2 f(2,1) = \frac{58}{5}\left(\frac{3}{5}\right)2 + \frac{18}{5} + \frac{24}{5}(2) + \frac{24}{5}\left(\frac{4}{5}\right)$

Simplify.

Comment: $\vec{u} = \langle a, b \rangle$, $\|\vec{u}\| = 1$

$$D_{\vec{u}}^2 f(x,y) = f_{xx}a^2 + 2f_{xy}ab + f_{yy}b^2$$