1 Operations

1.1 Associativity

Which of the following operations are associative? Justify your answer.

a) The operation $\star$ on $\mathbb{Z}$ defined by $a \star b = a - b$

b) The operation $\star$ on $\mathbb{R}$ defined by $a \star b = a + b + ab$

1.2 Commutativity

For each of the following sets with a binary operation, find **all** ordered pairs (a, b) of commuting elements.

a) $\mathbb{Q} \setminus \{0\}$ with $a \star b = \frac{a}{b}$

b) $\mathbb{Z}$ with $a \star b = a + b - ab$

c) $\mathbb{Z}$ with $a \star b = a - b$

2 Group of order 3

1. Can G be abelian?

2. Can G be non abelian? SSSSSSSSSSSSSS

3. Describe the operation on the group (Caley table)

3 Cyclic Groups

A Cyclic Group is a group that is generated by a single element. In other words there exists an element a such that for all $g \in G$ there exists $n \in \mathbb{N}$ such that $g = a^n$

1. Prove that Cyclic groups are abelian.

2. Are all abelian groups cyclic? Explain

4 Subgroups

4.1 Quaternions

The Quaternion Group Q_8 is a non abelian group with elements

$$\{1, -1, i, j, k, -i, -j, -k\}$$

Such that

$$i^2 = j^2 = k^2 = -1$$

and

$$ij = k, ji = -ij$$

List two subgroups of Q_8 and prove that they are a subgroups