Review:

Ex: Independence. $P(\text{success}) = \frac{1}{4}$.

① drill at 10 locations, $P(\text{at least 3 successes}) = 1 - P(1\,S) - P(2\,S) - P(0\,S)$

$= 1 - 0.75^{10} - \binom{10}{1} 0.25\; 0.75^{9} - \binom{10}{2} 0.25^{2}\; 0.75^{8}$

② Bankrupt if 10 drills, and no success.

$\Rightarrow$ $P(\text{bankrupt}) = 0.75^{10}$

③ P(the 5th success happen on the 18th drill)

$\binom{17}{4} 0.25^{5}\; 0.75^{13}$ ✓

④ $P(\text{success}) = 0.25$ $P(\text{more work}) = 0.15$

$P(\text{failure}) = 0.60$

20 locations, $P(4\,S,\ 1\,W,\ 15\,F)$

$\binom{20}{4}\binom{16}{1}\binom{15}{15} \rightarrow = \frac{20!}{4!\,15!\,1!}\; 0.25^{4}\; 0.15^{1}\; 0.6^{15}$

$\uparrow \binom{20}{4,1,15}$

§5.3. Hypergeometric Dist.

Ex: A deck of cards, 12 face cards, 40 number cards. Randomly pick 5, what's the prob of getting exacty 2 face cards?

$$\checkmark \quad \frac{\binom{12}{2}\binom{40}{3}}{\binom{52}{5}}$$

$$\binom{5}{2} \qquad \boxed{\frac{5!}{3!2!}} \frac{12}{52}\cdot\frac{11}{51}\cdot\frac{40}{50}\cdot\frac{39}{49}\cdot\frac{38}{48}$$

There are N objects of 2 types, N_1 of type 1, $N-N_1$ of type 2

Randomly choose n from N $(n<N)$

$X = \#$ of type 1 objects.

$$P(X=x) = \frac{\binom{N_1}{x}\binom{N-N_1}{n-x}}{\binom{N}{n}}, \quad x = \cdots$$

§ 5.5 Poisson Distribution.

$$f(x) = e^{-\lambda}\frac{\lambda^x}{x!}$$

$\lambda > 0$ constant.

$E(X) = Var(X) = \lambda$

λ = average in the interested interval.

$x = 0, 1, 2, 3, \ldots$

Ex. $\lambda = 2$.

$$f(0) = e^{-2}\frac{2^0}{0!} = e^{-2} \approx 0.135$$

$$f(1) = e^{-2}\frac{2^1}{1!} = 2e^{-2} \approx 0.270$$

$$f(2) = e^{-2}\frac{2^2}{2!} = 2e^{-2} \approx 0.270$$

$$f(3) = e^{-2}\frac{2^3}{3!} = \frac{4}{3}e^{-2} \approx 0.18$$

$$f(4) = e^{-2}\frac{2^4}{4!} = \frac{2}{3}e^{-2} \approx 0.09$$

$\vdots$

If λ is an integer.

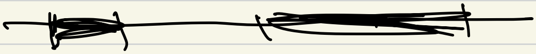
$f(\lambda - 1) = f(\lambda)$

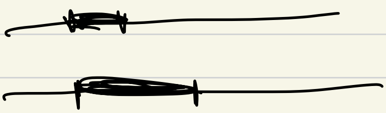
$f(x)$

Verify $\sum_{x=0}^{\infty} e^{-\lambda}\frac{\lambda^x}{x!} = 1$

$$\text{LHS} = e^{-\lambda}\sum_{x=0}^{\infty}\frac{\lambda^x}{x!} = e^{-\lambda}e^{\lambda} = 1 = \text{RHS}$$

$$1+\lambda+\frac{\lambda^2}{2!}+\frac{\lambda^3}{3!}$$

1  independent

2  P(a single event in a small interval) $\propto$ size of int.

3 P(> 1 in a small int.) negligible

poisson process.

$$f(x) = e^{-\lambda t}\frac{(\lambda t)^x}{x!}$$

$x = 0, 1, 2, \sim$ average

Ex: Typo $\sim$ Poisson with λ 3 typos each page.

① P(there is no typo in a random page)

$= e^{-3}$

② P(there are 2-4 typos in a random page)

$$= e^{-3}\left(\frac{3^2}{2!} + \frac{3^3}{3!} + \frac{3^4}{4!}\right)$$

③ P(there are 5 typos in a 2-page document)

$$= e^{-6}\frac{6^5}{5!}$$

$$2\left(e^{-3}\frac{3^0}{0!}e^{-3}\frac{3^5}{5!} + e^{-3}\frac{3^1}{1!}e^{-3}\frac{3^4}{4!} + e^{-3}\frac{3^2}{2!}e^{-3}\frac{3^3}{3!}\right)$$

$$= \left(\frac{3^5}{120} + \frac{3^5}{24} + \frac{3^5}{12}\right) \cdot 2$$

$$= 3^5 \left(\frac{1}{60} + \frac{1}{12} + \frac{1}{6} \right) = \frac{4}{15} \cdot 3^5$$

$$\frac{1+5+10}{60} \qquad \frac{16}{60} \qquad \boxed{\frac{4}{5} \cdot 3^4}$$

$$\frac{6^5}{5!} = \frac{3^5 \cdot 2^5 / 8}{120/8} = \frac{3^5 \cdot 4}{15}$$

(4) what's the prob the 6^{th} page is the 1^{st} page w/o typo?

$$(1-e^{-3})^5 \cdot e^{-3}$$

(5) typed a 100-page document,

P(there are 6 pages w/o typo)

$$= \binom{100}{6} (e^{-3})^6 (1-e^{-3})^{94}$$

Thm: If $X \sim Bin(n, p)$, when $n \to \infty$, $p \to 0$, and $np \xrightarrow[n\to\infty]{} \lambda$,

then $\binom{n}{x} p^x (1-p)^{n-x} \longrightarrow e^{-np} \frac{(np)^x}{x!}$

Approx: when n is big and p is small, and $np \sim O(1)$,

$B(n, p) \approx Poi(\lambda = np)$

Ex: production line, defective rate is 2 out of 1000. 4000 items were made, what's the prob there are 5 or 6 defective?

$X \sim Bin(4000, 0.002)$

$P(X=5 \text{ or } 6) = \binom{4000}{5} 0.002^5 \, 0.998^{3995} + \binom{4000}{6} 0.002^6 \, 0.998^{3994} =$

$$\lambda = 4000 * 0.002 = 8$$

$$P(X=5 \text{ or } 6) = e^{-8}\left(\frac{8^5}{5!} + \frac{8^6}{6!}\right) = 0.214$$

Bernoulli, Binomial, Geo, Neg. Bin

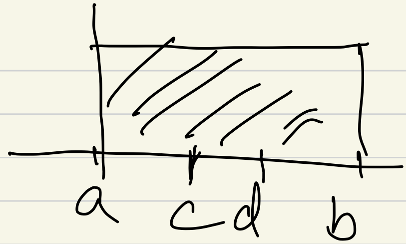
Multinomial

Hypergeometric

Poisson

chapter 6.

§6.1. Uniform Dist.



$$f(x) = \begin{cases} \frac{1}{b-a} & a \le x \le b \\ 0 & \text{elsewhere} \end{cases}$$

$$E(X) = \frac{a+b}{2}$$

$$Var(X) = E(X^2) - \left(\frac{a+b}{2}\right)^2$$

$$= \int_a^b x^2 \cdot \frac{1}{b-a} \, dx - \left(\frac{a+b}{2}\right)^2$$

$$= \frac{b^3 - a^3}{3(b-a)} - \frac{(a+b)^2}{4}$$

$$= \frac{b^2 + ab + a^2}{3} - \frac{a^2 + b^2 + 2ab}{4}$$

$$= \frac{(b-a)^2}{12}$$