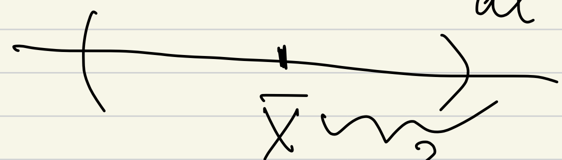
Review: Conf. int. for 1 popn μ.

case 1. normal underlying, σ^2 known.

$$\left(\bar{X} - z_{\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}},\ \bar{X} + \underbrace{z_{\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}}}\right)$$

margin of error at $100(1-\alpha)\%$ conf. level

$$\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}}$$



$$z_{\frac{\alpha}{2}}\sigma_{\bar{X}} = z_{\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}}$$

case 2. Large sample

$$\left(\bar{X} - z_{\frac{\alpha}{2}}\frac{s}{\sqrt{n}},\ \bar{X} + z_{\frac{\alpha}{2}}\frac{s}{\sqrt{n}}\right)$$ (use σ if it's known)

case 3. small sample, normal popn, σ^2 unknown.

$$\left(\bar{X} - t_{\frac{\alpha}{2}}(n-1)\frac{s}{\sqrt{n}},\ \bar{X} + t_{\frac{\alpha}{2}}(n-1)\frac{s}{\sqrt{n}}\right)$$

§9.6 later. §9.7 skipped.

§9.8 Conf. intervals for the diff of 2 means.

case 1. $X_1, X_2, \cdots X_{n_1} \sim N(\mu_1, \sigma_1^2)$
$Y_1, Y_2, \cdots Y_{n_2} \sim N(\mu_2, \sigma_2^2)$ } indep.

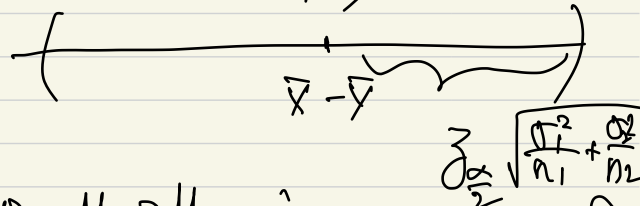
σ_1^2 & σ_2^2 are known.

Goal: To est $\mu_1 - \mu_2$

point estimator: $\bar{X} - \bar{Y}$

Sampling dist of $\bar{X} - \bar{Y}$: $N\left(\mu_1 - \mu_2, \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}\right)$

$\bar{X}$: $N\left(\mu_1, \frac{\sigma_1^2}{n_1}\right)$ $\bar{Y}$: $N\left(\mu_2, \frac{\sigma_2^2}{n_2}\right)$



$100(1-\alpha)\%$ CI for $\mu_1 - \mu_2$ is,

$$\left(\bar{X} - \bar{Y} - z_{\frac{\alpha}{2}}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}},\ \bar{X} - \bar{Y} + z_{\frac{\alpha}{2}}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}\right)$$

case 2. (large sample, $n_1 \geq 30$, $n_2 \geq 30$)

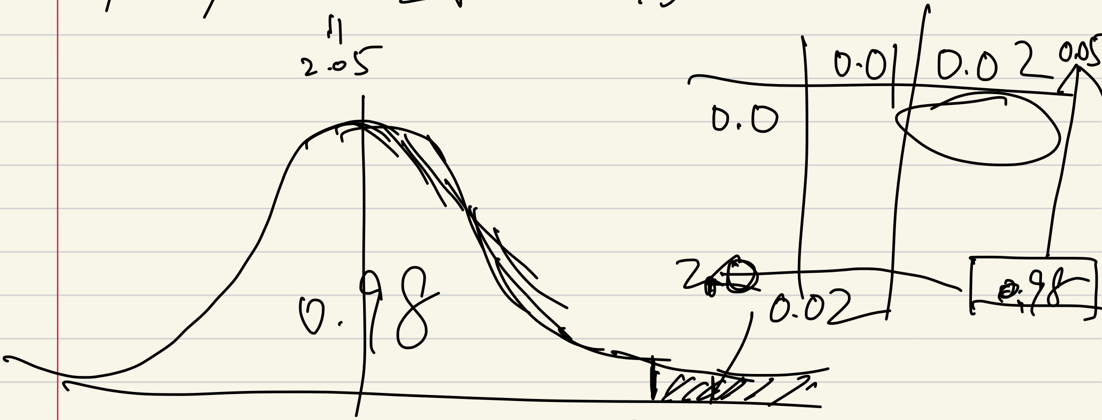
$$(\bar{X} - \bar{Y}) \pm z_{\frac{\alpha}{2}}\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}$$

Ex: engine type A: $n_1 = 50$, $\bar{X} = 36$ $\sigma_1 = 6$

B. $n_2 = 75$ $\bar{Y} = 42$ $\sigma_2 = 8$

Find 96% CI for $\mu_B - \mu_A$

$$(\bar{y}-\bar{x}) \pm \underset{\underset{2.05}{\parallel}}{z_{0.02}} \sqrt{\frac{36}{50}+\frac{64}{75}}$$



$$z_{0.02} = 2.05$$

$$= (42-36) \pm 2.05\sqrt{\frac{64}{75}+\frac{36}{50}} = 6 \pm 2.57$$

$$= (3.43, 8.57)$$

Case 3. Small samples, both population normal, variance unknown. but equal

$$X_1, X_2, \cdots X_{n_1} \sim N(\mu_1, \sigma_1^2) \qquad \sigma_1^2 = \sigma_2^2 = \sigma^2$$

$$Y_1, Y_2, \cdots Y_{n_2} \sim N(\mu_2, \sigma_2^2) \qquad \text{unknown}$$

point estimator for $\mu_1 - \mu_2$, $\bar{X} - \bar{Y}$.

$$\left.\begin{aligned} \bar{X} &\sim N\left(\mu_1, \frac{\sigma^2}{n_1}\right) \\ \bar{Y} &\sim N\left(\mu_2, \frac{\sigma^2}{n_2}\right) \end{aligned}\right\}$$

$$\bar{X} - \bar{Y} \sim N\left(\mu_1 - \mu_2,\ \sigma^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)\right)$$

$$\frac{(\bar{X} - \bar{Y}) - (\mu_1 - \mu_2)}{S_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \sim t\,(n_1 + n_2 - 2)$$

$$\frac{(\bar{X} - \bar{Y}) - (\mu_1 - \mu_2)}{\sigma\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \sim Z.$$

$$S_p^2 = \frac{\sum_{i=1}^{n_1}(X_i - \bar{X})^2 + \sum_{i=1}^{n_2}(y_i - \bar{y})^2}{n_1 + n_2 - 2} = \frac{(n_1-1)S_1^2 + (n_2-1)S_2^2}{n_1 + n_2 - 2}$$

pooled sample variance

$100(1-\alpha)\%$ CI for $\mu_1 - \mu_2$.

$$\left(\bar{X} - \bar{Y} - t_{\frac{\alpha}{2}} \cdot S_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}},\ \bar{X} - \bar{Y} + t_{\frac{\alpha}{2}} \cdot S_p \cdot \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}\right)$$

t has $(n_1 + n_2 - 2)$ d.o.f.

$$\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} = \sqrt{\frac{\sigma^2}{n_1} + \frac{\sigma^2}{n_2}}$$

$$= \sigma\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

S_p

Ex: downstream station: $n_1 = 12$, $\bar{x} = 3.11$, $s_1 = 0.771$

upperstream station: $n_2 = 10$, $\bar{y} = 2.04$, $s_2 = 0.448$

Assume both popu are normal, with equal variance.

Find a 90% conf. interval for the diff between 2 popu means.

$$s_p = \sqrt{\frac{11 s_1^2 + 9 s_2^2}{20}} = \sqrt{\frac{11 \times 0.771^2 + 9 \cdot 0.448^2}{20}}$$

$$= \sqrt{0.417} = 0.646$$

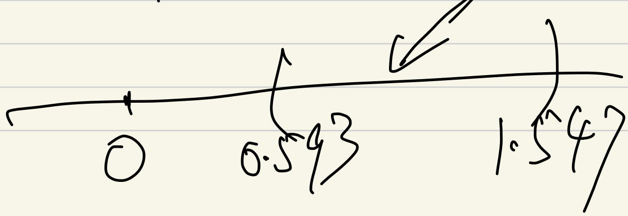
$$(\bar{x} - \bar{y}) \pm t_{0.05}^{(20)} \cdot s_p \cdot \sqrt{\frac{1}{12} + \frac{1}{10}}$$

$$= 1.07 \pm 1.725 \cdot 0.646 \sqrt{\frac{1}{12} + \frac{1}{10}}$$

$$= 1.07 \pm 0.477$$

$$= (0.593, 1.547)$$

$\mu_1 - \mu_2$



§9.9. Paired Observation.

Ex: 2 varieties of wheat. yields?

9 universities,

Each variety was planted on a plot of equal area in each university. And the yield was recorded:

univ	1	2	3	4	5	6	7	8	9
variety 1	38	23	35	41	44	29	37	31	38
2	45	25	31	38	50	33	36	40	43
diff	-7	-2	4	3	-6	-4	1	-9	-5

Assume the diff of yield $\sim$ Normal dist.

Find a 95% CI for the mean diff. of 2 varieties.

$$\bar{d} = -2.78 \qquad S_d = 4.58$$

95% C.I.: $\bar{d} \pm t_{0.025}^{(8)} \dfrac{S_d}{\sqrt{9}}$