Ex: weight of chickens $\sim N(4, 0.5^2)$

1. Randomly pick a chicken, $P(\text{weighs} > 4.4)$
$= P(Z > \frac{4.4-4}{0.5}) = P(Z > 0.8) \approx 0.2119$

2. Find a weight, such that 75% of chickens weighs less. $z_{0.25} = 0.675$ | or $\frac{x-4}{0.5} = 0.675$
$x = 4 + 0.675 * 0.5 = 4.3375$

3. Find a range, such that it includes all the middle 50% of the weights.
$(3.6625, 4.3375)$

Ex: The time till the next person who's going to come to the chicken farm $\sim$ exp. with average 30 minutes.

$$f(x) = \frac{1}{30} e^{-x/30} \qquad x > 0.$$

① what's the prob someone will walk in the next 10 minutes.

$$P(X<10) = \int_0^{10} \frac{1}{30} e^{-x/30} dx = -e^{-x/30}\Big|_0^{10}$$
$$= 1 - e^{-1/3}$$

② There hasn't been anyone visiting for 40 minutes, what's the prob there will be someone in the next 10 minutes?

$$= P(X<10) = 1 - e^{-1/3}$$

$$P(X>s+t \mid X>s) = P(X>t)$$

$$\left(\underbrace{P(X<s+t \mid X>s)}_{\parallel} = P(X<t) \right)$$

$$\frac{P(s<X<s+t)}{P(X>s)} = \frac{\int_s^{s+t} \frac{1}{\beta} e^{-x/\beta} dx}{\int_s^{\infty} \frac{1}{\beta} e^{-x/\beta} dx}$$

$$= \frac{e^{-s/\beta} - e^{-(s+t)/\beta}}{e^{-s/\beta}} = 1 - e^{-t/\beta}$$

$$= P(X<t)$$

Gamma Distribution (α, β)

$$f(x) = \frac{1}{\beta^{\alpha}\Gamma(\alpha)} x^{\alpha-1} e^{-x/\beta} \qquad x>0.$$

where $\Gamma(\alpha) = \int_0^{\infty} x^{\alpha-1} e^{-x} dx$

e.g. $\Gamma(1) = \int_0^{\infty} e^{-x} dx = 1$, $\Gamma(2) = \int_0^{\infty} x e^{-x} dx = 1$

$\Gamma(3) = \int_0^{\infty} x^2 e^{-x} dx = 2.$

$\vdots$

$\Gamma(n) = (n-1)!$ if n is an integer.

what if $\alpha = 1$?

$$f(x) = \frac{1}{\beta} e^{-x/\beta} \qquad x>0$$

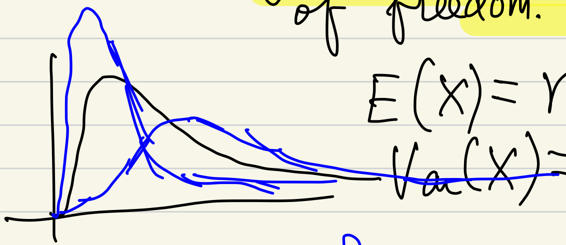
In general, $E(X) = \alpha\beta$, $Var(X) = \alpha\beta^2$

for $X \sim Gamma(\alpha, \beta)$

Let $\alpha = \frac{r}{2}$, $\beta = 2$, then Gamma(α, β) looks like:

$$f(x) = \frac{1}{\Gamma(\frac{r}{2})\, 2^{r/2}}\, x^{\frac{r}{2}-1} e^{-x/2} \quad x > 0$$

$\Rightarrow$ chi-square distribution with r degrees of freedom. $\chi^2(r)$



$E(X) = r$

$Var(X) = 2r$

Know χ^2 table !!

Chapter 7 results:

Review: $X_1, X_2, \cdots X_n$ independent.

$E(X_i) = \mu_i$, $\quad Var(X_1) = \sigma_i^2$, $i = 1, 2, \cdots n$

Let $Y = a_1 X_1 + a_2 X_2 + \cdots + a_n X_n$

then $E(Y) = a_1\mu_1 + a_2\mu_2 + \cdots + a_n\mu_n$

$Var(Y) = a_1^2\sigma_1^2 + a_2^2\sigma_2^2 + \cdots + a_n^2\sigma_n^2$

Thm 7.11

$X_1, X_2, \cdots X_n$ independent normal

r.v., $X_i \sim N(\mu_i, \sigma_i^2)$ $i = 1, 2, \cdots n$

Let $Y = a_1 X_1 + a_2 X_2 + \cdots + a_n X_n$

then $E(Y) = a_1\mu_1 + a_2\mu_2 + \cdots + a_n\mu_n$

$\mathrm{Var}(Y) = a_1^2\sigma_1^2 + a_2^2\sigma_2^2 + \cdots + a_n^2\sigma_n^2$

still true

$$Y \sim N\left(\sum_{i=1}^{n} a_i\mu_i,\ \sum_{i=1}^{n} a_i^2\sigma_i^2\right)$$

Ex: Univ. 1 graduates 1st job salary

$X \sim N(70, 8^2)$

Univ 2. $Y \sim N(50, 6^2)$

Ex: ① Randomly pick 1 graduates from each, P(combined salary > 100)

② Randomly pick 1 from each. P(they ~~diff~~ by > 15)

① $P(\underbrace{X+Y}_{\sim N(120,\,100)} > 100) = P\left(Z > \frac{100-120}{10}\right)$

$= P(Z > -2)$

$= 0.9772$

② $P(\underbrace{X-Y}_{\sim N(20,\,100)} > 15) + P(\underbrace{Y-X}_{\sim N(-20,\,100)} > 15)$

$= P\left(Z > \frac{15-20}{10}\right) + P\left(Z > \frac{15+20}{10}\right)$

$= P(Z > -0.5) + P(Z > 3.5)$

$= 0.6915 + 0$

③ Randomly pick 1 from A.

2 B

what's the prob the salary from 1 from A is higher than combined 2 from B?

$$P(X > Y_1 + Y_2) = P(\underbrace{X - Y_1 - Y_2}_{N(-30,\, 136)} > 0)$$

$$= P\left(Z > \frac{0-(-30)}{\sqrt{136}}\right)$$

$$= P(Z > 2.57) = 0.005$$

4 Randomly pick 1 from each, what's the prob the person from A makes more than double of the person from B.

$$P(\underbrace{X - 2Y}_{N(-30,\, 208)} > 0) = P\left(Z > \frac{30}{\sqrt{208}}\right)$$

$$= P(Z > 2.08)$$

$$= 0.0188$$

Thm $Z \sim N(0,1)$

then $Z^2 \sim \chi^2(1)$.

Thm 7.12. If $X_1, X_2, \cdots X_n$ are indep chi-square dist with d.o.f. $r_1, r_2, \cdots r_n$ respectively, then $Y = X_1 + X_2 + \cdots + X_n$

$$\sim \chi^2(r_1 + r_2 + \cdots + r_n)$$

Corollary: If $X_1, X_2, \cdots X_n$ are indep and all have the same dist $N(\mu, \sigma^2)$

(or: a random sample from $N(\mu, \sigma^2)$).

then $\sum_{i=1}^{n} \left(\frac{X_i - \mu}{\sigma} \right)^2 \sim \chi^2(n)$