$$\frac{\bar{X}-\mu}{\sigma/\sqrt{n}} \sim Z$$

Q2. a. $\sim N(\mu, 10^2)$

$n=12, \quad \bar{x} = 66.5$

95% CI for μ: $\quad \bar{x} \pm 1.96 \frac{10}{\sqrt{12}}$

$= 66.5 \pm 1.96 \frac{10}{\sqrt{12}}$

$= 66.5 \pm 5.66$

$= (60.84, \ 72.16)$

Q5 $\quad X_1, X_2, \cdots X_{100} \sim Exp(\beta = 20)$

a $\quad \underbrace{P(18 < \bar{X} < 23)} = P\left(\frac{18-20}{\sqrt{4}} < \frac{\bar{X}-20}{\sqrt{4}} < \frac{23-20}{\sqrt{4}}\right) \quad \mu = \beta = 20$

$\sigma^2 = \beta^2 = 400$

$\bar{X} \sim N(\mu, \frac{\sigma^2}{n}) \sim N(20, \frac{400}{100})$

$\sim N(20, 4)$

$= P(-1 < Z < 1.5) = \cdots$

b $\quad$ Find prob at least 30 of $X_1, X_2 \cdots X_{100}$ are smaller 10.

$p = P(X < 10) = \cancel{P\left(Z < \frac{10-20}{\sqrt{400}}\right)} = P(Z < -0.5)$

$= \int_0^{10} \frac{1}{20} e^{-x/20} dx = -e^{-x/20} \Big|_0^{10} = \cdots$

$Y \sim Bin(100, p) \sim N(100p, 100pq)$

$P(Y \geq 30) = P\left(Z \geq \frac{29.5 - 100p}{\sqrt{100pq}}\right) = \cdots$

Ex: measuring error $\sim N(\mu, \sigma^2)$

$n = 15, \quad \bar{x} = 2.5 \quad s = 0.2$

$H_0: \mu = 2.4 \qquad \alpha = 0.05$

$\boxed{H_1: \mu \neq 2.4}$

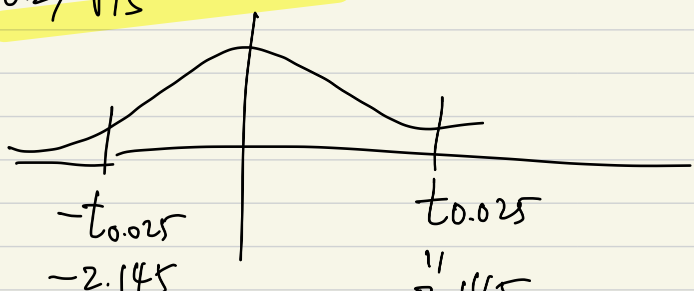
① What's the C for this test. (in terms of test stat.)

② p-value of the test.

under H_0, $\dfrac{\bar{x} - \mu_0}{s/\sqrt{n}} \sim t(n-1)$

$$\frac{\bar{x} - 2.4}{0.2/\sqrt{15}} \sim t(14)$$

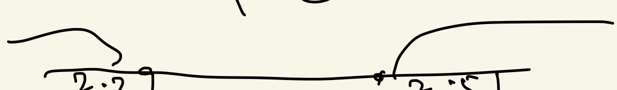
ⓐ



$$\left| \frac{\bar{x} - 2.4}{0.2/\sqrt{15}} \right| > 2.145 \qquad |\bar{x} - 2.4| > 0.11$$

$$C = \{ \bar{x} > 2.51 \text{ or } \bar{x} < 2.29 \}$$

$\bar{x}_{obs} = 2.5 \notin C$ can not rej H_0.

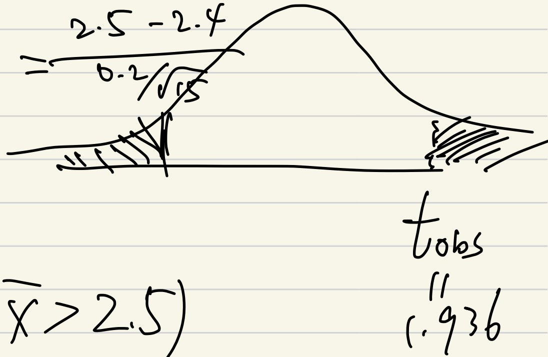


(b)

p-value:

$$t_{obs} = \frac{\bar{x}_{obs} - 2.4}{0.2/\sqrt{15}} = \frac{2.5 - 2.4}{0.2/\sqrt{15}}$$

$$= 1.936$$



$$\text{p-value} = 2 * P(\bar{x} > 2.5)$$

$$= 2 * P(t(14) > 1.936)$$

$$\in (0.05, 0.10)$$

$$> \alpha$$

cannot rej H_0.

Ex: manufacturer's claims: thread A. μ_A
thread B μ_B

average tensile strength. A exceeds B by at least 12 kg.

50 of each type were test.

$\bar{X}_A = 86.7$ $\bar{X}_B = 77.8$

$S_A = 6.28$ $S_B = 5.61$ $\alpha = 0.05$

Test the claim.

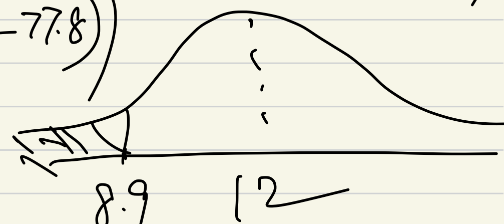
$H_0: \mu_A - \mu_B \geq 12 \quad (\mu_A - \mu_B = 12)$ $\quad \dfrac{\sigma_A^2}{n_1} + \dfrac{\sigma_B^2}{n_2}$

$H_1: \boxed{\mu_A - \mu_B < 12}$ ☆

under H_0, $\bar{X}_A - \bar{X}_B \sim N\left(12, \dfrac{S_A^2}{50} + \dfrac{S_B^2}{50}\right)$

$p\text{-value} = P(\bar{X}_A - \bar{X}_B < (86.7 - 77.8))$

$= P(\bar{X}_A - \bar{X}_B < 8.9)$



$$= P\left(Z < \frac{8.9 - 12}{\sqrt{\frac{6.28^2}{50} + \frac{5.61^2}{50}}}\right)$$

$= P(Z < -2.603) < 0.05$

$= 0.0047$

Ex: Material 1: $n=12$. $\bar{X}_1=85$, $S_1=4$

material 2: $n=10$ $\bar{X}_2=81$, $S_2=5$

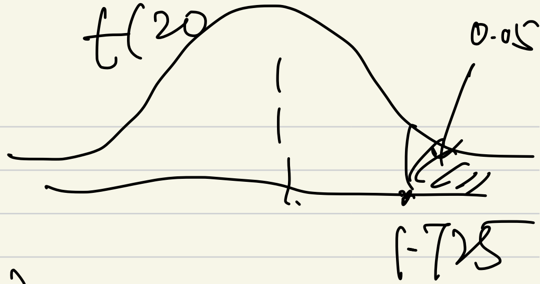
Can we conclude at $\alpha=0.05$ that abrasive wear of mat. 1 exceeds that of 2 by more than 2 units? Assume the popu's ~ normal with equal variance.

H_0: $\mu_1-\mu_2 \le 2$

H_1: $\mu_1-\mu_2 > 2$

under H_0. $$\frac{(\bar{X}_1-\bar{X}_2)-2}{S_p\sqrt{\frac{1}{12}+\frac{1}{10}}} \sim t(20)$$

$$S_p=\sqrt{\frac{11*4^2+9*5^2}{20}} = 4.478$$



$$C=\left\{\frac{\bar{X}_1-\bar{X}_2-2}{4.478\sqrt{\frac{1}{12}+\frac{1}{10}}}>1.725\right\}$$

$$=\left\{\bar{X}_1-\bar{X}_2>\underline{5.31}\right\}$$

$$(\bar{X}_1-\bar{X}_2)_{obs}=4\notin C.$$

$\Rightarrow$ can not conclude the diff is > 2.