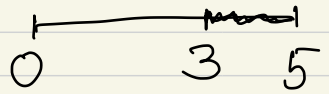


X ,
Ex: Waiting time for a bus $\sim U[0, 5]$

① $P(X > 3) = \frac{2}{5}$



② Took the bus 20 times.

$P(\text{at least 3 times, you had to wait } > 4 \text{ minutes})$

$$p(X > 4) = 0.2$$

$$Y \sim \text{Bin}(20, 0.2)$$

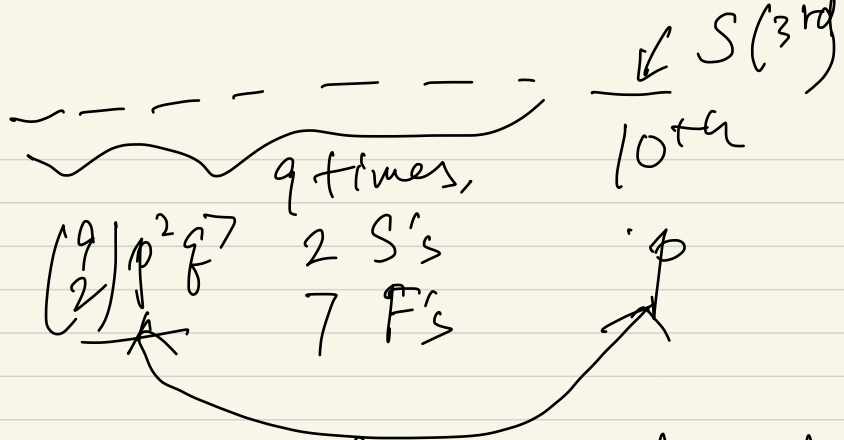
$$\begin{aligned} P(Y \geq 3) &= 1 - P(Y = 0, 1, 2) \\ &= 1 - 0.8^{20} - \binom{20}{1} 0.2 0.8^{19} - \binom{20}{2} 0.2^2 0.8^{18} \\ &= \boxed{} \end{aligned}$$

③ $P(\text{the 6th time is the 1st time you didn't have to wait } > 1 \text{ min})$

$$= \left(\frac{4}{5}\right)^5 \frac{1}{5}$$

④ $P(10^{\text{th}} \text{ time is the 3rd time that you didn't have to wait } > 1 \text{ min})$

$$= \binom{9}{2} \left(\frac{4}{5}\right)^7 \left(\frac{1}{5}\right)^3$$



Ex: clothes of 1 yard wide

The flaws occur \sim Poisson.
on average 2/yard.

① $P(\text{there are } 9 \text{ or } 10 \text{ on a } 4 \text{ yard piece}) = e^{-8} \frac{8^{10}}{10!} + e^{-8} \frac{8^9}{9!}$

② acceptable if it has 1 or 0 flaw. (1 yard long)

$$P(\text{acceptable}) = e^{-2} \left(\frac{2^0}{0!} + \frac{2^1}{1!} \right) = 3e^{-2} \approx 0.4$$

- ③ 30 1-yard pieces, 12 are acceptable. Randomly select 4 pieces, $P(2 \text{ acceptable})$

$$\frac{\binom{12}{2} \cdot \binom{18}{2}}{\binom{30}{4}}$$

120 pieces,
12 are good

- ④ You are inspecting these 1-yard pieces. What's the prob the 20th piece was the 5th accp. piece you find?

$$\binom{19}{4} \left(\frac{12}{30} \cdot \frac{11}{29} \cdot \frac{10}{28} \cdot \frac{9}{27} \cdot \frac{18}{26} \cdot \frac{17}{25} \cdots \frac{4}{12} \right) \cdot \frac{8}{11}$$

better

$$\frac{\binom{12}{4} \binom{18}{15}}{\binom{30}{19}}$$

$$\frac{8}{11}$$

Ex

X	-3	4	6
f(x)	0.3	0.2	0.5

$$E(X) = \sum x f(x) = -3 \cdot 0.3 + 4 \cdot 0.2 + 6 \cdot 0.5$$

$$= 2.9$$

$$Var(X) = E(X^2) - \mu^2$$

$$= (-3)^2 \cdot 0.3 + (4^2) \cdot 0.2 + (6^2) \cdot 0.5$$

$$= 2.7 + 3.2 + 18 - 2.9^2$$

$$= \dots$$

$$E(10 - 4X) = 10 - 4 \cdot 2.9$$

$$= 8.4$$

$$E(a + bX) = a + bE(X)$$

$$\text{Var}(a+bX) = b^2 \text{Var}(X)$$

$$\text{Var}(20-4X) = (-4)^2 \text{Var}(X) = 16 \cdot \text{Var}(X)$$

$$f(x) = c(x-1)^2 \quad 0 \leq x \leq 3$$

$$1 = \int_0^3 c(x-1)^2 dx = \frac{c}{3} (x-1)^3 \Big|_0^3$$

$$= \frac{c}{3} (8 - (-1)) = 3c$$

$$\Rightarrow c = \frac{1}{3}$$

$$0 \leq x \leq 3: F(x) = P(X \leq x) = \int_0^x \frac{1}{3}(t-1)^2 dt$$

$$= \frac{1}{9} (t-1)^3 \Big|_0^x = \frac{1}{9} (x-1)^3 + \frac{1}{9}$$

$$0 \leq x \leq 3$$

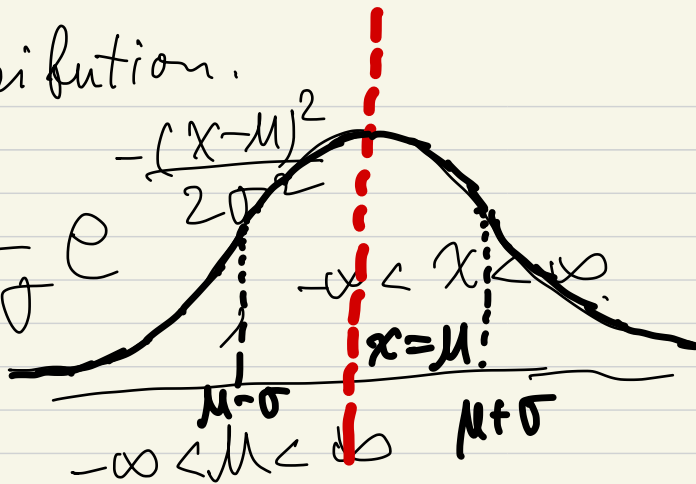
$$F(x) = \begin{cases} 0 & x < 0 \\ \frac{1}{9}(x+1)^3 + \frac{1}{9} & 0 \leq x \leq 3 \\ 1 & x > 3. \end{cases}$$

$$E(X) = \int_0^3 x \cdot \frac{(x+1)^2}{3} dx$$

9-10 pm
Tuesday

$N(\mu, \sigma^2)$ Normal Distribution.

$$f(x) = \frac{1}{\sqrt{2\pi} \sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$



$$E(X) = \mu$$

$$\text{Var}(X) = \sigma^2$$

example: $\mu = 5, \sigma = 2$

$$f(x) = \frac{1}{2\sqrt{2\pi}} e^{-\frac{(x-5)^2}{8}}$$

$-\infty < x < \infty$

$$P(X < 8) = \int_{-\infty}^8 \downarrow x$$

The Standard Normal Dist. $N(0, 1)$

$$f(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} \quad -\infty < z < \infty.$$

Table:

$P(Z \leq z)$ for $0 \leq z \leq 3.49$

