Cross Product.

only defined for vectors with 3 components.

$\left(\begin{matrix} V_3 \\ \mathbb{R}^3 \end{matrix} \right)$

$\vec{a} = \langle a_1, a_2, a_3 \rangle$

$\vec{b} = \langle b_1, b_2, b_3 \rangle$

$$\vec{a} \times \vec{b} = \langle a_2 b_3 - b_2 a_3,\ a_3 b_1 - b_3 a_1,\ a_1 b_2 - b_1 a_2 \rangle$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

12.4

② $\vec{a} = \langle 4, 3, -2 \rangle$ $\quad\quad$ $\vec{b} = \langle 2, -1, 1 \rangle$

find $\vec{a} \times \vec{b}$ $= \langle (3)(1) - \underbrace{(-1)(-2)}_{-2},\ \underset{-4}{(-2)(2)} - \underset{-4}{(1)(4)},\ \underset{-4}{(4)(-1)} - \underset{-6}{(2)(3)} \rangle$

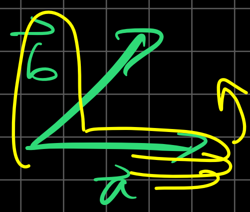
$= \langle 1, -8, -10 \rangle$

is new vector $\perp$ to $\vec{a}, \vec{b}$?

$\langle 1, -8, -10 \rangle \cdot \vec{a} = (1)(4) + (-8)(3) + (-10)(-2) = 0.$

$\langle 1, -8, -10 \rangle \cdot \vec{b} = 0$

for every $\vec{a}, \vec{b}$ $\quad \vec{a}\times\vec{b} \perp \vec{a}, \vec{b}$



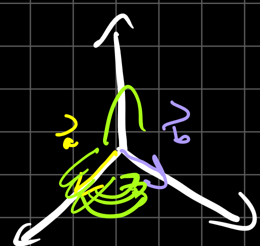
order of cross product determines if orthogonal vector is "up" or "down"

if we do $\vec{a}\times\vec{b}$ up

$\vec{b}\times\vec{a}$ down

In previous ex: $\vec{b}\times\vec{a} = \langle -1, 8, 10 \rangle = -\vec{a}\times\vec{b}$

Right hand Rule



$\vec{a}\times\vec{b}$

⑥ $\vec{a} = t\hat{\imath} + \cos t\,\hat{\jmath} + \sin t\,\hat{k}$

$\vec{b} = \hat{\imath} - \sin t\,\hat{\jmath} + \cos t\,\hat{k}$

$$\vec{a}\times\vec{b} = \left(t\hat{\imath} + \cos t\,\hat{\jmath} + \sin t\,\hat{k}\right)\times\left(\hat{\imath} - \sin t\,\hat{\jmath} + \cos t\,\hat{k}\right)$$

Try using distributive property. $\quad \vec{a}\times(\alpha\vec{b}) = \alpha(\vec{a}\times\vec{b})$

$$\vec{a}\times\vec{b} = (t\hat{\imath})\times(\hat{\imath}) - (t\hat{\imath})\times(\sin t\,\hat{\jmath}) + (t\hat{\imath})\times(\cos t\,\hat{k})$$

$+\cos t\,\hat{\jmath}\times\hat{\imath} - \cos t\,\hat{\jmath}\times\sin t\,\hat{\jmath} + \cos t\,\hat{\jmath}\times\cos t\,\hat{k}$

$+\sin t\,\hat{k}\times\hat{\imath} - \sin t\,\hat{k}\times\sin t\,\hat{\jmath} + \sin t\,\hat{k}\times\cos t\,\hat{k}$

$$= t\underbrace{(\hat{\imath}\times\hat{\imath})}_{\vec{0}} - t\sin t\underbrace{(\hat{\imath}\times\hat{\jmath})}_{\hat{k}} + t\cos t\underbrace{(\hat{\imath}\times\hat{k})}_{-\hat{\jmath}}$$

$$+\cos t\underbrace{(\hat{\jmath}\times\hat{\imath})}_{-\hat{k}} - \sin t\cos t\underbrace{(\hat{\jmath}\times\hat{\jmath})}_{\vec{0}} + \cos^2 t\underbrace{(\hat{\jmath}\times\hat{k})}_{\hat{\imath}}$$

$$+\sin t\underbrace{(\hat{k}\times\hat{\imath})}_{\hat{\jmath}} - \sin^2 t\underbrace{(\hat{k}\times\hat{\jmath})}_{-\hat{\imath}} + \sin t\cos t\underbrace{(\hat{k}\times\hat{k})}_{\vec{0}}$$

$$= -t\sin t\,\hat{k} - t\cos t\,\hat{\jmath} - \cos t\,\hat{k} + \cos^2 t\,\hat{\imath} + \sin t\,\hat{\jmath} + \sin^2 t\,\hat{\imath}$$

$$\boxed{= \hat{\imath} + (\sin t - t\cos t)\hat{\jmath} - (\cos t + t\sin t)\hat{k}}$$

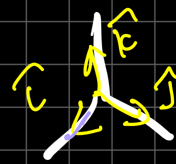
what is

$\vec{a}\times\vec{a} = \vec{0}$

$\vec{a}\times\alpha\vec{a} = \vec{0}$

$\vec{a}\parallel\vec{b}$

$\vec{a}\times\vec{b} = \vec{0}$



(11) $(\hat{\jmath}-\hat{k})\times(\hat{k}-\hat{\imath}) = \hat{\imath} - (-\hat{k}) - \vec{0} + (\hat{\jmath}) = \hat{\imath}+\hat{\jmath}+\hat{k}$

(13) $\vec{a}\cdot(\vec{b}\times\vec{c})$ ✓ Scalar   are any meaningful?

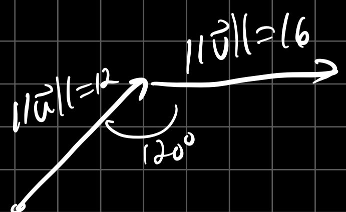
$\vec{a}\times(\vec{b}\times\vec{c})$ ✓ vector

$(\vec{a}\cdot\vec{b})\times(\vec{c}\cdot\vec{d})$ ✗ Scalar $\times$ scalar = bad.

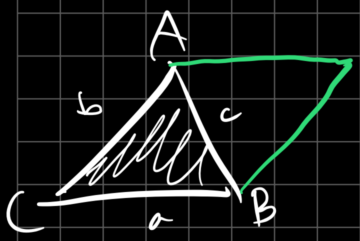
$(\vec{a}\times\vec{b})\cdot(\vec{c}\times\vec{d})$ ✓ scaler.

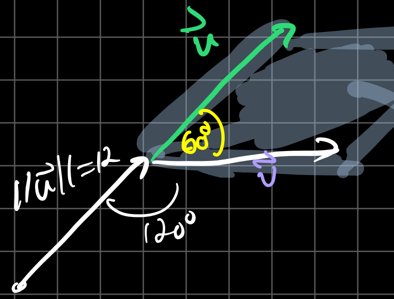
a $\vec{u}\cdot\vec{v} = \|\vec{u}\|\,\|\vec{v}\|\cos\theta$

(15)



find $\|\vec{u}\times\vec{v}\| = \underbrace{\|\vec{u}\|\,\|\vec{v}\|\sin\theta}_{\text{geometric of } \|\vec{u}\times\vec{v}\|}$





$\|\vec{u}\times\vec{v}\| = (12)(16)\sin(60^\circ)$

$= (12)(16)\frac{\sqrt{3}}{2}$

$= 96\sqrt{3}$

area of

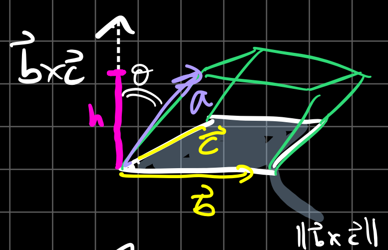
$\vec{u}\times\vec{v}$ into board (page).

Area $= \frac{1}{2}ab\sin C$

Area of ▱ $= ab\sin C$

(35) find volume of parallelepiped with adjacent edges PQ, PR, PS

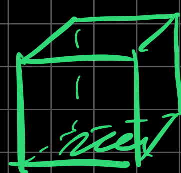
$P(-2,1,0)$ $Q(2,3,2)$ $R(1,4,-1)$ $S(3,6,1)$.



opposite planes parallel.

to find volume.

"area base" · height



Volume

$V = Ah$

$= \|\vec{b}\times\vec{c}\|\,h$ find from trigonometry.

$\cos\theta = \frac{h}{\|\vec{a}\|} \Rightarrow h = \|\vec{a}\|\cos\theta$

$= \|\vec{b}\times\vec{c}\|\,\|\vec{a}\|\,|\cos\theta|$

$= \left|\vec{a}\cdot(\vec{b}\times\vec{c})\right|$ scalar triple product.

Back to problem

$P(-2,1,0)$ $Q(2,3,2)$ $R(1,4,-1)$ $S(3,6,1)$.

$\vec{PQ} = \langle 2-(-2), 3-(1), 2-0\rangle = \langle 4,2,2\rangle$

$\vec{PR} = \langle 1-(-2), 4-1, -1-0\rangle = \langle 3,3,-1\rangle$

$\vec{PS} = \langle 3-(-2), 6-1, 1-0\rangle = \langle 5,5,1\rangle$

$$\text{Volume} = \left|\vec{PQ}\cdot(\vec{PR}\times\vec{PS})\right| = 16$$

$$\stackrel{?}{=} \left|\vec{PS}\cdot(\vec{PQ}\times\vec{PR})\right| = 16$$

$$\stackrel{?}{=} \left|\vec{PR}\cdot(\vec{PQ}\times\vec{PS})\right| = 16$$

What happens if one of the vectors is coplanar w/ others?

$h = 0$.

$$V = 0$$

$$\left|\vec{a}\cdot(\vec{b}\times\vec{c})\right| = 0$$

if this is true then $\vec{a},\vec{b},\vec{c}$ are coplanar.

Quiz open Thursday 12 am due Friday 11:59 pm