# Integral of Jump Processes

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$$X_t = X_0 + \underbrace{I_t}_{\int_0^t \sigma\, dB_t} + \underbrace{R_t}_{\int_0^t a\, ds} + \underbrace{J_t}_{\text{pure jump, right cts}, J_0=0}$$

记 $X_t$ CTS $= X_t^c = X_0 + I_t + R_t$

$$\Delta X_t = X_t - X_{t-} = \Delta J_t$$

考虑 $$\int_0^t f(s,\omega)\, dX_t = \int_0^t f(s,\omega)\, dX_t^c + \sum_{0<s\le t} f(s,\omega)\cdot \Delta J_s$$

$$= \int_0^t af\, ds + \int_0^t \sigma f\, dB_s + \sum_{0<s\le t} f(s,\omega)\cdot \Delta J_s$$

例 $X_t = M_t = \underbrace{N_t}_{\to J_t} - \underbrace{\lambda t}_{\to X_t^c}$ $\Rightarrow a(t,\omega) = -\lambda$, $\sigma(t,\omega) = 0$

$f = (t,\omega) = \Delta N_t$

则 $$\int_0^t f\, dX_t = \underbrace{\int_0^t f(-\lambda)\, ds}_{0} + \int_0^t f(0)\cdot dB_s + \sum_{0<s\le t} \underbrace{\Delta N_s}_{f(s,\omega)} \cdot \underbrace{\Delta N_s}_{\Delta J = \Delta N_t}$$

$$= \sum_{0<s\le t} (\Delta N_s)^2 = N_t$$

every jump 为 1.

martingale 积出来 非 martingale.

考虑 $f$ left-CTS

则 $\int_0^t f\, dX_t$ 也 martingale!

**Theorem 11.4.5.** *Assume that the jump process $X(s)$ of (11.4.1)–(11.4.3) is a martingale, the integrand $\Phi(s)$ is* left-continuous *and adapted, and*

$$\mathbb{E}\int_0^t \Gamma^2(s)\Phi^2(s)\, ds < \infty \text{ for all } t \ge 0.$$

*Then the stochastic integral $\int_0^t \Phi(s)\, dX(s)$ is also a martingale.*

The mathematical literature on integration with respect to jump processes gives a slightly more general version of Theorem 11.4.5 in which the integrand