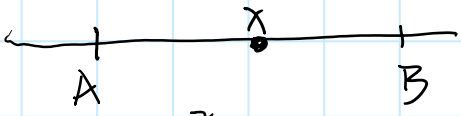
美丽的例题

2022年7月27日 20:14

求 $\mathbb{E}_{B_0=x}\tau$



$$Ans = \mathbb{E}_{B_0=x}\int_0^\tau ds$$

考虑 BVP Type 3 w/ $b=1$

$$\begin{cases} \mathcal{L}u + 1 = 0 \\ u(x) = 0, \quad x \in \partial D \end{cases} \iff \begin{cases} \mathcal{L}u + 1 = 0 \\ u(A) = 0 \ \&\& \ u(B) = 0 \end{cases}$$

在 1-D : $\mathcal{L}u = au_x + \frac{1}{2}\sigma^2 u_{xx}$

BM: $a=0, \sigma=1$

Pbm $\begin{cases} \frac{1}{2}\cdot 1\cdot u_{xx} + 1 = 0 \\ u(A) = 0, \quad u(B) = 0 \end{cases}$ ODE! $\rightsquigarrow$ 解得 $u = -x^2 + cx + d$.

代入 $\Rightarrow$ $\begin{cases} -A^2 + c\cdot A + d = 0 \\ -B^2 + c\cdot B + d = 0 \end{cases}$ $\Rightarrow (c,d) = (A+B, -AB)$

$\Rightarrow$ $u(x) = -x^2 + (A+B)x - AB$