Ito-Doeblin Formula

2022年8月10日 20:03

Theorem 11.5.1 (Itô-Doeblin formula for one jump process). *Let $X(t)$ be a jump process and $f(x)$ a function for which $f'(x)$ and $f''(x)$ are defined and continuous. Then*

$$f(X(t)) = f(X(0)) + \int_0^t f'(X(s))\,dX^c(s) + \frac{1}{2}\int_0^t f''(X(s))\,dX^c(s)\,dX^c(s) + \sum_{0<s\le t}[f(X(s)) - f(X(s-))]. \tag{11.5.4}$$

考虑 $f(t,x)$ s.t. f_t, f_x, f_{xx} 有定义且 CTS

则 $f(T,X_T) - f(0,X_0) = \int_0^T f_t(s,X_s)\,ds + \int_0^T f_x(s,X_s)\,dX_s^c + \frac{1}{2}\int_0^T f_{xx}(s,X_s)(dX_s^c)^2$

$$+ \sum_{0<s\le t}\left[f(s,X_s) - f(s,X_{s^-})\right]$$