Exam 2 2 hrs

on gradescope. open 10am close at 6pm.

Open book.

Cylindrical Coordinates

$x = r\cos\theta \quad y = r\sin\theta \quad z = z \quad x^2+y^2=r^2$

$\theta = \tan^{-1}\left(\frac{y}{x}\right).$

15.7

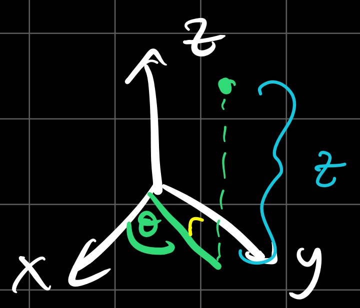
① $(4, \frac{\pi}{3}, -2)$ convert to rectangular $(2, 2\sqrt{3}, -2)$

r θ z

$z=-2$

$x = 4\cos\left(\frac{\pi}{3}\right), \; y = 4\sin\frac{\pi}{3} = 4\frac{\sqrt{3}}{2} = 2\sqrt{3}$

$x = 2$



⑨ $x^2 - x + y^2 + z^2 = 1$ write in cylindrical.

$r^2 - r\cos\theta + z^2 = 1$

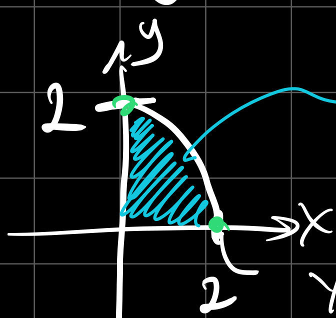
$z = x^2 - y^2 \rightarrow z = (r\cos\theta)^2 - (r\sin\theta)^2 = r^2\underbrace{(\cos^2\theta - \sin^2\theta)}_{\cos(2\theta)}$

$z = r^2\cos(2\theta)$

$z = \sqrt{x^2+y^2} \rightarrow z = r$

(19) $\iiint_E (x+y+z)\,dV$ E is solid in 1^{st} octant under $z = 4 - x^2 - y^2$

Set up using rectangular & cylindrical



$y=0$ and $z=0$

$0 \le r \le 2$

$0 \le \theta \le \frac{\pi}{2}$

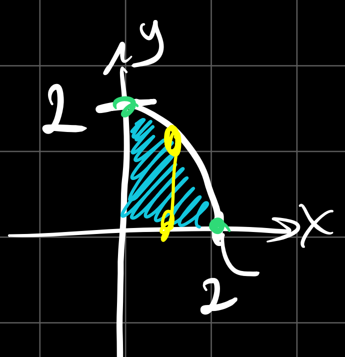
$0 \le z \le 4 - x^2 - y^2$ no good!!

$0 \le z \le 4 - r^2$

$$\iiint_E (x+y+z)\,dV = \int_0^{\frac{\pi}{2}} \int_0^{2} \int_0^{4-r^2} (r\cos\theta + r\sin\theta + z)\, r\,dz\,dr\,d\theta$$

θ r z

rectangular.



$0 \le x \le 2$

$0 \le y \le \sqrt{4-x^2}$

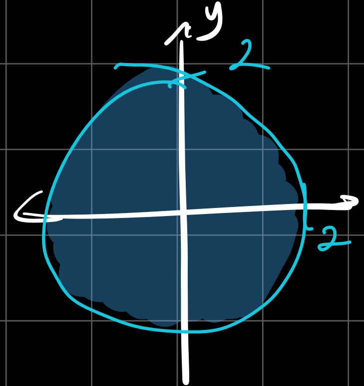
$0 \le z \le 4 - x^2 - y^2$

$$\iiint_E (x+y+z)\,dV = \int_0^2 \int_0^{\sqrt{4-x^2}} \int_0^{4-x^2-y^2} (x+y+z)\,dz\,dy\,dx$$

(29) $$\int_{-2}^{2} \int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}} \underbrace{\int_{\sqrt{x^2+y^2}}^{2} xz\,dz\,dx\,dy.}_{w=xz}$$

$$\left.\begin{aligned} -2 &\le y \le 2 \\ -\sqrt{4-y^2} &\le x \le \sqrt{4-y^2} \end{aligned}\right\} \rightarrow$$

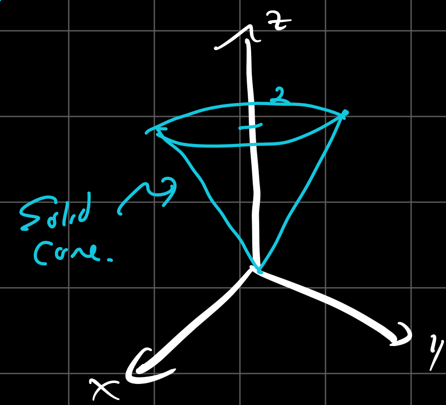
$$\underbrace{\sqrt{x^2+y^2}}_{r} \le z \le 2$$



$0 \le r \le 2$

$0 \le \theta \le 2\pi$

$r \le z \le 2$



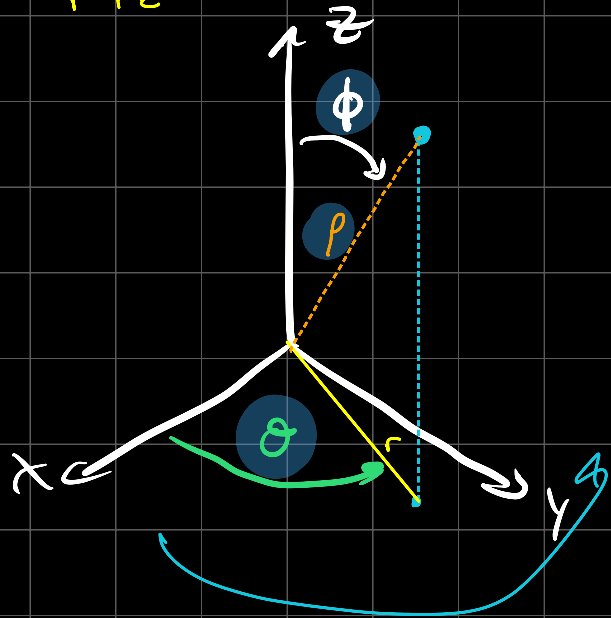
$$\int_{-2}^{2} \int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}} \int_{\sqrt{x^2+y^2}}^{2} xz\,dz\,dx\,dy = \int_0^{2\pi}\int_0^2\int_r^2 (r\cos\theta\, z)\, r\,dz\,dr\,d\theta$$

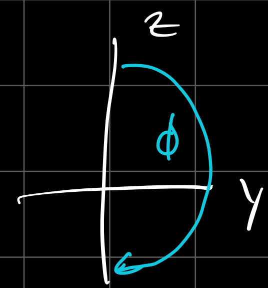
$$= \underbrace{\int_0^{2\pi} \cos\theta\,d\theta}_{0} \int_0^2 \int_r^2 r^2 z\,dz\,dr = 0.$$

15.8 Spherical

$x = \rho \sin\phi \cos\theta$ $\quad y = \rho \sin\phi \sin\theta$ $\quad z = \rho \cos\phi$

$\rho^2 = \underbrace{x^2+y^2}_{r^2}+z^2$ $\quad 0 \le \theta \le 2\pi$ $\quad 0 \le \phi \le \pi$





① $(6, \frac{\pi}{3}, \frac{\pi}{6})$ convert to cartesian/rectangular.
(ρ, θ, ϕ)

$x = 6\sin(\frac{\pi}{6})\cos(\frac{\pi}{3}) = 6(\frac{1}{2})(\frac{1}{2}) = \frac{6}{4} = \frac{3}{2}$

$y = 6\sin(\frac{\pi}{6})\sin(\frac{\pi}{3}) = 6(\frac{1}{2})(\frac{\sqrt{3}}{2}) = \frac{3\sqrt{3}}{2}$

$z = 6\cos\frac{\pi}{6} = 6(\frac{\sqrt{3}}{2}) = 3\sqrt{3}$

$(6, \frac{\pi}{3}, \frac{\pi}{6}) \to (\frac{3}{2}, \frac{3\sqrt{3}}{2}, 3\sqrt{3})$.

write in spherical!

⑩ $z = x^2 + y^2$

$\rho\cos\phi = (\rho\sin\phi\cos\theta)^2 + (\rho\sin\phi\sin\theta)^2$

$\rho\cos\phi = \rho^2\sin^2\phi\cos^2\theta + \rho^2\sin^2\phi\sin^2\theta$

$x^2+y^2+z^2 = 9$

$\rho^2 = 9$

$\rho = 3$

easy!!

$\rho\cos\phi = \rho^2\sin^2\phi$

$\rho^2\sin^2\phi - \rho\cos\phi = 0$

$\rho(\rho\sin^2\phi - \cos\phi) = 0$

(23) $\iiint_E (x^2+y^2)\,dV$

E is region between
$x^2+y^2+z^2=4$
$x^2+y^2+z^2=9$

$2 \le \rho \le 3$

$0 \le \theta \le 2\pi$

$0 \le \phi \le \pi$

$x^2+y^2 \rightarrow \rho^2\sin^2\phi\cos^2\theta + \rho^2\sin^2\phi\sin^2\theta = \rho^2\sin^2\phi$

$dV = \rho^2\sin\phi\, \underline{d\rho\, d\theta\, d\phi}$

change order.

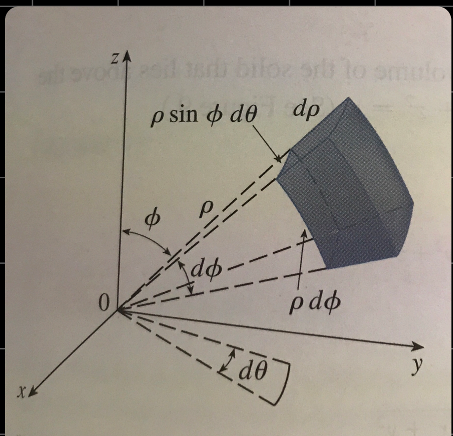


FIGURE 8

$$\iiint_E (x^2+y^2)\,dV = \int_0^{2\pi}\int_0^{\pi}\int_2^{3} (\rho^2\sin^2\phi)\,\rho^2\sin\phi\, d\rho\, d\phi\, d\theta$$

$$= \underbrace{\int_0^{2\pi} d\theta}_{2\pi} \int_0^{\pi} \sin^3\phi \, d\phi \underbrace{\int_2^3 \rho^4 \, d\rho}_{\left(\frac{3^5}{5}-\frac{2^5}{5}\right)} = 2\pi\left(\frac{4}{3}\right)\left(\frac{3^5}{5}-\frac{2^5}{5}\right)$$

$$\int_0^{\pi} \sin\phi \sin^2\phi \, d\phi = \int_0^{\pi} \sin\phi\,(1-\cos^2\phi)\, d\phi$$

$u = \cos\phi$, $du = -\sin\phi \, d\phi$

$$= -\int_1^{-1} (1-u^2)\,du = \int_{-1}^{1} (1-u^2)\,du$$

$$= \left(u - \frac{u^3}{3}\right)\Big|_{u=-1}^{u=1} = 1-\frac{1}{3}-\left(-1+\frac{1}{3}\right)$$

$$= 2 - \frac{2}{3} = \frac{4}{3}$$

(43) $$\int_{-2}^{2} \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_{2-\sqrt{4-x^2-y^2}}^{2+\sqrt{4-x^2-y^2}} (x^2+y^2+z^2)^{3/2} \, dz \, dy \, dx$$

$$\left.\begin{array}{l} -2 \le x \le 2 \\ -\sqrt{4-x^2} \le y \le \sqrt{4-x^2} \end{array}\right\}$$

$0 \le r \le 2$ not ρ. still need more work.

$0 \le \theta \le 2\pi$

in cylindrical:

$2-\sqrt{4-r^2} \le z \le 2+\sqrt{4-r^2}$

$$\underbrace{2-\sqrt{4-x^2-y^2}}_{\text{bottom.}} \le z \le \underbrace{2+\sqrt{4-x^2-y^2}}_{\text{Top}}$$

looks bad.

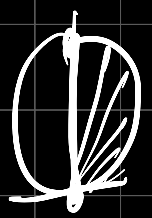
Continue to Spherical.

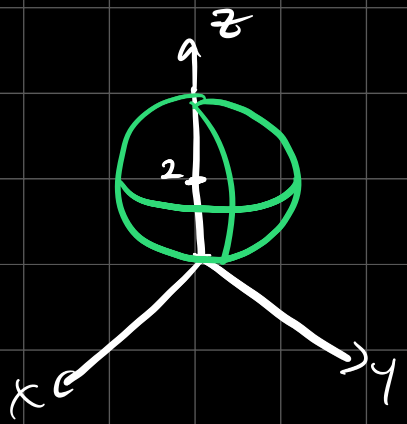
What is $z = 2+\sqrt{4-x^2-y^2}$

$$(z-2)^2 = \left(\sqrt{4-x^2-y^2}\right)^2$$

$$(z-2)^2 = 4-x^2-y^2 \rightarrow x^2+y^2+(z-2)^2 = 4$$

Sphere about $(0,0,2)$
with radius $= 2$.





$0 \le \theta \le 2\pi$

$0 \le \phi \le \frac{\pi}{2}$

radius depends on phi, ϕ.

$0 \le \rho \le$ function of ϕ

find this.

note $(x^2+y^2+z^2)^{3/2}$ integrand. $\rightarrow (\rho^2)^{3/2} = \rho^3$