12.5

(3) find line through $(2, 2.4, 3.5)$
parallel to $3\hat{i} + 2\hat{j} - \hat{k}$

Recall: vector equation of line $\vec{r}(t) = \underset{\text{initial}}{\vec{r}_0} + t\underset{\text{direction}}{\vec{v}}$

vector

$\vec{r}_0 = \langle 2, 2.4, 3.5 \rangle$

$\vec{v} = \langle 3, 2, -1 \rangle$

$$\vec{r}(t) = \langle 2, 2.4, 3.5 \rangle + t\langle 3, 2, -1 \rangle$$

$$= \langle 2 + 3t, 2.4 + 2t, 3.5 - t \rangle$$

Parametric equations:

$$x = 2 + 3t, \quad y = 2.4 + 2t, \quad z = 3.5 - t$$

Symmetric equations: (solve for t)

$$\frac{x-2}{3} = \frac{y-2.4}{2} = \frac{z-3.5}{-1}$$

(5) vector eqn of line through $(1, 0, 6)$ $\perp$ to $\underbrace{x + 3y + z = 5}_{\text{plane}}$.

$\vec{r}_0 = \langle 1, 0, 6 \rangle$

normal vector

$ax+by+cz=d$

$\vec{v}$ = normal vector to plane $= \langle 1,3,1\rangle$

$\vec{n} = \langle a,b,c\rangle$

$$\vec{r}(t) = \langle 1,0,6\rangle + t\langle 1,3,1\rangle = \langle 1+t,\ 3t,\ 6+t\rangle$$

Comment:

assume $\vec{n} = \langle a,b,c\rangle$ $\perp$ to a given plane

$\vec{r} = \langle x,y,z\rangle$ $\qquad$ $\vec{r}_0 = \langle x_0,y_0,z_0\rangle$

$\vec{r}-\vec{r}_0$ in the plane.

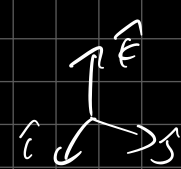
$\vec{n}\cdot(\vec{r}-\vec{r}_0) = 0$ $\qquad$ expand & simplify

$\boxed{ax+by+cz=d}$

d depends on x_0,y_0,z_0 a,b,c.

(10) line through $(2,1,0)$ $\perp$ to $\hat{i}+\hat{j}$ & $\hat{j}+\hat{k}$

find $\perp$ vector use $\times$.



$$(\hat{i}+\hat{j})\times(\hat{j}+\hat{k}) = \hat{k}-\hat{j}+\hat{i}$$
$$= \hat{i}-\hat{j}+\hat{k} = \vec{v} = \langle 1,-1,1\rangle$$

$\vec{r}_0 = \langle 2,1,0\rangle$

$$\boxed{\vec{r}(t) = \langle 2+t,\ 1-t,\ t \rangle}$$

(24) find equation of plane through $(5,3,5)$ with normal vector $2\hat{\imath}+\hat{\jmath}-\hat{k}$

recall: $\vec{n}\cdot(\vec{r}-\vec{r}_0) = 0$

$\vec{r} = \langle x, y, z \rangle$

$\vec{r}_0 = \langle x_0, y_0, z_0 \rangle$

$\vec{n} = \langle a, b, c \rangle$

$$\boxed{a(x-x_0)+b(y-y_0)+c(z-z_0)=0}$$

$$2(x-5)+1(y-3)+(-1)(z-5)=0$$

In standard form:

$$2x-10+y-3-z+5=0$$

$$2x+y-z=8$$

(30) find equation of the plane containing line:

$x = 1+t$
$y = 2-t$
$z = 4-3t$

and is parallel to $\underbrace{5x+2y+z}_{\text{normal} = \langle 5,2,1 \rangle}=1.$

$\vec{r}_0 = \langle 1,2,4 \rangle$

let $t=0$

point: $(1,2,4)$

$\vec{n}\cdot(\vec{r}-\vec{r}_0) = 0$

or

$a(x-x_0)+b(y-y_0)+c(z-z_0)=0$

plane: $5(x-1)+2(y-2)+1(z-4)=0.$

(67) $P_1: 3x+6y-3z=6$

which are $\parallel$ to each other? are any identical?

$P_2: 4x-12y+8z=5$

$P_3: 9y=1+3x+6z \rightarrow 3x-9y+6z=-1$

$P_4: z=x+2y-2 \rightarrow x+2y-z=2$

$\vec{n}_1=\langle 3,6,-3\rangle$ $\vec{n}_2=\langle 4,-12,8\rangle$

$\vec{n}_3=\langle 3,-9,6\rangle$ $\vec{n}_4=\langle 1,2,-1\rangle$

$\vec{n}_1 \parallel \vec{n}_4$

$\vec{n}_1\times\vec{n}_4=\vec{0}$

$\frac{4}{3}\vec{n}_3=\langle 4,-12,8\rangle=\vec{n}_2$

$\vec{n}_3\parallel\vec{n}_2$

$\vec{n}_3\times\vec{n}_2=\vec{0}$

$P_1: 3x+6y-3z=6$

$P_2: 4x-12y+8z=5$

$P_3: 9y=1+3x+6z \rightarrow 3x-9y+6z=-1$

$P_4: z=x+2y-2 \rightarrow x+2y-z=2$

Same!

Check to see if $P_2 = P_3$

$\frac{4}{3}(P_3)$: $4x - 12y + 8z = -\frac{4}{3}$

$P_2 = 4x - 12y + 8z = 5$

not identical

Question: if $\vec{n}$ fixed and d changes
what happens? we get a family of $\parallel$ planes

algebraically:

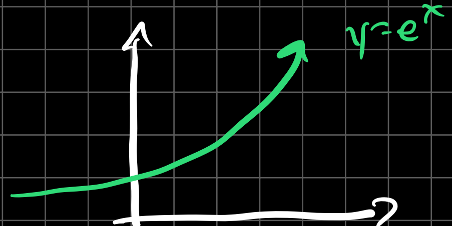
(82) ish $a(x-x_0) + b(y-y_0) + c(z-z_0) = 0$

$ax + by + cz = \boxed{ax_0 + by_0 + cz_0} = d$

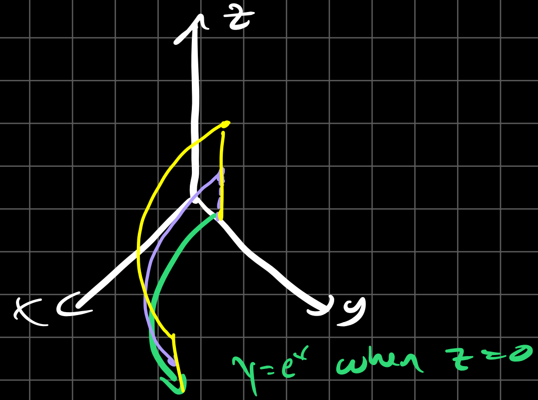
as d changes

12.6

(2) $y = e^x$ in $\mathbb{R}^2$



in $\mathbb{R}^3$ for each z we have $y = e^x$



Describe range: $\{(x,y,z) \in \mathbb{R}^3 \mid y > 0\}$.

element of.

?

:

$y > 0$ in $\mathbb{R}^3$

everything in front of xz plane.

(20) $x = y^2 - z^2$ p 837 - diagram

draw traces

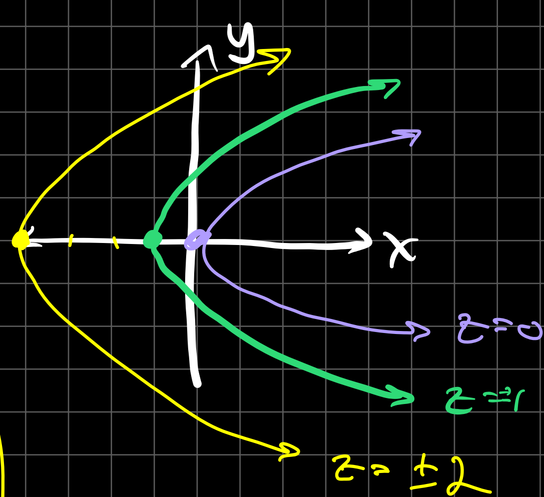
fix x, y or z draw other variables.

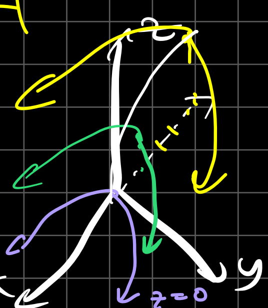
fix z - traces in z

$z = 0$ $x = y^2$

$z = 1$ $x = y^2 - 1$

$z = 2$ $x = y^2 - 4$





Traces in y

$$x = y^2 - z^2$$

$y=0$ $\quad x = -z^2$

$y=1$ $\quad x = 1 - z^2$

$y=2$ $\quad x = 4 - z^2$