Chapter 9. One and Two-Sample Estimation Problems

Estimation

Hypothesis Testing

μ, $\mu_1 - \mu_2$, p. $p_1 - p_2$, σ^2, $\frac{\sigma_1^2}{\sigma_2^2}$

sample from the population

$X_1, X_2, \ldots X_n \sim$

use $\overline{X}$ to est μ.

$$E(\bar{X}) = \mu, \qquad Var(\bar{X}) = \frac{\sigma^2}{n}$$

(these are true as long as $X_1, X_2 \cdots X_n$ is a random sample from the popn μ, σ^2)

① Case 1: $X_1, X_2, \cdots X_n \sim N(\mu, \sigma^2)$, σ^2 known

Goal: To est μ.

Point Estimator: $\bar{X}$

Sampling dist of $\bar{X}$: $\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$

$$\Updownarrow$$

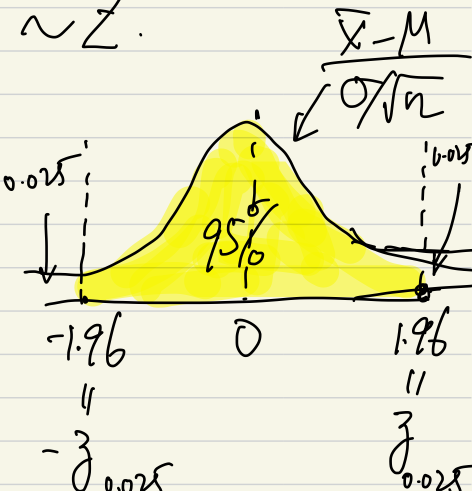
$$\frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \sim Z.$$

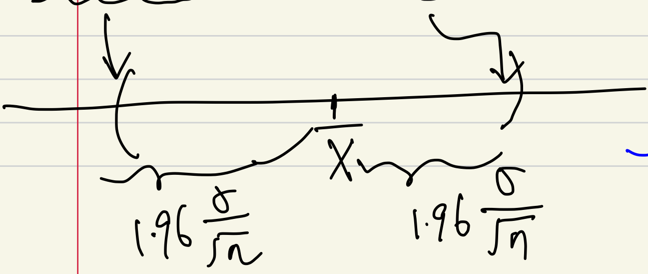
$$P\left(-1.96 \le \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \le 1.96\right) = 0.95$$

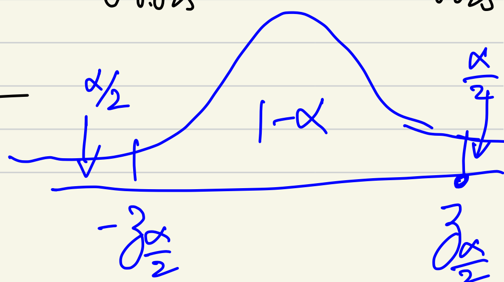
$$-1.96\frac{\sigma}{\sqrt{n}} \le \bar{x} - \mu \le 1.96\frac{\sigma}{\sqrt{n}}$$

$$-\bar{x} - 1.96\frac{\sigma}{\sqrt{n}} \le -\mu \le -\bar{x} + 1.96\frac{\sigma}{\sqrt{n}}$$

$$P\left(\underbrace{\bar{x} - 1.96\frac{\sigma}{\sqrt{n}}} \le \mu \le \underbrace{\bar{x} + 1.96\frac{\sigma}{\sqrt{n}}}\right) = 0.95$$







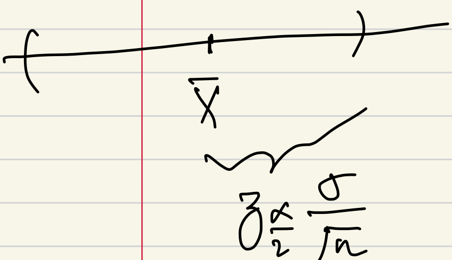
α: a small number like 0.01, 0.05 0.10

$1-\alpha$ close to 1.

$100(1-\alpha)\%$ confidence interval:

$$\left(\bar{X} - z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}},\ \ \bar{X} + \underbrace{z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}}\right)$$

margin of error (maximum error) of $\bar{X}$ at $100(1-\alpha)\%$ conf. level.



Ex: Assume zinc concentertration measures $\sim N(\mu, 0.3^2)$

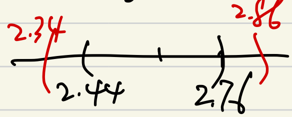
$n = 9$, $\bar{X} = 2.6$

① Find a 90% CI for μ

$$\bar{X} \pm z_{0.05} \frac{\sigma}{\sqrt{n}} = 2.6 \pm 1.645 \frac{0.3}{3} = 2.6 \pm 0.16\cancel{45}$$

$$= (2.44,\ 2.76)$$

② Find a 99% CI for μ.

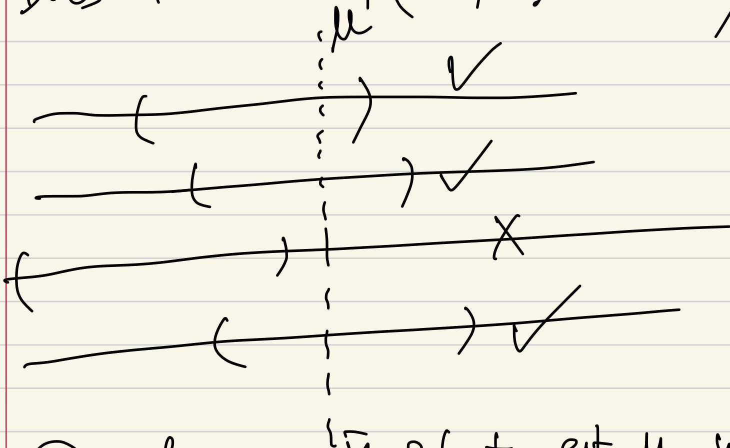


$$\bar{X} \pm z_{0.005} \frac{\sigma}{\sqrt{n}} = 2.6 \pm 2.57\cancel{6} \frac{0.3}{3}$$

$$= 2.6 \pm 0.26$$

$$= (2.34,\ 2.86)$$

Does it mean $P(2.34 < \mu < 2.86) = 0.99$?



③ When use $\bar{X} = 2.6$ to est μ, what's the m.o.e at 90% conf. level?

$$z_{0.05}\frac{\sigma}{\sqrt{n}} = 1.645 * \frac{0.3}{\sqrt{9}} = 0.1645$$

④ If we wish to have m.o.e < 0.1 at 90% confidence level, what need to be done?

$$z_{\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}} \le e$$

$$n \ge \frac{z_{\frac{\alpha}{2}}^2\sigma^2}{e^2} = \frac{1.645^2 \cdot 0.3^2}{0.1^2} = 25$$

$100(1-\alpha)\%$ 1-sided confidence interval.

$\begin{cases} \text{an upper bound at } 100(1-\alpha)\% \text{ conf. level} \\ \text{a lower bound — — —} \end{cases}$

5 find a 96% 1-sided conf. interval for μ with an upper bound.

$$\left(0,\ \bar{X} + z_\alpha \frac{\sigma}{\sqrt{n}}\right) = \left(0,\ 2.6 + 1.75 \frac{0.3}{3}\right)$$

$$= (0,\ 2.775)$$

Case 2. (large sample confidence interval)

$X_1, X_2, \cdots X_n \sim \mu, \sigma^2$.

CLT $\Rightarrow$ $\bar{X}$ is still approx normal.

n is large $\Rightarrow$ S is a good enough est for σ.

$$\frac{\bar{X}-\mu}{S/\sqrt{n}} \approx Z$$

$100(1-\alpha)\%$ CI for μ is:

$$\left(\bar{X} - z_{\frac{\alpha}{2}} \frac{S}{\sqrt{n}},\ \bar{X} + z_{\frac{\alpha}{2}} \frac{S}{\sqrt{n}}\right)$$ (if σ is availble use σ)

case 3. (small sample confidence interval).

$X_1, X_2 \cdots X_n \sim N(\mu, \sigma^2)$, σ^2 unknown

$$\frac{\bar{X}-\mu}{S/\sqrt{n}} \sim t(n-1)$$

$100(1-\alpha)\%$ CI for μ:

$$\left(\bar{X} - t_{\frac{\alpha}{2}}(n-1)\frac{S}{\sqrt{n}}, \quad \bar{X} + t_{\frac{\alpha}{2}}(n-1)\frac{S}{\sqrt{n}}\right)$$

Ex: mpg for a car model $\sim N(\mu, \sigma^2)$

$n=7$: 19.6, 20.4, 20.8, 19.6, 20.0, 20.4, 19.2

① Find a 95% CI for μ.

$\bar{X} = 20.0$ $\quad S = \cancel{1.386}\ 0.57$

$$\bar{X} \pm t_{0.025}(6)\frac{S}{\sqrt{n}} = 20.0 \pm 2.447\frac{0.57}{\sqrt{7}}$$
$$= 20.0 \pm 0.53$$
$$= (19.47, 20.53)$$

$$S = \sqrt{\frac{\sum_{i=1}^{n}(X_i - \bar{X})^2}{n-1}}$$

② Find a number C, so we can assert with 95% confidence, that $\mu \le C$.

$$C = \bar{X} + t_{0.05}(6)\frac{S}{\sqrt{7}} = 20.0 + 1.943\frac{0.57}{\sqrt{7}}$$