Monetary-fiscal policy interactions with debt maturities and primary market frictions

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Abstract

This paper studies the interactions between monetary and fiscal policies in the presence of debt maturities and primary market frictions. Under these conditions, the standard local determinacy requirements outlined in Leeper (1991) are modified: extending debt maturities within the traditional fiscal regime (AF/PM) can induce local equilibrium indeterminacy. This modification arises from two distinct channels: (i) an institutional channel, wherein longer maturities alleviate institutional frictions by reducing issuance costs; and (ii) a liquidity channel, wherein lengthened maturities mitigate rollover costs by relieving distributional pressures on primary dealers and the associated liquidity costs. As a result, our analysis reveals a policy trilemma: with primary market frictions, policymakers cannot simultaneously accommodate inflation, enhance fiscal adjustment and lengthen average debt maturity. By incorporating primary market frictions into the FTPL framework, our findings highlight the crucial role of debt maturities in macroeconomic policy design and the prevention of sunspot equilibria.

Keywords: Debt maturity; Fiscal theory of price level; Debt sustainability; Inflation.

JEL Classification: E31; E52; E62.

1 Introduction

In recent decades, governments worldwide have increasingly lengthened the average maturities of their debt portfolios. As reported by De Graeve and Mazzolini (2023), the average debt maturities across countries increased from 5.49 years in 1995 to 7.84 years in 2020. This trend, amplified by the aftermath of the 2008 Global Financial Crisis and further accelerated by fiscal measures implemented in response to the COVID-19 pandemic, underscores the critical importance policymakers assign to debt maturity management. Figure 1 illustrates this point by depicting the share of long-term debt relative to total debt and the weighted-average maturities (WAM) across OECD countries. The figure clearly shows substantial variation in maturities, with most countries issuing over half of their debt in the form of long-term bonds. Longer maturities are typically viewed as tools for enhancing fiscal sustainability by reducing rollover risks and stabilizing debt-servicing costs. Yet, despite their widespread adoption, the macroeconomic implications of these maturity extensions—especially their impact on fiscal and monetary policy interactions—remain insufficiently understood.

The theoretical literature on policy interactions, notably initiated by Leeper (1991) among others, establishes clear guidelines regarding the requirements under which monetary and fiscal policies interact to yield macroeconomic stability. Specifically, most of their works prescribe active/passive combinations of monetary and fiscal policies necessary for local equilibrium determinacy, implicitly abstracting from the role of debt maturity. Under this classical view, the maturity structure of government debt is typically regarded as peripheral and has been largely overlooked in analyses of local equilibrium outcomes and macroeconomic stability within the framework of fiscal theory of price level.[1]

However, existing works overlook practical realities present in financial markets — most notably, liquidity costs encountered by primary dealers in bond markets. These dealers, acting as intermediaries between the primary issuance and secondary trading of government bonds, need time to liquidate their bond portfolios auctioned from the government. As a result, the larger the auctioned amounts, the lower the price they are willing to pay.[2] Figure 2 empirically illustrates this friction by depicting the seasonally adjusted 10-year bond auction tail—a proxy for the bid-ask spread—alongside year-on-year changes in the debt-to-GDP ratio over time. The evident comovement

[1] Cochrane (2001) provides a thorough treatment of long-term bonds within the fiscal theory of the price level but does not grapple explicitly with determinacy issues.

[2] Bigio et al. (2023) documents the presence of significant liquidity costs in Spanish sovereign debt auctions. Bigio et al. (2019) reviews the three main economic reasons in literature are: inventory management of financial intermediaries, adverse selection and order processing costs.

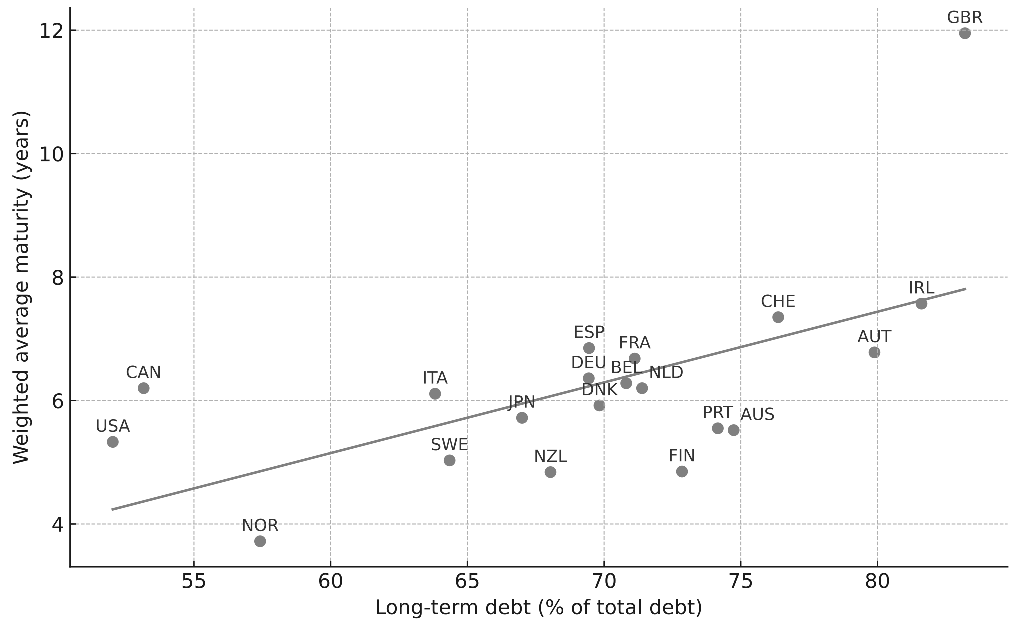

Figure 1: The Weighted Average Maturities across OECD Countries. *Notes*: The figure reproduces the long-term-debt share (long-term debt divided by total debt) and the weighted-average maturities (WAM) of OECD countries reported by De Graeve and Mazzolini (2023).

indicates a strong correlation between increased debt issuance and enlarged redistribution costs faced by primary dealers. The omission of such market frictions raises a critical question: Do Leeper (1991)'s prescriptions on policy coordination still hold when debt maturity is considered in such a realistic bond market setting?

In this paper, we directly address this gap by examining how extended government debt maturities influence the well-established monetary-fiscal interactions paradigm when primary market is frictional. To this end, we begin by constructing a standard New-Keynesian model, incorporating a government issuing long-term bonds for deficit financing and primary dealers trading in a frictional primary market. Specifically, the government issues a portfolio of bonds with maturities to finance its expenditures. At the same time, we formulate profit optimizing primary dealers, who purchase newly issued long-term government bonds from the fiscal authority in the primary market and later resell them to households in the secondary market. We also introduce primary market frictions similar to those in over-the-counter (OTC) markets, where dealers face challenges in unloading bond holdings in the secondary market, as outlined by Bigio et al. (2023). These frictions arise because dealers incur additional costs when liquidating more auctioned bonds. Consequently, it creates an extra price

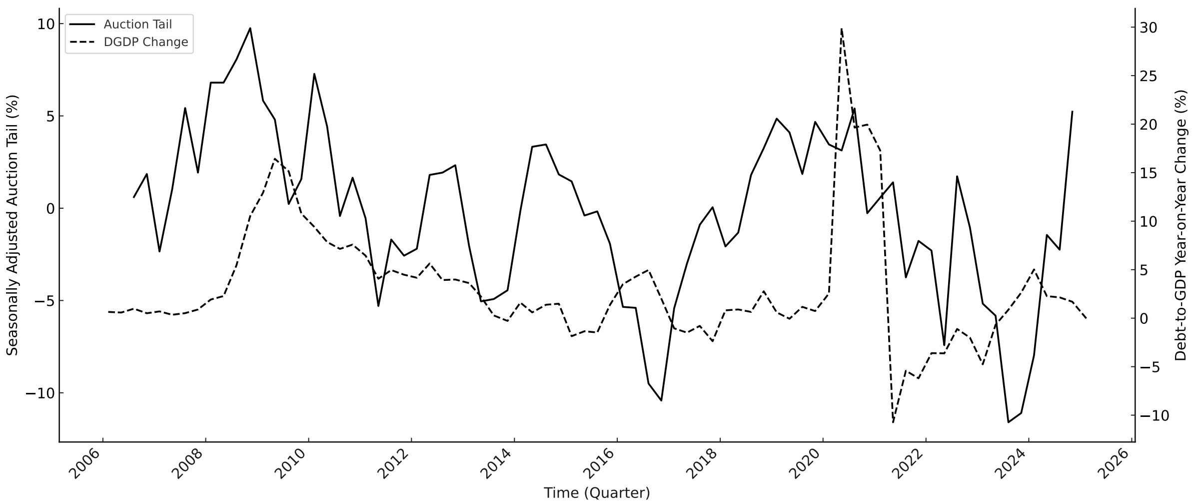

Figure 2: 10-Year Bond Auction Tail and DGDP Change. *Notes*: The figure depicts the seasonally adjusted auction tail and the year-on-year change in the debt-to-GDP ratio. The auction tail is calculated as the difference between the actual auction yield and the when-issued yield, capturing primary market frictions. Debt-to-GDP changes represent the year-on-year variation in government debt relative to GDP.

discrepancy between the purchase price in the primary market and the resale price in the secondary market, which can be referred to bid-ask spreads.[1] This gap is crucial for understanding how monetary and fiscal policy decisions—particularly those regarding debt issuance—impact bond yields and vice verse. Contrast to previous literature, this price discrepancy is endogenous in our model, influenced by both the scale of debt issuance and the average debt maturity, and results from primary dealers' profit-maximizing behavior under such liquidity costs.[2]

Under the above settings, we focus on the underlying mechanisms driving policy distortions. Our main findings are as follows:

1. In the absence of primary market frictions, the local equilibrium determinacy requirements remain unaffected by bond maturity. The canonical active/passive policy prescription from Leeper (1991) continues to hold, as long as fiscal policy targets the market value of bonds.

2. When primary market frictions are present, bond maturity significantly influ-

[1] Similar idea is also adopted by Cui et al. (2025), who investigates endogenous liquidity frictions in capital markets. They argue that this friction is driven by search-and-matching frictions, emphasizing the role of primary-secondary market segmentation.

[2] A notable exception is Andreolli (2021) that examines the effects of debt maturity on monetary policy. Our assumption is similar to his, but we perform our work with different debt maturity structures and in absence of secondary market frictions to facilitate the stability analysis.

ences policy interactions, where the traditional active/passive policy prescriptions no longer guarantee local equilibrium determinacy. Specifically, under primary market frictions, extending bond maturities requires either less passive fiscal policy (for active monetary policies) or more active fiscal policy (for passive monetary policies). Thus, the policy space in Regime M expands as maturity increases, whereas the policy space in Regime F contracts with longer maturities.

These results reveal two novel channels through which debt maturity affects monetary and fiscal policy interactions under primary market frictions. In a frictionless environment, fiscal targeting of the market value of outstanding debt fully neutralizes term premium effects, as debt sustainability hinges exclusively on the evolution of its market valuation. However, when primary-market frictions are introduced, extending debt maturity affects determinacy requirements via two distinct channels. The first, which we label the *Institutional Channel*, arises from the debt issuance costs—such as administrative expenses, informational asymmetries, and auction-related frictions like the winner's curse—which are diluted by reducing the fixed cost of auctions when maturities lengthen. Consequently, a longer maturity structure enhances primary dealers' willingness to bid higher prices, reducing the institutional friction component of bond yields. The second channel, termed the *Liquidity Channel*, captures how maturity extension alleviates liquidity pressures by decreasing the rollover size per period relative to dealers' limited balance sheet capacity. Specifically, longer maturities reduce the periodic quantity to rollover, reduce primary dealers' inventory costs and facilitate redistribution to preferred-habitat investors, thereby raising auction prices through diminished liquidity costs.

These two channels jointly determine bond pricing and both operate in the same directions. In particular, unlike the conventional term-premium channel—which depresses bond prices and thus raises debt-servicing costs—a maturity extension in a frictional primary market environment predominantly boosts prices through reductions in institutional and liquidity costs.[1] These mechanisms has critical implications for policy interactions. Under an active monetary/passive fiscal regime (Regime M), the reduction in institutional and liquidity costs due to maturity extension strengthens equilibrium determinacy without necessitating additional fiscal adjustments. Conversely, under an active fiscal/passive monetary regime (Regime F), extending maturity might erode fiscal anchoring effectiveness. Under this regime, the lowered liquidity-driven

[1] In this paper, we assume that the secondary market is complete without arbitrage, thus the term-premium-induced policy distortion is totally absorbed by the fiscal rule targeting the real value of debt instead of a nominal one. Which is also the core mechanism behind our first maturity-irrelevant result.

costs implicitly relax fiscal constraints, prompting the fiscal authority to overestimate the degree of its policy activeness. As a result, the price level is prone to be unanchored: its convergence hinges on self-fulfilling, non-fundamental disturbances and sunspot equilibria emerge.

Building on these insights, we further propose a policy trilemma for local stability while implementing fiscal and monetary policies: To avoid local instability, policymakers cannot simultaneously implement the following three measures: (1) accommodating higher inflation to erode real debt; (2) intensifying fiscal adjustment through higher taxation; (3) lengthening the average maturity of public debt. At most two measures can be pursued simultaneously; adopting all three inevitably induces local instability.

This trilemma directly engages the prevailing claim on understanding of how fiscal policy can eliminate self-fulfilling equilibria. Specifically, Lorenzoni and Werning (2019) demonstrate that pairing sufficiently aggressive fiscal policy with longer-maturity debt effectively precludes slow-moving debt crises and rules out equilibrium multiplicity. However, our findings critically qualify their conclusion by emphasizing the pivotal role of primary market frictions. When public debt sustainability is in question, central banks frequently adopt passive stances, accommodating higher inflation to partially erode the debt burden. From a local equilibrium perspective, this accommodation can inadvertently reopen the door to equilibrium indeterminacy. Therefore, we underscore that preserving macroeconomic stability necessitates explicit coordination between fiscal and monetary authorities, as well as internal alignment within fiscal policy itself.

Finally, we revisit the canonical framework of Leeper (1991) to further discuss the relevance of bond maturity within the fiscal theory of the price level (FTPL) and clarify the intuitive underpinnings of the policy interactions trilemma. Following Leeper (1991), we construct an endowment economy model, abstracting from production elements, and decompose intertemporal budget constraint which is the core of the fiscal theory. We highlight that debt maturity fundamentally affects the equilibrium through its impact on the stochastic discount factor (SDF) used for public debt valuation. Given fixed primary surpluses, extending bond maturities increases the effective discount factor and consequently raises expectations of future primary surpluses, creating conditions susceptible to self-fulfilling price expectations (sunspot equilibria). Crucially, we reinterpret the trilemma as fundamentally arising from imbalances within the intertemporal budget constraint: the three considered policy instruments simultaneously increase expected future revenues (the right-hand side), while leaving current debt obligations (the left-hand side) predetermined. Maintaining equilibrium therefore necessitates that at least one policy instrument moves inversely to the others, balancing expectations to yield a unique monetary policy trajectory and thereby eliminate belief-driven instabilities.

The remainder of this paper proceeds as follows. Section 2 reviews the relevant literature. Section 3 presents our model within a New-Keynesian framework, explicitly detailing the primary market optimization problem. Section 4 derives analytical results that specify the critical conditions under which local equilibrium is stable and presenting our policy trilemma proposition. Section 5 further demonstrates the core insights within the canonical framework of Leeper (1991). Section 6 concludes.

2 Literature Review

Interactions between fiscal and monetary policies have been extensively studied since the seminal work of Sargent et al. (1981), which introduced the notion of "unpleasant monetarist arithmetic," emphasizing inherent tensions between inflation control by monetary authorities and fiscal deficit accommodation. Building upon this foundational contribution, Leeper (1991) formalized these interactions through the Fiscal Theory of the Price Level (FTPL), distinguishing clearly between active monetary/passive fiscal regimes and passive monetary/active fiscal regimes. He showed that price level determinacy critically depends on the coordination of these policies. Subsequently, Sims (1997) and Woodford (1995, 1998) integrated monetary and fiscal policy dynamics using one-period debt frameworks. However, introducing long-term debt significantly changed the analysis of these interactions. Cochrane (2001) demonstrated that the maturity structure of government debt significantly influences inflation responses to fiscal shocks, highlighting that long-term debt allows smoother and more delayed price-level adjustments, aligning closely with observed inflation dynamics.

Recent literature has advanced on several critical fronts, notably the integration of debt management practices into fiscal-monetary interaction frameworks. Cochrane (2022) examined how partially-repaid long-term debt modifies immediate price-level reactions, leading to gradual inflation adjustments that better reflect actual policy practices. Studies by Leeper and Zhou (2021), Chen et al. (2022), Mao et al. (2023), Stangebye (2023), and Cao (2024) quantitatively analyze the impacts of extended debt maturities on inflation dynamics and fiscal sustainability constraints. Specifically, Mao et al. (2024) underscored the paradoxical outcome that the probability of shifting to passive monetary regimes under high debt levels can exacerbate fiscal burdens, highlighting the importance of managing debt levels effectively.

Another essential development involves incorporating market frictions into the analysis and highlighting the liquidity role of government debt. Cui (2016) modeled endogenous liquidity frictions within monetary-fiscal interactions, illustrating that while government bonds enhance investment efficiency, they simultaneously raise fiscal financing costs, implying an optimal long-run debt issuance strategy. Bassetto and Cui (2018) reconsidered FTPL implications in contexts where the government debt return rate may fall below the economic growth rate, emphasizing the role of liquidity premiums in distorting traditional policy prescriptions. Further, Cantore and Leonardi (2025) introduced a heterogeneous-agent New-Keynesian model, revealing a ”self-insurance demand” channel driven by households' preference for liquidity, which substantially alters fiscal multipliers and monetary policy transmission.

Moreover, recent studies explicitly address primary market frictions inherent in debt issuance, particularly liquidity, inventory, and issuance costs. Bigio et al. (2023) formalized these liquidity frictions, showing how large issuances lead to price discounts due to slow bond resale by intermediaries, directly impacting issuance costs and balance sheet capacity. Similarly, Vayanos and Vila (2021) incorporated search frictions, demonstrating constraints faced by primary dealers, resulting in higher yields during substantial auctions. Complementary insights from Boyarchenko et al. (2021), Tse and Xu (2021), and Cui et al. (2025) further clarified how these endogenous liquidity frictions influence dealer behavior, market segmentation and spreads. This paper extends the spirit of Bigio et al. (2023) by explicitly embedding liquidity costs into a New Keynesian framework, thereby analyzing how primary market frictions influence monetary-fiscal interactions and equilibrium determinacy within the debt maturity perspective.

Literature on debt maturity management also identifies trade-offs governments face between fiscal costs and macroeconomic stability. Greenwood et al. (2015) formalized the balance between short-term debt, offering liquidity benefits but subjecting governments to rollover risks, and long-term debt, which secures interest rates but incurs higher issuance costs. Conversely, Leeper et al. (2021) argued that lengthening debt maturity reduces inflation bias but exacerbates debt stabilization bias, advocating for an optimal maturity structure that balances these competing effects. Andreolli (2021) further emphasized that extensive reliance on long-term debt dampens monetary policy effectiveness via a ”financing channel,” introducing additional macroeconomic stability risks. Our paper contributes by highlighting how primary market liquidity frictions amplify these maturity trade-offs, distorting the dynamics between fiscal and monetary policies.

Finally, significant research has focused on how policy coordination affects equilibrium determinacy and vulnerability to self-fulfilling equilibria. Early studies by Bullard and Mitra (2002), Guo and Harrison (2004), and Schabert (2006) rigorously established necessary and sufficient conditions for local equilibrium determinacy, explicitly linking government budget structures with macroeconomic stability. Recent contributions by Bassetto and Cui (2018), Cho (2021), and Chen et al. (2022) have examined how low-interest-rate environments and regime-switching policy frameworks affect equilibrium uniqueness. Furthermore, Angeletos and Lian (2023) illustrated that minor informational frictions alone can restore equilibrium determinacy independently of traditional Taylor-rule conditions. Our work adds to this stream by explicitly embedding primary market liquidity frictions into a New Keynesian model with long-term government debt, thus providing novel insights into how these frictions modify established conditions for equilibrium determinacy and monetary-fiscal coordination.

3 Model Settings

In this section, we augment an otherwise canonical New Keynesian model, building on Galí (2015), by introducing two key features. First, we explicitly model a government that finances expenditures and debt service through lump-sum taxation and issuance of long-term bonds characterized by geometrically decaying coupon payments. Second, we incorporate profit-maximizing primary dealers who purchase newly issued government bonds and subsequently sell them to households within the same period on a complete secondary market. Importantly, we introduce frictions arising from primary dealers' market participation—referred to as *primary market frictions*—in line with the spirit in Bigio et al. (2023). Specifically, these liquidity constraints imply that primary dealers incur additional adjustment costs associated with liquidating their auctioned bond holdings, thereby generating a bid-ask spread between primary issuance and secondary market trading.

3.1 Households

There is a continuum of identical households of measure unity. The representative households enjoy consumption goods C_t and dislike supplying labor N_t. They maximize their life-time utility in the form:

$$\max_{\{C_t,H_t,B_t,M_t,\{D_t^{t-j}\}_{j=0}^{\infty}\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{C_t^{1-\sigma}}{1-\sigma} - \chi \frac{H_t^{1+\eta}}{1+\eta} \right\} \tag{3.1}$$

subject to

$$P_t C_t + B_t + Q_t^s D_t + P_t T_t = W_t H_t + R_{t-1} B_{t-1} + (1 + \delta Q_t^s) D_{t-1} + P_t \Pi_t \tag{3.2}$$

where households earn wages at the rate W_t, receive profits Π_t from monopolistic competitive firms, and pay lump-sum taxes T_t to the government. In each period t, they can invest in two types of assets. The first asset is nominal risk-free bonds B_t that yield a return rate R_t, issued by the central bank in zero net supply and used to conduct monetary policy. The second asset consists of perpetuities characterized by exponentially decaying coupon payments: one unit bond issued at date t provides a coupon payment of δ^{s-1} units at date $t+s$, where $\delta \in [0,1]$ controls the average maturity structure. Specifically, the average duration of this debt portfolio is $(1-\beta\delta)^{-1}$. Special cases emerge naturally: setting $\delta = 1$ yields a consol, while $\delta = 0$ corresponds precisely to a one-period discount bond. Consequently, in period t, the government raises $Q_t^s D_t$ through primary issuance and pays $(1+\delta Q_t^s)D_{t-1}$ to redeem bonds issued in the previous period on average.

The equilibrium conditions derived from household optimization yield the following Euler equations:

$$\mathbb{E}_t\left[\beta\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma}\frac{R_t}{\pi_{t+1}}\right] = 1, \tag{3.3}$$

$$R_t^D = \mathbb{E}_t\left\{\frac{1+\delta Q_{t+1}^s}{Q_t^s}\right\} = R_t, \tag{3.4}$$

where R_t denotes the short-term nominal interest rate, and R_t^D represents the expected return on long-term bonds. The first equation is a standard Euler equation, reflecting the Fisherian relationship that links nominal interest rates with real stochastic discount factors. The second equation characterizes the Euler equation specific to long-term bond holdings. Under the assumption of complete secondary markets, whereby arbitrage opportunities are absent, it must hold that $R_t = R_t^D$. Thus, the second equation embodies a no-arbitrage condition ensuring equivalence between returns on short-term and long-term assets in equilibrium.

Finally, we assume no-Ponzi game condition such that

$$\lim_{T\to\infty} \Lambda_{0,T} A_T \geq 0, \tag{3.5}$$

where $A_t \equiv \{B_{t-1}(1+R_{t-1}) + (1+\delta)Q_t^s D_{t-1}\}/P_t$ denotes the real financial wealth at the beginning of period t and $\Lambda_{0,t}$ is the stochastic discount factor. Combining the no-Ponzi game condition with the fact that it is not optimal for households to leave unspent wealth at the ending period yields the transversality condition given by:

$$\lim_{T\to\infty} \Lambda_{0,T} A_T = 0. \tag{3.6}$$

3.2 Firms

A representative final goods firm bundles intermediate goods with a CES technology:

$$Y_t = \left[\int_0^1 Y_t(i)^{1-\frac{1}{\epsilon}} di \right]^{\frac{\epsilon}{\epsilon-1}}, \tag{3.7}$$

where $Y_t(i)$ denotes the input of intermediate goods i and ϵ measure the elasticity of substitution between intermediate goods. Profit maximization yields the demand schedules below:

$$Y_t(i) = \left[\frac{P_t(i)}{P_t} \right]^{-\epsilon} Y_t. \tag{3.8}$$

A continuum of monopolistically competitive firms produce differentiated intermediate goods index by $i \in [0, 1]$ with the following constant return to scale technology:

$$Y_t(i) = N_t(i). \tag{3.9}$$

Following Calvo (1983), at any given period, each intermediate goods firm can reset the price of its goods with probability $1 - \theta$. Profit maximizing subject to the demand function (3.8) leads to the optimal price setting decision. After derivations, the inflation dynamics is described by the equation below:

$$\pi_t = \left[(1-\theta)(\pi_t^*)^{1-\epsilon} + \theta \right]^{\frac{1}{1-\epsilon}} \tag{3.10}$$

where $\pi_t^* = p_t^* / p_{t-1}$ is denoted as re-pricing inflation rate. We can also show that the inflation rate follows:

$$\pi_t^* = \frac{\epsilon}{\epsilon - 1} \frac{x_{1,t}}{x_{2.t}} \pi_t \tag{3.11}$$

where

$$x_{1,t} = C_t^{-\sigma} Y_t \frac{1}{\mu_t} + \theta\beta\mathbb{E}_t x_{1,t+1} \pi_{t+1}^{\epsilon} \tag{3.12}$$

$$x_{2,t} = C_t^{-\sigma} Y_t + \theta\beta\mathbb{E}_t x_{2,t+1} \pi_{t+1}^{\epsilon-1} \tag{3.13}$$

where $\mu_t = 1/w_t$ is the average price markup and $\mu = \epsilon/(\epsilon - 1)$ denotes its desired level.

3.3 Primary Dealers

Primary dealers purchase newly issued government bonds in the primary market and resell them to households on the secondary market at the price Q_t^s within the period. This activity incurs a financing cost Ψ_t, which depends on the quantity of newly issued bonds $d_t = \frac{Q_t D_t}{P_t}$. The resulting cost introduces an adjustment friction into the model.

Formally, the dealers' profit maximization problem is:

$$\max_{d_t} Q_t^s\, d_t - Q_t d_t - \Psi_t(d_t). \tag{3.14}$$

The first-order condition implies:

$$Q_t^s = Q_t + \psi_t(d_t), \tag{3.15}$$

where $\psi_t(d_t) \equiv \partial \Psi_t(d_t)/\partial d_t$ denotes the marginal financing cost per additional bond issued. At steady state, defining $\psi_t(d_t) = \psi$, we have a direct interpretation of the steady-state marginal cost as the price differential between primary and secondary markets, $\psi = Q^s - Q$. In our subsequent analysis, this steady-state differential captures *institutional frictions*, reflecting fixed frictions independent of issuance volume. For analytical convenience, we also introduce a net spread ratio, $\lambda \equiv Q^s/Q - 1$, which directly measures the primary-secondary market spread. Clearly, ψ and λ are proportionally related: absent institutional friction, both ψ and λ vanish, while their positive values reflect the presence and intensity of institutional frictions.

Next, we specify the primary market friction function. Following Andreolli (2021), we assume that it follows a CRRA form:

$$\Psi(d_t) = \Psi_0 \frac{d_t^{1+\Psi_1}}{1+\Psi_1} \exp\!\left(\frac{\nu_t}{\Psi}\right) \quad \text{with} \quad \Psi_0 \geq 0,\ \Psi_1 \geq 0, \tag{3.16}$$

where ν_t^{Ψ} is a shock to the financing friction, Ψ denotes the steady-state value of the friction's first derivative, and Ψ_0 and Ψ_1 serve as scaling parameters. The shock ν_t follows a standard AR(1) process:

$$\nu_t = \rho_\nu\, \nu_{t-1} + \epsilon_t^{\Psi}. \tag{3.17}$$

The virtue of of this specification lies in its tractability. Specifically, after first-order derivation with respect to d_t, equation (3.16) becomes:

$$\psi_t = \zeta d_t \exp\!\left(\frac{\nu_t}{\Psi}\right), \quad \psi > 0, \tag{3.18}$$

where $\zeta \equiv \Psi\Psi_1/\bar{d}$ characterizes the sensitivity of marginal liquidity costs to variations in bond issuance. Thus, we interpret ζ as the intensity of *liquidity frictions*. If $\zeta = 0$, primary dealers face only institutional frictions without incremental liquidity frictions.

Importantly, parameter Ψ_1 governs the curvature of marginal cost functions. A linear marginal cost ($\Psi_1 = 0$) implies constant marginal costs independent of issuance scale, whereas a positive Ψ_1 indicates convex marginal costs ($\psi'(d_t) > 0, \psi''(d_t) > 0$), reflecting escalating liquidity costs as issuance grows. In subsequent sections, we systematically examine how these frictional specifications influence macroeconomic stability outcomes.

3.4 Government

The government issues long-term debt on the primary market to finance its expenditures. It's budget constraint is represented as:

$$P_t T_t + Q_t D_t = P_t G_t + (1 + \delta Q_t) D_{t-1}, \tag{3.19}$$

where Q_t denotes the bond price in the primary market at time t and government spending G_t is exogenously given and invariant for simplicity. To analyze fiscal variables, we define the lowercase letter $\hat{x} = \frac{X_t}{Y}$ as the real value of variable X_t over steady state aggregate output Y, and proceed to rewrite equation (3.19) in real terms:

$$d_t + s_t = \frac{1 + \delta Q_t}{Q_{t-1}} \frac{d_{t-1}}{\pi_t}. \tag{3.20}$$

where $s_t = P_t(T_t - G_t)/Y$ represents the ratio of primary surplus over steady state GDP.

3.5 Fiscal and Monetary Policies

The primary surplus depends on the deviation of public debt from the steady-state level:

$$s_t = s + \phi_d (d_{t-1} - d), \tag{3.21}$$

where $s = T_t - G_t$ denotes the primary surplus and ϕ_d determines the responsiveness of fiscal policy to the change of government deficits.

The monetary authority conducts policy according to a simple Taylor rule, wherein the nominal interest rate, R_t, responds to deviations of expected inflation from the target π:

$$R_t = R \left(\frac{\pi_t}{\pi} \right)^{\phi_\pi}, \tag{3.22}$$

where $\phi_\pi > 1$ denotes the degree of interest rate responsiveness to inflation deviations, and π is the long-run inflation target set by central bank.

3.6 Equilibrium Conditions

Definition 1. *The decentralized equilibrium is an infinite sequence of allocations* $\{C_t, H_t, Y_t,$

$\pi_t, B_t, D_t, T_t\}$, prices $\{w_t, \mu_t, P_t, R_t, Q_t, Q_t^s, \psi_t\}$, and policies $\{\phi_d, \phi_\pi\}$ that satisfy the following conditions at each period:

1. *Households choose $\{C_t, H_t, B_t, D_t\}$ to maximize their lifetime utility, taking prices $\{w_t, P_t, R_t, Q_t^s\}$ and policies $\{\phi_d, \phi_\pi\}$as given;*
2. *Monopolistic intermediate goods firms produce $\{y_t(j)\}$ by choosing $\{H_t(j), p_t(j)\}$ to maximize their profits, taking wage as given;*
3. *Competitive final goods firms produce y_t to maximize profits, taking the price intermediate goods $p_t(j)$ as given;*
4. *The goods market clears: $Y_t = C_t + G$;*
5. *The labor market clears: $L_t = \int_0^1 L_t(j)dj$;*
6. *The short-term bonds market clears: $B_t^s = B_t^d$;*
7. *The long-term bonds market clears: $D_t^s = D_t^d$;*
8. *The government budget constraint is balanced: $P_tT_t + Q_tD_t = P_tG_t + (1 + \delta Q_t)D_{t-1}$.*

Letting non-fiscal allocations $\hat{y}_t \equiv log(Y_t/Y)$, $\hat{c}_t \equiv log(C_t/C)$,$\hat{\pi}_t \equiv log(\pi_t/\pi)$, $\hat{R}_t \equiv (R_t - R)/R$; fiscal allocations $\hat{d}_t \equiv (D_t - D)/Y$, $\hat{g}_t \equiv G_t/Y$; net prices $\hat{Q}_t \equiv log(Q_t/Q)$, $\hat{Q}_t^s \equiv log(Q_t^s/Q^s)$, and $\hat{\psi}_t \equiv log(\psi_t/\psi)$ now, the equilibrium around the zero-inflation steady state can be approximated by the following system:

$$\hat{y}_t = \hat{c}_t \tag{3.23}$$

$$\hat{c}_t = \hat{c}_{t+1} - \frac{1}{\sigma}(\hat{R}_t - \hat{\pi}_t) \tag{3.24}$$

$$\hat{\pi}_t = \kappa \hat{y}_t + \beta \hat{\pi}_{t+1} \tag{3.25}$$

$$\hat{Q}_t^s = \beta\delta\hat{Q}_{t+1}^s - \hat{R}_t, \tag{3.26}$$

$$\hat{Q}_t = (1+\lambda)\hat{Q}_t^s - \lambda\hat{\psi}_t, \tag{3.27}$$

$$\hat{d}_t + \hat{s}_t = (1/Q + \delta)\hat{d}_{t-1} - (1/Q + \delta)d^*\hat{\pi}_t + \delta d^*\hat{Q}_t - (1/Q + \delta)d^*\hat{Q}_{t-1}, \tag{3.28}$$

$$\hat{R}_t = \phi_\pi \hat{\pi}_t \tag{3.29}$$

$$\hat{s}_t = \phi_d \hat{d}_{t-1}, \tag{3.30}$$

where d^* is the long-run debt-to-GDP targets. Define $\lambda = Q^s/Q - 1$ as price tail, representing the steady state bid-ask spreads between primary and secondary market. Thus, we derive $1/Q = (1+\lambda)(\beta^{-1} - \delta)$, which is the steady-state value of inverse primary bond price Q_t. In addition, ϕ_π and ϕ_d are policy parameters controlled by monetary and fiscal authorities. Other parameters are the same as definitions in the standard New-Keynesian models.

In order to close the model, the above equilibrium conditions must be supplemented with an equation describing the functional form of $\hat{\psi}_t(\hat{d}_t)$, which in turn is determined by the relationship between the primary market friction $\hat{\psi}_t$ and the government bonds $\hat{d}_t$ assumed. Afterwards, we have two scenarios to analyze.

To begin with, in the case of no frictions, $\hat{\psi}_t$ is described by:

$$\hat{\psi}_t = \hat{v}_t, \tag{3.31}$$

which bond market frictions are totally exogenous.

Second, in the presence of primary market frictions, $\hat{\psi}_t$ adjusts endogenously with the change of bonds, thus we have:

$$\hat{\psi}_t = \zeta \hat{d}_t + \hat{v}_t, \tag{3.32}$$

where the frictions are endogenous with the issuance of public debt, and ζ represents the intensity of such liquidity frictions.

4 Aggregate (In)stability

Before analyzing the local stability around the steady states, we first derive the above equations into a self-closed system of first-order linear difference equations. The most simplified system is characterized by:[1]

$$AX_t = B\mathbb{E}_t X_{t+1}, \tag{4.1}$$

[1] See Appendix A.2 for details.

where the endogenous variable vector is $X_t = \left[\hat{y}_t, \hat{\pi}_t, \hat{Q}_t^s, \hat{d}_t\right]'$.

The coefficient matrix A is:

$$A = \begin{bmatrix} \sigma & \phi_\pi & 0 & 0 \\ -\kappa & 1 & 0 & 0 \\ 0 & \phi_\pi & 1 & 0 \\ 0 & 0 & -\gamma(1+\lambda) & \gamma_0 \end{bmatrix}; \quad B = \begin{bmatrix} \sigma & 1 & 0 & 0 \\ 0 & \beta & 0 & 0 \\ 0 & 0 & \beta\delta & 0 \\ 0 & \gamma & -\delta d^*(1+\lambda) & \gamma_1 \end{bmatrix}, \tag{4.2}$$

where $\gamma_0 = (1 + \zeta\lambda d^*)[(1+\lambda)/\beta - \lambda\delta] - \phi_d$ and $\gamma_1 = 1 + \zeta\delta\lambda d^*$.

As a result, equations (3.23)-(3.32) are finally simplified into a system of four linear equations. The system involves four jump variables $\hat{y}_t$, $\hat{\pi}_t$ and $\hat{Q}_t$ with one predetermined variables $\hat{d}_t$. For the system to satisfy the locally determinacy condition, it must exhibit exactly three explosive roots and one stable root.

Notably, the above two coefficient matrices exhibit special structural properties: when partitioned into 2×2 blocks, matrix A and matrix B can be transformed into two lower triangular matrices. As a result, the eigenvalues of the differential system can be characterized independently by the diagonal submatrices of A and B. Specifically, let $X_{i,j}$ denote the (i, j)-th block of matrix X after partitioning. Then, the expression for $A^{-1}B$ is given by:

$$A^{-1}B = \begin{bmatrix} A_{11}^{-1}B_{11} & 0 \\ A_{22}^{-1}\left(B_{21} - A_{21}A_{11}^{-1}B_{11}\right) & A_{22}^{-1}B_{22} \end{bmatrix}, \tag{4.3}$$

where $A_{i,j}$ and $B_{i,j}$ for $i, j = 1, 2$ are all 2×2 matrices. It is straight forward to show that the eigenvalues are exactly the eigenvalues of the diagonal matrices $A_{11}^{-1}B_{11}$ and $A_{22}^{-1}B_{22}$. Therefore, we have the following lemma:

Lemma 1. *The eigenvalues of the matrix $A^{-1}B$ are determined by the submatrices $A_{11}^{-1}B_{11}$ and $A_{22}^{-1}B_{22}$. More explicitly:*

1. *The first two eigenvalues depend exclusively on ϕ_π. If $\phi_\pi > 1$, both eigenvalues lie within the unit circle, corresponding to two unstable roots. Conversely, if $\phi_\pi \leq 1$, exactly one eigenvalue is stable, while the other becomes unstable.*

2. *The remaining two eigenvalues are governed by ϕ_d. These eigenvalues can be analytically derived as $\beta\delta < 1$ and γ_1/γ_0, where γ_0 itself is a function of ϕ_d.*

Proof. See Appendix A.4. □

4.1 Policy requirements for local stability

In this subsection, we analyze the fiscal and monetary policy requirements for local stability in our model. We first establish necessary and sufficient conditions ensuring the existence of a saddle-path equilibrium in a frictionless setting. By setting the primary market friction parameter $\psi = 0$ we have:

Proposition 1 (Frictionless Threshold for Local Stability). *Define the frictionless threshold as*

$$\phi^* = \beta^{-1} - 1.$$

In the absence of primary market frictions, the determinacy regions are:

1. *If $\phi_\pi > 1$, i.e.* active, *local stability requires $\phi_d > \phi^*$; otherwise, the equilibrium is explosive.*
2. *If $\phi_\pi < 1$, i.e.* passive, *local stability requires $\phi_d < \phi^*$; otherwise, the equilibrium is unstable (indeterminate).*

Proof. See Appendix A.5 □

The above frictionless analysis performs as a baseline scenario that highlights ϕ^* as the critical boundary distinguishing different fiscal policy regimes, reproducing the classical results in Leeper (1991). Notably, the threshold ϕ^* is independent of debt maturity—contrast to Cochrane (2001) who argues that debt maturity significantly matters because it allows fiscal authorities to trade off current versus future inflation via debt operations. The key to understand our maturity-irrelevant results is our fiscal policy rule (3.21), which directly targets the market value of government debt. This configuration endogenizes bond prices into fiscal adjustments. As a result, the minimal strength of fiscal responsiveness simplifies dramatically to a constant rate r^*, fully internalizing bond price fluctuations. This provides theoretical justification for models that replace long-term bonds with short-term bonds without introducing extra indeterminacy issues.

However, when primary market frictions are present — characterized by parameters $\psi > 0$, the conditions for achieving local equilibrium stability are explicitly influenced by the intensity of frictions and long-term fiscal targets.[1] We begin by establishing the necessary and sufficient conditions for saddle-path stability:

[1] According to our definitions, $\psi > 0$ is equivalent to $\lambda > 0$. The equivalence between $\psi \equiv Q^s - Q > 0$ and $\lambda \equiv Q^s/Q - 1 > 0$ clearly indicates a positive correlation.

Proposition 2 (Fiscal Requirements for Local Stability)**.** *Define the boundary condition as:*

$$\phi_F^* = \phi^* + \phi^I + \phi^L,$$

where $\phi^I = \lambda\left(\beta^{-1} - \delta\right)$ *and* $\phi^L = \zeta\, d^*(1+\lambda)\lambda\left(\beta^{-1} - \delta\right)$*. Under primary market frictions, the stability regions are:*

1. *If* $\phi_\pi > 1$*, i.e.* active, *local stability requires* $\phi_d > \phi_F^*$*; otherwise, the equilibrium is explosive.*

2. *If* $\phi_\pi < 1$*, i.e.* passive, *local stability requires* $\phi_d < \phi_F^*$*; otherwise, the equilibrium is unstable (indeterminate).*

Proof. See Appendix A.6. □

Algebraically, the boundary condition holds if and only if:

$$\phi_d = \underbrace{(\beta^{-1} - 1)}_{\text{Frictionless Threshold } (\phi^*)} + \underbrace{\lambda(\beta^{-1} - \delta)}_{\text{Institutional Channel } (\phi^I)} + \underbrace{\zeta d^*(1+\lambda)\left[\lambda\left(\beta^{-1} - \delta\right)\right]}_{\text{Liquidity Channel } (\phi^L)}, \tag{4.4}$$

where λ, ζ, δ, and d^* represent steady-state bid-ask spreads, marginal liquidity costs, average debt maturity, and long-run debt-GDP target respectively.

This equation displays two additional channels through which debt maturity affects the boundary conditions for ϕ_d in the presence of primary market frictions ($\phi > 0$ and $\lambda > 0$), with the presence of liquidity costs ($\zeta > 0$) activating the second channel.

The first channel, termed the *Institutional Channel*, captures market frictions arising primarily from fixed institutional and informational costs inherent in debt issuance. These costs, including administrative expenses, compliance fees, and uncertainties associated with primary auctions (notably, the winner's curse), remain largely invariant to the scale of issuance. Extending the maturity structure of debt issuance dilutes these fixed costs, as it reduces the frequency or the size of individual auctions, thereby diminishing the per-unit administrative burden.[1] Furthermore, longer maturities anchor bond valuations more closely to long-term fundamental factors, which mitigates informational asymmetries among bidders. This anchoring effect weakens the impact of *the*

[1]To see why, imagine the government must finance \$1 of debt. If it issues a 1-year bill, that full amount has to be refinanced every year, so ten auctions are required over a decade. By contrast, issuing a 10-year note postpones the refinancing until year 10, collapsing those ten auctions into one. Equivalently, lengthening the average maturity can be viewed as rolling over only a smaller fraction of the outstanding stock each period. Either way—fewer auctions or smaller rollovers—the fixed administrative cost per dollar raised falls.

winner's curse[1], resulting in less bid shading and, consequently, a lower institutional cost component in the government's borrowing costs.

The second channel, the *Liquidity Channel*, arises from liquidity frictions driven by the relative scale of debt issuance in comparison to dealers' finite balance sheet capacities, analogous to over-the-counter market frictions as illustrated in Duffie et al. (2005). Specifically, dealers incur elevated liquidity risks as larger debt issuances strain their limited capital and inventory management capabilities, leading to longer resale periods and consequently suppressed auction prices. Consequently, extending average debt maturity mitigates these liquidity pressures: holding rollover frequency constant, longer maturities reduce the relative magnitude of liquidity shocks, effectively lowering the dealers' unloading pressure. An alternative interpretation of this spread arises through the lens of asset-market segmentation driven by preferred-habitat investors, , as formalized by Ray et al. (2024), whose demand elasticity is typically low. These investors, which prominently include institutional entities such as pension funds and insurance companies, exhibit stable and duration-specific investment preferences. As a result, longer-term securities are more readily absorbed by these institutions, facilitating quicker redistribution following auctions, and substantially relieving inventory pressures on dealers. Collectively, these interactions serve to diminish primary dealers' balance sheet constraints, thereby reducing the liquidity-related costs associated with issuance.

It is worth noting that the supply-induced liquidity channel can effectively be viewed as an interaction term, equivalent to the scale-invariant institutional channel scaled by a liquidity-cost parameter $\zeta d^*(1+\lambda)$. Holding the institutional premium constant, any increase in liquidity-related costs (ζ) or steady-state debt objectives (d^*) directly exacerbates primary market frictions, intensifying supply-induced issuance costs for debt management. Conversely, if marginal liquidity frictions remain fixed, the spreads can be interpreted as institutional frictions adjusted by an elasticity factor. This elasticity depends critically on prevailing liquidity conditions, aggregate debt issuance levels, and dealers' market-making capacities. Consequently, heightened institutional frictions amplify marginal liquidity costs indirectly, rendering the aggregate fiscal responsiveness more sensitive to debt maturity changes and broader structural characteristics.

[1] Milgrom and Weber (1982) demonstrates that in common-value auctions the highest bid systematically embodies the most optimistic private signal, so the ex-post asset value is expected to underrun the price—an informational pitfall they term *the winner's curse*.

4.2 The role of primary market frictions and debt maturities

In this subsection, we examine how variations in primary market frictions and government debt maturities affect local equilibrium stability.

We begin by analyzing the role of primary market frictions. According to the analysis in subsection 4.1 , we find that both channels—the institutional channel and liquidity channel—are functions of the bid-ask spread parameter, λ. It can be demonstrated that each channel positively correlates with primary market frictions. This insight leads to the following proposition:

Proposition 3 (Primary Market Frictions and Local Stability). *Primary market frictions reshape the policy parameter space that supports a unique locally stable rational-expectations equilibrium. Specifically:*

1. *Under a monetary-led regime ($\phi_\pi > 1$), primary market frictions* ***contract*** *the stability region. A more aggressive fiscal feedback to debt dynamics is therefore required to ensure both debt sustainability and local stability.*

2. *Under a fiscal-led regime ($\phi_\pi < 1$), the same frictions* ***expand*** *the stability region, rendering local stability under a wide range of parameter configurations that would be unstable in a frictionless primary market.*

Proof. See Figure 3. □

Figure 3 shows how boundary conditions shift when primary market frictions increase from λ_1 to λ_2. In response to an active monetary policy, the threshold ϕ_F^* functions as a lower bound for fiscal policy responsiveness, enforcing more passive fiscal responses as primary market frictions intensify. In contrast, under passive monetary policy, ϕ_F^* represents an upper bound, implying less fiscal activeness is required since the ignorance of the intensified debt issuance costs act analogously to an active fiscal measure, effectively anchoring the price level through the rebalance of the government budget constraint.

Critically, overlooking primary market frictions can induce misleading policy assessments. Consider an illustrative case: suppose the government neglects the friction ($\lambda_1 = 0$), whereas frictions actually exist ($\lambda_2 > 0$). A sudden rise in public debt to finance increased expenditures destabilizes the economy. With monetary policy remaining active ($\phi_\pi > 1$), disregarding frictions means maintaining fiscal responsiveness at ϕ_{d,λ_1} instead of the necessary higher level, ϕ_{d,λ_2}. This underestimation of issuance costs

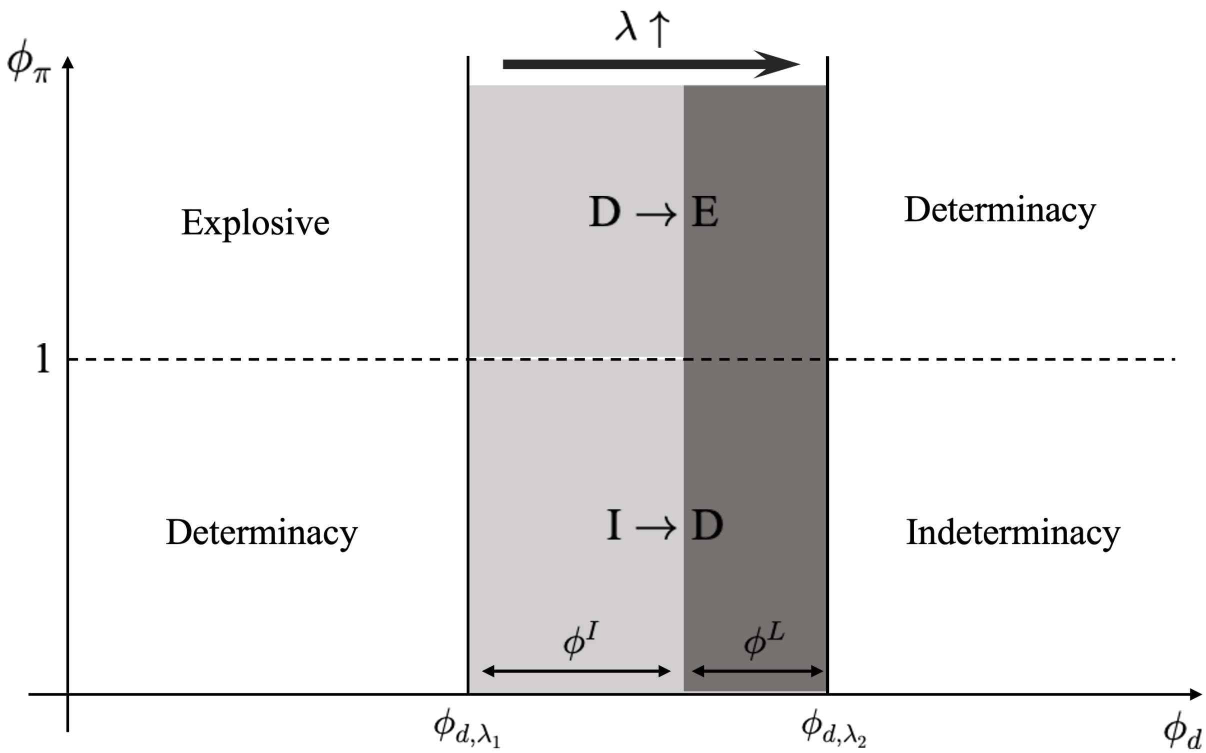

Figure 3: The Variation of Determinacy Regions. *Notes*: The figure displays an increase in primary market frictions from λ_1 to λ_2.

and overly optimistic assessment of fiscal sustainability can lead to explosive debt dynamics. Conversely, under fiscal-led regimes, ignoring primary market frictions poses less risk, as these frictions inherently expand the conditions supporting local stability.

Next, we investigate the impact of debt maturities. From equation (4.4), longer maturities reduce the fiscal responsiveness parameter ϕ_d, producing effects opposite to those of primary market frictions. We summarize this relationship with the following proposition:

Proposition 4 (Debt Maturities and Local Stability)**.** *In the presence of primary-market frictions:*

1. *Under a monetary-led regime ($\phi_\pi > 1$), extending debt maturity* ***preserves*** *stability.*

2. *Under a fiscal-led regime ($\phi_\pi < 1$), extending debt maturity can transform previously stable equilibria into* ***unstable (indeterminate)*** *ones.*

Proof. See Figure 4. □

Figure 4 illustrates how equilibrium determinacy conditions shift when government debt maturities extend from δ_1 to δ_2. Unlike the influence of primary market

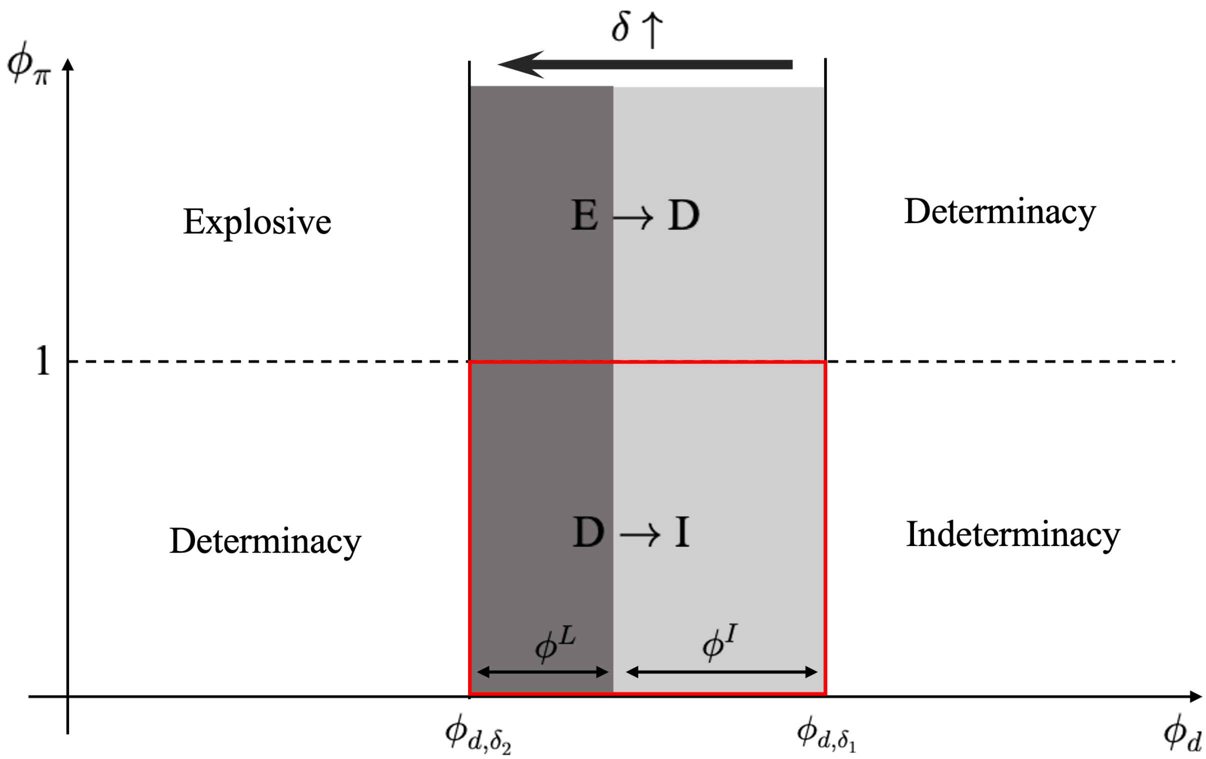

Figure 4: The Variation of Determinacy Regions. *Notes*: The figure displays an increase in debt maturity from δ_1 to δ_2.

frictions, extending debt maturities reduces the boundary fiscal responsiveness parameter ϕ_d. Under an active monetary policy regime, the fiscal authority is compelled to internalize primary-market issuance costs fully to maintain debt sustainability. Consequently, increasing debt maturities—by lowering immediate issuance costs—expands fiscal maneuverability, allowing the monetary authority greater latitude in anchoring the price level independently.

Conversely, under a fiscal-led regime, extending debt maturities erodes the fiscal policy's capacity to uniquely anchor the price level through frictional primary markets. In particular, lengthened maturities make the originally active fiscal stance relatively less active (or more passive) at critical boundary conditions. This shift renders fiscal policy susceptible to self-fulfilling, belief-driven equilibria (sunspots), creating multiple potential paths for the price level that satisfy local equilibrium conditions. Given that fiscal revenue trajectory is exogenous to the central bank, passive monetary policy conditions today's price level solely on expected future prices, resulting in equilibrium indeterminacy and macroeconomic instability.

Importantly, these conclusions critically depend on the presence of primary market frictions. If no friction exists, i.e. $\lambda = 0$, the impact of debt maturity on the requirements of fiscal policy disappears, reverting to classical results established by Leeper (1991). If the market is frictional, such frictions and debt maturities interactively modify permissible ranges for fiscal policy in opposite directions. As shown in Figure 4, longer maturities reduce frictional costs, thereby shifting the determinacy boundary outward under active monetary policy but inward under passive monetary policy. These results underscore the need to reconsider standard stability selection criteria and emphasize internal fiscal authority coordination—particularly regarding tax adjustments and debt maturity management—in economies characterized by frictional primary markets.

4.3 Local stability trilemma

In our model, policymakers typically have three policy instruments available to preserve debt sustainability and macroeconomic stability: (1) debt maturity extension, (2) monetary accommodation allowing higher inflation, and (3) fiscal adjustments via taxation. Each policy individually reshapes the boundary conditions for local equilibrium determinacy, as detailed in Proposition 2. However, when primary market frictions are present, achieving local stability implies that at most two of these instruments can simultaneously move in the same direction, while the third must shift inversely to sustain fiscal solvency and price determinacy. This critical insight leads to the following proposition:

Proposition 5 (Monetary-Fiscal Policy Interactions Trilemma)**.** *To avoid local instability, policymakers cannot simultaneously implement the following three measures:*

1. *Accommodating higher inflation to erode real debt,*
2. *Intensifying fiscal adjustment through higher taxation,*
3. *Lengthening the average maturity of public debt.*

At most two measures can be pursued simultaneously; adopting all three inevitably induces local instability.

Figure 4 illustrates the policy trilemma region. The area delineated by the red boundary signifies the transition from determinacy to indeterminacy. To escape from this trilemma, policymakers must reverse the direction of at least one measure. For instance, shortening debt maturities can diminish the indeterminacy region, restoring local equilibrium stability. Alternatively, reducing fiscal responsiveness does not alter the size of the trilemma area but makes fiscal policy more active, providing a unique fiscal anchor and eliminating sunspot equilibria. Finally, adopting an active monetary stance, wherein the monetary authority aggressively targets inflation and compels fiscal policy to ensure debt sustainability, can shift equilibrium into a determinacy region above the trilemma area, restoring stability.

The underlying logic of Proposition 5 proceeds as follows. First, monetary accommodation, characterized by a passive stance that permits higher inflation to erode real debt, eases short-run fiscal pressures. However, this approach weakens inflation anchoring, raising susceptibility to self-fulfilling inflationary expectations. Consequently, determinacy of the price level becomes tenuous, increasing local equilibrium instability risks.

Second, intensified fiscal adjustments, primarily through higher taxation, signal a credible commitment to debt sustainability. Aggressive fiscal tightening mitigates equilibrium instability by reassuring markets of future solvency. However, overly stringent fiscal policy can depress economic activity and raise borrowing costs, partially offsetting stability gains. Such excessive fiscal tightening may inadvertently contribute to weaker fundamentals and elevated inflation expectations, potentially fostering price indeterminacy.

Third, extending debt maturities reduces immediate refinancing risks and stabilizes fiscal outcomes by distributing repayment obligations over a longer horizon. While beneficial from saving the auction costs at the primary market, longer maturities increase bond price sensitivity to future policy expectations and interest rate fluctuations. Coupled with monetary accommodation that allows unchecked inflation, markets may perceive debt sustainability as reliant on inflation-driven erosion rather than credible fiscal adjustment. This perception enhances macroeconomic instability through heightened vulnerability to self-fulfilling expectations.

Collectively, these mechanisms clarify why simultaneous implementation of all three measures inevitably leads to local equilibrium indeterminacy. Specifically, combining fiscal tightening, monetary accommodation, and longer debt maturities conveys mutually reinforcing signals about the uncertainty of future fiscal revenue path and expectations on future inflation. This ambiguity generates infinitely many ways to balance the government budget, fostering conditions conducive to price-level indeterminacy.

5 Discussion: Insights from Leeper (1991)

5.1 An endowment economy In order to illustrate the intuition behind our main conclusions, we revisit this issue in the settings in Leeper (1991). Consider a cashless endowment economy populated by identical, infinitely-lived households who discount future utility at the constant factor $\beta \in (0, 1)$. Each period, households receive and fully consume a perishable endowment y, while paying lump-sum taxes τ_t to the government. Additionally, households can trade one-period discount bonds B_t, issued directly by the government at a gross nominal interest rate R_t, which exist in zero net supply, as well as long-term government bonds D_t with decaying coupon structure, traded with the primary dealers in the secondary market at the price Q_t^s.

The government employs the discount bonds solely for monetary policy purposes and finances a fixed and strictly positive level of expenditures g through lump-sum taxes and issuance of long-term bonds. This long-term bonds comprise perpetuities characterized by exponentially decaying coupon payments: one unit bond issued at date t provides a coupon payment of δ^{s-1} units at date $t+s$, where $\delta \in [0, 1]$ controls the average maturity structure. Consequently, in period t, the government raises $Q_t D_t$ through primary issuance and pays $(1 + \delta Q_t) D_{t-1}$ to redeem bonds issued in the previous period on average.

Primary dealers initially purchase newly issued government bonds and subsequently resell them to households in the secondary market within the same period.This intermediation process is subject to a frictional liquidity cost, as primary dealers face difficulties in quickly liquidating the bonds they have purchased at auction. Consequently, they are unwilling to pay the full secondary market price. This liquidity cost is captured by an exogenous spread ψ, which links the primary and secondary market bond prices through the following relationship:

$$Q_t^s = Q_t + \psi, \tag{5.1}$$

where Q_t is the primary issuance price. It is worth noting that, we model ψ explicitly as an exogenous friction without structural microfoundations in this model for convenience. However, this simplification doesn't alters the key insights in the full-fledged model.

In equilibrium, the non-policy block is governed by the following three equations:

$$\pi_{t+1} = \beta R_t, \tag{5.2}$$

$$R_t = \frac{1 + \delta(Q_{t+1} + \psi)}{Q_t + \psi}, \tag{5.3}$$

$$\frac{Q_t D_t}{P_t} + \tau_t = (1 + \delta Q_t) \frac{D_{t-1}}{P_t} + g, \tag{5.4}$$

For fiscal and monetary authorities, we assume that they each follow a simple feedback rule as in Leeper (1991):

$$s_t = \gamma_0 + \gamma(d_{t-1} - d^*), \tag{5.5}$$

$$R_t = \alpha_0 + \alpha(\pi_t - \pi^*), \tag{5.6}$$

where we assume $s_t = \tau_t - g$ as primary surplus and $d_t = Q_t B_t / P_t$ as the real market value for long-term bonds. α_0 and γ_0 affect the deterministic steady state but play no role in the following analysis while α and γ are monetary and fiscal policy parameters that control the equilibrium determinacy. Meanwhile, d^* and π^* denote long-run targets of real debt and inflation. Combining the above policy rules with the non-policy block, equation (5.2)-(5.6) characterize an equilibrium.

For comparison purpose, we first consider a baseline scenario where the bond maturity is zero and there is no friction on the secondary market, i.e., $\delta = 0$ and $\psi = 0$. Under this assumption, the government uses one-period bonds to balance its budget. After linearization, the system can be simplified into the following two equations, which replicates the results in Leeper (1991)[1]:

$$\tilde{\pi}_{t+1} = \alpha\beta\tilde{\pi}_t, \tag{5.7}$$

$$\tilde{d}_t = (\beta^{-1} - \gamma)\tilde{d}_{t-1}, \tag{5.8}$$

where $\tilde{\pi}_t$ and $\tilde{d}_{t-1}$ are variables deviating from their steady-state values.

Referring to the traditional classification of active/passive policies in Leeper (1991), we first define the following definition:

Definition 2 (The Stance of Monetary and Fiscal Policies)**.** *According to Leeper (1991), we define the stance of monetary and fiscal policies as follows:*

1. *Monetary policy is defined as active (passive) when $\alpha\beta > 1(\alpha\beta < 1)$;*

2. *Fiscal policy is defined as active (passive) when $\beta^{-1} - \gamma > 1(\beta^{-1} - \gamma < 1)$.*

Since the dynamic system consists of one predetermined variable and one jump variable, local equilibrium determinacy requires one stable root and one explosive root. According to the above definition, we draw the following lemma:

Lemma 2 (The Frictionless Local Stability Requirements)**.** *When government debt maturity is zero and primary market frictions are absent, locally stable equilibria occur exclusively*

[1]See B.1 for derivations in detail.

*under the traditional active/passive policy combination. Active monetary and fiscal policies jointly yield **explosive** equilibria, whereas passive monetary and fiscal policies jointly yield **indeterminate** equilibria.*

Lemma 2 acts as a benchmark interpreting the canonical active/passive prescriptions for local stability as in Leeper (1991).

5.2 The (ir)relevance of bond maturities We next consider a setting in which the government finances its budget through long-term bonds and explicitly accounts for bond maturity. By allowing $\delta > 0$ while maintaining $\psi = 0$ (thereby ruling out primary market frictions), the maturity terms in (5.4) and (5.3) cancel out.[1] As a result, the system collapses to the baseline dynamic system characterized by equation (5.7) and (5.8), yielding the following proposition:

Proposition 6 (The Irrelevance of Bond Maturities)**.** *In the absence of primary market frictions (i.e., $\psi = 0$), the local equilibrium determinacy is invariant to bond maturity (δ). The canonical active/passive prescription in Leeper (1991) continues to hold.*

The intuition behind this irrelevance rests on the no-arbitrage condition (5.3). Under the frictionless markets without risks, the yield to maturity on long-term bonds must equal that of discount bonds, whose interest rate is set directly by the monetary authority in this model. Consequently, extending average bond maturity merely alters the path of market price $\{Q_t\}_{t=0}^{\infty}$; the bond's overall return (which is also the fiscal cost) remains unchanged unless the monetary policy shifts the market interest rates. This mechanism grants monetary authorities full leverage over the cost of servicing government debt and, in turn, over the implementation of fiscal policy. Thus, when the fiscal instrument is the market value of debt, the canonical prescriptions in Leeper (1991) remain robust to changes in bond maturity, allowing bond maturities to be chosen arbitrarily without affecting equilibrium determinacy.

However, will the irrelevance property of bond maturity still holds while the secondary market is frictional? To answer this question, we assume a scenario where δ and ψ are both positive. In Appendix B.2, we have proved that the linearized dynamic system is characterized approximately by the following equations:

$$\tilde{\pi}_{t+1} = \alpha\beta\tilde{\pi}_t, \tag{5.9}$$

$$\tilde{d}_t = v(\delta,\psi)d^*\tilde{\pi}_t + \left\{[\beta^{-1} + v(\delta,\psi)] - \gamma\right\}\tilde{d}_{t-1}, \tag{5.10}$$

[1] See Appendix B.2 for detailed derivations

where we define $v(\delta, \psi)$ is a function of maturity δ and liquidity costs ψ, which represents a spread between primary and secondary markets for long-term bonds. We have shown in the appendix that the spread v increases with ψ and decreases with δ. Another property is that $v(\delta, 0) = 0$, which means that the maturity only matters when there are liquidity costs in the primary market.

It is easy to show that the Jacobian matrix is a lower-diagonal matrix, where the eigenvalues are $\alpha\beta$ and $[\beta^{-1} + v(\delta, \psi)] - \gamma$. Since we have one forward-looking and one predetermined variables, the above eigenvalues need to be one unstable root and one stable root. Thus, we have the following proposition:

Proposition 7 (The Role of Primary Market Frictions)**.** *While considering primary market frictions for primary dealers (i.e.,$\psi > 0$), the canonical active/passive policy prescriptions may not always deliver locally stable equilibria. In particular, while extending bond maturities (δ) under liquidity costs, local stability requires less passive fiscal policies (for active monetary policies) or more active fiscal policies (for passive monetary policies).*

In order to prove the above proposition, we start by rewriting the government's budget constraint in recursive form. Ensuring the transversality condition holds, which guarantees that agents willingly hold public debt, implies the present value of outstanding debt must converge to zero. Iterating forward and imposing this constraint on equation (5.4) yields the intertemporal budget constraint in real terms:

$$d_t = \mathbb{E}_t \sum_{s=0}^{\infty} \underbrace{\left\{ \prod_{j=0}^{s} \frac{\pi_{t+j+1}}{R_{t+j} + v(\delta, \psi)} \right\}}_{\text{Stochastic Discount Factor (SDF)}} \underbrace{(\tau_{t+s+1} - g)}_{\text{Primary Surplus}} \tag{5.11}$$

In this formulation, the an additional component $v(\delta, \psi)$ functions as a premium within the stochastic discount factor. With constant fiscal revenues, lengthening bond maturities reduces liquidity costs for primary dealers, increasing the SDF and thus raising the present value of future primary surpluses. To maintain the government budget constraint—particularly when real debt values are predetermined—fiscal policy must become more active or less passive, implying reduced tax responsiveness to debt variations. Ignoring such adjustments would yield surplus expectations persistently exceeding debt obligations, causing belief-driven (sunspot) equilibria.

Equation (5.11) also illustrates the previously discussed fiscal-monetary policy interactions trilemma (Proposition 5). To uphold government budget constraints, policymakers might consider inflating away debt, intensifying fiscal adjustments via taxation, or lengthening bond maturities. Each policy independently expands fiscal space, thereby increasing the RHS. Yet, as the current debt obligation (LHS) is predetermined, simultaneous implementation of all three measures violates equilibrium conditions. Only two policies can jointly move in the same direction, necessitating inverse adjustment of the third.

6 Conclusion

This paper begins by noting that in Leeper (1991)'s active/passive framework, debt maturity has traditionally not been a focal point. While extensive related literature exists on the analysis of monetary policy rules and fiscal revenue rules, the use of public debt maturity management has not received similar attention. As demonstrated in this paper, the impact of maturity variations on macroeconomic stability can be entirely offset by targeting the market value of debt through surplus adjustments. This property, which renders debt maturity irrelevant, has led to the common practice of substituting long-term bonds with one-period discount bonds, as it simplifies the analytical derivations for determining local determinacy requirements. Consequently, fiscal policy is often treated as a whole, represented solely by a fiscal (tax) rule. The main contribution of this paper is to show, within a simple and tractable framework, how this logic must be revised in an environment where primary bond market frictions are present. This revision implies that not only must monetary and fiscal authorities be coordinated, but internal coordination within fiscal authority itself is also crucial.

Although extending debt maturity is generally beneficial for maintaining macroeconomic stability in Regime M, shortening maturity can pose significant risks to debt sustainability. This particular issue has not been extensively explored in the current paper, primarily because recent global trends have focused on lengthening rather than shortening debt maturities. Nonetheless, our analysis implies that if fiscal authorities choose to shorten debt maturities, the feasible policy space within Regime M will shrink accordingly, increasing the likelihood of debt sustainability problems. Therefore, our findings lay a useful foundation for future discussions on proper policy interactions for fiscal consolidation.

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A Proofs for the New-Keynesian framework

A.1 Steady States Since our model integrates a textbook New-Keynesian framework for its non-fiscal elements, referring closely to Galí (2015), our focus here is exclusively on deriving the fiscal components central to our analysis. The fiscal part of our model is characterized by four fundamental equations:

First, the no-arbitrage condition linking short-term and long-term bonds traded in the secondary market is given by:

$$R_t = \frac{1 + \delta Q^s_{t+1}}{Q^s_t}. \tag{A.1}$$

Second, primary dealers' optimization yields the equilibrium spread between primary and secondary markets:

$$Q^s_t = Q_t + \psi_t. \tag{A.2}$$

Third, liquidity friction assumptions imply the following functional form for transaction costs:

$$\psi_t = \psi_0 d_t^{\psi_1}, \tag{A.3}$$

where ψ_0, ψ_1 are parameters governing liquidity costs.

Finally, the government budget constraint in real terms takes the form:

$$d_t + s_t = \frac{1 + \delta Q^s_t}{Q^s_{t-1}} \frac{d_{t-1}}{\pi_t}, \tag{A.4}$$

with $d_t \equiv \frac{Q_t D_t}{P_t}$ representing the real market value of outstanding long-term bonds, and s_t denoting the real primary surplus.

In a zero-inflation steady state ($\pi = 1$), equation (A.1) simplifies to yield:

$$Q^s = \frac{1}{\beta^{-1} - \delta}, \tag{A.5}$$

leveraging Fisher's steady-state condition $R = \beta^{-1}\pi$.

To facilitate analysis, we introduce the variable $\alpha_t = Q^s_t / Q_t$, representing the gross premium ratio between primary and secondary markets. Consequently, equation (A.2) is conveniently expressed as:

$$Q = \frac{Q^s}{\alpha}, \quad \text{and} \quad \psi = Q(\alpha - 1). \tag{A.6}$$

This substitution allows us to perform qualitative and quantitative assessments more efficiently, particularly because α is directly observable in empirical contexts.

Using the steady-state government budget constraint (A.4), we derive the relationship between steady-state surpluses s and the long-run debt target d^* as:

$$s = \left(Q^{-1} + \delta - 1\right) d^*, \tag{A.7}$$

where steady-state surplus s explicitly depends on the target debt level d^*.

Finally, from liquidity cost specification (A.3), we obtain a unique steady-state relationship between the liquidity cost and the long-term debt target:

$$\psi = \psi_0 (d^*)^{\psi_1}, \tag{A.8}$$

thereby uniquely determining the equilibrium debt level through liquidity cost parameters ψ_0, ψ_1.

A.2 Log-Linearizations for fiscal part After log-linearization, equation (A.1)-(A.3) can be summarized as follows:

$$\phi_\pi \hat{\pi}_t + \hat{Q}^s_t = \beta\delta \hat{Q}^s_{t+1}, \tag{A.9}$$

$$\hat{Q}_t = \alpha \hat{Q}^s_t - \lambda \hat{\psi}_t, \tag{A.10}$$

$$\hat{\psi}_t = \zeta \hat{d}_t, \tag{A.11}$$

where we substitute $\hat{R}_t$ with $\phi_\pi \hat{\pi}_t$ according to the monetary policy rule, and $\zeta = \psi_1 / d^*$ represents the intensity of liquidity costs.

The fourth equation (B.3) needs a little bit carefulness. First, we rewrite it in the form of difference, i.e., $\tilde{x} = x_t - x$:

$$\tilde{d}_t + \tilde{s}_t = \frac{1 + \delta Q_t}{Q_{t-1}} \frac{d_{t-1}}{\pi_t} - \frac{1 + \delta Q}{Q} d^* \tag{A.12}$$

where we linearize the equation around $\pi = 1$. Then, we define the linearized fiscal variables as $\hat{x}_t = \frac{x_t - x}{x} = \frac{\tilde{x}}{Y}$, which means the ratio over GDP. Hence, we have:

$$\hat{d}_t + \hat{s}_t = (1/Q + \delta)\hat{d}_{t-1} - (1/Q + \delta) d^* \hat{\pi}_t + \delta d^* \hat{Q}_t - (1/Q + \delta) d^* \hat{Q}_{t-1}. \tag{A.13}$$

Utilizing the fiscal rule $\hat{s}_t = \phi_d \hat{d}_{t-1}$, we finally derive:

$$\hat{d}_t = (1/Q + \delta - \phi_d)\hat{d}_{t-1} - (1/Q + \delta)d^* \hat{\pi}_t + \delta d^* \hat{Q}_t - (1/Q + \delta)d^* \hat{Q}_{t-1}. \tag{A.14}$$

The next step is to substitute equation (A.10) and (A.11) into the above function to eliminate $\hat{Q}_t$ and $\hat{\psi}_t$. Then, we get:

$$-[\alpha/\beta - (\alpha-1)\delta]d^* \alpha \hat{Q}^s_t + \gamma_0 \hat{d}_t = [\alpha/\beta - (\alpha-1)\delta]d^* \hat{\pi}_{t+1} - \delta d^* \alpha \hat{Q}^s_{t+1} + \gamma_1 \hat{d}_{t+1}. \tag{A.15}$$

where $\gamma_0 = [1 + \zeta d^*(\alpha - 1)][\alpha/\beta - (\alpha - 1)\delta]$ and $\gamma_1 = 1 + \delta\zeta d^*(\alpha - 1)$.

Finally, we rewrite the intertemporal government budget constraint by substituting α with $1 + \lambda$, we have:

$$-\gamma(1+\lambda)\hat{Q}^s_t + \gamma_0 \hat{d}_t = \gamma \hat{\pi}_{t+1} - \delta d^*(1+\lambda)\hat{Q}^s_{t+1} + \gamma_1 \hat{d}_{t+1}, \tag{A.16}$$

where $\gamma = [(1+\lambda)/\beta - \lambda\delta]d^*$ is an auxiliary parameter and $\gamma_0 = (1 + \zeta\lambda d^*)[(1 + \lambda)/\beta - \lambda\delta] - \phi_d$ and $\gamma_1 = 1 + \zeta\delta\lambda d^*$.

There are two things worth noting. First, we use the steady-state relation $1/Q + \delta = \alpha/\beta - (\alpha - 1)\delta$ to substitute Q with parameters. Second, we move the equation one period forward to facilitate the local equilibrium determinacy analysis. This method refers to Bullard and Mitra (2002), which proves that this movement doesn't change the determinacy properties.

A.3 Derivations for dynamic system

The derivation proceeds as follows[1]:

By substituting equations (3.23), (3.24), and (3.29), we derive a simplified dynamic IS equation:

$$\hat{y}_t = \hat{y}_{t+1} - \frac{1}{\sigma}(\phi_\pi \hat{\pi}_t - \hat{\pi}_{t+1}), \tag{A.17}$$

where $\hat{g}_t$, the exogenous government expenditure, is omitted for convenience. This exclusion does not affect the determinacy properties of the system.

Equation (3.25), together with (A.17), representing the non-fiscal component of the New Keynesian model, remains unchanged:

$$\hat{\pi}_t = \kappa \hat{y}_t + \beta \hat{\pi}_{t+1}. \tag{A.18}$$

Next, we substitute equation (3.29) into (3.26) to obtain the law of motion for Q^s_t:

[1] The exogenous process $\hat{g}_t$ and $\hat{v}_t$ are omitted for convenience. This exclusion does not affect the determinacy properties of the system.

$$\hat{Q}_t^s + \phi_\pi \hat{\pi}_t = \beta\delta \hat{Q}_{t+1}^s, \tag{A.19}$$

where $\beta\delta$ is the coefficient deciding the eigenvalue for Q_t.

Finally, we rewrite the intertemporal government budget constraint by combining equation (3.27) and (3.32):

$$-\gamma(1+\lambda)\hat{Q}_t^s + \gamma_0 \hat{d}_t = \gamma \hat{\pi}_{t+1} - \delta d^*(1+\lambda)\hat{Q}_{t+1}^s + \gamma_1 \hat{d}_{t+1}, \tag{A.20}$$

where $\gamma = [(1+\lambda)/\beta - \lambda\delta]d^*$ is an auxiliary parameter and $\gamma_0 = (1+\zeta\lambda d^*)[(1+\lambda)/\beta - \lambda\delta] - \phi_d$ and $\gamma_1 = 1 + \zeta\delta\lambda d^*$are coefficients controlling the sustainability of government debt.

Together, equations (A.17)-(A.20) form a system of four linear equations. The system involves four jump variables $\hat{y}_t, \hat{\pi}_t$ and $\hat{Q}_t$ with one predetermined variables $\hat{d}_t$. For the system to satisfy the determinacy condition, it must exhibit exactly three explosive roots and one stable root.

For the analysis of uniqueness, we move Equation (A.20) one time period forward and rearrange the above functions into the following vector form:[1]

$$AX_t = B\mathbb{E}_t X_{t+1}, \tag{A.21}$$

where the state vector is $X_t = \left[\hat{y}_t, \hat{\pi}_t, \hat{Q}_t^s, \hat{d}_t\right]'$.

A.4 Proofs for Lemma 1 To establish Lemma 1, we first examine the submatrix $A_{11}^{-1}B_{11}$. It is straightforward to recognize this submatrix as characterizing a standard New Keynesian framework. Thus, according to Bullard and Mitra (2002), the condition $\phi_\pi > 1$ emerges with two unstable roots as the familiar Taylor principle necessary for equilibrium determinacy. Otherwise, its a saddle with one stable root and one unstable root.

The remaining eigenvalues are determined by the submatrix $A_{22}^{-1}B_{22}$, where both A_{22} and B_{22} are lower-triangular matrices defined as follows:

$$A_{22} = \begin{bmatrix} 1 & 0 \\ -\gamma(1+\lambda) & \gamma_0 \end{bmatrix}, \tag{A.22}$$

and

$$B_{22} = \begin{bmatrix} \beta\delta & 0 \\ -\delta d^*(1+\lambda) & \gamma_1 \end{bmatrix}. \tag{A.23}$$

[1]This method refers to Bullard and Mitra (2002)

Consequently, the eigenvalues of interest are simply the ratios of the corresponding diagonal elements of B_{22} and A_{22}, yielding:

$$e_3 = \beta\delta < 0, \quad \text{and} \quad e_4 = \frac{\gamma_1}{\gamma_0} = \frac{1 + \delta\zeta\lambda d^*}{[(1+\lambda)\beta^{-1} - \lambda\delta]\,(1 + \zeta\lambda d^*) - \phi_d}. \tag{A.24}$$

A.5 Proofs for Definition 2 With zero primary market frictions, it follows immediately that $\gamma_0 = \beta^{-1} - \phi_d$ and $\gamma_1 = 1$. Thus, the condition for having exactly one stable and one unstable eigenvalue in the matrix $A_{22}^{-1}B_{22}$ is:

$$e \equiv \frac{\gamma_1}{\gamma_0} = \frac{1}{\beta^{-1} - \phi_d} > 1. \tag{A.25}$$

Otherwise, matrix $A_{22}^{-1}B_{22}$ exhibits two unstable eigenvalues.

The dynamic stability of the full system critically depends on the monetary and fiscal policy parameters ϕ_π and ϕ_d. Under active monetary policy ($\phi_\pi > 1$), the block $A_{11}^{-1}B_{11}$ contains two unstable eigenvalues. Therefore, fiscal policy must ensure one stable and one unstable eigenvalue in block $A_{22}^{-1}B_{22}$ to achieve local equilibrium determinacy. Conversely, under passive monetary policy ($\phi_\pi < 1$), the block $A_{11}^{-1}B_{11}$ produces one stable and one unstable root, necessitating an relatively active fiscal stance to guarantee determinacy. Both configurations yield three unstable eigenvalues and one stable eigenvalue, satisfying Blanchard and Kahn (1980) conditions.

A.6 Proofs for Proposition 1 Given positive primary market frictions, the parameters are explicitly derived as:

$$\gamma_0 = (1 + \zeta\lambda d^*)\left[(1+\lambda)\beta^{-1} - \lambda\delta\right] - \phi_d; \quad \gamma_1 = 1 + \zeta\delta\lambda d^*. \tag{A.26}$$

Substituting these expressions directly into the eigenvalue formula of matrix $A_{22}^{-1}B_{22}$ yields:

$$e = \frac{\gamma_1}{\gamma_0} = \frac{1 + \delta\zeta\lambda d^*}{[(1+\lambda)\beta^{-1} - \lambda\delta]\,(1 + \zeta\lambda d^*) - \phi_d}. \tag{A.27}$$

The sub-matrix $A_{22}^{-1}B_{22}$ exhibits one stable and one unstable root if and only if $e > 1$. Otherwise, it contains two unstable roots. Similar to the frictionless settings, the parameters ϕ_π and ϕ_d govern two eigenvalues of the dynamic system. Achieving three unstable eigenvalues and one stable eigenvalue satisfies the conditions outlined in Blanchard and Kahn (1980).

B Proofs for the Endowment Economy

B.1 Derivations for the equilibrium without primary market frictions

First, we substitute monetary policy (5.6) into the Euler's equation (5.2). Thus, we have:

$$\pi_{t+1} = \alpha\beta\pi_t, \tag{B.1}$$

Linearize this equation aroud steady states, we have:

$$\tilde{\pi}_{t+1} = \alpha\beta\tilde{\pi}_t. \tag{B.2}$$

Then, it is obvious that the government budget constraint (5.4) turns to be:

$$d_t + s_t = \frac{d_t}{Q_t\pi_t}, \tag{B.3}$$

and the no-arbitrage condition (5.3) degenerates into:

$$R_t = \frac{1}{Q_t}. \tag{B.4}$$

Therefore, we combine these two equations with fiscal policy (5.5) and linearize them, we have the other first-order difference equation:

$$\tilde{d}_t = (\beta^{-1} - \gamma)\tilde{d}_{t-1}. \tag{B.5}$$

Thus, equation (B.2) and (B.5) constitute the equilibrium system.

B.2 Derivations for the equilibrium under under primary market frictions

When $\psi = 0$, the no-arbitrage condition (5.3) becomes:

$$R_t = \frac{1 + \delta Q_{t+1}}{Q_t}, \tag{B.6}$$

Plug it into the government budget constraint (5.4), we have:

$$d_t + s_t = \frac{R_t d_t}{\pi_t}, \tag{B.7}$$

where δ is eliminate and the government budget turns to be the same as (B.3).

Next, we derive the equilibrium conditions for $\psi > 0$ and $\zeta > 0$.

Since the recursive function for $\tilde{\pi}_t$ is unchanged, we only focus on the derivations for $\tilde{d}_t$.

First, we rewrite the no-arbitrage condition as following:

$$R_t = \frac{1 + \delta Q_{t+1} + \delta\psi}{Q_t + \psi}, \tag{B.8}$$

and rearrange the equation in the form below:

$$\begin{aligned} \frac{1 + \delta Q_{t+1}}{Q_t} &= \left(R_t - \frac{\delta\psi}{Q_t + \psi} \right) \cdot \frac{Q_t + \psi}{Q_t} \\ &= R_t + \frac{R_t \psi}{Q_t} - \frac{\delta\psi}{Q_t} \\ &= R_t + \frac{(R_t - \delta)\psi}{Q_t}. \end{aligned}$$

In a neighborhood of the steady state, the above return rate can be approximated as:

$$R_t + \frac{(R_t - \delta)\psi}{Q_t} \approx R_t + \frac{(R - \delta)\psi}{Q} = R_t + v(\delta, \psi), \tag{B.9}$$

where we define $v(\delta, \psi)$ is a function of maturity and liquidity costs, which represents a spread between primary and secondary markets for long-term bonds. It can be easily shown that the spread v increases with ψ and decreases with δ.

Then, substitute it into the government budget constraint:

$$d_t + s_t = \left[R_{t-1} + v(\delta, \psi) \right] \frac{d_{t-1}}{\pi_t} \tag{B.10}$$

After linearization around the zero inflation steady states and plug in the fiscal rule, we can finally derive equation (5.10).