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Research Paper

The effects of a money-financed fiscal stimulus under fiscal stress[☆]

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ABSTRACT

This paper studies the local determinacy requirements and the effects of a money-financed fiscal stimulus under fiscal stress in a canonical New Keynesian model. We consider three alternative monetary policies and find that the money-financed policy adopted in Galí (2020) to keep zero-debt-increase (ZDI) leads to an unsustainable debt path, while introducing a debt growth target restores stability. A debt-targeting rule (DT) generates smaller instantaneous multipliers and larger cumulative multipliers with respect to the ZDI, whereas a mixed-targeting rule (MT) that takes both debt and inflation into consideration exaggerates the trade-off between short-run and long-run multipliers. Deficit financing decomposition shows that, relative to seigniorage, inflation and changes in the stochastic discount factor play more important roles. Moreover, welfare analysis implies that a sluggish money financing scheme causes extra welfare loss. Finally, we quantify the effects of money-financed fiscal measures in a COVID recession.

1. Introduction

Many economies have undertaken large scale fiscal stimulus in response to the recent pandemic. This causes government debt to surge in the last two years, reaching a five-decade high at 97% of GDP globally. While the economic outlook is still weak and uncertain, the debt-sustainability issue will significantly constrain governments' ability to employ fiscal measures to boost economies when in need. Therefore, an urgent question for policy makers around the world is how to finance fiscal stimulus in an era of fiscal stress.

Some unconventional policy options have been advocated in policy and academic circles. One of the most influential proposals suggests that countries with sovereign currency debt can overcome the hurdle of fiscal stress by printing money to finance fiscal stimulus.[1] They argue that government in these countries could print as much money as they need to spend, without causing an increase in the public debt level and higher taxes. Although it sounds appealing, a major criticism to this proposal is that money-financed fiscal stimulus may generate high inflation, which erodes the value of seigniorage revenue from money creation.[2] However, the debate so far lacks a formal quantitative assessment of the effects of money-financed fiscal stimulus under fiscal stress.[3] This paper fills the gap.

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[1] This idea at least dated back to Friedman (1948) and was recently developed by Bernanke et al. (2002), Reichlin et al. (2013), and Wray (2015).

[2] See for example Barro (2009) and Mankiw (2020).

[3] A notable exception is Galí (2020) that examines the effects of money-financed fiscal stimulus in a New Keynesian model. Our framework is almost identical to his, but we perform our analysis in the presence of fiscal stress, and with more general monetary policies. We provide more detailed comparison with Galí (2020) in Section 3.4.

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Our analytical framework is a canonical New Keynesian model with real money balance and government budget. In particular, we focus our analysis in the presence of fiscal stress, which is defined following Leeper and Walker (2012) as the fiscal authority unwilling or unable to collect additional tax revenue in response to rising government debt, and therefore the primary surplus follows an exogenous process.[4] We assume the monetary authority employs money supply instead of nominal interest rates as the policy instrument, and consider the following three money supply rules. First, the narrowest specification of money-finance policy that strictly adjusts seigniorage to keep real debt level unchanged in every period as in Galí (2020). Second, a money supply rule that targets government debt growth. This rule allows the monetary authority to react to debt growth and, as a result, partially finances fiscal stimulus through money creation. The introduction of government debt as a monetary policy target is also considered by Wallace (1984), among others. [5] Unlike the first money supply rule we consider, the second policy rule is more flexible that allows government debt to temporarily deviate from its long-run level. Third, a combination of both government debt growth and inflation as monetary policy targets. This money growth rule takes price stability into consideration besides debt stabilization, and is adopted by Schabert (2006).

Based on these specifications, we analytically characterize the determinacy requirements across different monetary policies and examine the impacts of a money-financed fiscal stimulus on macroeconomic dynamics, in particular the fiscal multipliers and debt-output ratios. Note that although many studies have examined the stability property of non-Ricardian fiscal policy regime with various monetary policy regimes, few has evaluated how the money-financing scheme affects the effectiveness of fiscal stimulus and the role played by seigniorage.

Four results emerge. First, we show that although a constant real debt level could be an equilibrium outcome, Galí (2020)'s monetary policy leads to an unsustainable debt path because it cannot anchor fiscal expectations alone without fiscal backing. Introducing a feedback component to debt growth in Galí (2020)'s monetary policy restores local determinacy and produces identical macroeconomic dynamics as in Galí (2020).

Second, a money growth policy that reacts to debt growth at a rate greater than the product of the steady state interest rate and income velocity of money yields local determinant equilibrium. This feedback rule allows the debt level to temporarily deviate from its long-run value, but is anchored in expectation. In such case, the instantaneous fiscal multiplier becomes lower because part of the deficits are temporarily financed by debt. But we also find that the cumulative fiscal multiplier increases since additional future money creation in response to higher interests of debt pushes up inflation and presses down real interest rates in the long run. These effects are greater with more aggressive reaction coefficients. Moreover, our results suggest that the government spending multiplier is larger than the tax multiplier, and is always above unity.

Third, we find that under a mixed targeting rule on inflation and debt, local equilibrium determinacy requires that when the money supply more aggressively targets inflation, it has to be more responsive to debt growth. Regarding policy effects, the trade-off between short-run and long-run multipliers exaggerates. In particular, the more hawkish monetary policy is to contain inflation, the lower instantaneous and higher long-run fiscal multipliers are. This is because tighter inflation control restricts the decline of real interest rates, and hence the aggregate demand in the short run. In the longer term, debt interest burden rises due to higher real interest rates. Those debt obligations are paid off by higher inflation, which stimulates future aggregate demand.

Fourth, we perform a decomposition of financing sources to quantitatively measure the contributions among seigniorage, current inflation and stochastic discount factor. We find that their relative importance varies across different monetary and fiscal policy configurations. Seigniorage is never the main source of a money-financed fiscal stimulus. Changes in the stochastic discount factor contributes the most in the case of tax cuts while current inflation is the most important contributor in the case of government spending increases. These results highlight the inflationary pressure after implementing the money-financed fiscal stimulus under fiscal stress and the impotence of monetary policy to generate significant real seigniorage revenue.

Welfare analysis shows that money-financed policies may be costly in welfare terms since they generates higher inflation. The DT and MT rules permit a temporary increase in debt level and lead to extra welfare loss relative to the ZDI policy. This implies that a sluggish money financing policy is less desirable.

Furthermore, we examine the performance of these policies against a COVID recession by introducing pandemic-related supply and demand shocks. We find that money-financed fiscal stimulus can always generate fiscal multipliers larger than unity, and improve social welfare.

The remainder of the paper is organized as follows. Section 2 discusses the related literature. Section 3 describes the model setup and derives the requirements of local equilibrium determinacy under different monetary policy rules. Section 4 quantitatively assesses the short-run and long-run effects of a money-financed fiscal stimulus, and decomposes the contribution of deficit financing among various sources. This section also conduct welfare analysis and a case study against pandemic shocks. Section 5 summarizes the main findings and offers concluding remarks.

[4] Leeper and Walker (2012) assumes that the fiscal policy remains non-Ricardian forever after reaching a fiscal limit while Leeper (2010) permits fiscal policy to switch between normal and stress regimes. We follow the former definition for tractability.

[5] See for example Schreft and Smith (2000), Bhattacharya and Kudoh (2002), and Kaas and Weinrich (2003).

2. Literature review

Our paper contributes to the literature that studies the effects of money-financed budget deficits. Friedman (1948) and Friedman (1969) pioneered this policy discussion and coined the term "helicopter money" that is considered as a classical example of budget deficit monetarization. This argument is future elaborated by Bernanke et al. (2002), Turner (2013), Reichlin et al. (2013), and Turner (2017). However, their analyses are qualitative and informal in nature. Palivos and Yip (1995) and Espinosa-Vega and Yip (2002, 1999) quantitatively assess the effectiveness of seigniorage-financed government spending, whereas Auerbach and Obstfeld (2005), Buiter (2014) and English et al. (2017) analyze the effects of a money-financed tax cut. Besides, Galí (2020), Tsuruga and Wake (2019) and Punzo and Rossi (2023) discuss the pros and cons of a money-financed fiscal stimulus relative to a debt-financed stimulus. In particular, our work is closely related to Galí (2020), who employs a textbook New-Keynesian model to evaluate the effects of a money-financed fiscal stimulus. We adopt his framework, but differ from his analysis along two critical dimensions. First, he maintains the assumption of sufficient fiscal space whereas we examine the economic consequences without it. Second, in contrast to a single monetary policy adopted in Galí (2020), we consider richer and more general policy specifications and find that a debt-targeting rule can yield local determinacy and produce higher fiscal multipliers.

Another relevant thread of literature discusses the impacts of fiscal policy on the conduct of monetary policy. Much of these works assume fiscal policy stabilizes debt and monetary policy controls inflation, whereas others emphasize the importance of policy coordination, such as Woodford (1998), Kim (2003), Linnemann and Schabert (2003), and Kim et al. (2023). In the context of fiscal stress, Leeper (2010), Leeper and Walker (2011), Davig and Leeper (2011) and Ouliaris and Rochon (2021) find that if future budget surpluses are unable to payoff government debt, then monetary policy has to allow higher inflation to devalue government debt. As a result, the effects of fiscal stimulus largely depend on the monetary and fiscal policy regimes.[6] We follow their specification of fiscal stress, but adopt the quantity of money as the policy instrument instead. This change allows us to describe how seigniorage revenue can be used to finance budget deficits. Similar settings include Sims (1994), Woodford (1995), Schmitt-Grohé and Uribe (2000) and Schabert (2006).[7]

Our paper also belongs to a broad literature that evaluates the effects of fiscal stimulus. Recently, much of efforts has been devoted to find the conditions for larger multipliers. The key factors include sticky prices and wages (e.g. Woodford (2011) and Dupor et al. (2019)), the existence of non-Ricardian households (e.g. Galí et al. (2007)), non-separable utility in consumption and hours (e.g. Bilbiie (2011)), a binding zero lower bound of nominal interest rates (e.g. Christiano et al. (2011)), an accommodating monetary policy regime (e.g. Davig and Leeper (2011) and Leeper et al. (2017)), and a money-financed-deficit scheme (e.g. Galí (2020)). On the contrary, Li and Tian (2018) finds that the output multiplier is not necessarily greater than unity if the monetary–fiscal expansion is associated with an anticipated spending reversal. Similarly, Mao et al. (2023) argues that a moderate inflation-driven switching probability reduces fiscal multipliers to be lower than unity when interacted with high government debt. Our paper finds that the money-financed fiscal stimulus is able to produce fiscal multipliers larger than one under fiscal stress.

3. Model and local equilibrium determinacy

In this section, we first introduce our model settings and policies, and then discuss the local equilibrium determinacy requirements. Our analytical framework is a canonical New Keynesian model with real money balance and a consolidated government budget, but abstract from capital accumulation. In the model, tax cut and government spending can be financed by issuing interest-bearing bonds, collecting revenue from taxes and seigniorage. The model is concluded by specifying a tax rule and a monetary policy. Although our model follows Galí (2020) closely, it differs from his by assuming a fiscal stress scenario, which excludes tax policy from responding to debt growth, and considering various money supply rules that target government debt and inflation.

3.1. Non-policy blocks

We first spell out the non-policy blocks of our model and derive their equilibrium conditions.

3.1.1. Households

The economy is populated by a unit mass of identical infinitely-lived households. The households maximize their lifetime utility given by:

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, L_t) - V(N_t), \tag{3.1}$$

where $\beta \equiv 1/(1+\rho)$ is the discount factor. C_t and N_t denote consumption and labor, respectively. $L_t \equiv M_t/P_t$ denotes holdings of real money balance.

[6] According to Leeper (1991)'s definition, a unique stable equilibrium requires monetary policy to be "passive" when fiscal policy is in the stress ("active") regime.

[7] However, they typically assume money growth is either exogenous or targets inflation, but do not allow it to finance budget deficits. In this regard, Schabert (2006) is a related paper that studies the determinant properties of different monetary instrument choices under active and passive fiscal policy regimes. Contrary to his analysis, we set money supply to follow a feedback rule in response to both government debt growth and inflation, and evaluate how the debt financing schemes affect the effectiveness of fiscal stimulus.

The sequential budget constraints are given by:

$$P_tC_t + B_t + M_t = B_{t-1}(1 + i_{t-1}) + M_{t-1} + W_tN_t + D_t - P_tT_t, \tag{3.2}$$

where W_t and D_t are the nominal wage rate and dividends from the ownership of firms. We assume no-Ponzi game condition such that

$$\lim_{T\to\infty} \Lambda_{0,T} A_T \geq 0, \tag{3.3}$$

where $A_t \equiv [B_{t-1}(1+i_{t-1})+M_{t-1}]/P_t$ denotes the real financial wealth at the beginning of period t and $\Lambda_{0,t}$ is the stochastic discount factor. Combining the no-Ponzi game condition with the fact that it is not optimal for households to leave unspent wealth at the ending period yields the transversality condition given by:

$$\lim_{T\to\infty} \Lambda_{0,T} A_T = 0. \tag{3.4}$$

The optimality conditions are given by:

$$U_{c,t} = \beta(1 + i_t)\frac{P_t}{P_{t-1}}U_{c,t+1}, \tag{3.5}$$

$$\frac{W_t}{P_t} = \frac{V_{n,t}}{U_{c,t}}, \tag{3.6}$$

$$h\left(\frac{L_t}{C_t}\right) = \frac{i_t}{1 + i_t}, \tag{3.7}$$

where $h(L/C) \equiv U_l/U_c$.

3.1.2. Firms

A representative final goods firm bundles intermediate goods with a CES technology:

$$Y_t = \left[\int_0^1 Y_t(i)^{1-\frac{1}{\epsilon}}\, di\right]^{\frac{\epsilon}{\epsilon-1}}, \tag{3.8}$$

where $Y_t(i)$ denotes the input of intermediate goods i and ϵ measure the elasticity of substitution between intermediate goods. Profit maximization yields the demand schedules below:

$$Y_t(i) = \left[\frac{P_t(i)}{P_t}\right]^{-\epsilon} Y_t. \tag{3.9}$$

A continuum of monopolistically competitive firms produce differentiated intermediate goods index by $i \in [0,1]$ with the following technology:

$$Y_t(i) = N_t(i)^{1-\alpha}. \tag{3.10}$$

Following Calvo (1983), at any given period, each intermediate goods firm can reset the price of its goods with probability $1-\theta$. Profit maximizing subject to the demand function (3.9) leads to the optimal price setting decision. After derivations, the aggregate inflation dynamics around a zero steady state inflation is described by the equation below:

$$\pi_t = \beta\pi_{t+1} - \lambda(\mu_t - \mu), \tag{3.11}$$

where $\mu_t \equiv log((1-\alpha)P_t/(W_tN_t^{\alpha}))$ is the log of average price markup, $\mu \equiv log M = log(\epsilon/(\epsilon-1))$ is the log of desired price markup, and $\lambda \equiv \frac{(1-\theta)(1-\beta\theta)(1-\alpha)}{\theta(1-\alpha+\alpha\epsilon)}$.

3.1.3. Government budget constraint

The policy block includes a consolidated government that combines the monetary and fiscal authorities.

It finances expenditures G_t with lump-sum taxes T_t, issuance of nominal debt B_t with a nominal interest rate i_t, and seigniorage $(M_t - M_{t-1})/P_t$. The consolidated budget constraint is given by:

$$\frac{G_t}{P_t} + B_{t-1}\frac{1 + i_{t-1}}{P_t} = \frac{T_t}{P_t} + \frac{B_t}{P_t} + \frac{\Delta M_t}{P_t}, \tag{3.12}$$

where we let P_t to denote the price level and $\Delta M_t \equiv M_t - M_{t-1}$.

We can write the level of seigniorage as a fraction of steady state output as:

$$\frac{\Delta M_t/P_t}{Y} = \frac{\Delta M_t/M_{t-1}}{P_t/P_{t-1}}\frac{L_{t-1}}{Y} \approx \chi\Delta m_t, \tag{3.13}$$

where $L_t \equiv M_t/P_t$ denotes the real money balance. This expression of seigniorage can be approximated by $\chi\Delta m_t$, where $m_t \equiv log M_t$ and $\chi \equiv L/Y$ is the inverse income velocity of money at steady state. Therefore (3.13) implies that for the same money growth rate Δm_t, a higher income velocity of money generates lower seigniorage.

We focus on equilibria around a zero-inflation, zero-seigniorage steady state. Let $b_t \equiv B_t/P_t$ to denote the real government debt. Denote $\hat{b}_t \equiv (b_t - b)/Y$, $\hat{g}_t \equiv (g_t - g)/Y$, and $\hat{\tau}_t \equiv (\tau_t - \tau)/Y$ to be the deviations of government debt, purchases, and taxes from their steady state values, as a fraction of steady state output. Linearize the budget constraint (3.12) to obtain:

$$\hat{b}_t = (1+\rho)\hat{b}_{t-1} + b(1+\rho)(\hat{i}_{t-1} - \hat{\pi}_t) + \hat{g}_t - \hat{\tau}_t - \chi \Delta m_t, \tag{3.14}$$

where $b \equiv b/y$ denotes the steady state debt level as a fraction of steady state output and $\hat{\pi}_t = log(P_t/P_{t-1})$. Define $(1+i)/\pi = 1+\rho$ in the steady state, so we have $i_t = log(1+i_t)/(1+\rho)$.

3.2. Fiscal policy

As in Galí (2020), we adopt a tax rule given by:

$$\hat{\tau}_t = \psi_b \hat{b}_{t-1} + \hat{\tau}^*_t, \tag{3.15}$$

where the tax responds to the deviations of the debt ratio from its long-run level, and is subject to exogenous shocks. In particular, we assume $\hat{\tau}^*_t = -\delta^t$ for a tax cut stimulus and $\hat{g}_t = \delta^t$ for a government spending increase. Combining (3.15) with government budget constraint (3.14) yields:

$$\hat{b}_t = (1+\rho-\psi_b)\hat{b}_{t-1} + b(1+\rho)(\hat{i}_{t-1} - \hat{\pi}_t) + \hat{g}_t - \chi \Delta m_t - \hat{\tau}^*_t. \tag{3.16}$$

Notice that Galí (2020) implicitly assumes fiscal space is ample by setting the tax feedback coefficient $\psi_b > \rho$. In contrast, we assume fiscal space is limited, and follow Leeper (2010) to set $\psi_b = 0$, which implies taxes do not respond to debt growth at all.[8] A novelty of our setup is that we fix the fiscal policy to represent fiscal stress and only use monetary policy rule to achieve local equilibrium determinacy. This is in contrast with the literature that often uses separate fiscal and monetary policy rules for determinacy.

3.3. Monetary policy

We follow Galí (2020) to assume the central bank has direct control over nominal money growth,[9] and consider three types of monetary policy: (1) a zero-debt-increase policy; (2) a targeting rule that money growth feedbacks to debt growth; (3) a mixed targeting rule that money growth feedbacks to debt growth and inflation.

3.3.1. Zero-debt-increase policy

First, we adopt the zero-debt-increase (ZDI) monetary policy used in Galí (2020), which tends to fill the gap of government budget with seignoirage and keeps the real debt unchanged every period. This policy shares similar ideas with a broad set of monetary policies in response to COVID shocks. For example, the Federal Reserve purchases unlimited quantities of treasury bonds, the Bank of England purchases 200 billion of gilts, and the European Central Bank directly buys 750 billion of Eurozone bonds. The essence of these monetary policies is to fully fund the stimulative packages. In a consolidated government budget constraint, fully money-funded fiscal stimulus results zero-debt-increase.

The particular form of the ZDI policy is obtained by plugging the equilibrium condition $\hat{b}_t = 0$ into the government budget constraint and solve for Δm_t. As a result, we have the following ZDI policy:

$$\Delta m_t = (1/\chi)[\delta^t + b(1+\rho)(\hat{i}_{t-1} - \pi_t)] \tag{3.17}$$

This policy not only permits us to assess how the presence of fiscal stress alters the outcome in Galí (2020), but also serves as a natural benchmark for our analysis since it represents an extreme scenario that the entire new budget deficit is financed by money creation.

3.3.2. Debt-targeting rule

Second, the debt-targeting (DT) monetary policy rule can be regarded as the central bank partially fund the fiscal deficits through money supply to stabilize government debt. It is originated from the idea that the Federal Reserve controls the bond-to-money ratio after the global financial crisis and European debt crisis to manage the economy. As Gokan and Turnovsky (2023) empirically finds in their paper, this rule seems to be a reasonable approximation to US's monetary policy particularly during periods of relative stability.[10] Therefore, we design our DT rule in the same way.

In particular, the policy rule is given by:

$$\Delta m_t = \varphi_b \hat{b}_{t-1} \tag{3.18}$$

where φ_b denotes the policy responsiveness coefficient, which determines the strength of the monetary authority's reaction to the deviations of the government debt from its long-run target level. This is a more general specification of monetarization policy relative to the ZDI policy in (3.17) since it allows real debt to temporarily deviate from its long-run target, but eventually returns to the steady state level.

[8] In fact (Leeper, 2010) assumes fiscal limit is expected to arrive at some future date and policy adjustments are uncertain. We focus on the extreme case $\psi_b = 0$ across all states and at all time in order to avoid compounding the effects of debt monetization policy.

[9] Sargent and Wallace (1976), Chowdhury and Schabert (2003) and Ascari and Ropele (2013) also adopt money supply as the monetary policy instrument.

[10] They show that during the period of post-financial crisis, the bond-to-money ratio is about 1.35 with a standard deviation of only 0.058.

3.3.3. Mixed-targeting rule

Finally, the mixed-targeting (MT) monetary policy rule introduces an inflation target to reflect the inflation mandate adopted by central banks. Schabert (2006) employs a money growth rule in the same form to analyze the determinacy requirements in an New-Keynesian model.

Specifically, the policy rule is given by:

$$\Delta m_t = \varphi_b \hat{b}_{t-1} + \varphi_\pi \pi_t \tag{3.19}$$

where φ_b and φ_π denote the policy feedback coefficients, which determine the strength of the central bank's responsiveness to government debt growth and inflation.

Note that Galí (2020) considers two deficit financing schemes. One employs money supply to keep the real debt level unaltered which he labels as "money-financed", and the other sets money supply to fix inflation at its steady state which he labels as "debt-financed". Our DT and MT monetary policy rules allow both debt and inflation to vary in response to fiscal stimulus, but with different strength. Hence, these two monetary policy specifications have more flexibility relative to those considered in Galí (2020). Another reason for us to design the targeting monetary policies is that, in our model neither "money-financed" nor "debt-financed" scheme delivers local determinacy under fiscal stress, so monetary policy has to react to debt growth. In addition, since the main purpose of using money to finance fiscal deficits is to avoid relying on further rises in the stock of government debt, a debt growth target in the monetary policy seems natural.

3.4. Local equilibrium determinacy

Before assessing the effects of a money-financed fiscal stimulus, we firstly analyze how fiscal stress affects the conduct of a money-financing policy by characterizing the range of policy coefficients that delivers local determinacy.

Our complete dynamic system is characterized by the conditions specified above,[11] which is to be solved for the paths of $\{\hat{y}_t, \hat{c}_t, \hat{\xi}_t, \hat{i}_t, \pi_t, \hat{l}_t, \hat{\mu}_t, \hat{b}_t, \Delta\hat{m}_t\}$, given two exogenous processes of $\{\hat{g}_t\}$ and $\{\hat{\tau}_t^*\}$. For the convenience of deriving the determinacy conditions analytically, it is useful to simplify the equations into a four-dimension difference system, which can be represented by the following vector form:

$$E_t X_{t+1} = B X_t + C V_t, \tag{3.20}$$

where the vector of state variable is $X_t = [\hat{y}_t, \pi_t, \hat{i}_t, \hat{b}_t]^T$ and the vector of exogenous process is $V_t = [\hat{g}_t, \hat{\tau}_t^*]^T$. In consequence, the local equilibrium determinacy depends on the coefficient matrix $\boldsymbol{B}$.

3.4.1. Zero-debt-increase policy

Let us first examine the ZDI money supply policy (3.17) employed in Galí (2020). We find that, although Galí (2020)'s monetary policy setting, which uses money growth to fix $\hat{b}_t = 0$, will be an equilibrium when fiscal space is ample, it actually leads to an unsustainable debt path under fiscal stress. To see this, we substitute the monetary policy (3.17) into the government budget constraint to obtain:

$$\hat{b}_t = (1 + \rho - \psi_b)\hat{b}_{t-1}. \tag{3.21}$$

Notice that when $\psi_b > \rho$, which is the assumption (Galí, 2020) maintains throughout his analysis, the equation above guarantees that the debt ratio is anchored in expectation and converges to its long-run level. Since we assume the government debt is unfunded, i.e. $\psi_b = 0$, in the presence of fiscal stress, the equation above becomes:

$$\hat{b}_t = (1 + \rho)\hat{b}_{t-1}, \tag{3.22}$$

Given a positive steady state interest rate $\rho = 1/\beta$, (3.22) implies the debt dynamics is explosive. This is in sharp contrast with Galí (2020) where the zero-debt-increase monetary policy (3.17) produces a local stable equilibrium. To understand the different results, notice that unlike Galí (2020), the presence of fiscal stress in our model constrains fiscal policy's ability to stabilize government debt. Therefore, monetary policy has to accommodate debt. However, the money supply rule (3.17) adopted in Galí (2020), which is obtained by imposing $\hat{b}_t = 0$ for all t, cannot anchor fiscal expectation. This is because given $\hat{b}_t = 0$ for all t, monetary policy (3.17) is one solution that satisfies the government's budget constraints, but not unique. It can be easily shown that any policy combining (3.17) with an extra term formulated by a subset of $\{\hat{b}_t\}$ leads to the same result. Therefore, we introduce a lagged debt target in the zero-debt-increase monetary policy to pin down fiscal expectation. In particular, the policy is given by:

$$\Delta m_t = (1/\chi)[\delta^t + b(1+\rho)(\hat{i}_{t-1} - \pi_t)] + \varphi_b \hat{b}_{t-1}, \tag{3.23}$$

where φ_b denotes the feedback coefficient of money growth to debt growth. We assume $\varphi_b > 0$, which indicates that the money supply responds positively to debt increase.

[11] The complete equilibrium conditions are listed in Appendix A.

Proposition 1. *Under zero-debt-increase monetary policy (3.23), the following conditions are necessary and sufficient for a unique and stable equilibrium.*

$$\left|1+\rho-\chi\varphi_b\right|<1 \tag{3.24}$$

$$b<\frac{\chi}{1+\rho}\left[2\eta+\sigma^{-1}+\frac{\kappa(1+2\eta)}{2\sigma(1+\beta)}\right] \tag{3.25}$$

Proof. See Appendix B.1 □

Note that, by substituting the ZDI rule into the government budget constraint, we find that the real debt path is independent to the equilibrium system. Thus, condition (3.24) ensures the fiscal expectation is pinned down. Rearrange terms in (3.24) to yield:

$$\rho\chi^{-1}<\varphi_b<(2+\rho)\chi^{-1}, \tag{3.26}$$

which gives a range of the value of the money supply feedback coefficient φ_b to ensure the sustainability of real debt. It shows that the convergent region depends on the steady state real interest rate ρ and the income velocity of money χ. Since both terms are positive, this condition implies the central bank must respond to deviations of real debt from its target with a "sufficient but moderate strength".[12] Introducing a lagged term of real debt $\varphi_b\hat{b}_{t-1}$ as a monetary policy target poses a "threat" of a more aggressive response to the deviation of debt target by the central bank, which pins down the fiscal expectation.

On the other hand, condition (3.25) keeps inflation stable. When monetary authority adjusts money supply to keep real debt unchanged, it has to give up the control of inflation. This specification is often referred to the passive monetary policy,[13] which must be coordinated with an active fiscal policy to pin down the price level. This requirement for fiscal policy is satisfied since the fiscal authority is not free to choose under fiscal stress. To understand this property, it is useful to consider a special case in the fully flexible price, i.e. $\kappa=\infty$. Rearrange the condition (3.25) to yield:

$$\frac{2\sigma(1+\beta)(w_1-\eta)}{1+2\eta}<\kappa=\infty \tag{3.27}$$

where w_1 is positively related to the target value of real debt. It is easy to see that any values of these structural parameters can satisfy this condition, so the local determinacy only concerns the convergence of real debt.

It is interesting to note that, paradoxically, if Proposition 1 is satisfied, the real debt $\hat{b}_t$ will stay at zero in all periods as long as the money supply adjusts "sufficiently but moderately" to meet the financial needs and, therefore, the equation $\Delta m_t=(1/\chi)[\delta^t+b(1+\rho)(\hat{i}_{t-1}-\pi_t)]$ will hold ex-post for all t. This is in contrast to Galí (2020), where the equilibrium outcome $\Delta m_t=(1/\chi)[\delta^t+b(1+\rho)(\hat{i}_{t-1}-\pi_t)]$ is regarded as a money-financed policy rule.

3.4.2. Debt-targeting rule

Deriving the local equilibrium determinacy requirements under the DT rule (3.18) gives rise to the following proposition:

Proposition 2. *Under the debt-targeting monetary policy, the necessary condition for a unique stable equilibrium is given by:*

$$\varphi_b>\rho\chi^{-1} \tag{3.28}$$

Proof. See Appendix B.2. □

To obtain intuition from Proposition 2, we consider the ultimate impact of the DT rule on money supply.[14] In particular, when there is a permanent increase in real debt on the scale $d\hat{b}$, the long term relationship between real debt and money supply is given by:

$$\chi d\Delta m=\rho d\hat{b} \tag{3.29}$$

where the equality is derived from the government budget constraint. Since the seigniorage is given $(\Delta M_t/P_t)/Y_t=\chi\Delta m_t$, Eq. (3.29) suggests that in equilibrium, the amount of seigniorage equals to the interests of debt.

Analogously, when implement the DT rule, we have:

$$d\Delta m=\varphi_b d\hat{b} \tag{3.30}$$

where φ_b measures the amount of money increase when debt increases. Note that condition (3.28) is equivalent to the coefficient of $d\hat{b}$ being greater than $\rho\chi^{-1}$. Thus, the relationship between money and real debt becomes $\chi d\Delta m>\rho d\hat{b}$, that is the real debt path will be convergent only if the monetary authority levies more seigniorage than the accumulation of debt interests. Moreover, we can see that the lower bound for φ_b to guarantee local determinacy is identical to that under zero-debt-increase monetary policy. This suggests that money supply must feedback at least modest to debt growth, otherwise debt path is explosive.

[12] Note that in Galí (2020)'s setup, determinacy does not require money supply to react to debt when fiscal space is ample.

[13] The definition of a passive (active) policy refers to Leeper (1991).

[14] Note that this proposition only states the necessary condition for local determinacy. The analytical characterization of the sufficient condition is much more complicated since the four-order dynamic system cannot be simplified as much as under the ZDI policy, and hence is beyond the scope of this paper.

Table 1
Parameter calibration.

Parameter	Description	Value
σ	Relative Risk Aversion	1
υ	Separability of Real Balances	0
β	Subjective Discount Factor	0.994
α	Index of Decreasing Returns to Labor	0.25
φ	Curvature of Labor Elasticity	5
γ	Scaling Parameter of Real Balance	0.0015
η	Semi-elasticity of Money Demand	7
b	Target Debt Ratio	2.4
ρ	Steady State Quarterly Interest Rate	0.005
χ	The Steady State Inverse Velocity	0.29
ϵ	Elasticity of Substitution among Goods	9
θ	Calvo Index of Price Rigidity	0.75
δ	Persistence of Shock	0.5
ψ_b	Debt Feedback Coefficient	0

3.4.3. Mixed-targeting rule

The following proposition summarizes the determinacy requirements for the MT rule (3.19):

Proposition 3. *Under the mixed-targeting monetary policy, the necessary condition for a unique stable equilibrium is given by:*

$$\chi\varphi_b - \rho(1-\varphi_\pi) > 0 \tag{3.31}$$

Proof. See Appendix B.3 □

It is obvious to see that the determinacy region under MT monetary policy hinges on the interplay between the desire to stabilize debt and inflation. In particular, when the money supply policy is more aggressive in fighting inflation, it has to be more responsive to debt growth as well, otherwise the government debt is unsustainable. This is because stronger inflation targeting rises real interest rates, which increase the interest burden of government debt, then the monetary authority has to create more money supply to payoff the debt interests.

Next, we consider the ultimate impact of MT policy rule on money supply. When the public debt increases in the size of $d\hat{b}$ permanently, the change of money supply $d\Delta m$ follows:

$$\begin{aligned} d\Delta m &= \varphi_\pi d\pi + \varphi_b d\hat{b} \\ &= (\varphi_\pi \frac{\rho}{\chi} + \varphi_b) d\hat{b} \end{aligned} \tag{3.32}$$

where the second equality makes use of the long run relationship between real debt and money supply implied by the law of motion for debt. Notice that the determinacy condition (3.31) is equivalent to the coefficient in the bracket in (3.32) being greater than the long run real rate $\rho\chi^{-1}$. Therefore, the local determinacy is guaranteed under the MT rule whenever central bank chooses φ_π and φ_b so that the seigniorage outnumbers the interest of public debt.

On the other hand, if the inflation permanently increase in the size of $d\pi$, the permanent change of money supply $d\Delta m$ follows:

$$\begin{aligned} d\Delta m &= \varphi_\pi d\pi + \varphi_b d\hat{b} \\ &= (\varphi_\pi + \varphi_b \frac{\chi}{\rho}) d\pi \end{aligned} \tag{3.33}$$

where the second equality also makes use of the long run relationship between real debt and money supply. Note that the determinacy condition (3.31) is equivalent to the coefficient in the bracket in (3.33) being greater than one, that is the money supply should increase more than one for one in the face of an increase in inflation. In this way, the supply of the real balance increases and guarantees the real rates eventually rise to reduce the interest burden for government debt, and thus keep the debt path sustainable.

4. Quantitative analysis

In this section, we quantitatively evaluate the effects of a fiscal stimulus under the three monetary policies specified above. We first discuss the stimulative effects of a 1% tax cut and a 1% spending increase, and also their welfare implications. Finally, we perform a case study to assess the macroeconomic and welfare impacts of money-financed fiscal stimulus in the presence of pandemic shocks.

4.1. Calibration

Our calibration strategy largely follows Galí (2020).[15] The calibrated parameter values are listed in Table 1. First, we set the risk aversion parameter $\sigma = 1$, which is equivalent to a log utility in consumption. We follow Galí (2020) to assume the real balance is separable in utility and set the elasticity of substitution $\upsilon = 0$.

We calculate that the US's post-GFC real interest rate (from 2009 Q3 to 2020 Q1) is about 0.59 on average.[16] Thus, we calibrate the steady state value of real rate ρ to match this average, that implies an annual real interest rate of 2.36% and the discount factor $\beta = 0.994$.

Second, the labor share is chosen to be 25%, which is a standard value used in the literature. We assume the Frisch elasticity of labor supply is 0.2, which indicates $\varphi = 5$. Following most of the New Keynesian literature, we choose the Calvo parameter $\theta = 0.75$, which suggests an average price duration of four quarters. The average markup $\mu = log\frac{\epsilon}{\epsilon-1}$ is assumed to be 12.5%, so $\epsilon = 9$. In terms of parameters related to money balance, we follow Galí (2020) to set the money demand elasticity $\eta = 7$. We employ data of GDP divided by currency in circulation to calculate the steady state velocity of money. In the period from 2009 Q3 to 2020 Q1, the mean value of money velocity is 3.44. Thus, we calibrate the scaling parameter for real balance $\gamma = 0.0015$, which implies the steady state value of Y/L consistent with the data and the inverse velocity of money $\chi = 0.29$.

The steady state government-debt-to-GDP ratio b is fixed at 2.4, which is consistent with a 60% target when quarterly GDP is used. This is the upper bound of debt-output ratio required by the Maastricht Treaty, which represents a high fiscal burden.[17] Due to the existence of fiscal stress, fiscal policy feedback coefficient is assumed to be $\psi_b = 0$, that is, the fiscal surplus is assumed to follow an exogenous process. The persistence of expansionary fiscal policy is set to be $\delta = 0.5$.

4.2. Impulse response functions

4.2.1. A money-financed tax cut

Fig. 1 demonstrates the dynamic response of macroeconomic variables of interest to an exogenous 1% tax cut, under the baseline calibration described above. The black lines display the responses under the ZDI policy,[18] while the blue lines with diamonds and the red lines with circles present the responses under the DT rule and the MT rule, respectively.[19]

It is clear from Fig. 1 that, a money-financed tax cut generates a higher rate of money growth initially under the ZDI policy. This causes a sharp decline in real rates because of the liquidity effect. Due to a surge in aggregate demand, output and consumption both rise by about 0.6% on impact, which is close to the outcome obtained by Galí (2020). The real interest costs of government outstanding debt fall, which leads to a slower growth of money supply in the following periods due to the monetary policy's strict debt growth targeting. Because the ZDI policy has a substantial expansionary effect on economic activities without rising government debt, the debt-output ratio falls. In the presence of fiscal stress, the fiscal theory of price level operates. Since the ZDI policy generates more future seniorage revenue, it requires a milder inflation to stabilize debt. The stronger liquidity effect of higher growth of money supply presses down the nominal interest rates.

The expansionary effects of a money-financed tax cut on output and consumption are dampened under a DT rule. This is because the deficits are partially financed by government debt initially and hence produce slower money growth. Less money growth in turn weakens the liquidity effect, leading to less decline of real rates and lower increase in output and consumption. Less seniorage indicates more unfunded debt, which drives up inflation. In this case, the fisher effect exceeds the liquidity effect, reflected by a rise of the nominal interest rates. Furthermore, higher debt and lower output growth imply a less decline of the debt-output ratio.

The above mechanism also works under a MT rule, but with an additional inflation targeting channel. While money growth is constrained by an inflation control consideration, the deficits are financed through more debt and less money supply relative to the DT rule. Therefore, the policy effects are closer to a debt-financed scheme rather than a money-financed one. As a result, we find that including an inflation target to the monetary growth rule does not alter the qualitative outcome, but attenuates the expansionary effects on output and consumption. Meanwhile, it is not surprising to observe that inflation path turns out to be higher and more persistent since less seigniorage is generated.

Note that a higher money growth coexist with a lower inflation under the ZDI policy. The key to understand this result is that in the presence of fiscal stress, the fiscal theory of price level operates, so that the price level is determined by fiscal behavior rather than money growth. Therefore, higher expected real fiscal resources imply lower inflation path. In our model, money growth is higher under the ZDI policy relative to under the other two policies. This generates more seigniorage revenues for the government, which in turn increases expected fiscal resources and reduces inflation.

To elaborate the argument above, it is useful to rewrite the government budget constraint as:

$$\frac{B_{t-1}}{P_t} = \frac{1}{1+i_{t-1}}\left(\frac{B_t}{P_{t+1}}\pi_{t+1} + \tau_t - g_t + \frac{\Delta M_t}{P_t}\right), \tag{4.1}$$

where we let τ_t and g_t to denote the real government taxes and purchases, respectively. $\pi_t \equiv P_t/P_{t-1}$ is the inflation rate.

[15] This permits a direct comparison between his results and ours. We also conduct sensitivity checks for key parameter values in Appendix C.

[16] Source: 10-Year Real Interest Rate, Federal Reserve Bank of St. Louis.

[17] We also examine the effects of a 120% debt-output ratio in Appendix C.

[18] We use Eq. (3.23) as the ZDI policy to achieve determinacy.

[19] We pick monetary policy coefficients $\phi_b = 0.5$ and $\phi_\pi = -1$ as the baseline, and check the sensitivity of alternative coefficient values in Section 4.3.2.

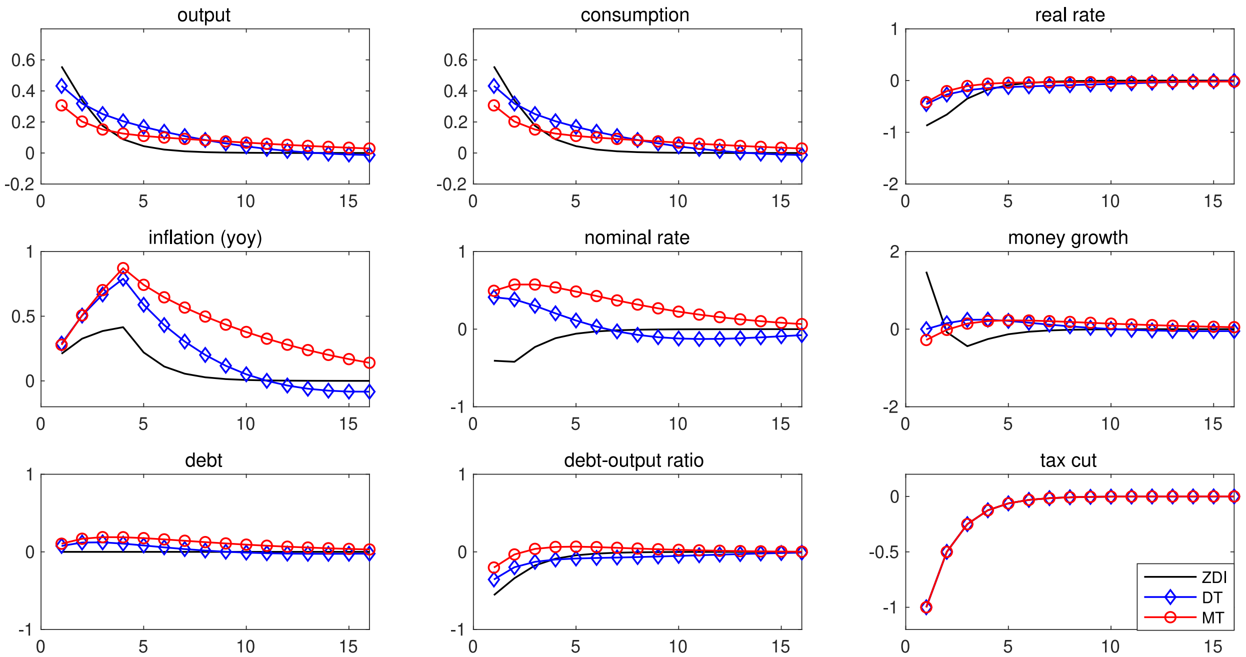


Fig. 1. Dynamic effects of a tax cut under various monetary policies. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

Iterate the above equation forward on B_t/P_{t-1} and impose the transversality condition to obtain the following intertemporal government budget constraint:

$$\frac{B_{t-1}}{P_t} = \sum_{j=0}^{\infty} \prod_{k=0}^{j} \frac{\pi_{t+k+1}}{1+i_{t+k-1}} \left(\tau_{t+j} - g_{t+j} + \frac{\Delta M_{t+j}}{P_{t+j}} \right), \tag{4.2}$$

which equates the real value of outstanding government debt to the present value of budget surplus and seigniorage. In response to a fiscal stimulus under fiscal stress, this expression suggests that seigniorage, current price level, and the real discount rate can adjust to restore the equilibrium, whose precise contributions in backing the deficits hinges on the specification of economic structure, monetary and fiscal policy.

Note that ZDI policy strictly finances the deficits with seigniorage, the left-hand side of Eq. (4.2) keeps unchanged at all periods. The only source of inflation comes from the liquidity effect, which lowers the real interest rate, enhances the initial wealth effect and results in an increase in aggregate demand. At the same time, the cost of debt services decreases. Thus, the inflation need not to rise to a high level while financing the fiscal stimulus.[20] On the other hand, when implement the DT rule, a sluggish response to debt generates less seigniorage and causes a temporal surge in debt-to-output ratio, which in turn needs a higher price level in the future to inflate away outstanding debt. Moreover, a delayed response to outstanding debt also generates more debt services to be paid since less liquidity is created and the real interest rate falls less than before. This also implies a higher inflation path.

4.2.2. A money-financed increase in government purchases

Fig. 2 shows the dynamic responses of key macroeconomic variables to an exogenous increase in government purchases. Again, The black lines display the responses under the ZDI policy, while the blue lines with diamonds and the red lines with circles display the responses under the DT rule and the MT rule respectively.

The transmission channels under money-financed government purchases are similar to the ones under tax cut. When the fiscal stimulus is implemented, the real rates drop because of the liquidity effect produced by higher money growth. Since government spending directly affects the aggregate demand, it further expands output and consumption beyond the liquidity channel. Consequently, an increase in government spending produces a larger decline in debt-to-output ratio and a higher inflation.

On the other hand, the responses of consumption diverge across various monetary policy rules due to the crowding-out effects. Quantitatively, households' consumption drops to the negative territory two periods after the implementation of fiscal stimulus under the DT and MT rules. This is because the weak and sluggish feedback of money growth to debt leads to slower money growth initially and hence higher real interest rates in the short-run. This in turn implies a crowding-out effect instead of a crowding-in effect under the ZDI policy.

20 The money growth even drops after the first period.

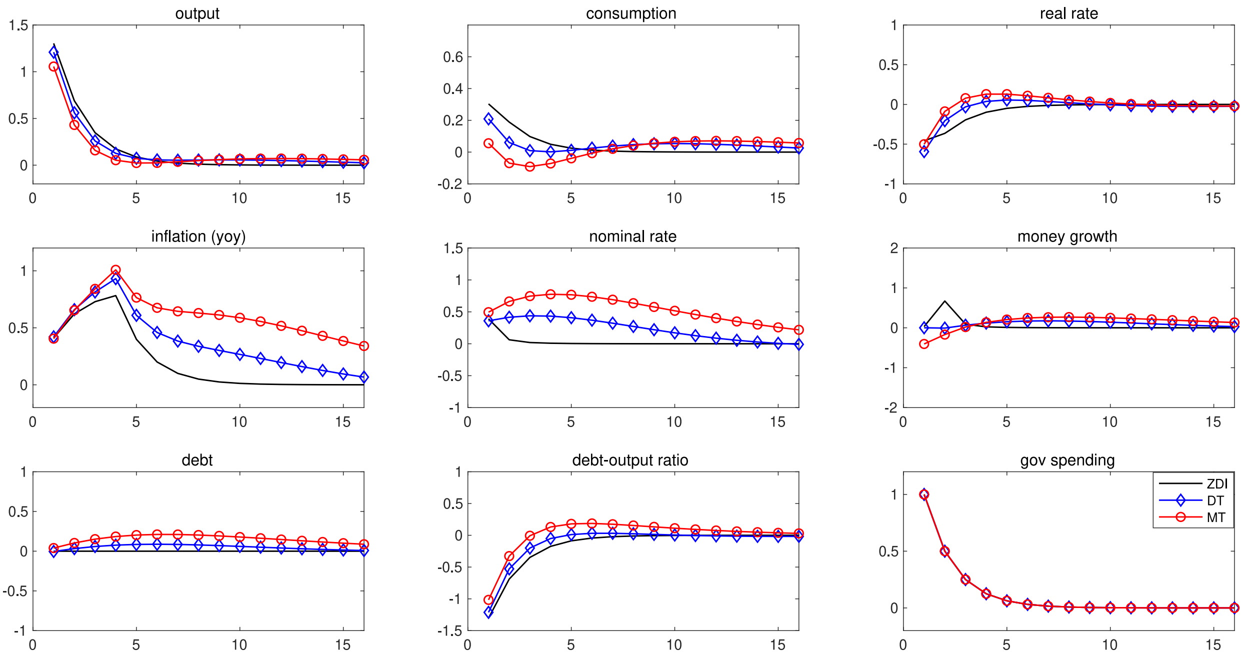


Fig. 2. Dynamic effects of an increase in government purchases under various monetary policies. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

Table 2
Fiscal multipliers under different policy rules.

	IFM	CFM
Tax cut		
Zero-debt-increase (ZDI)	0.5576	0.6244
Debt-targeting (DT)	0.4926	0.8239
Mixed-targeting (MT)	0.4284	0.9338
Government spending		
Zero-debt-increase (ZDI)	1.3033	1.3441
Debt-targeting (DT)	1.2442	1.4257
Mixed-targeting (MT)	1.1349	1.5480

4.3. Fiscal multipliers

4.3.1. Short run vs. long run

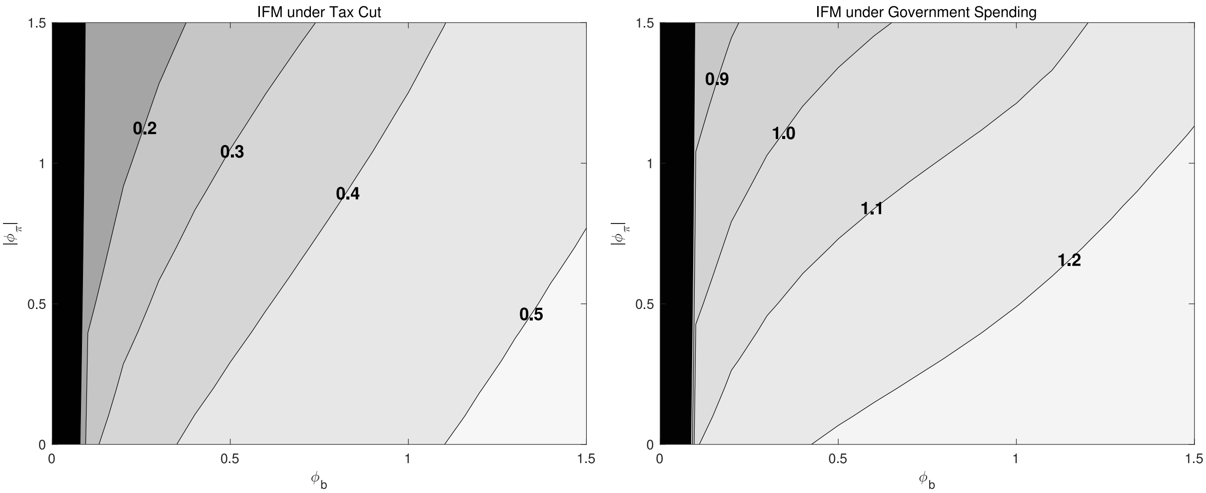
Next, we discuss the short-run and long-run effects of various momentary policy configurations. We follow Galí (2020) to use the instantaneous fiscal multiplier, $IFM = \hat{y}_1$, to measure the short-run effects of fiscal stimulus, and the cumulative multiplier, $CFM = (1-\delta)\sum_0^\infty \hat{y}_t$, to evaluate the long-run effects.

Table 2 shows the size of IFMs and CFMs across different monetary policies. It is worth noting that the IMFs are greater in the case of ZDI policy than that of DT and MT policies while the opposite is true for CFMs. This result holds for both tax cut and government spending increase. This is because money supply increases more initially under the ZDI policy, which produces lower real interest rates that boost more output immediately. On the other hand, allowing debt to increase implies higher money supply and lower real interest rate in later periods, which produces a larger fiscal multiplier in the long-run.

4.3.2. The strength of policy responsiveness

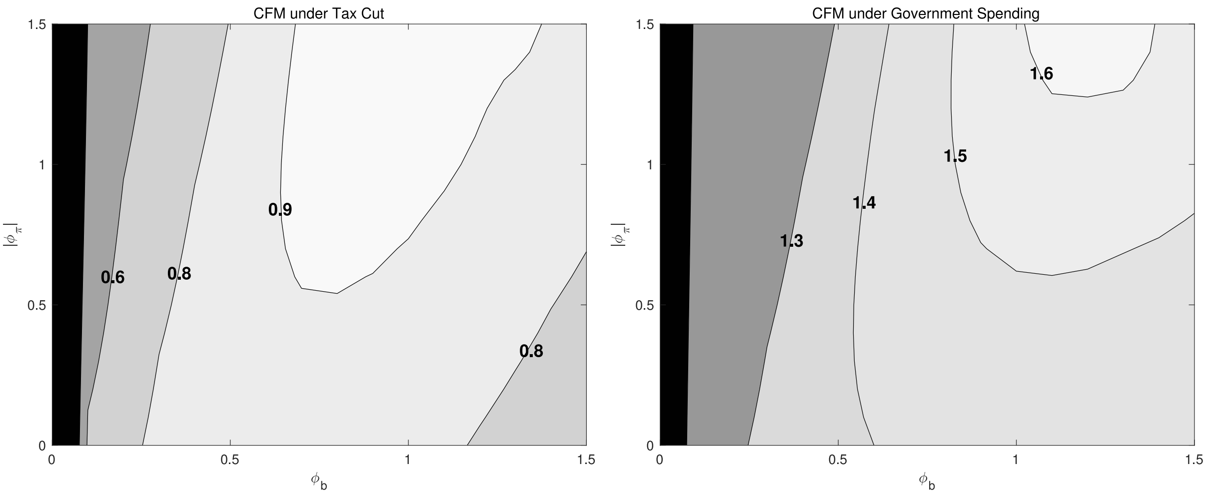
We then assess how the effects of fiscal stimulus differ when the responsiveness of monetary policy to debt growth and inflation, measured by ϕ_b and ϕ_π, vary.

Fig. 3 shows the instantaneous fiscal multipliers across different values of monetary policy coefficients. Three points worth mentioning. First, the fiscal multipliers increase with the responsiveness to debt. This is because a more rapid growth of money supply presses down the real rates and boosts the economy. Second, a more aggressive monetary policy reaction to inflation dampens the instantaneous stimulative effects but amplifies the mid-term multipliers since money growth is slower in the short-run, but stays positive for longer periods. Finally, the difference of short-run effects between tax cut and government spending is less sensitive to the coefficients, which generates government spending multipliers that are greater than one.



(a) IFMs under Tax Cut (b) IFMs under Gov. Spending

Fig. 3. Instantaneous fiscal multipliers: The role of policy response parameters. Note: The figures display the instantaneous output multipliers while implementing a feedback rule, as a function of the policy response parameters φ_b and φ_π. The black area is the unstable region leading to unsustainable debt path.



(a) CFMs under Tax Cut (b) CFMs under Gov. Spending

Fig. 4. Cumulative fiscal multipliers: The role of policy response parameters. Note: The figures display the cumulative output multipliers while implementing a feedback rule, as a function of the policy response parameters φ_b and φ_π. The black area is the unstable region leading to unsustainable debt path.

Fig. 4(a) displays the cumulative fiscal multipliers under tax cut or government spending as the two policy coefficients, ϕ_b and ϕ_π, vary. Two observations are worth making. First, there is a nonlinear relationship between ϕ_b and CFM, which suggests that the CFM peaks when the monetary policy responsiveness to debt is set to a moderate size. This is because the deficits may not be fully financed by seigniorage if monetary policy weakly responds to debt, and the real rates are higher. On the other hand, if the deficits are over-financed by money, the debt growth turns to be negative, and therefore the money supply will drop down in the future periods. This has a contractionary effect on output. Second, the size of CFM becomes larger when the money supply aggressively responds to inflation. This is because when the monetary authority targets on inflation, the money growth is slower, and more deficits is financed by debt. Consequently, more debt will be paid in the future with higher interest expenses. As a result, the amount of money supply surges in the long run, which produces larger CMFs.

Fig. 4(b) presents the CFMs of an increase in government purchases across different monetary policy coefficients. Compared to the outcome under a tax cut, the size of the CFM is greater, and the maximum is more than 1.6 whereas it is below 1 under tax cut. Also note that the relationship between CFM and ϕ_b is inverted U-shaped and CFM is monotonically increasing with ϕ_π. This pattern is consistent with the result under tax cut.

Table 3
Deficit financing decomposition.

	Seigniorage	SDF	Current inflation
Tax cut			
Zero-debt-increase (ZDI)	7.68%	66.95%	25.38%
Debt-targeting (DT)	13.67%	51.46%	34.87%
Mixed-targeting (MT)	29.93%	36.33%	33.74%
Government spending			
Zero-debt-increase (ZDI)	13.89%	36.47%	49.64%
Debt-targeting (DT)	24.11%	24.97%	50.92%
Mixed-targeting (MT)	44.53%	6.39%	49.08%

4.4. Decomposition of deficit financing sources

Although the nonlinear intertemporal government budget constraint (4.2) provides clear economic intuitions on the ultimate financing sources for budget deficits, it is not straightforward to quantitatively assess their relative contribution. To overcome this hurdle, we derive a linearized counterpart to (4.2) and conduct a numerical decomposition of budget deficit financing across various monetary policy configurations.

We iterate the linearized government budget constraint (3.14) on $\hat{b}_t$ and impose a limiting condition to obtain:

$$\hat{b}_{t-1} = \sum_{k=0}^{\infty} \frac{1}{(1+\rho)^{k+1}} \left[(\hat{\tau}_{t+k} - \hat{g}_{t+k}) + \chi \Delta m_{t+k} - b(1+\rho)(\hat{i}_{t-1+k} - \pi_{t+k}) \right]. \tag{4.3}$$

Since we assume the economy starts from its steady state and $\hat{b}_{t-1}$ and $\hat{i}_{t-1}$ are predetermined at time t, their deviation from the steady state equals to zero. Hence the expression above can be rewritten as:

$$\underbrace{\sum_{k=0}^{\infty} \frac{\hat{g}_{t+k} - \hat{\tau}_{t+k}}{(1+\rho)^{k+1}}}_{\text{PV of Budget Deficits}} = \underbrace{\sum_{k=0}^{\infty} \frac{\chi \Delta m_{t+k}}{(1+\rho)^{k+1}}}_{\text{PV of Seigniorage}} - \underbrace{\sum_{k=0}^{\infty} \frac{b(\hat{i}_{t+k} - \hat{\pi}_{t+k+1})}{(1+\rho)^{k+1}}}_{\text{PV of Real Discount Factor}} + \underbrace{b\pi_t}_{\text{Current Inflation}} . \tag{4.4}$$

This is the linearized counterpart to the intertemporal condition (4.2). The left-hand-side is the present value of budget deficits and the right-hand-side contains the present value of seigniorage and the real discount factor, plus current inflation. In the case of fiscal stress, a fiscal stimulus causes a permanent increase in the left-hand-side. Therefore, adjustments must occur through the right-hand-side. It is clear monetary policy can alter the value of the right-hand-side in three channels. First, it can generate higher seigniorage revenue. Second, it can produce surprise inflation. Third, it can lower the stochastic discount rate. These three measures either raise the present value of future government resources or decrease the real value of current outstanding debt to restore the equilibrium condition.

This linearized intertemporal expression is useful in showing how parameter and the steady state values affect the contribution of each financing channel analytically. As we can see from (4.4), seigniorage accounts for a greater share of budget deficit financing when the income velocity of money is lower, while current inflation and future discount rate play a more important role when the steady state debt level is higher.

Table 3 shows the numerical decomposition of deficit financing under different monetary policy specifications. We can see the most striking result is that seigniorage is never the major source for deficit financing. Changes in the real discount factor account for the most in tax-cut-led-deficit financing, while current inflation plays the most important role in spending-increase-led-deficit financing. The shares of financing through current inflation and seigniorage increase under the DT and MT rules relative to the ZDI policy. Moreover, under the DT rule, more responsiveness suggests less reliance on seigniorage but more reliance on changes of SDF. The reason is that lower debt growth leads to lower money growth, which implies less seigniorage revenue. Under the MT rule, the more aggressive monetary policy targets inflation, the more likely for seigniorage to play a significant role in financing deficit. This is because when inflation is contained, seigniorage has to assume more financing responsibility.

We then evaluate how the policy coefficients affect the relative contribution of each financing source. Fig. 5 shows the decomposition of deficit financing under various fiscal measures and monetary policy responsiveness. Two aspects worth noting. First, as the monetary authority pays more attention to debt control, that is a higher value of ϕ_b, the contribution of seigniorage shrinks while the SDF contributes more. The reason is that aggressive reactions of monetary policy to debt alleviates the fiscal burden in the future, which leads to less money creation and hence less seigniorage. However, since current inflation is less influenced by future path of financing, its contribution remains relative stable.

Second, as the responsiveness to inflation goes up, the seigniorage takes more responsibility to finance deficits. This is because aggressive reactions of monetary policy to inflation leads to a tighter money supply, which decreases the seigniorage but generates higher inflation. The relative contribution of current inflation shows little changes.

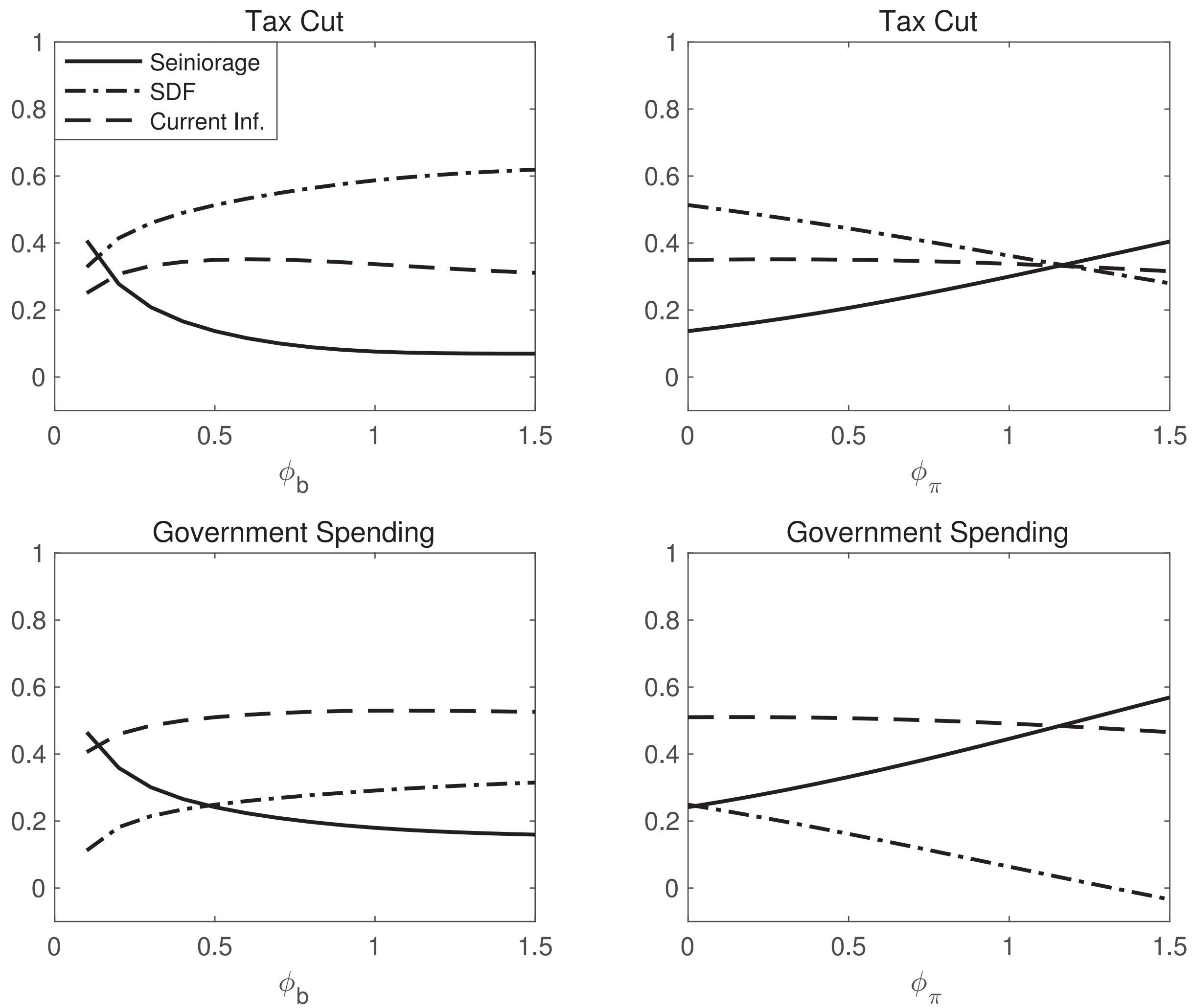


Fig. 5. Decomposition of financing sources: The role of policy response coefficients. Note: The above lines show the percentage contribution of each financing sources as a function of single coefficient values of ϕ_b or ϕ_π.

4.5. Welfare analysis

In addition to employ fiscal multipliers as a measure of effectiveness, we conduct welfare analysis to evaluate the desirability of different policies. We follow Schmitt-Grohé and Uribe (2007) to calculate the conditional compensating variation as a measure for welfare comparison. First, we solve the analytical solution by iterating the welfare function. Then, we employ second-order Taylor expansions to approximate the welfare function and calculate the quantitative results. The results imply how much consumption need to be compensated to households in order to keep the welfare unchanged.

In particular, we first define the welfare function as the lifetime utility of households given by:

$$W_0^i = E_0 \sum_{j=0}^{\infty} \beta^j \left[\frac{(C_t^i)^{1-\sigma}}{1-\sigma} - \Psi \frac{(N_t^i)^{1+\eta}}{1+\eta} + \gamma \log \left(L_t^i \right) \right], \quad i = ZDI, DT, MT \tag{4.5}$$

where W_0^i is the welfare under policy i conditional on the state in period 0. Next, we choose ZDI policy as the benchmark and calculate how much consumption compensation is needed under DT and MT such that the households can achieve the same level of welfare as the benchmark case. Therefore, we have the following relationship:

$$W_0^{ZDI} = E_0 \sum_{j=0}^{\infty} \beta^j \left[\frac{((1+\lambda^i)C_t^i)^{1-\sigma}}{1-\sigma} - \Psi \frac{(N_t^i)^{1+\eta}}{1+\eta} + \gamma \log \left(L_t^i \right) \right], \quad i = DT, MT \tag{4.6}$$

Table 4
Conditional compensating variation.

	Zero-debt-increase (ZDI)	Debt-targeting (DT)	Mixed-targeting (MT)
Tax Cut	0%	0.08%	0.40%
Gov. Spd.	0%	0.10%	0.80%

Table 5
Updated calibration.

Parameter	Description	Value
ξ_t	Households' demand shock	$(-7\%, -22\%, -21\%)$
η_t	Labor supply shock	$(-9\%, 17\%, 17\%)$
τ_t^*	Tax cut shock (of steady state tax)	$(60\%, 60\%, 60\%)$
g_t^*	Gov. spending shock (of GDP)	$(1.13\%, 1.13\%, 1.13\%)$

where λ^i measures the consumption compensation needed. It is obvious that there is a welfare loss when λ^i is positive. We can derive the analytical solution of λ^i as:

$$\lambda^i = exp\left\{(1-\beta)(W_t^{ZDI,*} - W_t^{i,*})\right\} - 1, \quad i = DT, MT \tag{4.7}$$

where W_t^* denotes the steady state of welfare.

We present our welfare comparison results in Table 4. We can see that the DT and MT policies are welfare inferior to the ZDI policy, which is in contrast with the ranking based on the cumulative fiscal multipliers (MT > DT > ZDI). The reason is that the DT and MT policies allow the debt to deviate from its steady state and be repaid gradually. This generates higher and more persistent inflation in the longer term, which offsets the welfare gain from higher output increase.

4.6. Money-financed fiscal stimulus against COVID shocks

The recent pandemic has struck the world economy and resulted in the deepest recession since the Great Depression. In response, a large amount of fiscal measures were implemented to fight against the negative supply and demand shocks caused by COVID. We extend our model by introducing a series of supply and demand shocks as well as fiscal stimulus to assess the policy effects.

4.6.1. Model and calibrations

To produce a "COVID recession" in the model, we follow Bhattarai et al. (2023) to introduce two corresponding shocks into households sector: one is an adverse demand shock capturing the "fear of unsafe consumption"; the other is a supply shock, representing the "lock out of work". Specifically, we modify the households' utility function as:

$$\sum_{T=0}^{\infty} \beta^t e^{\xi_t} \left[\frac{C_t^{1-\sigma}}{1-\sigma} - \Psi \frac{(1+\eta_t)N_t^{1+\varphi}}{1+\varphi} + \gamma log(L_t)\right] \tag{4.8}$$

where ξ_t denotes a preference shock and η_t is a labor supply shock.

We choose the size and duration of COVID shocks following Bhattarai et al. (2023), where the demand and supply shocks coexist for three periods, matching the dynamics of total hours and inflation reported by the U.S. Bureau of Labor Statistics (BLS). In terms of the stimulative measures, we calibrate each to an size of 3.4% of the GDP, which is equivalent to the amount of Coronavirus Aid, Relief, and Economic Security (CARES) Act.[21] These packages are also distributed in three periods[22] (see Table 5).

4.6.2. Dynamic effects

Fig. 6 shows the dynamic effects of output, inflation and government debt in response to the pandemic shocks, with and without fiscal stimulus. The effects of a money-financed tax cut are displayed in Fig. 6(a) whereas the effects of a money-financed government spending are presented in Fig. 6(b). We consider the following four responses to pandemic shocks: no stimulus (black dash line), ZDI policy (blue diamond line), DT rule (red circle line) and MT rule (orange X-shaped line).

There are three points worth noting. First, in the no stimulus case, it is evident that the COVID shocks generate a severe contraction that the output drops down more than 10% in the short-run, and inflation declines by nearly 7% as the aggregate demand falls. In the presence of fiscal stress, the monetary stabilizes the debt level.

Second, if the fiscal authority actively stimulate the economy through an exogenous tax cut, output does not drop as deeply as before. Because of its unfunded property, the money-financed stimulus is also inflationary, which avoids the price level dropping

[21] We abstract from the specific fiscal instruments of the CARES Act, and only consider its total monetary value since it provides a useful reference to examine what if the same amount of stimulus is implemented by a money-financed scheme.

[22] Thus, the tax cut amounts to a decrease in 60% of steady state taxation in each of the three periods under our calibration. Or equivalently, the government spending increases by 1.13 of GDP for each stimulating periods.

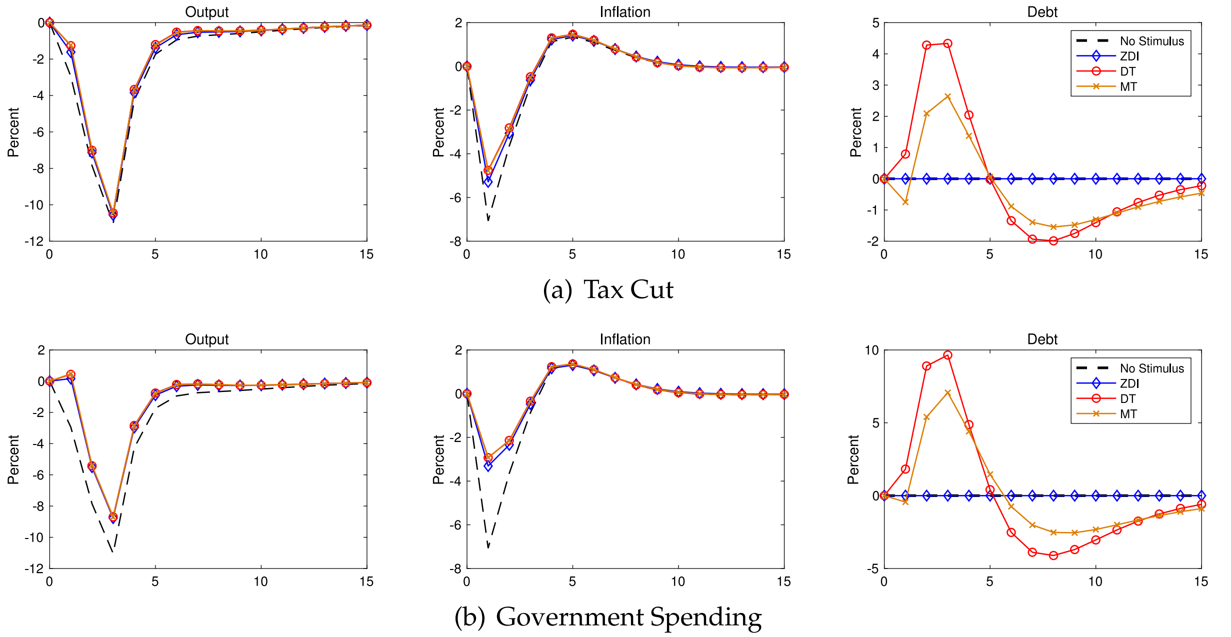


Fig. 6. Money-financed fiscal stimulus against COVID shock. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

Table 6
Fiscal multipliers against COVID shocks.

	IFM	CFM
Tax cut		
Zero-debt-increase (ZDI)	1.1782	1.2832
Debt-target (DT)	1.5615	1.6772
Mixed-target (MT)	1.5081	1.6557
Government spending		
Zero-debt-increase (ZDI)	2.7518	3.7385
Debt-target (DT)	3.0098	4.0182
Mixed-target (MT)	3.0187	4.0130

down to some extent. The effectiveness is shown to be more significant in the case of government spending increase. We can see from Fig. 6(b) that deflation rate reduces to half relative to that in the no-stimulus case.

Finally, the effects of fiscal stimulus on output and inflation under the DT and MT policies are greater relative to those under the ZDI policy. The main difference between them is the evolution of debt level. In MT case, the debt path is less fluctuated since the inflation target helps to print more money (and thus more seigniorage) to combat deflation, which indirectly finances the deficits and partly inflates the government liabilities away.

4.6.3. Fiscal multipliers

To quantitatively evaluate the effects of a money-financed policy intervention on output, we calculate the corresponding IFMs and CFMs as in baseline model. The only difference here is that we define $\hat{y}_t = \hat{y}_{i,t} - \hat{y}_{ns,t}$, where $i = ZDI, DT, MT$ and $\hat{y}_{ns,t}$ denotes the no-stimulus case.

Table 6 presents the IFMs and CFMs in response to the pandemic shocks. We can observe that these multipliers are all above unity regardless of fiscal measures and monetary policy rules. This highlights the effectiveness of money-financing policies against sudden supply and demand shocks. Moreover, government spending increase produces higher multipliers since it directly affects aggregate demand. In terms of financing policies, it is obvious that the ZDI policy performs worse relative to the other two policies. This is because strict debt targeting prevents inflation from surging, and is therefore less effective in a recession. Since MT rule takes inflation into consideration, its long-run effect (CFM) is muted.

4.6.4. Welfare effects

Finally, we study the welfare effects of alternative policies. As in the baseline model, we evaluate the relative welfare gains by calculating its compensation variation of consumption relative to the benchmark case without policy intervention, which represents

Table 7
Welfare gain against COVID shocks.

	Short-Run	Long-Run
Tax cut		
Zero-debt-increase (ZDI)	0.7409	0.0220
Debt-target (DT)	0.9471	0.0289
Mixed-target (MT)	0.9597	0.0287
Government spending		
Zero-debt-increase (ZDI)	1.3503	0.0357
Debt-target (DT)	1.5344	0.0411
Mixed-target (MT)	1.5512	0.0411

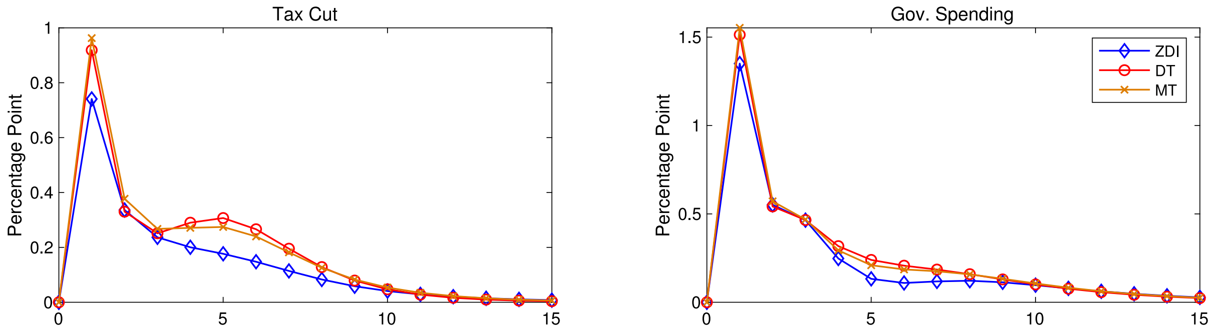


Fig. 7. Dynamics of short-run welfare gains.

the long-run welfare effects of policies. In addition, we compute their period by period welfare gains from period 0 to period t in units of consumption compensation relative to the no-stimulus case as a measurement for its dynamics in short-run. The corresponding results are listed in Table 7.

First and foremost, we find that the money-financed fiscal stimulus produces welfare gains in response to the COVID shocks. This Pareto improvement comes from the consequence that the COVID shocks are recessionary. Thus, the fiscal measures improve the social welfare by stimulating both output and inflation.

Second, we can see that the long-run effects show little difference among financing policies and fiscal measures. It is natural for our experiment because of the model settings that pandemic shocks is transitory with a duration of only three periods. Therefore, the recession does not last long. However, the short-run welfare improvement is significant. While using feedback rules, the welfare gain has been increased by more than 0.2 percentage points of consumption, relative to the ZDI policy. Moreover, fiscal instruments matter. For example, the welfare gain of ZDI policy is nearly doubled from a tax cut to a government spending stimulus.

Finally, we show that the dynamics of short-run welfare gains under different policy mix in Fig. 7. It is obvious that, in the first three periods, the combat to COVID shocks increases social welfare. After that, the dynamics of alternative policies diverge. The ZDI policy, which does not permit debt to rise, experience a sharp decline in welfare. On the other hand, DT and MT rules slow down this process mainly because of its delayed financing scheme, which relies more on inflation to finance debt. As a consequence, a sluggish deficits financing against the background of COVID is a better choice to stimulate the economy in terms of social welfare, although it is accompanied with a more persistent high inflation as what happens in the post-COVID era.

5. Conclusion

The recent pandemic causes the government debt in many economies to rise sharply and therefore hinders the ability of fiscal authorities from stimulating the economy. A heated discussion is sparked on whether a money-financed fiscal stimulus is an effective way to regain economic growth without jeopardizing debt sustainability. Our paper studies this question under a scenario with fiscal stress, where the government is unable or unwilling to increase fiscal revenue in response to a surge of debt. We find that, with a zero-debt-increased monetary policy proposed by Galí (2020), a unique locally bounded equilibrium can be obtained if the monetary policy additionally targets on the growth of debt. A more general debt-targeting rule allows a temporarily increase in debt level in response to fiscal stimulus, and faces a trade-off between short-run and long-run effects that leads to a smaller IMF and a larger CFM. Moreover, introducing an additional inflation target exaggerates this trade-off.

A concern for money-financed fiscal stimulus is that it may generate higher inflation, which erodes the value of seigniorage revenue from money creation. To analyze this issue, we conduct a decomposition of various financing sources. Our results show that seigniorage is not a major source of deficit financing. In contrast, changes in SDF and current inflation play more important roles. Welfare analysis suggests that a sluggish money financing scheme may cause extra welfare loss. Thus, policies that fully and immediately finances deficits at each period is desirable, although it is less effective to generate larger output multipliers in the long run.

Finally, we introduce a series of negative supply and demand shocks to represent COVID shocks, and evaluate the performance of alternative money-financed fiscal stimulus. We find that the effectiveness of fiscal measures is greater during economic downturns. Moreover, social welfare improves significantly. In sum, our analysis highlights the value and transmission channels of alternative financing schemes of fiscal stimulus when fiscal space is limited.

CRediT authorship contribution statement

Hao Jin: Conceptualization, Methodology, Software, Formal analysis, Writing – original draft, Writing – review & editing, Visualization, Supervision, Funding acquisition. **Junfeng Wang:** Conceptualization, Methodology, Software, Formal analysis, Writing – original draft, Writing – review & editing, Visualization.

Data availability

Data will be made available on request.

Appendix A. Equilibrium conditions

A.1. The full non-linear model

This subsection describes the full set of non-linear equilibrium conditions of the model.

First, the optimality conditions drawn from household's life-time utility maximization problem are (3.5), (3.6) and (3.7).

Second, the optimality conditions for intermediate firms under Calvo pricing can be re-written in stationary terms:

$$(\pi_t^*)^{1/\Theta} = \mathcal{M}\,\frac{x_{1,t}}{x_{2,t}}\,\pi_t^{1/\Theta}. \tag{A.1}$$

where π_t^* is the repricing inflation, and $x_{1,t}$, $x_{2,t}$ are auxiliary variables in their recursive form:

$$x_{1,t} = Y_t C_t^{-\sigma} mc_t + \beta\theta E_t x_{1,t+1} \pi_{t+1}^{\frac{1-\alpha+\epsilon}{1-\alpha}}, \tag{A.2}$$

$$x_{2,t} = Y_t C_t^{-\sigma} + \beta\theta E_t x_{2,t+1} \pi_{t+1}^{\epsilon}. \tag{A.3}$$

Meanwhile, the evolution of aggregate inflation rate is expressed by:

$$\pi_t^{1-\epsilon} = (1-\theta)(\pi_t^*)^{1-\epsilon} + \theta, \tag{A.4}$$

and the price dispersion evolves according to:

$$d_t^{\frac{1}{1-\alpha}} = (1-\theta)(\pi_t^*)^{-\frac{\epsilon}{1-\alpha}} \pi_t^{\frac{\epsilon}{1-\alpha}} + \theta \pi_t^{\frac{\epsilon}{1-\alpha}} d_{t-1}^{\frac{1}{1-\alpha}}. \tag{A.5}$$

For the final goods producer, we have the marginal cost:

$$mc_t = \frac{W_t}{(1-\alpha)N_t^{1-\alpha}}, \tag{A.6}$$

and we can also derive the aggregate production function as:

$$Y_t = \frac{N_t^{1-\alpha}}{d_t}. \tag{A.7}$$

Regarding the consolidated government, the government budget constraint can be expressed by:

$$G_t + B_{t-1}(1+r_{t-1}) = T_t + B_t + \Delta m_t \frac{L_t}{\pi_t}, \tag{A.8}$$

where $\Delta m_t = log M_t - log M_{t-1}$, and define $l_t = log L_t$ is the log real balance. Thus, the log money balance follows the relationship below:

$$\Delta m_t = l_t - l_{t-1} + \pi_t. \tag{A.9}$$

Besides, the tax rule follows a non-linear form:

$$\tau_t - \tau = \psi_b(b_t - b) + \hat{t}_t^*, \tag{A.10}$$

where the lowercase variables are ratios to output and $\hat{t}_t^* = -\delta^t$ is a deterministic tax cut shock.

On the other hand, the non-linearized monetary policies are:

$$b_t - b = 0, \tag{A.11}$$

$$\Delta m_t = \varphi_b(b_t - b), \tag{A.12}$$

$$\Delta m_t = \varphi_b(b_t - b) + \varphi_\pi(\pi_t - \pi). \tag{A.13}$$

representing ZDI, DT and MT policies respectively.

Finally, the final goods market clearing condition is given by:

$$Y_t = C_t + G_t. \tag{A.14}$$

where we define $g_t = G_t/Y$ as government spending to output ratio and it follows an exogeneous process $g_t = \delta^t$ while the government increases its spending.

As a result, our non-linear system is characterized by 22 equations: (3.5)–(3.7), (A.1)–(A.14) and the definitions of τ_t, g_t, b_t, $\hat{t}_t^*$ and l_t, which constitutes 22 variables: $\{w_t, C_t, N_t, i_t, \pi_t, L_t, l_t, \Delta m_t, mc_t, Y_t, \pi_t^*, x_{1,t}, x_{2,t}, G_t, g_t, d_t, B_t, b_t, r_t, T_t, \tau_t, \hat{t}_t^*\}$.

A.2. Simplified linearized model

Letting $\hat{y}_t \equiv log(Y_t/Y)$, $\hat{c}_t \equiv log(C_t/C)$, $l_t \equiv log(L_t/L)$, $\hat{\xi} \equiv log(U_{c,t}/U_c)$, $\hat{i}_t \equiv log(1+i_t)/(1+\rho)$, and with $\hat{g}_t = G_t/Y$, we linearize the equilibrium conditions around the steady state and simplify them to obtain[23]:

$$\hat{y}_t = \hat{c}_t + \hat{g}_t, \tag{A.15}$$

$$\hat{\xi}_t = E_t\hat{\xi}_{t+1} + (\hat{i}_t - E_t\pi_{t+1}), \tag{A.16}$$

$$\hat{\xi}_t = -\sigma\hat{c}_t + \nu\hat{l}_t, \tag{A.17}$$

$$\pi_t = \beta E_t\pi_{t+1} - \lambda\hat{\mu}_t, \tag{A.18}$$

$$\hat{\mu}_t = \hat{\xi}_t - \left(\frac{\alpha+\varphi}{1-\alpha}\right)\hat{y}_t, \tag{A.19}$$

$$\hat{l}_t = \hat{c}_t - \eta\hat{i}_t, \tag{A.20}$$

$$\hat{l}_{t-1} = \hat{l}_t + \pi_t - \Delta m_t, \tag{A.21}$$

$$\hat{b}_t = (1+\rho-\psi_b)\hat{b}_{t-1} + b(1+\rho)(\hat{i}_{t-1} - \pi_t) + \hat{g}_t - \hat{\tau}_t^* - \chi\Delta m_t. \tag{A.22}$$

where $\varphi \equiv NV_{nn}/V_n$, $\sigma \equiv -CU_{cc}/U_c$, $\nu \equiv LU_{cl}/U_c$, and $\eta \equiv \epsilon_{lc}/\rho$, with $\epsilon_{lc} \equiv -(1/h')(\rho/(1+\rho))V$ denoting the elasticity of substitution between consumption and real balances. In order to close the model, the equilibrium system is supplemented with specifications of monetary policies described in Section 3.3.

Our complete dynamic system is characterized by the Eqs. (A.15)–(A.22) and a monetary policy, which is to be solved for the paths of $\{\hat{y}_t, \hat{c}_t, \hat{\xi}_t, \hat{i}_t, \pi_t, \hat{l}_t, \hat{\mu}_t, \hat{b}_t, \Delta\hat{m}_t\}$, given two exogenous processes of $\{\hat{g}_t\}$ and $\{\hat{\tau}_t^*\}$. It is useful to rewrite (A.15) as:

$$\hat{c}_t = \hat{y}_t - \hat{g}_t, \tag{A.23}$$

and combine it with equations (A.16)–(A.17) to obtain dynamic IS curve (DIS):

$$\hat{y}_t - \hat{g}_t = E_t\hat{y}_{t+1} - \hat{g}_{t+1} - \frac{1}{\sigma}(\hat{i}_t - E_t\pi_{t+1}). \tag{A.24}$$

Similarly, we can combine (A.23) with Eqs. (A.18) and (A.19) to obtain New Keynesian Philips Curve (NKPC):

$$\pi_t = \beta\pi_{t+1} + \kappa\hat{y}_t - \lambda\sigma\hat{g}_t, \tag{A.25}$$

and combine (A.23) with Eqs. (A.18) and (A.19) to obtain the demand for real money balace:

$$\Delta m_t - \pi_t = \Delta(\hat{y}_t - \hat{g}_t) - \eta\Delta\hat{i}_t. \tag{A.26}$$

It is also useful to eliminate Δm_t by plugging one of the monetary policies illustrated in Section 3.3 into (A.26) and (A.22), and the resulting system is simplified into four difference equations, which can be represented by the vector form as:

$$E_tX_{t+1} = BX_t + CV_t, \tag{A.27}$$

where the vector of the state variables X_t and the vector of exogenous processes V_t are $[\hat{y}_t, \pi_t, \hat{i}_t, \hat{b}_t]^T$ and $[\hat{g}_t, \hat{\tau}_t^*]^T$ respectively.

In rewriting the equations in this form, it is to be understood that $E_t\hat{i}_{t+1} = \hat{i}_{t+1}$ and $E_t\hat{b}_{t+1} = \hat{b}_{t+1}$, and the local equilibrium determinacy depends on the 4×4 matrix B and is irrelevant to the coefficient matrix C for exogenous variables.

[23] Variables without a time subscript denote the steady state value of these variables.

Appendix B. Proof of propositions

B.1. Proof of Proposition 1

Given zero-debt-increase policy, we can rewrite the equation system as the following compact form given by:

$$\begin{bmatrix} E_t\left\{\hat{y}_{t+1}\right\} \\ E_t\left\{\pi_{t+1}\right\} \\ \hat{i}_{t+1} \\ \hat{b}_{t+1} \end{bmatrix} = B \begin{bmatrix} \hat{y}_t \\ \pi_t \\ \hat{i}_t \\ \hat{b}_t \end{bmatrix} + \upsilon\delta^t \tag{B.1}$$

and

$$B = \begin{bmatrix} 1+\kappa(\beta\sigma)^{-1} & w_2(\sigma\eta)^{-1} - (\beta\sigma)^{-1} & -w_1(\sigma\eta)^{-1} & -\varphi_b(\sigma\eta)^{-1} \\ -\kappa\beta^{-1} & \beta^{-1} & 0 & 0 \\ 0 & w_2\eta^{-1} & -w_1\eta^{-1} & -\varphi_b\eta^{-1} \\ 0 & 0 & 0 & \upsilon_b \end{bmatrix}. \tag{B.2}$$

where $\kappa = \lambda(\sigma + (\alpha+\varphi)(1-\alpha)^{-1})$, $w_1 = b(1+\rho)\chi^{-1} - \eta - \sigma^{-1}$, $w_2 = b(1+\rho)\chi^{-1} + 1 - \sigma^{-1}$, $\upsilon_b = 1+\rho-\chi\varphi_b$, denoting parameters relative to the price flexibility λ, target value of real debt b and the strength of the policy response to debt φ_b.

Since the linear system has two variables with expectation, $E_t\hat{y}_t$ and $E_t\pi_t$, Blanchard and Kahn (1980) states that two eigenvalues of the coefficient matrix (B.2) need to be outside unit circle and the other two inside unit circle to ensure a unique stable equilibrium. We characterize the conditions under which the zero-debt-increase monetary policy delivers local determinacy below.

The characteristic polynomial of the matrix (B.2) is

$$\begin{aligned} P(\lambda) = (\lambda - w_3)[\lambda^3 &- (1+\beta^{-1} - w_1\eta^{-1} + \kappa\beta^{-1}\sigma^{-1})\lambda^2 \\ &+ (\beta^{-1} - w_1\eta^{-1} - w_1\beta^{-1}\eta^{-1} + \kappa(1+\eta)\beta^{-1}\sigma^{-1}\eta^{-1})\lambda + w_1\beta^{-1}\eta^{-1}] \end{aligned} \tag{B.3}$$

Note that $w_3 > 0$ is one of the eigenvalues and the absolute value of w_3 must be less than one since the real debt path is an exogenous procedure, while the other three eigenvalues are the roots of the following polynomial:

$$\begin{aligned} p(\lambda) = \lambda^3 &- (1+\beta^{-1} - w_1\eta^{-1} + \kappa\beta^{-1}\sigma^{-1})\lambda^2 \\ &+ (\beta^{-1} - w_1\eta^{-1} - w_1\beta^{-1}\eta^{-1} + \kappa(1+\eta)\beta^{-1}\sigma^{-1}\eta^{-1})\lambda + w_1\beta^{-1}\eta^{-1} \end{aligned} \tag{B.4}$$

Obviously, $p(1) = \kappa\beta^{-1}\sigma^{-1}\eta^{-1} > 0$, $p(-\infty) < 0$ and $p(\infty) > 0$. According to Blanchard and Kahn (1980) condition, there should be necessarily one root inside the unit circle and the others outside the unit circle. Thus $p(-1) < 0$, which is equivalent to (3.25) and keeps an odd number of roots lies in the unit circle.

Next we need to rule out that there exists three roots whose absolute values are less than 1. By Schur–Cohn criterion,[24] all roots of polynomial $p(\lambda) = \lambda^3 + a_2\lambda^2 + a_1\lambda + a_0$ lie inside the unit circle if and only if:

$$\left|a_0 + a_2\right| < 1 + a_1 \tag{B.5}$$

and

$$\left|a_1 - a_0a_2\right| < 1 - a_0^2 \tag{B.6}$$

Plug the coefficients of (B.4) into the condition (B.6), we have:

$$\begin{aligned} LHS =&|\beta^{-1} - w_1\eta^{-1} - w_1\beta^{-1}\eta^{-1} \\ &+ \kappa(1+\eta)\beta^{-1}\sigma^{-1}\eta^{-1} + w_1\beta^{-1}\eta^{-1}(1+\beta^{-1} - w_1\eta^{-1} + \kappa\beta^{-1}\sigma^{-1})| \end{aligned} \tag{B.7}$$

$$RHS = 1 - w_1^2\beta^{-2}\eta^{-2} \tag{B.8}$$

We can easily show that RHS is negative when $|w_1| > \beta\eta$ and (B.6) cannot be satisfied. On the other hand, when $|w_1| < \beta\eta$, rearrange the polynomial in the absolute value notation of LHS, we have :

$$\begin{aligned} &\beta^{-1}(1 - w_1^2\eta^{-2}) - w_1\eta^{-1}(\beta^{-2} - 1) + \kappa\beta^{-1}\sigma{-}1\eta{-}1(w_1\beta^{-1} + 1 + \eta) \\ \geq&(\beta^{-1} - \beta) - (\beta^{-1} - \beta) + \kappa\beta^{-1}\sigma^{-1}\eta{-}1 \\ \geq&\kappa\beta^{-1}\sigma^{-1}\eta{-}1 \\ >&0 \end{aligned} \tag{B.9}$$

[24] Refer to LaSalle (2012).

Therefore, the inequality (B.6) turns to be:

$$(1-\beta)\beta^{-2}\eta - 2[w_1^2 + \eta(1+\beta)w_1 + \beta\eta^2] < 0 \tag{B.10}$$

Construct auxiliary function:

$$f(w_1) = w_1^2 + \eta(1+\beta)w_1 + \beta\eta^2 \tag{B.11}$$

The first order derivative of the auxiliary function (B.11) is:

$$f'(w_1) = 2w_1 + \eta(1+\beta) \geq -2\beta\eta + \eta(1+\beta) > 0 \tag{B.12}$$

where the second inequality uses the assumption $|w_1| < \beta\eta$. Obviously, the function (B.11) is increasing in $[-\beta\eta, \beta\eta]$ and the local minimum is $f(-\beta\eta) = 0$. Thus, $f(w_1) \geq 0$ holds when $|w_1| < \beta\eta$, which means the inequality (B.10) and therefore the condition (B.6) does not hold.

Above all, there must be two roots inside the unit circle while the other two outside. The system is locally determinant.

B.2. Proof of Proposition 2

Plugging in the debt-targeting rule (3.18) into the system yields the compact form $E_t X_{t+1} = BX_t + CV_t$, where coefficient matrix B is given by:

$$B = \begin{bmatrix} 1+\kappa(\beta\sigma)^{-1} & \sigma^{-1}v_1 - (\beta\sigma)^{-1} & \sigma^{-1}v_2 & -\varphi_b(\sigma\eta)^{-1} \\ -\kappa\beta^{-1} & \beta^{-1} & 0 & 0 \\ 0 & v_1 & v_2 & -\varphi_b\eta^{-1} \\ 0 & -b(1+\rho) & b(1+\rho) & v_b \end{bmatrix} \tag{B.13}$$

where $v_1 = \eta^{-1}(1-\sigma^{-1})$, $v_2 = 1 + (\sigma\eta)^{-1}$ and $v_b = 1 + \rho - \chi\varphi_b$ is an auxiliary variable related to the strength of the policy response φ_b.

Again, since the system has two expectations, determinacy requires two explosive eigenvalues, so we have the following result. The characteristic function of the matrix (B.13) follows the form:

$$p(\lambda) = \lambda^4 + a_3\lambda^3 + a_2\lambda^2 + a_1\lambda + a_0 \tag{B.14}$$

where,

$$a_0 = \beta^{-1}[v_b(1+\sigma^{-1}\eta^{-1}) + b(1+\rho)\eta^{-1}\varphi_b] \tag{B.15}$$

$$a_1 = -[\beta^{-1}(1+\sigma^{-1}\eta^{-1}+v_b) + (1+\beta^{-1})(v_b(1+\sigma^{-1}\eta^{-1}) + b(1+\rho)\eta^{-1}\varphi_b + \kappa\beta^{-1}\sigma^{-1}(1+\eta^{-1}))] \tag{B.16}$$

$$a_2 = (1+\beta^{-1}+\sigma^{-1}\eta^{-1}+\kappa\beta^{-1}\sigma^{-1}+v_b) + \beta^{-1}(1+\sigma^{-1}\eta^{-1}+v_b) + [v_b(1+\sigma^{-1}\eta^{-1}) + b(1+\rho)\eta^{-1}\varphi_b] + \kappa\beta^{-1}\sigma^{-1}(\eta^{-1}+v_b) \tag{B.17}$$

$$a_3 = -(2+\beta^{-1}+\sigma^{-1}\eta^{-1}+\kappa\beta^{-1}\sigma^{-1}+v_b) \tag{B.18}$$

According to the Blanchard and Kahn condition, the system is local determined if and only if there exists two roots lying in the unit circle. By model specification, we can easily show that the coefficients of (B.14) follows:

$$a_1, a_3 < 0 \tag{B.19}$$

and,

$$a_0, a_2 > 0 \tag{B.20}$$

Therefore, we have $p(-\infty) > 0$, $p(\infty) > 0$ and $p(-1) > 0$. If there exists two roots (or an even number of roots) lies in the unit circle, $p(1) > 0$ must hold. From (B.14) we have:

$$p(1) = \kappa\beta^{-1}\sigma^{-1}\eta^{-1}(\chi\varphi_b - \rho) > 0 \tag{B.21}$$

which is equal to Proposition 2.

B.3. Proof of Proposition 3

Analogous to previous two cases, combining the mixed-targeting rule with the system yields $E_t X_{t+1} = BX_t + CV_t$, where coefficient matrix B is given by:

$$B = \begin{bmatrix} 1+\kappa(\beta\sigma)^{-1} & (\sigma\eta)^{-1}(1-\sigma^{-1}) - (\beta\sigma)^{-1} & \sigma^{-1}\left(1+(\sigma\eta)^{-1}\right) & 0 \\ -\kappa\beta^{-1} & \beta^{-1} & 0 & 0 \\ 0 & v_1\eta^{-1} & 1+(\sigma\eta) & -v_b\eta^{-1} \\ 0 & v_2 & b(1+\rho) & v_b \end{bmatrix} \tag{B.22}$$

where $\upsilon_1 = 1 - \sigma^{-1} - \varphi_\pi$, $\upsilon_2 = -\chi\varphi_\pi - b(1+\rho)$ and $\upsilon_b = 1 + \rho - \chi\varphi_b$ are three auxiliary variables related to the strength of the policy response φ_π and φ_b.

The characteristic function of the matrix (B.22) follows the form:

$$p(\lambda) = \lambda^4 + a_3\lambda^3 + a_2\lambda^2 + a_1\lambda + a_0 \tag{B.23}$$

where,

$$a_0 = \beta^{-1}[\upsilon_b(1+\sigma^{-1}\eta^{-1}) + b(1+\rho)\eta^{-1}\varphi_b] \tag{B.24}$$

$$\begin{aligned} a_1 = &-[\beta^{-1}(1+\sigma^{-1}\eta^{-1}+\upsilon_b) + (1+\beta^{-1})(\upsilon_b(1+\sigma^{-1}\eta^{-1})) + b(1+\rho)\eta^{-1}\varphi_b \\ &+ \kappa\beta^{-1}\sigma^{-1}\upsilon_b(1+\eta^{-1}+\upsilon_1\eta^{-1}) + \chi\eta^{-1}\varphi_\pi\varphi_b] \end{aligned} \tag{B.25}$$

$$\begin{aligned} a_2 = &(1+\rho+\beta^{-1}+\sigma^{-1}\eta^{-1}+\kappa\beta^{-1}\sigma^{-1}+\upsilon_b) + \beta^{-1}(1+\sigma^{-1}\eta{-}1+\upsilon_b) + [\upsilon_b(1+ \\ &\sigma^{-1}\eta^{-1}) + b(1+\rho)\eta^{-1}\varphi_b] + \kappa\beta^{-1}\sigma^{-1}(\sigma^{-1}\eta^{-1}+\upsilon_b+\upsilon_1\eta^{-1}) \end{aligned} \tag{B.26}$$

$$a_3 = -(2+\beta^{-1}+\sigma^{-1}\eta^{-1}+\kappa\beta^{-1}\sigma^{-1}+\upsilon_b) \tag{B.27}$$

We can easily prove that, under model specification, the coefficients of (B.23) show the following properties:

$$a_1, a_3 < 0 \tag{B.28}$$

and,

$$a_0, a_2 > 0 \tag{B.29}$$

By Descartes' rule of signs,[25] all the four roots of Eq. (B.23) are positive. On the other hand, we can calculate the intercept:

$$p(0) = a_0 = \beta^{-1}[\upsilon_b(1+\sigma^{-1}\eta^{-1}) + b(1+\rho)\eta^{-1}\varphi_b] > 0 \tag{B.30}$$

According to the Blanchard and Kahn condition, if the local determinacy is satisfied, there are necessarily two roots inside the unit circle. Thus, the characteristic function (B.23) need to satisfy the condition:

$$p(1) = 1 + a_1 + a_2 + a_3 = -\kappa\beta^{-1}\sigma^{-1}\eta^{-1}[\chi\varphi_b - \rho(1-\varphi_\pi)] > 0 \tag{B.31}$$

which is equivalent to Proposition 3.

Appendix C. Sensitivity analysis

In this section, we analyze the sensitivity of the conclusion above regarding the effectiveness and mechanisms of fiscal policy. We focus on three parameters: those measuring the degree of price stickiness, velocity of money and the steady state level of debt.

C.1. Price stickiness

Firstly, we consider the effects of money-financed fiscal stimulus under different degree of price stickiness. As the analysis in Galí (2020), based on a separable real balances ($\upsilon = 0$), the policy effects depends on the price stickiness. In particular, if the price is flexible, the real effects is irrelevant to financing methods as a consequence of Ricardian equivalence. On the contrary, the irrelevance result no longer holds when the prices are sticky, and the sign and size of policy effects will be different under different degree of price stickiness.

Figs. 8 and 9 display the impulse response over time of macroeconomic variables of interest to an exogenous tax cut and an increase in government spending respectively. In each figure, we consider two state of price stickiness: a low level at $\theta = 1/4$, represented by blue diamonds, and a high level at $\theta = 3/4$, which is represented by red dots and in accord with the baseline calibration. As the figures make clear, a low price stickiness significantly dampens the instantaneous fiscal multipliers, independent of the fiscal measures and the monetary policy rules. The reason is obvious: When the price stickiness is low, there are more firms to set an over-average markup, and the inflation is higher. A higher inflation means a higher current inflation tax, which in turn reduces the current money growth and therefore raises the nominal interest rate. As a result, the increase in the nominal rate is greater than it in inflation, which finally pulls up the real rate and restrains the aggregate demand and output. Moreover, the difference between high and low level of price stickiness becomes smaller when using debt-target and mixed-target rule than zero-debt-increase policy. This comes from the difference of financing methods. When using debt-target rule, the government allows the debt to change temporarily and delays the timing of the money growth cut. As a result, the nominal rate will be less fluctuated and the maximum is delayed and smaller than ZDT. Thus, the real rate increases less and later, and the instantaneous difference of output turns to be smaller. It is similar to a mixed-target rule, but only different in the timing of money growth. If adding inflation to monetary policy targets, the money supply will be reduced in the first period instead of the next period, because it needs to adjust to control the inflation currently. Therefore the peaks of nominal and real rate happen one period earlier than DT, but still later than ZDI, which happens at the first month.

[25] See Barbeau (2003).

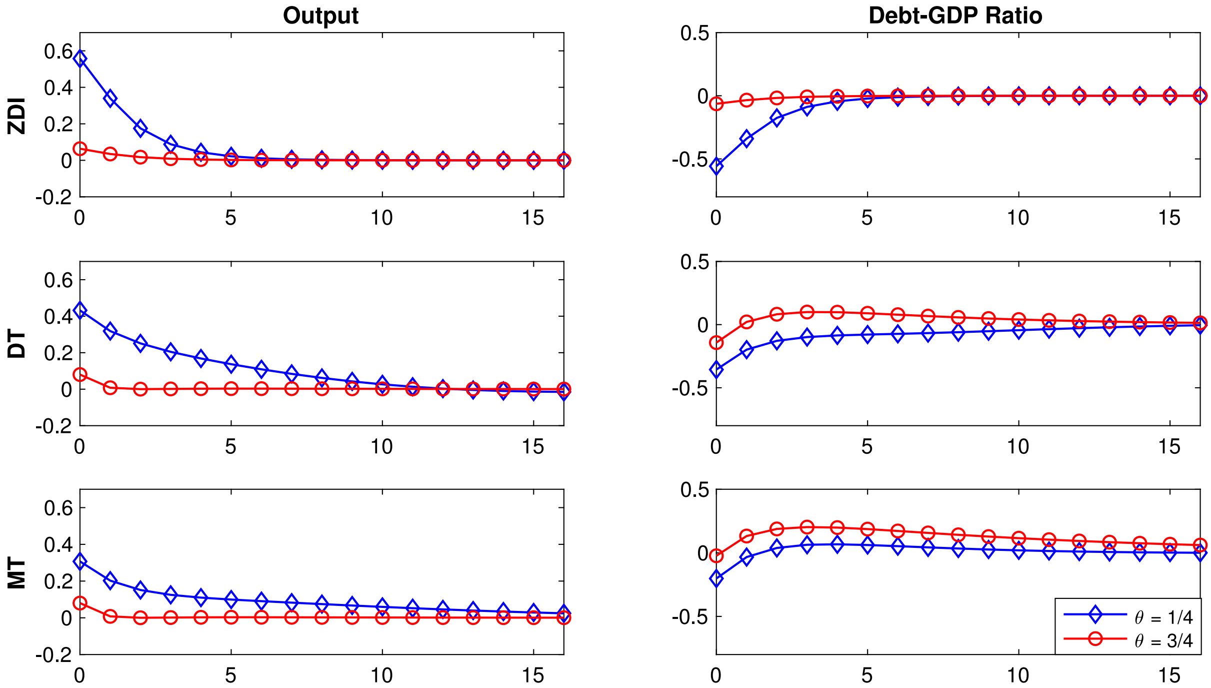


Fig. 8. Impulse response function under tax cut: the role of price stickiness parameter. Note: We use $\theta = 1/4$ for low stickiness in blue diamonds and $\theta = 3/4$ for high stickiness in red dots.

Figs. 10 and 11 show a simulation of financing decomposition under tax cut and government spending across a continuous number of price stickiness parameter. For every figure, we separate a bar into three fractions-yellow, red and blue, representing current inflation, SDF and seigniorage respectively. The proportion of the fraction of one bar means the percentage contribution to the financing regime. We can see from the figure that the inflation becomes a dominant contributor as the price stickiness decreases, while the SDF finances the most at a larger stikiness level. This is because a more flexible price pushes up price level, which inflates the total debt currently. And it also proves the conclusion in the IRF analysis that, when price is less sticky, a increase in real interest rate constrains the output and raises the future interest of government debt at the same time.

Fig. 12 displays the cumulative output multipliers for a tax cut and an increase in government purchases as a function of the index of price stickiness. The black line, blue diamonds and red dots represent zero-debt-increase policy, debt-target rule and mixed target rule respectively. Firstly, the CFM is increasing in the price stickiness θ, which is the same as in Galí (2020) without fiscal stress. It means that fiscal stress has nothing to do with the sign of the relationship between price stickiness and the effects of money financed fiscal stimulus. Secondly, when using a more flexible rule letting government debt changes temporarily, the fiscal multiplier becomes larger under a more sticky price. The reason is that when price is more sticky, the current inflation increases less and therefore SDF should finance more for the deficits. To do this, the future interest rates need to be cut, which reduces the interest expenditure for government debt and stimulates the aggregate demand and output in the long run.

C.2. Velocity of money

Apart from price stickiness, velocity of money is another parameter that is important to the effectiveness of monetary finance. As what we discuss in Section 3.1.3, if we approximate the seigniorage in the neighborhood of steady state, it can be expressed as a fraction of output. The linearization is: $seigniorage = \chi \Delta m_t$, and the multiplier χ represents the inverse velocity of money. Therefore, if χ decreases, i.e., the velocity of money increases, the quantity of seigniorage through the same amount of money creation becomes larger. Equivalently, it means that the issuance of money is more powerful for financing outstanding debt. Figs. 13 and 14 illustrates the impulse response for main macroeconomic variables of interests. We use blue diamonds to represent a smaller $\chi = 1/3$, which equals to a high level of money velocity, while red dots represents a bigger $\chi = 1$, which is equal to a low level velocity of money and shows the basic calibration in the baseline model. From the figures we can see that a high level of money velocity generates a bigger fiscal multiplier regardless of fiscal measures (tax cut or increase in government spending) and monetary policy rules. We find that, under a high level of velocity ($\chi = 1/3$), the money growth is significantly increased because of the contraction of seigniorage. As a result the nominal rates and real rates follow to drop down, which stimulates the aggregate demand and output. On the other hand, there is a difference between zero-debt-increase policy and the flexible rules. When implementing a more flexible rule, the gap of deficits can be financed partly by an increase in debt. Therefore the money growth rises gradually, which causes the interest

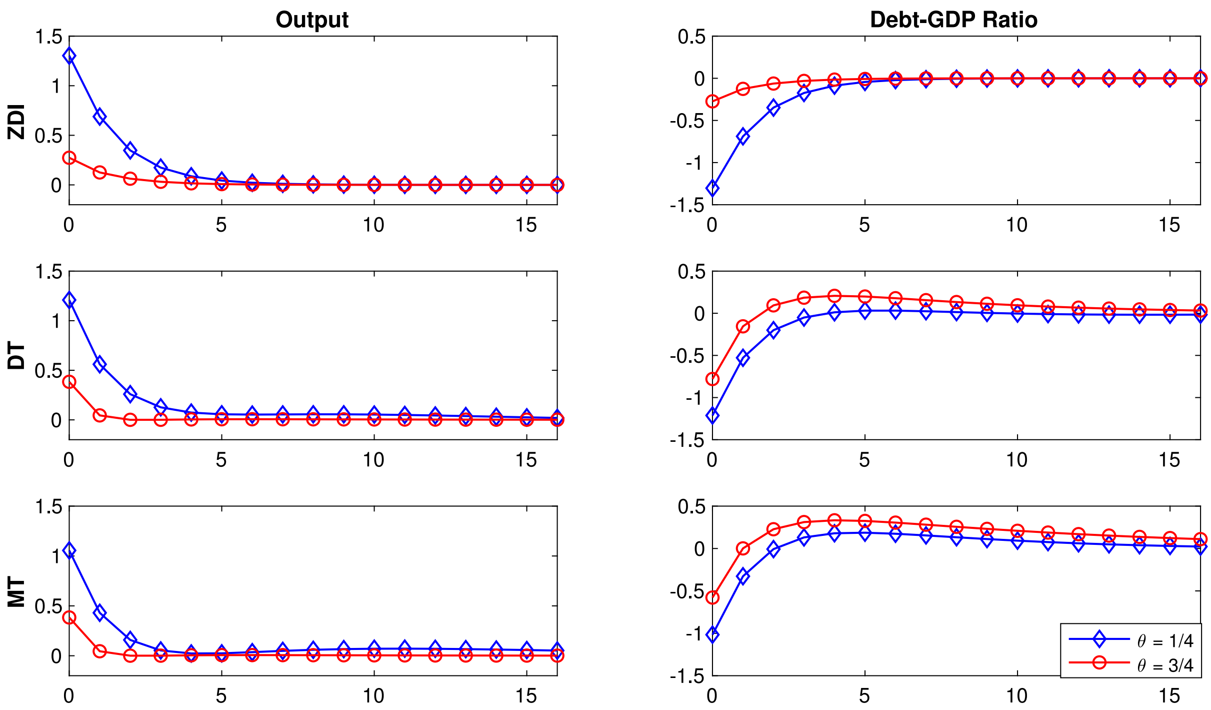


Fig. 9. Impulse response function under government spending: the role of price stickiness parameter. Note: We use $\theta = 1/4$ for low stickiness in blue diamonds and $\theta = 3/4$ for high stickiness in red dots.

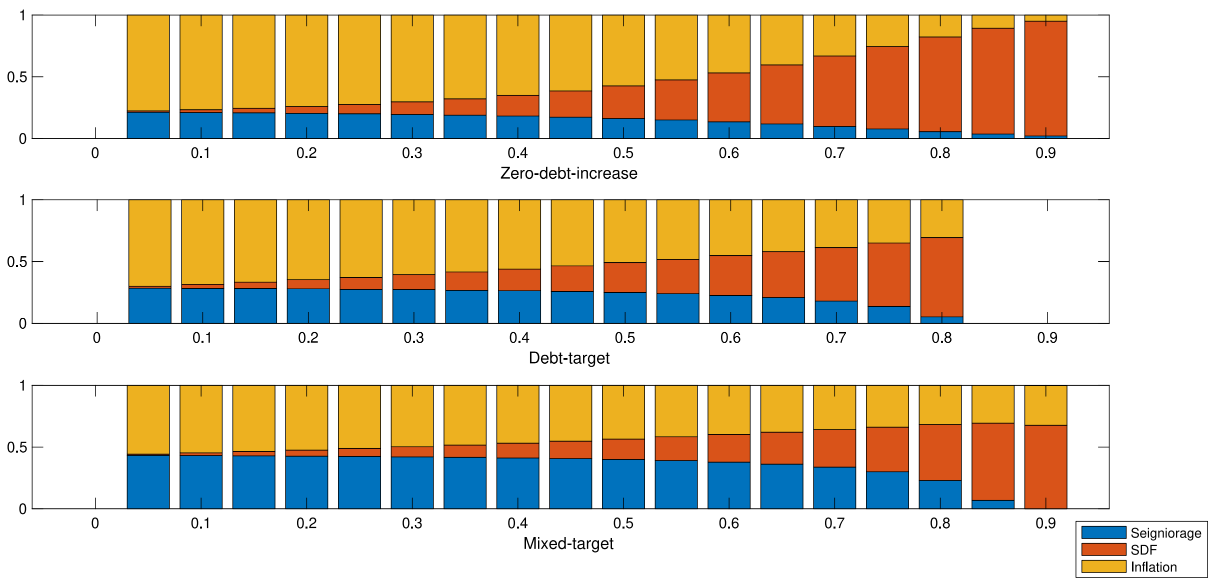


Fig. 10. Debt financing decomposition under tax cut: The role of price stickiness parameter. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

rates to experience a slower decrease instead of an abrupt drop. Thus, the output and consumption go up in a less extent than what happens in zero-debt-increase policy.

Figs. 15 and 16 displays the debt financing decomposition under different velocity of money. Similar to the decomposition for price stickiness, we use blue part, red part and yellow part to represent the contribution of seigniorage, SDF and current inflation respectively. The contribution is measured by percentage. Three observations are worth making. Firstly, the seigniorage never plays the most important role, especially when there is a tax cut fiscal stimulus. Essentially, the change of velocity of money is the change

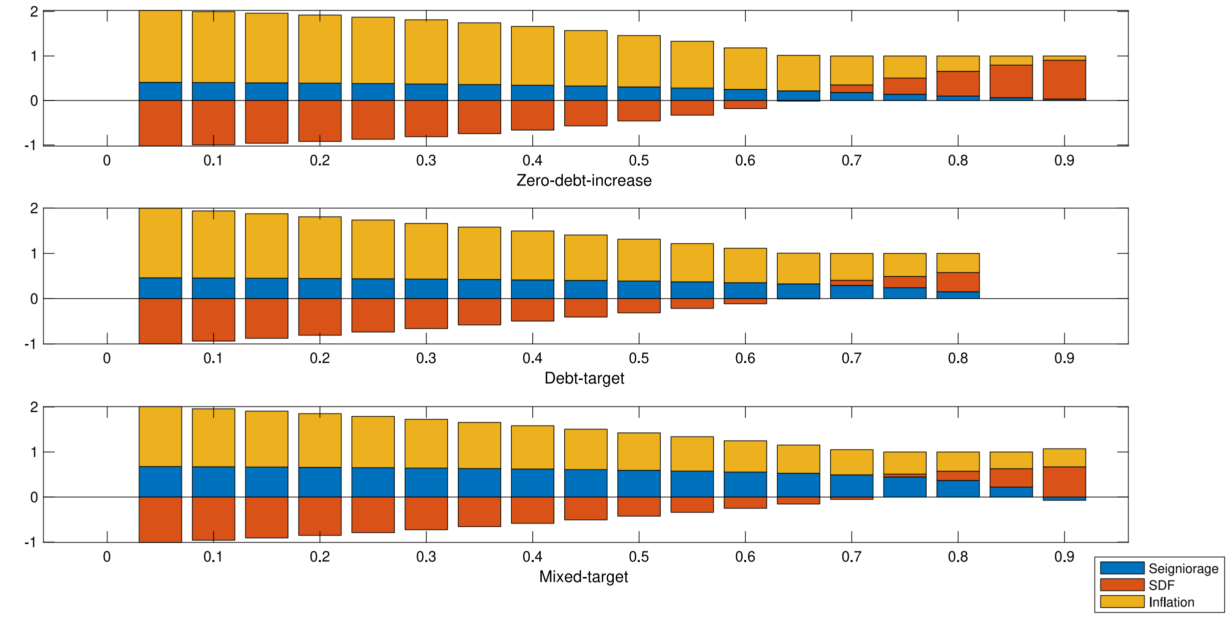


Fig. 11. Debt financing decomposition under government purchase: The role of price stickiness parameter. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

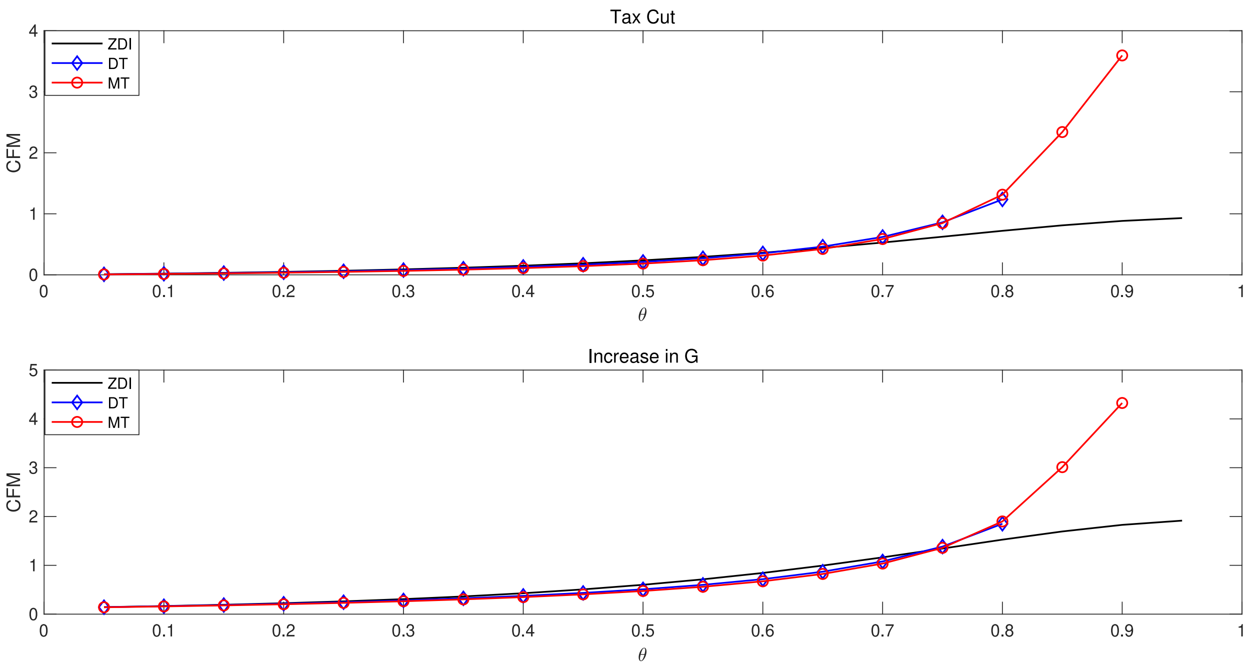


Fig. 12. Cumulative fiscal multipliers: The role of price stickiness.

of seigniorage multiplier. Therefore, the variation of money velocity affects the fiscal outcome less than other factors, which can be found in the IRF-the differece between high and low χ is smaller than the difference bewteen different price stickiness. Second, the SDF contributes the most to tax-cut fiscal stimulus and current inflation contributes the most to government-spending fiscal stimulus. This is because government spending is a more direct measure to influence the economy and it can generate an expansionary effect at the first period through an increase in aggregate demand. Thus, the current inflation raises more, which inflates the outstanding debt more. Thirdly, the financing from current inflation is an increase function of money velocity (an decrease function of χ equivalently). It means that the short run effects of fiscal stimulus may be larger when χ is smaller, which is proved by the figures above about IRFs.

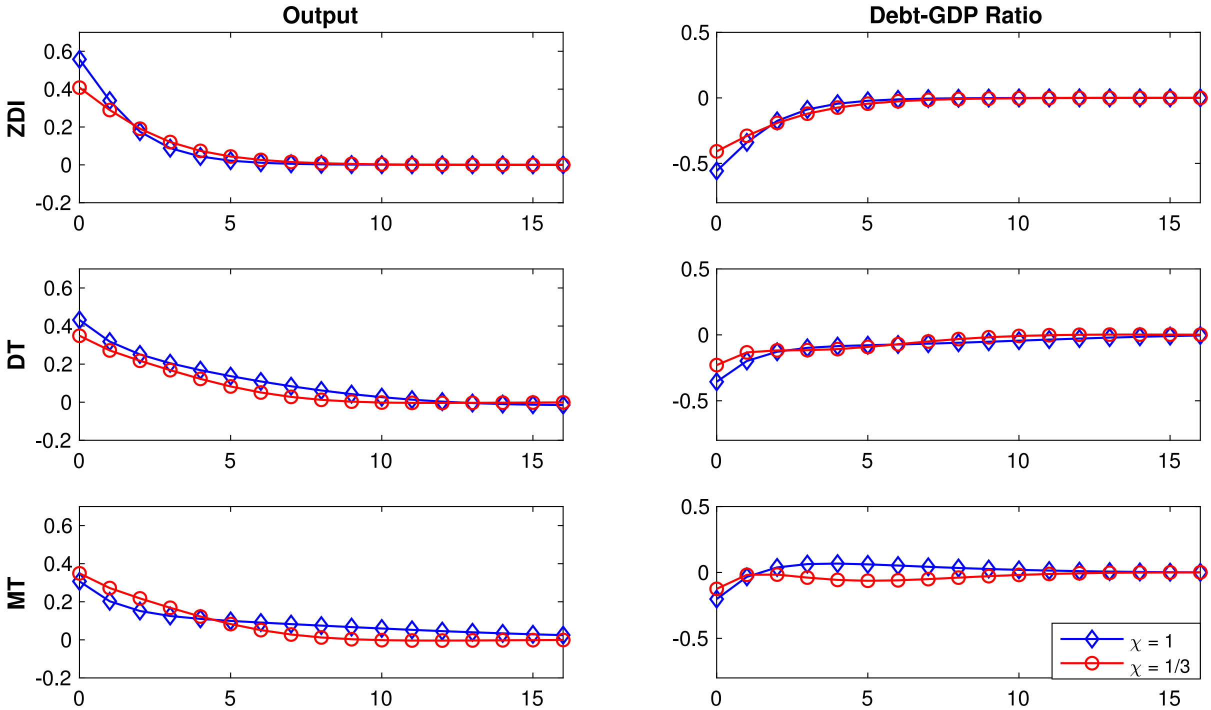


Fig. 13. Impulse response function under tax cut: The role of velocity of money. Note: We use $\chi = 1/3$ for high velocity in blue diamonds and $\chi = 1$ for low velocity in red dots.

Fig. 17 shows the long run effects of money-financed fiscal stimulus under different velocity of money. In particular, we use cumulative fiscal multiplier as a measure and compare the difference among three monetary rules. We use black lines to represents zero-debt-increase policy as a benchmark, while blue diamonds and red dots represents debt-target and mixed-target rules respectively. In the case of tax cut, the flexible rules generates a significantly larger cumulative fiscal multipliers than ZDT, while the same is true in the case of increase in government spending, but in a moderate extent. This is because the flexible rules permits a short-run increase in debts, which needs to be pay in a longer run to repay by cutting interests. The drops in SDF generates the long run effectiveness. On the other hand, the cumulative fiscal multipliers is decreasing in χ, which is the same as short run and means a lower velocity of money erodes both the long run and the short run fiscal multipliers.

C.3. Steady state level of debt

Finally, we discuss the influence of steady state level of debt on the effects of money-financed fiscal stimulus under fiscal stress. We compare two conditions: one is a medium level of debt-to-output level, which is the upper limit of Maastricht Treaty at 60% (i.e., $b = 2.4$), the other is a high level of debt-to-output level, which is reached by some leading developed countries, at 100% (i.e., $b = 4$). We analyze the difference of the short-run, long-run and financing decomposition on fiscal outcomes.

Fig. 18 displays the impulse response functions of several macroeconomic variables after a tax-cut fiscal stimulus. In particular, we use blue diamonds to represent the medium level($b = 2.4$), while the red dots represents the high level($b = 4$). We can see from the picture directly that the instantaneous fiscal multiplier turns to be less when the steady state level of debt is higher. The mechanism behind is worth discussing. First, we focus on the zero-debt-increase policy. When the government implements a tax cut, the money growth raises to finance the debt gap, keeping it unchanged. The quantity of newly issued money in this period is independent of steady state level of debt, but it in the next period be. If the debt burden is high in the steady state, the money creation will inflate more debt and save the interest in the next period. Therefore, the money growth in the next period will drop down and the expected inflation decreases. On the other hand, if the cut down of money growth in the second period realizes, the expected nominal rate will raise as well, which finally increases the expected real interest rate. As the discussion in Galí (2020), the aggregate demand and output are a function of current and expected real interest rate. Although the real rate in this period remains the same, the increase in next period will also reduce the output multipliers when the steady state debt-to-output level is large. Second, if the monetary authority implements a debt-target rule instead, the same is true but only different in size. Specifically, the money growth fluctuate more smoothly, but the instantaneous fiscal multiplier still reduces through the lower expectation real interest rate. We can also find that under a mixed-target rule, the fiscal measure generates even less multipliers because of the inflation controlling, which tightens the money supply. Moreover, the inflation becomes lower when the debt level is larger. This

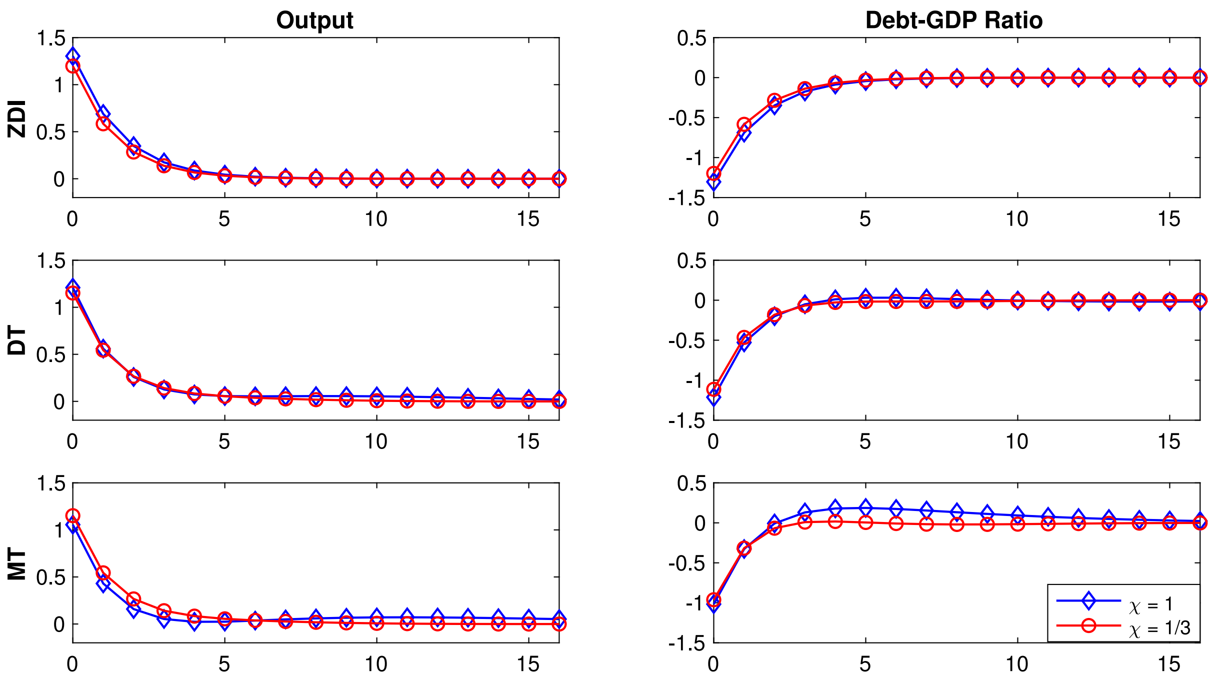


Fig. 14. Impulse response function under government spending: The role of velocity of money. Note: We use $\chi = 1/3$ for high velocity in blue diamonds and $\chi = 1$ for low velocity in red dots.

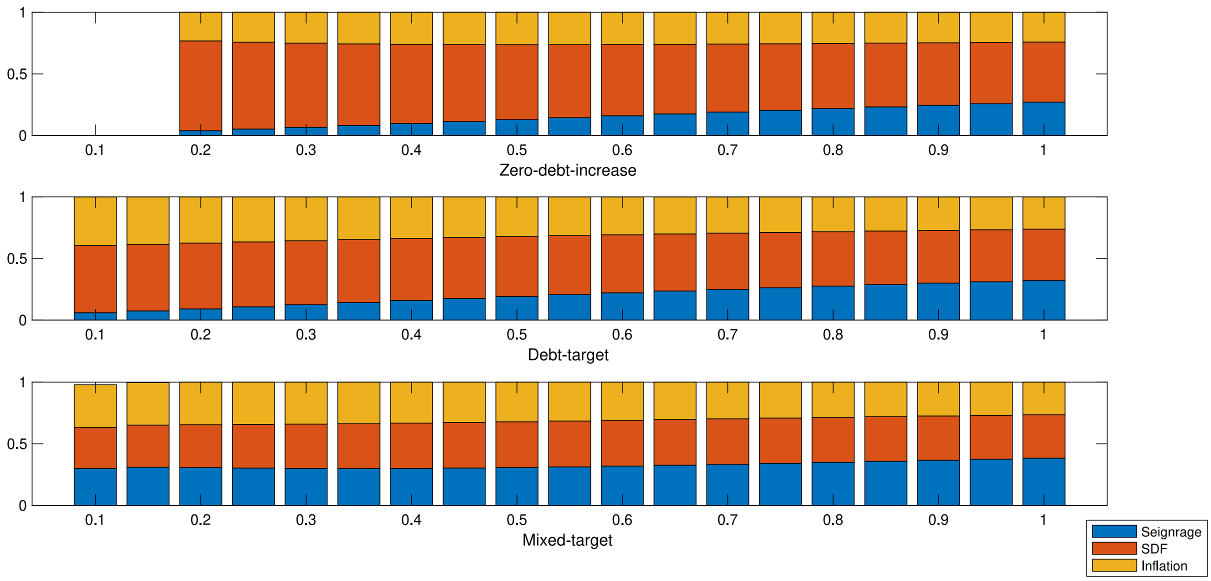


Fig. 15. Debt financing decomposition under tax cut: The role of velocity of money. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

is also related to the expectation of money growth. If the future money issuance is expected to be less, the inflation will not raises as much as before.

Fig. 19 displays the impulse response functions under government spending. As tax cut, the blue diamonds and red dots represents the medium level($b = 2.4$) and high level($b = 4$) respectively. Different from the tax-cut fiscal stimulus, an increase in government spending affects the aggregate output immediately. When the aggregate demand increases, the output and inflation raise up, though there is a crowd-out effect on consumption. The expanded inflation leads to more inflation tax under high debt level, which in turn means a low growth(even decrease) in money supply. This raises the real rate and the dampens the fiscal multipliers. Besides, higher interest rates aggravate the interest burden of debt, resulting in a higher money growth next period, which makes the economy

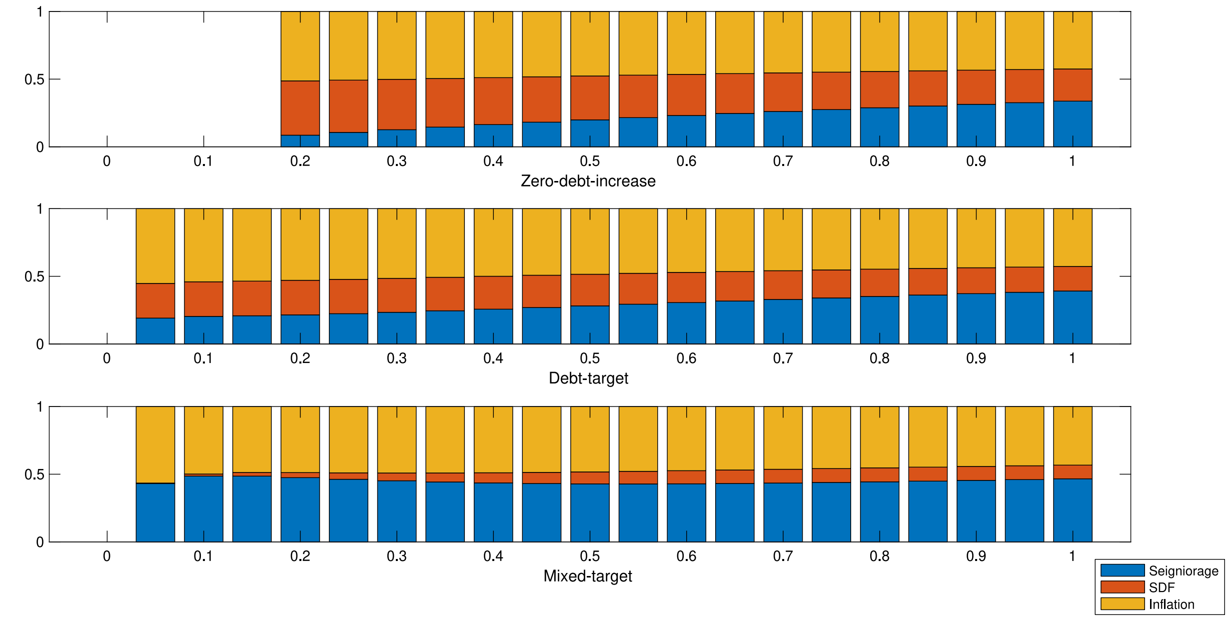


Fig. 16. Debt financing decomposition under government purchase: The role of velocity of money. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

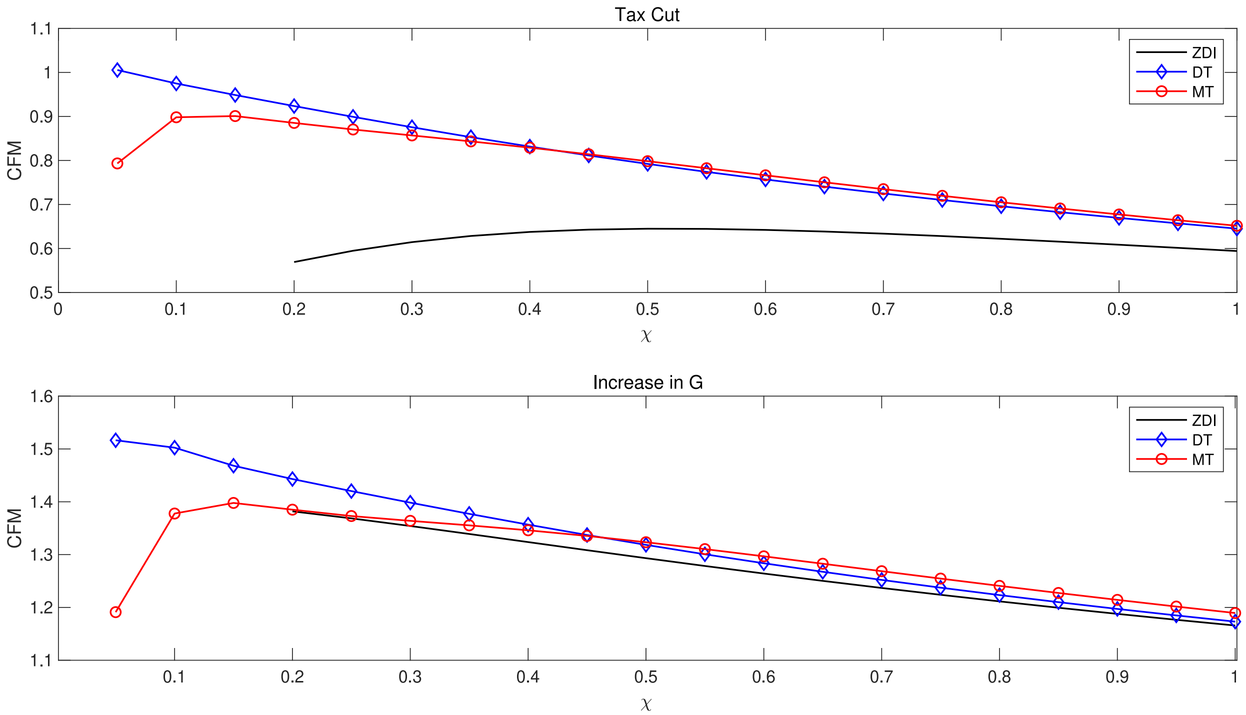


Fig. 17. Cumulative fiscal multipliers: The role of velocity of money.

more fluctuated. Similarly, the debt-target and mixed-target rule experience the same procedure, but a longer period of crowd-out effect. This is because the money growth is smoothed by the debt adjustment, which makes the real rate always higher than before. Moreover, it is worth noting that the instantaneous output multiplier may be less than 1 under mixed-target rule when steady state debt level is high, which is opposite to Galí (2020). As what we discuss before, adding an inflation target to monetary policy surely dampens the effectiveness of a money-financing regime.

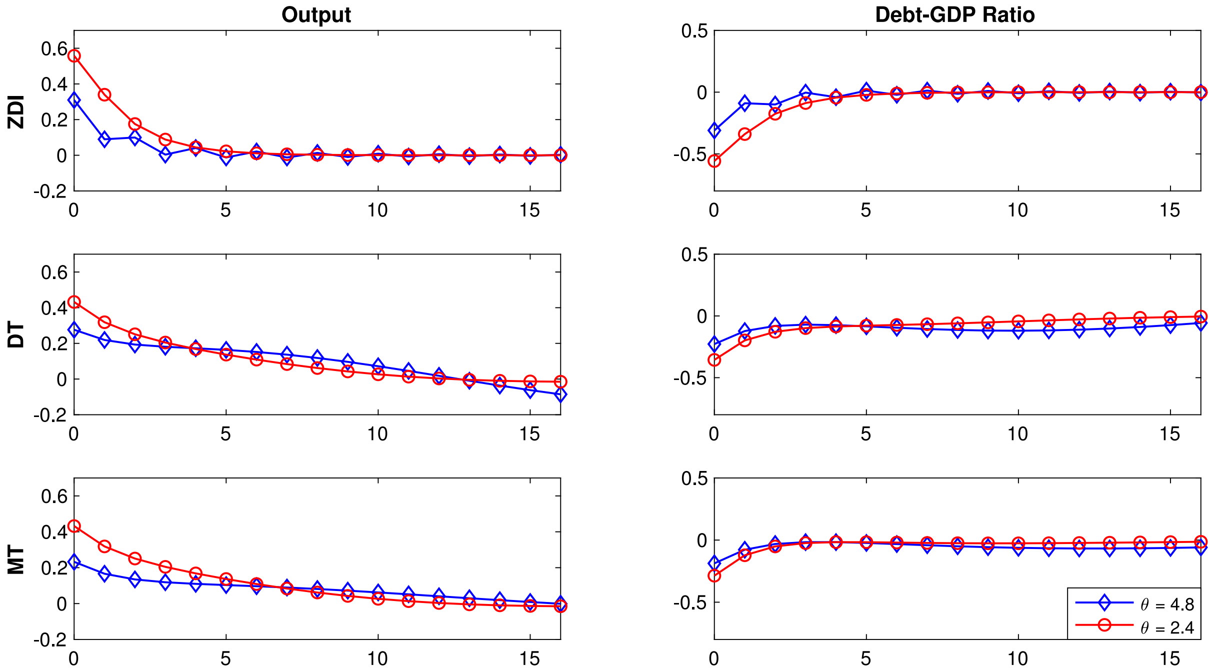


Fig. 18. Impulse response function under tax cut: The role of steady state level of debt. Note: We use $b = 2.4$ for low debt-to-output level in blue diamonds and $b = 4$ for high debt-to-output level in red dots.

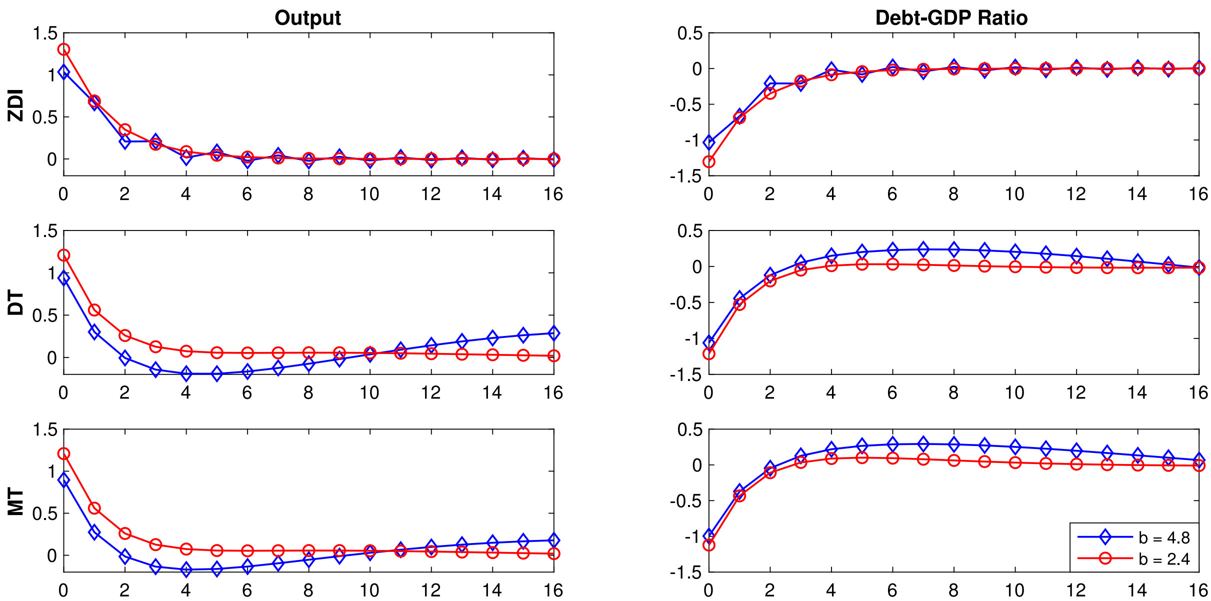


Fig. 19. Impulse response function under government spending: The role of steady state level of debt. Note: We use $b = 2.4$ for low debt-to-output level in blue diamonds and $b = 4$ for high debt-to-output level in red dots.

Figs. 20 and 21 shows the debt financing decomposition under tax cut and government spending across different steady state levels of debt respectively. We use blue, red and yellow part to represent the proportional contribution of seigniorage, SDF and current inflation to deficit financing. There are three observations worth making. First, it is obvious that the seigniorage dominates only when the debt level is small. Especially, when there is no debt, inflation tax(SDF and current inflation) does not works. All the payments of newly issued debt and their interests come from seigniorage. Second, the SDF plays the most important role in tax cut while the current inflation contributes the most when increase in government spending. This accords with the analysis about IRFs. When implementing a tax cut, the expected inflation, and therefore the interest rates, determines the fiscal multipliers. On the

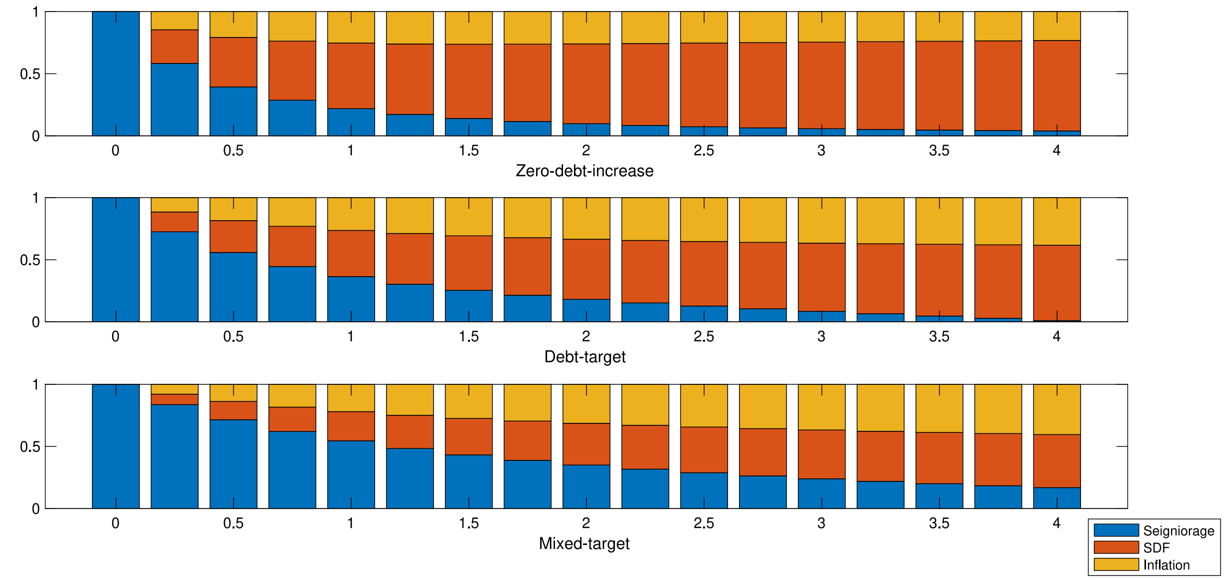


Fig. 20. Debt financing decomposition under tax cut: The role of steady state level of debt. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

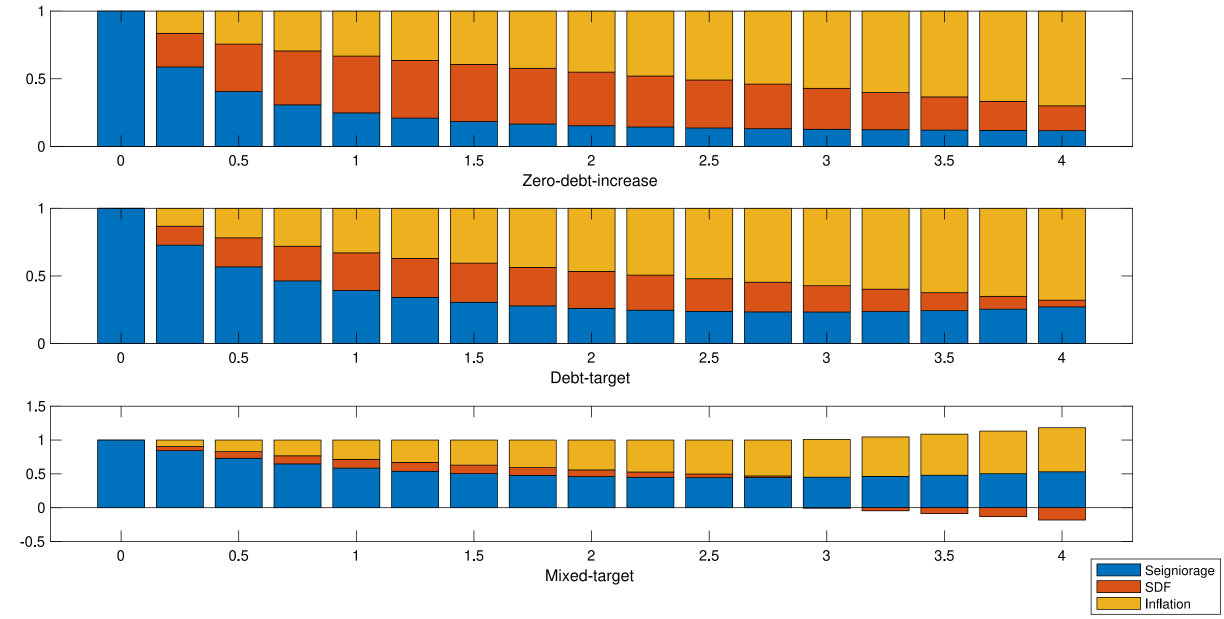


Fig. 21. Debt financing decomposition under government purchase: The role of steady state level of debt. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

contrary, if the government increases its spending, the current inflation expands a lot, which stimulates the economy and inflates the exited debt and interests. Finally, the inflation tax is a increase function of steady state level of debt. It means that the deficit financing will be solved by less money creation, and the inflation rises less when debt level is high.

Fig. 22 displays the cumulative fiscal multipliers under different steady state levels of debt. As before, we use black line to represents zero-debt-increase policy, blue diamonds to represents debt-target rule and red dots to represents the mixed-target rule. Three aspects are worth noting. First, the cumulative fiscal multipliers decrease in steady state level of debt, regardless of fiscal regime. This is originated from the heavy interests of debt, but also accompanied by a low level of inflation increment. Second, the flexible rules generate larger cumulative fiscal multipliers under tax cut while converge to the zero-debt-increase policy under government spending. This conclusion results from the difference of financing mechanism. According to financing decomposition above, SDF contributes the most to tax cut fiscal stimulus. Thus the interest rates need to be low in longer run under a flexible

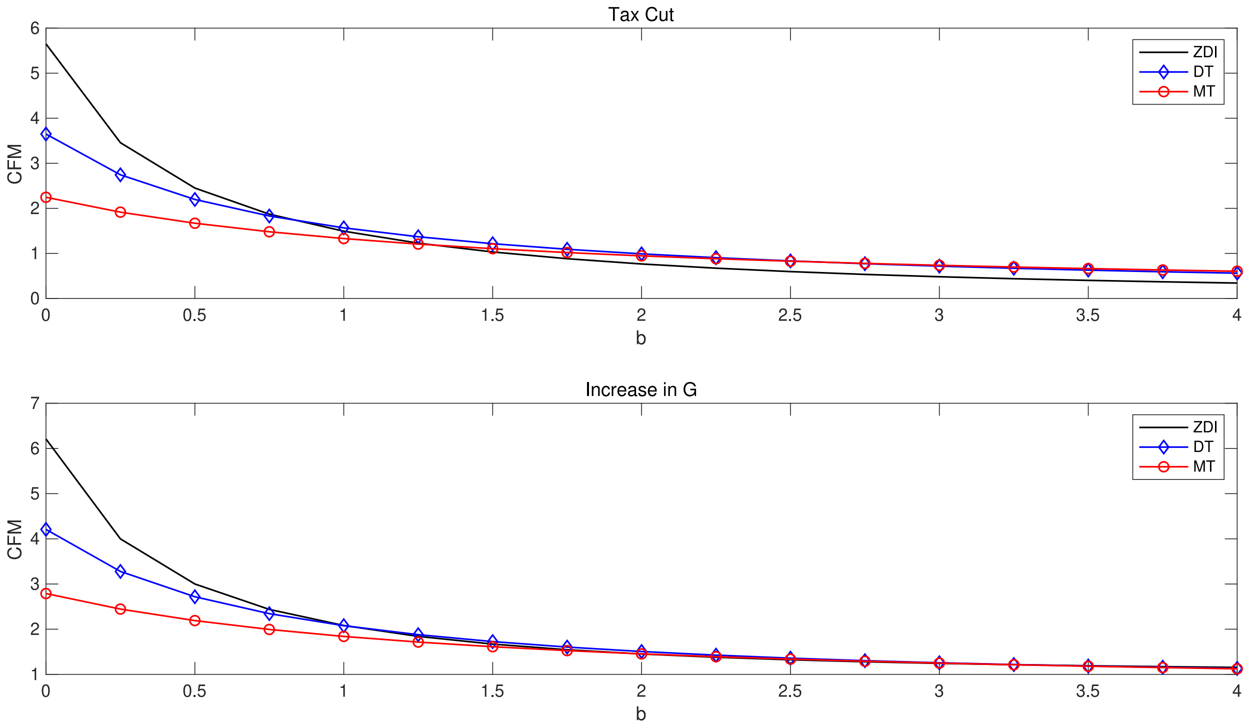


Fig. 22. Cumulative fiscal multipliers: The role of steady state level of debt.

rule to pay for the temporary increase of debt. The lower long run interest rate stimulates the long run output and raises the long run fiscal multipliers. On the other hand, if the government increases government spending, the main contribution is mainly from current inflation. Therefore, the long run effects are affected less by interest rate, and the CFMs converge to the same level regardless of the temporary financing arrangement.

References

Ascari, G., Ropele, T., 2013. Disinflation effects in a medium-scale new Keynesian model: money supply rule versus interest rate rule. Eur. Econ. Rev. 61, 77–100.
Auerbach, A.J., Obstfeld, M., 2005. The case for open-market purchases in a liquidity trap. Amer. Econ. Rev. 95 (1), 110–137.
Barbeau, E.J., 2003. Polynomials. Springer Science & Business Media.
Barro, R.J., 2009. Demand side voodoo economics. Econ. Voice 6 (2).
Bernanke, B.S., et al., 2002. Deflation: Making Sure 'It'Doesn't Happen Here. Remarks before the National Economists Club, Washington, DC 21.
Bhattacharya, J., Kudoh, N., 2002. Tight money policies and inflation revisited. Can. J. Econ./Rev. Can. d'Écon. 35 (2), 185–217.
Bhattarai, S., Lee, J.W., Yang, C., 2023. Redistribution and the Monetary–Fiscal policy mix. Quant. Econ. 14 (3), 817–853.
Bilbiie, F.O., 2011. Nonseparable preferences, frisch labor supply, and the consumption multiplier of government spending: One solution to a fiscal policy puzzle. J. Money Credit Bank. 43 (1), 221–251.
Blanchard, O.J., Kahn, C.M., 1980. The solution of linear difference models under rational expectations. Econometrica 1305–1311.
Buiter, W.H., 2014. The simple analytics of helicopter money: Why it works—always. Economics 8 (1).
Calvo, G.A., 1983. Staggered prices in a utility-maximizing framework. J. Monet. Econ. 12 (3), 383–398.
Chowdhury, I., Schabert, A., 2003. Assessing money supply rules.
Christiano, L., Eichenbaum, M., Rebelo, S., 2011. When is the government spending multiplier large? J. Polit. Econ. 119 (1), 78–121.
Davig, T., Leeper, E.M., 2011. Monetary–fiscal policy interactions and fiscal stimulus. Eur. Econ. Rev. 55 (2), 211–227.
Dupor, B., Li, J., Li, R., 2019. Sticky wages, private consumption, and fiscal multipliers. J. Macroecon. 62, 103157.
English, W.B., Erceg, C.J., Lopez-Salido, D., 2017. Money-financed fiscal programs: A cautionary tale.
Espinosa-Vega, M.A., Yip, C.K., 1999. Fiscal and monetary policy interactions in an endogenous growth model with financial intermediaries. Internat. Econom. Rev. 40 (3), 595–616.
Espinosa-Vega, M.A., Yip, C.K., 2002. Government financing in an endogenous growth model with financial market restrictions. Econom. Theory 20, 237–257.
Friedman, M., 1948. A monetary and fiscal framework for economic stability. Am. Econ. Rev. 38 (3), 245–264.
Friedman, M., 1969. Optimun quantity of money and other essays, the.
Galí, J., 2020. The effects of a money-financed fiscal stimulus. J. Monetary Econ. 115, 1–19.
Galí, J., López-Salido, J.D., Vallés, J., 2007. Understanding the effects of government spending on consumption. J. Eur. Econ. Assoc. 5 (1), 227–270.
Gokan, Y., Turnovsky, S.J., 2023. Government spending, debt management, and wealth and income inequality in a growing monetary economy. J. Money Credit Bank..
Kaas, L., Weinrich, G., 2003. Money and growth in a production economy with multiple assets. Macroecon. Dyn. 7 (5), 670–690.
Kim, S., 2003. Structural shocks and the fiscal theory of the price level in the sticky price model. Macroecon. Dyn. 7 (5), 759–782.
Kim, H., Shao, P., Zhang, S., 2023. Policy coordination and the effectiveness of fiscal stimulus. J. Macroecon. 75, 103489.

LaSalle, J.P., 2012. The Stability and Control of Discrete Processes. Vol. 62, Springer Science & Business Media.
Leeper, E.M., 1991. Equilibria under 'active'and 'passive'monetary and fiscal policies. J. Monet. Econ. 27 (1), 129–147.
Leeper, E.M., 2010. Monetary Science, Fiscal Alchemy. Technical Report, National Bureau of Economic Research.
Leeper, E.M., Traum, N., Walker, T.B., 2017. Clearing up the fiscal multiplier morass. Amer. Econ. Rev. 107 (8), 2409–2454.
Leeper, E.M., Walker, T.B., 2011. Fiscal limits in advanced economies. Econ. Pap.: J. Appl. Econ. Policy 30 (1), 33–47.
Leeper, E.M., Walker, T.B., 2012. Perceptions and Misperceptions of Fiscal Inflation. Technical Report, National Bureau of Economic Research.
Li, R., Tian, X., 2018. Spending reversals and fiscal multipliers under an interest rate peg. J. Money Credit Bank. 50 (4), 789–815.
Linnemann, L., Schabert, A., 2003. Fiscal policy in the new neoclassical synthesis. J. Money Credit Bank. 911–929.
Mankiw, N.G., 2020. A skeptic's guide to modern monetary theory. In: AEA Papers and Proceedings. Vol. 110, pp. 141–144.
Mao, R., Shen, W., Yang, S.-C.S., 2023. Uncertain policy regimes and government spending effects. Eur. Econ. Rev. 152, 104330.
Ouliaris, S., Rochon, C., 2021. Pre-and post-global financial crisis policy multipliers. J. Macroecon. 70, 103370.
Palivos, T., Yip, C.K., 1995. Government expenditure financing in an endogenous growth model: a comparison. J. Money Credit Bank. 27 (4), 1159–1178.
Punzo, C., Rossi, L., 2023. A money-financed fiscal stimulus: Redistribution and social welfare. J. Money Credit Bank. 55 (2–3), 595–617.
Reichlin, L., Turner, A., Woodford, M., 2013. Helicopter Money as a Policy Option. VoxEU. org, May 20.
Sargent, T.J., Wallace, N., 1976. Rational expectations and the theory of economic policy. J. Monet. Econ. 2 (2), 169–183.
Schabert, A., 2006. Central bank instruments, fiscal policy regimes, and the requirements for equilibrium determinacy. Rev. Econ. Dyn. 9 (4), 742–762.
Schmitt-Grohé, S., Uribe, M., 2000. Price level determinacy and monetary policy under a balanced-budget requirement. J. Monetary Econ. 45 (1), 211–246.
Schmitt-Grohé, S., Uribe, M., 2007. Optimal simple and implementable monetary and fiscal rules. J. Monet. Econ. 54 (6), 1702–1725.
Schreft, S.L., Smith, B.D., 2000. The evolution of cash transactions: some implications for monetary policy. J. Monetary Econ. 46 (1), 97–120.
Sims, C.A., 1994. A simple model for study of the determination of the price level and the interaction of monetary and fiscal policy. Econ. Theory 4 (3), 381–399.
Tsuruga, T., Wake, S., 2019. Money-financed fiscal stimulus: The effects of implementation lag. J. Econom. Dynam. Control 104, 132–151.
Turner, A., 2013. Debt, money, and mephistopheles: how do we get out of this mess?
Turner, A., 2017. Between debt and the devil. In: Between Debt and the Devil. Princeton University Press.
Wallace, N., 1984. Some of the choices for monetary policy. Q. Rev. 8 (Win).
Woodford, M., 1995. Price-level determinacy without control of a monetary aggregate. In: Carnegie-Rochester Conference Series on Public Policy. Vol. 43, Elsevier, pp. 1–46.
Woodford, M., 1998. Control of the public debt: a requirement for price stability? In: The Debt Burden and Its Consequences for Monetary Policy. Springer, pp. 117–158.
Woodford, M., 2011. Simple analytics of the government expenditure multiplier. Am. Econ. J.: Macroecon. 3 (1), 1–35.
Wray, L.R., 2015. Modern Money Theory: A Primer on Macroeconomics for Sovereign Monetary Systems. Springer.