

# Integration of OpenPyXL, .xlsx Geometry Spreadsheet and DiffDRR

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PicoRad -> MGH

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[DRR](#) is a PyTorch module that computes differentiable digitally reconstructed radiographs. The viewing angle for the DRR (known generally in computer graphics as the *camera pose*) is parameterized by the following parameters:

- SDR : Source-to-Detector radius (half of the source-to-detector distance)
- $\mathbf{R} \in \text{SO}(3)$  : a rotation
- $\mathbf{t} \in \mathbb{R}^3$  : a translation

#### 💡 Tip

[DiffDRR](#) can take a rotation parameterized in any of the following forms to move the detector plane:

- `axis_angle`
- `euler_angles` (note: also need to specify the `convention` for the Euler angles)
- `matrix`
- `quaternion`
- `rotation_6d` ([Zhou et al., 2019](#))
- `rotation_10d` ([Peretroukhin et al., 2021](#))
- `quaternion_adjugate` ([Hanson and Hanson, 2022](#))

If using Euler angles, the parameters are

- `alpha` : Azimuthal angle
- `beta` : Polar angle
- `gamma` : Plane rotation angle
- `bx` : X-dir translation
- `by` : Y-dir translation
- `bz` : Z-dir translation
- `convention` : Order of angles (e.g., `ZYX`)

(`bx`, `by`, `bz`) are translational parameters and (`alpha`, `beta`, `gamma`) are rotational parameters. The rotational parameters are detailed in [Spherical Coordinates Tutorial](#).

<https://vivekg.dev/DiffDRR/api/drr.html#drr>

`DRR` is a PyTorch module that compues differentiable digitally reconstructed radiographs. The viewing angle for the DRR (known generally in computer graphics as the *camera pose*) is parameterized by the following parameters:

- SDR : Source-to-Detector radius (half of the source-to-detector distance)
- $\mathbf{R} \in \text{SO}(3)$  : a rotation
- $\mathbf{t} \in \mathbb{R}^3$  : a translation

DRR

DRR (volume:numpy.ndarray, spacing:numpy.ndarray, sdr:float, height:int, delx:float, width:int|None=None, dely:float|None=None, x0:float=0.0, y0:float=0.0, p\_subsample:float|None=None, reshape:bool=True, reverse\_x\_axis:bool=False, patch\_size:int|None=None, bone\_attenuation\_multiplier:float=1.0)

DRR.forward

[source](#)

```
DRR.forward (rotation:torch.Tensor, translation:torch.Tensor,
              parameterization:str, convention:str=None,
              pose:pytorch3d.transforms.Transform3d=None,
              bone_attenuation_multiplier:float=None)
```

Generate DRR with rotational and translational parameters.

	Type	Default	Details
rotation	torch.Tensor		
translation	torch.Tensor		
parameterization	str		
convention	str	None	
pose	Transform3d	None	If you have a preformed pose, can pass it directly
bone_attenuation_multiplier	float	None	Contrast ratio of bone to soft tissue

PyTorch module that computes differentiable digitally reconstructed radiographs.

	Type	Default	Details
volume	np.ndarray		CT volume
spacing	np.ndarray		Dimensions of voxels in the CT volume
sdr	float		Source-to-detector radius for the C-arm (half of the source-to-detector distance)
height	int		Height of the rendered DRR
delx	float		X-axis pixel size
width	int   None	None	Width of the rendered DRR (if not provided, set to <code>height</code> )
dely	float   None	None	Y-axis pixel size (if not provided, set to <code>delx</code> )
x0	float	0.0	Principal point X-offset
y0	float	0.0	Principal point Y-offset
p_subsample	float   None	None	Proportion of pixels to randomly subsample
reshape	bool	True	Return DRR with shape (b, 1, h, w)
reverse_x_axis	bool	False	If pose includes reflection (in E(3) not SE(3)), reverse x-axis
patch_size	int   None	None	If the entire DRR can't fit in memory, render patches of the DRR in series
bone_attenuation_multiplier	float	1.0	Contrast ratio of bone to soft tissue

The forward pass of the `DRR` module generated DRRs from the input CT volume. The pose parameters (i.e., viewing angles) from which the DRRs are generated are passed to the forward call.

## How to use DiffDRR

<https://vivekg.dev/DiffDRR/tutorials/introduction.html>

In-depth tutorial of the DRR module's functionality

```
import matplotlib.pyplot as plt
import torch

from diffdr.data import load_example_ct
from diffdr.drr import DRR
from diffdr.visualization import plot_drr
```



## DRR Generation

We demonstrate the speed of DiffDRR by timing repeated DRR synthesis. Timing results are on a single NVIDIA RTX 2080 Ti GPU.

DiffDRR is implemented as a custom PyTorch module.

33.8 ms ± 427 µs per loop (mean ± std. dev. of 7 runs, 10 loops each)

All raytracing operations have been formulated in a vectorized function, enabling use of PyTorch's GPU support and autograd. This also means that DRR generation is available as a layer in deep learning frameworks.

### Tip

Rotations can be parameterized with numerous conventions (not just Euler angles). See [diffdr.drr](#) for more details.

```
# Read in the volume and get the isocenter
volume, spacing = load_example_ct()
bx, by, bz = torch.tensor(volume.shape) * torch.tensor(spacing) / 2

# Initialize the DRR module for generating synthetic X-rays
device = torch.device("cuda" if torch.cuda.is_available() else "cpu")
drr = DRR(
    volume, # The CT volume as a numpy array
    spacing, # Voxel dimensions of the CT
    sdr=300.0, # Source-to-detector radius (half of the source-to-detector distance)
    height=200, # Height of the DRR (if width is not separately provided, the generated image is square)
    delx=4.0, # Pixel spacing (in mm)
).to(device)

# Set the camera pose with rotations (yaw, pitch, roll) and translations (x, y, z)
rotations = torch.tensor([[torch.pi, 0.0, torch.pi / 2]], device=device)
translations = torch.tensor([[bx, by, bz]], device=device)
img = drr(rotations, translations, parameterization="euler_angles", convention="ZYX")
plot_drr(img, ticks=False)
plt.show()
```

<https://github.com/eigenvivek/DiffDRR>

```
import matplotlib.pyplot as plt
import torch

from diffdr.drr import DRR
from diffdr.data import load_example_ct
from diffdr.visualization import plot_drr
```

```
# Read in the volume and get the isocenter
volume, spacing = load_example_ct()
bx, by, bz = torch.tensor(volume.shape) * torch.tensor(spacing) / 2
```

```
# Initialize the DRR module for generating synthetic X-rays
device = "cuda" if torch.cuda.is_available() else "cpu"
drr = DRR(
    volume, # The CT volume as a numpy array
    spacing, # Voxel dimensions of the CT
    sdr=300.0, # Source-to-detector radius (half of the source-to-detector distance)
    height=200, # Height of the DRR (if width is not separately provided, the generated image is square)
    delx=4.0, # Pixel spacing (in mm)
).to(device)
```

```
# Set the camera pose with rotation (yaw, pitch, roll) and translation (x, y, z)
rotation = torch.tensor([[torch.pi, 0.0, torch.pi / 2]], device=device)
translation = torch.tensor([[bx, by, bz]], device=device)
```

# 📌 Also note that DiffDRR can take many representations of  $SO(3)$  📌

# For example, quaternions, rotation matrix, axis-angle, etc...

```
img = drr(rotation, translation, parameterization="euler_angles", convention="ZYX")
plot_drr(img, ticks=False)
plt.show()
```





## Tutorials

[Overview](#)

## 3D operators

[Fit Mesh](#)

[Bundle Adjustment](#)

## Rendering

[Render Textured Meshes](#)

[Render DensePose Meshes](#)

[Render Colored Pointclouds](#)

[Fit a Mesh with Texture via  
Rendering](#)

[Camera Position Optimization  
with Differentiable Rendering](#)

[Fit a volume via raymarching](#)



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In [ ]:

```
# Copyright (c) Meta Platforms, Inc. and affiliates. ALL rights reserved.
```

# Camera position optimization using differentiable rendering

In this tutorial we will learn the  $[x, y, z]$  position of a camera given a reference image using differentiable rendering.

We will first initialize a renderer with a starting position for the camera. We will then use this to generate an image, compute a loss with the reference image, and finally backpropagate through the entire pipeline to update the position of the camera.

This tutorial shows how to:

# <https://colab.research.google.com/github/BillWorstell/DiffDRR/blob/main/notebooks/tutorials/introduction.ipynb>

introduction.ipynb

PRO File Edit View Insert Runtime Tools Help Cannot save changes

+ Code + Text Copy to Drive

title: How to use DiffDRR description: In-depth tutorial of the DRR module's functionality output-file: introduction.html

<https://vivekg.dev/DiffDRR/>

install DiffDRR from PyPI:

```
1 !pip install diffdr
```

```
Collecting diffdr
Using cached diffdr-0.3.8-py3-none-any.whl (34.6 MB)
Collecting fastcore
Downloading fastcore-1.5.29-py3-none-any.whl (67 kB)
67.6/67.6 kB 3.1 MB/s eta 0:00:00
Collecting matplotlib
Downloading matplotlib-3.8.2-cp310-cp310-manylinux_2_17_x86_64.manylinux2014_x86_64.whl (11.6 MB)
11.6/11.6 MB 28.0 MB/s eta 0:00:00
Collecting torchvision
Downloading torchvision-0.16.2-cp310-cp310-manylinux1_x86_64.whl (6.8 MB)
6.8/6.8 MB 18.4 MB/s eta 0:00:00
```

```
1 device = "cuda" if torch.cuda.is_available() else "cpu"
2 drr = DRR(
3     volume,          # The CT volume as a numpy array
4     spacing,         # Voxel dimensions of the CT
5     sdr=300.0,       # Source-to-detector radius (half of the source-to-detector distance)
6     height=200,      # Height of the DRR (if width is not separately provided, the generated image is square)
7     delx=4.0,        # Pixel spacing (in mm)
8 ).to(device)
```

Set the camera pose with rotation (yaw, pitch, roll) and translation (x, y, z)

```
[25] 1 rotation = torch.tensor([[torch.pi, 0.0, torch.pi / 2]], device=device)
      2 translation = torch.tensor([[bx, by, bz]], device=device)
```

Also note that DiffDRR can take many representations of SO(3) 🤖

For example, quaternions, rotation matrix, axis-angle, etc...

```
[26] 1 img = drr(rotation, translation, parameterization="euler_angles", convention="ZYX")
      2 plot_drr(img, ticks=False)
      3 plt.show()
```

[https://colab.research.google.com/github/facebookresearch/pytorch3d/blob/stable/docs/tutorials/camera\\_position\\_optimization\\_with\\_differtable\\_rendering.ipynb#scrollTo=sEVdNGFwripM](https://colab.research.google.com/github/facebookresearch/pytorch3d/blob/stable/docs/tutorials/camera_position_optimization_with_differtable_rendering.ipynb#scrollTo=sEVdNGFwripM)

```
[2] 1 import os
      2 import sys
      3 import torch
      4 need_pytorch3d=False
      5 try:
      6     import pytorch3d
      7 except ModuleNotFoundError:
      8     need_pytorch3d=True
      9 if need_pytorch3d:
     10     if torch.__version__.startswith("2.1.") and sys.platform.startswith("linux"):
     11         # We try to install PyTorch3D via a released wheel.
     12         pyt_version_str=torch.__version__.split("+")[0].replace(".", "")
     13         version_str=".".join([
     14             f"py3{sys.version_info.minor}_cu",
     15             torch.version.cuda.replace(".", ""),
     16             f"_pyt{pyt_version_str}"
     17         ])
     18         !pip install fvcore iopath
     19         !pip install --no-index --no-cache-dir pytorch3d -f https://dl.fbaipublicfiles.com/pytorch3d/packaging/wheels/{version_str}/download.html
     20     else:
     21         # We try to install PyTorch3D from source.
     22         !pip install 'git+https://github.com/facebookresearch/pytorch3d.git@stable'
```



Get characteristics of volume

```
[39] 1 ic(volume.dtype)
      2 ic(volume.shape)
      3 ic(volume.size)
```

```
ic| volume.dtype: dtype('float32')
ic| volume.shape: (512, 512, 133)
ic| volume.size: 34865152
34865152
```

Get voxel spacing

```
[42] 1 ic(spacing)
```

```
ic| spacing: [0.703125, 0.703125, 2.5]
[0.703125, 0.703125, 2.5]
```

<https://github.com/BillWorstell/DiffDRR/blob/main/notebooks/tutorials/introduction.ipynb>



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	A	B	C	D	E	F	G	H	I	J	K
1		x1	y1	z1	b	S	I	I + b+Td	x2	y2	z2
2	Definition	x coordinate value at center of hole	y coordinate value at center of hole	z coordinate value at center of hole	distance from center of hole to center of heart	3D slope	length of collimator		x coordinate value at end of detector	y coordinate value at end of detector	z coordinate value at end of detector
3	1	277.002	70.37124	-70.0363	294.2572051	1.062292709	89.07020616	393.3274112	370.2627419	94.06375503	-93.61604779
4	2	242.8831	107.2608	-70.2602	274.6517344	1.130798044	82.70081149	367.3525458	324.8613206	143.4636051	-93.97451431
5	3	201.3326	136.87	-70.5519	253.4675193	1.258949218	75.77607567	339.2435949	269.4656704	183.1882482	-94.42740535
6	4	155.0181	158.4473	-70.9056	232.7310944	1.501315617	68.94258232	311.6736767	207.6003695	212.192757	-94.95683896
7	5	106.2709	171.9591	-71.3542	214.3708425	2.017211132	62.83224619	287.2030887	142.3763156	230.3820057	-95.59670705
8	6	56.74056	177.7703	-71.7895	199.9387482	3.523735899	57.97763045	267.9163786	76.03191225	238.2108293	-96.19737564

ReadGeometryXLS.ipynb

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```
[12] 2 print(wb.active.title)
      3 ws = wb.active
```

Coordinates

x coordinate value at center of hole

```
[13] 1 ic(ws.cell(2,2).value)
      2 x1=np.zeros(80)
      3 for i in range(3,83):
      4 | x1[i-3]=(ws.cell(i,2).value)
      5 print(x1)
```

ic| ws.cell(2,2).value: 'x coordinate value at center of hole'

[	277.002	242.8831	201.3326	155.0181	106.2709	56.74056
	7.351419	-41.7583	-89.0623	-132.361	-170.499	-201.258
	-221.948	-230.423	-225.927	-209.229	-182.273	-147.473
	-107.125	-63.1068	277.002	242.8831	201.3326	155.0181
	106.2709	56.74056	7.351419	-41.7583	-89.0623	-132.361

ReadGeometryXLS.ipynb

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y coordinate value at center of hole

```
[14] 1 ic(ws.cell(2,3).value)
      2 y1=np.zeros(80)
      3 for i in range(3,83):
      4 | y1[i-3]=(ws.cell(i,3).value)
      5 print(y1)
```

ic| ws.cell(2,3).value: 'y coordinate value at center of hole'

[	70.37124	107.2608	136.87	158.4473	171.9591	177.7703
	176.2461	167.3279	150.8126	126.955	95.57272	56.97774
	12.56229	-35.1111	-82.9365	-127.899	-167.656	-200.839
	-226.898	-245.785	70.37124	107.2608	136.87	158.4473

ReadGeometryXLS.ipynb

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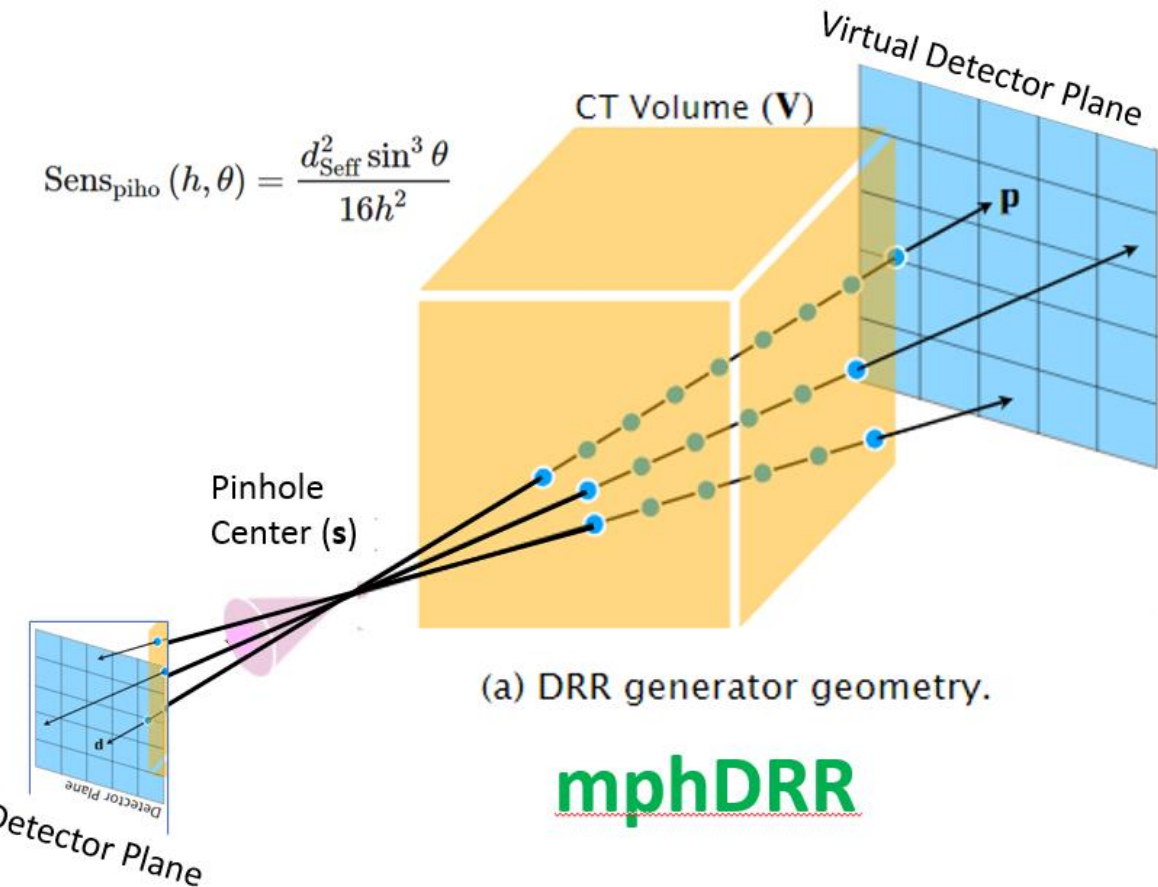
+ Code + Text

z coordinate value at center of hole

```
[15] 1 ic(ws.cell(2,4).value)
      2 z1=np.zeros(80)
      3 for i in range(3,83):
      4 | z1[i-3]=(ws.cell(i,4).value)
      5 print(z1)
```

ic| ws.cell(2,4).value: 'z coordinate value at center of hole'

[	-70.0363	-70.2602	-70.5519	-70.9056	-71.3542	-71.7895	-72.1241	-72.2656
	-72.1683	-71.8898	-71.5318	-71.1818	-70.8925	-70.7108	-70.5931	-70.5263
	-70.4918	-70.4707	-70.4467	-70.4085	-23.4255	-23.5096	-23.62	-23.7553
	-23.9071	-24.0565	-24.1728	-24.2224	-24.1883	-24.0912	-23.9678	-23.8485
	-23.7500	-23.6000	-23.6357	-23.6403	-23.5000	-23.5000	-23.5000	-23.5000



**mphDRR**

	Type	Default	Details
volume	np.ndarray		CT volume
spacing	np.ndarray		Dimensions of voxels in the CT volume
sdr	float		Source-to-detector radius for the C-arm (half of the source-to-detector distance)

Choose  $\text{sdr} = 5 * [\text{length of collimator}]$   
Magnification  $M=10$

$f_{\text{X}} = (\text{ATAN2}(B3, C3))$

	T	U	V
axis			pinhole azimuth (radians)
		pinhole range	pinhole elevation (radians)
4725		285.8010137	0.2487829888
1903		265.5128612	0.4158590487

$f_{\text{X}} = (\text{ATAN2}(U3, D3))$

	T	U	V	W
axis			pinhole azimuth (radians)	pinhole elevation (radians)
		pinhole range		
4725		285.8010137	0.2487829888	-0.2403169548
1903		265.5128612	0.4158590487	-0.2586912886

If using Euler angles, the parameters are

- **alpha** : Azimuthal angle
- **beta** : Polar angle
- **gamma** : Plane rotation angle
- **bx** : X-dir translation
- **by** : Y-dir translation
- **bz** : Z-dir translation
- **convention** : Order of angles (e.g., **ZYX**)

(bx, by, bz) are translational parameters and (alpha, beta, gamma) are rotational parameters. The rotational parameters are detailed in [Spherical Coordinates Tutorial](#).

$f_{\text{X}} = E3 + G3 + \text{Main!T3}$

B	C	D	E	F	G	H
x1	y1	z1	b	S	I	I + b+Td
x coordinate value at center of hole	y coordinate value at center of hole	z coordinate value at center of hole	distance from center of hole to center of heart	3D slope	length of collimator	
277.002	70.37124	-70.0363	294.2572051	1.062292709	89.07020616	393.3274112
242.8831	107.2608	-70.2602	274.6517344	1.130798044	82.70081149	367.3525458

$f_{\text{X}} = \text{SQRT}(\text{SUMSQ}(B3, C3))$

T	U
axis	pinhole range (radians)
725	285.8010137
903	265.5128612





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Length of Collimator

```
1 ic(ws.cell(2,7).value)
2 lcoll=np.zeros(80)
3 for i in range(3,83):
4 | lcoll[i-3]=(ws.cell(i,7).value)
5 print(lcoll)
```

Choose vsdr = 5 \* length of collimator

```
[ ] 1 vsdr=5.*lcoll
```

alpha: Azimuthal angle (radians)

```
1 ic(ws.cell(2,22).value)
2 alpha=np.zeros(80)
3 for i in range(3,83):
4 | alpha[i-3]=(ws.cell(i,22).value)
5 print(alpha)
```

ic| ws.cell(2,22).value: 'pinhole azimuth (radians)'

```
[ 0.24878299  0.41585905  0.59705381  0.79633737  1.01724582  1.26183823
 1.52910939  1.81536054  2.10423792  2.37703862  2.63068794  2.86570428
 3.08505283 -2.9903791  -2.78977101 -2.59291509 -2.39794155 -2.20416353
-2.01100013 -1.82212375 -1.64070200 -1.45850005 -1.27850301 -1.09633737
```



+ Code + Text

beta = altitude (radians)

```
1 ic(ws.cell(2,233).value)
2 beta=np.zeros(80)
3 for i in range(3,83):
4 | beta[i-3]=(ws.cell(i,23).value)
5 print(beta)
```

ic| ws.cell(2,233).value: None

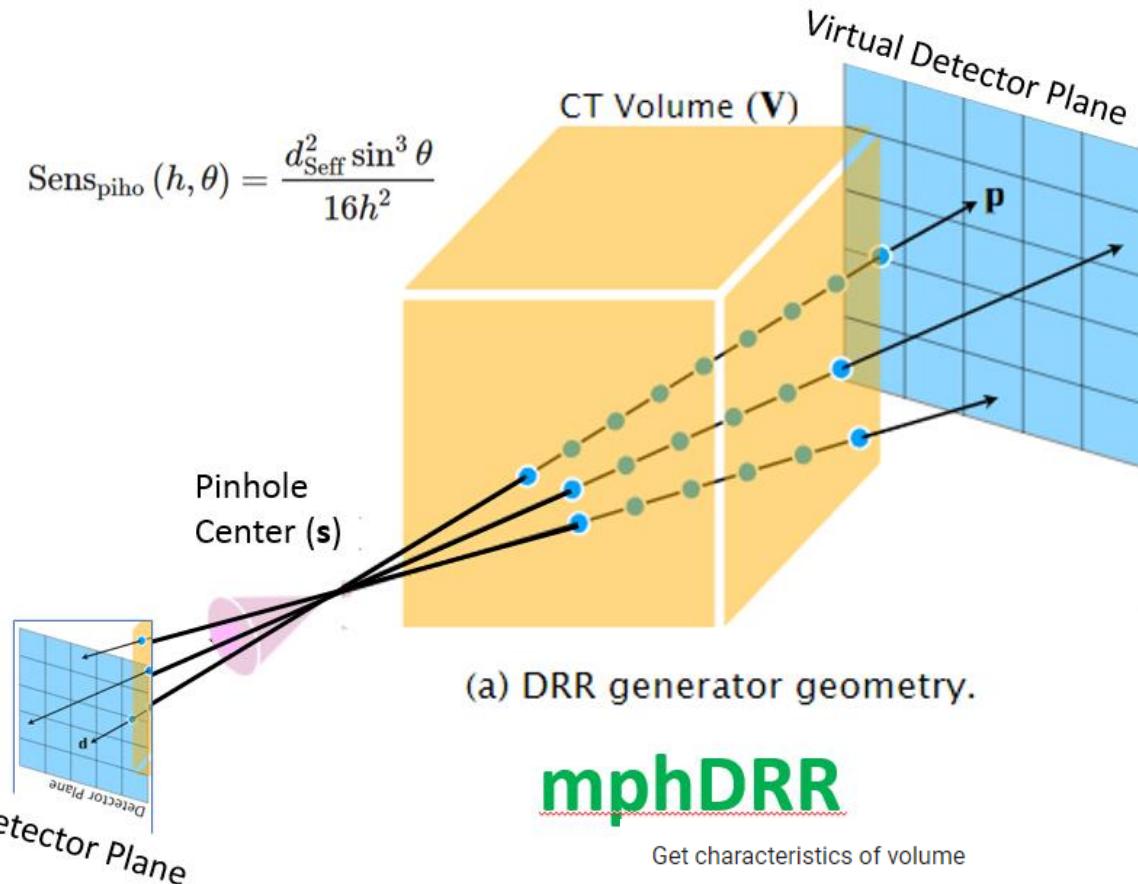
```
[-0.24031695 -0.25869129 -0.28207257 -0.30958931 -0.33932858 -0.36725782
-0.38812799 -0.39680203 -0.39084561 -0.37356937 -0.35082977 -0.32801569
-0.3087048  -0.29454762 -0.28531763 -0.28004146 -0.27730567 -0.27562572
-0.27371232 -0.27065554 -0.08178156 -0.0883138  -0.096719  -0.10675926
-0.11771907 -0.12820893 -0.13618629 -0.13953966 -0.13723464 -0.13060819
-0.1220143  -0.11352578 -0.10643635 -0.10125  -0.09789449 -0.09598459
-0.09499655 -0.09439062 -0.09370115 -0.09260112  0.08178156  0.0883138
```

W

pinhole  
elevation  
(radians)

```
-0.2403169548
-0.2586912886
-0.2820725687
-0.3095893101
-0.3393285829
```

$$\text{Sens}_{\text{piho}}(h, \theta) = \frac{d_{\text{Seff}}^2 \sin^3 \theta}{16h^2}$$



(a) DRR generator geometry.

## mphDRR

Get characteristics of volume

```
[39] 1 ic(volume.dtype)
      2 ic(volume.shape)
      3 ic(volume.size)

ic| volume.dtype: dtype('float32')
ic| volume.shape: (512, 512, 133)
ic| volume.size: 34865152
```

Get voxel spacing

```
[42] 1 ic(spacing)

ic| spacing: [0.703125, 0.703125, 2.5]
         [0.703125, 0.703125, 2.5]
```

Get size of image

```
1 ic(img.shape)

ic| img.shape: torch.Size([1, 1, 200, 200])
         torch.Size([1, 1, 200, 200])
```

Volume [X Y Z] is  
360 x 360 x 332.5 mm

<https://vivekg.dev/DiffDRR/api/drr.html#drr>

height	int		Height of the rendered DRR
delx	float		X-axis pixel size
width	int   None	None	Width of the rendered DRR (if not provided, set to <code>height</code> )
dely	float   None	None	Y-axis pixel size (if not provided, set to <code>delx</code> )
x0	float	0.0	Principal point X-offset
y0	float	0.0	Principal point Y-offset

Choose  $\text{sdr} = 5 * [\text{length of collimator}] \rightarrow \sim 450\text{mm}$   
Magnification  $M=10$ , Source-to-detector  $\rightarrow \sim 900\text{mm}$

Length of side of virtual detector crystal = 500 mm

height= 500  
delx =  $500/200 = 2.5$   
width = 500  
dely =  $500/200=2.5$   
x0=0  
y0=0

	s	
Ld		To
length of side of virtual detector crystal		th of or le detector crystal cr
33	50	
47	50	
38	50	
44	50	
17	50	

```
# Initialize the DRR module for generating synthetic X-rays
device = torch.device("cuda" if torch.cuda.is_available() else "cpu")
drr = DRR(
    volume, # The CT volume as a numpy array
    spacing, # Voxel dimensions of the CT
    sdr=300.0, # Source-to-detector radius (half of the source-to-detector distance)
    height=200, # Height of the DRR (if width is not seperately provided, the generated image is square)
    delx=4.0, # Pixel spacing (in mm)
).to(device)
# Set the camera pose with rotations (yaw, pitch, roll) and translations (x, y, z)
rotations = torch.tensor([[torch.pi, 0.0, torch.pi / 2]], device=device)
translations = torch.tensor([[bx, by, bz]], device=device)
img = drr(rotations, translations, parameterization="euler_angles", convention="ZYX")
```

```
# Initialize the DRR module for generating synthetic X-rays
device = torch.device("cuda" if torch.cuda.is_available() else "cpu")
drr = DRR(
    volume, # The CT volume as a numpy array
    spacing, # Voxel dimensions of the CT
    sdr=vsdr[i], # Source-to-virtual-detector radius (half of the source-to-virtual-detector distance)
    height=500, # Height of the DRR (if width is not seperately provided, the generated image is square)
    delx=2.5, # Pixel spacing (in mm)
).to(device)
# Set the camera pose with rotations (yaw, pitch, roll) and translations (x, y, z)
rotations = torch.tensor([alpha[i], beta[i], 0.], device=device)
translations = torch.tensor([[bx, by, bz]], device=device)
img = drr(rotations, translations, parameterization="euler_angles", convention="ZYX")
```

## module 1-80 version 008

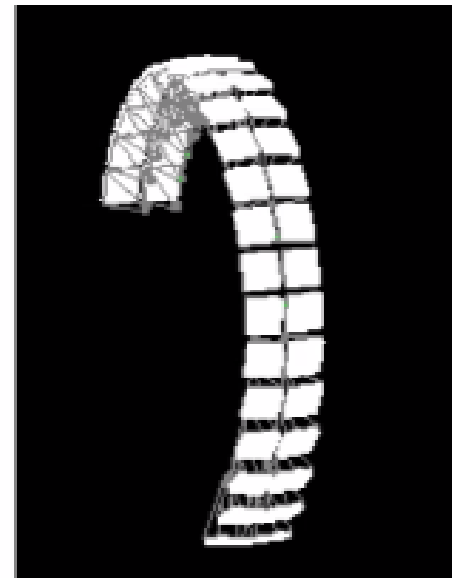
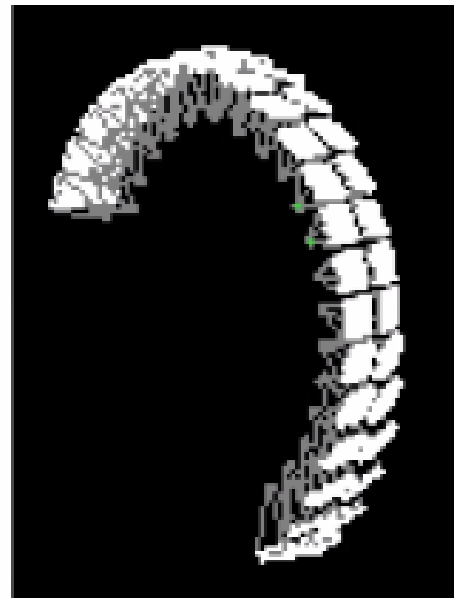
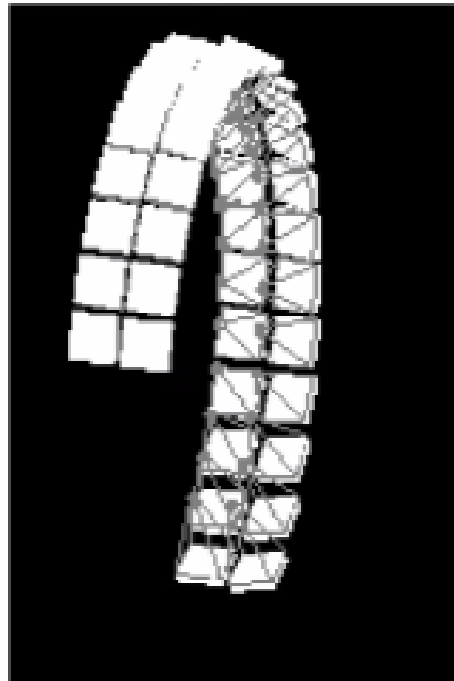


Figure 12: Left: module 41-80, central: module 1-40, right: module 21-60  
I

Issue remaining: Gate fails when involving 80 heads in a long macro.