A Differentiable Fast Forward Projector for Multi-pinhole SPECT and Co-registered CT

We are developing a Dynamic Cardiac SPECT system at the Massachusetts General Hospital with a stationary multiple-pinhole geometry, as illustrated in Figure 2a, and as shown under construction in Figure 2b. For ideal infinitesimal pinholes, there is a geometric correspondence (in the form of a simple inversion and projection transformation) that relates SPECT data tensors with cone-beam CT projections from an ideal point-like source at the pinhole position, as illustrated in Figure 1. We have used this to make displays of sets of virtual digital radiographs from each pinhole's perspective, projected through an arbitrary CT volume. By adjusting the scale of the virtual detector size and distance, we can image each camera's field of view (FOV) to observe the immediate context outside the FOV using co-registered images from other modalities, such as Dual Energy CT, as illustrated in Figure 2c.

We use the Open Source DiffDRR package, which provides PyTorch-based auto-differentiable X-ray rendering and GPU-accelerated DRR synthesis, mapping from volume CT onto synthetic C-ARM projection images DiffDRR is well-documented and provides example tutorials which we have slightly modified, and executed as JuPyter notebooks which we have executed on Google Colab and posted on GitHub. We have interfaced with our reference Geometry which is illustrated schematically in Figure 2a. DiffDRR reference input CT volumes are [500,500,133] x [0.7,0.7,2.5] mm as illustrated in Figure 2c from the DiffDRR documentation. Output Digitally Reconstructed Radiographs are each [250,250] as illustrated in Figure 2c., collected and reshaped as [4,250,20,250] for the display shown in Figure 2d. We scale the distance from pinhole to virtual detector as a fixed multiple of the collimator length, sufficient to place all virtual detectors well behind the co-registered CT volume as seen from their pinhole (set to 2X FOV in Figure 2c).

Figure 2d illustrates high-resolution displays showing each multi-pinhole SPECT camera's field of view (and beyond) across the CT volume, as a high spatial resolution X-ray attenuation line integral.

By using DiffDRR's implementation of DRR rendering as a PyTorch module, we inherit its interoperable with deep learning pipelines. DiffDRR's original design purpose of co-registering interventional X-ray radiography images with previously acquired volumetric CT images is similar to our purpose of co-registering our SPECT images with a previously acquired CT volume. In addition, this work provides a foundation for developing PyTorch variations on DiffDRR where we can use the same framework and 3D Siddon coefficients but add an additional sensitivity term (scaling like the inverse square of the pinhole to source voxel distance as in Figure 1b) to their kernel summation when forward projecting from a source volume to a synthetic multipinhole SPECT Emission image. Attenuation correction for quantitatively accurate multipinhole SPECT (which will also require scatter modeling) will require modification of the line integral kernel to include the exponentiated summation of attenuation coefficients for points on the line integral between the source and the pinhole. We are currently implementing these as our next steps.. For further documentation see https://vivekg.dev/DiffDRR/tutorials/introduction.html and https://github.com/BillWorstell/DiffDRR.