Chapter 1

Problem Solutions

1.1

(a) fcc: 8 corner atoms $\times$ 1/8 = 1 atom
6 face atoms $\times$ ½ = 3 atoms
Total of 4 atoms per unit cell

(b) bcc: 8 corner atoms $\times$ 1/8 = 1 atom
1 enclosed atom = 1 atom
Total of 2 atoms per unit cell

(c) Diamond: 8 corner atoms $\times$ 1/8 = 1 atom
6 face atoms $\times$ ½ = 3 atoms
4 enclosed atoms = 4 atoms
Total of 8 atoms per unit cell

1.2

(a) 4 Ga atoms per unit cell

$$\text{Density} = \frac{4}{\left(5.65x10^{-8}\right)^3} \Rightarrow$$

$$\text{Density of Ga} = 2.22x10^{22}\ cm^{-3}$$

4 As atoms per unit cell, so that

$$\text{Density of As} = 2.22x10^{22}\ cm^{-3}$$

(b)

8 Ge atoms per unit cell

$$\text{Density} = \frac{8}{\left(5.65x10^{-8}\right)^3} \Rightarrow$$

$$\text{Density of Ge} = 4.44x10^{22}\ cm^{-3}$$

1.3

(a) Simple cubic lattice; $a = 2r$

Unit cell vol $= a^3 = (2r)^3 = 8r^3$

1 atom per cell, so atom vol. $= (1)\left(\frac{4\pi r^3}{3}\right)$

Then

$$Ratio = \frac{\left(\frac{4\pi r^3}{3}\right)}{8r^3} \times 100\% \Rightarrow \quad Ratio = 52.4\%$$

(b) Face-centered cubic lattice

$$d = 4r = a\sqrt{2} \Rightarrow \quad a = \frac{d}{\sqrt{2}} = 2\sqrt{2}\, r$$

Unit cell vol $= a^3 = \left(2\sqrt{2}\, r\right)^3 = 16\sqrt{2}\, r^3$

4 atoms per cell, so atom vol. $= 4\left(\frac{4\pi r^3}{3}\right)$

Then

$$Ratio = \frac{4\left(\frac{4\pi r^3}{3}\right)}{16\sqrt{2}\, r^3} \times 100\% \Rightarrow \quad Ratio = 74\%$$

(c) Body-centered cubic lattice

$$d = 4r = a\sqrt{3} \Rightarrow a = \frac{4}{\sqrt{3}} r$$

Unit cell vol. $= a^3 = \left(\frac{4}{\sqrt{3}} r\right)^3$

2 atoms per cell, so atom vol. $= 2\left(\frac{4\pi r^3}{3}\right)$

Then

$$Ratio = \frac{2\left(\frac{4\pi r^3}{3}\right)}{\left(\frac{4r}{\sqrt{3}}\right)^3} \times 100\% \Rightarrow \quad Ratio = 68\%$$

(d) Diamond lattice

Body diagonal $= d = 8r = a\sqrt{3} \Rightarrow a = \frac{8}{\sqrt{3}} r$

Unit cell vol. $= a^3 = \left(\frac{8r}{\sqrt{3}}\right)^3$

8 atoms per cell, so atom vol. $8\left(\frac{4\pi r^3}{3}\right)$

Then

$$Ratio = \frac{8\left(\frac{4\pi r^3}{3}\right)}{\left(\frac{8r}{\sqrt{3}}\right)^3} \times 100\% \Rightarrow \quad Ratio = 34\%$$

1.4

From Problem 1.3, percent volume of fcc atoms is 74%; Therefore after coffee is ground,

Volume $= 0.74\ cm^3$

1.5

(a) $a = 5.43\ \mathring{A}$ From 1.3d, $a = \dfrac{8}{\sqrt{3}}r$

so that $r = \dfrac{a\sqrt{3}}{8} = \dfrac{(5.43)\sqrt{3}}{8} = 1.18\ \mathring{A}$

Center of one silicon atom to center of nearest neighbor $= 2r \Rightarrow \underline{2.36\ \mathring{A}}$

(b) Number density

$$= \frac{8}{\left(5.43x10^{-8}\right)^3} \Rightarrow \text{Density } = 5x10^{22}\ cm^{-3}$$

(c) Mass density

$$= \rho = \frac{N(At.Wt.)}{N_A} = \frac{\left(5x10^{22}\right)(28.09)}{6.02x10^{23}} \Rightarrow$$

$$\underline{\rho = 2.33\ grams/cm^3}$$

1.6

(a) $a = 2r_A = 2(1.02) = 2.04\ \mathring{A}$

Now

$$2r_A + 2r_B = a\sqrt{3} \Rightarrow 2r_B = 2.04\sqrt{3} - 2.04$$

so that $r_B = 0.747\ \mathring{A}$

(b) A-type; 1 atom per unit cell

$$\text{Density } = \frac{1}{\left(2.04x10^{-8}\right)^3} \Rightarrow$$

$$\text{Density(A)} = \underline{1.18x10^{23}\ cm^{-3}}$$

B-type: 1 atom per unit cell, so

$$\text{Density(B)} = \underline{1.18x10^{23}\ cm^{-3}}$$

1.7

(b)

$$a = 1.8 + 1.0 \Rightarrow \underline{a = 2.8\ \mathring{A}}$$

(c)

$$\text{Na: Density } = \frac{1/2}{\left(2.8x10^{-8}\right)^3}\ \underline{= 2.28x10^{22}\ cm^{-3}}$$

Cl: Density (same as Na) $\underline{= 2.28x10^{22}\ cm^{-3}}$

(d)
Na: At.Wt. = 22.99
Cl: At. Wt. = 35.45
So, mass per unit cell

$$= \frac{\frac{1}{2}(22.99) + \frac{1}{2}(35.45)}{6.02x10^{23}} = 4.85x10^{-23}$$

Then mass density is

$$\rho = \frac{4.85x10^{-23}}{\left(2.8x10^{-8}\right)^3} \Rightarrow$$

$$\underline{\rho = 2.21\ gm/cm^3}$$

1.8

(a) $a\sqrt{3} = 2(2.2) + 2(1.8) = 8\ \mathring{A}$

so that

$$\underline{a = 4.62\ \mathring{A}}$$

$$\underline{\text{Density of A}} = \frac{1}{\left(4.62x10^{-8}\right)^3} \Rightarrow \underline{1.01x10^{22}\ cm^{-3}}$$

$$\underline{\text{Density of B}} = \frac{1}{\left(4.62x10^{-8}\right)} \Rightarrow \underline{1.01x10^{22}\ cm^{-3}}$$

(b) Same as (a)
(c) Same material

1.9

(a) Surface density

$$= \frac{1}{a^2\sqrt{2}} = \frac{1}{\left(4.62x10^{-8}\right)^2\sqrt{2}} \Rightarrow$$

$$\underline{3.31x10^{14}\ cm^{-2}}$$

Same for A atoms and B atoms
(b) Same as (a)
(c) Same material

1.10

(a) Vol density $= \dfrac{1}{a_o^3}$

Surface density $= \dfrac{1}{a_o^2\sqrt{2}}$

(b) Same as (a)

1.11
Sketch

1.12
(a)

$$\left(\frac{1}{1},\frac{1}{3},\frac{1}{1}\right) \Rightarrow \underline{(313)}$$

(b)

$$\left(\frac{1}{4},\frac{1}{2},\frac{1}{4}\right) \Rightarrow \underline{(121)}$$

1.13
(a) Distance between nearest (100) planes is:

$$\underline{d = a = 5.63\ \overset{\circ}{A}}$$

(b)Distance between nearest (110) planes is:

$$d = \frac{1}{2}a\sqrt{2} = \frac{a}{\sqrt{2}} = \frac{5.63}{\sqrt{2}}$$

or

$$\underline{d = 3.98\ \overset{\circ}{A}}$$

(c) Distance between nearest (111) planes is:

$$d = \frac{1}{3}a\sqrt{3} = \frac{a}{\sqrt{3}} = \frac{5.63}{\sqrt{3}}$$

or

$$\underline{d = 3.25\ \overset{\circ}{A}}$$

1.14
(a)
Simple cubic: $a = 4.50\ \overset{\circ}{A}$
(i) (100) plane, surface density,

$$= \frac{1\ atom}{\left(4.50x10^{-8}\right)^2} \Rightarrow \underline{4.94x10^{14}\ cm^{-2}}$$

(ii) (110) plane, surface density,

$$= \frac{1\ atom}{\sqrt{2}\left(4.50x10^{-8}\right)^2} \Rightarrow \underline{3.49x10^{14}\ cm^{-2}}$$

(iii) (111) plane, surface density,

$$= \frac{3\left(\frac{1}{6}\right)atoms}{\frac{1}{2}\left(a\sqrt{2}\right)(x)} = \frac{\frac{1}{2}}{\frac{1}{2}\cdot a\sqrt{2}\cdot\frac{a\sqrt{3}}{\sqrt{2}}} = \frac{1}{\sqrt{3}a^2}$$

$$= \frac{1}{\sqrt{3}\left(4.50x10^{-8}\right)^2} \Rightarrow \underline{2.85x10^{14}\ cm^{-2}}$$

(b)
Body-centered cubic
(i) (100) plane, surface density,
Same as (a),(i); surface density $\underline{4.94x10^{14}\ cm^{-2}}$
(ii) (110) plane, surface density,

$$= \frac{2\ atoms}{\sqrt{2}\left(4.50x10^{-8}\right)^2} \Rightarrow \underline{6.99x10^{14}\ cm^{-2}}$$

(iii) (111) plane, surface density,
Same as (a),(iii), surface density $\underline{2.85x10^{14}\ cm^{-2}}$

(c)
Face centered cubic
(i) (100) plane, surface density

$$= \frac{2\ atoms}{\left(4.50x10^{-8}\right)^2} \Rightarrow \underline{9.88x10^{14}\ cm^{-2}}$$

(ii) (110) plane, surface density,

$$= \frac{2\ atoms}{\sqrt{2}\left(4.50x10^{-8}\right)^2} \Rightarrow \underline{6.99x10^{14}\ cm^{-2}}$$

(iii) (111) plane, surface density,

$$= \frac{\left(3\cdot\frac{1}{6}+3\cdot\frac{1}{2}\right)}{\frac{\sqrt{3}}{2}a^2} = \frac{4}{\sqrt{3}\left(4.50x10^{-8}\right)^2}$$

or $\underline{1.14x10^{15}\ cm^{-2}}$

1.15
(a)
(100) plane of silicon – similar to a fcc,

$$\text{surface density } = \frac{2\ atoms}{\left(5.43x10^{-8}\right)^2} \Rightarrow$$

$$\underline{6.78x10^{14}\ cm^{-2}}$$

(b)
(110) plane, surface density,

$$= \frac{4\ atoms}{\sqrt{2}\left(5.43x10^{-8}\right)^2} \Rightarrow \underline{9.59x10^{14}\ cm^{-2}}$$

(c)
(111) plane, surface density,

$$= \frac{4\ atoms}{\sqrt{3}\left(5.43x10^{-8}\right)^2} \Rightarrow \underline{7.83x10^{14}\ cm^{-2}}$$

1.16

$$d = 4r = a\sqrt{2}$$

then

$$a = \frac{4r}{\sqrt{2}} = \frac{4(2.25)}{\sqrt{2}} = 6.364\ \overset{\circ}{A}$$

(a)

$$\text{Volume Density } = \frac{4\ atoms}{\left(6.364x10^{-8}\right)^3} \Rightarrow$$

$$\underline{1.55x10^{22}\ cm^{-3}}$$

(b)
Distance between (110) planes,

$$= \frac{1}{2}a\sqrt{2} = \frac{a}{\sqrt{2}} = \frac{6.364}{\sqrt{2}} \Rightarrow$$

or

$\underline{4.50\ \overset{\circ}{A}}$

(c)
Surface density

$$= \frac{2\ atoms}{\sqrt{2}a^2} = \frac{2}{\sqrt{2}\left(6.364x10^{-8}\right)^2}$$

or

$\underline{3.49x10^{14}\ cm^{-2}}$

1.17

Density of silicon atoms $= 5x10^{22}\ cm^{-3}$ and 4 valence electrons per atom, so

Density of valence electrons $\underline{2x10^{23}\ cm^{-3}}$

1.18

Density of GaAs atoms

$$= \frac{8\ atoms}{\left(5.65x10^{-8}\right)^3} = 4.44x10^{22}\ cm^{-3}$$

An average of 4 valence electrons per atom,

Density of valence electrons $\underline{1.77x10^{23}\ cm^{-3}}$

1.19

(a) Percentage $= \dfrac{2x10^{16}}{5x10^{22}} x100\% \Rightarrow$

$\underline{4x10^{-5}\%}$

(b) Percentage $= \dfrac{1x10^{15}}{5x10^{22}} x100\% \Rightarrow$

$\underline{2x10^{-6}\%}$

1.20

(a) Fraction by weight $\approx \dfrac{\left(5x10^{16}\right)(30.98)}{\left(5x10^{22}\right)(28.06)} \Rightarrow$

$\underline{1.10x10^{-6}}$

(b) Fraction by weight

$$\approx \frac{\left(10^{18}\right)(10.82)}{\left(5x10^{16}\right)(30.98) + \left(5x10^{22}\right)(28.06)} \Rightarrow$$

$\underline{7.71x10^{-6}}$

1.21

Volume density $= \dfrac{1}{d^3} = 2x10^{15}\ cm^{-3}$

So

$d = 7.94x10^{-6}\ cm = 794\ \overset{\circ}{A}$

We have $a_o = 5.43\ \overset{\circ}{A}$

So

$$\frac{d}{a_o} = \frac{794}{5.43} \Rightarrow \underline{\frac{d}{a_o} = 146}$$

Chapter 2

Problem Solutions

2.1 Computer plot

2.2 Computer plot

2.3 Computer plot

2.4

For problem 2.2; Phase $= \dfrac{2\pi x}{\lambda} - \omega t =$ constant

Then

$$\frac{2\pi}{\lambda} \cdot \frac{dx}{dt} - \omega = 0 \text{ or } \underline{\frac{dx}{dt} = v_p = +\omega\left(\frac{\lambda}{2\pi}\right)}$$

For problem 2.3; Phase $= \dfrac{2\pi x}{\lambda} + \omega t =$ constant

Then

$$\frac{2\pi}{\lambda} \cdot \frac{dx}{dt} + \omega = 0 \text{ or } \underline{\frac{dx}{dt} = v_p = -\omega\left(\frac{\lambda}{2\pi}\right)}$$

2.5

$$E = h\nu = \frac{hc}{\lambda} \Rightarrow \lambda = \frac{hc}{E}$$

Gold: $E = 4.90\ eV = (4.90)\left(1.6x10^{-19}\right)\ J$

So

$$\lambda = \frac{\left(6.625x10^{-34}\right)\left(3x10^{10}\right)}{(4.90)\left(1.6x10^{-19}\right)} \Rightarrow 2.54x10^{-5}\ cm$$

or

$$\underline{\lambda = 0.254\ \mu m}$$

Cesium: $E = 1.90\ eV = (1.90)\left(1.6x10^{-19}\right)\ J$

So

$$\lambda = \frac{\left(6.625x10^{-34}\right)\left(3x10^{10}\right)}{(1.90)\left(1.6x10^{-19}\right)} \Rightarrow 6.54x10^{-5}\ cm$$

or

$$\underline{\lambda = 0.654\ \mu m}$$

2.6

(a) Electron: (i) K.E. $= T = 1\ eV = 1.6x10^{-19}\ J$

$$p = \sqrt{2mT} = \sqrt{2\left(9.11x10^{-31}\right)\left(1.6x10^{-19}\right)}$$

or

$$\underline{p = 5.4x10^{-25}\ kg - m/s}$$

$$\lambda = \frac{h}{p} = \frac{6.625x10^{-34}}{5.4x10^{-25}} \Rightarrow$$

or

$$\underline{\lambda = 12.3\ \overset{\circ}{A}}$$

(ii) K.E. $= T = 100\ eV = 1.6x10^{-17}\ J$

$$p = \sqrt{2mT} \Rightarrow \underline{p = 5.4x10^{-24}\ kg - m/s}$$

$$\lambda = \frac{h}{p} \Rightarrow \underline{\lambda = 1.23\ \overset{\circ}{A}}$$

(b) Proton: K.E. $= T = 1\ eV = 1.6x10^{-19}\ J$

$$p = \sqrt{2mT} = \sqrt{2\left(1.67x10^{-27}\right)\left(1.6x10^{-19}\right)}$$

or

$$\underline{p = 2.31x10^{-23}\ kg - m/s}$$

$$\lambda = \frac{h}{p} = \frac{6.625x10^{-34}}{2.31x10^{-23}} \Rightarrow$$

or

$$\underline{\lambda = 0.287\ \overset{\circ}{A}}$$

(c) Tungsten Atom: At. Wt. = 183.92

For $T = 1\ eV = 1.6x10^{-19}\ J$

$$p = \sqrt{2mT}$$

$$= \sqrt{2(183.92)\left(1.66x10^{-27}\right)\left(1.6x10^{-19}\right)}$$

or

$$\underline{p = 3.13x10^{-22}\ kg = m/s}$$

$$\lambda = \frac{h}{p} = \frac{6.625x10^{-34}}{3.13x10^{-22}} \Rightarrow$$

or

$$\underline{\lambda = 0.0212\ \overset{\circ}{A}}$$

(d) A 2000 kg traveling at 20 m/s:

$$p = mv = (2000)(20) \Rightarrow$$

or

$$\underline{p = 4x10^{4}\ kg - m/s}$$

$$\lambda = \frac{h}{p} = \frac{6.625x10^{-34}}{4x10^{4}} \Rightarrow$$

or

$$\underline{\lambda = 1.66x10^{-28}\ \overset{\circ}{A}}$$

2.7

$$E_{avg} = \frac{3}{2}kT = \frac{3}{2}(0.0259) \Rightarrow$$

or

$$\underline{E_{avg} = 0.01727\ eV}$$

Now

$$p_{avg} = \sqrt{2mE_{avg}}$$

$$= \sqrt{2\left(9.11x10^{-31}\right)(0.01727)\left(1.6x10^{-19}\right)}$$

or

$$\underline{p_{avg} = 7.1x10^{-26}\ kg-m/s}$$

Now

$$\lambda = \frac{h}{p} = \frac{6.625x10^{-34}}{7.1x10^{-26}} \Rightarrow$$

or

$$\underline{\lambda = 93.3\ \overset{\circ}{A}}$$

2.8

$$E_p = h\nu_p = \frac{hc}{\lambda_p}$$

Now

$$E_e = \frac{p_e^2}{2m} \text{ and } p_e = \frac{h}{\lambda_e} \Rightarrow E_e = \frac{1}{2m}\left(\frac{h}{\lambda_e}\right)^2$$

Set $E_p = E_e$ and $\lambda_p = 10\lambda_e$

Then

$$\frac{hc}{\lambda_p} = \frac{1}{2m}\left(\frac{h}{\lambda_e}\right)^2 = \frac{1}{2m}\left(\frac{10h}{\lambda_p}\right)^2$$

which yields

$$\lambda_p = \frac{100h}{2mc}$$

$$E_p = E = \frac{hc}{\lambda_p} = \frac{hc}{100h}\cdot 2mc = \frac{2mc^2}{100}$$

$$= \frac{2\left(9.11x10^{-31}\right)\left(3x10^8\right)^2}{100} \Rightarrow$$

So

$$\underline{E = 1.64x10^{-15}\ J = 10.3\ keV}$$

2.9

(a) $E = \frac{1}{2}mv^2 = \frac{1}{2}\left(9.11x10^{-31}\right)\left(2x10^4\right)^2$

or

$$E = 1.822x10^{-22}\ J \Rightarrow \underline{E = 1.14x10^{-3}\ eV}$$

Also

$$p = mv = \left(9.11x10^{-31}\right)\left(2x10^4\right) \Rightarrow$$

$$\underline{p = 1.822x10^{-26}\ kg-m/s}$$

Now

$$\lambda = \frac{h}{p} = \frac{6.625x10^{-34}}{1.822x10^{-26}} \Rightarrow$$

$$\underline{\lambda = 364\ \overset{\circ}{A}}$$

(b)

$$p = \frac{h}{\lambda} = \frac{6.625x10^{-34}}{125x10^{-10}} \Rightarrow$$

$$\underline{p = 5.3x10^{-26}\ kg-m/s}$$

Also

$$v = \frac{p}{m} = \frac{5.3x10^{-26}}{9.11x10^{-31}} = 5.82x10^4\ m/s$$

or

$$\underline{v = 5.82x10^6\ cm/s}$$

Now

$$E = \frac{1}{2}mv^2 = \frac{1}{2}\left(9.11x10^{-31}\right)\left(5.82x10^4\right)^2$$

or

$$E = 1.54x10^{-21}\ J \Rightarrow \underline{E = 9.64x10^{-3}\ eV}$$

2.10

(a) $E = h\nu = \frac{hc}{\lambda} = \frac{\left(6.625x10^{-34}\right)\left(3x10^8\right)}{1x10^{-10}}$

or

$$E = 1.99x10^{-15}\ J$$

Now

$$E = e\cdot V \Rightarrow 1.99x10^{-15} = \left(1.6x10^{-19}\right)V$$

so

$$\underline{V = 12.4x10^3\ V = 12.4\ kV}$$

(b) $p = \sqrt{2mE} = \sqrt{2\left(9.11x10^{-31}\right)\left(1.99x10^{-15}\right)}$

$$= 6.02x10^{-23}\ kg-m/s$$

Then

$$\lambda = \frac{h}{p} = \frac{6.625x10^{-34}}{6.02x10^{-23}} \Rightarrow \underline{\lambda = 0.11\ \overset{\circ}{A}}$$

2.11

(a) $\Delta p = \frac{\hbar}{\Delta x} = \frac{1.054x10^{-34}}{10^{-6}} \Rightarrow$

$\underline{\Delta p = 1.054x10^{-28} \ kg - m/s}$

(b)

$E = \frac{hc}{\lambda} = hc\left(\frac{p}{h}\right) = pc$

So

$\Delta E = c(\Delta p) = \left(3x10^{8}\right)\left(1.054x10^{-28}\right) \Rightarrow$

or

$\Delta E = 3.16x10^{-20} \ J \Rightarrow \ \underline{\Delta E = 0.198 \ eV}$

2.12

(a) $\Delta p = \frac{\hbar}{\Delta x} = \frac{1.054x10^{-34}}{12x10^{-10}} \Rightarrow$

$\underline{\Delta p = 8.78x10^{-26} \ kg - m/s}$

(b)

$\Delta E = \frac{1}{2} \cdot \frac{(\Delta p)^2}{m} = \frac{1}{2} \cdot \frac{\left(8.78x10^{-26}\right)^2}{5x10^{-29}} \Rightarrow$

$\Delta E = 7.71x10^{-23} \ J \Rightarrow \ \underline{\Delta E = 4.82x10^{-4} \ eV}$

2.13

(a) Same as 2.12 (a), $\underline{\Delta p = 8.78x10^{-26} \ kg - m/s}$

(b)

$\Delta E = \frac{1}{2} \cdot \frac{(\Delta p)^2}{m} = \frac{1}{2} \cdot \frac{\left(8.78x10^{-26}\right)^2}{5x10^{-26}} \Rightarrow$

$\Delta E = 7.71x10^{-26} \ J \Rightarrow \ \Delta E = 4.82x10^{-7} \ eV$

2.14

$\Delta p = \frac{\hbar}{\Delta x} = \frac{1.054x10^{-34}}{10^{-2}} = 1.054x10^{-32}$

$p = mv \Rightarrow \Delta v = \frac{\Delta p}{m} = \frac{1.054x10^{-32}}{1500} \Rightarrow$

or

$\underline{\Delta v = 7x10^{-36} \ m/s}$

2.15

(a) $\Delta p = \frac{\hbar}{\Delta x} = \frac{1.054x10^{-34}}{10^{-10}} \Rightarrow$

$\underline{\Delta p = 1.054x10^{-24} \ kg - m/s}$

(b) $\Delta t = \frac{1.054x10^{-34}}{(1)\left(1.6x10^{-19}\right)} \Rightarrow$

or

$\underline{\Delta t = 6.6x10^{-16} \ s}$

2.16

(a) If $\Psi_1(x,t)$ and $\Psi_2(x,t)$ are solutions to Schrodinger's wave equation, then

$$\frac{-\hbar^2}{2m} \cdot \frac{\partial^2 \Psi_1(x,t)}{\partial x^2} + V(x)\Psi_1(x,t) = j\hbar \frac{\partial \Psi_1(x,t)}{\partial t}$$

and

$$\frac{-\hbar^2}{2m} \cdot \frac{\partial^2 \Psi_2(x,t)}{\partial x^2} + V(x)\Psi_2(x,t) = j\hbar \frac{\partial \Psi_2(x,t)}{\partial t}$$

Adding the two equations, we obtain

$$\frac{-\hbar^2}{2m} \cdot \frac{\partial^2}{\partial x^2}\left[\Psi_1(x,t) + \Psi_2(x,t)\right]$$
$$+V(x)\left[\Psi_1(x,t) + \Psi_2(x,t)\right]$$
$$= j\hbar \frac{\partial}{\partial t}\left[\Psi_1(x,t) + \Psi_2(x,t)\right]$$

which is Schrodinger's wave equation. So $\Psi_1(x,t) + \Psi_2(x,t)$ is also a solution.

(b)

If $\Psi_1 \cdot \Psi_2$ were a solution to Schrodinger's wave equation, then we could write

$$\frac{-\hbar^2}{2m} \frac{\partial^2}{\partial x^2}\left(\Psi_1 \cdot \Psi_2\right) + V(x)\left(\Psi_1 \cdot \Psi_2\right)$$
$$= j\hbar \frac{\partial}{\partial t}\left(\Psi_1 \cdot \Psi_2\right)$$

which can be written as

$$\frac{-\hbar^2}{2m}\left[\Psi_1 \frac{\partial^2 \Psi_2}{\partial x^2} + \Psi_2 \frac{\partial^2 \Psi_1}{\partial x^2} + 2\frac{\partial \Psi_1}{\partial x} \cdot \frac{\partial \Psi_2}{\partial x}\right]$$
$$+V(x)\Psi_1 \cdot \Psi_2 = j\hbar\left[\Psi_1 \frac{\partial \Psi_2}{\partial t} + \Psi_2 \frac{\partial \Psi_1}{\partial t}\right]$$

Dividing by $\Psi_1 \cdot \Psi_2$ we find

$$\frac{-\hbar^2}{2m}\left[\frac{1}{\Psi_2} \cdot \frac{\partial^2 \Psi_2}{\partial x^2} + \frac{1}{\Psi_1} \cdot \frac{\partial^2 \Psi_1}{\partial x^2} + \frac{1}{\Psi_1 \Psi_2} \frac{\partial \Psi_1}{\partial x} \frac{\partial \Psi_2}{\partial x}\right]$$
$$+V(x) = j\hbar\left[\frac{1}{\Psi_2} \frac{\partial \Psi_2}{\partial t} + \frac{1}{\Psi_1} \frac{\partial \Psi_1}{\partial x}\right]$$

Since Ψ_1 is a solution, then

$$\frac{-\hbar^2}{2m}\cdot\frac{1}{\Psi_1}\cdot\frac{\partial^2\Psi_1}{\partial x^2}+V(x)=j\hbar\cdot\frac{1}{\Psi_1}\cdot\frac{\partial\Psi_1}{\partial t}$$

Subtracting these last two equations, we are left with

$$\frac{-\hbar^2}{2m}\left[\frac{1}{\Psi_2}\frac{\partial^2\Psi_2}{\partial x^2}+\frac{2}{\Psi_1\Psi_2}\frac{\partial\Psi_1}{\partial x}\frac{\partial\Psi_2}{\partial x}\right]$$

$$=j\hbar\frac{1}{\Psi_2}\frac{\partial\Psi_2}{\partial t}$$

Since Ψ_2 is also a solution, we may write

$$\frac{-\hbar^2}{2m}\frac{1}{\Psi_2}\frac{\partial^2\Psi_2}{\partial x^2}+V(x)=j\hbar\frac{1}{\Psi_2}\frac{\partial\Psi_2}{\partial t}$$

Subtracting these last two equations, we obtain

$$\frac{-\hbar^2}{2m}\cdot\frac{2}{\Psi_1\Psi_2}\cdot\frac{\partial\Psi_1}{\partial x}\cdot\frac{\partial\Psi_2}{\partial x}-V(x)=0$$

This equation is not necessarily valid, which means that $\Psi_1\Psi_2$ is, in general, not a solution to Schrodinger's wave equation.

2.17

$$\Psi(x,t)=A[\sin(\pi x)]\exp(-j\omega t)$$

$$\int_{-1}^{+1}|\Psi(x,t)|^2dx=1=|A|^2\int_{-1}^{+1}\sin^2(\pi x)dx$$

or

$$\left|A^2\right|\cdot\left[\frac{1}{2}x-\frac{1}{4\pi}\sin(2\pi x)\right]_{-1}^{+1}=1$$

which yields

$$\underline{\left|A^2\right|=1}\quad\text{or}\quad\underline{A=+1,-1,+j,-j}$$

2.18

$$\Psi(x,t)=A[\sin(n\pi x)]\exp(-j\omega t)$$

$$\int_{0}^{+1}|\Psi(x,t)|^2dx=1=|A|^2\int_{0}^{+1}\sin^2(n\pi x)dx$$

or

$$|A|^2\cdot\left[\frac{1}{2}x-\frac{1}{4n\pi}\sin(2n\pi x)\right]_{0}^{+1}=1$$

which yields

$$\underline{|A|^2=2}\quad\text{or}$$

$$\underline{\underline{A=+\sqrt{2},-\sqrt{2},+j\sqrt{2},-j\sqrt{2}}}$$

2.19

Note that $\int_0^\infty\Psi\cdot\Psi^*dx=1$

Function has been normalized

(a) Now

$$P=\int_0^{a_o/4}\left[\sqrt{\frac{2}{a_o}}\exp\left(\frac{-x}{a_o}\right)\right]^2dx$$

$$=\frac{2}{a_o}\int_0^{a_o/4}\exp\left(\frac{-2x}{a_o}\right)dx$$

$$=\frac{2}{a_o}\left(\frac{-a_o}{2}\right)\exp\left(\frac{-2x}{a_o}\right)\Bigg|_0^{a_o/4}$$

or

$$P=-1\left[\exp\left(\frac{-2a_o}{4a_o}\right)-1\right]=1-\exp\left(\frac{-1}{2}\right)$$

which yields

$$\underline{P=0.393}$$

(b)

$$P=\int_{a_o/4}^{a_o/2}\left(\sqrt{\frac{2}{a_o}}\exp\left(\frac{-x}{a_o}\right)\right)^2dx$$

$$=\frac{2}{a_o}\int_{a_o/4}^{a_o/2}\exp\left(\frac{-2x}{a_o}\right)dx$$

$$=\frac{2}{a_o}\left(\frac{-a_o}{2}\right)\exp\left(\frac{-2x}{a_o}\right)\Bigg|_{a_o/4}^{a_o/2}$$

or

$$P=-1\left[\exp(-1)-\exp\left(\frac{-1}{2}\right)\right]$$

which yields

$$\underline{P=0.239}$$

(c)

$$P=\int_0^{a_o}\left(\sqrt{\frac{2}{a_o}}\exp\left(\frac{-x}{a_o}\right)\right)^2dx$$

$$=\frac{2}{a_o}\int_0^{a_o}\exp\left(\frac{-2x}{a_o}\right)dx=\frac{2}{a_o}\left(\frac{-a_o}{2}\right)\exp\left(\frac{-2x}{a_o}\right)\Bigg|_0^{a_o}$$

or

$$P=-1[\exp(-2)-1]$$

which yields

$$\underline{P=0.865}$$

2.20

(a) $kx - \omega t = \text{constant}$

Then

$$k\frac{dx}{dt} - \omega = 0 \Rightarrow \frac{dx}{dt} = v_p = +\frac{\omega}{k}$$

or

$$v_p = \frac{1.5x10^{13}}{1.5x10^{9}} = 10^4 \ m/s$$

$$\underline{v_p = 10^6 \ cm/s}$$

(b)

$$k = \frac{2\pi}{\lambda} \Rightarrow \lambda = \frac{2\pi}{k} = \frac{2\pi}{1.5x10^9}$$

or

$$\underline{\lambda = 41.9 \ \overset{\circ}{A}}$$

Also

$$p = \frac{h}{\lambda} = \frac{6.625x10^{-34}}{41.9x10^{-10}} \Rightarrow$$

or

$$\underline{p = 1.58x10^{-25} \ kg - m/s}$$

Now

$$E = h\nu = \frac{hc}{\lambda} = \frac{\left(6.625x10^{-34}\right)\left(3x10^8\right)}{41.9x10^{-10}}$$

or

$$E = 4.74x10^{-17} \ J \Rightarrow \underline{E = 2.96x10^2 \ eV}$$

2.21

$$\underline{\psi(x) = A\exp\left[-j(kx + \omega t)\right]}$$

where

$$k = \frac{\sqrt{2mE}}{\hbar}$$

$$= \frac{\sqrt{2\left(9.11x10^{-31}\right)(0.015)\left(1.6x10^{-19}\right)}}{1.054x10^{-34}}$$

or

$$\underline{k = 6.27x10^8 \ m^{-1}}$$

Now

$$\omega = \frac{E}{\hbar} = \frac{(0.015)\left(1.6x10^{-19}\right)}{1.054x10^{-34}}$$

or

$$\underline{\omega = 2.28x10^{13} \ rad/s}$$

2.22

$$E = \frac{\hbar^2 n^2 \pi^2}{2ma^2} = \frac{\left(1.054x10^{-34}\right)^2 \pi^2 n^2}{2\left(9.11x10^{-31}\right)\left(100x10^{-10}\right)^2}$$

so

$$E = 6.018x10^{-22} n^2 \ (J)$$

or

$$E = 3.76x10^{-3} n^2 \ (eV)$$

Then

$$n = 1 \Rightarrow \underline{E_1 = 3.76x10^{-3} \ eV}$$

$$n = 2 \Rightarrow \underline{E_2 = 1.50x10^{-2} \ eV}$$

$$n = 3 \Rightarrow \underline{E_3 = 3.38x10^{-2} \ eV}$$

2.23

(a) $E = \dfrac{\hbar^2 n^2 \pi^2}{2ma^2}$

$$= \frac{\left(1.054x10^{-34}\right)^2 \pi^2 n^2}{2\left(9.11x10^{-31}\right)\left(12x10^{-10}\right)^2}$$

$$= 4.81x10^{-20} n^2 \ (J)$$

So

$$E_1 = 4.18x10^{-20} \ J \Rightarrow \underline{E_1 = 0.261 \ eV}$$

$$E_2 = 1.67x10^{-19} \ J \Rightarrow \underline{E_2 = 1.04 \ eV}$$

(b)

$$E_2 - E_1 = h\nu = \frac{hc}{\lambda} \Rightarrow \lambda = \frac{hc}{\Delta E}$$

or

$$\lambda = \frac{\left(6.625x10^{-34}\right)\left(3x10^8\right)}{1.67x10^{-19} - 4.18x10^{-20}} \Rightarrow$$

$$\lambda = 1.59x10^{-6} \ m$$

or

$$\underline{\lambda = 1.59 \ \mu m}$$

2.24

(a) For the infinite potential well

$$E = \frac{\hbar^2 n^2 \pi^2}{2ma^2} \Rightarrow n^2 = \frac{2ma^2 E}{\hbar^2 \pi^2}$$

so

$$n^2 = \frac{2\left(10^{-5}\right)\left(10^{-2}\right)^2\left(10^{-2}\right)}{\left(1.054x10^{-34}\right)^2 \pi^2} = 1.82x10^{56}$$

or

$$n = 1.35x10^{28}$$

(b)

$$\Delta E = \frac{\hbar^2\pi^2}{2ma^2}\left[(n+1)^2 - n^2\right]$$

$$= \frac{\hbar^2\pi^2}{2ma^2}(2n+1)$$

or

$$\Delta E = \frac{(1.054x10^{-34})^2\pi^2(2)(1.35x10^{28})}{2(10^{-5})(10^{-2})^2}$$

$$\Delta E = 1.48x10^{-30}\ J$$

Energy in the (n+1) state is $1.48x10^{-30}$ Joules larger than 10 mJ.

(c)

Quantum effects would not be observable.

2.25

For a neutron and n = 1:

$$E_1 = \frac{\hbar^2\pi^2}{2ma^2} = \frac{(1.054x10^{-34})\pi^2}{2(1.66x10^{-27})(10^{-14})^2}$$

or

$$E_1 = 2.06x10^6\ eV$$

For an electron in the same potential well:

$$E_1 = \frac{(1.054x10^{-34})^2\pi^2}{2(9.11x10^{-31})(10^{-14})^2}$$

or

$$E_1 = 3.76x10^9\ eV$$

2.26

Schrodinger's wave equation

$$\frac{\partial^2\psi(x)}{\partial x^2} + \frac{2m}{\hbar^2}(E - V(x))\psi(x) = 0$$

We know that

$$\psi(x) = 0 \text{ for } x \ge \frac{a}{2} \text{ and } x \le \frac{-a}{2}$$

$$V(x) = 0 \text{ for } \frac{-a}{2} \le x \le \frac{+a}{2}$$

so in this region

$$\frac{\partial^2\psi(x)}{\partial x^2} + \frac{2mE}{\hbar^2}\psi(x) = 0$$

Solution is of the form

$$\psi(x) = A\cos Kx + B\sin Kx$$

where $K = \sqrt{\dfrac{2mE}{\hbar^2}}$

Boundary conditions:

$$\psi(x) = 0 \text{ at } x = \frac{+a}{2},\ x = \frac{-a}{2}$$

So, first mode:

$$\psi_1(x) = A\cos Kx$$

where $K = \dfrac{\pi}{a}$ so $E_1 = \dfrac{\pi^2\hbar^2}{2ma^2}$

Second mode:

$$\psi_2(x) = B\sin Kx$$

where $K = \dfrac{2\pi}{a}$ so $E_2 = \dfrac{4\pi^2\hbar^2}{2ma^2}$

Third mode:

$$\psi_3(x) = A\cos Kx$$

where $K = \dfrac{3\pi}{a}$ so $E_3 = \dfrac{9\pi^2\hbar^2}{2ma^2}$

Fourth mode:

$$\psi_4(x) = B\sin Kx$$

where $K = \dfrac{4\pi}{a}$ so $E_4 = \dfrac{16\pi^2\hbar^2}{2ma^2}$

2.27

The 3-D wave equation in cartesian coordinates, for V(x,y,z) = 0

$$\frac{\partial^2\psi(x,y,z)}{\partial x^2} + \frac{\partial^2\psi(x,y,z)}{\partial y^2} + \frac{\partial^2\psi(x,y,z)}{\partial z^2} + \frac{2mE}{\hbar^2}\psi(x,y,z) = 0$$

Use separation of variables, so let

$$\psi(x,y,z) = X(x)Y(y)Z(z)$$

Substituting into the wave equation, we get

$$YZ\frac{\partial^2 X}{\partial x^2} + XZ\frac{\partial^2 Y}{\partial y^2} + XY\frac{\partial^2 Z}{\partial z^2} + \frac{2mE}{\hbar^2}XYZ = 0$$

Dividing by XYZ and letting $k^2 = \dfrac{2mE}{\hbar^2}$, we obtain

$$(1)\quad \frac{1}{X}\cdot\frac{\partial^2 X}{\partial x^2} + \frac{1}{Y}\cdot\frac{\partial^2 Y}{\partial y^2} + \frac{1}{Z}\cdot\frac{\partial^2 Z}{\partial z^2} + k^2 = 0$$

We may set

$$\frac{1}{X}\cdot\frac{\partial^2 X}{\partial x^2} = -k_x^2 \text{ so } \frac{\partial^2 X}{\partial x^2} + k_x^2 X = 0$$

Solution is of the form

$$X(x) = A\sin(k_x x) + B\cos(k_x x)$$

Boundary conditions: $X(0) = 0 \Rightarrow B = 0$

and $X(x = a) = 0 \Rightarrow k_x = \dfrac{n_x \pi}{a}$

where $n_x = 1, 2, 3, \ldots$

Similarly, let

$$\frac{1}{Y}\cdot\frac{\partial^2 Y}{\partial y^2} = -k_y^2 \text{ and } \frac{1}{Z}\cdot\frac{\partial^2 Z}{\partial z^2} = -k_z^2$$

Applying the boundary conditions, we find

$$k_y = \frac{n_y \pi}{a},\ n_y = 1, 2, 3, \ldots$$

$$k_z = \frac{n_z \pi}{a},\ n_z = 1, 2, 3, \ldots$$

From Equation (1) above, we have

$$-k_x^2 - k_y^2 - k_z^2 + k^2 = 0$$

or

$$k_x^2 + k_y^2 + k_z^2 = k^2 = \frac{2mE}{\hbar^2}$$

so that

$$E \Rightarrow E_{n_x n_y n_z} = \frac{\hbar^2 \pi^2}{2ma^2}\left(n_x^2 + n_y^2 + n_z^2\right)$$

2.28

For the 2-dimensional infinite potential well:

$$\frac{\partial^2 \psi(x,y)}{\partial x^2} + \frac{\partial^2 \psi(x,y)}{\partial y^2} + \frac{2mE}{\hbar^2}\psi(x,y) = 0$$

Let $\psi(x,y) = X(x)Y(y)$

Then substituting,

$$Y\frac{\partial^2 X}{\partial x^2} + X\frac{\partial^2 Y}{\partial y^2} + \frac{2mE}{\hbar^2}XY = 0$$

Divide by XY

So

$$\frac{1}{X}\cdot\frac{\partial^2 X}{\partial x^2} + \frac{1}{Y}\cdot\frac{\partial^2 Y}{\partial y^2} + \frac{2mE}{\hbar^2} = 0$$

Let

$$\frac{1}{X}\cdot\frac{\partial^2 X}{\partial x^2} = -k_x^2$$

or

$$\frac{\partial^2 X}{\partial x^2} + k_x^2 X = 0$$

Solution is of the form:

$$X = A\sin(k_x x) + B\cos(k_x x)$$

But $X(x = 0) = 0 \Rightarrow B = 0$

So

$$X = A\sin(k_x x)$$

Also, $X(x = a) = 0 \Rightarrow k_x a = n_x \pi$

Where $n_x = 1, 2, 3, \ldots$

So that $k_x = \dfrac{n_x \pi}{a}$

We can also define

$$\frac{1}{Y}\cdot\frac{\partial^2 Y}{\partial y^2} = -k_y^2$$

Solution is of the form

$$Y = C\sin(k_y y) + D\cos(k_y y)$$

But

$$Y(y = 0) = 0 \Rightarrow D = 0$$

and

$$Y(y = b) = 0 \Rightarrow k_y b = n_y \pi$$

so that

$$k_y = \frac{n_y \pi}{b}$$

Now

$$-k_x^2 - k_y^2 + \frac{2mE}{\hbar^2} = 0$$

which yields

$$E \Rightarrow E_{n_x n_y} = \frac{\hbar^2 \pi^2}{2m}\left(\frac{n_x^2}{a^2} + \frac{n_y^2}{b^2}\right)$$

Similarities: energy is quantized
Difference: now a function of 2 integers

2.29

(a) Derivation of energy levels exactly the same as in the text.

(b) $\Delta E = \dfrac{\hbar^2 \pi^2}{2ma^2}\left(n_2^2 - n_1^2\right)$

For $n_2 = 2$, $n_1 = 1$

Then

$$\Delta E = \frac{3\hbar^2 \pi^2}{2ma^2}$$

(i) $a = 4\ \overset{\circ}{A}$

$$\Delta E = \frac{3\left(1.054x10^{-34}\right)^2 \pi^2}{2\left(1.67x10^{-27}\right)\left(4x10^{-10}\right)^2} \Rightarrow$$

$$\underline{\Delta E = 3.85x10^{-3}\ eV}$$

(ii) $a = 0.5\ cm$

$$\Delta E = \frac{3\left(1.054x10^{-34}\right)^2 \pi^2}{2\left(1.67x10^{-27}\right)\left(0.5x10^{-2}\right)^2} \Rightarrow$$

$$\underline{\Delta E = 2.46x10^{-17}\ eV}$$

2.30

(a) For region II, $x > 0$

$$\frac{\partial^2 \psi_2(x)}{\partial x^2} + \frac{2m}{\hbar^2}(E - V_O)\psi_2(x) = 0$$

General form of the solution is

$$\psi_2(x) = A_2 \exp(jK_2 x) + B_2 \exp(-jK_2 x)$$

where

$$K_2 = \sqrt{\frac{2m}{\hbar^2}(E - V_O)}$$

Term with B_2 represents incident wave, and term with A_2 represents the reflected wave.

Region I, $x < 0$

$$\frac{\partial^2 \psi_1(x)}{\partial x^2} + \frac{2mE}{\hbar^2}\psi_1(x) = 0$$

The general solution is of the form

$$\psi_1(x) = A_1 \exp(jK_1 x) + B_1 \exp(-jK_1 x)$$

where

$$K_1 = \sqrt{\frac{2mE}{\hbar^2}}$$

Term involving B_1 represents the transmitted wave, and the term involving A_1 represents the reflected wave; but if a particle is transmitted into region I, it will not be reflected so that $A_1 = 0$.

Then

$$\underline{\psi_1(x) = B_1 \exp(-jK_1 x)}$$

$$\underline{\psi_2(x) = A_2 \exp(jK_2 x) + B_2 \exp(-jK_2 x)}$$

(b)

Boundary conditions:

(1) $\psi_1(x = 0) = \psi_2(x = 0)$

(2) $\left.\dfrac{\partial \psi_1(x)}{\partial x}\right|_{x=0} = \left.\dfrac{\partial \psi_2(x)}{\partial x}\right|_{x=0}$

Applying the boundary conditions to the solutions, we find

$$B_1 = A_2 + B_2$$

$$K_2 A_2 - K_2 B_2 = -K_1 B_1$$

Combining these two equations, we find

$$A_2 = \left(\frac{K_2 - K_1}{K_2 + K_1}\right)B_2 \quad \text{and} \quad B_1 = \left(\frac{2K_2}{K_2 + K_1}\right)B_2$$

The reflection coefficient is

$$R = \frac{A_2 A_2^*}{B_2 B_2^*} \Rightarrow \quad \underline{R = \left(\frac{K_2 - K_1}{K_2 + K_1}\right)^2}$$

The transmission coefficient is

$$T = 1 - R \Rightarrow \quad \underline{T = \frac{4K_1 K_2}{\left(K_1 + K_2\right)^2}}$$

2.31

In region II, $x > 0$, we have

$$\psi_2(x) = A_2 \exp(-K_2 x)$$

where

$$K_2 = \sqrt{\frac{2m(V_O - E)}{\hbar^2}}$$

For $V_O = 2.4\ eV$ and $E = 2.1\ eV$

$$K_2 = \left\{\frac{2\left(9.11x10^{-31}\right)(2.4 - 2.1)\left(1.6x10^{-19}\right)}{\left(1.054x10^{-34}\right)^2}\right\}^{1/2}$$

or

$$K_2 = 2.81x10^9\ m^{-1}$$

Probability at x compared to x = 0, given by

$$P = \left|\frac{\psi_2(x)}{\psi_2(0)}\right|^2 = \exp(-2K_2 x)$$

(a) For $x = 12\ \overset{\circ}{A}$

$$P = \exp\left[-2\left(2.81x10^9\right)\left(12x10^{-10}\right)\right] \Rightarrow$$

$$\underline{P = 1.18x10^{-3} = 0.118\,\%}$$

(b) For $x = 48\ A^\circ$

$$P = \exp\left[-2\left(2.81x10^9\right)\left(48x10^{-10}\right)\right] \Rightarrow$$

$$\underline{P = 1.9x10^{-10}\ \%}$$

2.32

For $V_O = 6\ eV$, $E = 2.2\ eV$

We have that

$$T = 16\left(\frac{E}{V_O}\right)\left(1-\frac{E}{V_O}\right)\exp(-2K_2 a)$$

where

$$K_2 = \sqrt{\frac{2m(V_O - E)}{\hbar^2}}$$

$$= \left\{\frac{2(9.11x10^{-31})(6-2.2)(1.6x10^{-19})}{(1.054x10^{-34})^2}\right\}^{1/2}$$

or

$$K_2 = 9.98x10^9 \ m^{-1}$$

For $a = 10^{-10} \ m$

$$T = 16\left(\frac{2.2}{6}\right)\left(1-\frac{2.2}{6}\right)\exp\left[-2(9.98x10^9)(10^{-10})\right]$$

or

$$\underline{T = 0.50}$$

For $a = 10^{-9} \ m$

$$\underline{T = 7.97x10^{-9}}$$

2.33

Assume that Equation [2.62] is valid:

$$T = 16\left(\frac{E}{V_O}\right)\left(1-\frac{E}{V_O}\right)\exp(-2K_2 a)$$

(a) For $m = (0.067)m_o$

$$K_2 = \sqrt{\frac{2m(V_O - E)}{\hbar^2}}$$

$$= \left\{\frac{2(0.067)(9.11x10^{-31})(0.8-0.2)(1.6x10^{-19})}{(1.054x10^{-34})^2}\right\}^{1/2}$$

or

$$K_2 = 1.027x10^9 \ m^{-1}$$

Then

$$T = 16\left(\frac{0.2}{0.8}\right)\left(1-\frac{0.2}{0.8}\right)\exp\left[-2(1.027x10^9)(15x10^{-10})\right]$$

or

$$\underline{T = 0.138}$$

(b) For $m = (1.08)m_o$

$$K_2 = \left\{\frac{2(1.08)(9.11x10^{-31})(0.8-0.2)(1.6x10^{-19})}{(1.054x10^{-34})^2}\right\}^{1/2}$$

or

$$K_2 = 4.124x10^9 \ m^{-1}$$

Then

$$T = 3\exp\left[-2(4.124x10^9)(15x10^{-10})\right]$$

or

$$\underline{T = 1.27x10^{-5}}$$

2.34

$V_O = 10x10^6 \ eV$, $E = 3x10^6 \ eV$, $a = 10^{-14} \ m$

and $m = 1.67x10^{-27} \ kg$

Now

$$K_2 = \sqrt{\frac{2m(V_O - E)}{\hbar^2}}$$

$$= \left\{\frac{2(1.67x10^{-27})(10-3)(10^6)(1.6x10^{-19})}{(1.054x10^{-34})^2}\right\}^{1/2}$$

or

$$K_2 = 5.80x10^{14} \ m^{-1}$$

So

$$T = 16\left(\frac{3}{10}\right)\left(1-\frac{3}{10}\right)\exp\left[-2(5.80x10^{14})(10^{-14})\right]$$

or

$$\underline{T = 3.06x10^{-5}}$$

2.35

Region I, $V = 0 \ (x < 0)$; Region II, $V = V_O \ (0 < x < a)$; Region III, $V = 0 \ (x > a)$.

(a) Region I;

$$\psi_1(x) = A_1\exp(jK_1x) + B_1\exp(-jK_1x)$$

(incident) (reflected)

Region II;

$$\psi_2(x) = A_2\exp(K_2x) + B_2\exp(-K_2x)$$

Region III;

$$\psi_3(x) = A_3\exp(jK_1x) + B_3\exp(-jK_1x)$$

(b)

In region III, the B_3 term represents a reflected wave. However, once a particle is transmitted into region III, there will not be a reflected wave which means that $B_3 = 0$.

(c)

Boundary conditions:

For $x = 0$: $\psi_1 = \psi_2 \Rightarrow A_1 + B_1 = A_2 + B_2$

$$\frac{d\psi_1}{dx} = \frac{d\psi_2}{dx} \Rightarrow jK_1A_1 - jK_1B_2 = K_2A_2 - K_2B_2$$

For $x = a$: $\psi_2 = \psi_3 \Rightarrow$

$$A_2\exp(K_2a) + B_2\exp(-K_2a) = A_3\exp(jK_1a)$$

And also

$$\frac{d\psi_2}{dx} = \frac{d\psi_3}{dx} \Rightarrow$$

$$K_2 A_2 \exp(K_2 a) - K_2 B_2 \exp(-K_2 a)$$
$$= jK_1 A_3 \exp(jK_1 a)$$

Transmission coefficient is defined as

$$T = \frac{A_3 A_3^*}{A_1 A_1^*}$$

so from the boundary conditions, we want to solve for A_3 in terms of A_1. Solving for A_1 in terms of A_3, we find

$$A_1 = \frac{+jA_3}{4K_1K_2}\{(K_2^2 - K_1^2)[\exp(K_2 a) - \exp(-K_2 a)]$$
$$-2jK_1K_2[\exp(K_2 a) + \exp(-K_2 a)]\ \}\exp(jK_2 a)$$

We then find that

$$A_1 A_1^* = \frac{A_3 A_3^*}{(4K_1K_2)^2}\{(K_2^2 - K_1^2)[\exp(K_2 a)$$
$$-\exp(-K_2 a)]^2$$
$$+4K_1^2K_2^2[\exp(K_2 a) + \exp(-K_2 a)]^2\ \}$$

We have

$$K_2 = \sqrt{\frac{2m(V_O - E)}{\hbar^2}}$$

and since $V_O >> E$, then $K_2 a$ will be large so that

$$\exp(K_2 a) >> \exp(-K_2 a)$$

Then we can write

$$A_1 A_1^* = \frac{A_3 A_3^*}{(4K_1K_2)^2}\{(K_2^2 - K_1^2)[\exp(K_2 a)]^2$$
$$+4K_1^2K_2^2[\exp(K_2 a)]^2\}$$

which becomes

$$A_1 A_1^* = \frac{A_3 A_3^*}{(4K_1K_2)^2}(K_2^2 + K_1^2)\exp(2K_2 a)$$

Substituting the expressions for K_1 and K_2, we find

$$K_1^2 + K_2^2 = \frac{2mV_O}{\hbar^2}$$

and

$$K_1^2K_2^2 = \left[\frac{2m(V_O - E)}{\hbar^2}\right]\left[\frac{2mE}{\hbar^2}\right]$$

$$= \left(\frac{2m}{\hbar^2}\right)(V_O = E)(E)$$

or

$$K_1^2K_2^2 = \left(\frac{2m}{\hbar^2}\right)^2 V_O\left(1 - \frac{E}{V_O}\right)(E)$$

Then

$$A_1 A_1^* = \frac{A_3 A_3^*\left(\frac{2mV_O}{\hbar^2}\right)^2 \exp(2K_2 a)}{16\left[\left(\frac{2m}{\hbar^2}\right)^2 V_O\left(1 - \frac{E}{V_O}\right)(E)\right]}$$

$$= \frac{A_3 A_3^*}{16\left(\frac{E}{V_O}\right)\left(1 - \frac{E}{V_O}\right)\exp(-2K_2 a)}$$

or finally

$$T = \frac{A_3 A_3^*}{A_1 A_1^*} = 16\left(\frac{E}{V_O}\right)\left(1 - \frac{E}{V_O}\right)\exp(-2K_2 a)$$

2.36

Region I: $V = 0$

$$\frac{\partial^2\psi_1}{\partial x^2} + \frac{2mE}{\hbar^2}\psi_1 = 0 \Rightarrow$$

$$\psi_1 = A_1 \exp(jK_1 x) + B_1 \exp(-jK_1 x)$$

(incident wave) (reflected wave)

where $K_1 = \sqrt{\frac{2mE}{\hbar^2}}$

Region II: $V = V_1$

$$\frac{\partial^2\psi_2}{\partial x^2} + \frac{2m(E - V_1)}{\hbar^2}\psi_2 = 0 \Rightarrow$$

$$\psi_2 = A_2 \exp(jK_2 x) + B_2 \exp(-jK_2 x)$$

(transmitted wave) (reflected wave)

where $K_2 = \sqrt{\frac{2m(E - V_1)}{\hbar^2}}$

Region III: $V = V_2$

$$\frac{\partial^2\psi_3}{\partial x^2} + \frac{2m(E - V_2)}{\hbar^2}\psi_3 = 0 \Rightarrow$$

$$\psi_3 = A_3 \exp(jK_3 x)$$

(transmitted wave)

where $K_3 = \sqrt{\frac{2m(E - V_2)}{\hbar^2}}$

There is no reflected wave in region III.

The transmission coefficient is defined as

$$T = \frac{v_3}{v_1}\cdot\frac{A_3A_3^*}{A_1A_1^*} = \frac{K_3}{K_1}\cdot\frac{A_3A_3^*}{A_1A_1^*}$$

From boundary conditions, solve for A_3 in terms of A_1. The boundary conditions are:

$x = 0$: $\psi_1 = \psi_2 \Rightarrow A_1 + B_1 = A_2 + B_2$

$$\frac{\partial\psi_1}{\partial x} = \frac{\partial\psi_2}{\partial x} \Rightarrow K_1A_1 - K_1B_1 = K_2A_2 - K_2B_2$$

$x = a$: $\psi_2 = \psi_3 \Rightarrow$

$$A_2\exp(jK_2a) + B_2\exp(-jK_2a) = A_3\exp(jK_3a)$$

$$\frac{\partial\psi_2}{\partial x} = \frac{\partial\psi_3}{\partial x} \Rightarrow$$

$$K_2A_2\exp(jK_2a) - K_2B_2\exp(-jK_2a) = K_3A_3\exp(jK_3a)$$

But $K_2a = 2n\pi \Rightarrow$

$$\exp(jK_2a) = \exp(-jK_2a) = 1$$

Then, eliminating B_1, A_2, B_2 from the above equations, we have

$$T = \frac{K_3}{K_1}\cdot\frac{4K_1^2}{(K_1+K_3)^2} \Rightarrow T = \frac{4K_1K_3}{(K_1+K_3)^2}$$

2.37

(a) Region I: Since $V_O > E$, we can write

$$\frac{\partial^2\psi_1}{\partial x^2} - \frac{2m(V_O - E)}{\hbar^2}\psi_1 = 0$$

Region II: $V = 0$, so

$$\frac{\partial^2\psi_2}{\partial x^2} + \frac{2mE}{\hbar^2}\psi_2 = 0$$

Region III: $V \to \infty \Rightarrow \psi_3 = 0$

The general solutions can be written, keeping in mind that ψ_1 must remain finite for $x < 0$, as

$$\psi_1 = B_1\exp(+K_1x)$$

$$\psi_2 = A_2\sin(K_2x) + B_2\cos(K_2x)$$

$$\psi_3 = 0$$

where

$$K_1 = \sqrt{\frac{2m(V_O - E)}{\hbar^2}} \quad\text{and}\quad K_2 = \sqrt{\frac{2mE}{\hbar^2}}$$

(b)

Boundary conditions:

$x = 0$: $\psi_1 = \psi_2 \Rightarrow B_1 = B_2$

$$\frac{\partial\psi_1}{\partial x} = \frac{\partial\psi_2}{\partial x} \Rightarrow K_1B_1 = K_2A_2$$

$x = a$: $\psi_2 = \psi_3 \Rightarrow$

$$A_2\sin K_2a + B_2\cos K_2a = 0$$

or

$$B_2 = -A_2\tan K_2a$$

(c)

$$K_1B_1 = K_2A_2 \Rightarrow A_2 = \left(\frac{K_1}{K_2}\right)B_1$$

and since $B_1 = B_2$, then

$$A_2 = \left(\frac{K_1}{K_2}\right)B_2$$

From $B_2 = -A_2\tan K_2a$, we can write

$$B_2 = -\left(\frac{K_1}{K_2}\right)B_2\tan K_2a$$

which gives

$$1 = -\left(\frac{K_1}{K_2}\right)\tan K_2a$$

In turn, this equation can be written as

$$1 = -\sqrt{\frac{V_O - E}{E}}\tan\left[\sqrt{\frac{2mE}{\hbar^2}}\cdot a\right]$$

or

$$\sqrt{\frac{E}{V_O - E}} = -\tan\left[\sqrt{\frac{2mE}{\hbar^2}}\cdot a\right]$$

This last equation is valid only for specific values of the total energy E. The energy levels are quantized.

2.38

$$E_n = \frac{-m_oe^4}{(4\pi\in_o)^2 2\hbar^2n^2}\ (J)$$

$$= \frac{m_oe^3}{(4\pi\in_o)^2 2\hbar^2n^2}\ (eV)$$

$$= \frac{-(9.11x10^{-31})(1.6x10^{-19})^3}{\left[4\pi(8.85x10^{-12})\right]^2 2(1.054x10^{-34})^2n^2} \Rightarrow$$

$$E_n = \frac{-13.58}{n^2}\ (eV)$$

Then

$$n = 1 \Rightarrow \quad \underline{E_1 = -13.58\ eV}$$
$$n = 2 \Rightarrow \quad \underline{E_2 = -3.395\ eV}$$
$$n = 3 \Rightarrow \quad \underline{E_3 = -1.51\ eV}$$
$$n = 4 \Rightarrow \quad \underline{E_4 = -0.849\ eV}$$

2.39

We have

$$\psi_{100} = \frac{1}{\sqrt{\pi}} \cdot \left(\frac{1}{a_o}\right)^{3/2} \exp\left(\frac{-r}{a_o}\right)$$

and

$$P = 4\pi r^2 \psi_{100} \psi_{100}^* = 4\pi r^2 \cdot \frac{1}{\pi} \cdot \left(\frac{1}{a_o}\right)^3 \exp\left(\frac{-2r}{a_o}\right)$$

or

$$P = \frac{4}{(a_o)^3} \cdot r^2 \exp\left(\frac{-2r}{a_o}\right)$$

To find the maximum probability

$$\frac{dP(r)}{dr} = 0$$

$$= \frac{4}{(a_o)^3}\left\{r^2\left(\frac{-2}{a_o}\right)\exp\left(\frac{-2r}{a_o}\right) + 2r\exp\left(\frac{-2r}{a_o}\right)\right\}$$

which gives

$$0 = \frac{-r}{a_o} + 1 \Rightarrow \quad \underline{r = a_o}$$

or $r = a_o$ is the radius that gives the greatest probability.

2.40

ψ_{100} is independent of θ and ϕ, so the wave equation in spherical coordinates reduces to

$$\frac{1}{r^2} \cdot \frac{\partial}{\partial r}\left(r^2 \frac{\partial \psi}{\partial r}\right) + \frac{2m_o}{\hbar^2}\left(E - V(r)\right)\psi = 0$$

where

$$V(r) = \frac{-e^2}{4\pi \in_o r} = \frac{-\hbar^2}{m_o a_o r}$$

For

$$\psi_{100} = \frac{1}{\sqrt{\pi}} \cdot \left(\frac{1}{a_o}\right)^{3/2} \exp\left(\frac{-r}{a_o}\right) \Rightarrow$$

$$\frac{d\psi_{100}}{dr} = \frac{1}{\sqrt{\pi}} \cdot \left(\frac{1}{a_o}\right)^{3/2}\left(\frac{-1}{a_o}\right)\exp\left(\frac{-r}{a_o}\right)$$

Then

$$r^2 \frac{d\psi_{100}}{dr} = \frac{-1}{\sqrt{\pi}} \cdot \left(\frac{1}{a_o}\right)^{5/2} r^2 \exp\left(\frac{-r}{a_o}\right)$$

so that

$$\frac{d}{dr}\left(r^2 \frac{d\psi_{100}}{dr}\right)$$

$$= \frac{-1}{\sqrt{\pi}} \cdot \left(\frac{1}{a_o}\right)^{5/2}\left[2r\exp\left(\frac{-r}{a_o}\right) - \left(\frac{r^2}{a_o}\right)\exp\left(\frac{-r}{a_o}\right)\right]$$

Substituting into the wave equation, we have

$$\frac{-1}{r^2\sqrt{\pi}} \cdot \left(\frac{1}{a_o}\right)^{5/2}\left[2r\exp\left(\frac{-r}{a_o}\right) - \frac{r^2}{a_o}\exp\left(\frac{-r}{a_o}\right)\right]$$

$$+ \frac{2m_o}{\hbar^2}\left[E + \frac{\hbar^2}{m_o a_o r}\right] \cdot \left(\frac{1}{\sqrt{\pi}}\right) \cdot \left(\frac{1}{a_o}\right)^{3/2} \exp\left(\frac{-r}{a_o}\right) = 0$$

where

$$E = E_1 = \frac{-m_o e^4}{(4\pi \in_o)^2 \cdot 2\hbar^2} \Rightarrow \quad E_1 = \frac{-\hbar^2}{2m_o a_o^2}$$

Then the above equation becomes

$$\frac{1}{\sqrt{\pi}} \cdot \left(\frac{1}{a_o}\right)^{3/2}\left[\exp\left(\frac{-r}{a_o}\right)\right]\left\{\frac{-1}{r^2 a_o}\left[2r - \frac{r^2}{a_o}\right]\right.$$

$$\left. + \frac{2m_o}{\hbar^2}\left(\frac{-\hbar^2}{2m_o a_o} + \frac{\hbar^2}{m_o a_o r}\right)\right\} = 0$$

or

$$\frac{1}{\sqrt{\pi}} \cdot \left(\frac{1}{a_o}\right)^{3/2}\left[\exp\left(\frac{-r}{a_o}\right)\right]$$

$$\times\left\{\frac{-2}{a_o r} + \frac{1}{a_o^2} + \left(\frac{-1}{a_o^2} + \frac{2}{a_o r}\right)\right\} = 0$$

which gives 0 = 0, and shows that ψ_{100} is indeed a solution of the wave equation.

2.41

All elements from Group I column of the periodic table. All have one valence electron in the outer shell.

Chapter 3

Problem Solutions

3.1 If a_o were to increase, the bandgap energy would decrease and the material would begin to behave less like a semiconductor and more like a metal. If a_o were to decrease, the bandgap energy would increase and the material would begin to behave more like an insulator.

3.2
Schrodinger's wave equation

$$\frac{-\hbar^2}{2m}\cdot\frac{\partial^2\Psi(x,t)}{\partial x^2}+V(x)\cdot\Psi(x,t)=j\hbar\frac{\partial\Psi(x,t)}{\partial t}$$

Let the solution be of the form

$$\Psi(x,t)=u(x)\exp\left[j\left(kx-\left(\frac{E}{\hbar}\right)t\right)\right]$$

Region I, $V(x)=0$, so substituting the proposed solution into the wave equation, we obtain

$$\frac{-\hbar^2}{2m}\cdot\frac{\partial}{\partial x}\left\{jku(x)\exp\left[j\left(kx-\left(\frac{E}{\hbar}\right)t\right)\right]+\frac{\partial u(x)}{\partial x}\exp\left[j\left(kx-\left(\frac{E}{\hbar}\right)t\right)\right]\right\}$$
$$=j\hbar\left(\frac{-jE}{\hbar}\right)\cdot u(x)\exp\left[j\left(kx-\left(\frac{E}{\hbar}\right)t\right)\right]$$

which becomes

$$\frac{-\hbar^2}{2m}\left\{(jk)^2u(x)\exp\left[j\left(kx-\left(\frac{E}{\hbar}\right)t\right)\right]+2jk\frac{\partial u(x)}{\partial x}\exp\left[j\left(kx-\left(\frac{E}{\hbar}\right)t\right)\right]+\frac{\partial^2u(x)}{\partial x^2}\exp\left[j\left(kx-\left(\frac{E}{\hbar}\right)t\right)\right]\right\}$$
$$=+Eu(x)\exp\left[j\left(kx-\left(\frac{E}{\hbar}\right)t\right)\right]$$

This equation can then be written as

$$-k^2u(x)+2jk\frac{\partial u(x)}{\partial x}+\frac{\partial^2u(x)}{\partial x^2}+\frac{2mE}{\hbar^2}\cdot u(x)=0$$

Setting $u(x)=u_1(x)$ for region I, this equation becomes

$$\frac{d^2u_1(x)}{dx^2}+2jk\frac{du_1(x)}{dx}-\left(k^2-\alpha^2\right)u_1(x)=0$$

where

$$\alpha^2=\frac{2mE}{\hbar^2}$$

Q.E.D.

In region II, $V(x)=V_O$. Assume the same form of the solution

$$\Psi(x,t)=u(x)\exp\left[j\left(kx-\left(\frac{E}{\hbar}\right)t\right)\right]$$

Substituting into Schrodinger's wave equation, we obtain

$$\frac{-\hbar^2}{2m}\left\{(jk)^2u(x)\exp\left[j\left(kx-\left(\frac{E}{\hbar}\right)t\right)\right]+2jk\frac{\partial u(x)}{\partial x}\exp\left[j\left(kx-\left(\frac{E}{\hbar}\right)t\right)\right]+\frac{\partial^2u(x)}{\partial x^2}\exp\left[j\left(kx-\left(\frac{E}{\hbar}\right)t\right)\right]\right\}$$
$$+V_Ou(x)\exp\left[j\left(kx-\left(\frac{E}{\hbar}\right)t\right)\right]$$
$$=Eu(x)\exp\left[j\left(kx-\left(\frac{E}{\hbar}\right)t\right)\right]$$

This equation can be written as

$$-k^2u(x)+2jk\frac{\partial u(x)}{\partial x}+\frac{\partial^2u(x)}{\partial x^2}-\frac{2mV_O}{\hbar^2}u(x)+\frac{2mE}{\hbar^2}u(x)=0$$

Setting $u(x)=u_2(x)$ for region II, this equation becomes

$$\frac{d^2u_2(x)}{dx^2}+2jk\frac{du_2(x)}{dx}-\left(k^2-\alpha^2+\frac{2mV_O}{\hbar^2}\right)u_2(x)=0$$

where

$$\alpha^2=\frac{2mE}{\hbar^2}$$

Q.E.D.

3.3
We have

$$\frac{d^2u_1(x)}{dx^2} + 2jk\frac{du_1(x)}{dx} - \left(k^2 - \alpha^2\right)u_1(x) = 0$$

The proposed solution is

$$u_1(x) = A\exp\left[j(\alpha - k)x\right] + B\exp\left[-j(\alpha + k)x\right]$$

The first derivative is

$$\frac{du_1(x)}{dx} = j(\alpha - k)A\exp\left[j(\alpha - k)x\right]$$
$$-j(\alpha + k)B\exp\left[-j(\alpha + k)x\right]$$

and the second derivative becomes

$$\frac{d^2u_1(x)}{dx^2} = \left[j(\alpha - k)\right]^2 A\exp\left[j(\alpha - k)x\right]$$
$$+\left[j(\alpha + k)\right]^2 B\exp\left[-j(\alpha + k)x\right]$$

Substituting these equations into the differential equation, we find

$$-(\alpha - k)^2 A\exp\left[j(\alpha - k)x\right]$$
$$-(\alpha + k)^2 B\exp\left[-j(\alpha + k)x\right]$$
$$+2jk\{j(\alpha - k)A\exp\left[j(\alpha - k)x\right]$$
$$-j(\alpha + k)B\exp\left[-j(\alpha + k)x\right]\}$$
$$-\left(k^2 - \alpha^2\right)\{A\exp\left[j(\alpha - k)x\right]$$
$$+B\exp\left[-j(\alpha + k)x\right]\} = 0$$

Combining terms, we have

$$\left\{-\left(\alpha^2 - 2\alpha k + k^2\right) - 2k(\alpha - k)\right.$$
$$\left.-\left(k^2 - \alpha^2\right)\right\}A\exp\left[j(\alpha - k)x\right]$$
$$+\left\{-\left(\alpha^2 + 2\alpha k + k^2\right) + 2k(\alpha + k)\right.$$
$$\left.-\left(k^2 - \alpha^2\right)\right\}B\exp\left[-j(\alpha + k)x\right] = 0$$

We find that

$0 = 0$ Q.E.D.

For the differential equation in $u_2(x)$ and the proposed solution, the procedure is exactly the same as above.

3.4
We have the solutions

$$u_1(x) = A\exp\left[j(\alpha - k)x\right] + B\exp\left[-j(\alpha + k)x\right]$$

for $0 < x < a$

$$u_2(x) = C\exp\left[j(\beta - k)x\right] + D\exp\left[-j(\beta + k)x\right]$$

for $-b < x < 0$

The boundary conditions:

$$u_1(0) = u_2(0)$$

which yields

$$\underline{A + B - C - D = 0}$$

Also

$$\left.\frac{du_1}{dx}\right|_{x=0} = \left.\frac{du_2}{dx}\right|_{x=0}$$

which yields

$$\underline{(\alpha - k)A - (\alpha + k)B - (\beta - k)C + (\beta + k)D = 0}$$

The third boundary condition is

$$u_1(a) = u_2(-b)$$

which gives

$$A\exp\left[j(\alpha - k)a\right] + B\exp\left[-j(\alpha + k)a\right]$$
$$= C\exp\left[j(\beta - k)(-b)\right] + D\exp\left[-j(\beta + k)(-b)\right]$$

This becomes

$$\underline{A\exp\left[j(\alpha - k)a\right] + B\exp\left[-j(\alpha + k)a\right]}$$
$$\underline{-C\exp\left[-j(\beta - k)b\right] - D\exp\left[j(\beta + k)b\right] = 0}$$

The last boundary condition is

$$\left.\frac{du_1}{dx}\right|_{x=a} = \left.\frac{du_2}{dx}\right|_{x=-b}$$

which gives

$$j(\alpha - k)A\exp\left[j(\alpha - k)a\right]$$
$$-j(\alpha + k)B\exp\left[-j(\alpha + k)a\right]$$
$$= j(\beta - k)C\exp\left[j(\beta - k)(-b)\right]$$
$$-j(\beta + k)D\exp\left[-j(\beta + k)(-b)\right]$$

This becomes

$$\underline{(\alpha - k)A\exp\left[j(\alpha - k)a\right]}$$
$$\underline{-(\alpha + k)B\exp\left[-j(\alpha + k)a\right]}$$
$$\underline{-(\beta - k)C\exp\left[-j(\beta - k)b\right]}$$
$$\underline{+(\beta + k)D\exp\left[j(\beta + k)b\right] = 0}$$

3.5 Computer plot

3.6 Computer plot

3.7

$$P'\frac{\sin\alpha a}{\alpha a} + \cos\alpha a = \cos ka$$

Let $ka = y$, $\alpha a = x$

Then

$$P'\frac{\sin x}{x} + \cos x = \cos y$$

Consider $\frac{d}{dy}$ of this function

$$\frac{d}{dy}\left\{\left[P'\cdot(x)^{-1}\cdot\sin x\right]+\cos x\right\}=-\sin y$$

We obtain

$$P'\left\{(-1)(x)^{-2}\sin x\frac{dx}{dy}+(x)^{-1}\cos x\frac{dx}{dy}\right\}$$

$$-\sin x\frac{dx}{dy}=-\sin y$$

Then

$$\frac{dx}{dy}\left\{P'\left[\frac{-1}{x^2}\sin x+\frac{\cos x}{x}\right]-\sin x\right\}=-\sin y$$

For $y=ka=n\pi$, $n=0,1,2,\ldots$

$$\Rightarrow \sin y=0$$

So that, in general, then

$$\frac{dx}{dy}=0=\frac{d(\alpha a)}{d(ka)}=\frac{d\alpha}{dk}$$

And

$$\alpha=\sqrt{\frac{2mE}{\hbar^2}}\Rightarrow\frac{d\alpha}{dk}=\frac{1}{2}\left(\frac{2mE}{\hbar^2}\right)^{-1/2}\left(\frac{2m}{\hbar^2}\right)\frac{dE}{dk}$$

This implies that

$$\underline{\frac{d\alpha}{dk}=0=\frac{dE}{dk}}\ \text{for}\ \underline{k=\frac{n\pi}{a}}$$

3.8

$$f(\alpha a)=9\frac{\sin\alpha a}{\alpha a}+\cos\alpha a=\cos ka$$

(a) $ka=\pi\Rightarrow\cos ka=-1$

1st point: $\alpha a=\pi$: 2nd point: $\alpha a=1.66\pi$

(2nd point by trial and error)

Now

$$\alpha a=a\sqrt{\frac{2mE}{\hbar^2}}\Rightarrow E=\left(\frac{\alpha a}{a}\right)^2\cdot\frac{\hbar^2}{2m}$$

So

$$E=\frac{(\alpha a)^2}{\left(5x10^{-10}\right)^2}\cdot\frac{\left(1.054x10^{-34}\right)^2}{2\left(9.11x10^{-31}\right)}\Rightarrow$$

$$E=(\alpha a)^2\left[2.439x10^{-20}\right]\ (J)$$

or

$$E=(\alpha a)^2(0.1524)\quad(eV)$$

So

$$\alpha a=\pi\Rightarrow E_1=1.504\ eV$$

$$\alpha a=1.66\pi\Rightarrow E_2=4.145\ eV$$

Then

$$\underline{\Delta E=2.64\ eV}$$

(b)

$ka=2\pi\Rightarrow\cos ka=+1$

1st point: $\alpha a=2\pi$

2nd point: $\alpha a=2.54\pi$

Then

$$E_3=6.0165\ eV$$

$$E_4=9.704\ eV$$

so

$$\underline{\Delta E=3.69\ eV}$$

(c)

$ka=3\pi\Rightarrow\cos ka=-1$

1st point: $\alpha a=3\pi$

2nd point: $\alpha a=3.44\pi$

Then

$$E_5=13.537\ eV$$

$$E_6=17.799\ eV$$

so

$$\underline{\Delta E=4.26\ eV}$$

(d)

$ka=4\pi\Rightarrow\cos ka=+1$

1st point: $\alpha a=4\pi$

2nd point: $\alpha a=4.37\pi$

Then

$$E_7=24.066\ eV$$

$$E_8=28.724\ eV$$

so

$$\underline{\Delta E=4.66\ eV}$$

3.9

(a) $0<ka<\pi$

For $ka=0\Rightarrow\cos ka=+1$

By trial and error: 1st point: $\alpha a=0.822\pi$

2nd point: $\alpha a=\pi$

From Problem 3.8, $E=(\alpha a)^2(0.1524)\quad(eV)$

Then

$$E_1=1.0163\ eV$$

$$E_2=1.5041\ eV$$

so

$$\underline{\Delta E=0.488\ eV}$$

(b)

$\pi<ka<2\pi$

Using results of Problem 3.8

1st point: $\alpha a=1.66\pi$

2nd point: $\alpha a=2\pi$

Then

$E_3 = 4.145\ eV$

$E_4 = 6.0165\ eV$

so

$\underline{\Delta E = 1.87\ eV}$

(c)
$2\pi < ka < 3\pi$

1st point: $\alpha a = 2.54\pi$

2nd point: $\alpha a = 3\pi$

Then

$E_5 = 9.704\ eV$

$E_6 = 13.537\ eV$

so

$\underline{\Delta E = 3.83\ eV}$

(d)
$3\pi < ka < 4\pi$

1st point: $\alpha a = 3.44\pi$

2nd point: $\alpha a = 4\pi$

Then

$E_7 = 17.799\ eV$

$E_8 = 24.066\ eV$

so

$\underline{\Delta E = 6.27\ eV}$

3.10

$$6\frac{\sin \alpha a}{\alpha a} + \cos \alpha a = \cos ka$$

Forbidden energy bands

(a) $ka = \pi \Rightarrow \cos ka = -1$

1st point: $\alpha a = \pi$

2nd point: $\alpha a = 1.56\pi$ (By trial and error)

From Problem 3.8, $E = (\alpha a)^2 (0.1524)\ eV$

Then

$E_1 = 1.504\ eV$

$E_2 = 3.660\ eV$

so

$\underline{\Delta E = 2.16\ eV}$

(b)
$ka = 2\pi \Rightarrow \cos ka = +1$

1st point: $\alpha a = 2\pi$

2nd point: $\alpha a = 2.42\pi$

Then

$E_3 = 6.0165\ eV$

$E_4 = 8.809\ eV$

so

$\underline{\Delta E = 2.79\ eV}$

(c)
$ka = 3\pi \Rightarrow \cos ka = -1$

1st point: $\alpha a = 3\pi$

2nd point: $\alpha a = 3.33\pi$

Then

$E_5 = 13.537\ eV$

$E_6 = 16.679\ eV$

so

$\underline{\Delta E = 3.14\ eV}$

(d)
$ka = 4\pi \Rightarrow \cos ka = +1$

1st point: $\alpha a = 4\pi$

2nd point: $\alpha a = 4.26\pi$

Then

$E_7 = 24.066\ eV$

$E_8 = 27.296\ eV$

so

$\underline{\Delta E = 3.23\ eV}$

3.11

Allowed energy bands

Use results from Problem 3.10.

(a)
$0 < ka < \pi$

1st point: $\alpha a = 0.759\pi$ (By trial and error)

2nd point: $\alpha a = \pi$

We have

$E = (\alpha a)^2 (0.1524)\ eV$

Then

$E_1 = 0.8665\ eV$

$E_2 = 1.504\ eV$

so

$\underline{\Delta E = 0.638\ eV}$

(b)
$\pi < ka < 2\pi$

1st point: $\alpha a = 1.56\pi$

2nd point: $\alpha a = 2\pi$

Then

$E_3 = 3.660\ eV$

$E_4 = 6.0165\ eV$

so

$\underline{\Delta E = 2.36\ eV}$

(c)
$2\pi < ka < 3\pi$

1st point: $\alpha a = 2.42\pi$

2nd point: $\alpha a = 3\pi$

Then

$E_5 = 8.809\ eV$

$E_6 = 13.537\ eV$

so

$$\underline{\Delta E = 4.73\ eV}$$

(d)

$3\pi < ka < 4\pi$

1st point: $\alpha a = 3.33\pi$

2nd point: $\alpha a = 4\pi$

Then

$E_7 = 16.679\ eV$

$E_8 = 24.066\ eV$

so

$$\underline{\Delta E = 7.39\ eV}$$

3.12

$$T = 100K\ ;\quad E_g = 1.170 - \frac{(4.73x10^{-4})(100)^2}{636+100} \Rightarrow$$

$$\underline{E_g = 1.164\ eV}$$

$$T = 200K \Rightarrow \underline{E_g = 1.147\ eV}$$

$$T = 300K \Rightarrow \underline{E_g = 1.125\ eV}$$

$$T = 400K \Rightarrow \underline{E_g = 1.097\ eV}$$

$$T = 500K \Rightarrow \underline{E_g = 1.066\ eV}$$

$$T = 600K \Rightarrow \underline{E_g = 1.032\ eV}$$

3.13

The effective mass is given by

$$m^* = \left(\frac{1}{\hbar^2}\cdot\frac{d^2E}{dk^2}\right)^{-1}$$

We have that

$$\frac{d^2E}{dk^2}(curve\ A) > \frac{d^2E}{dk^2}(curve\ B)$$

so that

$$\underline{m^*(curve\ A) < m^*(curve\ B)}$$

3.14

The effective mass for a hole is given by

$$m_p^* = \left(\frac{1}{\hbar^2}\cdot\left|\frac{d^2E}{dk^2}\right|\right)^{-1}$$

We have that

$$\left|\frac{d^2E}{dk^2}\right|(curve\ A) > \left|\frac{d^2E}{dk^2}\right|(curve\ B)$$

so that

$$\underline{m_p^*(curve\ A) < m_p^*(curve\ B)}$$

3.15

Points A, B: $\dfrac{\partial E}{\partial k} < 0 \Rightarrow$ velocity in –x direction;

Points C, D: $\dfrac{\partial E}{\partial x} > 0 \Rightarrow$ velocity in +x direction;

Points A, D; $\dfrac{\partial^2 E}{\partial k^2} < 0 \Rightarrow$ negative effective mass;

Points B, C; $\dfrac{\partial^2 E}{\partial k^2} > 0 \Rightarrow$ positive effective mass;

3.16

$$E - E_C = \frac{k^2\hbar^2}{2m}$$

At $k = 0.1\left(A^\circ\right)^{-1} \Rightarrow \dfrac{1}{k} = 10\ A^\circ = 10^{-9}\ m$

So

$k = 10^{+9}\ m^{-1}$

For A:

$$(0.07)\left(1.6x10^{-19}\right) = \frac{\left(10^9\right)^2\left(1.054x10^{-34}\right)^2}{2m}$$

which yields

$$m = 4.96x10^{-31}\ kg$$

so

$$\underline{curve\ A;\ \frac{m}{m_o} = 0.544}$$

For B:

$$(0.7)\left(1.6x10^{-19}\right) = \frac{\left(10^9\right)\left(1.054x10^{-34}\right)^2}{2m}$$

which yields

$$m = 4.96x10^{-32}\ kg$$

so

$$\underline{Curve\ B:\ \frac{m}{m_o} = 0.0544}$$

3.17

$$E_V - E = \frac{k^2\hbar^2}{2m}$$

$$k = 0.1\left(A^*\right)^{-1} \Rightarrow 10^9\ m^{-1}$$

For Curve A:

$$(0.08)\left(1.6x10^{-19}\right) = \frac{\left(10^9\right)^2\left(1.054x10^{-34}\right)^2}{2m}$$

which yields

$$m = 4.34x10^{-31}\ kg \Rightarrow \frac{m}{m_o} = 0.476$$

For Curve B:

$$(0.4)\left(1.6x10^{-19}\right) = \frac{\left(10^9\right)^2\left(1.054x10^{-34}\right)^2}{2m}$$

which yields

$$m = 8.68x10^{-32}\ kg \Rightarrow \frac{m}{m_o} = 0.0953$$

3.18

(a) $E = h\nu$

Then

$$\nu = \frac{E}{h} = \frac{(1.42)\left(1.6x10^{-19}\right)}{\left(6.625x10^{-34}\right)} \Rightarrow$$

$$\nu = 3.43x10^{14}\ Hz$$

(b)

$$\lambda = \frac{c}{\nu} = \frac{3x10^8}{3.43x10^{14}} = 8.75x10^{-7}\ m$$

or

$$\lambda = 0.875\ \mu m$$

3.19

(c) Curve A: Effective mass is a constant
Curve B: Effective mass is positive around $k = 0$, and is negative around $k = \pm\frac{\pi}{2}$.

3.20

$$E = E_O - E_1\cos\left[\alpha\left(k - k_O\right)\right]$$

$$\frac{dE}{dk} = \left(-E_1\right)(-\alpha)\sin\left[\alpha\left(k - k_O\right)\right]$$

$$= +E_1\alpha\sin\left[\alpha\left(k - k_O\right)\right]$$

So

$$\frac{d^2E}{dk^2} = E_1\alpha^2\cos\left[\alpha\left(k - k_O\right)\right]$$

Then

$$\left.\frac{d^2E}{dk^2}\right|_{k=k_O} = E_1\alpha^2$$

We have

$$\frac{1}{m^*} = \frac{1}{\hbar^2}\cdot\frac{d^2E}{dk^2} = \frac{E_1\alpha^2}{\hbar^2}$$

or

$$m^* = \frac{\hbar^2}{E_1\alpha^2}$$

3.21

For the 3-dimensional infinite potential well, $V(x) = 0$ when $0 < x < a$, $0 < y < a$, and $0 < z < a$. In this region, the wave equation is

$$\frac{\partial^2\psi(x,y,z)}{\partial x^2} + \frac{\partial^2\psi(x,y,z)}{\partial y^2} + \frac{\partial^2\psi(x,y,z)}{\partial z^2} + \frac{2mE}{\hbar^2}\psi(x,y,z) = 0$$

Use separation of variables technique, so let

$$\psi(x,y,z) = X(x)Y(y)Z(z)$$

Substituting into the wave equation, we have

$$YZ\frac{\partial^2X}{\partial x^2} + XZ\frac{\partial^2Y}{\partial y^2} + XY\frac{\partial^2Z}{\partial z^2} + \frac{2mE}{\hbar^2}\cdot XYZ = 0$$

Dividing by XYZ, we obtain

$$\frac{1}{X}\cdot\frac{\partial^2X}{\partial x^2} + \frac{1}{Y}\cdot\frac{\partial^2Y}{\partial y^2} + \frac{1}{Z}\cdot\frac{\partial^2Z}{\partial z^2} + \frac{2mE}{\hbar^2} = 0$$

Let

$$\frac{1}{X}\cdot\frac{\partial^2X}{\partial x^2} = -k_x^2 \Rightarrow \frac{\partial^2X}{\partial x^2} + k_x^2X = 0$$

The solution is of the form

$$X(x) = A\sin k_xx + B\cos k_xx$$

Since $\psi(x,y,z) = 0$ at $x = 0$, then $X(0) = 0$ so that $B \equiv 0$.

Also, $\psi(x,y,z) = 0$ at $x = a$, then $X(a) = 0$ so we must have $k_xa = n_x\pi$, where $n_x = 1, 2, 3, \ldots$

Similarly, we have

$$\frac{1}{Y}\cdot\frac{\partial^2Y}{\partial y^2} = -k_y^2 \quad\text{and}\quad \frac{1}{Z}\cdot\frac{\partial^2Z}{\partial z^2} = -k_z^2$$

From the boundary conditions, we find

$k_y a = n_y \pi$ and $k_z a = n_z \pi$

where $n_y = 1, 2, 3, \ldots$ and $n_z = 1, 2, 3, \ldots$

From the wave equation, we have

$$-k_x^2 - k_y^2 - k_z^2 + \frac{2mE}{\hbar^2} = 0$$

The energy can then be written as

$$E = \frac{\hbar^2}{2m}\left(n_x^2 + n_y^2 + n_z^2\right)\left(\frac{\pi}{a}\right)^2$$

3.22

The total number of quantum states in the 3-dimensional potential well is given (in k-space) by

$$g_T(k)dk = \frac{\pi k^2 dk}{\pi^3} \cdot a^3$$

where

$$k^2 = \frac{2mE}{\hbar^2}$$

We can then write

$$k = \frac{1}{\hbar} \cdot \sqrt{2mE}$$

Taking the differential, we obtain

$$dk = \frac{1}{\hbar} \cdot \sqrt{2m} \cdot \frac{1}{2} \cdot \frac{1}{\sqrt{E}} \cdot dE = \frac{1}{\hbar} \cdot \sqrt{\frac{m}{2E}} \cdot dE$$

Substituting these expressions into the density of states function, we obtain

$$g_T(E)dE = \frac{\pi a^3}{\pi^3}\left(\frac{2mE}{\hbar^2}\right) \cdot \frac{1}{\hbar} \cdot \sqrt{\frac{m}{2E}} \cdot dE$$

Noting that

$$\hbar = \frac{h}{2m}$$

this density of states function can be simplified and written as

$$g_T(E)dE = \frac{4\pi a^3}{h^3}(2m)^{3/2} \cdot \sqrt{E} \cdot dE$$

Dividing by a^3 will yield the density of states, so that

$$g(E) = \frac{4\pi(2m)^{3/2}}{h^3} \cdot \sqrt{E}$$

3.23

$$g_C(E) = \frac{4\pi\left(2m_n^*\right)^{3/2}}{h^3}\sqrt{E - E_C}$$

Now

$$g_T = \frac{4\pi\left(2m_n^*\right)^{3/2}}{h^3} \int_{E_C}^{E_C + kT} \sqrt{E - E_C} \cdot dE$$

$$= \frac{4\pi\left(2m_n^*\right)^{3/2}}{h^3}\left(\frac{2}{3}\right)\left(E - E_C\right)^{3/2} \Big|_{E_C}^{E_C + kT}$$

$$= \frac{4\pi\left(2m_n^*\right)^{3/2}}{h^3}\left(\frac{2}{3}\right)(kT)^{3/2}$$

Then

$$g_T = \frac{4\pi\left[2(0.067)\left(9.11x10^{-31}\right)\right]^{3/2}}{\left(6.625x10^{-34}\right)^3}\left(\frac{2}{3}\right) \times \left[(0.0259)\left(1.6x10^{-19}\right)\right]^{3/2}$$

or

$$g_T = 3.28x10^{23}\ m^{-3} = 3.28x10^{17}\ cm^{-3}$$

3.24

$$g_V(E) = \frac{4\pi\left(2m_p^*\right)^{3/2}}{h^3}\sqrt{E_V - E}$$

Now

$$g_T = \frac{4\pi\left(2m_p^*\right)^{3/2}}{h^3} \int_{E_V - kT}^{E_V} \sqrt{E_V - E} \cdot dE$$

$$= \frac{4\pi\left(2m_p^*\right)^{3/2}}{h^3}\left(\frac{-2}{3}\right)\left(E_V - E\right)^{3/2} \Big|_{E_V - kT}^{E_V}$$

$$= \frac{4\pi\left(2m_p^*\right)^{3/2}}{h^3}\left(\frac{2}{3}\right)(kT)^{3/2}$$

$$g_T = \frac{4\pi\left[2(0.48)\left(9.11x10^{-31}\right)\right]^{3/2}}{\left(6.625x10^{-34}\right)^3}\left(\frac{2}{3}\right) \times \left[(0.0259)\left(1.6x10^{-19}\right)\right]^{3/2}$$

or

$$g_T = 6.29x10^{24}\ m^{-3} = 6.29x10^{18}\ cm^{-3}$$

3.25

(a) $$g_C(E) = \frac{4\pi\left(2m_n^*\right)^{3/2}}{h^3}\sqrt{E - E_C}$$

$$= \frac{4\pi\left[2(1.08)\left(9.11x10^{-31}\right)\right]^{3/2}}{\left(6.625x10^{-34}\right)^3}\left(1.6x10^{-19}\right)^{1/2}\sqrt{E - E_C}$$

$$= 4.77x10^{46}\sqrt{E - E_C} \quad m^{-3}J^{-1}$$

or

$$g_C(E) = 7.63x10^{21}\sqrt{E - E_C}\ cm^{-3}eV^{-1}$$

Then

E	g_C
$E_C + 0.05\ eV$	$1.71x10^{21}\ cm^{-3}eV^{-1}$
$E_C + 0.10\ eV$	$2.41x10^{21}$
$E_C + 0.15\ eV$	$2.96x10^{21}$
$E_C + 0.20\ eV$	$3.41x10^{21}$

(b) $g_V(E) = \dfrac{4\pi\left(2m_p^*\right)^{3/2}}{h^3}\sqrt{E_V - E}$

$$= \frac{4\pi\left[2(0.56)\left(9.11x10^{-31}\right)\right]^{3/2}}{\left(6.625x10^{-34}\right)^3}\left(1.6x10^{-19}\right)^{1/2}\sqrt{E_V - E}$$

$$= 1.78x10^{46}\sqrt{E_V - E}\quad m^{-3}J^{-1}$$

$$g_V(E) = 2.85x10^{21}\sqrt{E_V - E}\ cm^{-3}eV^{-1}$$

E	$g_V(E)$
$E_V - 0.05\ eV$	$0.637x10^{21}\ cm^{-3}\ eV^{-1}$
$E_V - 0.10\ eV$	$0.901x10^{21}$
$E_V - 0.15\ eV$	$1.10x10^{21}$
$E_V - 0.20\ eV$	$1.27x10^{21}$

3.26

$$\frac{g_C}{g_V} = \frac{\left(m_n^*\right)^{3/2}}{\left(m_p^*\right)^{3/2}} \Rightarrow \underline{\frac{g_C}{g_V} = \left(\frac{m_n^*}{m_p^*}\right)^{3/2}}$$

3.27

Computer Plot

3.28

$$\frac{g_i!}{N_i!\left(g_i - N_i\right)!} = \frac{10!}{8!(10-8)!}$$

$$= \frac{(10)(9)(8!)}{(8!)(2!)} = \frac{(10)(9)}{(2)(1)} \Rightarrow \underline{= 45}$$

3.29

(a) $f(E) = \dfrac{1}{1 + \exp\left[\dfrac{\left(E_C + kT\right) - E_C}{kT}\right]}$

$$= \frac{1}{1 + \exp(1)} \Rightarrow \underline{f(E) = 0.269}$$

(b)

$$1 - f(E) = 1 - \frac{1}{1 + \exp\left[\dfrac{\left(E_V - kT\right) - E_V}{kT}\right]}$$

$$= 1 - \frac{1}{1 + \exp(-1)} \Rightarrow \underline{1 - f(E) = 0.269}$$

3.30

$$f(E) = \frac{1}{1 + \exp\left(\dfrac{E - E_F}{kT}\right)}$$

(a) $E - E_F = kT$, $f(E) = \dfrac{1}{1 + \exp(1)} \Rightarrow$

$\underline{f(E) = 0.269}$

(b) $E - E_F = 5kT$, $f(E) = \dfrac{1}{1 + \exp(5)} \Rightarrow$

$\underline{f(E) = 6.69x10^{-3}}$

(c) $E - E_F = 10kT$, $f(E) = \dfrac{1}{1 + \exp(10)} \Rightarrow$

$\underline{f(E) = 4.54x10^{-5}}$

3.31

$$1 - f(E) = 1 - \frac{1}{1 + \exp\left(\dfrac{E - E_F}{kT}\right)}$$

or

$$1 - f(E) = \frac{1}{1 + \exp\left(\dfrac{E_F - E}{kT}\right)}$$

(a) $E_F - E = kT$, $\underline{1 - f(E) = 0.269}$

(b) $E_F - E = 5kT$, $\underline{1 - f(E) = 6.69x10^{-3}}$

(c) $E_F - E = 10kT$, $\underline{1 - f(E) = 4.54x10^{-5}}$

3.32

(a) $T = 300K \Rightarrow kT = 0.0259\ eV$

$$f(E) = \frac{1}{1 + \exp\left(\dfrac{E - E_F}{kT}\right)} \approx \exp\left[\frac{-\left(E - E_F\right)}{kT}\right]$$

E	$f(E)$
E_C	$6.43x10^{-5}$
$E_C+(1/2)kT$	$3.90x10^{-5}$
E_C+kT	$2.36x10^{-5}$
$E_C+(3/2)kT$	$1.43x10^{-5}$
E_C+2kT	$0.87x10^{-5}$

(b) $T=400K \Rightarrow kT=0.03453$

E	$f(E)$
E_C	$7.17x10^{-4}$
$E_C+(1/2)kT$	$4.35x10^{-4}$
E_C+kT	$2.64x10^{-4}$
$E_C+(3/2)kT$	$1.60x10^{-4}$
E_C+2kT	$0.971x10^{-4}$

3.33

$$E_n=\frac{\hbar^2 n^2\pi^2}{2ma^2}=\frac{(1.054x10^{-34})^2 n^2\pi^2}{2(9.11x10^{-31})(10x10^{-10})^2}$$

or

$$E_n=6.018x10^{-20}n^2\ J=0.376n^2\ eV$$

For $n=4 \Rightarrow E_4=6.02\ eV$,

For $n=5 \Rightarrow E_5=9.40\ eV$.

As a 1st approximation for $T>0$, assume the probability of $n=5$ state being occupied is the same as the probability of $n=4$ state being empty. Then

$$1-\frac{1}{1+\exp\left(\frac{E_4-E_F}{kT}\right)}=\frac{1}{1+\exp\left(\frac{E_5-E_F}{kT}\right)}$$

$$\Rightarrow \frac{1}{1+\exp\left(\frac{E_F-E_4}{kT}\right)}=\frac{1}{1+\exp\left(\frac{E_5-E_F}{kT}\right)}$$

or

$$E_F-E_4=E_5-E_F \Rightarrow E_F=\frac{E_4+E_5}{2}$$

Then

$$E_F=\frac{6.02+9.40}{2} \Rightarrow \underline{E_F=7.71\ eV}$$

3.34

(a) For 3-Dimensional infinite potential well,

$$E=\frac{\hbar^2\pi^2}{2ma^2}\left(n_x^2+n_y^2+n_z^2\right)$$

$$=\frac{(1.054x10^{-34})^2\pi^2}{2(9.11x10^{-31})(10^{-9})^2}\left(n_x^2+n_y^2+n_z^2\right)$$

$$=0.376\left(n_x^2+n_y^2+n_z^2\right)\ eV$$

For 5 electrons, energy state corresponding to $n_x n_y n_z=221=122$ contains both an electron and an empty state, so

$$E_F=(0.376)\left(2^2+2^2+1^2\right)\Rightarrow$$

$$\underline{E_F=3.384\ eV}$$

(b) For 13 electrons, energy state corresponding to $n_x n_y n_z=323=233$ contains both an electron and an empty state, so

$$E_F=(0.376)\left(2^2+3^2+3^2\right)\Rightarrow$$

$$\underline{E_F=8.272\ eV}$$

3.35

The probability of a state at $E_1=E_F+\Delta E$ being occupied is

$$f_1(E_1)=\frac{1}{1+\exp\left(\frac{E_1-E_F}{kT}\right)}=\frac{1}{1+\exp\left(\frac{\Delta E}{kT}\right)}$$

The probability of a state at $E_2=E_F-\Delta E$ being empty is

$$1-f_2(E_2)=1-\frac{1}{1+\exp\left(\frac{E_2-E_F}{kT}\right)}$$

$$=1-\frac{1}{1+\exp\left(\frac{-\Delta E}{kT}\right)}=\frac{\exp\left(\frac{-\Delta E}{kT}\right)}{1+\exp\left(\frac{-\Delta E}{kT}\right)}$$

or

$$1-f_2(E_2)=\frac{1}{1+\exp\left(\frac{+\Delta E}{kT}\right)}$$

Hence, we have that

$f_1(E_1)=1-f_2(E_2)$ Q.E.D.

3.36

(a) At energy E_1, we want

$$\frac{\dfrac{1}{\exp\left(\dfrac{E_1-E_F}{kT}\right)}-\dfrac{1}{1+\exp\left(\dfrac{E_1-E_F}{kT}\right)}}{\dfrac{1}{1+\exp\left(\dfrac{E_1-E_F}{kT}\right)}}=0.01$$

This expression can be written as

$$\frac{1+\exp\left(\dfrac{E_1-E_F}{kT}\right)}{\exp\left(\dfrac{E_1-E_F}{kT}\right)}-1=0.01$$

$$\Rightarrow 1=(0.01)\exp\left(\frac{E_1-E_F}{kT}\right)$$

or

$$E_1=E_F+kT\ln(100)$$

Then

$$\underline{E_1=E_F+4.6kT}$$

(b)

At $E_1=E_F+4.6kT$,

$$f(E_1)=\frac{1}{1+\exp\left(\dfrac{E_1-E_F}{kT}\right)}=\frac{1}{1+\exp\left(\dfrac{4.6kT}{kT}\right)}$$

which yields

$$\underline{f(E_1)=0.00990\approx 0.01}$$

3.37

(a) $E_F=6.25\ eV$, $T=300K$, At $E=6.50\ eV$

$$f(E)=\frac{1}{1+\exp\left(\dfrac{6.50-6.25}{0.0259}\right)}=6.43x10^{-5}$$

or

$$\underline{6.43x10^{-3}\%}$$

(b)

$$T=950K\Rightarrow kT=(0.0259)\left(\frac{950}{300}\right)$$

or

$$kT=0.0820\ eV$$

Then

$$f(E)=\frac{1}{1+\exp\left(\dfrac{6.50-6.25}{0.0820}\right)}=0.0453$$

or $\underline{4.53\%}$

(c) $$1-0.01=\frac{1}{1+\exp\left(\dfrac{-0.30}{kT}\right)}=0.99$$

Then

$$1+\exp\left(\frac{-0.30}{kT}\right)=\frac{1}{0.99}=1.0101$$

which can be written as

$$\exp\left(\frac{+0.30}{kT}\right)=\frac{1}{0.0101}=99$$

Then

$$\frac{0.30}{kT}=\ln(99)\Rightarrow kT=\frac{0.30}{\ln(99)}=0.06529$$

So

$$\underline{T=756K}$$

3.38

(a)

$$f(E)=\frac{1}{1+\exp\left(\dfrac{7.15-7.0}{0.0259}\right)}=0.00304$$

or

$$\underline{0.304\%}$$

(b)

At $T=1000K\Rightarrow kT=0.08633\ eV$

Then

$$f(E)=\frac{1}{1+\exp\left(\dfrac{7.15-7.0}{0.08633}\right)}=0.1496$$

or $\underline{14.96\%}$

(c)

$$f(E)=\frac{1}{1+\exp\left(\dfrac{6.85-7.0}{0.0259}\right)}=0.997$$

or

$$\underline{99.7\%}$$

(d)

At $E=E_F$, $\underline{f(E)=\frac{1}{2}}$ for all temperatures.

3.39

For $E=E_1$,

$$f(E)=\frac{1}{1+\exp\left(\dfrac{E_1-E_F}{kT}\right)}\approx\exp\left[\frac{-(E_1-E_F)}{kT}\right]$$

Then

$$f(E_1) = \exp\left(\frac{-0.30}{0.0259}\right) \Rightarrow \underline{f(E_1) = 9.3x10^{-6}}$$

For $E = E_2$, $E_F - E_2 = 1.12 - 0.3 = 0.82\ eV$

$$1 - f(E) = 1 - \frac{1}{1 + \exp\left(\frac{-0.82}{0.0259}\right)}$$

or

$$1 - f(E) \approx 1 - \left[1 - \exp\left(\frac{-0.82}{0.0259}\right)\right]$$

$$= \exp\left(\frac{-0.82}{0.0259}\right) \Rightarrow \underline{1 - f(E) = 1.78x10^{-14}}$$

(b)
For $E_F - E_2 = 0.4 \Rightarrow E_1 - E_F = 0.72\ eV$
At $E = E_1$,

$$f(E) = \exp\left[\frac{-(E_1 - E_F)}{kT}\right] = \exp\left(\frac{-0.72}{0.0259}\right)$$

so

$$\underline{f(E) = 8.45x10^{-13}}$$

At $E = E_2$,

$$1 - f(E) = \exp\left[\frac{-(E_F - E_2)}{kT}\right] = \exp\left(\frac{-0.4}{0.0259}\right)$$

so

$$\underline{1 - f(E) = 1.96x10^{-7}}$$

3.40

(a) At $E = E_1$,

$$f(E) = \exp\left[\frac{-(E - E_F)}{kT}\right] = \exp\left(\frac{-0.30}{0.0259}\right)$$

or

$$\underline{f(E) = 9.3x10^{-6}}$$

At $E = E_2$, then
$E_F - E_2 = 1.42 - 0.3 = 1.12\ eV$,
So

$$1 - f(E) = \exp\left[\frac{-(E_F - E_2)}{kT}\right] = \exp\left(\frac{-1.12}{0.0259}\right)$$

or

$$\underline{1 - f(E) = 1.66x10^{-19}}$$

(b)
For $E_F - E_2 = 0.4 \Rightarrow E_1 - E_F = 1.02\ eV$,
At $E = E_1$,

$$f(E) = \exp\left[\frac{-(E_1 - E_F)}{kT}\right] = \exp\left(\frac{-1.02}{0.0259}\right)$$

or

$$\underline{f(E) = 7.88x10^{-18}}$$

At $E = E_2$,

$$1 - f(E) = \exp\left[\frac{-(E_F - E_2)}{kT}\right] = \exp\left(\frac{-0.4}{0.0259}\right)$$

or

$$\underline{1 - f(E) = 1.96x10^{-7}}$$

3.41

$$f(E) = \left[1 + \exp\left(\frac{E - E_F}{kT}\right)\right]^{-1}$$

so

$$\frac{df(E)}{dE} = (-1)\left[1 + \exp\left(\frac{E - E_F}{kT}\right)\right]^{-2} \times\left(\frac{1}{kT}\right)\exp\left(\frac{E - E_F}{kT}\right)$$

or

$$\frac{df(E)}{dE} = \frac{\frac{-1}{kT}\exp\left(\frac{E - E_F}{kT}\right)}{\left[1 + \exp\left(\frac{E - E_F}{kT}\right)\right]^2}$$

(a) $T = 0$, For

$$E < E_F \Rightarrow \exp(-\infty) = 0 \Rightarrow \frac{df}{dE} = 0$$

$$E > E_F \Rightarrow \exp(+\infty) = +\infty \Rightarrow \frac{df}{dE} = 0$$

At $E = E_F \Rightarrow \dfrac{df}{dE} \to -\infty$

3.42

(a) At $E = E_{midgap}$,

$$f(E) = \frac{1}{1 + \exp\left(\frac{E - E_F}{kT}\right)} = \frac{1}{1 + \exp\left(\frac{E_g}{2kT}\right)}$$

Si: $E_g = 1.12\ eV$,

$$f(E) = \frac{1}{1 + \exp\left[\frac{1.12}{2(0.0259)}\right]}$$

or

$$\underline{f(E) = 4.07x10^{-10}}$$

Ge: $E_g = 0.66\ eV$,

$$f(E) = \frac{1}{1 + \exp\left[\frac{0.66}{2(0.0259)}\right]}$$

or

$$\underline{f(E) = 2.93x10^{-6}}$$

GaAs: $E_g = 1.42\ eV$,

$$f(E) = \frac{1}{1 + \exp\left[\frac{1.42}{2(0.0259)}\right]}$$

or

$$\underline{f(E) = 1.24x10^{-12}}$$

(b)
Using results of Problem 3.35, the answers to part (b) are exactly the same as those given in part (a).

3.43

$$f(E) = 10^{-6} = \frac{1}{1 + \exp\left(\frac{0.55}{kT}\right)}$$

Then

$$1 + \exp\left(\frac{0.55}{kT}\right) = \frac{1}{10^{-6}} = 10^{+6} \Rightarrow$$

$$\exp\left(\frac{0.55}{kT}\right) \approx 10^{+6} \Rightarrow \left(\frac{0.55}{kT}\right) = \ln\left(10^{6}\right)$$

or

$$kT = \frac{0.55}{\ln\left(10^{6}\right)} \Rightarrow \underline{T = 461K}$$

3.44
At $E = E_2$, $f(E_2) = 0.05$
So

$$0.05 = \frac{1}{1 + \exp\left(\frac{E_2 - E_F}{kT}\right)}$$

Then

$$\frac{E_2 - E_F}{kT} = \ln(19)$$

By symmetry, at $E = E_1$, $1 - f(E_1) = 0.05$,
So

$$\frac{E_F - E_1}{kT} = \ln(19)$$

Then

$$\frac{E_2 - E_1}{kT} = 2\ln(19)$$

(a)
At $T = 300K$, $kT = 0.0259\ eV$

$$E_2 - E_1 = \Delta E = (0.0259)(2)\ln(19) \Rightarrow$$
$$\underline{\Delta E = 0.1525\ eV}$$

(b)
At $T = 500K$, $kT = 0.04317\ eV$

$$E_2 - E_1 = \Delta E = (0.04317)(2)\ln(19) \Rightarrow$$
$$\underline{\Delta E = 0.254\ eV}$$

Chapter 4

Problem Solutions

4.1

$$n_i^2 = N_C N_V \exp\left(\frac{-E_g}{kT}\right)$$

(a) Silicon

$T(°K)$	$kT \ (eV)$	$n_i\left(cm^{-3}\right)$
200	0.01727	$7.68x10^4$
400	0.03453	$2.38x10^{12}$
600	0.0518	$9.74x10^{14}$

(b) Germanium (c) GaAs

$T(°K)$	$n_i\left(cm^{-3}\right)$	$n_i\left(cm^{-3}\right)$
200	$2.16x10^{10}$	1.38
400	$8.60x10^{14}$	$3.28x10^9$
600	$3.82x10^{16}$	$5.72x10^{12}$

4.2

$$n_i^2 = N_C N_V \exp\left(\frac{-E_g}{kT}\right)$$

$$\left(10^{12}\right)^2 = \left(2.8x10^{19}\right)\left(1.04x10^{19}\right)\left(\frac{T}{300}\right)^3 \exp\left(\frac{-1.12}{kT}\right)$$

Then

$$\exp\left(\frac{1.12}{kT}\right) = \left(2.912x10^{14}\right)\left(\frac{T}{300}\right)^3$$

By trial and error

$$\underline{T = 381K}$$

4.3

Computer Plot

4.4

$$n_i^2 = N_{CO} N_{VO} \cdot (T)^3 \cdot \exp\left(\frac{-E_g}{kT}\right)$$

So

$$\frac{n_i^2(T_2)}{n_i^2(T_1)} = \left(\frac{T_2}{T_1}\right)^3 \exp\left[-E_g\left(\frac{1}{kT_2} - \frac{1}{kT_1}\right)\right]$$

At $T_2 = 300K \Rightarrow kT = 0.0259\ eV$

At $T_1 = 200K \Rightarrow kT = 0.01727\ eV$

Then

$$\left(\frac{5.83x10^7}{1.82x10^2}\right)^2 = \left(\frac{300}{200}\right)^3 \exp\left[-E_g\left(\frac{1}{0.0259} - \frac{1}{0.01727}\right)\right]$$

or

$$1.026x10^{11} = 3.375\exp\left[(19.29)E_g\right]$$

which yields

$$\underline{E_g = 1.25\ eV}$$

For $T = 300K$,

$$\left(5.83x10^7\right)^2 = \left(N_{CO}N_{VO}\right)(300)^3 \exp\left(\frac{-1.25}{0.0259}\right)$$

or

$$\underline{N_{CO}N_{VO} = 1.15x10^{29}}$$

4.5

(a) $g_C f_F \propto \sqrt{E - E_C}\,\exp\left[\frac{-(E - E_F)}{kT}\right]$

$$\propto \sqrt{E - E_C}\,\exp\left[\frac{-(E - E_C)}{kT}\right]\exp\left[\frac{-(E_C - E_F)}{kT}\right]$$

Let $E - E_C \equiv x$

Then

$$g_C f_F \propto \sqrt{x}\,\exp\left(\frac{-x}{kT}\right)$$

Now, to find the maximum value

$$\frac{d(g_C f_F)}{dx} \propto \frac{1}{2}x^{-1/2}\exp\left(\frac{-x}{kT}\right) - \frac{1}{kT}\cdot x^{1/2}\exp\left(\frac{-x}{kT}\right) = 0$$

This yields

$$\frac{1}{2x^{1/2}} = \frac{x^{1/2}}{kT} \Rightarrow x = \frac{kT}{2}$$

Then the maximum value occurs at

$$\underline{E = E_C + \frac{kT}{2}}$$

(b)

$$g_V(1 - f_F) \propto \sqrt{E_V - E}\,\exp\left[\frac{-(E_F - E)}{kT}\right]$$

$$\propto \sqrt{E_V - E}\,\exp\left[\frac{-(E_F - E_V)}{kT}\right]\exp\left[\frac{-(E_V - E)}{kT}\right]$$

Let $E_V - E \equiv x$

Then

$$g_V(1-f_F) \propto \sqrt{x}\exp\left(\frac{-x}{kT}\right)$$

To find the maximum value

$$\frac{d\left[g_V(1-f_F)\right]}{dx} \propto \frac{d}{dx}\left[\sqrt{x}\exp\left(\frac{-x}{kT}\right)\right] = 0$$

Same as part (a). Maximum occurs at

$$x = \frac{kT}{2} = E_V - E$$

or

$$\underline{E = E_V - \frac{kT}{2}}$$

4.6

$$\frac{n(E_1)}{n(E_2)} = \frac{\sqrt{E_1 - E_C}\exp\left[\frac{-(E_1-E_C)}{kT}\right]}{\sqrt{E_2 - E_C}\exp\left[\frac{-(E_2-E_C)}{kT}\right]}$$

where

$$E_1 = E_C + 4kT \quad \text{and} \quad E_2 = E_C + \frac{kT}{2}$$

Then

$$\frac{n(E_1)}{n(E_2)} = \frac{\sqrt{4kT}}{\sqrt{\frac{kT}{2}}}\exp\left[\frac{-(E_1-E_2)}{kT}\right]$$

$$= 2\sqrt{2}\exp\left[-\left(4-\frac{1}{2}\right)\right] = 2\sqrt{2}\exp(-3.5)$$

or

$$\underline{\frac{n(E_1)}{n(E_2)} = 0.0854}$$

4.7
Computer Plot

4.8

$$\frac{n_i^2(A)}{n_i^2(B)} = \frac{\exp\left(\frac{-E_{gA}}{kT}\right)}{\exp\left(\frac{-E_{gB}}{kT}\right)} = \exp\left[\frac{-(E_{gA}-E_{gB})}{kT}\right]$$

or

$$\frac{n_i(A)}{n_i(B)} = \exp\left[\frac{-(E_{gA}-E_{gB})}{2kT}\right]$$

$$= \exp\left[\frac{-(1-1.2)}{2(0.0259)}\right] = \exp\left[\frac{+0.20}{2(0.0259)}\right]$$

or

$$\underline{\frac{n_i(A)}{n_i(B)} = 47.5}$$

4.9
Computer Plot

4.10

$$E_{Fi} - E_{midgap} = \frac{3}{4}kT\ln\left(\frac{m_p^*}{m_n^*}\right)$$

Silicon: $m_p^* = 0.56m_O$, $m_n^* = 1.08m_O$

$$\underline{E_{Fi} - E_{midgap} = -0.0128\ eV}$$

Germanium: $m_p^* = 0.37m_O$, $m_n^* = 0.55m_O$

$$\underline{E_{Fi} - E_{midgap} = -0.0077\ eV}$$

Gallium Arsenide: $m_p^* = 0.48m_O$, $m_n^* = 0.067m_O$

$$\underline{E_{Fi} - E_{midgap} = +0.038\ eV}$$

4.11

(a) $E_{Fi} - E_{midgap} = \frac{3}{4}kT\ln\left(\frac{m_p^*}{m_n^*}\right)$

$$= \frac{3}{4}(0.0259)\ln\left(\frac{1.4}{0.62}\right) \Rightarrow$$

$$\underline{E_{Fi} - E_{midgap} = +0.0158\ eV}$$

(b)

$$E_{Fi} - E_{midgap} = \frac{3}{4}(0.0259)\ln\left(\frac{0.25}{1.10}\right) \Rightarrow$$

$$\underline{E_{Fi} - E_{midgap} = -0.0288\ eV}$$

4.12

$$E_{Fi} - E_{midgap} = \frac{1}{2}(kT)\ln\left(\frac{N_V}{N_C}\right)$$

$$= \frac{1}{2}(kT)\ln\left(\frac{1.04x10^{19}}{2.8x10^{19}}\right) = -0.495(kT)$$

$T(°K)$	$kT\ (eV)$	$E_{Fi} - E_{midgap}\ (eV)$
200	0.01727	−0.0085
400	0.03453	−0.017
600	0.0518	−0.0256

4.13

Computer Plot

4.14

Let $g_C(E) = K = \text{constant}$

Then,

$$n_O = \int_{E_C}^{\infty} g_C(E) f_F(E) dE$$

$$= K \int_{E_C}^{\infty} \frac{1}{1+\exp\left(\frac{E-E_F}{kT}\right)} dE$$

$$\approx K \int_{E_C}^{\infty} \exp\left[\frac{-(E-E_F)}{kT}\right] dE$$

Let

$$\eta = \frac{E-E_F}{kT} \text{ so that } dE = kT \cdot d\eta$$

We can write

$$E - E_F = (E_C - E_F) - (E_C - E)$$

so that

$$\exp\left[\frac{-(E-E_F)}{kT}\right] = \exp\left[\frac{-(E_C-E_F)}{kT}\right] \cdot \exp(-\eta)$$

The integral can then be written as

$$n_O = K \cdot kT \cdot \exp\left[\frac{-(E_C-E_F)}{kT}\right] \int_0^{\infty} \exp(-\eta) d\eta$$

which becomes

$$\underline{n_O = K \cdot kT \cdot \exp\left[\frac{-(E_C-E_F)}{kT}\right]}$$

4.15

Let $g_C(E) = C_1(E - E_C)$ for $E \geq E_C$

$$n_O = \int_{E_C}^{\infty} g_C(E) f_F(E) dE$$

$$= C_1 \int_{E_C}^{\infty} \frac{(E-E_C)}{1+\exp\left(\frac{E-E_F}{kT}\right)} dE$$

or

$$n_O \approx C_1 \int_{E_C}^{\infty} (E-E_C) \exp\left[\frac{-(E-E_F)}{kT}\right] dE$$

Let

$$\eta = \frac{E-E_C}{kT} \text{ so that } dE = kT \cdot d\eta$$

We can write

$$(E - E_F) = (E - E_C) + (E_C - E_F)$$

Then

$$n_O = C_1 \exp\left[\frac{-(E_C-E_F)}{kT}\right] \times \int_{E_C}^{\infty} (E-E_C) \exp\left[\frac{-(E-E_C)}{kT}\right] dE$$

or

$$= C_1 \exp\left[\frac{-(E_C-E_F)}{kT}\right] \times \int_0^{\infty} (kT)\eta[\exp(-\eta)](kT) d\eta$$

We find that

$$\int_0^{\infty} \eta \exp(-\eta) d\eta = \frac{e^{-\eta}}{1}(-\eta - 1)\Big|_0^{\infty} = +1$$

So

$$\underline{n_O = C_1 (kT)^2 \exp\left[\frac{-(E_C-E_F)}{kT}\right]}$$

4.16

We have $\frac{r_1}{a_O} = \in_r \left(\frac{m_O}{m^*}\right)$

For Germanium, $\in_r = 16$, $m^* = 0.55 m_O$

Then

$$r_1 = (16)\left(\frac{1}{0.55}\right) a_O = 29(0.53)$$

so

$$\underline{r_1 = 15.4\ \mathring{A}}$$

The ionization energy can be written as

$$E = \left(\frac{m^*}{m_O}\right)\left(\frac{\in_O}{\in_S}\right)^2 (13.6) \quad eV$$

$$= \frac{0.55}{(16)^2}(13.6) \Rightarrow \underline{E = 0.029\ eV}$$

4.17

We have $\frac{r_1}{a_o} = \in_r \left(\frac{m_o}{m^*}\right)$

For GaAs, $\in_r = 13.1$, $m^* = 0.067 m_o$

Then

$$r_1 = (13.1)\left(\frac{1}{0.067}\right)(0.53)$$

or

$$\underline{r_1 = 104\ \overset{\circ}{A}}$$

The ionization energy is

$$E = \left(\frac{m^*}{m_o}\right)\left(\frac{\in_o}{\in_S}\right)^2 (13.6) = \frac{0.067}{(13.1)^2}(13.6)$$

or

$$\underline{E = 0.0053\ eV}$$

4.18

(a) $p_O = \frac{n_i^2}{n_O} = \frac{(1.5x10^{10})^2}{5x10^4} \Rightarrow$

$\underline{p_O = 4.5x10^{15}\ cm^{-3}}$, $p_O > n_O \Rightarrow$ p-type

(b)

$$E_{Fi} - E_F = kT \ln\left(\frac{p_O}{n_i}\right)$$

$$= (0.0259)\ln\left(\frac{4.5x10^{15}}{1.5x10^{10}}\right)$$

or

$$\underline{E_{Fi} - E_F = 0.3266\ eV}$$

4.19

$$p_O = N_V \exp\left[\frac{-(E_F - E_V)}{kT}\right]$$

$$= 1.04x10^{19} \exp\left(\frac{-0.22}{0.0259}\right)$$

so

$$\underline{p_O = 2.13x10^{15}\ cm^{-3}}$$

Assuming

$$E_C - E_F = 1.12 - 0.22 = 0.90\ eV$$

Then

$$n_O = N_C \exp\left[\frac{-(E_C - E_F)}{kT}\right]$$

$$= 2.8x10^{18} \exp\left(\frac{-0.90}{0.0259}\right)$$

or

$$\underline{n_O = 2.27x10^4\ cm^{-3}}$$

4.20

(a) $T = 400K \Rightarrow kT = 0.03453\ eV$

$$N_C = 4.7x10^{17}\left(\frac{400}{300}\right)^{3/2} = 7.24x10^{17}\ cm^{-3}$$

Then

$$n_O = N_C \exp\left[\frac{-(E_C - E_F)}{kT}\right]$$

$$= 7.24x10^{17} \exp\left(\frac{-0.25}{0.03453}\right)$$

or

$$\underline{n_O = 5.19x10^{14}\ cm^{-3}}$$

Also

$$N_V = 7x10^{18}\left(\frac{400}{300}\right)^{3/2} = 1.08x10^{19}\ cm^{-3}$$

and

$$E_F - E_V = 1.42 - 0.25 = 1.17\ eV$$

Then

$$p_O = 1.08x10^{19} \exp\left(\frac{-1.17}{0.03453}\right)$$

or

$$\underline{p_O = 2.08x10^4\ cm^{-3}}$$

(b)

$$E_C - E_F = kT \ln\left(\frac{N_C}{n_O}\right)$$

$$= (0.0259)\ln\left(\frac{4.7x10^{17}}{5.19x10^{14}}\right)$$

or $\underline{E_C - E_F = 0.176\ eV}$

Then

$$E_F - E_V = 1.42 - 0.176 = 1.244\ eV$$

and

$$p_O = (7x10^{18})\exp\left(\frac{-1.244}{0.0259}\right)$$

or $\underline{p_O = 9.67x10^{-3}\ cm^{-3}}$

4.21

$$p_O = N_V \exp\left[\frac{-(E_F - E_V)}{kT}\right]$$

or

$$E_F - E_V = kT \ln\left(\frac{N_V}{p_O}\right)$$

$$= (0.0259)\ln\left(\frac{1.04x10^{19}}{10^{15}}\right) = 0.24\ eV$$

Then

$$E_C - E_F = 1.12 - 0.24 = 0.88\ eV$$

So

$$n_O = N_C \exp\left[\frac{-(E_C - E_F)}{kT}\right]$$

$$= 2.8x10^{19} \exp\left(\frac{-0.88}{0.0259}\right)$$

or

$$\underline{n_O = 4.9x10^4\ cm^{-3}}$$

4.22

(a) $p_O = n_i \exp\left(\dfrac{E_{Fi} - E_F}{kT}\right)$

$$1.5x10^{10} \exp\left(\frac{0.35}{0.0259}\right)$$

or

$$\underline{p_O = 1.11x10^{16}\ cm^{-3}}$$

(b)

From Problem 4.1, $n_i(400K) = 2.38x10^{12}\ cm^{-3}$

$$kT = (0.0259)\left(\frac{400}{300}\right) = 0.03453\ eV$$

Then

$$E_{Fi} - E_F = kT \ln\left(\frac{p_O}{n_i}\right)$$

$$= (0.03453)\ln\left(\frac{1.11x10^{16}}{2.38x10^{12}}\right)$$

or

$$\underline{E_{Fi} - E_F = 0.292\ eV}$$

(c)

From (a)

$$n_O = \frac{n_i^2}{p_O} = \frac{(1.5x10^{10})^2}{1.11x10^{16}}$$

or

$$\underline{n_O = 2.03x10^4\ cm^{-3}}$$

From (b)

$$n_O = \frac{n_i^2}{p_O} = \frac{(2.38x10^{12})^2}{1.11x10^{16}}$$

or

$$\underline{n_O = 5.10x10^8\ cm^{-3}}$$

4.23

(a) $p_O = n_i \exp\left(\dfrac{E_{Fi} - E_F}{kT}\right)$

$$= (1.8x10^6)\exp\left(\frac{0.35}{0.0259}\right)$$

or

$$\underline{p_O = 1.33x10^{12}\ cm^{-3}}$$

(b) From Problem 4.1,

$n_i(400K) = 3.28x10^9\ cm^{-3}$, $kT = 0.03453\ eV$

Then

$$E_{Fi} - E_F = kT \ln\left(\frac{p_O}{n_i}\right)$$

$$= (0.03453)\ln\left(\frac{1.33x10^{12}}{3.28x10^9}\right)$$

or

$$\underline{E_{Fi} - E_F = 0.207\ eV}$$

(c) From (a)

$$n_O = \frac{n_i^2}{p_O} = \frac{(1.8x10^6)^2}{1.33x10^{12}}$$

or

$$\underline{n_O = 2.44\ cm^{-3}}$$

From (b)

$$n_O = \frac{(3.28x10^9)^2}{1.33x10^{12}}$$

or

$$\underline{n_O = 8.09x10^6\ cm^{-3}}$$

4.24

For silicon, $T = 300K$, $E_F = E_V$

$$\eta' = \frac{E_V - E_F}{kT} = 0 \Rightarrow F_{1/2}(\eta') = 0.60$$

We can write

$$p_O = \frac{2}{\sqrt{\pi}} N_V F_{1/2}(\eta') = \frac{2}{\sqrt{\pi}}\left(1.04x10^{19}\right)(0.60)$$

or

$$p_O = 7.04x10^{18}\ cm^{-3}$$

4.25

Silicon, $T = 300K$, $n_O = 5x10^{19}\ cm^{-3}$

We have

$$n_O = \frac{2}{\sqrt{\pi}} N_C F_{1/2}(\eta_F)$$

or

$$5x10^{19} = \frac{2}{\sqrt{\pi}}\left(2.8x10^{19}\right)F_{1/2}(\eta_F)$$

which gives

$$F_{1/2}(\eta_F) = 1.58$$

Then

$$\eta_F = 1.3 = \frac{E_F - E_C}{kT}$$

or $E_F - E_C = (1.3)(0.0259) \Rightarrow$

$$E_C - E_F = -0.034\ eV$$

4.26

For the electron concentration

$$n(E) = g_C(E) f_F(E)$$

The Boltzmann approximation applies so

$$n(E) = \frac{4\pi\left(2m_n^*\right)^{3/2}}{h^3}\sqrt{E - E_C}\exp\left[\frac{-(E - E_F)}{kT}\right]$$

or

$$n(E) = \frac{4\pi\left(2m_n^*\right)^{3/2}}{h^3}\exp\left[\frac{-(E_C - E_F)}{kT}\right] \times\sqrt{kT}\sqrt{\frac{E - E_C}{kT}}\exp\left[\frac{-(E - E_C)}{kT}\right]$$

Define

$$x = \frac{E - E_C}{kT}$$

Then

$$n(E) \to n(x) = K\sqrt{x}\exp(-x)$$

To find maximum $n(E) \to n(x)$, set

$$\frac{dn(x)}{dx} = 0 = K\left[\frac{1}{2}x^{-1/2}\exp(-x) + x^{1/2}(-1)\exp(-x)\right]$$

or

$$0 = Kx^{-1/2}\exp(-x)\left[\frac{1}{2} - x\right]$$

which yields

$$x = \frac{1}{2} = \frac{E - E_C}{kT} \Rightarrow \quad E = E_C + \frac{1}{2}kT$$

For the hole concentration

$$p(E) = g_V(E)\left[1 - f_F(E)\right]$$

From the text, using the Maxwell-Boltzmann approximation, we can write

$$p(E) = \frac{4\pi\left(2m_p^*\right)^{3/2}}{h^3}\sqrt{E_V - E}\exp\left[\frac{-(E_F - E)}{kT}\right]$$

or

$$p(E) = \frac{4\pi\left(2m_p^*\right)^{3/2}}{h^3}\exp\left[\frac{-(E_F - E_V)}{kT}\right] \times\sqrt{kT}\sqrt{\frac{E_V - E}{kT}}\exp\left[\frac{-(E_V - E)}{kT}\right]$$

Define $x' = \dfrac{E_V - E}{kT}$

Then

$$p(x') = K'\sqrt{x'}\exp(-x')$$

To find the maximum of $p(E) \to p(x')$, set $\dfrac{dp(x')}{dx'} = 0$. Using the results from above, we find the maximum at

$$E = E_V - \frac{1}{2}kT$$

4.27

(a) Silicon: We have

$$n_O = N_C\exp\left[\frac{-(E_C - E_F)}{kT}\right]$$

We can write

$$E_C - E_F = (E_C - E_d) + (E_d - E_F)$$

For

$$E_C - E_d = 0.045\ eV,\ E_d - E_F = 3kT$$

$$n_O = \left(2.8x10^{19}\right)\exp\left[\frac{-0.045}{0.0259} - 3\right]$$

$$= \left(2.8x10^{19}\right)\exp(-4.737)$$

or

$$n_O = 2.45x10^{17}\ cm^{-3}$$

We also have

$$p_O = N_V \exp\left[\frac{-(E_F - E_V)}{kT}\right]$$

Again, we can write

$$E_F - E_V = (E_F - E_a) + (E_a - E_V)$$

For

$$E_F - E_a = 3kT\,,\; E_a - E_V = 0.045\ eV$$

Then

$$p_O = \left(1.04x10^{19}\right)\exp\left[-3 - \frac{0.045}{0.0259}\right]$$

$$= \left(1.04x10^{19}\right)\exp(-4.737)$$

or

$$\underline{p_O = 9.12x10^{16}\ cm^{-3}}$$

(b)
GaAs: Assume $E_C - E_d = 0.0058\ eV$
Then

$$n_O = \left(4.7x10^{17}\right)\exp\left[\frac{-0.0058}{0.0259} - 3\right]$$

$$= \left(4.7x10^{17}\right)\exp(-3.224)$$

or

$$\underline{n_O = 1.87x10^{16}\ cm^{-3}}$$

Assume $E_a - E_V = 0.0345\ eV$
Then

$$p_O = \left(7x10^{18}\right)\exp\left[\frac{-0.0345}{0.0259} - 3\right]$$

$$= \left(7x10^{18}\right)\exp(-4.332)$$

or

$$\underline{p_O = 9.20x10^{16}\ cm^{-3}}$$

4.28
Computer Plot

4.29
(a) Ge:

$$p_O = \frac{N_a - N_d}{2} + \sqrt{\left(\frac{N_a - N_d}{2}\right)^2 + n_i^2}$$

Then

$$p_O = \frac{10^{13}}{2} + \sqrt{\left(\frac{10^{13}}{2}\right)^2 + \left(2.4x10^{13}\right)^2}$$

or

$$\underline{p_O = 2.95x10^{13}\ cm^{-3}}$$

and

$$n_O = \frac{n_i^2}{p_O} = \frac{\left(2.4x10^{13}\right)^2}{2.95x10^{13}} \Rightarrow$$

$$\underline{n_O = 1.95x10^{13}\ cm^{-3}}$$

(b)

$$n_O = \frac{N_d - N_a}{2} + \sqrt{\left(\frac{N_d - N_a}{2}\right)^2 + n_i^2}$$

Then

$$n_O = \frac{5x10^{15}}{2} + \sqrt{\left(\frac{5x10^{15}}{2}\right)^2 + \left(2.4x10^{13}\right)^2}$$

or

$$\underline{n_O \approx 5x10^{15}\ cm^{-3}}$$

and

$$p_O = \frac{n_i^2}{n_O} = \frac{\left(2.4x10^{13}\right)^2}{5x10^{15}} \Rightarrow$$

$$\underline{p_O = 1.15x10^{11}\ cm^{-3}}$$

4.30
For the donor level

$$\frac{n_d}{N_d} = \frac{1}{1 + \frac{1}{2}\exp\left(\frac{E_d - E_F}{kT}\right)}$$

$$= \frac{1}{1 + \frac{1}{2}\exp\left(\frac{0.20}{0.0259}\right)}$$

or

$$\underline{\frac{n_d}{N_d} = 8.85x10^{-4}}$$

And

$$f_F(E) = \frac{1}{1 + \exp\left(\frac{E - E_F}{kT}\right)}$$

Now

$$E - E_F = (E - E_C) + (E_C - E_F)$$

or

$$E - E_F = kT + 0.245$$

Then

$$f_F(E) = \frac{1}{1 + \exp\left(1 + \frac{0.245}{0.0259}\right)} \Rightarrow$$

$$\underline{f_F(E) = 2.87x10^{-5}}$$

4.31

(a) $n_O = N_d = 2x10^{15}\ cm^{-3}$

$$p_O = \frac{n_i^2}{n_O} = \frac{\left(1.5x10^{10}\right)^2}{2x10^{15}} \Rightarrow$$

$p_O = 1.125x10^{5}\ cm^{-3}$

(b)

$p_O = N_a = 10^{16}\ cm^{-3}$

$$n_O = \frac{n_i^2}{p_O} = \frac{\left(1.5x10^{10}\right)^2}{10^{16}} \Rightarrow$$

$n_O = 2.25x10^{4}\ cm^{-3}$

(c)

$n_O = p_O = n_i = 1.5x10^{10}\ cm^{-3}$

(d)

$T = 400K \Rightarrow kT = 0.03453\ eV$

$$n_i^2 = \left(2.8x10^{19}\right)\left(1.04x10^{19}\right)\left(\frac{400}{300}\right)^3 \exp\left(\frac{-1.12}{0.03453}\right)$$

or

$n_i = 2.38x10^{12}\ cm^{-3}$

$$p_O = \frac{N_a}{2} + \sqrt{\left(\frac{N_a}{2}\right)^2 + n_i^2}$$

$$= 5x10^{13} + \sqrt{\left(5x10^{13}\right)^2 + \left(2.38x10^{12}\right)^2}$$

or

$p_O = 1.0x10^{14}\ cm^{-3}$

Also

$$n_O = \frac{n_i^2}{p_O} = \frac{\left(2.38x10^{12}\right)^2}{10^{14}} \Rightarrow$$

$n_O = 5.66x10^{10}\ cm^{-3}$

(e)

$T = 500K \Rightarrow kT = 0.04317\ eV$

$$n_i^2 = \left(2.8x10^{19}\right)\left(1.04x10^{19}\right)\left(\frac{500}{300}\right)^3 \exp\left(\frac{-1.12}{0.04317}\right)$$

or

$n_i = 8.54x10^{13}\ cm^{-3}$

Now

$$n_O = \frac{N_d}{2} + \sqrt{\left(\frac{N_d}{2}\right)^2 + n_i^2}$$

$$= 5x10^{13} + \sqrt{\left(5x10^{13}\right)^2 + \left(8.54x10^{13}\right)^2}$$

or

$n_O = 1.49x10^{14}\ cm^{-3}$

Also

$$p_O = \frac{n_i^2}{n_O} = \frac{\left(8.54x10^{13}\right)^2}{1.49x10^{14}} \Rightarrow$$

$p_O = 4.89x10^{13}\ cm^{-3}$

4.32

(a) $n_O = N_d = 2x10^{15}\ cm^{-3}$

$$p_O = \frac{n_i^2}{n_O} = \frac{\left(1.8x10^{6}\right)^2}{2x10^{15}} \Rightarrow$$

$p_O = 1.62x10^{-3}\ cm^{-3}$

(b)

$p_O = N_a = 10^{16}\ cm^{-3}$

$$n_O = \frac{n_i^2}{p_O} = \frac{\left(1.8x10^{6}\right)^2}{10^{16}} \Rightarrow$$

$n_O = 3.24x10^{-4}\ cm^{-3}$

(c)

$n_O = p_O = n_i = 1.8x10^{6}\ cm^{-3}$

(d)

$kT = 0.03453\ eV$

$$n_i^2 = \left(4.7x10^{17}\right)\left(7x10^{18}\right)\left(\frac{400}{300}\right)^3 \exp\left(\frac{-1.42}{0.03453}\right)$$

or

$n_i = 3.28x10^{9}\ cm^{-3}$

Now

$p_O = N_a = 10^{14}\ cm^{-3}$

and

$$n_O = \frac{n_i^2}{p_O} = \frac{\left(3.28x10^{9}\right)^2}{10^{14}} \Rightarrow$$

$n_O = 1.08x10^{5}\ cm^{-3}$

(e)

$kT = 0.04317\ eV$

$$n_i^2 = \left(4.7x10^{17}\right)\left(7x10^{18}\right)\left(\frac{500}{300}\right)^3 \exp\left(\frac{-1.42}{0.04317}\right)$$

or

$n_i = 2.81x10^{11}\ cm^{-3}$

Now

$$\underline{n_O = N_d = 10^{14}\ cm^{-3}}$$

Also

$$p_O = \frac{n_i^2}{n_O} = \frac{(2.81x10^{11})^2}{10^{14}} \Rightarrow$$

$$\underline{p_O = 7.90x10^8\ cm^{-3}}$$

4.33

(a) $N_a > N_d \Rightarrow$ p-type

(b) Si:

$$p_O = N_a - N_d = 2.5x10^{13} - 1x10^{13}$$

or

$$\underline{p_O = 1.5x10^{13}\ cm^{-3}}$$

Then

$$n_O = \frac{n_i^2}{p_O} = \frac{(1.5x10^{10})^2}{1.5x10^{13}} \Rightarrow$$

$$\underline{n_O = 1.5x10^7\ cm^{-3}}$$

Ge:

$$p_O = \frac{N_a - N_d}{2} + \sqrt{\left(\frac{N_a - N_d}{2}\right)^2 + n_i^2}$$

$$= \left(\frac{1.5x10^{13}}{2}\right) + \sqrt{\left(\frac{1.5x10^{13}}{2}\right)^2 + (2.4x10^{13})^2}$$

or

$$\underline{p_O = 3.26x10^{13}\ cm^{-3}}$$

Then

$$n_O = \frac{n_i^2}{p_O} = \frac{(2.4x10^{13})^2}{3.26x10^{13}} \Rightarrow$$

$$\underline{n_O = 1.77x10^{13}\ cm^{-3}}$$

GaAs:

$$\underline{p_O = 1.5x10^{13}\ cm^{-3}}$$

And

$$n_O = \frac{n_i^2}{p_O} = \frac{(1.8x10^6)^2}{1.5x10^{13}} \Rightarrow$$

$$\underline{n_O = 0.216\ cm^{-3}}$$

4.34

For $T = 450K$

$$n_i^2 = (2.8x10^{19})(1.04x10^{19})\left(\frac{450}{300}\right)^3 \times \exp\left[\frac{-1.12}{(0.0259)(450/300)}\right]$$

or

$$\underline{n_i = 1.72x10^{13}\ cm^{-3}}$$

(a)

$N_a > N_d \Rightarrow$ p-type

(b)

$$p_O = \frac{N_a - N_d}{2} + \sqrt{\left(\frac{N_a - N_d}{2}\right)^2 + n_i^2}$$

$$= \frac{1.5x10^{15} - 8x10^{14}}{2} + \sqrt{\left(\frac{1.5x10^{15} - 8x10^{14}}{2}\right)^2 + (1.72x10^{13})^2}$$

or

$$\underline{p_O \approx N_a - N_d = 7x10^{14}\ cm^{-3}}$$

Then

$$n_O = \frac{n_i^2}{p_O} = \frac{(1.72x10^{13})^2}{7x10^{14}} \Rightarrow$$

$$\underline{n_O = 4.23x10^{11}\ cm^{-3}}$$

(c)

Total ionized impurity concentration

$$N_I = N_a + N_d = 1.5x10^{15} + 8x10^{14}$$

or

$$\underline{N_I = 2.3x10^{15}\ cm^{-3}}$$

4.35

$$n_O = \frac{n_i^2}{p_O} = \frac{(1.5x10^{10})^2}{2x10^5} \Rightarrow$$

$$\underline{n_O = 1.125x10^{15}\ cm^{-3}}$$

$n_O > p_O \Rightarrow$ n-type

4.36

$$kT = (0.0259)\left(\frac{200}{300}\right) = 0.01727\ eV$$

$$n_i^2 = (4.7x10^{17})(7x10^{18})\left(\frac{200}{300}\right)^3 \times \exp\left[\frac{-1.42}{0.01727}\right]$$

or

$$\underline{n_i = 1.38\ cm^{-3}}$$

Now

$$n_o p_o = n_i^2 \Rightarrow 5p_o^2 = n_i^2$$

or\

$$p_o = \frac{n_i}{\sqrt{5}} \Rightarrow \underline{p_o = 0.617\ cm^{-3}}$$

And

$$n_o = 5p_o \Rightarrow \underline{n_o = 3.09\ cm^{-3}}$$

4.37
Computer Plot

4.38
Computer Plot

4.39
Computer Plot

4.40
n-type, so majority carrier = electrons

$$n_o = \frac{N_d}{2} + \sqrt{\left(\frac{N_d}{2}\right)^2 + n_i^2}$$

$$= 10^{13} + \sqrt{(10^{13})^2 + (2x10^{13})^2}$$

or

$$\underline{n_o = 3.24x10^{13}\ cm^{-3}}$$

Then

$$p_o = \frac{n_i^2}{n_o} = \frac{(2x10^{13})^2}{3.24x10^{13}} \Rightarrow$$

$$\underline{p_o = 1.23x10^{13}\ cm^{-3}}$$

4.41

(a) $N_d > N_a \Rightarrow$ n-type

$$n_o = N_d - N_a = 2x10^{16} - 1x10^{16}$$

or

$$\underline{n_o = 1x10^{16}\ cm^{-3}}$$

Then

$$p_o = \frac{n_i^2}{n_o} = \frac{(1.5x10^{10})^2}{10^{16}} \Rightarrow$$

$$\underline{p_o = 2.25x10^4\ cm^{-3}}$$

(b)

$N_a > N_d \Rightarrow$ p-type

$$p_o = N_a - N_d = 3x10^{16} - 2x10^{15}$$

or

$$\underline{p_o = 2.8x10^{16}\ cm^{-3}}$$

Then

$$n_o = \frac{n_i^2}{p_o} = \frac{(1.5x10^{10})^2}{2.8x10^{16}} \Rightarrow$$

$$\underline{n_o = 8.04x10^3\ cm^{-3}}$$

4.42

(a) $n_o < n_i \Rightarrow$ p-type

(b) $\underline{n_o = 4.5x10^4\ cm^{-3}} \Rightarrow$ electrons: minority carrier

$$p_o = \frac{n_i^2}{n_o} = \frac{(1.5x10^{10})^2}{4.5x10^4} \Rightarrow$$

$\underline{p_o = 5x10^{15}\ cm^{-3}} \Rightarrow$ holes: majority carrier

(c)

$$p_o = N_a - N_d$$

so

$$5x10^{15} = N_a - 5x10^{15} \Rightarrow \underline{N_a = 10^{16}\ cm^{-3}}$$

Acceptor impurity concentration,

$\underline{N_d = 5x10^{15}\ cm^{-3}}$ Donor impurity concentration

4.43

$$E_{Fi} - E_F = kT\ln\left(\frac{p_o}{n_i}\right)$$

For Germanium:

$T(°K)$	$kT(eV)$	$n_i(cm^{-3})$
200	0.01727	$2.16x10^{10}$
400	0.03454	$8.6x10^{14}$
600	0.0518	$3.82x10^{16}$

$$p_O = \frac{N_a}{2} + \sqrt{\left(\frac{N_a}{2}\right)^2 + n_i^2} \text{ and } N_a = 10^{15} \ cm^{-3}$$

$T(°K)$	$p_O(cm^{-3})$	$E_{Fi} - E_F \ (eV)$
200	$1.0x10^{15}$	0.1855
400	$1.49x10^{15}$	0.01898
600	$3.87x10^{16}$	0.000674

4.44

$$E_F - E_{Fi} = kT \ln\left(\frac{n_O}{n_i}\right)$$

For Germanium,

$$T = 300K \Rightarrow n_i = 2.4x10^{13} \ cm^{-3}$$

$$n_O = \frac{N_d}{2} + \sqrt{\left(\frac{N_d}{2}\right)^2 + n_i^2}$$

$N_d(cm^{-3})$	$n_O(cm^{-3})$	$E_F - E_{Fi} \ (eV)$
10^{14}	$1.05x10^{14}$	0.0382
10^{16}	10^{16}	0.156
10^{18}	10^{18}	0.2755

4.45

$$n_O = \frac{N_d}{2} + \sqrt{\left(\frac{N_d}{2}\right)^2 + n_i^2}$$

Now

$$n_i = 0.05 n_O$$

so

$$n_O = 1.5x10^{15} + \sqrt{\left(1.5x10^{15}\right)^2 + \left[(0.05)n_O\right]^2}$$

which yields

$$n_O = 3.0075x10^{15} \ cm^{-3}$$

Then

$$n_i = 1.504x10^{14} \ cm^{-3}$$

We have

$$n_i^2 = N_C N_V \exp\left(\frac{-E_g}{kT}\right)$$

so

$$\left(1.504x10^{14}\right)^2 = \left(4.7x10^{17}\right)\left(7x10^{18}\right)\left(\frac{T}{300}\right)^3 \times \exp\left[\frac{-1.42}{(0.0259)(T/300)}\right]$$

By trial and error

$$T \approx 762K$$

4.46

Computer Plot

4.47

Computer Plot

4.48

(a) $E_{Fi} - E_{midgap} = \dfrac{3}{4} kT \ln\left(\dfrac{m_p^*}{m_n^*}\right)$

$$= \frac{3}{4}(0.0259)\ln(10) \Rightarrow$$

$$E_{Fi} - E_{midgap} = +0.0447 \ eV$$

(b)

Impurity atoms to be added so

$$E_{midgap} - E_F = 0.45 \ eV$$

(i) p-type, so add acceptor impurities

(ii) $E_{Fi} - E_F = 0.0447 + 0.45 = 0.4947 \ eV$

$$p_O = n_i \exp\left(\frac{E_{Fi} - E_F}{kT}\right) = 10^5 \exp\left(\frac{0.4947}{0.0259}\right)$$

or

$$p_O = N_a = 1.97x10^{13} \ cm^{-3}$$

4.49

$$n_O = N_d - N_a = N_C \exp\left[\frac{-(E_C - E_F)}{kT}\right]$$

so

$$N_d = 5x10^{15} + 2.8x10^{19} \exp\left(\frac{-0.215}{0.0259}\right)$$

$$= 5x10^{15} + 6.95x10^{15}$$

so

$$N_d = 1.2x10^{16} \ cm^{-3}$$

4.50

(a) $p_O = N_a = N_V \exp\left[\dfrac{-(E_F - E_V)}{kT}\right]$

or

$$\exp\left[\frac{+(E_F - E_V)}{kT}\right] = \frac{N_V}{N_a} = \frac{1.04x10^{19}}{7x10^{15}}$$

$$= 1.49x10^3$$

Then

$$E_F - E_V = (0.0259)\ln(1.49x10^3)$$

or

$$\underline{E_F - E_V = 0.189\ eV}$$

(b)

If $E_F - E_V = 0.1892 - 0.0259 = 0.1633\ eV$

Then

$$N_a = 1.04x10^{19}\exp\left(\frac{-0.1633}{0.0259}\right)$$

$$= 1.90x10^{16}\ cm^{-3}$$

so that

$$\Delta N_a = 1.90x10^{16} - 7x10^{15} \Rightarrow$$

$$\underline{\Delta N_a = 1.2x10^{16}\ cm^{-3}}$$

Acceptor impurities to be added

4.51

(a) $E_F - E_{Fi} = kT\ln\left(\frac{N_d}{n_i}\right) = (0.0259)\ln\left(\frac{10^{15}}{1.5x10^{10}}\right)$

or

$$\underline{E_F - E_{Fi} = 0.2877\ eV}$$

(b)

$$E_{Fi} - E_F = kT\ln\left(\frac{N_a}{n_i}\right) = 0.2877\ eV$$

(c)

For (a), $\underline{n_O = N_d = 10^{15}\ cm^{-3}}$

For (b)

$$n_O = \frac{n_i^2}{p_O} = \frac{(1.5x10^{10})^2}{10^{15}} \Rightarrow$$

$$\underline{n_O = 2.25x10^5\ cm^{-3}}$$

4.52

$$E_{Fi} - E_F = kT\ln\left(\frac{p_O}{n_i}\right)$$

$$= (0.0259)\ln\left(\frac{p_O}{n_i}\right) = 0.45\ eV$$

Then

$$p_O = (1.8x10^6)\exp\left(\frac{0.45}{0.0259}\right) \Rightarrow$$

$$\underline{p_O = 6.32x10^{13}\ cm^{-3}}$$

Now

$p_O < N_a$, Donors must be added

$$p_O = N_a - N_d \Rightarrow N_d = N_a - p_O$$

so

$$N_d = 10^{15} - 6.32x10^{13} \Rightarrow$$

$$\underline{N_d = 9.368x10^{14}\ cm^{-3}}$$

4.53

(a) $E_F - E_{Fi} = kT\ln\left(\frac{N_d}{n_i}\right)$

$$= (0.0259)\ln\left(\frac{2x10^{15}}{1.5x10^{10}}\right) \Rightarrow$$

$$\underline{E_F - E_{Fi} = 0.3056\ eV}$$

(b)

$$E_{Fi} - E_F = kT\ln\left(\frac{N_a}{n_i}\right)$$

$$= (0.0259)\ln\left(\frac{10^{16}}{1.5x10^{10}}\right) \Rightarrow$$

$$\underline{E_{Fi} - E_F = 0.3473\ eV}$$

(c)

$$\underline{E_F = E_{Fi}}$$

(d)

$kT = 0.03453\ eV$, $n_i = 2.38x10^{12}\ cm^{-3}$

$$E_{Fi} - E_F = kT\ln\left(\frac{p_O}{n_i}\right)$$

$$= (0.03453)\ln\left(\frac{10^{14}}{2.38x10^{12}}\right) \Rightarrow$$

$$\underline{E_{Fi} - E_F = 0.1291\ eV}$$

(e)

$kT = 0.04317\ eV$, $n_i = 8.54x10^{13}\ cm^{-3}$

$$E_F - E_{Fi} = kT\ln\left(\frac{n_O}{n_i}\right)$$

$$= (0.04317)\ln\left(\frac{1.49x10^{14}}{8.54x10^{13}}\right) \Rightarrow$$

$$\underline{E_F - E_{Fi} = 0.0024\ eV}$$

4.54

(a) $E_F - E_{Fi} = kT\ln\left(\frac{N_d}{n_i}\right)$

$$= (0.0259)\ln\left(\frac{2x10^{15}}{1.8x10^{6}}\right) \Rightarrow$$

$$\underline{E_F - E_{Fi} = 0.5395\ eV}$$

(b)

$$E_{Fi} - E_F = kT\ln\left(\frac{N_a}{n_i}\right)$$

$$= (0.0259)\ln\left(\frac{10^{16}}{1.8x10^{6}}\right) \Rightarrow$$

$$\underline{E_{Fi} - E_F = 0.5811\ eV}$$

(c)

$$\underline{E_F = E_{Fi}\ E_{Fi} -}$$

(d)

$kT = 0.03453\ eV$, $n_i = 3.28x10^9\ cm^{-3}$

$$E_{Fi} - E_F = (0.03453)\ln\left(\frac{10^{14}}{3.28x10^{9}}\right) \Rightarrow$$

$$\underline{E_{Fi} - E_F = 0.3565\ eV}$$

(e)

$kT = 0.04317\ eV$, $n_i = 2.81x10^{11}\ cm^{-3}$

$$E_F - E_{Fi} = kT\ln\left(\frac{n_O}{n_i}\right)$$

$$= (0.04317)\ln\left(\frac{10^{14}}{2.81x10^{11}}\right) \Rightarrow$$

$$\underline{E_F - E_{Fi} = 0.2536\ eV}$$

4.55

p-type

$$E_{Fi} - E_F = kT\ln\left(\frac{p_O}{n_i}\right)$$

$$= (0.0259)\ln\left(\frac{5x10^{15}}{1.5x10^{10}}\right) \Rightarrow$$

$$\underline{E_{Fi} - E_F = 0.3294\ eV}$$

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Chapter 5

Problem Solutions

5.1

(a) $\underline{n_O = 10^{16}\ cm^{-3}}$

and

$$p_O = \frac{n_i^2}{n_O} = \frac{(1.8x10^6)^2}{10^{16}} \Rightarrow$$

$$\underline{p_O = 3.24x10^{-4}\ cm^{-3}}$$

(b)

$$J = e\mu_n n_O \mathrm{E}$$

For GaAs doped at $N_d = 10^{16}\ cm^{-3}$,

$$\mu_n \approx 7500\ cm^2 / V - s$$

Then

$$J = (1.6x10^{-19})(7500)(10^{16})(10)$$

or

$$\underline{J = 120\ A / cm^2}$$

(b) (i) $\underline{p_O = 10^{16}\ cm^{-3}}$, $\underline{n_O = 3.24x10^{-4}\ cm^{-3}}$

(ii) For GaAs doped at $N_a = 10^{16}\ cm^{-3}$,

$$\mu_p \approx 310\ cm^2 / V - s$$

$$J = e\mu_p p_O \mathrm{E}$$

$$= (1.6x10^{-19})(310)(10^{16})(10) \Rightarrow$$

$$\underline{J = 4.96\ A / cm^2}$$

5.2

(a) $V = IR \Rightarrow 10 = (0.1R) \Rightarrow$

$$\underline{R = 100\ \Omega}$$

(b)

$$R = \frac{L}{\sigma A} \Rightarrow \sigma = \frac{L}{RA} \Rightarrow$$

$$\sigma = \frac{10^{-3}}{(100)(10^{-3})} \Rightarrow$$

$$\underline{\sigma = 0.01\ (\Omega - cm)^{-1}}$$

(c)

$$\sigma \approx e\mu_n N_d$$

or

$$0.01 = (1.6x10^{-19})(1350)N_d$$

or

$$\underline{N_d = 4.63x10^{13}\ cm^{-3}}$$

(d)

$$\sigma \approx e\mu_p p_O \Rightarrow$$

$$0.01 = (1.6x10^{-19})(480)p_O$$

or

$$p_O = 1.30x10^{14}\ cm^{-3} = N_a - N_d = N_a - 10^{15}$$

or

$$\underline{N_a = 1.13x10^{15}\ cm^{-3}}$$

Note: For the doping concentrations obtained, the assumed mobility values are valid.

5.3

(a) $R = \dfrac{\rho L}{A} = \dfrac{L}{\sigma A}$ and $\sigma \approx e\mu_n N_d$

For $N_d = 5x10^{16}\ cm^{-3}$, $\mu_n \approx 1100\ cm^2 / V - s$

Then

$$R = \frac{0.1}{(1.6x10^{-19})(1100)(5x10^{16})(100)(10^{-4})^2}$$

or

$$\underline{R = 1.136x10^4\ \Omega}$$

Then

$$I = \frac{V}{R} = \frac{5}{1.136x10^4} \Rightarrow \underline{I = 0.44\ mA}$$

(b)

In this case

$$\underline{R = 1.136x10^3\ \Omega}$$

Then

$$I = \frac{V}{R} = \frac{5}{1.136x10^3} \Rightarrow \underline{I = 4.4\ mA}$$

(c)

$$\mathrm{E} = \frac{V}{L}$$

For (a), $\mathrm{E} = \dfrac{5}{0.10} = 50\ V / cm$

And

$$v_d = \mu_n \mathrm{E} = (1100)(50) \text{ or } \underline{v_d = 5.5x10^4\ cm / s}$$

For (b), $\mathrm{E} = \dfrac{V}{L} = \dfrac{5}{0.01} = 500\ V / cm$

And

$$v_d = (1100)(500) \Rightarrow \underline{v_d = 5.5x10^5\ cm / s}$$

5.4

(a) GaAs:

$$R = \frac{\rho L}{A} = \frac{V}{I} = \frac{10}{20} = 0.5\ k\Omega = \frac{L}{\sigma A}$$

Now

$$\sigma \approx e\mu_p N_a$$

For $N_a = 10^{17}\ cm^{-3}$, $\mu_p \approx 210\ cm^2/V-s$

Then

$$\sigma = \left(1.6x10^{-19}\right)(210)\left(10^{17}\right) = 3.36\ (\Omega - cm)^{-1}$$

So

$$L = R\sigma A = (500)(3.36)\left(85x10^{-8}\right)$$

or

$$\underline{L = 14.3\ \mu m}$$

(b) Silicon

For $N_a = 10^{17}\ cm^{-3}$, $\mu_p \approx 310\ cm^2/V-s$

Then

$$\sigma = \left(1.6x10^{-19}\right)(310)\left(10^{17}\right) = 4.96\ (\Omega - cm)^{-1}$$

So

$$L = R\sigma A = (500)(4.96)\left(85x10^{-8}\right)$$

or

$$\underline{L = 21.1\ \mu m}$$

5.5

(a) $\mathrm{E} = \frac{V}{L} = \frac{3}{1} = 3\ V/cm$

$$v_d = \mu_n \mathrm{E} \Rightarrow \mu_n = \frac{v_d}{\mathrm{E}} = \frac{10^4}{3}$$

or

$$\underline{\mu_n = 3333\ cm^2/V-s}$$

(b)

$$v_d = \mu_n \mathrm{E} = (800)(3)$$

or

$$\underline{v_d = 2.4x10^3\ cm/s}$$

5.6

(a) Silicon: For $\mathrm{E} = 1\ kV/cm$,

$$v_d = 1.2x10^6\ cm/s$$

Then

$$t_t = \frac{d}{v_d} = \frac{10^{-4}}{1.2x10^6} \Rightarrow \underline{t_t = 8.33x10^{-11}\ s}$$

For GaAs, $v_d = 7.5x10^6\ cm/s$

Then

$$t_t = \frac{d}{v_d} = \frac{10^{-4}}{7.5x10^6} \Rightarrow \underline{t_t = 1.33x10^{-11}\ s}$$

(b)

Silicon: For $\mathrm{E} = 50\ kV/cm$,

$$v_d = 9.5x10^6\ cm/s$$

Then

$$t_t = \frac{d}{v_d} = \frac{10^{-4}}{9.5x10^6} \Rightarrow \underline{t_t = 1.05x10^{-11}\ s}$$

GaAs, $v_d = 7x10^6\ cm/s$

Then

$$t_t = \frac{d}{v_d} = \frac{10^{-4}}{7x10^6} \Rightarrow \underline{t_t = 1.43x10^{-11}\ s}$$

5.7

For an intrinsic semiconductor,

$$\sigma_i = en_i\left(\mu_n + \mu_p\right)$$

(a)

For $N_d = N_a = 10^{14}\ cm^{-3}$,

$$\mu_n = 1350\ cm^2/V-s,\ \mu_p = 480\ cm^2/V-s$$

Then

$$\sigma_i = \left(1.6x10^{-19}\right)\left(1.5x10^{10}\right)(1350 + 480)$$

or

$$\underline{\sigma_i = 4.39x10^{-6}\ (\Omega - cm)^{-1}}$$

(b)

For $N_d = N_a = 10^{18}\ cm^{-3}$,

$$\mu_n \approx 300\ cm^2/V-s,\ \mu_p \approx 130\ cm^2/V-s$$

Then

$$\sigma_i = \left(1.6x10^{-19}\right)\left(1.5x10^{10}\right)(300 + 130)$$

or

$$\underline{\sigma_i = 1.03x10^{-6}\ (\Omega - cm)^{-1}}$$

5.8

(a) GaAs

$$\sigma \approx e\mu_p p_O \Rightarrow 5 = \left(1.6x10^{-19}\right)\mu_p p_O$$

From Figure 5.3, and using trial and error, we find

$$\underline{p_O \approx 1.3x10^{17}\ cm^{-3},\ \mu_p \approx 240\ cm^2/V-s}$$

Then

$$n_O = \frac{n_i^2}{p_O} = \frac{\left(1.8x10^6\right)^2}{1.3x10^{17}} \quad \text{or} \quad \underline{n_O = 2.49x10^{-5}\ cm^{-3}}$$

(b) Silicon:

$$\sigma = \frac{1}{\rho} \approx e\mu_n n_O$$

or

$$n_O = \frac{1}{\rho e\mu_n} = \frac{1}{(8)\left(1.6x10^{-19}\right)(1350)}$$

or

$$\underline{n_O = 5.79x10^{14}\ cm^{-3}}$$

and

$$p_O = \frac{n_i^2}{n_O} = \frac{\left(1.5x10^{10}\right)^2}{5.79x10^{14}} \Rightarrow \underline{p_O = 3.89x10^5\ cm^{-3}}$$

Note: For the doping concentrations obtained in part (b), the assumed mobility values are valid.

5.9

$$\sigma_i = en_i\left(\mu_n + \mu_p\right)$$

Then

$$10^{-6} = \left(1.6x10^{-19}\right)(1000+600)n_i$$

or

$$\underline{n_i(300K) = 3.91x10^9\ cm^{-3}}$$

Now

$$n_i^2 = N_C N_V \exp\left(\frac{-E_g}{kT}\right)$$

or

$$E_g = kT\ln\left(\frac{N_C N_V}{n_i^2}\right) = (0.0259)\ln\left[\frac{\left(10^{19}\right)^2}{\left(3.91x10^9\right)^2}\right]$$

or

$$\underline{E_g = 1.122\ eV}$$

Now

$$n_i^2(500K) = \left(10^{19}\right)^2 \exp\left[\frac{-1.122}{(0.0259)(500/300)}\right]$$

$$= 5.15x10^{26}$$

or

$$\underline{n_i(500K) = 2.27x10^{13}\ cm^{-3}}$$

Then

$$\sigma_i = \left(1.6x10^{-19}\right)\left(2.27x10^{13}\right)(1000+600)$$

so

$$\underline{\sigma_i(500K) = 5.81x10^{-3}\ (\Omega - cm)^{-1}}$$

5.10

(a) (i) Silicon: $\sigma_i = en_i\left(\mu_n + \mu_p\right)$

$$\sigma_i = \left(1.6x10^{-19}\right)\left(1.5x10^{10}\right)(1350+480)$$

or

$$\underline{\sigma_i = 4.39x10^{-6}\ (\Omega - cm)^{-1}}$$

(ii) Ge:

$$\sigma_i = \left(1.6x10^{-19}\right)\left(2.4x10^{13}\right)(3900+1900)$$

or

$$\underline{\sigma_i = 2.23x10^{-2}\ (\Omega - cm)^{-1}}$$

(iii) GaAs:

$$\sigma_i = \left(1.6x10^{-19}\right)\left(1.8x10^6\right)(8500+400)$$

or

$$\underline{\sigma_i = 2.56x10^{-9}\ (\Omega - cm)^{-1}}$$

(b) $R = \dfrac{L}{\sigma A}$

(i) $R = \dfrac{200x10^{-4}}{\left(4.39x10^{-6}\right)\left(85x10^{-8}\right)} \Rightarrow$

$$\underline{R = 5.36x10^9\ \Omega}$$

(ii) $R = \dfrac{200x10^{-4}}{\left(2.23x10^{-2}\right)\left(85x10^{-8}\right)} \Rightarrow$

$$\underline{R = 1.06x10^6\ \Omega}$$

(iii) $R = \dfrac{200x10^{-4}}{\left(2.56x10^{-9}\right)\left(85x10^{-8}\right)} \Rightarrow$

$$\underline{R = 9.19x10^{12}\ \Omega}$$

5.11

(a) $\rho = 5 = \dfrac{1}{e\mu_n N_d}$

Assume $\mu_n = 1350\ cm^2/V-s$

Then

$$N_d = \frac{1}{\left(1.6x10^{-19}\right)(1350)(5)} \Rightarrow$$

$$\underline{N_d = 9.26x10^{14}\ cm^{-3}}$$

(b)

$T = 200K \rightarrow T = -75C$

$T = 400K \rightarrow T = 125C$

From Figure 5.2,

$T = -75C$, $N_d = 10^{15}\ cm^{-3} \Rightarrow$

$\mu_n \approx 2500\ cm^2/V-s$

$T = 125C$, $N_d = 10^{15}\ cm^{-3} \Rightarrow$

$\mu_n \approx 700\ cm^2/V-s$

Assuming $n_O = N_d = 9.26x10^{14}\ cm^{-3}$ over the temperature range,

For $T = 200K$,

$$\rho = \frac{1}{(1.6x10^{-19})(2500)(9.26x10^{14})} \Rightarrow$$

$$\underline{\rho = 2.7\ \Omega - cm}$$

For $T = 400K$,

$$\rho = \frac{1}{(1.6x10^{-19})(700)(9.26x10^{14})} \Rightarrow$$

$$\underline{\rho = 9.64\ \Omega - cm}$$

5.12

Computer plot

5.13

(a) $\mathrm{E} = 10\ V/cm \Rightarrow |v_d| = \mu_n \mathrm{E}$

$v_d = (1350)(10) \Rightarrow \quad v_d = 1.35x10^4\ cm/s$

so

$$T = \frac{1}{2} m_n^* v_d^2 = \frac{1}{2}(1.08)(9.11x10^{-31})(1.35x10^2)^2$$

or

$$\underline{T = 8.97x10^{-27}\ J \Rightarrow 5.6x10^{-8}\ eV}$$

(b)

$\mathrm{E} = 1\ kV/cm$,

$v_d = (1350)(1000) = 1.35x10^6\ cm/s$

Then

$$T = \frac{1}{2}(1.08)(9.11x10^{-31})(1.35x10^4)^2$$

or

$$\underline{T = 8.97x10^{-23}\ J \Rightarrow 5.6x10^{-4}\ eV}$$

5.14

(a) $n_i^2 = N_C N_V \exp\left(\frac{-E_g}{kT}\right)$

$$= (2x10^{19})(1x10^{19})\exp\left(\frac{-1.10}{0.0259}\right)$$

$$= 7.18x10^{19} \Rightarrow n_i = 8.47x10^9\ cm^{-3}$$

For $N_d = 10^{14}\ cm^{-3} >> n_i \Rightarrow n_O = 10^{14}\ cm^{-3}$

Then

$$J = \sigma\mathrm{E} = e\mu_n n_O \mathrm{E}$$

$$= (1.6x10^{-19})(1000)(10^{14})(100)$$

or

$$\underline{J = 1.60\ A/cm^2}$$

(b)

A 5% increase is due to a 5% increase in electron concentration. So

$$n_O = 1.05x10^{14} = \frac{N_d}{2} + \sqrt{\left(\frac{N_d}{2}\right)^2 + n_i^2}$$

We can write

$$(1.05x10^{14} - 5x10^{13})^2 = (5x10^{13})^2 + n_i^2$$

so

$$n_i^2 = 5.25x10^{26}$$

$$= (2x10^{19})(1x10^{19})\left(\frac{T}{300}\right)^3 \exp\left(\frac{-E_g}{kT}\right)$$

which yields

$$2.625x10^{-12} = \left(\frac{T}{300}\right)^3 \exp\left(\frac{-1.10}{kT}\right)$$

By trial and error, we find

$$\underline{T = 456K}$$

5.15

(a) $\sigma = e\mu_n n_O + e\mu_p p_O$ and $n_O = \frac{n_i^2}{p_O}$

Then

$$\sigma = \frac{e\mu_n n_i^2}{p_O} + e\mu_p p_O$$

To find the minimum conductivity,

$$\frac{d\sigma}{dp_O} = 0 = \frac{(-1)e\mu_n n_i^2}{p_O^2} + e\mu_p \Rightarrow$$

which yields

$$\underline{p_O = n_i\left(\frac{\mu_n}{\mu_p}\right)^{1/2}} \quad \text{(Answer to part (b))}$$

Substituting into the conductivity expression

$$\sigma = \sigma_{\min} = \frac{e\mu_n n_i^2}{\left[n_i(\mu_n/\mu_p)^{1/2}\right]} + e\mu_p\left[n_i(\mu_n/\mu_p)^{1/2}\right]$$

which simplifies to

$$\sigma_{\min} = 2en_i\sqrt{\mu_n\mu_p}$$

The intrinsic conductivity is defined as

$$\sigma_i = en_i(\mu_n + \mu_p) \Rightarrow en_i = \frac{\sigma_i}{\mu_n + \mu_p}$$

The minimum conductivity can then be written as

$$\sigma_{\min} = \frac{2\sigma_i\sqrt{\mu_n\mu_p}}{\mu_n + \mu_p}$$

5.16

$$\sigma = e\mu n_i = \frac{1}{\rho}$$

Now

$$\frac{1/\rho_1}{1/\rho_2} = \frac{1/50}{1/5} = \frac{5}{50} = 0.10 = \frac{\exp\left(\frac{-E_g}{2kT_1}\right)}{\exp\left(\frac{-E_g}{2kT_2}\right)}$$

or

$$0.10 = \exp\left[-E_g\left(\frac{1}{2kT_1} - \frac{1}{2kT_2}\right)\right]$$

$$kT_1 = 0.0259$$

$$kT_2 = (0.0259)\left(\frac{330}{300}\right) = 0.02849$$

$$\frac{1}{2kT_1} = 19.305 \quad , \quad \frac{1}{2kT_2} = 17.550$$

Then

$$E_g(19.305 - 17.550) = \ln(10)$$

or

$$E_g = 1.312\ eV$$

5.17

$$\frac{1}{\mu} = \frac{1}{\mu_1} + \frac{1}{\mu_2} + \frac{1}{\mu_3}$$

$$= \frac{1}{2000} + \frac{1}{1500} + \frac{1}{500}$$

$$= 0.00050 + 0.000667 + 0.0020$$

or

$$\mu = 316\ cm^2/V-s$$

5.18

$$\mu_n = (1300)\left(\frac{T}{300}\right)^{-3/2} = (1300)\left(\frac{300}{T}\right)^{+3/2}$$

(a)
At $T = 200K$, $\mu_n = (1300)(1.837) \Rightarrow$

$$\mu_n = 2388\ cm^2/V-s$$

(b)
At $T = 400K$, $\mu_n = (1300)(0.65) \Rightarrow$

$$\mu_n = 844\ cm^2/V-s$$

5.19

$$\frac{1}{\mu} = \frac{1}{\mu_1} + \frac{1}{\mu_2} = \frac{1}{250} + \frac{1}{500} = 0.006$$

Then

$$\mu = 167\ cm^2/V-s$$

5.20
Computer plot

5.21
Computer plot

5.22

$$J_n = eD_n\frac{dn}{dx} = eD_n\left(\frac{5x10^{14} - n(0)}{0.01 - 0}\right)$$

$$0.19 = (1.6x10^{-19})(25)\left(\frac{5x10^{14} - n(0)}{0.010}\right)$$

Then

$$\frac{(0.19)(0.010)}{(1.6x10^{-19})(25)} = 5x10^{14} - n(0)$$

which yields

$$n(0) = 0.25x10^{14}\ cm^{-3}$$

5.23

$$J = eD_n\frac{dn}{dx} = eD_n\frac{\Delta n}{\Delta x}$$

$$= (1.6x10^{-19})(25)\left(\frac{10^{16} - 10^{15}}{0 - 0.10}\right)$$

or

$$|J| = 0.36\ A/cm^2$$

For $A = 0.05\ cm^2$

$$I = AJ = (0.05)(0.36) \Rightarrow \quad I = 18\ mA$$

5.24

$$J_n = eD_n \frac{dn}{dx} = eD_n \frac{\Delta n}{\Delta x}$$

so

$$-400 = \left(1.6x10^{-19}\right) D_n \left(\frac{10^{17} - 6x10^{16}}{0 - 4x10^{-4}}\right)$$

or

$$-400 = D_n(-16)$$

Then

$$\underline{D_n = 25\ cm^2 / s}$$

5.25

$$J = -eD_p \frac{dp}{dx}$$

$$= -eD_p \frac{d}{dx}\left[10^{16}\left(1 - \frac{x}{L}\right)\right] = -eD_p\left(\frac{-10^{16}}{L}\right)$$

$$= \frac{\left(1.6x10^{-19}\right)(10)\left(10^{16}\right)}{10x10^{-4}}$$

or

$$\underline{J = 16\ A / cm^2} = \underline{\text{constant at all three points}}$$

5.26

$$J_p(x = 0) = -eD_p \frac{dp}{dx}\Big|_{x=0}$$

$$= -eD_p \frac{10^{15}}{\left(-L_p\right)} = \frac{\left(1.6x10^{-19}\right)(10)\left(10^{15}\right)}{5x10^{-4}}$$

or

$$J_p(x = 0) = 3.2\ A / cm^2$$

Now

$$J_n(x = 0) = eD_n \frac{dn}{dx}\Big|_{x=0}$$

$$= eD_n\left(\frac{5x10^{14}}{L_n}\right) = \frac{\left(1.6x10^{-19}\right)(25)\left(5x10^{14}\right)}{10^{-3}}$$

or

$$J_n(x = 0) = 2\ A / cm^2$$

Then

$$J = J_p(x = 0) + J_n(x = 0) = 3.2 + 2$$

or

$$\underline{J = 5.2\ A / cm^2}$$

5.27

$$J_p = -eD_p \frac{dp}{dx} = -eD_p \frac{d}{dp}\left[10^{15} \exp\left(\frac{-x}{22.5}\right)\right]$$

Distance x is in μm, so $22.5 \to 22.5x10^{-4}\ cm$.
Then

$$J_p = -eD_p\left(10^{15}\right)\left(\frac{-1}{22.5x10^{-4}}\right)\exp\left(\frac{-x}{22.5}\right)$$

$$= \frac{+\left(1.6x10^{-19}\right)(48)\left(10^{15}\right)}{22.5x10^{-4}}\exp\left(\frac{-x}{22.5}\right)$$

or

$$\underline{J_p = 3.41\exp\left(\frac{-x}{22.5}\right)\quad A / cm^2}$$

5.28

$$J_n = e\mu_n n \mathrm{E} + eD_n \frac{dn}{dx}$$

or

$$-40 = \left(1.6x10^{-19}\right)(960)\left[10^{16} \exp\left(\frac{-x}{18}\right)\right]\mathrm{E}$$

$$+\left(1.6x10^{-19}\right)(25)\left(10^{16}\right)\left(\frac{-1}{18x10^{-4}}\right)\exp\left(\frac{-x}{18}\right)$$

Then

$$-40 = 1.536\left[\exp\left(\frac{-x}{18}\right)\right]\mathrm{E} - 22.2\exp\left(\frac{-x}{18}\right)$$

Then

$$\mathrm{E} = \frac{22.2\exp\left(\frac{-x}{18}\right) - 40}{1.536\exp\left(\frac{-x}{18}\right)} \Rightarrow$$

$$\underline{\mathrm{E} = 14.5 - 26\exp\left(\frac{+x}{18}\right)}$$

5.29

$$J_T = J_{n,drf} + J_{p,dif}$$

(a) $J_{p,dif} = -eD_p \dfrac{dp}{dx}$ and

$$p(x) = 10^{15}\exp\left(\frac{-x}{L}\right) \quad \text{where} \quad L = 12\ \mu m$$

so

$$J_{p,dif} = -eD_p\left(10^{15}\right)\left(\frac{-1}{L}\right)\exp\left(\frac{-x}{L}\right)$$

or

$$J_{p,dif} = \frac{(1.6x10^{-19})(12)(10^{15})}{12x10^{-4}} \exp\left(\frac{-x}{12}\right)$$

or

$$\underline{J_{p,dif} = +1.6\exp\left(\frac{-x}{L}\right) \quad A/cm^2}$$

(b)

$$J_{n,drf} = J_T - J_{p,dif}$$

or

$$\underline{J_{n,drf} = 4.8 - 1.6\exp\left(\frac{-x}{L}\right)}$$

(c)

$$J_{n,drf} = e\mu_n n_O \mathrm{E}$$

Then

$$(1.6x10^{-19})(1000)(10^{16})\mathrm{E} = 4.8 - 1.6\exp\left(\frac{-x}{L}\right)$$

which yields

$$\underline{\mathrm{E} = \left[3 - 1\times\exp\left(\frac{-x}{L}\right)\right] \quad V/cm}$$

5.30

(a) $J = e\mu_n n(x)\mathrm{E} + eD_n \dfrac{dn(x)}{dx}$

Now $\mu_n = 8000\ cm^2/V-s$ so that

$$D_n = (0.0259)(8000) = 207\ cm^2/s$$

Then

$$100 = (1.6x10^{-19})(8000)(12)n(x) + (1.6x10^{-19})(207)\frac{dn(x)}{dx}$$

which yields

$$100 = 1.54x10^{-14}n(x) + 3.31x10^{-17}\frac{dn(x)}{dx}$$

Solution is of the form

$$n(x) = A + B\exp\left(\frac{-x}{d}\right)$$

so that

$$\frac{dn(x)}{dx} = \frac{-B}{d}\exp\left(\frac{-x}{d}\right)$$

Substituting into the differential equation, we have

$$100 = (1.54x10^{-14})\left[A + B\exp\left(\frac{-x}{d}\right)\right] - \frac{(3.31x10^{-17})}{d}B\exp\left(\frac{-x}{d}\right)$$

This equation is valid for all x, so

$$100 = 1.54x10^{-14}A$$

or

$$A = 6.5x10^{15}$$

Also

$$1.54x10^{-14}B\exp\left(\frac{-x}{d}\right) - \frac{(3.31x10^{-17})}{d}B\exp\left(\frac{-x}{d}\right) = 0$$

which yields

$$\underline{d = 2.15x10^{-3}\ cm}$$

At $x = 0$, $e\mu_n n(0)\mathrm{E} = 50$

so that

$$50 = (1.6x10^{-19})(8000)(12)(A + B)$$

which yields $B = -3.24x10^{15}$

Then

$$\underline{n(x) = 6.5x10^{15} - 3.24x10^{15}\exp\left(\frac{-x}{d}\right) \quad cm^{-3}}$$

(b)

At $x = 0$, $n(0) = 6.5x10^{15} - 3.24x10^{15}$

Or

$$\underline{n(0) = 3.26x10^{15}\ cm^{-3}}$$

At $x = 50\ \mu m$,

$$n(50) = 6.5x10^{15} - 3.24x10^{15}\exp\left(\frac{-50}{21.5}\right)$$

or

$$\underline{n(50) = 6.18x10^{15}\ cm^{-3}}$$

(c)

At $x = 50\ \mu m$, $J_{drt} = e\mu_n n(50)\mathrm{E}$

$$= (1.6x10^{-19})(8000)(6.18x10^{15})(12)$$

or

$$\underline{J_{drf}(x = 50) = 94.9\ A/cm^2}$$

Then

$$J_{dif}(x = 50) = 100 - 94.9 \Rightarrow$$

$$\underline{J_{dif}(x = 50) = 5.1\ A/cm^2}$$

5.31

$$n = n_i \exp\left(\frac{E_F - E_{Fi}}{kT}\right)$$

(a) $E_F - E_{Fi} = ax + b$, $b = 0.4$

$0.15 = a\left(10^{-3}\right) + 0.4$ so that $a = -2.5x10^2$

Then

$$E_F - E_{Fi} = 0.4 - 2.5x10^2 x$$

So

$$\underline{n = n_i \exp\left(\frac{0.4 - 2.5x10^2 x}{kT}\right)}$$

(b)

$$J_n = eD_n \frac{dn}{dx}$$

$$= eD_n n_i \left(\frac{-2.5x10^2}{kT}\right)\exp\left(\frac{0.4 - 2.5x10^2 x}{kT}\right)$$

Assume $T = 300K$, $kT = 0.0259\ eV$, and

$n_i = 1.5x10^{10}\ cm^{-3}$

Then

$$J_n = \frac{-\left(1.6x10^{-19}\right)(25)\left(1.5x10^{10}\right)\left(2.5x10^2\right)}{(0.0259)} \times \exp\left(\frac{0.4 - 2.5x10^2 x}{0.0259}\right)$$

or

$$J_n = -5.79x10^{-4} \exp\left(\frac{0.4 - 2.5x10^2 x}{0.0259}\right)$$

(i) At $x = 0$, $\underline{J_n = -2.95x10^3\ A/cm^2}$

(ii) At $x = 5\ \mu m$, $\underline{J_n = -23.7\ A/cm^2}$

5.32

(a) $J_n = e\mu_n n\text{E} + eD_n \dfrac{dn}{dx}$

$$-80 = \left(1.6x10^{-19}\right)(1000)\left(10^{16}\right)\left(1 - \frac{x}{L}\right)\text{E} + \left(1.6x10^{-19}\right)(25.9)\left(\frac{-10^{16}}{L}\right)$$

where $L = 10x10^{-4} = 10^{-3}\ cm$

We find

$$-80 = 1.6\text{E} - 1.6\left(\frac{x}{10^{-3}}\right)\text{E} - 41.44$$

or

$$80 = 1.6\left(\frac{x}{L} - 1\right)\text{E} + 41.44$$

Solving for the electric field, we find

$$\underline{\text{E} = \frac{38.56}{\left(\frac{x}{L} - 1\right)}}$$

(b)

For $J_n = -20\ A/cm^2$

$$20 = 1.6\left(\frac{x}{L} - 1\right)\text{E} + 41.44$$

Then

$$\underline{\text{E} = \frac{21.44}{\left(1 - \frac{x}{L}\right)}}$$

5.33

(a) $J = e\mu_n n\text{E} + eD_n \dfrac{dn}{dx}$

Let $n = N_d = N_{do} \exp(-\alpha x)$, $J = 0$

Then

$$0 = \mu_n N_{do}\left[\exp(-\alpha x)\right]\text{E} + D_n N_{do}(-\alpha)\exp(-\alpha x)$$

or

$$0 = \text{E} + \frac{D_n}{\mu_n}(-\alpha)$$

Since $\dfrac{D_n}{\mu_n} = \dfrac{kT}{e}$

So

$$\underline{\text{E} = \alpha\left(\frac{kT}{e}\right)}$$

(b)

$$V = -\int_0^{1/\alpha} \text{E}dx = -\alpha\left(\frac{kT}{e}\right)\int_0^{1/\alpha} dx$$

$$= -\left[\alpha\left(\frac{kT}{e}\right)\right]\cdot\left(\frac{1}{\alpha}\right) \text{ so that } \underline{V = -\left(\frac{kT}{e}\right)}$$

5.34

From Example 5.5

$$\text{E}_x = \frac{(0.0259)\left(10^{19}\right)}{\left(10^{16} - 10^{19} x\right)} = \frac{(0.0259)\left(10^3\right)}{\left(1 - 10^3 x\right)}$$

$$V = -\int_0^{10^{-4}} \text{E}_x dx = -(0.0259)\left(10^3\right)\int_0^{10^{-4}} \frac{dx}{\left(1 - 10^3 x\right)}$$

$$= -(0.0259)\left(10^3\right)\left(\frac{-1}{10^3}\right)\ln\left[1-10^3 x\right]_0^{10^{-4}}$$

$$= (0.0259)\left[\ln(1-0.1)-\ln(1)\right]$$

or

$$\underline{V = -2.73\ mV}$$

5.35

From Equation [5.40]

$$\mathrm{E}_x = -\left(\frac{kT}{e}\right)\left(\frac{1}{N_d(x)}\right)\cdot\frac{dN_d(x)}{dx}$$

Now

$$1000 = -(0.0259)\left(\frac{1}{N_d(x)}\right)\cdot\frac{dN_d(x)}{dx}$$

or

$$\frac{dN_d(x)}{dx} + 3.86x10^4\, N_d(x) = 0$$

Solution is of the form

$$N_d(x) = A\exp(-\alpha x)$$

and

$$\frac{dN_d(x)}{dx} = -A\alpha\exp(-\alpha x)$$

Substituting into the differential equation

$$-A\alpha\exp(-\alpha x) + 3.86x10^4 A\exp(-\alpha x) = 0$$

which yields

$$\underline{\alpha = 3.86x10^4\ cm^{-1}}$$

At $x = 0$, the actual value of $N_d(0)$ is arbitrary.

5.36

(a) $J_n = J_{drf} + J_{dif} = 0$

$$J_{dif} = eD_n\frac{dn}{dx} = eD_n\frac{dN_d(x)}{dx}$$

$$= \frac{eD_n}{(-L)}\cdot N_{do}\exp\left(\frac{-x}{L}\right)$$

We have

$$D_n = \mu_n\left(\frac{kT}{e}\right) = (6000)(0.0259) = 155.4\ cm^2/s$$

Then

$$J_{dif} = \frac{-\left(1.6x10^{-19}\right)(155.4)\left(5x10^{16}\right)}{\left(0.1x10^{-4}\right)}\exp\left(\frac{-x}{L}\right)$$

or

$$\underline{J_{dif} = -1.24x10^5\exp\left(\frac{-x}{L}\right)\quad A/cm^2}$$

(b)

$$0 = J_{drf} + J_{dif}$$

Now

$$J_{drf} = e\mu_n n\mathrm{E}$$

$$= \left(1.6x10^{-19}\right)(6000)\left(5x10^{16}\right)\left[\exp\left(\frac{-x}{L}\right)\right]\mathrm{E}$$

$$= 48\mathrm{E}\exp\left(\frac{-x}{L}\right)$$

We have

$$J_{drf} = -J_{dif}$$

so

$$48\mathrm{E}\exp\left(\frac{-x}{L}\right) = 1.24x10^5\exp\left(\frac{-x}{L}\right)$$

which yields

$$\underline{\mathrm{E} = 2.58x10^3\ V/cm}$$

5.37

Computer Plot

5.38

(a) $D = \mu\left(\frac{kT}{e}\right) = (925)(0.0259)$

so

$$\underline{D = 23.96\ cm^2/s}$$

(b)

For $D = 28.3\ cm^2/s$

$$\mu = \frac{28.3}{0.0259} \Rightarrow \underline{\mu = 1093\ cm^2/V-s}$$

5.39

We have $L = 10^{-1}\ cm = 10^{-3}\ m$,

$$W = 10^{-2}\ cm = 10^{-4}\ m,\ d = 10^{-3}\ cm = 10^{-5}\ m$$

(a)

We have

$$p = 10^{16}\ cm^{-3} = 10^{22}\ m^{-3},\ I_x = 1\ mA = 10^{-3}\ A$$

Then

$$V_H = \frac{I_x B_z}{epd} = \frac{\left(10^{-3}\right)\left(3.5x10^{-2}\right)}{\left(1.6x10^{-19}\right)\left(10^{22}\right)\left(10^{-5}\right)}$$

or

$$\underline{V_H = 2.19\ mV}$$

(b)

$$E_H = \frac{V_H}{W} = \frac{2.19x10^{-3}}{10^{-2}}$$

or

$$\underline{E_H = 0.219\ V/cm}$$

5.40

(a) $$V_H = \frac{-I_x B_z}{ned} = \frac{-(250x10^{-6})(5x10^{-2})}{(5x10^{21})(1.6x10^{-19})(5x10^{-5})}$$

or

$$\underline{V_H = -0.3125\ mV}$$

(b)

$$E_H = \frac{V_H}{W} = \frac{-0.3125x10^{-3}}{2x10^{-2}} \Rightarrow$$

$$\underline{E_H = -1.56x10^{-2}\ V/cm}$$

(c)

$$\mu_n = \frac{I_x L}{enV_x Wd}$$

$$= \frac{(250x10^{-6})(10^{-3})}{(1.6x10^{-19})(5x10^{21})(0.1)(2x10^{-4})(5x10^{-5})}$$

or

$$\underline{\mu_n = 0.3125\ m^2/V-s = 3125\ cm^2/V-s}$$

5.41

(a) V_H = positive $\Rightarrow$ p-type

(b)

$$V_H = \frac{I_x B_z}{epd} \Rightarrow p = \frac{I_x B_z}{eV_H d}$$

$$= \frac{(0.75x10^{-3})(10^{-1})}{(1.6x10^{-19})(5.8x10^{-3})(10^{-5})}$$

or

$$\underline{p = 8.08x10^{21}\ m^{-3} = 8.08x10^{15}\ cm^{-3}}$$

(c)

$$\mu_p = \frac{I_x L}{epV_x Wd}$$

$$= \frac{(0.75x10^{-3})(10^{-3})}{(1.6x10^{-19})(8.08x10^{21})(15)(10^{-4})(10^{-5})}$$

or

$$\underline{\mu_p = 3.87x10^{-2}\ m^2/V-s = 387\ cm^2/V-s}$$

5.42

(a) $V_H = E_H W = -(16.5x10^{-3})(5x10^{-2})$

or

$$\underline{V_H = -0.825\ mV}$$

(b)

V_H = negative $\Rightarrow$ <u>n-type</u>

(c)

$$n = \frac{-I_x B_z}{edV_H}$$

$$= \frac{-(0.5x10^{-3})(6.5x10^{-2})}{(1.6x10^{-19})(5x10^{-5})(-0.825x10^{-3})}$$

or

$$\underline{n = 4.92x10^{21}\ m^{-3} = 4.92x10^{15}\ cm^{-3}}$$

(d)

$$\mu_n = \frac{I_x L}{enV_x Wd}$$

$$= \frac{(0.5x10^{-3})(0.5x10^{-2})}{(1.6x10^{-19})(4.92x10^{21})(1.25)(5x10^{-4})(5x10^{-5})}$$

or

$$\underline{\mu_n = 0.102\ m^2/V-s = 1020\ cm^2/V-s}$$

5.43

(a) V_H = negative $\Rightarrow$ <u>n-type</u>

(b) $n = \dfrac{-I_x B_z}{edV_H} \Rightarrow \underline{n = 8.68x10^{14}\ cm^{-3}}$

(c) $\mu_n = \dfrac{I_x L}{enV_x Wd} \Rightarrow \underline{\mu_n = 8182\ cm^2/V-s}$

(d) $\sigma = \dfrac{1}{\rho} = e\mu_n n = (1.6x10^{-19})(8182)(8.68x10^{14})$

or $\underline{\rho = 0.88\ (\Omega-cm)}$

Chapter 6

Problem Solutions

6.1

n-type semiconductor, low-injection so that

$$R' = \frac{\delta p}{\tau_{pO}} = \frac{5x10^{13}}{10^{-6}}$$

or

$$\underline{R' = 5x10^{19}\ cm^{-3}s^{-1}}$$

6.2

(a) $R_{nO} = \dfrac{n_O}{\tau_{nO}}$

and

$$n_O = \frac{n_i^2}{p_O} = \frac{\left(10^{10}\right)^2}{10^{16}} = 10^4\ cm^{-3}$$

Then

$$R_{nO} = \frac{10^4}{2x10^{-7}} \Rightarrow \underline{R_{nO} = 5x10^{10}\ cm^{-3}s^{-1}}$$

(b)

$$R_n = \frac{\delta n}{\tau_{nO}} = \frac{10^{12}}{2x10^{-7}} \text{ or } R_n = 5x10^{18}\ cm^{-3}s^{-1}$$

so

$$\Delta R_n = R_n - R_{nO} = 5x10^{18} - 5x10^{10} \Rightarrow$$

$$\underline{\Delta R_n \approx 5x10^{18}\ cm^{-3}s^{-1}}$$

6.3

(a) Recombination rates are equal

$$\frac{n_O}{\tau_{nO}} = \frac{p_O}{\tau_{pO}}$$

$$n_O = N_d = 10^{16}\ cm^{-3}$$

$$p_O = \frac{n_i^2}{n_O} = \frac{\left(1.5x10^{10}\right)^2}{10^{16}} = 2.25x10^4\ cm^{-3}$$

So

$$\frac{10^{16}}{\tau_{nO}} = \frac{2.25x10^4}{20x10^{-6}}$$

or

$$\underline{\tau_{nO} = 8.89x10^{+6}\ s}$$

(b) Generation Rate = Recombination Rate

So

$$G = \frac{2.25x10^4}{20x10^{-6}} \Rightarrow \underline{G = 1.125x10^9\ cm^{-3}s^{-1}}$$

(c)

$$\underline{R = G = 1.125x10^9\ cm^{-3}s^{-1}}$$

6.4

(a) $E = h\nu = \dfrac{hc}{\lambda} = \dfrac{\left(6.625x10^{-34}\right)\left(3x10^8\right)}{6300x10^{-10}}$

or

$E = 3.15x10^{-19}\ J$ This is the energy of 1 photon.

Now

$1\ W = 1\ J/s \Rightarrow$ $3.17x10^{18}$ photons/s

Volume = $(1)(0.1) = 0.1\ cm^{+3}$

Then

$$g = \frac{3.17x10^{18}}{0.1} \Rightarrow$$

$$\underline{g = 3.17x10^{19}\ e-h\ pairs/cm^3 - s}$$

(b)

$$\delta n = \delta p = g\tau = \left(3.17x10^{19}\right)\left(10x10^{-6}\right)$$

or

$$\underline{\delta n = \delta p = 3.17x10^{14}\ cm^{-3}}$$

6.5

We have

$$\frac{\partial p}{\partial t} = -\nabla \bullet F_p^+ + g_p - \frac{p}{\tau_p}$$

and

$$J_p = e\mu_p p\mathrm{E} - eD_p \nabla p$$

The hole particle current density is

$$F_p^+ = \frac{J_p}{(+e)} = \mu_p p\mathrm{E} - D_p \nabla p$$

Now

$$\nabla \bullet F_p^+ = \mu_p \nabla \bullet (p\mathrm{E}) - D_p \nabla \bullet \nabla p$$

We can write

$$\nabla \bullet (p\mathrm{E}) = \mathrm{E} \bullet \nabla p + p\nabla \bullet \mathrm{E}$$

and

$$\nabla \bullet \nabla p = \nabla^2 p$$

so

$$\nabla \bullet F_p^+ = \mu_p(\mathrm{E} \bullet \nabla p + p\nabla \bullet \mathrm{E}) - D_p \nabla^2 p$$

Then

$$\frac{\partial p}{\partial t} = -\mu_p(\mathrm{E} \bullet \nabla p + p\nabla \bullet \mathrm{E}) + D_p \nabla^2 p + g_p - \frac{p}{\tau_p}$$

We can then write

$$D_p \nabla^2 p - \mu_p(\mathrm{E} \bullet \nabla p + p\nabla \bullet \mathrm{E}) + g_p - \frac{p}{\tau_p} = \frac{\partial p}{\partial t}$$

6.6

From Equation [6.18]

$$\frac{\partial p}{\partial t} = -\nabla \bullet F_p^+ + g_p - \frac{p}{\tau_p}$$

For steady-state, $\frac{\partial p}{\partial t} = 0$

Then

$$0 = -\nabla \bullet F_p^+ + g_p - R_p$$

and for a one-dimensional case,

$$\frac{dF_p^+}{dx} = g_p - R_p = 10^{20} - 2x10^{19} \Rightarrow$$

$$\underline{\frac{dF_p^+}{dx} = 8x10^{19}\ cm^{-3}s^{-1}}$$

6.7

From Equation [6.18],

$$0 = -\frac{dF_p^+}{dx} + 0 - 2x10^{19}$$

or

$$\underline{\frac{dF_p^+}{dx} = -2x10^{19}\ cm^{-3}s^{-1}}$$

6.8

We have the continuity equations

(1) $$D_p \nabla^2(\delta p) - \mu_p\left[\mathrm{E} \bullet \nabla(\delta p) + p\nabla \bullet \mathrm{E}\right] + g_p - \frac{p}{\tau_p} = \frac{\partial(\delta p)}{\partial t}$$

and

(2) $$D_n \nabla^2(\delta n) + \mu_n\left[\mathrm{E} \bullet \nabla(\delta n) + n\nabla \bullet \mathrm{E}\right] + g_n - \frac{n}{\tau_n} = \frac{\partial(\delta n)}{\partial t}$$

By charge neutrality

$$\delta n = \delta p \equiv \delta n \Rightarrow \nabla(\delta n) = \nabla(\delta p)$$

and $\nabla^2(\delta n) = \nabla^2(\delta p)$ and $\frac{\partial(\delta n)}{\partial t} = \frac{\partial(\delta p)}{\partial t}$

Also

$$g_n = g_p \equiv g\ ,\ \frac{p}{\tau_p} = \frac{n}{\tau_n} \equiv R$$

Then we can write

(1) $$D_p \nabla^2(\delta n) - \mu_p\left[\mathrm{E} \bullet \nabla(\delta n) + p\nabla \bullet \mathrm{E}\right] + g - R = \frac{\partial(\delta n)}{\partial t}$$

and

(2) $$D_n \nabla^2(\delta n) + \mu_n\left[\mathrm{E} \bullet \nabla(\delta n) + n\nabla \bullet \mathrm{E}\right] + g - R = \frac{\partial(\delta n)}{\partial t}$$

Multiply Equation (1) by $\mu_n n$ and Equation (2) by $\mu_p p$, and then add the two equations.

We find

$$\left(\mu_n n D_p + \mu_p p D_n\right)\nabla^2(\delta n) + \mu_n \mu_p (p - n)\mathrm{E} \bullet \nabla(\delta n) + \left(\mu_n n + \mu_p p\right)(g - R) = \left(\mu_n n + \mu_p p\right)\frac{\partial(\delta n)}{\partial t}$$

Divide by $\left(\mu_n n + \mu_p p\right)$, then

$$\left(\frac{\mu_n n D_p + \mu_p p D_n}{\mu_n n + \mu_p p}\right)\nabla^2(\delta n) + \left[\frac{\mu_n \mu_p (p - n)}{\mu_n n + \mu_p p}\right]\mathrm{E} \bullet \nabla(\delta n) + (g - R) = \frac{\partial(\delta n)}{\partial t}$$

Define

$$D' = \frac{\mu_n n D_p + \mu_p p D_n}{\mu_n n + \mu_p p} = \frac{D_n D_p (n + p)}{D_n n + D_p p}$$

and $$\mu' = \frac{\mu_n \mu_p (p - n)}{\mu_n n + \mu_p p}$$

Then we have

$$\underline{D' \nabla^2(\delta n) + \mu' \mathrm{E} \bullet \nabla(\delta n) + (g - R) = \frac{\partial(\delta n)}{\partial t}}$$

Q.E.D.

6.9

For Ge: $T = 300K$, $n_i = 2.4x10^{13}\ cm^{-3}$

$$n = \frac{N_d}{2} + \sqrt{\left(\frac{N_d}{2}\right)^2 + n_i^2}$$

$$= 10^{13} + \sqrt{\left(10^{13}\right)^2 + \left(2.4x10^{13}\right)^2}$$

or

$$n = 3.6x10^{13}\ cm^{-3}$$

Also

$$p = \frac{n_i^2}{n} = \frac{\left(2.4x10^{13}\right)^2}{3.6x10^{13}} = 1.6x10^{13}\ cm^{-3}$$

We have

$$\mu_n = 3900\ ,\ \mu_p = 1900$$

$$D_n = 101\ ,\ D_p = 49.2$$

Now

$$D' = \frac{D_n D_p (n+p)}{D_n n + D_p p}$$

$$= \frac{(101)(49.2)\left(3.6x10^{13} + 1.6x10^{13}\right)}{(101)\left(3.6x10^{13}\right) + (49.2)\left(1.6x10^{13}\right)}$$

or

$$\underline{D' = 58.4\ cm^2/s}$$

Also

$$\mu' = \frac{\mu_n \mu_p (p-n)}{\mu_n n + \mu_p p}$$

$$= \frac{(3900)(1900)\left(1.6x10^{13} - 3.6x10^{13}\right)}{(3900)\left(3.6x10^{13}\right) + (1900)\left(1.6x10^{13}\right)}$$

or

$$\underline{\mu' = -868\ cm^2/V-s}$$

Now

$$\frac{n}{\tau_n} = \frac{p}{\tau_p} \Rightarrow \frac{3.6x10^{13}}{\tau_n} = \frac{1.6x10^{13}}{24\ \mu s}$$

which yields

$$\underline{\tau_n = 54\ \mu s}$$

6.10

$$\sigma = e\mu_n n + e\mu_p p$$

With excess carriers present

$$n = n_O + \delta n \quad \text{and} \quad p = p_O + \delta p$$

For an n-type semiconductor, we can write

$$\delta n = \delta p \equiv \delta p$$

Then

$$\sigma = e\mu_n\left(n_O + \delta p\right) + e\mu_p\left(p_O + \delta p\right)$$

or

$$\sigma = e\mu_n n_O + e\mu_p p_O + e\left(\mu_n + \mu_p\right)(\delta p)$$

so

$$\underline{\Delta\sigma = e\left(\mu_n + \mu_p\right)(\delta p)}$$

In steady-state, $\delta p = g'\tau$

So that

$$\underline{\Delta\sigma = e\left(\mu_n + \mu_p\right)\left(g'\tau_{pO}\right)}$$

6.11

n-type, so that minority carriers are holes. Uniform generation throughout the sample means we have

$$g' - \frac{\delta p}{\tau_{pO}} = \frac{\partial(\delta p)}{\partial t}$$

Homogeneous solution is of the form

$$(\delta p)_H = A\exp\left(\frac{-t}{\tau_{pO}}\right)$$

and the particular solution is

$$(\delta p)_P = g'\tau_{pO}$$

so that the total solution is

$$(\delta p) = g'\tau_{pO} + A\exp\left(\frac{-t}{\tau_{pO}}\right)$$

At $t = 0$, $\delta p = 0$ so that

$$0 = g'\tau_{pO} + A \Rightarrow A = -g'\tau_{pO}$$

Then

$$\delta p = g'\tau_{pO}\left[1 - \exp\left(\frac{-t}{\tau_{pO}}\right)\right]$$

The conductivity is

$$\sigma = e\mu_n n_O + e\mu_p p_O + e\left(\mu_n + \mu_p\right)(\delta p)$$

$$\approx e\mu_n n_O + e\left(\mu_n + \mu_p\right)(\delta p)$$

so

$$\sigma = \left(1.6x10^{-19}\right)(1000)\left(5x10^{16}\right)$$

$$+\left(1.6x10^{-19}\right)(1000+420)\left(5x10^{21}\right)\left(10^{-7}\right)$$

$$\times\left[1 - \exp\left(\frac{-t}{\tau_{pO}}\right)\right]$$

Then

$$\sigma = 8 + 0.114\left[1 - \exp\left(\frac{-t}{\tau_{pO}}\right)\right]$$

where $\tau_{pO} = 10^{-7}\ s$

6.12
n-type GaAs:

$$\Delta\sigma = e(\mu_n + \mu_p)(\delta p)$$

In steady-state, $\delta p = g'\tau_{pO}$. Then

$$\Delta\sigma = (1.6x10^{-19})(8500 + 400)(2x10^{21})(2x10^{-7})$$

or

$$\Delta\sigma = 0.57\ (\Omega - cm)^{-1}$$

The steady-state excess carrier recombination rate

$$R' = g' = 2x10^{21}\ cm^{-3}s^{-1}$$

6.13
For $t < 0$, steady-state, so

$$\delta p(0) = g'\tau_{pO} = (5x10^{21})(3x10^{-7}) \Rightarrow$$

$$\delta p(0) = 1.5x10^{15}\ cm^{-3}$$

Now

$$\sigma = e\mu_n n_O + e(\mu_n + \mu_p)(\delta p)$$

For $t \geq 0$, $\delta p = \delta p(0)\exp(-t/\tau_{pO})$

Then

$$\sigma = (1.6x10^{-19})(1350)(5x10^{16})$$
$$+(1.6x10^{-19})(1350 + 480)(1.5x10^{15})\exp(-t/\tau_{pO})$$

or

$$\sigma = 10.8 + 0.439\exp(-t/\tau_{pO})$$

We have that

$$I = AJ = A\sigma E = \frac{A\sigma V}{L}$$

so

$$I = \frac{(10^{-4})(5)}{(0.10)}[10.8 + 0.439\exp(-t/\tau_{pO})]$$

or

$$I = [54 + 2.20\exp(-t/\tau_{pO})]\ mA$$

where

$$\tau_{pO} = 3x10^{-7}\ s$$

6.14
(a) p-type GaAs,

$$D_n\nabla^2(\delta n) + \mu_n E \bullet \nabla(\delta n) + g' - \frac{\delta n}{\tau_{nO}} = \frac{\partial(\delta n)}{\partial t}$$

Uniform generation rate, so that

$\nabla(\delta n) = \nabla^2(\delta n) = 0$, then

$$g' - \frac{\delta n}{\tau_{nO}} = \frac{\partial(\delta n)}{\partial t}$$

The solution is of the form

$$\delta n = g'\tau_{nO}[1 - \exp(-t/\tau_{nO})]$$

Now

$$R_n' = \frac{\delta n}{\tau_{nO}} = g'[1 - \exp(-t/\tau_{nO})]$$

(b)
Maximum value at steady-state, $n_O = 10^{14}\ cm^{-3}$
So

$$(\delta n)_O = g'\tau_{nO} \Rightarrow \tau_{nO} = \frac{(\delta n)_O}{g'} = \frac{10^{14}}{10^{20}}$$

or

$$\tau_{nO} = 10^{-6}\ s$$

(c)
Determine t at which

(i) $\delta n = (0.75)x10^{14}\ cm^{-3}$

We have

$$0.75x10^{14} = 10^{14}[1 - \exp(-t/\tau_{nO})]$$

which yields

$$t = \tau_{nO}\ln\left(\frac{1}{1 - 0.75}\right) \Rightarrow t = 1.39\ \mu s$$

(ii) $\delta n = 0.5x10^{14}\ cm^{-3}$

We find

$$t = \tau_{nO}\ln\left(\frac{1}{1 - 0.5}\right) \Rightarrow t = 0.693\ \mu s$$

(iii) $\delta n = 0.25x10^{14}\ cm^{-3}$

We find

$$t = \tau_{nO}\ln\left(\frac{1}{1 - 0.25}\right) \Rightarrow t = 0.288\ \mu s$$

6.15
(a)

$$p_O = \frac{n_i^2}{n_O} = \frac{(1.5x10^{10})^2}{10^{15}} 2.25x10^4\ cm^{-3}$$

Then

$$R_{pO} = \frac{p_O}{\tau_{pO}} \Rightarrow \tau_{pO} = \frac{p_O}{R_{pO}} = \frac{2.25x10^4}{10^{11}}$$

or

$$\tau_{pO} = 2.25x10^{-7}\ s$$

Now

$$R'_p = \frac{\delta p}{\tau_{pO}} = \frac{10^{14}}{2.25x10^{-7}} \Rightarrow$$

or

$$R'_p = 4.44x10^{20}\ cm^{-3}s^{-1}$$

Recombination rate increases by the factor

$$\frac{R'_p}{R_{pO}} = \frac{4.44x10^{20}}{10^{11}} \Rightarrow \frac{R'_p}{R_{pO}} = 4.44x10^9$$

(b)

From part (a), $\tau_{pO} = 2.25x10^{-7}\ s$

6.16

Silicon, n-type. For $0 \le t \le 10^{-7}\ s$

$$\delta p = g'\tau_{pO}\left[1-\exp(-t/\tau_{pO})\right]$$
$$= \left(2x10^{20}\right)\left(10^{-7}\right)\left[1-\exp(-t/\tau_{pO})\right]$$

or

$$\delta p = 2x10^{13}\left[1-\exp(-t/\tau_{pO})\right]$$

At $t = 10^{-7}\ s$,

$$\delta p\left(10^{-7}\right) = 2x10^{13}\left[1-\exp(-1)\right]$$

or

$$\delta p\left(10^{-7}\right) = 1.26x10^{13}\ cm^{-3}$$

For $t > 10^{-7}\ s$,

$$\delta p = \left(1.26x10^{13}\right)\exp\left[\frac{-\left(t-10^{-7}\right)}{\tau_{pO}}\right]$$

where

$$\tau_{pO} = 10^{-7}\ s$$

6.17

(a) For $0 < t < 2x10^{-6}\ s$

$$\delta n = g'\tau_{nO}\left[1-\exp(-t/\tau_{nO})\right]$$
$$= \left(10^{20}\right)\left(10^{-6}\right)\left[1-\exp[-t/\tau_{nO}]\right]$$

or

$$\delta n = 10^{14}\left[1-\exp(-t/\tau_{nO})\right]$$

where $\tau_{nO} = 10^{-6}\ s$

At $t = 2x10^{-6}\ s$

$$\delta n(2\ \mu s) = \left(10^{14}\right)\left[1-\exp(-2/1)\right]$$

or

$$\delta n(2\ \mu s) = 0.865x10^{14}\ cm^{-3}$$

For $t > 2x10^{-6}\ s$

$$\delta n = 0.865x10^{14}\exp\left[\frac{-\left(t-2x10^{-6}\right)}{\tau_{nO}}\right]$$

(b) (i) At $t = 0$, $\delta n = 0$

(ii) At $t = 2x10^{-6}\ s$, $\delta n = 0.865x10^{14}\ cm^{-3}$

(iii) At $t \to \infty$, $\delta n = 0$

6.18

p-type, minority carriers are electrons

In steady-state, $\dfrac{\partial(\delta n)}{\partial t} = 0$, then

(a)

$$D_n\frac{d^2(\delta n)}{dx^2} - \frac{\delta n}{\tau_{nO}} = 0$$

or

$$\frac{d^2(\delta n)}{dx^2} - \frac{\delta n}{L_n^2} = 0$$

Solution is of the form

$$\delta n = A\exp(-x/L_n) + B\exp(+x/L_n)$$

But $\delta n = 0$ as $x \to \infty$ so that $B \equiv 0$.

At $x = 0$, $\delta n = 10^{13}\ cm^{-3}$

Then

$$\delta n = 10^{13}\exp(-x/L_n)$$

Now

$$L_n = \sqrt{D_n\tau_{nO}}\ \text{, where } D_n = \mu_n\left(\frac{kT}{e}\right)$$

or

$$D_n = (0.0259)(1200) = 31.1\ cm^2/s$$

Then

$$L_n = \sqrt{(31.1)\left(5x10^{-7}\right)} \Rightarrow$$

or

$$L_n = 39.4\ \mu m$$

(b)

$$J_n = eD_n \frac{d(\delta n)}{dx} = \frac{eD_n(10^{13})}{(-L_n)} \exp(-x/L_n)$$

$$= \frac{-(1.6x10^{-19})(31.1)(10^{13})}{39.4x10^{-4}} \exp(-x/L_n)$$

or

$$\underline{J_n = -12.6 \exp(-x/L_n) \quad mA/cm^2}$$

6.19

(a) p-type silicon, $p_{pO} = 10^{14}\ cm^{-3}$ and

$$n_{pO} = \frac{n_i^2}{p_{pO}} = \frac{(1.5x10^{10})^2}{10^{14}} = 2.25x10^6\ cm^{-3}$$

(b) Excess minority carrier concentration

$\delta n = n_p - n_{pO}$

At $x = 0$, $n_p = 0$ so that

$\delta n(0) = 0 - n_{pO} = -2.25x10^6\ cm^{-3}$

(c) For the one-dimensional case,

$$D_n \frac{d^2(\delta n)}{dx^2} - \frac{\delta n}{\tau_{nO}} = 0$$

or

$$\frac{d^2(\delta n)}{dx^2} - \frac{\delta n}{L_n^2} = 0 \quad \text{where} \quad L_n^2 = D_n \tau_{nO}$$

The general solution is of the form

$\delta n = A \exp(-x/L_n) + B \exp(+x/L_n)$

For $x \to \infty$, δn remains finite, so that $B = 0$.
Then the solution is

$$\underline{\delta n = -n_{pO} \exp(-x/L_n)}$$

6.20

p-type so electrons are the minority carriers

$$D_n \nabla^2(\delta n) + \mu_n \mathrm{E} \bullet \nabla(\delta n) + g' - \frac{\delta n}{\tau_{nO}} = \frac{\partial(\delta n)}{\partial t}$$

For steady state, $\frac{\partial(\delta n)}{\partial t} = 0$ and for $x > 0$,
$g' = 0$, $\mathrm{E} = 0$, so we have

$$D_n \frac{d^2(\delta n)}{dx^2} - \frac{\delta n}{\tau_{nO}} = 0 \quad \text{or} \quad \frac{d^2(\delta n)}{dx^2} - \frac{\delta n}{L_n^2} = 0$$

where $L_n^2 = D_n \tau_{nO}$

The solution is of the form

$\delta n = A \exp(-x/L_n) + B \exp(+x/L_n)$

The excess concentration δn must remain finite, so that $B = 0$. At $x = 0$, $\delta n(0) = 10^{15}\ cm^{-3}$, so the solution is

$\delta n = 10^{15} \exp(-x/L_n)$

We have that $\mu_n = 1050\ cm^2/V-s$, then

$$D_n = \mu_n \left(\frac{kT}{e}\right) = (1050)(0.0259) = 27.2\ cm^2/s$$

Then

$$L_n = \sqrt{D_n \tau_{nO}} = \sqrt{(27.2)(8x10^{-7})} \Rightarrow$$

$L_n = 46.6\ \mu m$

(a)

Electron diffusion current density at $x = 0$

$$J_n = eD_n \frac{d(\delta n)}{dx}\bigg|_{x=0}$$

$$= eD_n \frac{d}{dx}\left[10^{15} \exp(-x/L_n)\right]_{x=0}$$

$$= \frac{-eD_n(10^{15})}{L_n} = \frac{-(1.6x10^{-19})(27.2)(10^{15})}{46.6x10^{-4}}$$

or

$$\underline{J_n = -0.934\ A/cm^2}$$

Since $\delta p = \delta n$, excess holes diffuse at the same rate as excess electrons, then

$$\underline{J_p(x = 0) = +0.934\ A/cm^2}$$

(b)

At $x = L_n$,

$$J_n = eD_n \frac{d(\delta n)}{dx}\bigg|_{x=L_n} = \frac{eD_n(10^{15})}{(-L_n)} \exp(-1)$$

$$= \frac{-(1.6x10^{-19})(27.2)(10^{15})}{46.6x10^{-4}} \exp(-1)$$

or

$$\underline{J_n = -0.344\ A/cm^2}$$

Then

$$\underline{J_p = +0.344\ A/cm^2}$$

6.21

n-type, so we have

$$D_p \frac{d^2(\delta p)}{dx^2} - \mu_p \mathrm{E}_O \frac{d(\delta p)}{dx} - \frac{\delta p}{\tau_{pO}} = 0$$

Assume the solution is of the form

$\delta p = A \exp(sx)$

Then

$$\frac{d(\delta p)}{dx} = As\exp(sx)\,,\quad \frac{d^2(\delta p)}{dx^2} = As^2\exp(sx)$$

Substituting into the differential equation

$$D_p As^2\exp(sx) - \mu_p \mathrm{E}_O As\exp(sx) - \frac{A\exp(sx)}{\tau_{pO}} = 0$$

or

$$D_p s^2 - \mu_p \mathrm{E}_O s - \frac{1}{\tau_{pO}} = 0$$

Dividing by D_p

$$s^2 - \frac{\mu_p}{D_p}\mathrm{E}_O s - \frac{1}{L_p^2} = 0$$

The solution for s is

$$s = \frac{1}{2}\left[\frac{\mu_p}{D_p}\mathrm{E}_O \pm \sqrt{\left(\frac{\mu_p}{D_p}\mathrm{E}_O\right)^2 + \frac{4}{L_p^2}}\right]$$

This can be rewritten as

$$s = \frac{1}{L_p}\left[\frac{\mu_p L_p \mathrm{E}_O}{2D_p} \pm \sqrt{\left(\frac{\mu_p L_p \mathrm{E}_O}{2D_p}\right)^2 + 1}\right]$$

We may define

$$\beta \equiv \frac{\mu_p L_p \mathrm{E}_O}{2D_p}$$

Then

$$s = \frac{1}{L_p}\left[\beta \pm \sqrt{1+\beta^2}\right]$$

In order that $\delta p = 0$ for $x > 0$, use the minus sign for $x > 0$ and the plus sign for $x < 0$. Then the solution is

$$\underline{\delta p(x) = A\exp(s_- x)} \text{ for } x > 0$$

$$\underline{\delta p(x) = A\exp(s_+ x)} \text{ for } x < 0$$

where

$$\underline{s_\pm = \frac{1}{L_p}\left[\beta \pm \sqrt{1+\beta^2}\right]}$$

6.22
Computer Plot

6.23
(a) From Equation [6.55],

$$D_n \frac{d^2(\delta n)}{dx^2} + \mu_n \mathrm{E}_O \frac{d(\delta n)}{dx} - \frac{\delta n}{\tau_{nO}} = 0$$

or

$$\frac{d^2(\delta n)}{dx^2} + \frac{\mu_n}{D_n}\mathrm{E}_O \frac{d(\delta n)}{dx} - \frac{\delta n}{L_n^2} = 0$$

We have that

$$\frac{D_n}{\mu_n} = \left(\frac{kT}{e}\right) \text{ so we can define}$$

$$\frac{\mu_n}{D_n}\mathrm{E}_O = \frac{\mathrm{E}_O}{(kT/e)} \equiv \frac{1}{L'}$$

Then we can write

$$\frac{d^2(\delta n)}{dx^2} + \frac{1}{L'}\cdot\frac{d(\delta n)}{dx} - \frac{\delta n}{L_n^2} = 0$$

Solution will be of the form

$$\delta n = \delta n(0)\exp(-\alpha x) \quad \text{where} \quad \alpha > 0$$

Then

$$\frac{d(\delta n)}{dx} = -\alpha(\delta n) \quad \text{and} \quad \frac{d^2(\delta n)}{dx^2} = \alpha^2(\delta n)$$

Substituting into the differential equation, we have

$$\alpha^2(\delta n) + \frac{1}{L'}\cdot[-\alpha(\delta n)] - \frac{\delta n}{L_n^2} = 0$$

or

$$\alpha^2 - \frac{\alpha}{L'} - \frac{1}{L_n^2} = 0$$

which yields

$$\alpha = \frac{1}{L_n}\left\{\frac{L_n}{2L'} + \sqrt{\left(\frac{L_n}{2L'}\right)^2 + 1}\right\}$$

Note that if $\mathrm{E}_O = 0$, $L' \to \infty$, then $\alpha = \dfrac{1}{L_n}$

(b)

$$L_n = \sqrt{D_n \tau_{nO}} \quad \text{where} \quad D_n = \mu_n\left(\frac{dT}{e}\right)$$

or

$$D_n = (1200)(0.0259) = 31.1\ cm^2/s$$

Then

$$L_n = \sqrt{(31.1)(5x10^{-7})} = 39.4\ \mu m$$

For $\mathrm{E}_O = 12\ V/cm$, then

$$L' = \frac{(kT/e)}{E_O} = \frac{0.0259}{12} = 21.6x10^{-4}\ cm$$

Then

$$\alpha = 5.75x10^{2}\ cm^{-1}$$

(c)
Force on the electrons due to the electric field is in the negative x-direction. Therefore, the effective diffusion of the electrons is reduced and the concentration drops off faster with the applied electric field.

6.24

p-type so the minority carriers are electrons, then

$$D_n \nabla^2(\delta n) + \mu_n E \bullet \nabla(\delta n) + g' - \frac{\delta n}{\tau_{nO}} = \frac{\partial(\delta n)}{\partial t}$$

Uniform illumination means that
$\nabla(\delta n) = \nabla^2(\delta n) = 0$. For $\tau_{nO} = \infty$, we are left with

$$\frac{d(\delta n)}{dt} = g' \quad \text{which gives} \quad \delta n = g't + C_1$$

For $t < 0$, $\delta n = 0$ which means that $C_1 = 0$. Then

$$\underline{\delta n = G'_O t} \quad \text{for} \quad 0 \le t \le T$$

For $t > T$, $g' = 0$ so we have $\dfrac{d(\delta n)}{dt} = 0$

Or

$$\underline{\delta n = G'_O T} \quad \text{(No recombination)}$$

6.25

n-type so minority carriers are holes, then

$$D_p \nabla^2(\delta p) - \mu_p E \bullet \nabla(\delta p) + g' - \frac{\delta p}{\tau_{pO}} = \frac{\partial(\delta p)}{\partial t}$$

We have $\tau_{pO} = \infty$, $E = 0$, $\dfrac{\partial(\delta p)}{\partial t} = 0$ (steady state). Then we have

$$D_p \frac{d^2(\delta p)}{dx^2} + g' = 0 \quad \text{or} \quad \frac{d^2(\delta p)}{dx^2} = -\frac{g'}{D_p}$$

For $-L < x < +L$, $g' = G'_O$ = constant. Then

$$\frac{d(\delta p)}{dx} = -\frac{G'_O}{D_p}x + C_1 \quad \text{and}$$

$$\delta p = -\frac{G'_O}{2D_p}x^2 + C_1 x + C_2$$

For $L < x < 3L$, $g' = 0$ so we have

$$\frac{d^2(\delta p)}{dx^2} = 0 \quad \text{so that} \quad \frac{d(\delta p)}{dx} = C_3 \quad \text{and}$$

$$\delta p = C_3 x + C_4$$

For $-3L < x < -L$, $g' = 0$ so that

$$\frac{d^2(\delta p)}{dx^2} = 0, \quad \frac{d(\delta p)}{dx} = C_5, \quad \text{and}$$

$$\delta p = C_5 x + C_6$$

The boundary conditions are
(1) $\delta p = 0$ at $x = +3L$; (2) $\delta p = 0$ at $x = -3L$;
(3) δp continuous at $x = +L$; (4) δp continuous at $x = -L$; The flux must be continuous so that
(5) $\dfrac{d(\delta p)}{dx}$ continuous at $x = +L$; (6) $\dfrac{d(\delta p)}{dx}$ continuous at $x = -L$.

Applying these boundary conditions, we find

$$\delta p = \frac{G'_O}{2D_p}\left(5L^2 - x^2\right) \quad \text{for} \quad -L < x < +L$$

$$\delta p = \frac{G'_O L}{D_p}(3L - x) \quad \text{for} \quad L < x < 3L$$

$$\delta p = \frac{G'_O L}{D_p}(3L + x) \quad \text{for} \quad -3L < x < -L$$

6.26

$$\mu_p = \frac{d}{E_O t} = \frac{0.75}{\left(\frac{2.5}{1}\right)\left(160x10^{-6}\right)} = 1875\ cm^2/V-s$$

Then

$$D_p = \frac{\left(\mu_p E_O\right)^2 (\Delta t)^2}{16 t_O}$$

$$= \frac{\left[(1875)\left(\frac{2.5}{1}\right)\right]^2 \left(75.5x10^{-6}\right)^2}{16\left(160x10^{-6}\right)}$$

which gives

$$D_p = 48.9\ cm^2/s$$

From the Einstein relation,

$$\frac{D_p}{\mu_p} = \frac{kT}{e} = \frac{48.9}{1875} = 0.02608\ V$$

6.27

Assume that $f(x,t) = (4\pi Dt)^{-1/2}\exp\left(\frac{-x^2}{4Dt}\right)$
is the solution to the differential equation

$$D\left(\frac{\partial^2 f}{\partial x^2}\right) = \frac{\partial f}{\partial t}$$

To prove: we can write

$$\frac{\partial f}{\partial x} = (4\pi Dt)^{-1/2}\left(\frac{-2x}{4Dt}\right)\exp\left(\frac{-x^2}{4Dt}\right)$$

and

$$\frac{\partial^2 f}{\partial x^2} = (4\pi Dt)^{-1/2}\left(\frac{-2x}{4Dt}\right)^2\exp\left(\frac{-x^2}{4Dt}\right)$$
$$+(4\pi Dt)^{-1/2}\left(\frac{-2}{4Dt}\right)\exp\left(\frac{-x^2}{4Dt}\right)$$

Also

$$\frac{\partial f}{\partial t} = (4\pi Dt)^{-1/2}\left(\frac{-x^2}{4D}\right)\left(\frac{-1}{t^2}\right)\exp\left(\frac{-x^2}{4Dt}\right)$$
$$+(4\pi D)^{-1/2}\left(\frac{-1}{2}\right)t^{-3/2}\exp\left(\frac{-x^2}{4Dt}\right)$$

Substituting the expressions for $\frac{\partial^2 f}{\partial x^2}$ and $\frac{\partial f}{\partial t}$ into the differential equation, we find $0 = 0$, Q.E.D.

6.28

Computer Plot

6.29

n-type

$$\delta n = \delta p = g'\tau_{pO} = (10^{21})(10^{-6}) = 10^{15}\ cm^{-3}$$

We have $n_O = 10^{16}\ cm^{-3}$

$$p_O = \frac{n_i^2}{n_O} = \frac{(1.5x10^{10})^2}{10^{16}} = 2.25x10^4\ cm^{-3}$$

Now

$$E_{Fn} - E_{Fi} = kT\ln\left(\frac{n_O + \delta n}{n_i}\right)$$
$$= (0.0259)\ln\left(\frac{10^{16} + 10^{15}}{1.5x10^{10}}\right)$$

or

$$\underline{E_{Fn} - E_{Fi} = 0.3498\ eV}$$

and

$$E_{Fi} - E_{Fp} = kT\ln\left(\frac{p_O + \delta p}{n_i}\right)$$
$$= (0.0259)\ln\left(\frac{2.25x10^4 + 10^{15}}{1.5x10^{10}}\right)$$

or

$$\underline{E_{Fi} - E_{Fp} = 0.2877\ eV}$$

6.30

(a) p-type

$$E_{Fi} - E_F = kT\ln\left(\frac{p_O}{n_i}\right)$$
$$= (0.0259)\ln\left(\frac{5x10^{15}}{1.5x10^{10}}\right)$$

or

$$\underline{E_{Fi} - E_F = 0.3294\ eV}$$

(b)

$$\delta n = \delta p = 5x10^{14}\ cm^{-3}$$

and

$$n_O = \frac{(1.5x10^{10})^2}{5x10^{15}} = 4.5x10^4\ cm^{-3}$$

Then

$$E_{Fn} - E_{Fi} = kT\ln\left(\frac{n_O + \delta n}{n_i}\right)$$
$$= (0.0259)\ln\left(\frac{4.5x10^4 + 5x10^{14}}{1.5x10^{10}}\right)$$

or

$$\underline{E_{Fn} - E_{Fi} = 0.2697\ eV}$$

and

$$E_{Fi} - E_{Fp} = kT\ln\left(\frac{p_O + \delta p}{n_i}\right)$$
$$= (0.0259)\ln\left(\frac{5x10^{15} + 5x10^{14}}{1.5x10^{10}}\right)$$

or

$$\underline{E_{Fi} - E_{Fp} = 0.3318\ eV}$$

6.31

n-type GaAs; $n_O = 5x10^{16}\ cm^{-3}$

$$p_O = \frac{n_i^2}{n_O} = \frac{\left(1.8x10^6\right)^2}{5x10^{16}} = 6.48x10^{-5}\ cm^{-3}$$

We have

$$\delta n = \delta p = (0.1)N_d = 5x10^{15}\ cm^{-3}$$

(a)

$$E_{Fn} - E_{Fi} = kT\ln\left(\frac{n_O + \delta n}{n_i}\right)$$

$$= (0.0259)\ln\left(\frac{5x10^{16} + 5x10^{15}}{1.8x10^6}\right)$$

or

$$\underline{E_{Fn} - E_{Fi} = 0.6253\ eV}$$

We have

$$E_F - E_{Fi} = kT\ln\left(\frac{N_d}{n_i}\right)$$

$$= (0.0259)\ln\left(\frac{5x10^{16}}{1.8x10^6}\right)$$

or

$$\underline{E_F - E_{Fi} = 0.6228\ eV}$$

Now

$$E_{Fn} - E_F = \left(E_{Fn} - E_{Fi}\right) - \left(E_F - E_{Fi}\right)$$
$$= 0.6253 - 0.6228$$

so

$$\underline{E_{Fn} - E_F = 0.0025\ eV}$$

(b)

$$E_{Fi} - E_{Fp} = kT\ln\left(\frac{p_O + \delta p}{n_i}\right)$$

$$= (0.0259)\ln\left(\frac{5x10^{15}}{1.8x10^6}\right)$$

or

$$\underline{E_{Fi} - E_{Fp} = 0.5632\ eV}$$

6.32

Quasi-Fermi level for minority carrier electrons

$$E_{Fn} - E_{Fi} = kT\ln\left(\frac{n_O + \delta n}{n_i}\right)$$

We have

$$\delta n = \left(10^{14}\right)\left(\frac{x}{50\ \mu m}\right)$$

Neglecting the minority carrier electron concentration

$$E_{Fn} - E_{Fi} = kT\ln\left[\frac{\left(10^{14}\right)(x)}{(50\ \mu m)\left(1.8x10^6\right)}\right]$$

We find

$x(\mu m)$	$E_{Fn} - E_{Fi}\quad (eV)$
0	–0.581
1	+0.361
2	+0.379
10	+0.420
20	+0.438
50	+0.462

Quasi-Fermi level for holes: we have

$$E_{Fi} - E_{Fp} = kT\ln\left(\frac{p_O + \delta p}{n_i}\right)$$

We have $p_O = 10^{16}\ cm^{-3}$, $\delta p = \delta n$

We find

$x(\mu m)$	$E_{Fi} - E_{Fp}\quad (eV)$
0	+0.58115
50	+0.58140

6.33

(a) We can write

$$E_{Fi} - E_F = kT\ln\left(\frac{p_O}{n_i}\right)$$

and

$$E_{Fi} - E_{Fp} = kT\ln\left(\frac{p_O + \delta p}{n_i}\right)$$

so that

$$\left(E_{Fi} - E_{Fp}\right) - \left(E_{Fi} - E_F\right) = E_F - E_{Fp}$$
$$= kT\ln\left(\frac{p_O + \delta p}{n_i}\right) - kT\ln\left(\frac{p_O}{n_i}\right)$$

or

$$E_F - E_{Fp} = kT\ln\left(\frac{p_O + \delta p}{p_O}\right) = (0.01)kT$$

Then

$$\frac{p_O + \delta p}{p_O} = \exp(0.01) = 1.010 \Rightarrow$$

$$\frac{\delta p}{p_O} = 0.010 \Rightarrow \text{ low-injection, so that}$$

$$\underline{\delta p = 5x10^{12}\ cm^{-3}}$$

(b)

$$E_{Fn} - E_{Fi} \approx kT \ln\left(\frac{\delta p}{n_i}\right)$$

$$= (0.0259)\ln\left(\frac{5x10^{12}}{1.5x10^{10}}\right)$$

or

$$\underline{E_{Fn} - E_{Fi} = 0.1505\ eV}$$

6.34
Computer Plot

6.35
Computer Plot

6.36
(a)

$$R = \frac{C_n C_p N_t\left(np - n_i^2\right)}{C_n(n + n') + C_p(p + p')}$$

$$= \frac{\left(np - n_i^2\right)}{\tau_{pO}(n + n') + \tau_{nO}(p + p')}$$

For $n = p = 0$

$$R = \frac{-n_i^2}{\tau_{pO}n_i + \tau_{nO}n_i} \Rightarrow \underline{R = \frac{-n_i}{\tau_{pO} + \tau_{nO}}}$$

(b)
We had defined the net generation rate as
$g - R = g_O + g' - (R_O + R')$ where
$g_O = R_O$ since these are the thermal equilibrium generation and recombination rates. If $g' = 0$, then $g - R = -R'$ and $R' = \dfrac{-n_i}{\tau_{pO} + \tau_{nO}}$ so that

$g - R = +\dfrac{n_i}{\tau_{pO} + \tau_{nO}}$. Thus a negative recombination rate implies a net positive generation rate.

6.37
We have that

$$R = \frac{C_n C_p N_t\left(np - n_i^2\right)}{C_n(n + n') + C_p(p + p')}$$

$$= \frac{\left(np - n_i^2\right)}{\tau_{pO}(n + n_i) + \tau_{nO}(p + n_i)}$$

If $n = n_O + \delta n$ and $p = p_O + \delta n$, then

$$R = \frac{(n_O + \delta n)(p_O + \delta n) - n_i^2}{\tau_{pO}(n_O + \delta n + n_i) + \tau_{nO}(p_O + \delta n + n_i)}$$

$$= \frac{n_O p_O + \delta n(n_O + p_O) + (\delta n)^2 - n_i^2}{\tau_{pO}(n_O + \delta n + n_i) + \tau_{nO}(p_O + \delta n + n_i)}$$

If $\delta n << n_i$, we can neglect the $(\delta n)^2$; also
$n_O p_O = n_i^2$
Then

$$R = \frac{\delta n(n_O + p_O)}{\tau_{pO}(n_O + n_i) + \tau_{nO}(p_O + n_i)}$$

(a)
For n-type, $n_O >> p_O$, $n_O >> n_i$
Then

$$\underline{\frac{R}{\delta n} = \frac{1}{\tau_{pO}} = 10^{+7}\ s^{-1}}$$

(b)
Intrinsic, $n_O = p_O = n_i$
Then

$$\frac{R}{\delta n} = \frac{2n_i}{\tau_{pO}(2n_i) + \tau_{nO}(2n_i)}$$

or

$$\frac{R}{\delta n} = \frac{1}{\tau_{pO} + \tau_{nO}} = \frac{1}{10^{-7} + 5x10^{-7}} \Rightarrow$$

$$\underline{\frac{R}{\delta n} = 1.67x10^{+6}\ s^{-1}}$$

(c)
p-type, $p_O >> n_O$, $p_O >> n_i$
Then

$$\underline{\frac{R}{\delta n} = \frac{1}{\tau_{nO}} = \frac{1}{5x10^{-7}} = 2x10^{+6}\ s^{-1}}$$

6.38

(a) From Equation [6.56],

$$D_p \frac{d^2(\delta p)}{dx^2} + g' - \frac{\delta p}{\tau_{pO}} = 0$$

Solution is of the form

$$\delta p = g'\tau_{pO} + A\exp(-x/L_p) + B\exp(+x/L_p)$$

At $x = \infty$, $\delta p = g'\tau_{pO}$, so that $B \equiv 0$,

Then

$$\delta p = g'\tau_{pO} + A\exp(-x/L_p)$$

We have

$$D_p \frac{d(\delta p)}{dx}\Big|_{x=0} = s(\delta p)\big|_{x=0}$$

We can write

$$\frac{d(\delta p)}{dx}\Big|_{x=0} = \frac{-A}{L_p} \quad \text{and} \quad (\delta p)\big|_{x=0} = g'\tau_{pO} + A$$

Then

$$\frac{-AD_p}{L_p} = s(g'\tau_{pO} + A)$$

Solving for A we find

$$A = \frac{-sg'\tau_{pO}}{\dfrac{D_p}{L_p} + s}$$

The excess concentration is then

$$\delta p = g'\tau_{pO}\left[1 - \frac{s}{(D_p/L_p) + s} \cdot \exp\left(\frac{-x}{L_p}\right)\right]$$

where

$$L_p = \sqrt{D_p \tau_{pO}} = \sqrt{(10)(10^{-7})} = 10^{-3}\ cm$$

Now

$$\delta p = (10^{21})(10^{-7})\left[1 - \frac{s}{(10/10^{-3}) + s}\exp\left(\frac{-x}{L_p}\right)\right]$$

or

$$\underline{\delta p = 10^{14}\left[1 - \frac{s}{10^4 + s}\exp\left(\frac{-x}{L_p}\right)\right]}$$

(i) $s = 0$, $\underline{\delta p = 10^{14}\ cm^{-3}}$

(ii) $s = 2000\ cm/s$,

$$\underline{\delta p = 10^{14}\left[1 - 0.167\exp\left(\frac{-x}{L_p}\right)\right]}$$

(iii) $s = \infty$, $\underline{\delta p = 10^{14}\left[1 - \exp\left(\frac{-x}{L_p}\right)\right]}$

(b) (i) $s = 0$, $\underline{\delta p(0) = 10^{14}\ cm^{-3}}$

(ii) $s = 2000\ cm/s$, $\underline{\delta p(0) = 0.833x10^{14}\ cm^{-3}}$

(iii) $s = \infty$, $\underline{\delta p(0) = 0}$

6.39

$$L_n = \sqrt{D_n \tau_{nO}} = \sqrt{(25)(5x10^{-7})} = 35.4x10^{-4}\ cm$$

(a)

At $x = 0$, $g'\tau_{nO} = (2x10^{21})(5x10^{-7}) = 10^{15}\ cm^{-3}$

Or $\delta n_O = g'\tau_{nO} = 10^{15}\ cm^{-3}$

For $x > 0$

$$D_n \frac{d^2(\delta n)}{dx^2} - \frac{\delta n}{\tau_{nO}} = 0 \Rightarrow \frac{d^2(\delta n)}{dx^2} - \frac{\delta n}{L_n^2} = 0$$

Solution is of the form

$$\delta n = A\exp(-x/L_n) + B\exp(+x/L_n)$$

At $x = 0$, $\delta n = \delta n_O = A + B$

At $x = W$,

$$\delta n = 0 = A\exp(-W/L_n) + B\exp(+W/L_n)$$

Solving these two equations, we find

$$A = \frac{-\delta n_O \exp(+2W/L_n)}{1 - \exp(2W/L_n)}$$

$$B = \frac{\delta n_O}{1 - \exp(2W/L_n)}$$

Substituting into the general solution, we find

$$\delta n = \frac{\delta n_O}{[\exp(+W/L_n) - \exp(-W/L_n)]} \times \{\exp[+(W-x)/L_n] - \exp[-(W-x)/L_n]\}$$

or

$$\underline{\delta n = \frac{\delta n_O \sinh[(W-x)/L_n]}{\sinh[W/L_n]}}$$

where

$\underline{\delta n_O = 10^{15}\ cm^{-3}}$ and $\underline{L_n = 35.4\ \mu m}$

(b)

If $\tau_{nO} = \infty$, we have

$$\frac{d^2(\delta n)}{dx^2} = 0$$

so the solution is of the form

$$\delta n = Cx + D$$

Applying the boundary conditions, we find

$$\delta n = \delta n_O \left(1 - \frac{x}{W}\right)$$

6.40

For $\tau_{pO} = \infty$, we have

$$\frac{d^2(\delta p)}{dx^2} = 0 \quad \text{so that} \quad \frac{d(\delta p)}{dx} = A \quad \text{and}$$

$$\delta p = Ax + B$$

At $x = W$

$$-D_p \frac{d(\delta p)}{dx}\Big|_{x=W} = s \cdot (\delta p)\big|_{x=W}$$

or

$$-D_p A = s(AW + B)$$

which yields

$$B = \frac{-A}{s}\left(D_p + sW\right)$$

At $x = 0$, the flux of excess holes is

$$10^{19} = -D_p \frac{d(\delta p)}{dx}\Big|_{x=0} = -D_p A$$

so that

$$A = \frac{-10^{19}}{10} = -10^{18} \ cm^{-4}$$

and

$$B = \frac{10^{18}}{s}(10 + sW) = 10^{18}\left(\frac{10}{s} + W\right)$$

The solution is now

$$\delta p = 10^{18}\left(W - x + \frac{10}{s}\right)$$

(a)

For $s = \infty$,

$$\delta p = 10^{18}\left(20x10^{-4} - x\right) cm^{-3}$$

(b)

For $s = 2x10^3 \ cm/s$

$$\delta p = 10^{18}\left(70x10^{-4} - x\right) cm^{-3}$$

6.41

For $-W < x < 0$,

$$D_n \frac{d^2(\delta n)}{dx^2} + G'_O = 0$$

so that

$$\frac{d(\delta n)}{dx} = -\frac{G'_O}{D_p} x + C_1$$

and

$$\delta n = -\frac{G'_O}{2D_n} x^2 + C_1 x + C_2$$

For $0 < x < W$,

$$\frac{d^2(\delta n)}{dx^2} = 0, \text{ so } \frac{d(\delta n)}{dx} = C_3, \ \delta n = C_3 x + C_4$$

The boundary conditions are:

(1) $s = 0$ at $x = -W$, so that $\frac{d(\delta n)}{dx}\Big|_{x=-W} = 0$

(2) $s = \infty$ at $x = +W$, so that $\delta n(W) = 0$

(3) δn continuous at $x = 0$

(4) $\frac{d(\delta n)}{dx}$ continuous at $x = 0$

Applying the boundary conditions, we find

$$C_1 = C_3 = -\frac{G'_O W}{D_n}, \quad C_2 = C_4 = +\frac{G'_O W^2}{D_n}$$

Then, for $-W < x < 0$

$$\delta n = \frac{G'_O}{2D_n}\left(-x^2 - 2Wx + 2W^2\right)$$

and for $0 < x < +W$

$$\delta n = \frac{G'_O W}{D_n}(W - x)$$

6.42

Computer Plot

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Chapter 7

Problem Solutions

7.1

$$V_{bi} = V_t \ln\left(\frac{N_a N_d}{n_i^2}\right)$$

where $V_t = 0.0259\ V$ and $n_i = 1.5x10^{10}\ cm^{-3}$

We find

(a)

For $N_d = 10^{15}\ cm^{-3}$	$V_{bi}(V)$
$(i)\quad N_a = 10^{15}\ cm^{-3}$	$0.575\ V$
$(ii)\quad N_a = 10^{16}\ cm^{-3}$	0.635
$(iii)\quad N_a = 10^{17}\ cm^{-3}$	0.695
$(iv)\quad N_a = 10^{18}\ cm^{-3}$	0.754

(b)

For $N_d = 10^{18}\ cm^{-3}$	$V_{bi}(V)$
$(i)\quad N_a = 10^{15}\ cm^{-3}$	$0.754\ V$
$(ii)\quad N_a = 10^{16}\ cm^{-3}$	0.814
$(iii)\quad N_a = 10^{17}\ cm^{-3}$	0.874
$(iv)\quad N_a = 10^{18}\ cm^{-3}$	0.933

7.2

Si: $n_i = 1.5x10^{10}\ cm^{-3}$

Ge: $n_i = 2.4x10^{13}\ cm^{-3}$

GaAs: $n_i = 1.8x10^{6}\ cm^{-3}$

$$V_{bi} = V_t \ln\left(\frac{N_a N_d}{n_i^2}\right) \quad \text{and} \quad V_t = 0.0259\ V$$

(a)

$N_d = 10^{14}\ cm^{-3}$, $N_a = 10^{17}\ cm^{-3}$

Then

$\underline{Si: V_{bi} = 0.635\ V}$, $\underline{Ge: V_{bi} = 0.253\ V}$,

$\underline{GaAs: V_{bi} = 1.10\ V}$

(b)

$N_d = 5x10^{16}\ cm^{-3}$, $N_a = 5x10^{16}\ cm^{-3}$

Then

$\underline{Si: V_{bi} = 0.778\ V}$, $\underline{Ge: V_{bi} = 0.396\ V}$,

$\underline{GaAs: V_{bi} = 1.25\ V}$

(c)

$N_d = 10^{17}\ cm^{-3}$, $N_a = 10^{17}\ cm^{-3}$

Then

$\underline{Si: V_{bi} = 0.814\ V}$, $\underline{Ge: V_{bi} = 0.432\ V}$,

$\underline{GaAs: V_{bi} = 1.28\ V}$

7.3

Computer Plot

7.4

Computer Plot

7.5

(a) n-side:

$$E_F - E_{Fi} = kT \ln\left(\frac{N_d}{n_i}\right)$$

$$= (0.0259)\ln\left(\frac{5x10^{15}}{1.5x10^{10}}\right)$$

or

$$\underline{E_F - E_{Fi} = 0.3294\ eV}$$

p-side:

$$E_{Fi} - E_F = kT \ln\left(\frac{N_a}{n_i}\right)$$

$$= (0.0259)\ln\left(\frac{10^{17}}{1.5x10^{10}}\right)$$

or

$$\underline{E_{Fi} - E_F = 0.4070\ eV}$$

(b)

$$V_{bi} = 0.3294 + 0.4070$$

or

$$\underline{V_{bi} = 0.7364\ V}$$

(c)

$$V_{bi} = V_t \ln\left(\frac{N_a N_d}{n_i^2}\right)$$

$$= (0.0259)\ln\left[\frac{(10^{17})(5x10^{15})}{(1.5x10^{10})^2}\right]$$

or

$$\underline{V_{bi} = 0.7363\ V}$$

(d)

$$x_n = \left[\frac{2 \in V_{bi}}{e}\left(\frac{N_a}{N_d}\right)\left(\frac{1}{N_a + N_d}\right)\right]^{1/2}$$

$$= \left[\frac{2(11.7)(8.85x10^{-14})(0.736)}{1.6x10^{-19}}\right.$$

$$\left.\times\left(\frac{10^{17}}{5x10^{15}}\right)\left(\frac{1}{10^{17} + 5x10^{15}}\right)\right]^{1/2}$$

or

$$\underline{x_n = 0.426\ \mu m}$$

Now

$$x_p = \left[\frac{2(11.7)(8.85x10^{-14})(0.736)}{1.6x10^{-19}}\right.$$

$$\left.\times\left(\frac{5x10^{15}}{10^{17}}\right)\left(\frac{1}{10^{17} + 5x10^{15}}\right)\right]^{1/2}$$

or

$$\underline{x_p = 0.0213\ \mu m}$$

We have

$$|E_{max}| = \frac{eN_d x_n}{\in}$$

$$= \frac{(1.6x10^{-19})(5x10^{15})(0.426x10^{-4})}{(11.7)(8.85x10^{-14})}$$

or

$$\underline{|E_{max}| = 3.29x10^4\ V/cm}$$

7.6

(a) n-side

$$E_F - E_{Fi} = (0.0259)\ln\left(\frac{2x10^{16}}{1.5x10^{10}}\right) \Rightarrow$$

$$\underline{E_F - E_{Fi} = 0.3653\ eV}$$

p-side

$$E_{Fi} - E_F = (0.0259)\ln\left(\frac{2x10^{16}}{1.5x10^{10}}\right) \Rightarrow$$

$$\underline{E_{Fi} - E_F = 0.3653\ eV}$$

(b)

$$V_{bi} = 0.3653 + 0.3653 \Rightarrow$$

$$\underline{V_{bi} = 0.7306\ V}$$

(c)

$$V_{bi} = V_t \ln\left(\frac{N_a N_d}{n_i^2}\right)$$

$$= (0.0259)\ln\left[\frac{(2x10^{16})(2x10^{16})}{(1.5x10^{10})^2}\right]$$

or

$$\underline{V_{bi} = 0.7305\ V}$$

(d)

$$x_n = \left[\frac{2(11.7)(8.85x10^{-14})(0.7305)}{1.6x10^{-19}}\right.$$

$$\left.\times\left(\frac{2x10^{16}}{2x10^{16}}\right)\left(\frac{1}{2x10^{16} + 2x10^{16}}\right)\right]^{1/2}$$

or

$$\underline{x_n = 0.154\ \mu m}$$

By symmetry

$$\underline{x_p = 0.154\ \mu m}$$

Now

$$|E_{max}| = \frac{eN_d x_n}{\in}$$

$$= \frac{(1.6x10^{-19})(2x10^{16})(0.154x10^{-4})}{(11.7)(8.85x10^{-14})}$$

or

$$\underline{|E_{max}| = 4.76x10^4\ V/cm}$$

7.7

(b) $n_O = N_C \exp\left[\frac{-(E_C - E_F)}{kT}\right]$

$$= 2.8x10^{19}\exp\left(\frac{-0.21}{0.0259}\right)$$

or

$$\underline{n_O = N_d = 8.43x10^{15}\ cm^{-3}} \quad \text{(n-region)}$$

$$p_O = N_V \exp\left[\frac{-(E_F - E_V)}{kT}\right]$$

$$= 1.04x10^{19}\exp\left(\frac{-0.18}{0.0259}\right)$$

or

$$\underline{p_O = N_a = 9.97x10^{15}\ cm^{-3}} \quad \text{(p-region)}$$

(c)

$$V_{bi} = V_t \ln\left(\frac{N_a N_d}{n_i^2}\right)$$

$$= (0.0259)\ln\left[\frac{(9.97x10^{15})(8.43x10^{15})}{(1.5x10^{10})^2}\right]$$

or

$$\underline{V_{bi} = 0.690\ V}$$

7.8

(a) GaAs: $V_{bi} = 1.20\ V$, $n_i = 1.8x10^6\ cm^{-3}$

$$x_p = 0.2W = 0.2(x_n + x_p)$$

or

$$\frac{x_p}{x_n} = 0.25$$

Also

$$N_d x_n = N_a x_p \Rightarrow \frac{x_p}{x_n} = \frac{N_d}{N_a} = 0.25$$

Now

$$V_{bi} = V_t \ln\left(\frac{N_a N_d}{n_i^2}\right)$$

or

$$1.20 = (0.0259)\ln\left(\frac{0.25N_a^2}{n_i^2}\right)$$

Then

$$\frac{0.25N_a^2}{n_i^2} = \exp\left(\frac{1.20}{0.0259}\right)$$

or

$$N_a = 2n_i \exp\left[\frac{1.20}{2(0.0259)}\right]$$

or

$$\underline{N_a = 4.14x10^{16}\ cm^{-3}}$$

(b)

$$N_d = 0.25N_a \Rightarrow \underline{N_d = 1.04x10^{16}\ cm^{-3}}$$

(c)

$$x_n = \left[\frac{2\in V_{bi}}{e}\left(\frac{N_a}{N_d}\right)\left(\frac{1}{N_a + N_d}\right)\right]^{1/2}$$

$$= \left[\frac{2(13.1)(8.85x10^{-14})(1.20)}{1.6x10^{-19}} \times \left(\frac{4}{1}\right)\left(\frac{1}{4.14x10^{16} + 1.04x10^{16}}\right)\right]^{1/2}$$

or

$$\underline{x_n = 0.366\ \mu m}$$

(d)

$$x_p = 0.25x_n \Rightarrow \underline{x_p = 0.0916\ \mu m}$$

(e)

$$\mathrm{E}_{\max} = \frac{eN_d x_n}{\in} = \frac{eN_a x_p}{\in}$$

$$= \frac{(1.6x10^{-19})(1.04x10^{16})(0.366x10^{-4})}{(13.1)(8.85x10^{-14})}$$

or

$$\underline{\mathrm{E}_{\max} = 5.25x10^4\ V / cm}$$

7.9

(a) $$V_{bi} = (0.0259)\ln\left[\frac{(10^{16})(10^{15})}{(1.5x10^{10})^2}\right]$$

or

$$\underline{V_{bi} = 0.635\ V}$$

(b)

$$x_n = \left[\frac{2\in V_{bi}}{e}\left(\frac{N_a}{N_d}\right)\left(\frac{1}{N_a + N_d}\right)\right]^{1/2}$$

$$= \left[\frac{2(11.7)(8.85x10^{-14})(0.635)}{1.6x10^{-19}} \times \left(\frac{10^{16}}{10^{15}}\right)\left(\frac{1}{10^{16} + 10^{15}}\right)\right]^{1/2}$$

or $\underline{x_n = 0.864\ \mu m}$

Now

$$x_p = \left[\frac{2\in V_{bi}}{e}\left(\frac{N_d}{N_a}\right)\left(\frac{1}{N_a + N_d}\right)\right]^{1/2}$$

$$= \left[\frac{2(11.7)(8.85x10^{-14})(0.635)}{1.6x10^{-19}} \times \left(\frac{10^{15}}{10^{16}}\right)\left(\frac{1}{10^{16} + 10^{15}}\right)\right]^{1/2}$$

or $\underline{x_p = 0.0864\ \mu m}$

(c)

$$\mathrm{E}_{\max} = \frac{eN_d x_n}{\in}$$

$$= \frac{(1.6x10^{-19})(10^{15})(0.864x10^{-4})}{(11.7)(8.85x10^{-14})}$$

or

$$\underline{\mathrm{E}_{\max} = 1.34x10^4\ V / cm}$$

7.10

$$V_{bi} = V_t \ln\left(\frac{N_a N_d}{n_i^2}\right) \quad \text{and}$$

$$n_i^2 = N_C N_V \exp\left(\frac{-E_g}{kT}\right)$$

We can write

$$N_C N_V = N_{CO} N_{VO}\left(\frac{T}{300}\right)^3$$

Now

$$V_{bi} = V_t\left[\ln(N_a N_d) - \ln(n_i^2)\right]$$

$$= V_t\left[\ln(N_a N_d) - \ln(N_{CO}N_{VO}) - \ln\left(\frac{T}{300}\right)^3 + \frac{E_g}{kT}\right]$$

or

$$V_{bi} = V_t\left[\ln\left(\frac{N_a N_d}{N_{CO}N_{VO}}\right) - 3\ln\left(\frac{T}{300}\right) + \frac{E_g}{kT}\right]$$

or

$$0.40 = (0.0250)\left(\frac{T}{300}\right) \times \left[\ln\left[\frac{(5x10^{15})(10^{16})}{(2.8x10^{19})(1.04x10^{19})}\right] - 3\ln\left(\frac{T}{300}\right) + \frac{1.12}{(0.0259)(T/300)}\right]$$

Then

$$15.44 = \left(\frac{T}{300}\right)\left[-15.58 - 3\ln\left(\frac{T}{300}\right) + \frac{43.24}{(T/300)}\right]$$

By trial and error

$$\underline{T = 490K}$$

7.11

(a) $V_{bi} = V_t \ln\left(\dfrac{N_a N_d}{n_i^2}\right)$

$$= (0.0259)\ln\left[\frac{(5x10^{17})(10^{17})}{(1.5x10^{10})^2}\right]$$

or

$$\underline{V_{bi} = 0.8556\, V}$$

(b)

For a 1% change in V_{bi}, assume that the change is due to n_i^2, where the major dependence on temperature is given by

$$n_i^2 \propto \exp\left(\frac{-E_g}{kT}\right)$$

Now

$$\frac{V_{bi}(T_2)}{V_{bi}(T_1)} = \frac{\ln\left[\dfrac{N_a N_d}{n_i^2(T_2)}\right]}{\ln\left[\dfrac{N_a N_d}{n_i^2(T_1)}\right]}$$

$$= \frac{\ln(N_a N_d) - \ln\left[n_i^2(T_2)\right]}{\ln(N_a N_d) - \ln\left[n_i^2(T_1)\right]}$$

$$= \frac{\ln(N_a N_d) - \ln(N_C N_V) - \left(\dfrac{-E_g}{kT_2}\right)}{\ln(N_a N_d) - \ln(N_C N_V) - \left(\dfrac{-E_g}{kT_1}\right)}$$

$$= \left\{\ln\left[(5x10^{17})(10^{17})\right] - \ln\left[(2.8x10^{19})(1.04x10^{19})\right] + \frac{E_g}{kT_2}\right\}$$

$$/\left\{\ln\left[(5x10^{17})(10^{17})\right] - \ln\left[(2.8x10^{19})(1.04x10^{19})\right] + \frac{E_g}{kT_1}\right\}$$

or

$$\frac{V_{bi}(T_2)}{V_{bi}(T_1)} = \frac{79.897 - 88.567 + \dfrac{E_g}{kT_2}}{79.897 - 88.567 + \dfrac{E_g}{kT_1}}$$

We can write

$$0.990 = \frac{-8.67 + \dfrac{E_g}{kT_2}}{-8.67 + \dfrac{1.12}{0.0259}} = \frac{-8.67 + \dfrac{E_g}{kT_2}}{34.57}$$

so that

$$\frac{E_g}{kT_2} = 42.90 = \frac{1.12}{(0.0259)\left(\dfrac{T_2}{300}\right)}$$

We then find

$$\underline{T_2 = 302.4K}$$

7.12

(b) For $N_d = 10^{16}\ cm^{-3}$,

$$E_F - E_{Fi} = kT\ln\left(\frac{N_d}{n_i}\right)$$

$$= (0.0259)\ln\left(\frac{10^{16}}{1.5x10^{10}}\right)$$

or

$$\underline{E_F - E_{Fi} = 0.3473\ eV}$$

For $N_d = 10^{15}\ cm^{-3}$,

$$E_F - E_{Fi} = (0.0259)\ln\left(\frac{10^{15}}{1.5x10^{10}}\right)$$

or

$$\underline{E_F - E_{Fi} = 0.2877\ eV}$$

Then

$$V_{bi} = 0.3473 - 0.2877$$

or

$$\underline{V_{bi} = 0.0596\ V}$$

7.13

(a) $V_{bi} = V_t\ln\left(\frac{N_a N_d}{n_i^2}\right)$

$$= (0.0259)\ln\left[\frac{(10^{16})(10^{12})}{(1.5x10^{10})^2}\right]$$

or

$$\underline{V_{bi} = 0.456\ V}$$

(b)

$$x_n = \left[\frac{2(11.7)(8.85x10^{-14})(0.456)}{1.6x10^{-19}} \times\left(\frac{10^{12}}{10^{16}}\right)\left(\frac{1}{10^{16}+10^{12}}\right)\right]^{1/2}$$

or

$$\underline{x_n = 2.43x10^{-7}\ cm}$$

(c)

$$x_p = \left[\frac{2(11.7)(8.85x10^{-14})(0.456)}{1.6x10^{-19}} \times\left(\frac{10^{16}}{10^{12}}\right)\left(\frac{1}{10^{16}+10^{12}}\right)\right]^{1/2}$$

or

$$\underline{x_p = 2.43x10^{-3}\ cm}$$

(d)

$$|E_{max}| = \frac{eN_d x_n}{\in}$$

$$= \frac{(1.6x10^{-19})(10^{16})(2.43x10^{-7})}{(11.7)(8.85x10^{-14})}$$

or

$$\underline{|E_{max}| = 3.75x10^2\ V/cm}$$

7.14

Assume Silicon, so

$$L_D = \left(\frac{\in kT}{e^2 N_d}\right)^{1/2}$$

$$= \left[\frac{(11.7)(8.85x10^{-14})(0.0259)(1.6x10^{-19})}{(1.6x10^{-19})^2 N_d}\right]^{1/2}$$

or

$$L_D = \left(\frac{1.676x10^5}{N_d}\right)^{1/2}$$

(a) $N_d = 8x10^{14}\ cm^{-3}$, $\underline{L_D = 0.1447\ \mu m}$

(b) $N_d = 2.2x10^{16}\ cm^{-3}$, $\underline{L_D = 0.02760\ \mu m}$

(c) $N_d = 8x10^{17}\ cm^{-3}$, $\underline{L_D = 0.004577\ \mu m}$

Now

(a) $V_{bi} = 0.7427\ V$

(b) $V_{bi} = 0.8286\ V$

(c) $V_{bi} = 0.9216\ V$

Also

$$x_n = \left[\frac{2(11.7)(8.85x10^{-14})(V_{bi})}{1.6x10^{-19}} \times\left(\frac{8x10^{17}}{N_d}\right)\left(\frac{1}{8x10^{17}+N_d}\right)\right]^{1/2}$$

Then

(a) $\underline{x_n = 1.096\ \mu m}$

(b) $\underline{x_n = 0.2178\ \mu m}$

(c) $\underline{x_n = 0.02731\ \mu m}$

Now

(a) $\underline{\frac{L_D}{x_n} = 0.1320}$

(b) $\dfrac{L_D}{x_n} = 0.1267$

(c) $\dfrac{L_D}{x_n} = 0.1677$

7.15
Computer Plot

7.16

(a) $V_{bi} = V_t \ln\left(\dfrac{N_a N_d}{n_i^2}\right)$

$$= (0.0259)\ln\left[\frac{(2x10^{16})(2x10^{15})}{(1.5x10^{10})^2}\right]$$

or

$V_{bi} = 0.671\ V$

(b)

$$W = \left[\frac{2\in(V_{bi}+V_R)}{e}\left(\frac{N_a+N_d}{N_a N_d}\right)\right]^{1/2}$$

$$= \left\{\frac{2(11.7)(8.85x10^{-14})(V_{bi}+V_R)}{1.6x10^{-19}} \times \left[\frac{2x10^{16}+2x10^{15}}{(2x10^{16})(2x10^{15})}\right]\right\}^{1/2}$$

or

$$W = \left[7.12x10^{-9}(V_{bi}+V_R)\right]^{1/2}$$

For $V_R = 0$, $W = 0.691x10^{-4}\ cm$

For $V_R = 8\ V$, $W = 2.48x10^{-4}\ cm$

(c)

$$E_{max} = \frac{2(V_{bi}+V_R)}{W}$$

For $V_R = 0$, $E_{max} = 1.94x10^4\ V/cm$

For $V_R = 8\ V$, $E_{max} = 7.0x10^4\ V/cm$

7.17

(a) $V_{bi} = V_t \ln\left(\dfrac{N_a N_d}{n_i^2}\right)$

$$= (0.0259)\ln\left[\frac{(5x10^{17})(10^{17})}{(1.5x10^{10})^2}\right]$$

or

$V_{bi} = 0.856\ V$

(b)

$$x_n = \left[\frac{2\in(V_{bi}+V_R)}{e}\left(\frac{N_a}{N_d}\right)\left(\frac{1}{N_a+N_d}\right)\right]^{1/2}$$

$$= \left[\frac{2(11.7)(8.85x10^{-14})(5.856)}{1.6x10^{-19}} \times \left(\frac{5x10^{17}}{1x10^{17}}\right)\left(\frac{1}{5x10^{17}+1x10^{17}}\right)\right]^{1/2}$$

or

$x_n = 0.251\ \mu m$

Also

$$x_p = \left[\frac{2\in(V_{bi}+V_R)}{e}\left(\frac{N_d}{N_a}\right)\left(\frac{1}{N_a+N_d}\right)\right]^{1/2}$$

$$= \left[\frac{2(11.7)(8.85x10^{-14})(5.856)}{1.6x10^{-19}} \times \left(\frac{1x10^{17}}{5x10^{17}}\right)\left(\frac{1}{5x10^{17}+1x10^{17}}\right)\right]^{1/2}$$

or

$x_p = 0.0503\ \mu m$

Also

$W = x_n + x_p$

or

$W = 0.301\ \mu m$

(c)

$$E_{max} = \frac{2(V_{bi}+V_R)}{W} = \frac{2(5.856)}{0.301x10^{-4}}$$

or

$E_{max} = 3.89x10^5\ V/cm$

(d)

$$C_T = \frac{\in A}{W} = \frac{(11.7)(8.85x10^{-14})(10^{-4})}{0.301x10^{-4}}$$

or

$C_T = 3.44\ pF$

7.18

(a) $V_{bi} = V_t \ln\left(\dfrac{N_a N_d}{n_i^2}\right)$

$$= (0.0259)\ln\left[\frac{50N_a^2}{(1.5x10^{10})^2}\right]$$

We can write

$$\exp\left(\frac{0.752}{0.0259}\right) = \frac{50N_a^2}{\left(1.5x10^{10}\right)^2}$$

or

$$N_a = \frac{1.5x10^{10}}{\sqrt{50}}\exp\left[\frac{0.752}{2(0.0259)}\right]$$

and

$$\underline{N_a = 4.28x10^{15}\ cm^{-3}}$$

Then

$$\underline{N_d = 2.14x10^{17}\ cm^{-3}}$$

(b)

$$x_p \approx W \approx \left[\frac{2\in\left(V_{bi}+V_R\right)}{e}\cdot\left(\frac{1}{N_a}\right)\right]^{1/2}$$

$$= \left[\frac{2(11.7)\left(8.85x10^{-14}\right)(10.752)}{\left(1.6x10^{-19}\right)\left(4.28x10^{15}\right)}\right]^{1/2}$$

or

$$\underline{x_p = 1.80\ \mu m}$$

(c)

$$C' \approx \left[\frac{e\in N_a}{2\left(V_{bi}+V_R\right)}\right]^{1/2}$$

$$= \left[\frac{\left(1.6x10^{-19}\right)(11.7)\left(8.85x10^{-14}\right)\left(4.28x10^{15}\right)}{2(10.752)}\right]^{1/2}$$

or

$$\underline{C' = 5.74x10^{-9}\ F/cm^2}$$

7.19

(a) Neglecting change in V_{bi}

$$\frac{C'(2N_a)}{C'(N_a)} = \left\{\frac{\left[\dfrac{2}{(2N_a+N_d)}\right]}{\left(\dfrac{1}{N_a+N_d}\right)}\right\}^{1/2}$$

For a $n^+p \Rightarrow N_d >> N_a$

Then

$$\frac{C'(2N_a)}{C'(N_a)} = \sqrt{2} = 1.414$$

so a <u>41.4% change</u>.

(b)

$$\frac{V_{bi}(2N_a)}{V_{bi}(N_a)} = \frac{kT\ln\left(\dfrac{2N_aN_d}{n_i^2}\right)}{kT\ln\left(\dfrac{N_aN_d}{n_i^2}\right)}$$

$$= \frac{kT\ln 2 + kT\ln\left(\dfrac{N_aN_d}{n_i^2}\right)}{kT\ln\left(\dfrac{N_aN_d}{n_i^2}\right)}$$

So we can write this as

$$\frac{V_{bi}(2N_a)}{V_{bi}(N_a)} = \frac{kT\ln 2 + V_{bi}(N_a)}{V_{bi}(N_a)}$$

so

$$\Delta V_{bi} = kT\ln 2 = (0.0259)\ln 2$$

or

$$\underline{\Delta V_{bi} = 17.95\ mV}$$

17.20

(a)

$$\frac{W(A)}{W(B)} = \frac{\left[\dfrac{2\in\left(V_{biA}+V_R\right)}{e}\left(\dfrac{N_a+N_{dA}}{N_aN_{dA}}\right)\right]^{1/2}}{\left[\dfrac{2\in\left(V_{biB}+V_R\right)}{e}\left(\dfrac{N_a+N_{dB}}{N_aN_{dB}}\right)\right]^{1/2}}$$

or

$$\frac{W(A)}{W(B)} = \left[\frac{\left(V_{biA}+V_R\right)}{\left(V_{biB}+V_R\right)}\cdot\frac{\left(N_a+N_{dA}\right)}{\left(N_a+N_{dB}\right)}\cdot\left(\frac{N_{dB}}{N_{dA}}\right)\right]^{1/2}$$

We find

$$V_{biA} = (0.0259)\ln\left[\frac{\left(10^{18}\right)\left(10^{15}\right)}{\left(1.5x10^{10}\right)^2}\right] = 0.7543\ V$$

$$V_{biB} = (0.0259)\ln\left[\frac{\left(10^{18}\right)\left(10^{16}\right)}{\left(1.5x10^{10}\right)^2}\right] = 0.8139\ V$$

So we find

$$\frac{W(A)}{W(B)} = \left[\left(\frac{5.7543}{5.8139}\right)\left(\frac{10^{18}+10^{15}}{10^{18}+10^{16}}\right)\left(\frac{10^{16}}{10^{15}}\right)\right]^{1/2}$$

or

$$\underline{\frac{W(A)}{W(B)} = 3.13}$$

(b)

$$\frac{\mathrm{E}(A)}{\mathrm{E}(B)}=\frac{\dfrac{2(V_{biA}+V_R)}{W(A)}}{\dfrac{2(V_{biB}+V_R)}{W(B)}}=\frac{W(B)}{W(A)}\cdot\frac{(V_{biA}+V_R)}{(V_{biB}+V_R)}$$

$$=\left(\frac{1}{3.13}\right)\left(\frac{5.7543}{5.8139}\right)$$

or

$$\frac{\mathrm{E}(A)}{\mathrm{E}(B)}=0.316$$

(c)

$$\frac{C_j'(A)}{C_j'(B)}=\frac{\left[\dfrac{\in N_aN_{dA}}{2(V_{biA}+V_R)(N_a+N_{dA})}\right]^{1/2}}{\left[\dfrac{\in N_aN_{dB}}{2(V_{biB}+V_R)(N_a+N_{dB})}\right]^{1/2}}$$

$$=\left[\left(\frac{N_{dA}}{N_{dB}}\right)\left(\frac{V_{biB}+V_R}{V_{biA}+V_R}\right)\left(\frac{N_a+N_{dB}}{N_a+N_{dA}}\right)\right]^{1/2}$$

$$=\left[\left(\frac{10^{15}}{10^{16}}\right)\left(\frac{5.8139}{5.7543}\right)\left(\frac{10^{18}+10^{16}}{10^{18}+10^{15}}\right)\right]^{1/2}$$

or

$$\frac{C_j'(A)}{C_j'(B)}=0.319$$

17.21

(a) $$V_{bi}=(0.0259)\ln\left[\frac{(4x10^{15})(4x10^{17})}{(1.5x10^{10})^2}\right]\Rightarrow$$

$$V_{bi}=0.766\ V$$

Now

$$|\mathrm{E}_{\max}|=\left[\frac{2e(V_{bi}+V_R)}{\in}\left(\frac{N_aN_d}{N_a+N_d}\right)\right]^{1/2}$$

so

$$(3x10^5)^2=\left[\frac{2(1.6x10^{-19})}{(11.7)(8.85x10^{-14})}\right](V_{bi}+V_R)\times\left[\frac{(4x10^{15})(4x10^{17})}{4x10^{15}+4x10^{17}}\right]$$

or

$$9x10^{10}=1.22x10^9(V_{bi}+V_R)\Rightarrow$$

$$V_{bi}+V_R=73.77\ V$$

and

$$V_R=73\ V$$

(b)

$$V_{bi}=(0.0259)\ln\left[\frac{(4x10^{16})(4x10^{17})}{(1.5x10^{10})^2}\right]\Rightarrow$$

$$V_{bi}=0.826\ V$$

$$(3x10^5)^2=\left[\frac{2(1.6x10^{-19})}{(11.7)(8.85x10^{-14})}\right](V_{bi}+V_R)\times\left[\frac{(4x10^{16})(4x10^{17})}{4x10^{16}+4x10^{17}}\right]$$

which yields

$$V_{bi}+V_R=8.007\ V$$

and

$$V_R=7.18\ V$$

(c)

$$V_{bi}=(0.0259)\ln\left[\frac{(4x10^{17})(4x10^{17})}{(1.5x10^{10})^2}\right]\Rightarrow$$

$$V_{bi}=0.886\ V$$

$$(3x10^5)^2=\left[\frac{2(1.6x10^{-19})}{(11.7)(8.85x10^{-14})}\right](V_{bi}+V_R)\times\left[\frac{(4x10^{17})(4x10^{17})}{4x10^{17}+4x10^{17}}\right]$$

which yields

$$V_{bi}+V_R=1.456\ V$$

and

$$V_R=0.570\ V$$

17.22

(a) We have

$$\frac{C_j(0)}{C_j(10)}=\frac{\left[\dfrac{\in N_aN_d}{2V_{bi}(N_a+N_d)}\right]^{1/2}}{\left[\dfrac{\in N_aN_d}{2(V_{bi}+V_R)(N_a+N_d)}\right]^{1/2}}$$

or

$$\frac{C_j(0)}{C_j(10)}=3.13=\left(\frac{V_{bi}+V_R}{V_{bi}}\right)^{1/2}$$

For $V_R = 10\ V$, we find

$$(3.13)^2 V_{bi} = V_{bi} + 10$$

or

$$\underline{V_{bi} = 1.14\ V}$$

(b)

$$x_p = 0.2W = 0.2(x_p + x_n)$$

Then

$$\frac{x_p}{x_n} = 0.25 = \frac{N_d}{N_a}$$

Now

$$V_{bi} = V_t \ln\left(\frac{N_a N_d}{n_i^2}\right) \Rightarrow$$

so

$$1.14 = (0.0259)\ln\left[\frac{0.25 N_a^2}{\left(1.8x10^6\right)^2}\right]$$

We can then write

$$N_a = \frac{1.8x10^6}{\sqrt{0.25}} \exp\left[\frac{1.14}{2(0.0259)}\right]$$

or

$$\underline{N_a = 1.3x10^{16}\ cm^{-3}}$$

and

$$\underline{N_d = 3.25x10^{15}\ cm^{-3}}$$

7.23

$$V_{bi} = (0.0259)\ln\left[\frac{\left(10^{16}\right)\left(5x10^{16}\right)}{\left(1.8x10^6\right)^2}\right]$$

or

$$V_{bi} = 1.20\ V$$

Now

$$\frac{C_j'(V_{R1})}{C_j'(V_{R2})} = \frac{\left[\dfrac{1}{V_{bi} + V_{R1}}\right]^{1/2}}{\left[\dfrac{1}{V_{bi} + V_{R2}}\right]^{1/2}} = \left[\frac{V_{bi} + V_{R2}}{V_{bi} + V_{R1}}\right]^{1/2}$$

So

$$(3)^2 = \frac{1.20 + V_{R2}}{1.20 + 1} \Rightarrow$$

$$\underline{V_{R2} = 18.6\ V}$$

7.24

$$C' = \left[\frac{e \in N_a N_d}{2(V_{bi} + V_R)(N_a + N_d)}\right]^{1/2}$$

and

$$V_{bi} = (0.0259)\ln\left[\frac{\left(10^{18}\right)\left(10^{15}\right)}{\left(1.5x10^{10}\right)^2}\right]$$

or

$$V_{bi} = 0.754\ V$$

For $N_a >> N_d$, we have

$$C' = \left[\frac{\left(1.6x10^{-19}\right)(11.7)\left(8.85x10^{-14}\right)\left(10^{15}\right)}{2(V_{bi} + V_R)}\right]^{1/2}$$

or

$$C' = \left[\frac{8.28x10^{-17}}{V_{bi} + V_R}\right]^{1/2}$$

For $V_R = 1\ V$, $C' = 6.87x10^{-9}\ F/cm^2$

For $V_R = 10\ V$, $C' = 2.77x10^{-9}\ F/cm^2$

If $A = 6x10^{-4}\ cm^2$, then

For $V_R = 1\ V$, $C = 4.12\ pF$

For $V_R = 10\ V$, $C = 1.66\ pF$

The resonant frequency is given by

$$f_O = \frac{1}{2\pi\sqrt{LC}}$$

so that

For $V_R = 1\ V$, $\underline{f_O = 1.67\ MHz}$

For $V_R = 10\ V$, $\underline{f_O = 2.63\ MHz}$

7.25

$$\left|\mathrm{E}_{max}\right| = \frac{eN_d x_n}{\in}$$

For a p^+n junction,

$$x_n \approx \left[\frac{2 \in (V_{bi} + V_R)}{eN_d}\right]^{1/2}$$

so that

$$\left|\mathrm{E}_{max}\right| = \left[\frac{2eN_d}{\in}(V_{bi} + V_R)\right]^{1/2}$$

Assuming that $V_{bi} << V_R$, then

$$N_d = \frac{\in E_{\max}^2}{2eV_R} = \frac{(11.7)(8.85x10^{-14})(10^6)^2}{2(1.6x10^{-19})(10)}$$

or

$$\underline{N_d = 3.24x10^{17}\ cm^{-3}}$$

7.26

$$x_n = 0.1W = 0.1(x_n + x_p)$$

which yields

$$\frac{x_p}{x_n} = \frac{N_d}{N_a} = 9$$

We can write

$$V_{bi} = V_t \ln\left(\frac{N_a N_d}{n_i^2}\right)$$

$$= (0.0259)\ln\left[\frac{9N_a^2}{(1.5x10^{10})^2}\right]$$

We also have

$$C_j' = \frac{C_T}{A} = \frac{3.5x10^{-12}}{5.5x10^{-4}} = 6.36x10^{-9}\ F/cm^2$$

so

$$6.36x10^{-9} = \left[\frac{e \in N_a N_d}{2(V_{bi}+V_R)(N_a+N_d)}\right]^{1/2}$$

Which becomes

$$4.05x10^{-17} = \frac{(1.6x10^{-19})(11.7)(8.85x10^{-14})N_a(9N_a)}{2(V_{bi}+V_R)(N_a+9N_a)}$$

or

$$4.05x10^{-17} = \frac{7.46x10^{-32}\,N_a}{(V_{bi}+V_R)}$$

If $V_R = 1.2\ V$, then by iteration we find

$$\underline{N_a = 9.92x10^{14}\ cm^{-3}}$$

$$\underline{V_{bi} = 0.632\ V}$$

$$\underline{N_d = 8.93x10^{15}\ cm^{-3}}$$

7.27

(a) $V_{bi} = V_t \ln\left(\frac{N_a N_d}{n_i^2}\right)$

$$= (0.0259)\ln\left[\frac{(5x10^{15})(10^{14})}{(1.5x10^{10})^2}\right]$$

or

$$\underline{V_{bi} = 0.557\ V}$$

(b)

$$x_p = \left[\frac{2\in V_{bi}}{e}\left(\frac{N_d}{N_a}\right)\left(\frac{1}{N_a+N_d}\right)\right]^{1/2}$$

$$= \left[\frac{2(11.7)(8.85x10^{-14})(0.557)}{1.6x10^{-19}} \times \left(\frac{10^{14}}{5x10^{15}}\right)\left(\frac{1}{10^{14}+5x10^{15}}\right)\right]^{1/2}$$

or

$$\underline{x_p = 5.32x10^{-6}\ cm}$$

Also

$$x_n = \left[\frac{2\in V_{bi}}{e}\left(\frac{N_a}{N_d}\right)\left(\frac{1}{N_a+N_d}\right)\right]^{1/2}$$

$$= \left[\frac{2(11.7)(8.85x10^{-14})(0.557)}{1.6x10^{-19}} \times \left(\frac{5x10^{15}}{10^{14}}\right)\left(\frac{1}{10^{14}+5x10^{15}}\right)\right]^{1/2}$$

or

$$\underline{x_n = 2.66x10^{-4}\ cm}$$

(c)

For $x_n = 30\ \mu m$, we have

$$30x10^{-4} = \left[\frac{2(11.7)(8.85x10^{-14})(V_{bi}+V_R)}{1.6x10^{-19}} \times \left(\frac{5x10^{15}}{10^{14}}\right)\left(\frac{1}{10^{14}+5x10^{15}}\right)\right]^{1/2}$$

which becomes

$$9x10^{-6} = 1.27x10^{-7}(V_{bi}+V_R)$$

We find

$$\underline{V_R = 70.3\ V}$$

7.28

An n^+p junction with $N_a = 10^{14}\ cm^{-3}$,

(a)

A one-sided junction and assume $V_R >> V_{bi}$, then

$$x_p = \left[\frac{2\in V_R}{eN_a}\right]^{1/2}$$

so

$$\left(50x10^{-4}\right)^2 = \frac{2(11.7)\left(8.85x10^{-14}\right)V_R}{\left(1.6x10^{-19}\right)\left(10^{14}\right)}$$

which yields

$$\underline{V_R = 193\ V}$$

(b)

$$\frac{x_p}{x_n} = \frac{N_d}{N_a} \Rightarrow x_n = x_p\left(\frac{N_a}{N_d}\right)$$

so

$$x_n = \left(50x10^{-4}\right)\left(\frac{10^{14}}{10^{16}}\right) \Rightarrow$$

or

$$\underline{x_n = 0.5\ \mu m}$$

(c)

$$E_{max} = \frac{2\left(V_{bi} + V_R\right)}{W} = \frac{2(193)}{50.5x10^{-4}}$$

or

$$\underline{E_{max} = 7.72x10^4\ V/cm}$$

7.29

(a) $V_{bi} = (0.0259)\ln\left[\frac{\left(10^{18}\right)\left(5x10^{15}\right)}{\left(1.5x10^{10}\right)^2}\right]$

or

$$V_{bi} = 0.796\ V$$

$$C = AC' = A\left[\frac{e \in N_a N_d}{2\left(V_{bi} + V_R\right)\left(N_a + N_d\right)}\right]^{1/2}$$

$$= \left(5x10^{-5}\right)\left[\frac{\left(1.6x10^{-19}\right)(11.7)\left(8.85x10^{-14}\right)}{2\left(V_{bi} + V_R\right)} \times \frac{\left(10^{18}\right)\left(5x10^{15}\right)}{\left(10^{18} + 5x10^{15}\right)}\right]^{1/2}$$

or

$$C = \left(5x10^{-5}\right)\left[\frac{4.121x10^{-16}}{\left(V_{bi} + V_R\right)}\right]^{1/2}$$

For $V_R = 0$, $\underline{C = 1.14\ pF}$

For $V_R = 3\ V$, $\underline{C = 0.521\ pF}$

For $V_R = 6\ V$, $\underline{C = 0.389\ pF}$

We can write

$$\left(\frac{1}{C}\right)^2 = \frac{1}{A^2}\left[\frac{2\left(V_{bi} + V_R\right)\left(N_a + N_d\right)}{e \in N_a N_d}\right]$$

For the p^+n junction

$$\left(\frac{1}{C}\right)^2 \approx \frac{1}{A^2}\left[\frac{2\left(V_{bi} + V_R\right)}{e \in N_d}\right]$$

so that

$$\frac{\Delta(1/C)^2}{\Delta V_R} = \frac{1}{A^2} \cdot \frac{2}{e \in N_d}$$

We have

For $V_R = 0$, $\left(\frac{1}{C}\right)^2 = 7.69x10^{23}$

For $V_R = 6\ V$, $\left(\frac{1}{C}\right)^2 = 6.61x10^{24}$

Then, for $\Delta V_R = 6\ V$,

$$\Delta(1/C)^2 = 5.84x10^{24}$$

We find

$$N_d = \frac{2}{A^2 e \in} \cdot \frac{1}{\left(\frac{\Delta(1/C)^2}{\Delta V_R}\right)}$$

$$= \frac{2}{\left(5x10^{-5}\right)^2\left(1.6x10^{-19}\right)(11.7)\left(8.85x10^{-14}\right)} \times \frac{1}{\left(\frac{5.84x10^{24}}{6}\right)}$$

so that

$$\underline{N_d = 4.96x10^{15} \approx 5x10^{15}\ cm^{-3}}$$

Now, for a straight line

$$y = mx + b$$

$$m = \frac{\Delta(1/C)^2}{\Delta V_R} = \frac{5.84x10^{24}}{6}$$

At $V_R = 0$, $\left(\frac{1}{C}\right)^2 = 7.69x10^{23} = b$

Then

$$\left(\frac{1}{C}\right)^2 = \left(\frac{5.84x10^{24}}{6}\right) \cdot V_R + 7.69x10^{23}$$

Now, at $\left(\frac{1}{C}\right)^2 = 0$,

$$0 = \left(\frac{5.84x10^{24}}{6}\right) \cdot V_R + 7.69x10^{23}$$

which yields

$$\underline{V_R = -V_{bi} = -0.790\ V}$$

or

$$V_{bi} \approx 0.796\ V$$

(b)

$$V_{bi} = (0.0259)\ln\left[\frac{(10^{18})(6x10^{16})}{(1.5x10^{10})^2}\right]$$

or

$$V_{bi} = 0.860\ V$$

$$C = (5x10^{-5})\left[\frac{(1.6x10^{-19})(11.7)(8.85x10^{-14})}{2(V_{bi}+V_R)} \times \frac{(10^{18})(6x10^{16})}{(10^{18}+6x10^{16})}\right]^{1/2}$$

or

$$C = (5x10^{-5})\left[\frac{4.689x10^{-15}}{V_{bi}+V_R}\right]^{1/2}$$

Then

For $V_R = 0$, $\underline{C = 3.69\ pF}$

For $V_R = 3\ V$, $\underline{C = 1.74\ pF}$

For $V_R = 6\ V$, $\underline{C = 1.31\ pF}$

7.30

$$C' = \frac{C}{A} = \frac{1.3x10^{-12}}{10^{-5}} = 1.3x10^{-7}\ F/cm^2$$

(a) For a one-sided junction

$$C' = \left[\frac{e \in N_L}{2(V_{bi}+V_R)}\right]^{1/2}$$

where N_L is the doping concentration in the low-doped region.

We have $V_{bi} + V_R = 0.95 + 0.05 = 1.00\ V$

Then

$$(1.3x10^{-7})^2 = \frac{(1.6x10^{-19})(11.7)(8.85x10^{-14})N_L}{2(1)}$$

which yields

$$\underline{N_L = 2.04x10^{17}\ cm^{-3}}$$

(b)

$$V_{bi} = V_t \ln\left(\frac{N_L N_H}{n_i^2}\right)$$

where N_H is the doping concentration in the high-doped region.

So

$$0.95 = (0.0259)\ln\left[\frac{(2.04x10^{17})N_H}{(1.5x10^{10})^2}\right]$$

which yields

$$\underline{N_H = 9.38x10^{18}\ cm^{-3}}$$

7.31

Computer Plot

7.32

(a) $V_{bi} = V_t \ln\left(\dfrac{N_{aO}N_{dO}}{n_i^2}\right)$

(c) p-region

$$\frac{d\mathrm{E}}{dx} = \frac{\rho(x)}{\in} = \frac{-eN_{aO}}{\in}$$

or

$$\mathrm{E} = \frac{-eN_{aO}x}{\in} + C_1$$

We have

$$\mathrm{E} = 0 \text{ at } x = -x_p \Rightarrow C_1 = \frac{-eN_{aO}x_p}{\in}$$

Then for $\underline{-x_p < x < 0}$

$$\underline{\mathrm{E} = \frac{-eN_{aO}}{\in}(x + x_p)}$$

n-region, $0 < x < x_O$

$$\frac{d\mathrm{E}_1}{dx} = \frac{\rho(x)}{\in} = \frac{eN_{dO}}{2\in}$$

or

$$\mathrm{E}_1 = \frac{eN_{dO}x}{2\in} + C_2$$

n-region, $x_O < x < x_n$

$$\frac{d\mathrm{E}_2}{dx} = \frac{\rho(x)}{\in} = \frac{eN_{dO}}{\in}$$

or

$$\mathrm{E}_2 = \frac{eN_{dO}x}{\in} + C_3$$

We have $\mathrm{E}_2 = 0$ at $x = x_n$, then

$$C_3 = \frac{-eN_{dO}x_n}{\in}$$

so that for $\underline{x_O < x < x_n}$

$$\underline{\mathrm{E}_2 = \frac{-eN_{dO}}{\in}(x_n - x)}$$

We also have

$E_2 = E_1$ at $x = x_O$

Then

$$\frac{eN_{dO}x_O}{2\in} + C_2 = \frac{-eN_{dO}}{\in}\left(x_n - x_O\right)$$

or

$$C_2 = \frac{-eN_{dO}}{\in}\left(x_n - \frac{x_O}{2}\right)$$

Then, for $0 < x < x_O$,

$$E_1 = \frac{eN_{dO}x}{2\in} - \frac{eN_{dO}}{\in}\left(x_n - \frac{x_O}{2}\right)$$

7.33

(a) $\dfrac{d^2\phi(x)}{dx^2} = \dfrac{-\rho(x)}{\in} = \dfrac{-dE(x)}{dx}$

For $-2 < x < -1\ \mu m$, $\rho(x) = +eN_d$

So

$$\frac{dE}{dx} = \frac{eN_d}{\in} \Rightarrow E = \frac{eN_d x}{\in} + C_1$$

At $x = -2\ \mu m \equiv -x_O$, $E = 0$

So

$$0 = \frac{-eN_d x_O}{\in} + C_1 \Rightarrow C_1 = \frac{eN_d x_O}{\in}$$

Then

$$E = \frac{eN_d}{\in}\left(x + x_O\right)$$

At $x = 0$, $E(0) = E(x = -1\ \mu m)$, so

$$E(0) = \frac{eN_d}{\in}(-1+2)x10^{-4}$$

$$= \frac{\left(1.6x10^{-19}\right)\left(5x10^{15}\right)}{(11.7)\left(8.85x10^{-14}\right)}\left(1x10^{-4}\right)$$

which yields

$$E(0) = 7.73x10^{4}\ V/cm$$

(c)

Magnitude of potential difference is

$$|\phi| = \int E dx = \frac{eN_d}{\in}\int\left(x + x_O\right)dx$$

$$= \frac{eN_d}{\in}\left(\frac{x^2}{2} + x_O \cdot x\right) + C_2$$

Let $\phi = 0$ at $x = -x_O$, then

$$0 = \frac{eN_d}{\in}\left(\frac{x_O^2}{2} - x_O^2\right) + C_2 \Rightarrow C_2 = \frac{eN_d x_O^2}{2\in}$$

Then we can write

$$|\phi| = \frac{eN_d}{2\in}\left(x + x_O\right)^2$$

At $x = -1\ \mu m$

$$|\phi_1| = \frac{\left(1.6x10^{-19}\right)\left(5x10^{15}\right)}{2(11.7)\left(8.85x10^{-14}\right)}\left[(-1+2)x10^{-4}\right]$$

or

$$|\phi_1| = 3.86\ V$$

Potential difference across the intrinsic region

$$|\phi_i| = E(0)\cdot d = \left(7.73x10^4\right)\left(2x10^{-4}\right)$$

or

$$|\phi_i| = 15.5\ V$$

By symmetry, potential difference across the p-region space charge region is also $3.86\ V$. The total reverse-bias voltage is then

$$V_R = 2(3.86) + 15.5 \Rightarrow V_R = 23.2\ V$$

7.34

(a) For the linearly graded junction,

$\rho(x) = eax$,

Then

$$\frac{dE}{dx} = \frac{\rho(x)}{\in} = \frac{eax}{\in}$$

Now

$$E = \int \frac{eax}{\in}dx = \frac{ea}{\in}\cdot\frac{x^2}{2} + C_1$$

At $x = +x_O$ and $x = -x_O$, $E = 0$

So

$$0 = \frac{ea}{\in}\left(\frac{x_O^2}{2}\right) + C_1 \Rightarrow C_1 = \frac{-ea}{\in}\left(\frac{x_O^2}{2}\right)$$

Then

$$E = \frac{ea}{2\in}\left(x^2 - x_O^2\right)$$

(b)

$$\phi(x) = -\int E dx = \frac{-ea}{2\in}\left[\frac{x^3}{3} - x_O^2 \cdot x\right] + C_2$$

Set $\phi = 0$ at $x = -x_O$, then

$$0 = \frac{-ea}{2\in}\left[\frac{-x_O^3}{3} + x_O^3\right] + C_2 \Rightarrow C_2 = \frac{eax_O^3}{3\in}$$

Then

$$\phi(x) = \frac{-ea}{2\in}\left(\frac{x^3}{3} - x_O^2 \cdot x\right) + \frac{eax_O^3}{3\in}$$

7.35

We have that

$$C' = \left[\frac{ea \in^2}{12(V_{bi} + V_R)} \right]^{1/3}$$

then

$$\left(7.2x10^{-9}\right)^3 = \left[\frac{a\left(1.6x10^{-19}\right)\left[(11.7)\left(8.85x10^{-14}\right)\right]^2}{12(0.7 + 3.5)} \right]$$

which yields

$$\underline{a = 1.1x10^{20}\ cm^{-4}}$$

Chapter 8

Problem Solutions

8.1
In the forward bias
$$I_f \approx I_S \exp\left(\frac{eV}{kT}\right)$$
Then
$$\frac{I_{f1}}{I_{f2}} = \frac{I_S}{I_S} \cdot \frac{\exp\left(\frac{eV_1}{kT}\right)}{\exp\left(\frac{eV_2}{kT}\right)} = \exp\left[\frac{e}{kT}(V_1 - V_2)\right]$$
or
$$V_1 - V_2 = \left(\frac{kT}{e}\right)\ln\left(\frac{I_{f1}}{I_{f2}}\right)$$
(a)
$$\text{For } \frac{I_{f1}}{I_{f2}} = 10 \Rightarrow \underline{V_1 - V_2 = 59.9\ mV \approx 60\ mV}$$
(b)
$$\text{For } \frac{I_{f1}}{I_{f2}} = 100 \Rightarrow \underline{V_1 - V_2 = 119.3\ mV \approx 120 mV}$$

8.2
$$I = I_S\left[\exp\left(\frac{eV}{kT}\right) - 1\right]$$
or we can write this as
$$\frac{I}{I_S} + 1 = \exp\left(\frac{eV}{kT}\right)$$
so that
$$V = \left(\frac{kT}{e}\right)\ln\left(\frac{I}{I_S} + 1\right)$$
In reverse bias, I is negative, so at
$\frac{I}{I_S} = -0.90$, we have
$$V = (0.0259)\ln(1 - 0.90) \Rightarrow$$
or
$$V = -59.6\ mV$$

8.3
Computer Plot

8.4
The cross-sectional area is
$$A = \frac{I}{J} = \frac{10x10^{-3}}{20} = 5x10^{-4}\ cm^2$$
We have
$$J \approx J_S \exp\left(\frac{V_D}{V_t}\right) \Rightarrow\ 20 = J_S \exp\left(\frac{0.65}{0.0259}\right)$$
so that
$$J_S = 2.52x10^{-10}\ A/cm^2$$
We can write
$$J_S = en_i^2\left[\frac{1}{N_a} \cdot \sqrt{\frac{D_n}{\tau_{nO}}} + \frac{1}{N_d} \cdot \sqrt{\frac{D_p}{\tau_{pO}}}\right] \backslash$$
We want
$$\frac{\frac{1}{N_a} \cdot \sqrt{\frac{D_n}{\tau_{nO}}}}{\frac{1}{N_a} \cdot \sqrt{\frac{D_n}{\tau_{nO}}} + \frac{1}{N_d} \cdot \sqrt{\frac{D_p}{\tau_{pO}}}} = 0.10$$
or
$$\frac{\frac{1}{N_a} \cdot \sqrt{\frac{25}{5x10^{-7}}}}{\frac{1}{N_a} \cdot \sqrt{\frac{25}{5x10^{-7}}} + \frac{1}{N_d} \cdot \sqrt{\frac{10}{5x10^{-7}}}}$$
$$= \frac{7.07x10^3}{7.07x10^3 + \frac{N_a}{N_d}\left(4.47x10^3\right)} = 0.10$$
which yields
$$\frac{N_a}{N_d} = 14.24$$
Now
$$J_S = 2.52x10^{-10} = \left(1.6x10^{-19}\right)\left(1.5x10^{10}\right)^2$$
$$\times\left[\frac{1}{(14.24)N_d} \cdot \sqrt{\frac{25}{5x10^{-7}}} + \frac{1}{N_d} \cdot \sqrt{\frac{10}{5x10^{-7}}}\right]$$
We find
$$N_d = 7.1x10^{14}\ cm^{-3}$$
and
$$\underline{N_a = 1.01x10^{16}\ cm^{-3}}$$

8.5
(a)

$$\frac{J_n}{J_n+J_p}=\frac{\dfrac{eD_n n_{pO}}{L_n}}{\dfrac{eD_n n_{pO}}{L_n}+\dfrac{eD_p p_{nO}}{L_p}}$$

$$=\frac{\sqrt{\dfrac{D_n}{\tau_{nO}}}\cdot\dfrac{n_i^2}{N_a}}{\sqrt{\dfrac{D_n}{\tau_{nO}}}\cdot\dfrac{n_i^2}{N_a}+\sqrt{\dfrac{D_p}{\tau_{pO}}}\cdot\dfrac{n_i^2}{N_d}}$$

$$=\frac{1}{1+\sqrt{\dfrac{D_p\tau_{nO}}{D_n\tau_{pO}}}\cdot\left(\dfrac{N_a}{N_d}\right)}$$

We have

$$\frac{D_p}{D_n}=\frac{\mu_p}{\mu_n}=\frac{1}{2.4}\quad\text{and}\quad\frac{\tau_{nO}}{\tau_{pO}}=\frac{1}{0.1}$$

so

$$\frac{J_n}{J_n+J_p}=\frac{1}{1+\sqrt{\dfrac{1}{2.4}\cdot\dfrac{1}{0.1}}\left(\dfrac{N_a}{N_d}\right)}$$

or

$$\underline{\frac{J_n}{J_n+J_p}=\frac{1}{1+(2.04)\left(\dfrac{N_a}{N_d}\right)}}$$

(b)
Using Einstein's relation, we can write

$$\frac{J_n}{J_n+J_p}=\frac{\dfrac{e\mu_n}{L_n}\cdot\dfrac{n_i^2}{N_a}}{\dfrac{e\mu_n}{L_n}\cdot\dfrac{n_i^2}{N_a}+\dfrac{e\mu_p}{L_p}\cdot\dfrac{n_i^2}{N_d}}$$

$$=\frac{e\mu_n N_d}{e\mu_n N_d+\dfrac{L_n}{L_p}\cdot e\mu_p N_a}$$

We have

$$\sigma_n=e\mu_n N_d\quad\text{and}\quad\sigma_p=e\mu_p N_a$$

Also

$$\frac{L_n}{L_p}=\sqrt{\frac{D_n\tau_{nO}}{D_p\tau_{pO}}}=\sqrt{\frac{2.4}{0.1}}=4.90$$

Then

$$\underline{\frac{J_n}{J_n+J_p}=\frac{(\sigma_n/\sigma_p)}{(\sigma_n/\sigma_p)+4.90}}$$

8.6
For a silicon p^+n junction,

$$I_S=Aen_i^2\cdot\frac{1}{N_d}\sqrt{\frac{D_p}{\tau_{pO}}}$$

$$=\left(10^{-4}\right)\left(1.6x10^{-19}\right)\left(1.5x10^{10}\right)^2\cdot\frac{1}{10^{16}}\sqrt{\frac{12}{10^{-7}}}$$

or

$$\underline{I_S=3.94x10^{-15}\ A}$$

Then

$$I_D=I_S\exp\left(\frac{V_D}{V_t}\right)=\left(3.94x10^{-15}\right)\exp\left(\frac{0.50}{0.0259}\right)$$

or

$$\underline{I_D=9.54x10^{-7}\ A}$$

8.7
We want

$$\frac{J_n}{J_n+J_p}=0.95$$

$$=\frac{\dfrac{eD_n n_{pO}}{L_n}}{\dfrac{eD_n n_{pO}}{L_n}+\dfrac{eD_p p_{nO}}{L_p}}=\frac{\dfrac{D_n}{L_nN_a}}{\dfrac{D_n}{L_nN_a}+\dfrac{D_p}{L_pN_d}}$$

$$=\frac{\dfrac{D_n}{L_n}}{\dfrac{D_n}{L_n}+\dfrac{D_p}{L_p}\cdot\dfrac{N_a}{N_d}}$$

We obtain

$$L_n=\sqrt{D_n\tau_{nO}}=\sqrt{(25)\left(0.1x10^{-6}\right)}\Rightarrow$$

$$L_n=15.8\ \mu m$$

$$L_p=\sqrt{D_p\tau_{pO}}=\sqrt{(10)\left(0.1x10^{-6}\right)}\Rightarrow$$

$$L_p=10\ \mu m$$

Then

$$0.95 = \frac{\dfrac{25}{15.8}}{\dfrac{25}{15.8} + \dfrac{10}{10} \cdot \left(\dfrac{N_a}{N_d}\right)}$$

which yields

$$\frac{N_a}{N_d} = 0.083$$

8.8

(a) p-side: $E_{Fi} - E_F = kT \ln\left(\dfrac{N_a}{n_i}\right)$

$$= (0.0259) \ln\left(\frac{5x10^{15}}{1.5x10^{10}}\right) \Rightarrow$$

$$E_{Fi} - E_F = 0.329\ eV$$

Also

n-side: $E_F - E_{Fi} = kT \ln\left(\dfrac{N_d}{n_i}\right)$

$$= (0.0259) \ln\left(\frac{10^{17}}{1.5x10^{10}}\right) \Rightarrow$$

$$E_F - E_{Fi} = 0.407\ eV$$

(b)
We can find

$$D_n = (1250)(0.0259) = 32.4\ cm^2 / s$$

$$D_p = (420)(0.0259) = 10.9\ cm^2 / s$$

Now

$$J_S = en_i^2 \left[\frac{1}{N_a}\sqrt{\frac{D_n}{\tau_{nO}}} + \frac{1}{N_d}\sqrt{\frac{D_p}{\tau_{pO}}}\right]$$

$$= \left(1.6x10^{-19}\right)\left(1.5x10^{10}\right)^2$$

$$\times \left[\frac{1}{5x10^{15}}\sqrt{\frac{32.4}{10^{-6}}} + \frac{1}{10^{17}}\sqrt{\frac{10.9}{10^{-7}}}\right]$$

or

$$J_S = 4.48x10^{-11}\ A / cm^2$$

Then

$$I_S = AJ_S = \left(10^{-4}\right)\left(4.48x10^{-11}\right)$$

or

$$I_S = 4.48x10^{-15}\ A$$

We find

$$I = I_S \exp\left(\frac{V_D}{V_t}\right)$$

$$= \left(4.48x10^{-15}\right) \exp\left(\frac{0.5}{0.0259}\right)$$

or

$$I = 1.08\ \mu A$$

(c)
The hole current is proportional to

$$I_P \propto en_i^2 \cdot A \cdot \frac{1}{N_d}\sqrt{\frac{D_p}{\tau_{pO}}}$$

$$= \left(1.6x10^{-19}\right)\left(1.5x10^{10}\right)^2 \left(10^{-4}\right)\left(\frac{1}{10^{17}}\right)\sqrt{\frac{10.9}{10^{-7}}}$$

or

$$I_P \propto 3.76x10^{-16}\ A$$

Then

$$\frac{I_P}{I} = \frac{3.76x10^{-16}}{4.48x10^{-15}} \Rightarrow \frac{I_P}{I} = 0.0839$$

8.9

$$I = I_S\left[\exp\left(\frac{V_a}{V_t}\right) - 1\right]$$

For a p^+n diode,

$$I_S = A\left(\frac{eD_p p_{nO}}{L_p}\right) = A\left(e\sqrt{\frac{D_p}{\tau_{pO}}} \cdot \frac{n_i^2}{N_d}\right)$$

$$= \left(10^{-4}\right)\left[\left(1.6x10^{-19}\right)\sqrt{\frac{10}{10^{-6}}} \cdot \frac{\left(2.4x10^{13}\right)^2}{10^{16}}\right]$$

or

$$I_S = 2.91x10^{-9}\ A$$

(a)
For $V_a = +0.2\ V$,

$$I = \left(2.91x10^{-9}\right)\left[\exp\left(\frac{0.2}{0.0259}\right) - 1\right]$$

or

$$I = 6.55\ \mu A$$

(b)
For $V_a = -0.2\ V$,

$$I = \left(2.91x10^{-9}\right)\left[\exp\left(\frac{-0.2}{0.0259}\right) - 1\right]$$

$$\approx -2.91x10^{-9}\ A$$

or $I = -I_S = -2.91\ nA$

8.10

For an n^+p silicon diode

$$I_S = Aen_i^2 \cdot \frac{1}{N_a}\sqrt{\frac{D_n}{\tau_{nO}}}$$

$$= \frac{(10^{-4})(1.6x10^{-19})(1.5x10^{10})^2}{10^{16}}\sqrt{\frac{25}{10^{-6}}}$$

or

$$\underline{I_S = 1.8x10^{-15}\ A}$$

(a)

For $V_a = 0.5\ V$

$$I_D = I_S \exp\left(\frac{V_a}{V_t}\right) = (1.8x10^{-15})\exp\left(\frac{0.5}{0.0259}\right)$$

or

$$\underline{I_D = 4.36x10^{-7}\ A}$$

(b)

For $V_a = -0.5\ V$

$$\underline{I_D = -I_S = -1.8x10^{-15}\ A}$$

8.11

(a) We find

$$D_p = \mu_p\left(\frac{kT}{e}\right) = (480)(0.0259) = 12.4\ cm^2/s$$

and

$$L_p = \sqrt{D_p\tau_{pO}} = \sqrt{(12.4)(0.1x10^{-6})} \Rightarrow$$

$$L_p = 11.1\ \mu m$$

Also

$$p_{nO} = \frac{n_i^2}{N_d} = \frac{(1.5x10^{10})^2}{10^{15}} = 2.25x10^5\ cm^{-3}$$

Then

$$J_{pO} = \frac{eD_p p_{nO}}{L_p} = \frac{(1.6x10^{-19})(12.4)(2.25x10^5)}{(11.1x10^{-4})}$$

or

$$J_{pO} = 4.02x10^{-10}\ A/cm^2$$

For $A = 10^{-4}\ cm^2$, then

$$\underline{I_{pO} = 4.02x10^{-14}\ A}$$

(b)

We have

$$D_n = \mu_n\left(\frac{kT}{e}\right) = (1350)(0.0259) = 35\ cm^2/s$$

and

$$L_n = \sqrt{D_n\tau_{nO}} = \sqrt{(35)(0.4x10^{-6})} \Rightarrow$$

$$L_n = 37.4\ \mu m$$

Also

$$n_{pO} = \frac{n_i^2}{N_a} = \frac{(1.5x10^{10})^2}{5x10^{15}} = 4.5x10^4\ cm^{-3}$$

Then

$$J_{nO} = \frac{eD_n n_{pO}}{L_n} = \frac{(1.6x10^{-19})(35)(4.5x10^4)}{(37.4x10^{-4})}$$

or

$$J_{nO} = 6.74x10^{-11}\ A/cm^2$$

For $A = 10^{-4}\ cm^2$, then

$$\underline{I_{nO} = 6.74x10^{-15}\ A}$$

(c)

$$V_{bi} = V_t \ln\left(\frac{N_a N_d}{n_i^2}\right)$$

$$= (0.0259)\ln\left[\frac{(5x10^{15})(10^{15})}{(1.5x10^{10})^2}\right]$$

or

$$V_{bi} = 0.617\ V$$

Then for

$$V_a = \frac{1}{2}V_{bi} = 0.309\ V$$

We find

$$p_n = p_{nO}\exp\left(\frac{eV_a}{kT}\right)$$

$$= (2.25x10^5)\exp\left(\frac{0.309}{0.0259}\right)$$

or

$$\underline{p_n = 3.42x10^{10}\ cm^{-3}}$$

(d)

The total current is

$$I = (I_{pO} + I_{nO})\exp\left(\frac{eV_a}{kT}\right)$$

$$= (4.02x10^{-14} + 6.74x10^{-15})\exp\left(\frac{0.309}{0.0259}\right)$$

or

$$\underline{I = 7.13x10^{-9}\ A}$$

The hole current is

$$I_p = I_{pO} \exp\left(\frac{eV_a}{kT}\right) \exp\left[\frac{-(x - x_n)}{L_p}\right]$$

The electron current is given by

$$I_n = I - I_p$$

$$= 7.13x10^{-9} - (4.02x10^{-14})$$

$$\times \exp\left(\frac{0.309}{0.0259}\right) \exp\left[\frac{-(x - x_n)}{L_p}\right]$$

At $x = x_n + \frac{1}{2} L_p$

$$I_n = 7.13x10^{-9} - (6.10x10^{-9}) \exp\left(\frac{-1}{2}\right)$$

or

$$\underline{I_n = 3.43x10^{-9} \ A}$$

8.12

(a) The excess hole concentration is given by

$$\delta p_n = p_n - p_{nO}$$

$$= p_{nO}\left[\exp\left(\frac{eV_a}{kT}\right) - 1\right] \exp\left(\frac{-x}{L_p}\right)$$

We find

$$p_{nO} = \frac{n_i^2}{N_d} = \frac{(1.5x10^{10})^2}{10^{16}} = 2.25x10^4 \ cm^{-3}$$

and

$$L_p = \sqrt{D_p \tau_{pO}} = \sqrt{(8)(0.01x10^{-6})} \Rightarrow$$

$$L_p = 2.83 \ \mu m$$

Then

$$\delta p_n = (2.25x10^4)$$

$$\times \left[\exp\left(\frac{0.610}{0.0259}\right) - 1\right] \exp\left(\frac{-x}{2.83x10^{-4}}\right)$$

or

$$\underline{\delta p_n = 3.81x10^{14} \exp\left(\frac{-x}{2.83x10^{-4}}\right) cm^{-3}}$$

(b)

We have

$$J_p = -eD_p \frac{d(\delta p_n)}{dx}$$

$$= \frac{eD_p (3.81x10^{14})}{(2.83x10^{-4})} \exp\left(\frac{-x}{2.83x10^{-4}}\right)$$

At $x = 3x10^{-4} \ cm$,

$$J_p = \frac{(1.6x10^{-19})(8)(3.81x10^{14})}{2.83x10^{-4}} \exp\left(\frac{-3}{2.83}\right)$$

or

$$\underline{J_p = 0.597 \ A/cm^2}$$

(c)

We have

$$J_{nO} = \frac{eD_n n_{pO}}{L_n} \exp\left(\frac{eV_a}{kT}\right)$$

We can determine that

$$n_{pO} = 4.5x10^3 \ cm^{-3} \text{ and } L_n = 10.7 \ \mu m$$

Then

$$J_{nO} = \frac{(1.6x10^{-19})(23)(4.5x10^3)}{10.7x10^{-4}} \exp\left(\frac{0.610}{0.0259}\right)$$

or

$$J_{nO} = 0.262 \ A/cm^2$$

We can also find

$$\underline{J_{pO} = 1.72 \ A/cm^2}$$

Then, at $x = 3 \ \mu m$,

$$J_n(3 \ \mu m) = J_{nO} + J_{pO} - J_p(3 \ \mu m)$$

$$= 0.262 + 1.72 - 0.597$$

or

$$\underline{J_n(3 \ \mu m) = 1.39 \ A/cm^2}$$

8.13

(a) From Problem 8.9 (Ge diode)

Low injection means

$$p_n(0) = (0.1)N_d = 10^{15} \ cm^{-3}$$

Now

$$p_{nO} = \frac{n_i^2}{N_d} = \frac{(2.4x10^{13})^2}{10^{16}} = 5.76x10^{10} \ cm^{-3}$$

We have

$$p_n(0) = p_{nO} \exp\left(\frac{V_a}{V_t}\right)$$

or

$$V_a = V_t \ln\left[\frac{p_n(0)}{p_{nO}}\right]$$

$$= (0.0259) \ln\left(\frac{10^{15}}{5.76x10^{10}}\right)$$

or

$$\underline{V_a = 0.253 \ V}$$

(b)

For Problem 8.10 (Si diode)

$$n_p(0) = (0.1)N_a = 10^{15}\ cm^{-3}$$

$$n_{pO} = \frac{n_i^2}{N_a} = \frac{(1.5x10^{10})^2}{10^{16}} = 2.25x10^4\ cm^{-3}$$

Then

$$V_a = V_t \ln\left[\frac{n_p(0)}{n_{pO}}\right]$$

$$= (0.0259)\ln\left(\frac{10^{15}}{2.25x10^4}\right)$$

or

$$\underline{V_a = 0.635\ V}$$

8.14

The excess electron concentration is given by

$$\delta n_p = n_p - n_{pO}$$

$$= n_{pO}\left[\exp\left(\frac{eV_a}{kT}\right) - 1\right]\exp\left(\frac{-x}{L_n}\right)$$

The total number of excess electrons is

$$N_p = A\int_0^\infty \delta n_p dx$$

We may note that

$$\int_0^\infty \exp\left(\frac{-x}{L_n}\right)dx = -L_n \exp\left(\frac{-x}{L_n}\right)\Bigg|_0^\infty = L_n$$

Then

$$N_p = AL_n n_{pO}\left[\exp\left(\frac{eV_a}{kT}\right) - 1\right]$$

We can find

$$D_n = 35\ cm^2/s \quad \text{and} \quad L_n = 59.2\ \mu m$$

Also

$$n_{pO} = \frac{n_i^2}{N_a} = \frac{(1.5x10^{10})^2}{8x10^{15}} = 2.81x10^4\ cm^{-3}$$

Then

$$N_p = (10^{-3})(59.2x10^{-4})(2.81x10^4) \times\left[\exp\left(\frac{eV_a}{kT}\right) - 1\right]$$

or

$$N_p = 0.166\left[\exp\left(\frac{eV_a}{kT}\right) - 1\right]$$

Then we find the total number of excess electrons in the p-region to be:

(a) $V_a = 0.3\ V$, $\underline{N_p = 1.78x10^4}$

(b) $V_a = 0.4\ V$, $\underline{N_p = 8.46x10^5}$

(c) $V_a = 0.5\ V$, $\underline{N_p = 4.02x10^7}$

Similarly, the total number of excess holes in the n-region is found to be:

$$N_n = AL_p p_{nO}\left[\exp\left(\frac{eV_a}{kT}\right) - 1\right]$$

We find that

$$D_p = 12.4\ cm^2/s \quad \text{and} \quad L_p = 11.1\ \mu m$$

Also

$$p_{nO} = \frac{(1.5x10^{10})^2}{10^{16}} = 2.25x10^4\ cm^{-3}$$

Then

$$N_n = (2.50x10^{-2})\left[\exp\left(\frac{eV_a}{kT}\right) - 1\right]$$

So

(a) $V_a = 0.3\ V$, $\underline{N_n = 2.68x10^3}$

(b) $V_a = 0.4\ V$, $\underline{N_n = 1.27x10^5}$

(c) $V_a = 0.5\ V$, $\underline{N_n = 6.05x10^6}$

8.15

$$I \propto n_i^2 \exp\left(\frac{eV_a}{kT}\right) \propto \exp\left(\frac{-E_g}{kT}\right)\exp\left(\frac{eV_a}{kT}\right)$$

Then

$$I \propto \exp\left(\frac{eV_a - E_g}{kT}\right)$$

so

$$\frac{I_1}{I_2} = \frac{\exp\left(\frac{eV_{a1} - E_{g1}}{kT}\right)}{\exp\left(\frac{eV_{a2} - E_{g2}}{kT}\right)}$$

or

$$\frac{I_1}{I_2} = \exp\left(\frac{eV_{a1} - eV_{a2} - E_{g1} + E_{g2}}{kT}\right)$$

We have

$$\frac{10x10^{-3}}{10x10^{-6}} = \exp\left(\frac{0.255 - 0.32 - 0.525 + E_{g2}}{0.0259}\right)$$

or

$$10^3 = \exp\left(\frac{E_{g2} - 0.59}{0.0259}\right)$$

Then

$$E_{g2} = 0.59 + (0.0259)\ln\left(10^3\right)$$

which yields

$$\underline{E_{g2} = 0.769\ eV}$$

8.16

(a) We have

$$I_S = Aen_i^2\left[\frac{1}{N_a}\sqrt{\frac{D_n}{\tau_{nO}}} + \frac{1}{N_d}\sqrt{\frac{D_p}{\tau_{pO}}}\right]$$

which can be written in the form

$$I_S = C'n_i^2$$

$$= C'N_{CO}N_{VO}\left(\frac{T}{300}\right)^3 \exp\left(\frac{-E_g}{kT}\right)$$

or

$$\underline{I_S = CT^3 \exp\left(\frac{-E_g}{kT}\right)}$$

(b)

Taking the ratio

$$\frac{I_{S2}}{I_{S1}} = \left(\frac{T_2}{T_1}\right)^3 \cdot \frac{\exp\left(\frac{-E_g}{kT_2}\right)}{\exp\left(\frac{-E_g}{kT_1}\right)}$$

$$= \left(\frac{T_2}{T_1}\right)^3 \cdot \exp\left[+E_g\left(\frac{1}{kT_1} - \frac{1}{kT_2}\right)\right]$$

For $T_1 = 300K$, $kT_1 = 0.0259$, $\frac{1}{kT_1} = 38.61$

For $T_2 = 400K$, $kT_2 = 0.03453$, $\frac{1}{kT_2} = 28.96$

(i) Germanium, $E_g = 0.66\ eV$

$$\frac{I_{S2}}{I_{S1}} = \left(\frac{400}{300}\right)^3 \exp\left[(0.66)(38.61 - 28.96)\right]$$

or

$$\underline{\frac{I_{S2}}{I_{S1}} = 1383}$$

(ii) Silicon, $E_g = 1.12\ eV$

$$\frac{I_{S2}}{I_{S1}} = \left(\frac{400}{300}\right)^3 \cdot \exp\left[(1.12)(38.61 - 28.96)\right]$$

or

$$\underline{\frac{I_{S2}}{I_{S1}} = 1.17x10^5}$$

8.17

Computer Plot

8.18

One condition:

$$\left|\frac{I_f}{I_r}\right| = \frac{J_S \exp\left(\frac{eV_a}{kT}\right)}{J_S} = \exp\left(\frac{eV_a}{kT}\right) = 10^4$$

or

$$\frac{kT}{e} = \frac{V_a}{\ln\left(10^4\right)} = \frac{0.5}{\ln\left(10^4\right)}$$

or

$$\frac{kT}{e} = 0.05429 = (0.0259)\left(\frac{T}{300}\right)$$

which yields

$$T = 629K$$

Second conditon:

$$I_S = A\left(\frac{eD_n n_{pO}}{L_n} + \frac{eD_p p_{nO}}{L_p}\right)$$

$$= Aen_i^2\left(\frac{D_n}{L_n N_a} + \frac{D_p}{L_p N_d}\right)$$

$$= AeN_C N_V\left[\frac{1}{N_a}\sqrt{\frac{D_n}{\tau_{nO}}} + \frac{1}{N_d}\sqrt{\frac{D_p}{\tau_{pO}}}\right]\exp\left(\frac{-E_g}{kT}\right)$$

which becomes

$$10^{-6} = \left(10^{-4}\right)\left(1.6x10^{-19}\right)\left(2.8x10^{19}\right)\left(1.04x10^{19}\right)$$

$$\times\left(\frac{1}{5x10^{18}}\sqrt{\frac{25}{10^{-7}}} + \frac{1}{10^{15}}\sqrt{\frac{10}{10^{-7}}}\right)\exp\left(\frac{-E_g}{kT}\right)$$

or

$$\exp\left(\frac{+E_g}{kT}\right) = 4.66x10^{10}$$

For $E_g = 1.10\ eV$,

$$kT = \frac{E_g}{\ln\left(4.66x10^{10}\right)} = \frac{1.10}{\ln\left(4.66x10^{10}\right)}$$

or

$$kT = 0.04478\ eV = (0.0259)\left(\frac{T}{300}\right)$$

Then

$$T = 519K$$

This second condition yields a smaller temperature, so the maximum temperature is

$$\underline{T = 519K}$$

8.19

(a) We can write for the n-region

$$\frac{d^2(\delta p_n)}{dx^2} - \frac{\delta p_n}{L_p^2} = 0$$

The general solution is

$$\delta p_n = A\exp(+x/L_p) + B\exp(-x/L_p)$$

The boundary condition at $x = x_n$ gives

$$\delta p_n(x_n) = p_{nO}\left[\exp\left(\frac{V_a}{V_t}\right) - 1\right]$$

$$= A\exp(+x_n/L_p) + B\exp(-x_n/L_p)$$

and the boundary condition at $x = x_n + W_n$ gives

$$\delta p_n(x_n + W_n) = 0$$

$$= A\exp[(x_n + W_n)/L_p] + B\exp[-(x_n + W_n)/L_p]$$

From this equation, we have

$$A = -B\exp[-2(x_n + W_n)/L_p]$$

Then, from the first boundary condition, we obtain

$$p_{nO}\left[\exp\left(\frac{V_a}{V_t}\right) - 1\right]$$

$$= B\exp[-(x_n + 2W_n)/L_p] + B\exp(-x_n/L_p)$$

$$= B\exp(-x_n/L_p)[1 - \exp(-2W_n/L_p)]$$

We then obtain

$$B = \frac{p_{nO}\left[\exp\left(\frac{V_a}{V_t}\right) - 1\right]}{\exp(-x_n/L_p)[1 - \exp(-2W_n/L_p)]}$$

which can be written in the form

$$B = \frac{p_{nO}\left[\exp\left(\frac{V_a}{V_t}\right) - 1\right]\cdot\exp[(x_n + W_n)/L_p]}{\exp(W_n/L_p) - \exp(-W_n/L_p)}$$

Also

$$A = \frac{-p_{nO}\left[\exp\left(\frac{V_a}{V_t}\right) - 1\right]\cdot\exp[-(x_n + W_n)/L_p]}{\exp(W_n/L_p) - \exp(-W_n/L_p)}$$

The solution can now be written as

$$\delta p_n = \frac{p_{nO}\left[\exp\left(\frac{V_a}{V_t}\right) - 1\right]}{2\sinh\left(\frac{W_n}{L_p}\right)}$$

$$\times\left\{\exp\left[\frac{(x_n + W_n - x)}{L_p}\right] - \exp\left[\frac{-(x_n + W_n - x)}{L_p}\right]\right\}$$

or finally,

$$\delta p_n = p_{nO}\left[\exp\left(\frac{V_a}{V_t}\right) - 1\right]\cdot\frac{\sinh\left(\frac{x_n + W_n - x}{L_p}\right)}{\sinh\left(\frac{W_n}{L_p}\right)}$$

(b)

$$J_p = -eD_p\frac{d(\delta p_n)}{dx}\bigg|_{x=x_n}$$

$$= \frac{-eD_p p_{nO}\left[\exp\left(\frac{V_a}{V_t}\right) - 1\right]}{\sinh\left(\frac{W_n}{L_p}\right)}$$

$$\times\left(\frac{-1}{L_p}\right)\cosh\left(\frac{x_n + W_n - x}{L_p}\right)\bigg|_{x=x_n}$$

Then

$$J_p = \frac{eD_p p_{nO}}{L_p}\coth\left(\frac{W_n}{L_p}\right)\cdot\left[\exp\left(\frac{V_a}{V_t}\right) - 1\right]$$

8.20

$$I_D \propto n_i^2\exp\left(\frac{V_D}{V_t}\right)$$

For the temperature range $300 \le T \le 320K$, neglect the change in N_C and N_V

So

$$I_D \propto \exp\left(\frac{-E_g}{kT}\right)\cdot\exp\left(\frac{eV_D}{kT}\right)$$

$$\propto \exp\left[\frac{-(E_g - eV_D)}{kT}\right]$$

Taking the ratio of currents, but maintaining I_D a constant, we have

$$1 = \frac{\exp\left[\frac{-(E_g - eV_{D1})}{kT_1}\right]}{\exp\left[\frac{-(E_g - eV_{D2})}{kT_2}\right]} \Rightarrow$$

$$\frac{E_g - eV_{D1}}{kT_1} = \frac{E_g - eV_{D2}}{kT_2}$$

We have

$T = 300K$, $V_{D1} = 0.60\ V$ and

$$kT_1 = 0.0259\ eV\ ,\ \frac{kT_1}{e} = 0.0259\ V$$

$T = 310K$

$$kT_2 = 0.02676\ eV\ ,\ \frac{kT_2}{e} = 0.02676\ V$$

$T = 320K$

$$kT_3 = 0.02763\ eV\ ,\ \frac{kT_3}{e} = 0.02763\ V$$

So, for $\underline{T = 310K}$,

$$\frac{1.12 - 0.60}{0.0259} = \frac{1.12 - V_{D2}}{0.02676}$$

which yields

$$\underline{V_{D2} = 0.5827\ V}$$

For $\underline{T = 320K}$,

$$\frac{1.12 - 0.60}{0.0259} = \frac{1.12 - V_{D3}}{0.02763}$$

which yields

$$\underline{V_{D3} = 0.5653\ V}$$

8.21

Computer Plot

8.22

$$g_d = \frac{e}{kT} \cdot I_D = \frac{2x10^{-3}}{0.0259}$$

or

$$\underline{g_d = 0.0772\ S}$$

Also

$$C_d = \frac{1}{2}\left(\frac{e}{kT}\right)\left(I_{pO}\tau_{pO} + I_{nO}\tau_{nO}\right)$$

We have

$$\tau_{pO} = \tau_{nO} = 10^{-6}\ s$$

$$I_{pO} + I_{nO} = 2x10^{-3}\ A$$

Then

$$C_d = \frac{\left(2x10^{-3}\right)\left(10^{-6}\right)}{2(0.0259)} \Rightarrow$$

$$\underline{C_d = 3.86x10^{-8}\ F}$$

Then

$$Y = g_d + j\omega C_d$$

or

$$\underline{Y = 0.0772 + j\omega\left(3.86x10^{-8}\right)}$$

8.23 For a p^+n diode

$$g_d = \frac{I_{DQ}}{V_t}\quad ,\quad C_d = \frac{I_{DQ}\tau_{pO}}{2V_t}$$

Now

$$g_d = \frac{10^{-3}}{0.0259} = 3.86x10^{-2}\ S$$

and

$$C_d = \frac{\left(10^{-3}\right)\left(10^{-7}\right)}{2(0.0259)} = 1.93x10^{-9}\ F$$

Now

$$Z = \frac{1}{Y} = \frac{1}{g_d + j\omega C_d} = \frac{g_d - j\omega C_d}{g_d^2 + \omega^2 C_d^2}$$

We have $\omega = 2\pi f$,

We find:

$f = 10\ kHz$: $\underline{Z = 25.9 - j0.0814}$

$f = 100\ kHz$: $\underline{Z = 25.9 - j0.814}$

$f = 1\ MHz$: $\underline{Z = 23.6 - j7.41}$

$f = 10\ MHz$: $\underline{Z = 2.38 - j7.49}$

8.24

(b)

Two capacitances will be equal at some forward-bias voltage.

For a forward-bias voltage, the junction capacitance is

$$C_j = A\left[\frac{e \in N_a N_d}{2\left(V_{bi} - V_a\right)\left(N_a + N_d\right)}\right]^{1/2}$$

The diffusion capacitance is

$$C_d = \left(\frac{1}{2V_t}\right)\left(I_{pO}\tau_{pO} + I_{nO}\tau_{nO}\right)$$

where

$$I_{pO} = \frac{Aen_i^2}{N_d}\sqrt{\frac{D_p}{\tau_{pO}}} \cdot \exp\left(\frac{V_a}{V_t}\right)$$

and

$$I_{nO} = \frac{Aen_i^2}{N_a}\sqrt{\frac{D_n}{\tau_{nO}}} \cdot \exp\left(\frac{V_a}{V_t}\right)$$

We find

$$D_p = (320)(0.0259) = 8.29\ cm^2/s$$

$$D_n = (850)(0.0259) = 22.0\ cm^2/s$$

and

$$V_{bi} = (0.0259)\ln\left[\frac{(10^{17})(5x10^{15})}{(1.5x10^{10})^2}\right] = 0.7363\ V$$

Now, we obtain

$$C_j = (10^{-4})\left[\frac{(1.6x10^{-19})(11.7)(8.85x10^{-14})}{2(V_{bi}-V_a)} \times \frac{(5x10^{15})(10^{17})}{(5x10^{15}+10^{17})}\right]^{1/2}$$

or

$$C_j = (10^{-4})\left[\frac{3.945x10^{-16}}{(V_{bi}-V_a)}\right]^{1/2}$$

We also obtain

$$I_{pO} = \frac{(10^{-4})(1.6x10^{-19})(1.5x10^{10})^2}{10^{17}}\sqrt{\frac{8.29}{10^{-7}}} \times \exp\left(\frac{V_a}{V_t}\right)$$

or

$$I_{pO} = 3.278x10^{-16}\exp\left(\frac{V_a}{V_t}\right)$$

Also

$$I_{nO} = \frac{(10^{-4})(1.6x10^{-19})(1.5x10^{10})^2}{5x10^{15}}\sqrt{\frac{22}{10^{-6}}} \times \exp\left(\frac{V_a}{V_t}\right)$$

or

$$I_{nO} = 3.377x10^{-15}\exp\left(\frac{V_a}{V_t}\right)$$

We can now write

$$C_d = \frac{1}{2(0.0259)}\left[(3.278x10^{-16})(10^{-7}) + (3.377x10^{-15})(10^{-6})\right] \cdot \exp\left(\frac{V_a}{V_t}\right)$$

or

$$C_d = 6.583x10^{-20} \cdot \exp\left(\frac{V_a}{V_t}\right)$$

We want to set $C_j = C_d$

So

$$(10^{-4})\left[\frac{3.945x10^{-16}}{0.7363-V_a}\right]^{1/2} = 6.583x10^{-20} \cdot \exp\left(\frac{V_a}{0.0259}\right)$$

By trial and error, we find

$$\underline{V_a = 0.463\ V}$$

At this voltage,

$$\underline{C_j = C_d \approx 3.8\ pF}$$

8.25

For a p^+n diode, $I_{pO} >> I_{nO}$, then

$$C_d = \left(\frac{1}{2V_t}\right)(I_{pO}\tau_{pO})$$

Now

$$\frac{\tau_{pO}}{2V_t} = 2.5x10^{-6}\ F/A$$

Then

$$\tau_{pO} = 2(0.0259)(2.5x10^{-6})$$

or

$$\underline{\tau_{pO} = 1.3x10^{-7}\ s}$$

At $1\ mA$,

$$C_d = (2.5x10^{-6})(10^{-3}) \Rightarrow$$

$$\underline{C_d = 2.5x10^{-9}\ F}$$

8.26

(a) $C_d = \frac{1}{2}\left(\frac{e}{kT}\right)A(I_{pO}\tau_{pO} + I_{nO}\tau_{nO})$

For a one-sided n^+p diode, $I_{nO} >> I_{pO}$, then

$$C_d = \frac{1}{2}\left(\frac{e}{kT}\right)A(I_{nO}\tau_{nO})$$

so

$$10^{-12} = \frac{1}{2}\left(\frac{1}{0.0259}\right)\left(10^{-3}\right)\left(I_{nO}\right)\left(10^{-7}\right)$$

or

$$\underline{I_{nO} = I_D = 0.518\ mA}$$

(b)

$$I_{nO} = A\frac{eD_n n_{pO}}{L_n}\exp\left(\frac{V_a}{V_t}\right)$$

We find

$$L_n = \sqrt{D_n \tau_{nO}} = 15.8\ \mu m \quad \text{and}$$

$$n_{pO} = \frac{n_i^2}{N_a} = 2.25x10^4\ cm^{-3}$$

Then

$$0.518x10^{-3} = \frac{\left(1.6x10^{-19}\right)(25)\left(2.25x10^4\right)\left(10^{-3}\right)}{15.8x10^{-4}}\exp\left(\frac{V_a}{V_t}\right)$$

or

$$0.518x10^{-3} = 5.70x10^{-14}\exp\left(\frac{V_a}{0.0259}\right)$$

We find

$$\underline{V_a = 0.594\ V}$$

(c)

$$g_d = \left(\frac{e}{kT}\right)I_D = \frac{1}{r_d} \Rightarrow$$

$$r_d = \frac{0.0259}{0.518x10^{-3}}$$

or

$$\underline{r_d = 50\ \Omega}$$

8.27

(a) p-region

$$R_p = \frac{\rho_p L}{A} = \frac{L}{\sigma_p A} = \frac{L}{A\left(e\mu_p N_a\right)}$$

so

$$R_p = \frac{0.2}{\left(10^{-2}\right)\left(1.6x10^{-19}\right)(480)\left(10^{16}\right)}$$

or

$$R_p = 26\ \Omega$$

n-region

$$R_n = \frac{\rho_n L}{A} = \frac{L}{\sigma_n A} = \frac{L}{A\left(e\mu_n N_d\right)}$$

so

$$R_n = \frac{0.10}{\left(10^{-2}\right)\left(1.6x10^{-19}\right)(1350)\left(10^{15}\right)}$$

or

$$R_n = 46.3\ \Omega$$

The total series resistance is

$$R = R_p + R_n = 26 + 46.3 \Rightarrow$$

$$\underline{R = 72.3\ \Omega}$$

(b)

$$V = IR \Rightarrow 0.1 = I(72.3)$$

or

$$\underline{I = 1.38\ mA}$$

8.28

$$R = \frac{\rho_n L(n)}{A(n)} + \frac{\rho_p L(p)}{A(p)}$$

$$= \frac{(0.2)\left(10^{-2}\right)}{2x10^{-5}} + \frac{(0.1)\left(10^{-2}\right)}{2x10^{-5}}$$

or

$$\underline{R = 150\ \Omega}$$

We can write

$$V = I_D R + V_t \ln\left(\frac{I_D}{I_S}\right)$$

(a) (i) $I_D = 1\ mA$

$$V = \left(10^{-3}\right)(150) + (0.0259)\ln\left(\frac{10^{-3}}{10^{-10}}\right)$$

or

$$\underline{V = 0.567\ V}$$

(ii) $I_D = 10\ mA$

$$V = \left(10x10^{-3}\right)(150) + (0.0259)\ln\left(\frac{10x10^{-3}}{10^{-10}}\right)$$

or $\underline{V = 1.98\ V}$

(b)

For $R = 0$

(i) $I_D = 1\ mA$

$$V = (0.0259)\ln\left(\frac{10^{-3}}{10^{-10}}\right) \Rightarrow$$

$$\underline{V = 0.417\ V}$$

(ii) $I_D = 10\ mA$

$$V = (0.0259)\ln\left(\frac{10x10^{-3}}{10^{-10}}\right) \Rightarrow$$

$$\underline{V = 0.477\ V}$$

8.29

$$r_d = 48\ \Omega = \frac{1}{g_d} \Rightarrow g_d = 0.0208$$

We have

$$g_d = \frac{e}{kT} \cdot I_D \Rightarrow I_D = (0.0208)(0.0259)$$

or

$$\underline{I_D = 0.539\ mA}$$

Also

$$I_D = I_S \exp\left(\frac{V_a}{V_t}\right) \Rightarrow V_a = V_t \ln\left(\frac{I_D}{I_S}\right)$$

so

$$V_a = (0.0259)\ln\left(\frac{0.539x10^{-3}}{2x10^{-11}}\right) \Rightarrow$$

$$\underline{V_a = 0.443\ V}$$

8.30

(a) $$\frac{1}{r_d} = \frac{dI_D}{dV_a} = I_S\left(\frac{1}{V_t}\right)\exp\left(\frac{V_a}{V_t}\right)$$

or

$$\frac{1}{r_d} = \frac{10^{-13}}{0.0259} \cdot \exp\left(\frac{0.020}{0.0259}\right)$$

which yields

$$\underline{r_d = 1.2x10^{11}\ \Omega}$$

(b)

For $V_a = -0.020\ V$,

$$\frac{1}{r_d} = \frac{10^{-13}}{0.0259} \cdot \exp\left(\frac{-0.020}{0.0259}\right)$$

or

$$\underline{r_d = 5.6x10^{11}\ \Omega}$$

8.31

Ideal reverse-saturation current density

$$J_S = \frac{eD_n n_{pO}}{L_n} + \frac{eD_p p_{nO}}{L_p}$$

We find

$$n_{pO} = \frac{n_i^2}{N_a} = \frac{(1.8x10^6)^2}{10^{16}} = 3.24x10^{-4}\ cm^{-3}$$

and

$$p_{nO} = \frac{(1.8x10^6)^2}{10^{16}} = 3.24x10^{-4}\ cm^{-3}$$

Also

$$L_n = \sqrt{D_n \tau_{nO}} = \sqrt{(200)(10^{-8})} = 14.2\ \mu m$$

$$L_p = \sqrt{D_p \tau_{pO}} = \sqrt{(6)(10^{-8})} = 2.45\ \mu m$$

Then

$$J_S = \frac{(1.6x10^{-19})(200)(3.24x10^{-4})}{14.2x10^{-4}} + \frac{(1.6x10^{-19})(6)(3.24x10^{-4})}{2.45x10^{-4}}$$

so

$$\underline{J_S = 8.57x10^{-18}\ A/cm^2}$$

Reverse-biased generation current density

$$J_{gen} = \frac{en_i W}{2\tau_O}$$

We have

$$V_{bi} = V_t \ln\left(\frac{N_a N_d}{n_i^2}\right)$$

$$= (0.0259)\ln\left[\frac{(10^{16})(10^{16})}{(1.8x10^6)^2}\right]$$

or

$$V_{bi} = 1.16\ V$$

And

$$W = \left[\frac{2\in(V_{bi}+V_R)}{e}\left(\frac{N_a+N_d}{N_a N_d}\right)\right]^{1/2}$$

$$= \left[\frac{2(13.1)(8.85x10^{-14})(1.16+5)}{1.6x10^{-19}} \times \left[\frac{10^{16}+10^{16}}{(10^{16})(10^{16})}\right]\right]^{1/2}$$

or

$$W = 1.34x10^{-4}\ cm$$

Then

$$J_{gen} = \frac{(1.6x10^{-19})(1.8x10^6)(1.34x10^{-4})}{2(10^{-8})}$$

or

$$\underline{J_{gen} = 1.93x10^{-9}\ A/cm^2}$$

Generation current dominates in GaAs reverse-biased junctions.

8.32

(a) We can write

$$J_S = en_i^2\left[\frac{1}{N_a}\sqrt{\frac{D_n}{\tau_{nO}}}+\frac{1}{N_d}\sqrt{\frac{D_p}{\tau_{pO}}}\right]$$

$$= n_i^2\left(1.6x10^{19}\right)\left[\frac{1}{10^{16}}\sqrt{\frac{25}{5x10^{-7}}} + \frac{1}{10^{16}}\sqrt{\frac{10}{5x10^{-7}}}\right]$$

or

$$J_S = n_i^2\left(1.85x10^{-31}\right)$$

We also have

$$J_{gen} = \frac{en_iW}{2\tau_O}$$

For $V_{bi}+V_R = 5\ V$, we find $W = 1.14x10^{-4}\ cm$

So

$$J_{gen} = \frac{\left(1.6x10^{-19}\right)\left(1.14x10^{-4}\right)n_i}{2\left(5x10^{-7}\right)}$$

or

$$J_{gen} = n_i\left(1.82x10^{-17}\right)$$

When $J_S = J_{gen}$,

$$1.85x10^{-31}n_i = 1.82x10^{-17}$$

which yields

$$n_i = 9.88x10^{13}\ cm^{-3}$$

We have

$$n_i^2 = N_C N_V \exp\left(\frac{-E_g}{kT}\right)$$

Then

$$\left(9.88x10^{13}\right)^2 = \left(2.8x10^{19}\right)\left(1.04x10^{19}\right)\left(\frac{T}{300}\right)^3 \times\exp\left[\frac{-1.12}{(0.0259)(T/300)}\right]$$

By trial and error, we find

$$\underline{T = 505K}$$

At this temperature

$$J_S = J_{gen} = \left(1.82x10^{-17}\right)\left(9.88x10^{13}\right)\Rightarrow$$

$$\underline{J_S = J_{gen} = 1.8x10^{-3}\ A/cm^2}$$

(b)

$$J_S\exp\left(\frac{V_a}{V_t}\right) = J_{gen}\exp\left(\frac{V_a}{2V_t}\right)$$

At $T = 300K$

$$J_S = \left(1.5x10^{10}\right)^2\left(1.85x10^{-31}\right)$$

or

$$J_S = 4.16x10^{-11}\ A/cm^2$$

and

$$J_{gen} = \left(1.5x10^{10}\right)\left(1.82x10^{-17}\right)\Rightarrow$$

or

$$J_{gen} = 2.73x10^{-7}\ A/cm^2$$

Then we can write

$$\exp\left(\frac{V_a}{2V_t}\right) = \frac{J_{gen}}{J_S} = \frac{2.73x10^{-7}}{4.16x10^{-11}} = 6.56x10^3$$

so that

$$V_a = 2(0.0259)\ln\left(6.56x10^3\right)\Rightarrow$$

$$\underline{V_a = 0.455\ V}$$

8.33

(a) We can write

$$J_S = en_i^2\left[\frac{1}{N_a}\sqrt{\frac{D_n}{\tau_{nO}}}+\frac{1}{N_d}\sqrt{\frac{D_p}{\tau_{pO}}}\right]$$

We find

$$D_n = (3000)(0.0259) = 77.7\ cm^2/s$$

$$D_p = (200)(0.0259) = 5.18\ cm^2/s$$

Then

$$J_S = \left(1.6x10^{-19}\right)\left(1.8x10^6\right)^2\left[\frac{1}{10^{17}}\sqrt{\frac{77.7}{10^{-8}}} + \frac{1}{10^{17}}\sqrt{\frac{5.18}{10^{-8}}}\right]$$

or

$$J_S = 5.75x10^{-19}\ A/cm^2$$

so

$$I_S = AJ_S = \left(10^{-3}\right)\left(5.75x10^{-19}\right)$$

or

$$\underline{I_S = 5.75x10^{-22}\ A}$$

We also have

$$I_{gen} = \frac{en_iWA}{2\tau_O}$$

Now

$$V_{bi} = V_t \ln\left(\frac{N_a N_d}{n_i^2}\right)$$

$$= (0.0259)\ln\left[\frac{(10^{17})(10^{17})}{(1.8x10^6)^2}\right]$$

or

$$V_{bi} = 1.28\ V$$

Also

$$W = \left[\frac{2\in(V_{bi}+V_R)}{e}\left(\frac{N_a+N_d}{N_a N_d}\right)\right]^{1/2}$$

$$= \left[\frac{2(13.1)(8.85x10^{-14})(1.28+5)}{1.6x10^{-19}} \times\left(\frac{10^{17}+10^{17}}{(10^{17})(10^{17})}\right)\right]^{1/2}$$

or

$$W = 0.427x10^{-4}\ cm$$

so

$$I_{gen} = \frac{(1.6x10^{-19})(1.8x10^6)(0.427x10^{-4})(10^{-3})}{2(10^{-8})}$$

or

$$\underline{I_{gen} = 6.15x10^{-13}\ A}$$

The total reverse-bias current

$$I_R = I_S + I_{gen} = 5.75x10^{-22} + 6.15x10^{-13}$$

or

$$\underline{I_R \approx 6.15x10^{-13}\ A}$$

Forward Bias: Ideal diffusion current

For $V_a = 0.3\,V$

$$I_D = I_S \exp\left(\frac{V_a}{V_t}\right) = (5.75x10^{-22})\exp\left(\frac{0.3}{0.0259}\right)$$

or

$$\underline{I_D = 6.17x10^{-17}\ A}$$

For $V_a = 0.5\,V$

$$I_D = (5.75x10^{-22})\exp\left(\frac{0.5}{0.0259}\right)$$

or

$$\underline{I_D = 1.39x10^{-13}\ A}$$

Recombination current

For $V_a = 0.3\,V$:

$$W = \left[\frac{2(13.1)(8.85x10^{-14})(1.28-0.3)}{1.6x10^{-19}}\left(\frac{2x10^{17}}{10^{34}}\right)\right]^{1/2}$$

or

$$W = 0.169x10^{-4}\ cm$$

Then

$$I_{rec} = \frac{en_i WA}{2\tau_O}\exp\left(\frac{V_a}{2V_t}\right)$$

$$= \frac{(1.6x10^{-19})(1.8x10^6)(0.169x10^{-4})(10^{-3})}{2(10^{-8})} \times\exp\left[\frac{0.3}{2(0.0259)}\right]$$

or

$$\underline{I_{rec} = 7.96x10^{-11}\ A}$$

For $V_a = 0.5\,V$

$$W = \left[\frac{2(13.1)(8.85x10^{-14})(1.28-0.5)}{1.6x10^{-19}}\left(\frac{2x10^{17}}{10^{34}}\right)\right]^{1/2}$$

or

$$W = 0.150x10^{-4}\ cm$$

Then

$$I_{rec} = \frac{(1.6x10^{-19})(1.8x10^6)(0.15x10^{-4})(10^{-3})}{2(10^{-8})} \times\exp\left[\frac{0.5}{2(0.0259)}\right]$$

or

$$\underline{I_{rec} = 3.36x10^{-9}\ A}$$

Total forward-bias current:

For $V_a = 0.3\,V$;

$$I_D = 6.17x10^{-17} + 7.96x10^{-11}$$

or

$$\underline{I_D \approx 7.96x10^{-11}\ A}$$

For $V_a = 0.5\,V$

$$I_D = 1.39x10^{-13} + 3.36x10^{-9}$$

or

$$\underline{I_D \approx 3.36x10^{-9}\ A}$$

(b)

Reverse-bias; ratio of generation to ideal diffusion current:

$$\frac{I_{gen}}{I_S} = \frac{6.15x10^{-13}}{5.75x10^{-22}}$$

Ratio $\underline{= 1.07x10^{9}}$

Forward bias: Ratio of recombination to ideal diffusion current:

For $V_a = 0.3\,V$

$$\frac{I_{rec}}{I_D} = \frac{7.96x10^{-11}}{6.17x10^{-17}}$$

Ratio $\underline{= 1.29x10^{6}}$

For $V_a = 0.5\,V$

$$\frac{I_{rec}}{I_D} = \frac{3.36x10^{-9}}{1.39x10^{-13}}$$

Ratio $\underline{= 2.42x10^{4}}$

8.34

Computer Plot

8.35

Computer Plot

8.36

Computer Plot

8.37

We have that

$$R = \frac{np - n_i^2}{\tau_{pO}(n+n') + \tau_{nO}(p+p')}$$

Let $\tau_{pO} = \tau_{nO} = \tau_O$ and $n' = p' = n_i$

We can write

$$n = n_i \exp\left(\frac{E_{Fn} - E_{Fi}}{kT}\right)$$

$$p = n_i \exp\left(\frac{E_{Fi} - E_{Fp}}{kT}\right)$$

We also have

$$\left(E_{Fn} - E_{Fi}\right) + \left(E_{Fi} - E_{Fp}\right) = eV_a$$

so that

$$\left(E_{Fi} - E_{Fp}\right) = eV_a - \left(E_{Fn} - E_{Fi}\right)$$

Then

$$p = n_i \exp\left[\frac{eV_a - \left(E_{Fn} - E_{Fi}\right)}{kT}\right]$$

$$= n_i \exp\left(\frac{eV_a}{kT}\right)\cdot \exp\left[\frac{-\left(E_{Fn} - E_{Fi}\right)}{kT}\right]$$

Define

$$\eta_a = \frac{eV_a}{kT} \quad \text{and} \quad \eta = \left(\frac{E_{Fn} - E_{Fi}}{kT}\right)$$

Then the recombination rate can be written as

$$R = \frac{\left(n_i e^{\eta}\right)\left(n_i e^{\eta_a}\cdot e^{-\eta}\right) - n_i^2}{\tau_O\left[n_i e^{\eta} + n_i + n_i e^{\eta_a}\cdot e^{-\eta} + n_i\right]}$$

or

$$R = \frac{n_i\left(e^{\eta_a} - 1\right)}{\tau_O\left(2 + e^{\eta} + e^{\eta_a}\cdot e^{-\eta}\right)}$$

To find the maximum recombination rate, set

$$\frac{dR}{d\eta} = 0$$

$$= \frac{n_i\left(e^{\eta_a} - 1\right)}{\tau_O}\cdot\frac{d}{dx}\left[2 + e^{\eta} + e^{\eta_a}\cdot e^{-\eta}\right]^{-1}$$

or

$$0 = \frac{n_i\left(e^{\eta_a} - 1\right)}{\tau_O}\cdot(-1)\left[2 + e^{\eta} + e^{\eta_a}\cdot e^{-\eta}\right]^{-2}$$

$$\times\left[e^{\eta} - e^{\eta_a}\cdot e^{-\eta}\right]$$

which simplifies to

$$0 = \frac{-n_i\left(e^{\eta_a} - 1\right)}{\tau_O}\cdot\frac{\left[e^{\eta} - e^{\eta_a}\cdot e^{-\eta}\right]}{\left[2 + e^{\eta} + e^{\eta_a}\cdot e^{-\eta}\right]^2}$$

The denominator is not zero, so we have

$$e^{\eta} - e^{\eta_a}\cdot e^{-\eta} = 0 \Rightarrow$$

$$e^{2\eta} = e^{\eta_a} \Rightarrow \eta = \frac{1}{2}\eta_a$$

Then the maximum recombination rate becomes

$$R_{\max} = \frac{n_i\left(e^{\eta_a} - 1\right)}{\tau_O\left[2 + e^{\eta_a/2} + e^{\eta_a}\cdot e^{-\eta_a/2}\right]}$$

$$= \frac{n_i\left(e^{\eta_a} - 1\right)}{\tau_O\left[2 + e^{\eta_a/2} + e^{\eta_a/2}\right]}$$

or

$$R_{\max} = \frac{n_i\left(e^{\eta_a} - 1\right)}{2\tau_O\left(e^{\eta_a/2} + 1\right)}$$

which can be written as

$$R_{\max} = \frac{n_i\left[\exp\left(\frac{eV_a}{kT}\right) - 1\right]}{2\tau_O\left[\exp\left(\frac{eV_a}{2kT}\right) + 1\right]}$$

If $V_a >> \left(\frac{kT}{e}\right)$, then we can neglect the (-1) term in the numerator and the (+1) term in the denominator so we finally have

$$\underline{R_{\max} = \frac{n_i}{2\tau_O}\exp\left(\frac{eV_a}{2kT}\right)}$$

Q.E.D.

8.38

We have

$$J_{gen} = \int_0^W eG\,dx$$

In this case, $G = g' = 4x10^{19}\ cm^{-3}s^{-1}$, that is a constant through the space charge region. Then

$$J_{gen} = eg'W$$

We find

$$V_{bi} = V_t \ln\left(\frac{N_a N_d}{n_i^2}\right)$$

$$= (0.0259)\ln\left[\frac{(5x10^{15})(5x10^{15})}{(1.5x10^{10})^2}\right] = 0.659\ V$$

and

$$W = \left[\frac{2\in(V_{bi}+V_R)}{e}\left(\frac{N_a+N_d}{N_aN_d}\right)\right]^{1/2}$$

$$= \left[\frac{2(11.7)(8.85x10^{-14})(0.659+10)}{1.6x10^{-19}}\right.$$

$$\left.\times\left(\frac{5x10^{15}+5x10^{15}}{(5x10^{15})(5x10^{15})}\right)\right]^{1/2}$$

or

$$W = 2.35x10^{-4}\ cm$$

Then

$$J_{gen} = (1.6x10^{-19})(4x10^{19})(2.35x10^{-4})$$

or

$$\underline{J_{gen} = 1.5x10^{-3}\ A/cm^2}$$

8.39

$$J_S = en_i^2\left[\frac{1}{N_a}\sqrt{\frac{D_n}{\tau_{nO}}} + \frac{1}{N_d}\sqrt{\frac{D_p}{\tau_{pO}}}\right]$$

$$= (1.6x10^{-19})(1.5x10^{10})^2\left[\frac{1}{3x10^{16}}\sqrt{\frac{18}{10^{-7}}}\right.$$

$$\left.+\frac{1}{10^{18}}\sqrt{\frac{6}{10^{-7}}}\right]$$

or

$$J_S = 1.64x10^{-11}\ A/cm^2$$

Now

$$J_D = J_S \exp\left(\frac{V_D}{V_t}\right)$$

Also

$$J = 0 = J_G - J_D$$

or

$$0 = 25x10^{-3} - 1.64x10^{-11}\exp\left(\frac{V_D}{V_t}\right)$$

which yields

$$\exp\left(\frac{V_D}{V_t}\right) = 1.52x10^9$$

or

$$V_D = V_t \ln(1.52x10^9)$$

so

$$\underline{V_D = 0.548\ V}$$

8.40

$$V_B = \frac{\in E_{crit}^2}{2eN_B}$$

or

$$30 = \frac{(11.7)(8.85x10^{-14})(4x10^5)^2}{2(1.6x10^{-19})N_B}$$

which yields

$$\underline{N_B = N_d = 1.73x10^{16}\ cm^{-3}}$$

8.41

For the breakdown voltage, we need $N_d = 3x10^{15}\ cm^{-3}$ and for this doping, we find $\mu_p = 430\ cm^2/V-s$. Then

$$D_p = (430)(0.0259) = 11.14\ cm^2/s$$

For the p^+n junction,

$$J_S = en_i^2 \cdot \frac{1}{N_d}\sqrt{\frac{D_p}{\tau_{pO}}}$$

$$= \frac{(1.6x10^{-19})(1.5x10^{10})^2}{3x10^{15}}\sqrt{\frac{11.14}{10^{-7}}}$$

or

$$J_S = 1.27x10^{-10}\ A/cm^2$$

Then

$$I = J_S A \exp\left(\frac{V_a}{V_t}\right)$$

$$2x10^{-3} = \left(1.27x10^{-10}\right) A \exp\left(\frac{0.65}{0.0259}\right)$$

Finally

$$\underline{A = 1.99x10^{-4}\ cm^2}$$

8.42

GaAs, n^+p, and $N_a = 10^{16}\ cm^{-3}$

From Figure 8.25

$$\underline{V_B \approx 75\ V}$$

8.43

$$\mathrm{E}_{\max} = \frac{eN_d x_n}{\in}$$

We can write

$$x_n = \frac{\mathrm{E}_{\max} \in}{eN_d}$$

$$= \frac{\left(4x10^5\right)(11.7)\left(8.85x10^{-14}\right)}{\left(1.6x10^{-19}\right)\left(5x10^{16}\right)}$$

or

$$x_n = 5.18x10^{-5}\ cm$$

We find

$$V_{bi} = (0.0259)\ln\left[\frac{\left(5x10^{16}\right)\left(5x10^{16}\right)}{\left(1.5x10^{10}\right)^2}\right] = 0.778\ V$$

Now

$$x_n = \left[\frac{2\in\left(V_{bi}+V_R\right)}{e}\left(\frac{N_a}{N_d}\right)\left(\frac{1}{N_a+N_d}\right)\right]^{1/2}$$

or

$$\left(5.18x10^{-5}\right)^2 = \left[\frac{2(11.7)\left(8.85x10^{-14}\right)}{1.6x10^{-19}}\right.$$

$$\left.\times\left(V_{bi}+V_R\right)\left(\frac{5x10^{16}}{5x10^{16}}\right)\left(\frac{1}{5x10^{16}+5x10^{16}}\right)\right]$$

which yields

$$2.68x10^{-9} = 1.29x10^{-10}\left(V_{bi}+V_R\right)$$

so

$$V_{bi}+V_R = 20.7 \Rightarrow \underline{V_R = 19.9\ V}$$

8.44

For a silicon p^+n junction with

$N_d = 5x10^{15}\ cm^{-3}$ and $V_B \approx 100\ V$

Neglecting V_{bi} compared to V_B

$$x_n \approx \left[\frac{2\in V_B}{eN_d}\right]^{1/2}$$

$$= \left[\frac{2(11.7)\left(8.85x10^{-14}\right)(100)}{\left(1.6x10^{-19}\right)\left(5x10^{15}\right)}\right]^{1/2}$$

or

$$\underline{x_n(\min) = 5.09\ \mu m}$$

8.45

We find

$$V_{bi} = (0.0259)\ln\left[\frac{\left(10^{18}\right)\left(10^{18}\right)}{\left(1.5x10^{10}\right)^2}\right] = 0.933\ V$$

Now

$$\mathrm{E}_{\max} = \frac{eN_d x_n}{\in}$$

so

$$10^6 = \frac{\left(1.6x10^{-19}\right)\left(10^{18}\right)x_n}{(11.7)\left(8.85x10^{-14}\right)}$$

which yields

$$x_n = 6.47x10^{-6}\ cm$$

Now

$$x_n = \left[\frac{2\in\left(V_{bi}+V_R\right)}{e}\left(\frac{N_a}{N_d}\right)\left(\frac{1}{N_a+N_d}\right)\right]^{1/2}$$

Then

$$\left(6.47x10^{-6}\right)^2 = \left[\frac{2(11.7)\left(8.85x10^{-14}\right)}{1.6x10^{-19}}\right.$$

$$\left.\times\left(V_{bi}+V_R\right)\left(\frac{10^{18}}{10^{18}}\right)\left(\frac{1}{10^{18}+10^{18}}\right)\right]$$

which yields

$$V_{bi}+V_R = 6.468\ V$$

or

$$\underline{V_R = 5.54\ V}$$

8.46

Assume silicon: For an n^+p junction

$$x_p = \left[\frac{2\in\left(V_{bi}+V_R\right)}{eN_a}\right]^{1/2}$$

Assume $V_{bi} << V_R$

(a)

For $x_p = 75\ \mu m$

Then

$$\left(75x10^{-4}\right)^2 = \frac{2(11.7)\left(8.85x10^{-14}\right)V_R}{\left(1.6x10^{-19}\right)\left(10^{15}\right)}$$

which yields $V_R = 4.35x10^3\ V$

(b)

For $x_p = 150\ \mu m$, we find

$V_R = 1.74x10^4\ V$

From Figure 8.25, the breakdown voltage is approximately 300 V. So, in each case, breakdown is reached first.

8.47

Impurity gradient

$$a = \frac{2x10^{18}}{2x10^{-4}} = 10^{22}\ cm^{-4}$$

From the figure

$V_B = 15\ V$

8.48

(a) If $\frac{I_R}{I_F} = 0.2$

Then we have

$$erf\sqrt{\frac{t_s}{\tau_{pO}}} = \frac{I_F}{I_F + I_R} = \frac{1}{1 + \frac{I_R}{I_F}} = \frac{1}{1 + 0.2}$$

or

$$erf\sqrt{\frac{t_S}{\tau_{pO}}} = 0.833$$

We find

$$\sqrt{\frac{t_S}{\tau_{pO}}} = 0.978 \Rightarrow \underline{\frac{t_S}{\tau_{pO}} = 0.956}$$

(b)

If $\frac{I_R}{I_F} = 1.0$, then

$$erf\sqrt{\frac{t_S}{\tau_{pO}}} = \frac{1}{1+1} = 0.5$$

which yields

$$\underline{\frac{t_S}{\tau_{pO}} = 0.228}$$

8.49

We want

$$\frac{t_S}{\tau_{pO}} = 0.2$$

Then

$$erf\sqrt{\frac{t_S}{\tau_{pO}}} = \frac{1}{1 + \frac{I_R}{I_F}} = erf\sqrt{0.2}$$

where

$$erf\sqrt{0.2} = erf(0.447) = 0.473$$

We obtain

$$\frac{I_R}{I_F} = \frac{1}{0.473} - 1 \Rightarrow \underline{\frac{I_R}{I_F} = 1.11}$$

We have

$$erf\sqrt{\frac{t_2}{\tau_{pO}}} + \frac{\exp\left(\frac{-t_2}{\tau_{pO}}\right)}{\sqrt{\pi\left(\frac{t_2}{\tau_{pO}}\right)}} = 1 + (0.1)\left(\frac{I_R}{I_F}\right) = 1.11$$

By trial and error,

$$\underline{\frac{t_2}{\tau_{pO}} = 0.65}$$

8.50

$C_j = 18\ pF$ at $V_R = 0$

$C_j = 4.2\ pF$ at $V_R = 10\ V$

We have $\tau_{nO} = \tau_{pO} = 10^{-7}\ s$, $I_F = 2\ mA$

And $I_R \approx \frac{V_R}{R} = \frac{10}{10} = 1\ mA$

So

$$t_S \approx \tau_{pO} \ln\left(1 + \frac{I_F}{I_R}\right) = \left(10^{-7}\right)\ln\left(1 + \frac{2}{1}\right)$$

or

$$t_S = 1.1x10^{-7}\ s$$

Also

$$C_{avg} = \frac{18 + 4.2}{2} = 11.1\ pF$$

The time constant is

$$\tau_S = RC_{avg} = \left(10^4\right)\left(11.1x10^{-12}\right) = 1.11x10^{-7}\ s$$

Now

Turn-off time $= t_S + \tau_S = (1.1 + 1.11) \times 10^{-7}\ s$

Or

$$\underline{2.21x10^{-7}\ s}$$

8.51

$$V_{bi} = (0.0259)\ln\left[\frac{\left(5x10^{19}\right)^2}{\left(1.5x10^{10}\right)^2}\right] = 1.14\ V$$

We find

$$W = \left[\frac{2\in\left(V_{bi} - V_a\right)}{e}\left(\frac{N_a + N_d}{N_a N_d}\right)\right]^{1/2}$$

$$= \left[\frac{2(11.7)\left(8.85x10^{-14}\right)(1.14 - 0.40)}{1.6x10^{-19}} \times \left(\frac{5x10^{19} + 5x10^{19}}{\left(5x10^{19}\right)^2}\right)\right]^{1/2}$$

which yields

$$\underline{W = 6.19x10^{-7}\ cm = 61.9\ \overset{\circ}{A}}$$

8.52

Sketch

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Chapter 9

Problem Solutions

9.1

(a) We have

$$e\phi_n = eV_t \ln\left(\frac{N_C}{N_d}\right)$$

$$= (0.0259)\ln\left(\frac{2.8x10^{19}}{10^{16}}\right) = 0.206\ eV$$

(c)

$$\phi_{BO} = \phi_m - \chi = 4.28 - 4.01$$

or

$$\underline{\phi_{BO} = 0.27\ V}$$

and

$$V_{bi} = \phi_{BO} - \phi_n = 0.27 - 0.206$$

or

$$V_{bi} = 0.064\ V$$

Also

$$x_d = \left[\frac{2\in V_{bi}}{eN_d}\right]^{1/2}$$

$$= \left[\frac{2(11.7)(8.85x10^{-14})(0.064)}{(1.6x10^{-19})(10^{16})}\right]^{1/2}$$

or

$$\underline{x_d = 9.1x10^{-6}\ cm}$$

Then

$$|\mathrm{E}_{max}| = \frac{eN_d x_d}{\in}$$

$$= \frac{(1.6x10^{-19})(10^{16})(9.1x10^{-6})}{(11.7)(8.85x10^{-14})}$$

or

$$\underline{|\mathrm{E}_{max}| = 1.41x10^4\ V/cm}$$

(d)

Using the figure, $\underline{\phi_{Bn} = 0.55\ V}$

So

$$V_{bi} = \phi_{Bn} - \phi_n = 0.55 - 0.206$$

or

$$V_{bi} = 0.344\ V$$

We then find

$\underline{x_d = 2.11x10^{-5}\ cm}$ and $\underline{\mathrm{E}_{max} = 3.26x10^4\ V/cm}$

9.2

(a) $\phi_{BO} = \phi_m - \chi = 5.1 - 4.01$

or

$$\underline{\phi_{BO} = 1.09\ V}$$

(b)

$$\phi_n = V_t \ln\left(\frac{N_C}{N_d}\right)$$

$$= (0.0259)\ln\left(\frac{2.8x10^{19}}{10^{15}}\right) = 0.265\ V$$

Then

$$V_{bi} = \phi_{BO} - \phi_n = 1.09 - 0.265$$

or

$$\underline{V_{bi} = 0.825\ V}$$

(c)

$$W = x_d = \left[\frac{2\in V_{bi}}{eN_d}\right]^{1/2}$$

$$= \left[\frac{2(11.7)(8.85x10^{-14})(0.825)}{(1.6x10^{-19})(10^{15})}\right]^{1/2}$$

or

$$\underline{W = 1.03x10^{-4}\ cm}$$

(d)

$$|\mathrm{E}_{max}| = \frac{eN_d x_d}{\in}$$

$$= \frac{(1.6x10^{-19})(10^{15})(1.03x10^{-4})}{(11.7)(8.85x10^{-14})}$$

or

$$\underline{|\mathrm{E}_{max}| = 1.59x10^4\ V/cm}$$

9.3

(a) Gold on n-type GaAs

$\chi = 4.07\ V$ and $\phi_m = 5.1\ V$

$$\phi_{BO} = \phi_m - \chi = 5.1 - 4.07$$

and

$$\underline{\phi_{BO} = 1.03\ V}$$

(b)

$$\phi_n = V_t \ln\left(\frac{N_C}{N_d}\right)$$

$$= (0.0259)\ln\left(\frac{4.7x10^{17}}{5x10^{16}}\right)$$

or

$$\underline{\phi_n = 0.0580\ V}$$

(c)

$$V_{bi} = \phi_{BO} - \phi_n = 1.03 - 0.058$$

or

$$\underline{V_{bi} = 0.972\ V}$$

(d)

$$x_d = \left[\frac{2\in(V_{bi}+V_R)}{eN_d}\right]^{1/2}$$

$$= \left[\frac{2(13.1)(8.85x10^{-14})(0.972+5)}{(1.6x10^{-19})(5x10^{16})}\right]^{1/2}$$

or

$$\underline{x_d = 0.416\ \mu m}$$

(e)

$$|E_{max}| = \frac{eN_d x_d}{\in}$$

$$= \frac{(1.6x10^{-19})(5x10^{16})(0.416x10^{-4})}{(13.1)(8.85x10^{-14})}$$

or

$$\underline{|E_{max}| = 2.87x10^5\ V/cm}$$

9.4

$\phi_{Bn} = 0.86\ V$ and $\phi_n = 0.058\ V$ (Problem 9.3)

Then

$$V_{bi} = \phi_{Bn} - \phi_n = 0.86 - 0.058$$

or

$$\underline{V_{bi} = 0.802\ V}$$

and

$$x_d = \left[\frac{2\in(V_{bi}+V_R)}{eN_d}\right]^{1/2}$$

$$= \left[\frac{2(13.1)(8.85x10^{-14})(0.802+5)}{(1.6x10^{-19})(5x10^{16})}\right]^{1/2}$$

or]

$$\underline{x_d = 0.410\ \mu m}$$

Also

$$|E_{max}| = \frac{eN_d x_d}{\in}$$

$$= \frac{(1.6x10^{-19})(5x10^{16})(0.410x10^{-4})}{(13.1)(8.85x10^{-14})}$$

or

$$\underline{|E_{max}| = 2.83x10^5\ V/cm}$$

9.5

Gold, n-type silicon junction. From the figure,

$$\phi_{Bn} = 0.81\ V$$

For $N_d = 5x10^{15}\ cm^{-3}$, we have

$$\phi_n = V_t \ln\left(\frac{N_C}{N_d}\right)$$

$$= (0.0259)\ln\left(\frac{2.8x10^{19}}{5x10^{15}}\right) = \phi_n = 0.224\ V$$

Then

$$V_{bi} = 0.81 - 0.224 = 0.586\ V$$

(a)

Now

$$C' = \left[\frac{e\in N_d}{2(V_{bi}+V_R)}\right]^{1/2}$$

$$= \left[\frac{(1.6x10^{-19})(11.7)(8.85x10^{-14})(5x10^{15})}{2(0.586+4)}\right]^{1/2}$$

or

$$C' = 9.50x10^{-9}\ F/cm^2$$

For $A = 5x10^{-4}\ cm^2$, $C = C'A$

So

$$\underline{C = 4.75\ pF}$$

(b)

For $N_d = 5x10^{16}\ cm^{-3}$, we find

$$\phi_n = (0.0259)\ln\left(\frac{2.8x10^{19}}{5x10^{16}}\right) = 0.164\ V$$

Then

$$V_{bi} = 0.81 - 0.164 = 0.646\ V$$

Now

$$C' = \left[\frac{(1.6x10^{-19})(11.7)(8.85x10^{-14})(5x10^{16})}{2(0.646+4)}\right]^{1/2}$$

or

$$C' = 2.99x10^{-8}\ F/cm^2$$

and

$$C = C'A$$

so

$$\underline{C = 15\ pF}$$

9.6

(a) From the figure, $\underline{V_{bi} = 0.90\ V}$

(b) We find

$$\frac{\Delta\left(\frac{1}{C'}\right)^2}{\Delta V_R} = \frac{3x10^{15} - 0}{2 - (-0.9)} = 1.03x10^{15}$$

and

$$1.03x10^{15} = \frac{2}{e \in N_d}$$

Then we can write

$$N_d = \frac{2}{(1.6x10^{-19})(13.1)(8.85x10^{-14})(1.03x10^{15})}$$

or

$$\underline{N_d = 1.05x10^{16}\ cm^{-3}}$$

(c)

$$\phi_n = V_t \ln\left(\frac{N_C}{N_d}\right)$$

$$= (0.0259)\ln\left(\frac{4.7x10^{17}}{1.05x10^{16}}\right)$$

or

$$\underline{\phi_n = 0.0985\ V}$$

(d)

$$\phi_{Bn} = V_{bi} + \phi_n = 0.90 + 0.0985$$

or

$$\underline{\phi_{Bn} = 0.9985\ V}$$

9.7

From the figure, $\phi_{Bn} = 0.55\ V$

(a)

$$\phi_n = V_t \ln\left(\frac{N_C}{N_d}\right)$$

$$= (0.0259)\ln\left(\frac{2.8x10^{19}}{10^{16}}\right) = 0.206\ V$$

Then

$$V_{bi} = \phi_{Bn} - \phi_n = 0.55 - 0.206$$

or

$$\underline{V_{bi} = 0.344\ V}$$

We find

$$x_d = \left[\frac{2 \in V_{bi}}{eN_d}\right]^{1/2}$$

$$= \left[\frac{2(11.7)(8.85x10^{-14})(0.344)}{(1.6x10^{-19})(10^{16})}\right]^{1/2}$$

or

$$\underline{x_d = 0.211\ \mu m}$$

Also

$$|E_{max}| = \frac{eN_d x_d}{\in}$$

$$= \frac{(1.6x10^{-19})(10^{16})(0.211x10^{-4})}{(11.7)(8.85x10^{-14})}$$

or

$$\underline{|E_{max}| = 3.26x10^4\ V/cm}$$

(b)

$$\Delta\phi = \sqrt{\frac{eE}{4\pi \in}} = \left[\frac{(1.6x10^{-19})(3.26x10^4)}{4\pi(11.7)(8.85x10^{-14})}\right]^{1/2}$$

or

$$\underline{\Delta\phi = 20.0\ mV}$$

Also

$$x_m = \sqrt{\frac{e}{16\pi \in E}}$$

$$= \left[\frac{(1.6x10^{-19})}{16\pi(11.7)(8.85x10^{-14})(3.26x10^4)}\right]^{1/2}$$

or

$$\underline{x_m = 0.307x10^{-6}\ cm}$$

(c)

For $V_R = 4\ V$

$$x_d = \left[\frac{2(11.7)(8.85x10^{-14})(0.344 + 4)}{(1.6x10^{-19})(10^{16})}\right]^{1/2}$$

or

$$\underline{x_d = 0.75\ \mu m}$$

and

$$|E_{max}| = \frac{(1.6x10^{-19})(10^{16})(0.75x10^{-4})}{(11.7)(8.85x10^{-14})}$$

or

$$\underline{|E_{max}| = 1.16x10^5\ V/cm}$$

We find

$$\Delta\phi = \sqrt{\frac{eE}{4\pi \in}} \Rightarrow \underline{\Delta\phi = 37.8\ mV}$$

and

$$x_m = \sqrt{\frac{e}{16\pi \in \mathrm{E}}} \Rightarrow \underline{x_m = 0.163x10^{-6}\ cm}$$

9.8
We have

$$-\phi(x) = \frac{-e}{16\pi \in x} - \mathrm{E}x$$

or

$$e\phi(x) = \frac{e^2}{16\pi \in x} + \mathrm{E}ex$$

Now

$$\frac{d(e\phi(x))}{dx} = 0 = \frac{-e^2}{16\pi \in x^2} + \mathrm{E}e$$

Solving for x^2, we find

$$x^2 = \frac{e}{16\pi \in \mathrm{E}}$$

or

$$\underline{x = x_m = \sqrt{\frac{e}{16\pi \in \mathrm{E}}}}$$

Substituting this value of $x_m = x$ into the equation for the potential, we find

$$\Delta\phi = \frac{e}{16\pi \in \sqrt{\frac{e}{16\pi \in \mathrm{E}}}} + \mathrm{E}\sqrt{\frac{e}{16\pi \in \mathrm{E}}}$$

which yields

$$\underline{\Delta\phi = \sqrt{\frac{e\mathrm{E}}{4\pi \in}}}$$

9.9
Gold, n-type GaAs, from the figure $\phi_{Bn} = 0.87\ V$
(a)

$$\phi_n = V_t \ln\left(\frac{N_C}{N_d}\right)$$

$$= (0.0259)\ln\left(\frac{4.7x10^{17}}{5x10^{16}}\right) = 0.058\ V$$

Then

$$V_{bi} = \phi_{Bn} - \phi_n = 0.87 - 0.058$$

or

$$\underline{V_{bi} = 0.812\ V}$$

Also

$$x_d = \left[\frac{2 \in V_{bi}}{eN_d}\right]^{1/2}$$

$$= \left[\frac{2(13.1)(8.85x10^{-14})(0.812)}{(1.6x10^{-19})(5x10^{16})}\right]^{1/2}$$

or

$$\underline{x_d = 0.153\ \mu m}$$

Then

$$|\mathrm{E}_{max}| = \frac{eN_d x_d}{\in}$$

$$= \left[\frac{(1.6x10^{-19})(5x10^{16})(0.153x10^{-4})}{(13.1)(8.85x10^{-14})}\right]$$

or

$$\underline{|\mathrm{E}_{max}| = 1.06x10^5\ V/cm}$$

(b)
We want $\Delta\phi$ to be 7% or ϕ_{Bn},
So

$$\Delta\phi = (0.07)(0.87) = 0.0609\ V$$

Now

$$\Delta\phi = \sqrt{\frac{e\mathrm{E}}{4\pi \in}} \Rightarrow \mathrm{E} = \frac{(\Delta\phi^2)(4\pi \in)}{e}$$

so

$$\mathrm{E} = \frac{(0.0609)^2(4\pi)(13.1)(8.85x10^{-14})}{1.6x10^{-19}}$$

or

$$\mathrm{E}_{max} = 3.38x10^5\ V/cm$$

Now

$$\mathrm{E}_{max} = \frac{eN_d x_d}{\in} \Rightarrow x_d = \frac{\in \mathrm{E}}{eN_d}$$

so

$$x_d = \frac{(13.1)(8.85x10^{-14})(3.38x10^5)}{(1.6x10^{-19})(5x10^{16})}$$

or

$$x_d = 0.49\ \mu m$$

Then

$$x_d = 0.49x10^{-4} = \left[\frac{2 \in (V_{bi} + V_R)}{eN_d}\right]^{1/2}$$

or we can write

$$(V_{bi} + V_R) = \frac{eN_d x_d^2}{2 \in}$$

$$= \frac{(1.6x10^{-19})(5x10^{16})(0.49x10^{-4})^2}{2(13.1)(8.85x10^{-14})}$$

or

$$V_{bi} + V_R = 8.28\ V = 0.812 + V_R$$

or

$$\underline{V_R = 7.47\ V}$$

9.10

Computer Plot

9.11

(a) $\phi_{BO} = \phi_m - \chi = 5.2 - 4.07$

or

$$\underline{\phi_{BO} = 1.13\ V}$$

(b)

We have

$$\left(E_g - e\phi_O - e\phi_{Bn}\right) = \frac{1}{eD_{it}}\sqrt{2e \in N_d\left(\phi_{Bn} - \phi_n\right)}$$

$$- \frac{\in_i}{eD_{it}\delta}\left[\phi_m - \left(\chi + \phi_{Bn}\right)\right]$$

which becomes

$$e\left(1.43 - 0.60 - \phi_{Bn}\right)$$

$$= \frac{1}{e\left(\dfrac{10^{13}}{e}\right)}\left[2\left(1.6x10^{-19}\right)(13.1)\left(8.85x10^{-14}\right)\right.$$

$$\left.\times\left(10^{16}\right)\left(\phi_{Bn} - 0.10\right)\right]^{1/2}$$

$$- \frac{\left(8.85x10^{-14}\right)}{e\left(\dfrac{10^{13}}{e}\right)\left(25x10^{-8}\right)}\left[5.2 - \left(4.07 + \phi_{Bn}\right)\right]$$

or

$$0.83 - \phi_{Bn}$$

$$= 0.038\sqrt{\phi_{Bn} = 0.10} - 0.221\left(1.13 - \phi_{Bn}\right)$$

We then find

$$\underline{\phi_{Bn} = 0.858\ V}$$

(c)

If $\phi_m = 4.5\ V$, then

$$\phi_{BO} = \phi_m - \chi = 4.5 - 4.07$$

or

$$\underline{\phi_{BO} = 0.43\ V}$$

From part (b), we have

$$0.83 - \phi_{Bn}$$

$$= 0.038\sqrt{\phi_{Bn} - 0.10} - 0.221\left[4.5 - \left(4.07 + \phi_{Bn}\right)\right]$$

We then find

$$\underline{\phi_{Bn} = 0.733\ V}$$

With interface states, the barrier height is less sensitive to the metal work function.

9.12

We have that

$$\left(E_g - e\phi_O - e\phi_{Bn}\right)$$

$$= \frac{1}{eD_{it}}\sqrt{2e \in N_d\left(\phi_{Bn} - \phi_n\right)}$$

$$- \frac{\in_i}{eD_{it}\delta}\left[\phi_m - \left(\chi + \phi_{Bn}\right)\right]$$

Let $eD_{it} = D'_{it}\left(cm^{-2}eV^{-1}\right)$. Then we can write

$$e(1.12 - 0.230 - 0.60)$$

$$= \frac{1}{D'_{it}}\left[2\left(1.6x10^{-19}\right)(11.7)\left(8.85x10^{-14}\right)\right.$$

$$\left.\times\left(5x10^{16}\right)(0.60 - 0.164)\right]^{1/2}$$

$$- \frac{\left(8.85x10^{-14}\right)}{D'_{it}\left(20x10^{-8}\right)}\left[4.75 - (4.01 + 0.60)\right]$$

We find that

$$\underline{D'_{it} = 4.97x10^{11}\ cm^{-2}eV^{-1}}$$

9.13

(a) $\phi_n = V_t \ln\left(\dfrac{N_C}{N_d}\right)$

$$= (0.0259)\ln\left(\frac{2.8x10^{19}}{10^{16}}\right)$$

or

$$\underline{\phi_n = 0.206\ V}$$

(b)

$$V_{bi} = \phi_{Bn} - \phi_n = 0.89 - 0.206$$

or

$$\underline{V_{bi} = 0.684\ V}$$

(c)

$$J_{ST} = A^* T^2 \exp\left(\frac{-e\phi_{Bn}}{kT}\right)$$

For silicon, $A^* = 120\ A / cm^2 / ^\circ K^2$

Then

$$J_{ST} = (120)(300)^2 \exp\left(\frac{-0.89}{0.0259}\right)$$

or

$$\underline{J_{ST} = 1.3x10^{-8}\ A / cm^2}$$

(d)

$$J_n = J_{ST}\exp\left(\frac{eV_a}{kT}\right)$$

or

$$V_a = V_t\ln\left(\frac{J_n}{J_{ST}}\right) = (0.0259)\ln\left(\frac{2}{1.3x10^{-8}}\right)$$

or

$$\underline{V_a = 0.488\ V}$$

9.14

(a) From the figure, $\phi_{Bn} = 0.68\ V$

Then

$$J_{ST} = A^*T^2\exp\left(\frac{-\phi_{Bn}}{V_t}\right)$$

$$= (120)(300)^2\exp\left(\frac{-0.68}{0.0259}\right)$$

or

$$J_{ST} = 4.28x10^{-5}\ A/cm^2$$

For $I = 10^{-3}\ A \Rightarrow J_n = \dfrac{10^{-3}}{5x10^{-4}} = 2\ A/cm^2$

We have

$$V_a = V_t\ln\left(\frac{J_n}{J_{ST}}\right)$$

$$= (0.0259)\ln\left(\frac{2}{4.28x10^{-5}}\right)$$

or

$$\underline{V_a = 0.278\ V}$$

For $I = 10\ mA \Rightarrow J_n = 20\ A/cm^2$

And

$$V_a = (0.0259)\ln\left(\frac{20}{4.28x10^{-5}}\right)$$

or

$$\underline{V_a = 0.338\ V}$$

For $I = 100\ mA \Rightarrow J_n = 200\ A/cm^2$

And

$$V_a = (0.0259)\ln\left(\frac{200}{4.28x10^{-5}}\right)$$

or

$$\underline{V_a = 0.398\ V}$$

(b)

For $T = 400K$, $\phi_{Bn} = 0.68\ V$

Now

$$J_{ST} = (120)(400)^2\exp\left[\frac{-0.68}{(0.0259)(400/300)}\right]$$

or

$$J_{ST} = 5.39x10^{-2}\ A/cm^2$$

For $I = 1\ mA$,

$$V_a = (0.0259)\left(\frac{400}{300}\right)\ln\left(\frac{2}{5.39x10^{-2}}\right)$$

or

$$\underline{V_a = 0.125\ V}$$

For $I = 10\ mA$,

$$V_a = (0.0259)\left(\frac{400}{300}\right)\ln\left[\frac{20}{5.39x10^{-2}}\right]$$

or

$$\underline{V_a = 0.204\ V}$$

For $I = 100\ mA$,

$$V_a = (0.0259)\left(\frac{400}{300}\right)\ln\left(\frac{200}{5.39x10^{-2}}\right)$$

or

$$\underline{V_a = 0.284\ V}$$

9.15

(a) From the figure, $\phi_{Bn} = 0.86\ V$

$$J_{ST} = A^*T^2\exp\left(\frac{-\phi_{Bn}}{V_t}\right)$$

$$= (1.12)(300)^2\exp\left(\frac{-0.86}{0.0259}\right)$$

or

$$J_{ST} = 3.83x10^{-10}\ A/cm^2$$

Now

$$J_n = J_{ST}\exp\left(\frac{V_a}{V_t}\right)$$

and we can write, for $J_n = 5\ A/cm^2$

$$V_a = V_t\ln\left(\frac{J_n}{J_{ST}}\right)$$

$$= (0.0259)\ln\left(\frac{5}{3.83x10^{-10}}\right)$$

or

$$\underline{V_a = 0.603\ V}$$

(b)
For $J_n = 10\ A/cm^2$

$$V_a = (0.0259)\ln\left(\frac{10}{3.83x10^{-10}}\right) = 0.621\ V$$

so

$$\Delta V_a = 0.621 - 0.603 \Rightarrow$$

$$\underline{\Delta V_a = 18\ mV}$$

9.16
Computer Plot

9.17
From the figure, $\phi_{Bn} = 0.86\ V$

$$J_{ST} = A^* T^2 \exp\left(\frac{-\phi_{Bn}}{V_t}\right)\exp\left(\frac{\Delta\phi}{V_t}\right)$$

$$= (120)(300)^2 \exp\left(\frac{-0.68}{0.0259}\right)\exp\left(\frac{\Delta\phi}{V_t}\right)$$

or

$$J_{ST} = 4.28x10^{-5}\exp\left(\frac{\Delta\phi}{V_t}\right)$$

We have

$$\Delta\phi = \sqrt{\frac{e\mathrm{E}}{4\pi \in}}$$

Now

$$\phi_n = V_t \ln\left(\frac{N_C}{N_d}\right)$$

$$= (0.0259)\ln\left(\frac{2.8x10^{19}}{10^{16}}\right) = 0.206\ V$$

and

$$V_{bi} = \phi_{Bn} - \phi_n = 0.68 - 0.206 = 0.474\ V$$

(a)
We find for $V_R = 2\ V$,

$$x_d = \left[\frac{2 \in (V_{bi} + V_R)}{eN_d}\right]^{1/2}$$

$$= \left[\frac{2(11.7)(8.85x10^{-14})(2.474)}{(1.6x10^{-19})(10^{16})}\right]^{1/2}$$

or

$$x_d = 0.566\ \mu m$$

Then

$$|\mathrm{E}_{\max}| = \frac{eN_d x_d}{\in}$$

$$= \frac{(1.6x10^{-19})(10^{16})(0.566x10^{-4})}{(11.7)(8.85x10^{-14})}$$

or

$$|\mathrm{E}_{\max}| = 8.75x10^4\ V/cm$$

Now

$$\Delta\phi = \left[\frac{(1.6x10^{-19})(8.75x10^4)}{4\pi(11.7)(8.85x10^{-14})}\right]^{1/2}$$

or

$$\Delta\phi = 0.0328\ V$$

Then

$$J_{R1} = 4.28x10^{-5}\exp\left(\frac{0.0328}{0.0259}\right)$$

or

$$J_{R1} = 1.52x10^{-4}\ A/cm^2$$

For $A = 10^{-4}\ cm^2$, then

$$\underline{I_{R1} = 1.52x10^{-8}\ A}$$

(b)
For $V_R = 4\ V$,

$$x_d = \left[\frac{2(11.7)(8.85x10^{-14})(4.474)}{(1.6x10^{-19})(10^{16})}\right]^{1/2}$$

or

$$x_d = 0.761\ \mu m$$

$$|\mathrm{E}_{\max}| = \frac{(1.6x10^{-19})(10^{16})(0.761x10^{-4})}{(11.7)(8.85x10^{-14})}$$

or

$$|\mathrm{E}_{\max}| = 1.18x10^5\ V/cm$$

and

$$\Delta\phi = \left[\frac{(1.6x10^{-19})(1.18x10^5)}{4\pi(11.7)(8.85x10^{-14})}\right]^{1/2}$$

or

$$\Delta\phi = 0.0381\ V$$

Now

$$J_{R2} = 4.28x10^{-5}\exp\left(\frac{0.0381}{0.0259}\right)$$

or

$$J_{R2} = 1.86x10^{-4}\ A/cm^2$$

Finally,

$$\underline{I_{R2} = 1.86x10^{-8}\ A}$$

9.18
We have that

$$J_{s\to m}^{-} = \int_{E_C}^{\infty} v_x dn$$

The incremental electron concentration is given by

$$dn = g_C(E) f_F(E) dE$$

We have

$$g_C(E) = \frac{4\pi (2m_n^*)^{3/2}}{h^3}\sqrt{E - E_C}$$

and, assuming the Boltzmann approximation

$$f_F(E) = \exp\left[\frac{-(E - E_F)}{kT}\right]$$

Then

$$dn = \frac{4\pi (2m_n^*)^{3/2}}{h^3}\sqrt{E - E_C}\cdot \exp\left[\frac{-(E - E_F)}{kT}\right] dE$$

If the energy above E_C is kinetic energy, then

$$\frac{1}{2}m_n^* v^2 = E - E_C$$

We can then write

$$\sqrt{E - E_C} = v\sqrt{\frac{m_n^*}{2}}$$

and

$$dE = \frac{1}{2}m_n^*\cdot 2v dv = m_n^* v dv$$

We can also write

$$E - E_F = (E - E_C) + (E_C - E_F)$$
$$= \frac{1}{2}m_n^* v^2 + e\phi_n$$

so that

$$dn = 2\left(\frac{m_n^*}{h}\right)^3 \exp\left(\frac{-e\phi_n}{kT}\right)\cdot \exp\left(\frac{-m_n^* v^2}{2kT}\right)\cdot 4\pi v^2 dv$$

We can write

$$v^2 = v_x^2 + v_y^2 + v_z^2$$

The differential volume element is

$$4\pi v^2 dv = dv_x dv_y dv_z$$

The current is due to all x-directed velocities that are greater than v_{Ox} and for all y- and z-directed velocities. Then

$$J_{s\to m}^{-} = 2\left(\frac{m_n^*}{h}\right)^3 \exp\left(\frac{-e\phi_n}{kT}\right)$$
$$\times \int_{v_{Ox}}^{\infty} v_x \exp\left(\frac{-m_n^* v_x^2}{2kT}\right) dv_x$$
$$\times \int_{-\infty}^{+\infty} \exp\left(\frac{-m_n^* v_y^2}{2kT}\right) dv_y \times \int_{-\infty}^{+\infty} \exp\left(\frac{-m_n^* v_z^2}{2kT}\right) dv_z$$

We can write that

$$\frac{1}{2}m_n^* v_{Ox}^2 = e(V_{bi} - V_a)$$

Make a change of variables:

$$\frac{m_n^* v_x^2}{2kT} = \alpha^2 + \frac{e(V_{bi} - V_a)}{kT}$$

or

$$v_x^2 = \frac{2kT}{m_n^*}\left[\alpha^2 + \frac{e(V_{bi} - V_a)}{kT}\right]$$

Taking the differential, we find

$$v_x dv_x = \left(\frac{2kT}{m_n^*}\right)\alpha d\alpha$$

We may note that when $v_x = v_{Ox}$, $\alpha = 0$.
Other change of variables:

$$\frac{m_n^* v_y^2}{2kT} = \beta^2 \Rightarrow v_y = \left(\frac{2kT}{m_n^*}\right)^{1/2}\cdot\beta$$

$$\frac{m_n^* v_z^2}{2kT} = \gamma^2 \Rightarrow v_z = \left(\frac{2kT}{m_n^*}\right)^{1/2}\cdot\gamma$$

Substituting the new variables, we have

$$J_{s\to m}^{-} = 2\left(\frac{m_n^*}{h}\right)^3\cdot\left(\frac{2kT}{m_n^*}\right)^2 \exp\left(\frac{-e\phi_n}{kT}\right)$$
$$\times \exp\left[\frac{-e(V_{bi} - V_a)}{kT}\right]\cdot\int_0^{\infty}\alpha\exp(-\alpha^2)d\alpha$$
$$\times\int_{-\infty}^{+\infty}\exp(-\beta^2)d\beta\cdot\int_{-\infty}^{+\infty}\exp(-\gamma^2)d\gamma$$

9.19
For the Schottky diode,

$$J_{ST} = 3x10^{-8}\ A/cm^2\ ,\ A = 5x10^{-4}\ cm^2$$

For $I = 1\ mA$,

$$J = \frac{10^{-3}}{5x10^{-4}} = 2\ A/cm^2$$

We have

$$V_a = V_t \ln\left(\frac{J}{J_{ST}}\right)$$

$$= (0.0259)\ln\left(\frac{2}{3x10^{-8}}\right)$$

or

$$\underline{V_a = 0.467\ V} \ \ \text{(Schottky diode)}$$

For the pn junction, $J_S = 3x10^{-12}\ A/cm^2$

Then

$$V_a = (0.0259)\ln\left(\frac{2}{3x10^{-12}}\right)$$

or

$$\underline{V_a = 0.705\ V} \ \ \text{(pn junction diode)}$$

9.20

For the pn junction diode,

$J_S = 5x10^{-12}\ A/cm^2$, $A = 8x10^{-4}\ cm^2$

For $I = 1.2\ mA$,

$$J = \frac{1.2x10^{-3}}{8x10^{-4}} = 1.5\ A/cm^2$$

Then

$$V_a = V_t \ln\left(\frac{J}{J_S}\right)$$

$$= (0.0259)\ln\left(\frac{1.5}{5x10^{-12}}\right) = 0.684\ V$$

For the Schottky diode, the applied voltage will be less, so

$V_a = 0.684 - 0.265 = 0.419\ V$

We have

$$I = AJ_{ST}\exp\left(\frac{V_a}{V_t}\right)$$

so

$$1.2x10^{-3} = A\left(7x10^{-8}\right)\exp\left(\frac{0.419}{0.0259}\right)$$

which yields

$$\underline{A = 1.62x10^{-3}\ cm^2}$$

9.21

(a) Diodes in parallel:

We can write

$$I_S = I_{ST}\exp\left(\frac{V_{as}}{V_t}\right) \ \ \text{(Schottky diode)}$$

and

$$I_{PN} = I_S\exp\left(\frac{V_{apn}}{V_t}\right) \ \ \text{(pn junction diode)}$$

We have $I_S + I_{PN} = 0.5x10^{-3}\ A$, $V_{as} = V_{apn}$

Then

$$0.5x10^{-3} = \left(I_{ST} + I_S\right)\exp\left(\frac{V_a}{V_t}\right)$$

or

$$V_a = V_t \ln\left(\frac{0.5x10^{-3}}{I_S + I_{ST}}\right)$$

$$= (0.0259)\ln\left(\frac{0.5x10^{-3}}{5x10^{-8} + 10^{-12}}\right) = 0.239\ V$$

Now

$$I_S = 5x10^{-8}\exp\left(\frac{0.239}{0.0259}\right)$$

or

$$\underline{I_S \approx 0.5x10^{-3}\ A} \ \ \text{(Schottky diode)}$$

and

$$I_{PN} = 10^{-12}\exp\left(\frac{0.239}{0.0259}\right)$$

or

$$\underline{I_{PN} = 1.02x10^{-8}\ A} \ \ \text{(pn junction diode)}$$

(b) Diodes in Series:

We obtain,

$$V_{as} = (0.0259)\ln\left(\frac{0.5x10^{-3}}{5x10^{-8}}\right)$$

or

$$\underline{V_{as} = 0.239\ V} \ \ \text{(Schottky diode)}$$

and

$$V_{apn} = (0.0259)\ln\left(\frac{0.5x10^{-3}}{10^{-12}}\right)$$

or

$$\underline{V_{apn} = 0.519\ V} \ \ \text{(pn junction diode)}$$

9.22

(a) For $I = 0.8\ mA$, we find

$$J = \frac{0.8x10^{-3}}{7x10^{-4}} = 1.14\ A/cm^2$$

We have

$$V_a = V_t \ln\left(\frac{J}{J_S}\right)$$

For the pn junction diode,

$$V_a = (0.0259)\ln\left(\frac{1.14}{3x10^{-12}}\right)$$

or

$$\underline{V_a = 0.691\ V}$$

For the Schottky diode,

$$V_a = (0.0259)\ln\left(\frac{1.14}{4x10^{-8}}\right)$$

or

$$\underline{V_a = 0.445\ V}$$

(b)
For the pn junction diode,

$$J_S \propto n_i^2 \propto \left(\frac{T}{300}\right)^3 \exp\left(\frac{-E_g}{kT}\right)$$

Then

$$\frac{J_S(400)}{J_S(300)}$$

$$= \left(\frac{400}{300}\right)^3 \exp\left[\frac{-E_g}{(0.0259)(400/300)} + \frac{E_g}{0.0259}\right]$$

or

$$= 2.37\exp\left[\frac{1.12}{0.0259} - \frac{1.12}{0.03453}\right]$$

We find

$$\frac{J_S(400)}{J_S(300)} = 1.16x10^5$$

Now

$$I = (7x10^{-4})(1.16x10^5)(3x10^{-12})\exp\left(\frac{0.691}{0.03453}\right)$$

or

$$\underline{I = 120\ mA}$$

For the Schottky diode

$$J_{ST} \propto T^2 \exp\left(\frac{-e\phi_{BO}}{kT}\right)$$

Now

$$\frac{J_{ST}(400)}{J_{ST}(300)}$$

$$= \left(\frac{400}{300}\right)^2 \exp\left[\frac{-\phi_{BO}}{(0.0259)(400/300)} + \frac{\phi_{BO}}{0.0259}\right]$$

or

$$= 1.78\exp\left[\frac{0.82}{0.0259} - \frac{0.82}{0.03453}\right]$$

We obtain

$$\frac{J_{ST}(400)}{J_{ST}(300)} = 4.85x10^3$$

and so

$$I = (7x10^{-4})(4.85x10^3)(4x10^{-8})\exp\left(\frac{0.445}{0.03453}\right)$$

or

$$\underline{I = 53.7\ mA}$$

9.23
Computer Plot

9.24
We have

$$R_C = \frac{\left(\frac{kT}{e}\right)\cdot\exp\left(\frac{e\phi_{Bn}}{kT}\right)}{A^*T^2}$$

which can be rewritten as

$$\ln\left[\frac{R_C A^* T^2}{(kT/e)}\right] = \frac{e\phi_{Bn}}{kT}$$

so

$$\phi_{Bn} = \left(\frac{kT}{e}\right)\cdot\ln\left[\frac{R_C A^* T^2}{(kT/e)}\right]$$

$$= (0.0259)\ln\left[\frac{(10^{-5})(120)(300)^2}{0.0259}\right]$$

or

$$\underline{\phi_{Bn} = 0.216\ V}$$

9.25
(b) We need $\phi_n = \phi_m - \chi_s = 4.2 - 4.0 = 0.20\ V$
And

$$\phi_n = V_t \ln\left(\frac{N_C}{N_d}\right)$$

or

$$0.20 = (0.0259)\ln\left(\frac{2.8x10^{19}}{N_d}\right)$$

which yields

$$\underline{N_d = 1.24x10^{16}\ cm^{-3}}$$

(c)
Barrier height $\underline{= 0.20\ V}$

9.26
We have that

$$\mathrm{E} = \frac{-eN_d}{\in}(x_n - x)$$

Then

$$\phi = -\int \mathrm{E}dx = \frac{eN_d}{\in}\left(x_n \cdot x - \frac{x^2}{2}\right) + C_2$$

Let $\phi = 0$ at $x = 0 \Rightarrow C_2 = 0$
So

$$\phi = \frac{eN_d}{\in}\left(x_n \cdot x - \frac{x^2}{2}\right)$$

At $x = x_n$, $\phi = V_{bi}$, so

$$\phi = V_{bi} = \frac{eN_d}{\in} \cdot \frac{x_n^2}{2}$$

or

$$x_n = \sqrt{\frac{2 \in V_{bi}}{eN_d}}$$

Also

$$V_{bi} = \phi_{BO} - \phi_n$$

where

$$\phi_n = V_t \ln\left(\frac{N_C}{N_d}\right)$$

For

$$\phi = \frac{\phi_{BO}}{2} = \frac{0.70}{2} = 0.35\ V$$

we have

$$0.35 = \frac{(1.6x10^{-19})N_d}{(11.7)(8.85x10^{-14})}\left[x_n(50x10^{-8}) - \frac{(50x10^{-8})^2}{2}\right]$$

or

$$0.35 = 7.73x10^{-14}\ N_d(x_n - 25x10^{-8})$$

We have

$$x_n = \left[\frac{2(11.7)(8.85x10^{-14})V_{bi}}{(1.6x10^{-19})N_d}\right]^{1/2}$$

and

$$V_{bi} = 0.70 - \phi_n$$

By trial and error,

$$\underline{N_d = 3.5x10^{18}\ cm^{-3}}$$

9.27

(b) $\phi_{BO} = \phi_p = V_t \ln\left(\dfrac{N_V}{N_a}\right)$

$$= (0.0259)\ln\left(\frac{1.04x10^{19}}{5x10^{16}}\right) \Rightarrow$$

$$\underline{\phi_{BO} = 0.138\ V}$$

9.28
Sketches

9.29
Sketches

9.30
Electron affinity rule

$$\Delta E_C = e(\chi_n - \chi_p)$$

For GaAs, $\chi = 4.07$; and for AlAs, $\chi = 3.5$,
If we assume a linear extrapolation between GaAs and AlAs, then for

$$Al_{0.3}Ga_{0.7}As \Rightarrow \chi = 3.90$$

Then

$$|E_C| = 4.07 - 3.90 \Rightarrow$$

$$\underline{|E_C| = 0.17\ eV}$$

9.31
Consider an n-P heterojunction in thermal equilibrium. Poisson's equation is

$$\frac{d^2\phi}{dx^2} = -\frac{\rho(x)}{\in} = -\frac{d\mathrm{E}}{dx}$$

In the n-region,

$$\frac{d\mathrm{E}_n}{dx} = \frac{\rho(x)}{\in_n} = \frac{eN_{dn}}{\in_n}$$

For uniform doping, we have

$$\mathrm{E}_n = \frac{eN_{dn}x}{\in_n} + C_1$$

The boundary condition is
$\mathrm{E}_n = 0$ at $x = -x_n$, so we obtain

$$C_1 = \frac{eN_{dn}x_n}{\in_n}$$

Then

$$\mathrm{E}_n = \frac{eN_{dn}}{\in_n}(x + x_n)$$

In the P-region,

$$\frac{d\mathrm{E}_P}{dx} = -\frac{eN_{aP}}{\in_P}$$

which gives

$$\mathrm{E}_P = -\frac{eN_{aP}x}{\in_P} + C_2$$

We have the boundary condition that
$\mathrm{E}_P = 0$ at $x = x_P$ so that

$$C_2 = \frac{eN_{aP}x_P}{\in_P}$$

Then

$$\mathrm{E}_P = \frac{eN_{aP}}{\in_P}(x_P - x)$$

Assuming zero surface charge density at $x = 0$, the electric flux density D is continuous, so
$\in_n \mathrm{E}_n(0) = \in_P \mathrm{E}_P(0)$
which yields

$$N_{dn}x_n = N_{aP}x_P$$

We can determine the electric potential as

$$\phi_n(x) = -\int \mathrm{E}_n dx$$

$$= -\left[\frac{eN_{dn}x^2}{2\in_n} + \frac{eN_{dn}x_n x}{\in_n}\right] + C_3$$

Now

$$V_{bin} = |\phi_n(0) - \phi_n(-x_n)|$$

$$= C_3 - \left[C_3 - \frac{eN_{dn}x_n^2}{2\in_n} + \frac{eN_{dn}x_n^2}{\in_n}\right]$$

or

$$V_{bin} = \frac{eN_n x_n^2}{2\in_n}$$

Similarly on the P-side, we find

$$V_{biP} = \frac{eN_{aP}x_p^2}{2\in_P}$$

We have that

$$V_{bi} = V_{bin} + V_{biP} = \frac{eN_{dn}x_n^2}{2\in_n} + \frac{eN_{aP}x_P^2}{2\in_P}$$

We can write

$$x_P = x_n\left(\frac{N_{dn}}{N_{aP}}\right)$$

Substituting and collecting terms, we find

$$V_{bi} = \left[\frac{e\in_P N_{dn}N_{aP} + e\in_n N_{dn}^2}{2\in_n\in_P N_{aP}}\right]\cdot x_n^2$$

Solving for x_n, we have

$$x_n = \left[\frac{2\in_n\in_P N_{aP}V_{bi}}{eN_{dn}(\in_P N_{aP} + \in_n N_{dn})}\right]^{1/2}$$

Similarly on the P-side, we have

$$x_P = \left[\frac{2\in_n\in_P N_{dn}V_{bi}}{eN_{aP}(\in_P N_{aP} + \in_n N_{dn})}\right]^{1/2}$$

The total space charge width is then

$$W = x_n + x_P$$

Substituting and collecting terms, we obtain

$$W = \left[\frac{2\in_n\in_P V_{bi}(N_{aP} + N_{dn})}{eN_{dn}N_{aP}(\in_n N_{dn} + \in_P N_{aP})}\right]^{1/2}$$

Chapter 10

Problem Solutions

10.1
Sketch

10.2
Sketch

10.3

(a) $|I_S| = \dfrac{eD_n A_{BE} n_{BO}}{x_B}$

$= \dfrac{(1.6x10^{-19})(20)(10^{-4})(10^4)}{10^{-4}}$

or $\underline{I_S = 3.2x10^{-14}\ A}$

(b)

(i) $i_C = 3.2x10^{-14}\exp\left(\dfrac{0.5}{0.0259}\right) \Rightarrow$

$\underline{i_C = 7.75\ \mu A}$

(ii) $i_C = 3.2x10^{-14}\exp\left(\dfrac{0.6}{0.0259}\right) \Rightarrow$

$\underline{i_C = 0.368\ mA}$

(iii) $i_C = 3.2x10^{-14}\exp\left(\dfrac{0.7}{0.0259}\right) \Rightarrow$

$\underline{i_C = 17.5\ mA}$

10.4

(a) $\beta = \dfrac{\alpha}{1-\alpha} = \dfrac{0.9920}{1-0.9920} \Rightarrow \underline{\beta = 124}$

(b) From 10.3b

(i) For $i_C = 7.75\ \mu A$; $i_B = \dfrac{i_C}{\beta} = \dfrac{7.75}{124} \Rightarrow$

$\underline{i_B = 0.0625\ \mu A}$,

$i_E = \left(\dfrac{1+\beta}{\beta}\right)\cdot i_C = \left(\dfrac{125}{124}\right)(7.75) \Rightarrow$

$\underline{i_E = 7.81\ \mu A}$

(ii) For $i_C = 0.368\ mA$, $\underline{i_B = 2.97\ \mu A}$,

$\underline{i_E = 0.371\ mA}$

(iii) For $i_C = 17.5\ mA$, $\underline{i_B = 0.141\ mA}$,

$\underline{i_E = 17.64\ mA}$

10.5

(a) $\beta = \dfrac{i_C}{i_B} = \dfrac{510}{6} \Rightarrow \underline{\beta = 85}$

$\alpha = \dfrac{\beta}{1+\beta} = \dfrac{85}{86} \Rightarrow \underline{\alpha = 0.9884}$

$i_E = i_C + i_B = 510 + 6 \Rightarrow \underline{i_E = 516\ \mu A}$

(b)

$\beta = \dfrac{2.65}{0.05} \Rightarrow \underline{\beta = 53}$

$\alpha = \dfrac{53}{54} \Rightarrow \underline{\alpha = 0.9815}$

$i_E = 2.65 + 0.05 \Rightarrow \underline{i_E = 2.70\ mA}$

10.6

(c) For $i_B = 0.05\ mA$,

$i_C = \beta i_B = (100)(0.05) \Rightarrow \underline{i_C = 5\ mA}$

We have

$v_{CE} = V_{CC} - i_C R = 10 - (5)(1)$

or

$\underline{v_{CE} = 5\ V}$

10.7

(b) $V_{CC} = I_C R + V_{CB} + V_{BE}$

so

$10 = I_C(2) + 0 + 0.6$

or

$\underline{I_C = 4.7\ mA}$

10.8

(a)

$$n_{pO} = \frac{n_i^2}{N_B} = \frac{(1.5x10^{10})^2}{10^{16}} = 2.25x10^4\ cm^{-3}$$

At $x = 0$,

$$n_p(0) = n_{pO}\exp\left(\frac{V_{BE}}{V_t}\right)$$

or we can write

$$V_{BE} = V_t \ln\left(\frac{n_p(0)}{n_{pO}}\right)$$

We want $n_p(0) = 10\% \times 10^{16} = 10^{15}\ cm^{-3}$,
So

$$V_{BE} = (0.0259)\ln\left(\frac{10^{15}}{2.25x10^4}\right)$$

or

$$\underline{V_{BE} = 0.635\ V}$$

(b)
At $x' = 0$,

$$p_n(0) = p_{nO}\exp\left(\frac{V_{BE}}{V_t}\right)$$

where

$$p_{nO} = \frac{n_i^2}{N_E} = \frac{(1.5x10^{10})^2}{10^{17}} = 2.25x10^3\ cm^{-3}$$

Then

$$p_n(0) = 2.25x10^3\exp\left(\frac{0.635}{0.0259}\right) \Rightarrow$$

$$\underline{p_n(0) = 10^{14}\ cm^{-3}}$$

(c)
From the B-C space charge region,

$$x_{p1} = \left[\frac{2\in(V_{bi}+V_{R1})}{e}\left(\frac{N_C}{N_B}\right)\left(\frac{1}{N_C+N_B}\right)\right]^{1/2}$$

We find

$$V_{bi1} = (0.0259)\ln\left[\frac{(10^{16})(10^{15})}{(1.5x10^{10})^2}\right] = 0.635\ V$$

Then

$$x_{p1} = \left[\frac{2(11.7)(8.85x10^{-14})(0.635+3)}{1.6x10^{-19}} \times\left(\frac{10^{15}}{10^{16}}\right)\left(\frac{1}{10^{15}+10^{16}}\right)\right]^{1/2}$$

or

$$x_{p1} = 0.207\ \mu m$$

We find

$$V_{bi2} = (0.0259)\ln\left[\frac{(10^{17})(10^{16})}{(1.5x10^{10})^2}\right] = 0.754\ V$$

Then

$$x_{p2} = \left[\frac{2(11.7)(8.85x10^{-14})(0.754-0.635)}{1.6x10^{-19}} \times\left(\frac{10^{17}}{10^{16}}\right)\left(\frac{1}{10^{17}+10^{16}}\right)\right]^{1/2}$$

or

$$x_{p2} = 0.118\ \mu m$$

Now

$$x_B = x_{BO} - x_{p1} - x_{p2} = 1.10 - 0.207 - 0.118$$

or

$$\underline{x_B = 0.775\ \mu m}$$

10.9

(a) $p_{EO} = \dfrac{n_i^2}{N_E} = \dfrac{(1.5x10^{10})^2}{5x10^{17}} \Rightarrow$

$$\underline{p_{EO} = 4.5x10^2\ cm^{-3}}$$

$$n_{BO} = \frac{n_i^2}{N_B} = \frac{(1.5x10^{10})^2}{10^{16}} \Rightarrow$$

$$\underline{n_{BO} = 2.25x10^4\ cm^{-3}}$$

$$p_{CO} = \frac{n_i^2}{N_C} = \frac{(1.5x10^{10})^2}{10^{15}} \Rightarrow$$

$$\underline{p_{CO} = 2.25x10^5\ cm^{-3}}$$

(b)

$$n_B(0) = n_{BO}\exp\left(\frac{V_{BE}}{V_t}\right) = (2.25x10^4)\exp\left(\frac{0.625}{0.0259}\right)$$

or

$$\underline{n_B(0) = 6.80x10^{14}\ cm^{-3}}$$

Also

$$p_E(0) = p_{EO}\exp\left(\frac{V_{BE}}{V_t}\right) = (4.5x10^2)\exp\left(\frac{0.625}{0.0259}\right)$$

or

$$\underline{p_E(0) = 1.36x10^{13}\ cm^{-3}}$$

10.10

(a) $n_{EO} = \dfrac{n_i^2}{N_E} = \dfrac{(1.5x10^{10})^2}{10^{18}} \Rightarrow$

$$\underline{n_{EO} = 2.25x10^2\ cm^{-3}}$$

$$p_{BO} = \frac{n_i^2}{N_B} = \frac{(1.5x10^{10})^2}{5x10^{16}} \Rightarrow$$

$\underline{p_{BO} = 4.5x10^{3}\ cm^{-3}}$

$$n_{CO} = \frac{n_i^2}{N_C} = \frac{\left(1.5x10^{10}\right)^2}{10^{15}} \Rightarrow$$

$$\underline{n_{CO} = 2.25x10^{5}\ cm^{-3}}$$

(b)

$$p_B(0) = p_{BO}\exp\left(\frac{V_{EB}}{V_t}\right)$$

$$= \left(4.5x10^{3}\right)\exp\left(\frac{0.650}{0.0259}\right)$$

or

$$\underline{p_B(0) = 3.57x10^{14}\ cm^{-3}}$$

Also

$$n_E(0) = n_{EO}\exp\left(\frac{V_{EB}}{V_t}\right)$$

$$= \left(2.25x10^{2}\right)\exp\left(\frac{0.650}{0.0259}\right)$$

or

$$\underline{n_E(0) = 1.78x10^{13}\ cm^{-3}}$$

10.11

We have

$$\frac{d(\delta n_B)}{dx} = \frac{n_{BO}}{\sinh\left(\frac{x_B}{L_B}\right)}\left\{\left[\exp\left(\frac{V_{BE}}{V_t}\right) - 1\right]\right.$$

$$\left.\times\left(\frac{-1}{L_B}\right)\cosh\left(\frac{x_B - x}{L_B}\right) - \frac{1}{L_B}\cosh\left(\frac{x}{L_B}\right)\right\}$$

At $x = 0$,

$$\frac{d(\delta n_B)}{dx}\Big|(0) = \frac{-n_{BO}}{L_B\sinh\left(\frac{x_B}{L_B}\right)}\left\{\left[\exp\left(\frac{V_{BE}}{V_t}\right) - 1\right]\right.$$

$$\left.\times\cosh\left(\frac{x_B}{L_B}\right) + 1\right\}$$

At $x = x_B$,

$$\frac{d(\delta n_B)}{dx}\Big|(x_B) = \frac{-n_{BO}}{L_B\sinh\left(\frac{x_B}{L_B}\right)}$$

$$\times\left\{\left[\exp\left(\frac{V_{BE}}{V_t}\right) - 1\right] + \cosh\left(\frac{x_B}{L_B}\right)\right\}$$

Taking the ratio,

$$\frac{\frac{d(\delta n_B)}{dx}\Big|(x_B)}{\frac{d(\delta n_B)}{dx}\Big|(0)}$$

$$= \frac{\left[\exp\left(\frac{V_{BE}}{V_t}\right) - 1\right] + \cosh\left(\frac{x_B}{L_B}\right)}{\left[\exp\left(\frac{V_{BE}}{V_t}\right) - 1\right]\cosh\left(\frac{x_B}{L_B}\right) + 1}$$

$$\approx \underline{\frac{1}{\cosh\left(\frac{x_B}{L_B}\right)}}$$

(a) For $\frac{x_B}{L_B} = 0.1 \Rightarrow Ratio = \underline{0.9950}$

(b) For $\frac{x_B}{L_B} = 1.0 \Rightarrow Ratio = \underline{0.648}$

(c) For $\frac{x_B}{L_B} = 10 \Rightarrow Ratio = \underline{9.08x10^{-5}}$

10.12

In the base of the transistor, we have

$$D_B\frac{d^2(\delta n_B(x))}{dx^2} - \frac{\delta n_B(x)}{\tau_{BO}} = 0$$

or

$$\frac{d^2(\delta n_B(x))}{dx^2} - \frac{\delta n_B(x)}{L_B^2} = 0$$

where $L_B = \sqrt{D_B\tau_{BO}}$

The general solution to the differential equation is of the form,

$$\delta n_B(x) = A\exp\left(\frac{x}{L_B}\right) + B\exp\left(\frac{-x}{L_B}\right)$$

From the boundary conditions, we have

$$\delta n_B(0) = A + B = n_B(0) - n_{BO}$$

$$= n_{BO}\left[\exp\left(\frac{V_{BE}}{V_t}\right) - 1\right]$$

Also

$$\delta n_B(x_B) = A\exp\left(\frac{x_B}{L_B}\right) + B\exp\left(\frac{-x_B}{L_B}\right) = -n_{BO}$$

From the first boundary condition, we can write

$$A = n_{BO}\left[\exp\left(\frac{V_{BE}}{V_t}\right) - 1\right] - B$$

Substituting into the second boundary condition equation, we find

$$B\left[\exp\left(\frac{x_B}{L_B}\right)-\exp\left(\frac{-x_B}{L_B}\right)\right]$$
$$= n_{BO}\left[\exp\left(\frac{V_{BE}}{V_t}\right)-1\right]\cdot\exp\left(\frac{x_B}{L_B}\right)+n_{BO}$$

which can be written as

$$B=\frac{n_{BO}\left[\exp\left(\frac{V_{BE}}{V_t}\right)-1\right]\cdot\exp\left(\frac{x_B}{L_B}\right)+n_{BO}}{2\sinh\left(\frac{x_B}{L_B}\right)}$$

We then find

$$A=\frac{-n_{BO}\left[\exp\left(\frac{V_{BE}}{V_t}\right)-1\right]\cdot\exp\left(\frac{-x_B}{L_B}\right)-n_{BO}}{2\sinh\left(\frac{x_B}{L_B}\right)}$$

10.13

In the base of the pnp transistor, we have

$$D_B\frac{d^2(\delta p_B(x))}{dx^2}-\frac{\delta p_B(x)}{\tau_{BO}}=0$$

or

$$\frac{d^2(\delta p_B(x))}{dx^2}-\frac{\delta p_B(x)}{L_B^2}=0$$

where $L_B=\sqrt{D_B\tau_{BO}}$

The general solution is of the form

$$\delta p_B(x)=A\exp\left(\frac{x}{L_B}\right)+B\exp\left(\frac{-x}{L_B}\right)$$

From the boundary conditions, we can write

$$\delta p_B(0)=A+B=p_B(0)-p_{BO}$$
$$=p_{BO}\left[\exp\left(\frac{V_{EB}}{V_t}\right)-1\right]$$

Also

$$\delta p_B(x_B)=A\exp\left(\frac{x_B}{L_B}\right)+B\exp\left(\frac{-x_B}{L_B}\right)=-p_{BO}$$

From the first boundary condition equation, we find

$$A=p_{BO}\left[\exp\left(\frac{V_{EB}}{V_t}\right)-1\right]-B$$

Substituting into the second boundary equation

$$B=\frac{p_{BO}\left[\exp\left(\frac{V_{EB}}{V_t}\right)-1\right]\cdot\exp\left(\frac{x_B}{L_B}\right)+p_{BO}}{2\sinh\left(\frac{x_B}{L_B}\right)}$$

and then we obtain

$$A=\frac{-p_{BO}\left[\exp\left(\frac{V_{EB}}{V_t}\right)-1\right]\cdot\exp\left(\frac{-x_B}{L_B}\right)-p_{BO}}{2\sinh\left(\frac{x_B}{L_B}\right)}$$

Substituting the expressions for A and B into the general solution and collecting terms, we obtain

$$\delta p_B(x)=p_{BO}$$
$$\times\left\{\frac{\left[\exp\left(\frac{V_{EB}}{V_t}\right)-1\right]\cdot\sinh\left(\frac{x_B-x}{L_B}\right)-\sinh\left(\frac{x}{L_B}\right)}{\sinh\left(\frac{x_B}{L_B}\right)}\right\}$$

10.14

For the idealized straight line approximation, the total minority carrier concentration is given by

$$n_B(x)=n_{BO}\left[\exp\left(\frac{V_{BE}}{V_t}\right)\right]\cdot\left(\frac{x_B-x}{x_B}\right)$$

The excess concentration is

$$\delta n_B=n_B(x)-n_{BO}$$

so for the idealized case, we can write

$$\delta n_{BO}(x)=n_{BO}\left\{\left[\exp\left(\frac{V_{BE}}{V_t}\right)\right]\cdot\left(\frac{x_B-x}{x_B}\right)-1\right\}$$

At $x=\frac{1}{2}x_B$, we have

$$\delta n_{BO}\left(\frac{1}{2}x_B\right)=n_{BO}\left\{\frac{1}{2}\left[\exp\left(\frac{V_{BE}}{V_t}\right)\right]-1\right\}$$

For the actual case, we have

$$\delta n_B\left(\frac{1}{2}x_B\right)=n_{BO}$$
$$\times\left\{\frac{\left[\exp\left(\frac{V_{BE}}{V_t}\right)-1\right]\cdot\sinh\left(\frac{x_B}{2L_B}\right)-\sinh\left(\frac{x_B}{2L_B}\right)}{\sinh\left(\frac{x_B}{L_B}\right)}\right\}$$

(a) For $\dfrac{x_B}{L_B} = 0.1$, we have

$$\sinh\left(\frac{x_B}{2L_B}\right) = 0.0500208$$

and

$$\sinh\left(\frac{x_B}{L_B}\right) = 0.100167$$

Then

$$\frac{\delta n_{BO}\left(\frac{1}{2}x_B\right) - \delta n_B\left(\frac{1}{2}x_B\right)}{\delta n_{BO}\left(\frac{1}{2}x_B\right)}$$

$$= \frac{\left[\exp\left(\frac{V_{BE}}{V_t}\right)\right]\cdot(0.50 - 0.49937) - 1.0 + 0.99875}{\frac{1}{2}\exp\left(\frac{V_{BE}}{V_t}\right) - 1}$$

which becomes

$$= \frac{(0.00063)\exp\left(\frac{V_{BE}}{V_t}\right) - (0.00125)}{\frac{1}{2}\exp\left(\frac{V_{BE}}{V_t}\right) - 1}$$

If we assume that $\exp\left(\frac{V_{BE}}{V_t}\right) >> 1$, then we find that the ratio is

$$\frac{0.00063}{0.50} = 0.00126 \Rightarrow \underline{0.126\%}$$

(b)

For $\dfrac{x_B}{L_B} = 1.0$, we have

$$\sinh\left(\frac{x_B}{2L_B}\right) = 0.5211$$

and

$$\sinh\left(\frac{x_B}{L_B}\right) = 1.1752$$

Then

$$\frac{\delta n_{BO}\left(\frac{1}{2}x_B\right) - \delta n_B\left(\frac{1}{2}x_B\right)}{\delta n_{BO}\left(\frac{1}{2}x_B\right)}$$

$$= \frac{\left[\exp\left(\frac{V_{BE}}{V_t}\right)\right](0.50 - 0.4434) - 1.0 + 0.8868}{\frac{1}{2}\exp\left(\frac{V_{BE}}{V_t}\right) - 1}$$

which becomes

$$\frac{(0.0566)\exp\left(\frac{V_{BE}}{V_t}\right) - (0.1132)}{\frac{1}{2}\exp\left(\frac{V_{BE}}{V_t}\right) - 1}$$

Assuming that $\exp\left(\frac{V_{BE}}{V_t}\right) >> 1$

Then the ratio is

$$= \frac{0.0566}{0.50} = 0.1132 \Rightarrow \underline{11.32\%}$$

10.15

The excess hole concentration at $x = 0$ is

$$\delta p_B(0) = p_{BO}\left[\exp\left(\frac{V_{EB}}{V_t}\right) - 1\right] = 8x10^{14}\ cm^{-3}$$

and the excess hole concentration at $x = x_B$ is

$$\delta p_B(x_B) = -p_{BO} = -2.25x10^4\ cm^{-3}$$

From the results of problem 10.13, we can write

$$\delta p(x) = p_{BO}$$

$$\times\left\{\frac{\left[\exp\left(\frac{V_{EB}}{V_t}\right) - 1\right]\cdot\sinh\left(\frac{x_B - x}{L_B}\right) - \sinh\left(\frac{x}{L_B}\right)}{\sinh\left(\frac{x_B}{L_B}\right)}\right\}$$

or

$$\delta p_B(x) =$$

$$\frac{(8x10^{14})\sinh\left(\frac{x_B - x}{L_B}\right) - (2.25x10^4)\sinh\left(\frac{x}{L_B}\right)}{\sinh\left(\frac{x_B}{L_B}\right)}$$

Let $x_B = L_B = 10\ \mu m$, so that

$$\sinh\left(\frac{x_B}{L_B}\right) = 1.1752$$

Then, we can find $\delta p_B(x)$ for (a) the ideal linear approximation and for (b) the actual distribution as follow:

x	$(a)\ \delta p_B$	$(b)\ \delta p_B$
0	$8x10^{14}$	$8x10^{14}$
$0.25L_B$	$6x10^{14}$	$5.6x10^{14}$
$0.50L_B$	$4x10^{14}$	$3.55x10^{14}$
$0.75L_B$	$2x10^{14}$	$1.72x10^{14}$
$1.0L_B$	$-2.25x10^4$	$-2.25x10^4$

(c)
For the ideal case when $x_B << L_B$, then $J(0) = J(x_B)$, so that

$$\frac{J(x_B)}{J(0)} = 1$$

For the case when $x_B = L_B = 10\ \mu m$

$$J(0) = \frac{eD_B}{\sinh\left(\frac{x_B}{L_B}\right)}\frac{d}{dx}\left\{\left(8x10^{14}\right)\sinh\left(\frac{x_B - x}{L_B}\right) - \left(2.25x10^4\right)\sinh\left(\frac{x}{L_B}\right)\right\}\Big|_{x=0}$$

or

$$J(0) = \frac{eD_B}{\sinh(1)}\left\{\frac{-1}{L_B}\left(8x10^{14}\right)\cosh\left(\frac{x_B - x}{L_B}\right) - \frac{1}{L_B}\left(2.25x10^4\right)\cosh\left(\frac{x}{L_B}\right)\right\}\Big|_{x=0}$$

which becomes

$$= \frac{-eD_B}{L_B\sinh(1)}\cdot\left\{\left(8x10^{14}\right)\cosh(1) + \left(2.25x10^4\right)\cosh(0)\right\}$$

We find

$$J(0) = \frac{-\left(1.6x10^{-19}\right)(10)}{\left(10x10^{-4}\right)(1.175)} \times\left[\left(8x10^{14}\right)(1.543) + \left(2.25x10^4\right)(1)\right]$$

or

$$J(0) = -1.68\ A/cm^2$$

Now

$$J(x_B) = \frac{-eD_B}{L_B\sinh(1)}\left\{\left(8x10^{14}\right)\cosh(0) + \left(2.25x10^4\right)\cosh(1)\right\}$$

or

$$= \frac{-\left(1.6x10^{-19}\right)(10)}{\left(10x10^{-4}\right)(1.175)} \times\left[\left(8x10^{14}\right)(1) + \left(2.25x10^4\right)(1.543)\right]$$

We obtain

$$J(x_B) = -1.089\ A/cm^2$$

Then

$$\frac{J(x_B)}{J(0)} = \frac{-1.089}{-1.68} \Rightarrow \frac{J(x_B)}{J(0)} = 0.648$$

10.16
(a) npn transistor biased in saturation

$$D_B\frac{d^2(\delta n_B(x))}{dx^2} - \frac{\delta n_B(x)}{\tau_{BO}} = 0$$

or

$$\frac{d^2(\delta n_B(x))}{dx^2} - \frac{\delta n_B(x)}{L_B^2} = 0$$

where $L_B = \sqrt{D_B\tau_{BO}}$

The general solution is of the form

$$\delta n_B(x) = A\exp\left(\frac{x}{L_B}\right) + B\exp\left(\frac{-x}{L_B}\right)$$

If $x_B << L_B$, then also $x << L_B$ so that

$$\delta n_B(x) \approx A\left(1 + \frac{x}{L_B}\right) + B\left(1 - \frac{x}{L_B}\right)$$

$$= (A + B) + (A - B)\left(\frac{x}{L_B}\right)$$

which can be written as

$$\delta n_B(x) = C + D\left(\frac{x}{L_B}\right)$$

The boundary conditions are

$$\delta n_B(0) = C = n_{BO}\left[\exp\left(\frac{V_{BE}}{V_t}\right) - 1\right]$$

and

$$\delta n_B(x_B) = C + D\left(\frac{x_B}{L_B}\right) = n_{BO}\left[\exp\left(\frac{V_{BC}}{V_t}\right) - 1\right]$$

Then the coefficient D can be written as

$$D = \left(\frac{L_B}{x_B}\right)\left\{ n_{BO}\left[\exp\left(\frac{V_{BC}}{V_t}\right) - 1\right] - n_{BO}\left[\exp\left(\frac{V_{BE}}{V_t}\right) - 1\right]\right\}$$

The excess electron concentration is then given by

$$\delta n_B(x) = n_{BO}\left\{\left[\exp\left(\frac{V_{BE}}{V_t}\right) - 1\right]\cdot\left(1 - \frac{x}{L_B}\right) + \left[\exp\left(\frac{V_{BC}}{V_t}\right) - 1\right]\cdot\left(\frac{x}{x_B}\right)\right\}$$

(b)
The electron diffusion current density is

$$J_n = eD_B \frac{d(\delta n_B(x))}{dx}$$

$$= eD_B n_{BO}\left\{\left[\exp\left(\frac{V_{BE}}{V_t}\right) - 1\right]\cdot\left(\frac{-1}{x_B}\right) + \left[\exp\left(\frac{V_{BC}}{V_t}\right) - 1\right]\cdot\left(\frac{1}{x_B}\right)\right\}$$

or

$$J_n = -\frac{eD_B n_{BO}}{x_B}\left\{\exp\left(\frac{V_{BE}}{V_t}\right) - \exp\left(\frac{V_{BC}}{V_t}\right)\right\}$$

(c)
The total excess charge in the base region is

$$Q_{nB} = -e\int_0^{x_B} \delta n_B(x)dx$$

$$= -en_{BO}\left\{\left[\exp\left(\frac{V_{BE}}{V_t}\right) - 1\right]\cdot\left(x - \frac{x^2}{2x_B}\right) + \left[\exp\left(\frac{V_{BC}}{V_t}\right) - 1\right]\cdot\left(\frac{x^2}{2x_B}\right)\right\}\Bigg|_0^{x_B}$$

which yields

$$Q_{nB} = \frac{-en_{BO}x_B}{2}\left\{\left[\exp\left(\frac{V_{BE}}{V_t}\right) - 1\right] + \left[\exp\left(\frac{V_{BC}}{V_t}\right) - 1\right]\right\}$$

10.17

(a) Extending the results of problem 10.16 to a pnp transistor, we can write

$$J_p = \frac{eD_B p_{BO}}{x_B}\left[\exp\left(\frac{V_{EB}}{V_t}\right) - \exp\left(\frac{V_{CB}}{V_t}\right)\right]$$

We have

$$p_{BO} = \frac{n_i^2}{N_B} = \frac{\left(1.5x10^{10}\right)^2}{10^{17}} = 2.25x10^3\ cm^{-3}$$

Then

$$165 = \frac{\left(1.6x10^{-19}\right)(10)\left(2.25x10^3\right)}{0.7x10^{-4}} \times\left[\exp\left(\frac{0.75}{0.0259}\right) - \exp\left(\frac{V_{CB}}{V_t}\right)\right]$$

or

$$3.208x10^{12} = 3.768x10^{12} - \exp\left(\frac{V_{CB}}{V_t}\right)$$

which yields

$$V_{CB} = (0.0259)\ln\left(0.56x10^{12}\right) \Rightarrow$$

$$V_{CB} = 0.70\ V$$

(b)

$$V_{EC}(sat) = V_{EB} - V_{CB} = 0.75 - 0.70 \Rightarrow$$

$$V_{EC}(sat) = 0.05\ V$$

(c)
Again, extending the results of problem 10.16 to a pnp transistor, we can write

$$Q_{pB} = \frac{ep_{BO}x_B}{2}\left\{\left[\exp\left(\frac{V_{EB}}{V_t}\right) - 1\right] + \left[\exp\left(\frac{V_{CB}}{V_t}\right) - 1\right]\right\}$$

$$= \frac{\left(1.6x10^{-19}\right)\left(2.25x10^3\right)\left(0.7x10^{-4}\right)}{2} \times\left[3.768x10^{12} + 0.56x10^{12}\right]$$

or

$$Q_{pB} = 5.45x10^{-8}\ C/cm^2$$

or

$$\frac{Q_{pB}}{e} = 3.41x10^{11}\ holes/cm^2$$

(d)
In the collector, we have

$$\delta n_p(x) = n_{PO}\left[\exp\left(\frac{V_{CB}}{V_t}\right) - 1\right]\cdot\exp\left(\frac{-x}{L_C}\right)$$

The total number of excess electrons in the collector is

$$N_{Coll} = \int_0^\infty \delta n_P(x)dx$$

$$= -n_{PO}L_C\left[\exp\left(\frac{V_{CB}}{V_t}\right)-1\right]\cdot\exp\left(\frac{-x}{L_C}\right)\Big|_0^\infty$$

$$= n_{PO}L_C\left[\exp\left(\frac{V_{CB}}{V_t}\right)-1\right]$$

We have

$$n_{PO} = \frac{n_i^2}{N_C} = \frac{\left(1.5x10^{10}\right)^2}{5x10^{15}} = 4.5x10^4 \ cm^{-3}$$

Then the total number of electrons is

$$N_{Coll} = \left(4.5x10^4\right)\left(35x10^{-4}\right)\left(0.56x10^{12}\right)$$

or

$$\underline{N_{Coll} = 8.82x10^{13} \ electrons/cm^2}$$

10.18

(b) $n_{BO} = \dfrac{n_i^2}{N_B} = \dfrac{\left(1.5x10^{10}\right)^2}{10^{17}} = 2.25x10^3 \ cm^{-3}$

and

$$p_{CO} = \frac{n_i^2}{N_C} = \frac{\left(1.5x10^{10}\right)^2}{7x10^{15}} = 3.21x10^4 \ cm^{-3}$$

At $x = x_B$,

$$n_B(x_B) = n_{BO}\exp\left(\frac{V_{BC}}{V_t}\right)$$

$$= \left(2.25x10^3\right)\exp\left(\frac{0.565}{0.0259}\right)$$

or

$$n_B(x_B) = 6.7x10^{12} \ cm^{-3}$$

At $x'' = 0$,

$$p_C(0) = p_{CO}\exp\left(\frac{V_{BC}}{V_t}\right)$$

$$= \left(3.21x10^4\right)\exp\left(\frac{0.565}{0.0259}\right)$$

or

$$p_C(0) = 9.56x10^{13} \ cm^{-3}$$

(c)
From the B-C space-charge region,

$$V_{bi1} = (0.0259)\ln\left[\frac{\left(10^{17}\right)\left(7x10^{15}\right)}{\left(1.5x10^{10}\right)^2}\right] = 0.745 \ V$$

Then

$$x_{p1} = \left\{\frac{2(11.7)\left(8.85x10^{-14}\right)(0.745-0.565)}{1.6x10^{-19}} \times\left(\frac{7x10^{15}}{10^{17}}\right)\left(\frac{1}{7x10^{15}+10^{17}}\right)\right\}^{1/2}$$

or

$$x_{p1} = 1.23x10^{-6} \ cm$$

From the B-E space-charge region,

$$V_{bi2} = (0.0259)\ln\left[\frac{\left(10^{19}\right)\left(10^{17}\right)}{\left(1.5x10^{10}\right)^2}\right] = 0.933 \ V$$

Then

$$x_{p2} = \left\{\frac{2(11.7)\left(8.85x10^{-14}\right)(0.933+2)}{1.6x10^{-19}} \times\left(\frac{10^{19}}{10^{17}}\right)\left(\frac{1}{10^{19}+10^{17}}\right)\right\}^{1/2}$$

or

$$x_{p2} = 1.94x10^{-5} \ cm$$

Now

$$x_B = x_{BO} - x_{p1} - x_{p2} = 1.20 - 0.0123 - 0.194$$

or

$$\underline{x_B = 0.994 \ \mu m}$$

10.19

Low injection limit is reached when $p_C(0) = (0.10)N_C$, so that

$$p_C(0) = (0.10)\left(5x10^{14}\right) = 5x10^{13} \ cm^{-3}$$

We have

$$p_{CO} = \frac{n_i^2}{N_C} = \frac{\left(1.5x10^{10}\right)^2}{5x10^{14}} = 4.5x10^5 \ cm^{-3}$$

Also

$$p_C(0) = p_{CO}\exp\left(\frac{V_{CB}}{V_t}\right)$$

or

$$V_{CB} = V_t\ln\left(\frac{p_C(0)}{p_{CO}}\right)$$

$$= (0.0259)\ln\left(\frac{5x10^{13}}{4.5x10^5}\right)$$

or

$$\underline{V_{CB} = 0.48 \ V}$$

10.20

(a)

$$\alpha = \frac{J_{nC}}{J_{nE} + J_R + J_{pE}}$$

$$= \frac{1.18}{1.20 + 0.20 + 0.10} \Rightarrow \underline{\alpha = 0.787}$$

(b)

$$\gamma = \frac{J_{nE}}{J_{nE} + J_{pE}}$$

$$= \frac{1.20}{1.20 + 0.10} \Rightarrow \underline{\gamma = 0.923}$$

(c)

$$\alpha_T = \frac{J_{nC}}{J_{nE}} = \frac{1.18}{1.20} \Rightarrow \underline{\alpha_T = 0.983}$$

(d)

$$\delta = \frac{J_{nE} + J_{pE}}{J_{nE} + J_R + J_{pE}}$$

$$= \frac{1.20 + 0.10}{1.20 + 0.20 + 0.10} \Rightarrow \underline{\delta = 0.867}$$

(e)

$$\beta = \frac{\alpha}{1-\alpha} = \frac{0.787}{1 - 0.787}$$

or

$$\underline{\beta = 3.69}$$

10.21

$$n_{BO} = \frac{n_i^2}{N_B} = \frac{\left(1.5x10^{10}\right)^2}{10^{17}} = 2.25x10^3 \ cm^{-3}$$

Then

$$n_B(0) = n_{BO} \exp\left(\frac{V_{BE}}{V_t}\right)$$

$$= \left(2.25x10^3\right)\exp\left(\frac{0.50}{0.0259}\right)$$

or

$$\underline{n_B(0) = 5.45x10^{11} \ cm^{-3}}$$

As a good approximation,

$$I_C = \frac{eD_B A n_B(0)}{x_B}$$

$$= \frac{\left(1.6x10^{-19}\right)(20)\left(10^{-3}\right)\left(5.45x10^{11}\right)}{10^{-4}}$$

or

$$\underline{I_C = 17.4 \ \mu A}$$

(b)

Base transport factor

$$\alpha_T = \frac{1}{\cosh\left(x_B / L_B\right)}$$

We find

$$L_B = \sqrt{D_B \tau_{BO}} = \sqrt{(20)\left(10^{-7}\right)} = 1.41x10^{-3} \ cm$$

so that

$$\alpha_T = \frac{1}{\cosh(1/14.1)} \Rightarrow \underline{\alpha_T = 0.9975}$$

Emitter injection efficiency

Assuming $D_E = D_B$, $x_B = x_E$, and $L_E = L_B$; then

$$\gamma = \frac{1}{1 + \frac{N_B}{N_E}} = \frac{1}{1 + \frac{10^{17}}{10^{18}}} \Rightarrow \underline{\gamma = 0.909}$$

Then

$$\alpha = \gamma \alpha_T \delta = (0.909)(0.9975)(1) \Rightarrow$$

$$\underline{\alpha = 0.9067}$$

and

$$\beta = \frac{\alpha}{1-\alpha} = \frac{0.9067}{1 - 0.9067} \Rightarrow \underline{\beta = 9.72}$$

For $I_E = 1.5 \ mA$,

$$I_C = \alpha I_E = (0.9067)(1.5) \Rightarrow \underline{I_C = 1.36 \ mA}$$

(c)

For $I_B = 2 \ \mu A$,

$$I_C = \beta I_B = (9.72)(2) \Rightarrow \underline{I_C = 19.4 \ \mu A}$$

10.22

(a) We have

$$J_{nE} = \frac{eD_B n_{BO}}{L_B}\left\{\frac{1}{\sinh\left(\frac{x_B}{L_B}\right)} + \frac{\left[\exp\left(\frac{V_{BE}}{V_t}\right) - 1\right]}{\tanh\left(\frac{x_B}{L_B}\right)}\right\}$$

We find that

$$n_{BO} = \frac{n_i^2}{N_B} = \frac{\left(1.5x10^{10}\right)^2}{5x10^{16}} = 4.5x10^3 \ cm^{-3}$$

and

$$L_B = \sqrt{D_B \tau_{BO}} = \sqrt{(15)\left(5x10^{-8}\right)} = 8.66x10^{-4} \ cm$$

Then

$$J_{nE} = \frac{(1.6x10^{-19})(15)(4.5x10^{3})}{8.66x10^{-4}} \times \left\{ \frac{1}{\sinh\left(\frac{0.70}{8.66}\right)} + \frac{\exp\left(\frac{0.60}{0.0259}\right)}{\tanh\left(\frac{0.70}{8.66}\right)} \right\}$$

or

$$\underline{J_{nE} = 1.79 \ A/cm^2}$$

We also have

$$J_{pE} = \frac{eD_E p_{EO}}{L_E}\left[\exp\left(\frac{V_{BE}}{V_t}\right) - 1\right] \cdot \frac{1}{\tanh\left(\frac{x_E}{L_E}\right)}$$

Also

$$p_{EO} = \frac{n_i^2}{N_E} = \frac{(1.5x10^{10})^2}{10^{18}} = 2.25x10^2 \ cm^{-3}$$

and

$$L_E = \sqrt{D_E \tau_{EO}} = \sqrt{(8)(10^{-8})} = 2.83x10^{-4} \ cm$$

Then

$$J_{pE} = \frac{(1.6x10^{-19})(8)(2.25x10^{2})}{2.83x10^{-4}} \times \left[\exp\left(\frac{0.60}{0.0259}\right) - 1\right] \cdot \frac{1}{\tanh\left(\frac{0.8}{2.83}\right)}$$

or

$$\underline{J_{pE} = 0.0425 \ A/cm^2}$$

We can find

$$J_{nC} = \frac{eD_B n_{BO}}{L_B}\left\{ \frac{\left[\exp\left(\frac{0.60}{0.0259}\right) - 1\right]}{\sinh\left(\frac{x_B}{L_B}\right)} + \frac{1}{\tanh\left(\frac{x_B}{L_B}\right)} \right\}$$

$$= \frac{(1.6x10^{-19})(15)(4.5x10^{3})}{8.66x10^{-4}} \times \left\{ \frac{\exp\left(\frac{0.60}{0.0259}\right)}{\sinh\left(\frac{0.7}{8.66}\right)} + \frac{1}{\tanh\left(\frac{0.7}{8.66}\right)} \right\}$$

or

$$\underline{J_{nC} = 1.78 \ A/cm^2}$$

The recombination current is

$$J_R = J_{rO} \exp\left(\frac{eV_{BE}}{2kT}\right)$$

$$= (3x10^{-8})\exp\left(\frac{0.60}{2(0.0259)}\right)$$

or

$$\underline{J_R = 3.22x10^{-3} \ A/cm^2}$$

(b)
Using the calculated currents, we find

$$\gamma = \frac{J_{nE}}{J_{nE} + J_{pE}} = \frac{1.79}{1.79 + 0.0425} \Rightarrow$$

$$\underline{\gamma = 0.977}$$

We find

$$\alpha_T = \frac{J_{nC}}{J_{nE}} = \frac{1.78}{1.79} \Rightarrow \quad \underline{\alpha_T = 0.994}$$

and

$$\delta = \frac{J_{nE} + J_{pE}}{J_{nE} + J_R + J_{pE}} = \frac{1.79 + 0.0425}{1.79 + 0.00322 + 0.0425}$$

or

$$\underline{\delta = 0.998}$$

Then

$$\alpha = \gamma\alpha_T\delta = (0.977)(0.994)(0.998) \Rightarrow$$

$$\underline{\alpha = 0.969}$$

Now

$$\beta = \frac{\alpha}{1-\alpha} = \frac{0.969}{1 - 0.969} \Rightarrow \quad \underline{\beta = 31.3}$$

10.23

(a) $$\gamma = \frac{1}{1 + \frac{N_B}{N_E} \cdot \frac{D_E}{D_B} \cdot \frac{x_B}{x_E}} \approx 1 - \frac{N_B}{N_E} \cdot \frac{D_E}{D_B} \cdot \frac{x_B}{x_E}$$

or

$$\gamma \approx 1 - K \cdot \frac{N_B}{N_E}$$

(i)

$$\frac{\gamma(B)}{\gamma(A)} = \frac{1 - \frac{2N_{BO}}{N_E} \cdot K}{1 - \frac{N_{BO}}{N_E} \cdot K}$$

$$\approx \left(1 - \frac{2N_{BO}}{N_E} \cdot K\right)\left(1 + \frac{N_{BO}}{N_E} \cdot K\right)$$

$$\approx 1 - \frac{2N_{BO}}{N_E} \cdot K + \frac{N_{BO}}{N_E} \cdot K$$

or

$$\frac{\gamma(B)}{\gamma(A)} \approx 1 - \frac{N_{BO}}{N_E} \cdot \frac{D_E}{D_B} \cdot \frac{x_B}{x_E}$$

(ii)

$$\frac{\gamma(C)}{\gamma(A)} = 1$$

(b) (i)

$$\frac{\alpha_T(B)}{\alpha_T(A)} = 1$$

(ii)

$$\frac{\alpha_T(C)}{\alpha_T(A)} = \frac{\left(1 - \frac{1}{2} \cdot \frac{(x_{BO}/2)}{L_B}\right)^2}{\left(1 - \frac{1}{2} \cdot \frac{x_{BO}}{L_B}\right)^2}$$

$$\approx \frac{\left(1 - \frac{x_{BO}}{2L_B}\right)}{\left(1 - \frac{x_{BO}}{L_B}\right)} \approx \left(1 - \frac{x_{BO}}{2L_B}\right)\left(1 + \frac{x_{BO}}{L_B}\right)$$

$$\approx 1 - \frac{x_{BO}}{2L_B} + \frac{x_{BO}}{L_B}$$

or

$$\frac{\alpha_T(C)}{\alpha_T(A)} \approx 1 + \frac{x_{BO}}{2L_B}$$

(c) Neglect any change in space charge width. Then

$$\delta = \frac{1}{1 + \frac{J_{rO}}{J_{sO}} \cdot \exp\left(\frac{-V_{BE}}{2V_t}\right)}$$

$$\approx 1 - \frac{J_{rO}}{J_{sO}} \cdot \exp\left(\frac{-V_{BE}}{2V_t}\right)$$

(i)

$$\frac{\delta(B)}{\delta(A)} = \frac{1 - \frac{K}{J_{sOB}}}{1 - \frac{K}{J_{sOA}}} \approx \left(1 - \frac{K}{J_{sOB}}\right)\left(1 + \frac{K}{J_{sOA}}\right)$$

$$\approx 1 - \frac{K}{J_{sOB}} + \frac{K}{J_{sOA}}$$

Now

$$J_{sO} \propto n_{BO} = \frac{n_i^2}{N_B}$$

so

$$\frac{\delta(B)}{\delta(A)} \approx 1 - \frac{2N_{BO}K}{C} + \frac{N_{BO}K}{C} = 1 - \frac{N_{BO}K}{C}$$

Then

$$\frac{\delta(B)}{\delta(A)} \approx 1 - \frac{J_{rO}\exp\left(\frac{-V_{BE}}{2V_t}\right)}{\left(\frac{eD_B n_{BO}}{x_B}\right)}$$

(ii)
We find

$$\frac{\delta(C)}{\delta(A)} \approx 1 + \frac{J_{rO}\exp\left(\frac{-V_{BE}}{2V_t}\right)}{\left(\frac{eD_B n_{BO}}{x_B}\right)}$$

(d)
Device C has the largest β. Base transport factor as well as the recombination factor increases.

10.24

(a)

$$\gamma = \frac{1}{1 + \frac{N_B}{N_E} \cdot \frac{D_E}{D_B} \cdot \frac{x_B}{x_E}} = \frac{1}{1 + K \cdot \frac{N_B}{N_E}}$$

or

$$\gamma \approx 1 - K \cdot \frac{N_B}{N_E}$$

(i) Then

$$\frac{\gamma(B)}{\gamma(A)} = \frac{1 - K \cdot \frac{N_B}{2N_{EO}}}{1 - K \cdot \frac{N_B}{N_{EO}}}$$

$$\approx \left(1 - K \cdot \frac{N_B}{2N_{EO}}\right) \cdot \left(1 + K \cdot \frac{N_B}{N_{EO}}\right)$$

$$\approx 1 - K \cdot \frac{N_B}{2N_{EO}} + K \cdot \frac{N_B}{N_{EO}}$$

or

$$= 1 + K \cdot \frac{N_B}{2N_{EO}}$$

or

$$\frac{\gamma(B)}{\gamma(A)} = 1 + \frac{N_B}{2N_{EO}} \cdot \frac{D_E}{D_B} \cdot \frac{x_B}{x_E}$$

(ii)
Now

$$\gamma = \frac{1}{1 + K' \cdot \dfrac{x_B}{x_E}} \approx 1 - K' \cdot \frac{x_B}{x_E}$$

Then

$$\frac{\gamma(C)}{\gamma(A)} = \frac{1 - K' \cdot \dfrac{x_B}{(x_{EO}/2)}}{1 - K' \cdot \dfrac{x_B}{x_{EO}}}$$

$$\approx \left(1 - K' \cdot \frac{2x_B}{x_{EO}}\right) \cdot \left(1 + K' \cdot \frac{x_B}{x_{EO}}\right)$$

$$\approx 1 - 2K' \cdot \frac{x_B}{x_{EO}} + K' \cdot \frac{x_B}{x_{EO}}$$

$$= 1 - K' \cdot \frac{x_B}{x_{EO}}$$

or

$$\frac{\gamma(C)}{\gamma(A)} = 1 - \frac{N_B}{N_E} \cdot \frac{D_E}{D_B} \cdot \frac{x_B}{x_{EO}}$$

(b)

$$\alpha_T = 1 - \frac{1}{2}\left(\frac{x_B}{L_B}\right)^2$$

so
(i)

$$\frac{\alpha_T(B)}{\alpha_T(A)} = 1$$

and
(ii)

$$\frac{\alpha_T(C)}{\alpha_T(A)} = 1$$

(c)
Neglect any change in space charge width

$$\delta = \frac{1}{1 + \dfrac{J_{rO}}{J_{SO}} \exp\left(\dfrac{-V_{BE}}{2V_t}\right)}$$

$$= \frac{1}{1 + \dfrac{k}{J_{SO}}} \approx 1 - \frac{k}{J_{SO}}$$

(i)

$$\frac{\delta(B)}{\delta(A)} = \frac{1 - \dfrac{k}{J_{SOB}}}{1 - \dfrac{k}{J_{SOA}}} \approx \left(1 - \frac{k}{J_{SOB}}\right)\left(1 + \frac{k}{J_{SOA}}\right)$$

$$\approx 1 - \frac{k}{J_{SOB}} + \frac{k}{J_{SOA}}$$

Now

$$J_{SO} \propto \frac{1}{N_E x_E}$$

so
(i)

$$\frac{\delta(B)}{\delta(A)} = 1 - k'(2N_{EO}) + k'(N_{EO})$$

or

$$\frac{\delta(B)}{\delta(A)} = 1 - k' \cdot (N_{EO})$$

(recombination factor decreases)
(ii)
We have

$$\frac{\delta(C)}{\delta(A)} = 1 - k'' \cdot \left(\frac{x_{EO}}{2}\right) + k'' \cdot (x_{EO})$$

or

$$\frac{\delta(C)}{\delta(A)} = 1 + \frac{1}{2} k'' \cdot x_{EO}$$

(recombination factor increases)

10.25
(b)

$$n_{BO} = \frac{n_i^2}{N_B} = \frac{(1.5x10^{10})^2}{10^{17}} = 2.25x10^3 \ cm^{-3}$$

Then

$$n_B(0) = n_{BO} \exp\left(\frac{V_{BC}}{V_t}\right)$$

$$= (2.25x10^3)\exp\left(\frac{0.6}{0.0259}\right) = 2.59x10^{13} \ cm^{-3}$$

Now

$$J_{nC} = \frac{eD_B n_B(0)}{x_B}$$

$$= \frac{(1.6x10^{-19})(20)(2.59x10^{13})}{10^{-4}}$$

or

$$J_{nC} = 0.829 \ A/cm^2$$

Assuming a long collector,

$$J_{pC} = \frac{eD_C p_{nO}}{L_C} \exp\left(\frac{V_{BC}}{V_t}\right)$$

where

$$p_{nO} = \frac{n_i^2}{N_C} = \frac{\left(1.5x10^{10}\right)^2}{10^{16}} = 2.25x10^4 \ cm^{-3}$$

and

$$L_C = \sqrt{D_C \tau_{CO}} = \sqrt{(15)\left(2x10^{-7}\right)} = 1.73x10^{-3} \ cm$$

Then

$$J_{pC} = \frac{\left(1.6x10^{-19}\right)(15)\left(2.25x10^4\right)}{1.73x10^{-3}} \exp\left(\frac{0.6}{0.0259}\right)$$

or

$$J_{pC} = 0.359 \ A/cm^2$$

The collector current is

$$I_C = \left(J_{nC} + J_{pC}\right) \cdot A = (0.829 + 0.359)\left(10^{-3}\right)$$

or

$$\underline{I_C = 1.19 \ mA}$$

The emitter current is

$$I_E = J_{nC} \cdot A = (0.829)\left(10^{-3}\right)$$

or

$$\underline{I_E = 0.829 mA}$$

10.26

(a)

$$\alpha_T = \frac{1}{\cosh\left(x_B/L_B\right)} \qquad \beta = \frac{\alpha_T}{1-\alpha_T}$$

x_B/L_B	α_T	β
0.01	0.99995	19,999
0.10	0.995	199
1.0	0.648	1.84
10.0	0.0000908	≈ 0

(b)

For $D_E = D_B$, $L_E = L_B$, $x_E = x_B$, we have

$$\gamma = \frac{1}{1 + \left(p_{EO}/n_{BO}\right)} = \frac{1}{1 + \left(N_B/N_C\right)}$$

and

$$\beta = \frac{\gamma}{1-\gamma}$$

N_B/N_E	γ	β
0.01	0.990	99
0.10	0.909	9.99
1.0	0.50	1.0
10.0	0.0909	0.10

(c)

For $x_B/L_B < 0.10$, the value of β is unreasonably large, which means that the base transport factor is not the limiting factor. For $x_B/L_B > 1.0$, the value of β is very small, which means that the base transport factor will probably be the limiting factor.

If $N_B/N_E < 0.01$, the emitter injection efficiency is probably not the limiting factor. If, however, $N_B/N_E > 0.01$, then the current gain is small and the emitter injection efficiency is probably the limiting factor.

10.27

We have

$$J_{sO} = \frac{eD_B n_{BO}}{L_B \tanh\left(x_B/L_B\right)}$$

Now

$$n_{BO} = \frac{n_i^2}{N_B} = \frac{\left(1.5x10^{10}\right)^2}{10^{17}} = 2.25x10^3 \ cm^{-3}$$

and

$$L_B = \sqrt{D_B \tau_{BO}} = \sqrt{(25)\left(10^{-7}\right)} = 15.8x10^{-4} \ cm$$

Then

$$J_{sO} = \frac{\left(1.6x10^{-19}\right)(25)\left(2.25x10^3\right)}{\left(15.8x10^{-4}\right)\tanh\left(0.7/15.8\right)}$$

or

$$J_{sO} = 1.3x10^{-10} \ A/cm^2$$

Now

$$\delta = \frac{1}{1 + \frac{J_{rO}}{J_{sO}} \cdot \exp\left(\frac{-V_{BE}}{2V_t}\right)}$$

$$= \frac{1}{1 + \frac{2x10^{-9}}{1.3x10^{-10}} \cdot \exp\left(\frac{-V_{BE}}{2(0.0259)}\right)}$$

or

(a)

$$\delta = \frac{1}{1 + (15.38)\exp\left(\dfrac{-V_{BE}}{0.0518}\right)}$$

and
(b)

$$\beta = \frac{\delta}{1-\delta}$$

Now

V_{BE}	δ	β
0.20	0.755	3.08
0.40	0.993	142
0.60	0.99986	7,142

(c)
If $V_{BE} < 0.4\ V$, the recombination factor is likely the limiting factor in the current gain.

10.28

For $\beta = 120 = \dfrac{\alpha}{1-\alpha} \Rightarrow \alpha = \dfrac{\beta}{1+\beta}$

So

$$\alpha = \frac{120}{121} = 0.9917$$

Now

$$\alpha = \gamma\alpha_T\delta = 0.9917 = (0.998)x^2$$

where

$$x = \alpha_T = \gamma = 0.9968$$

We have

$$\alpha_T = \frac{1}{\cosh\left(\dfrac{x_B}{L_B}\right)} = 0.9968$$

which means

$$\frac{x_B}{L_B} = 0.0801$$

We find

$$L_B = \sqrt{D_B\tau_{BO}} = \sqrt{(25)(10^{-7})} = 15.8\ \mu m$$

Then

$$x_B(\max) = (0.0801)(15.8) \Rightarrow$$

$$\underline{x_B(\max) = 1.26\ \mu m}$$

We also have

$$\gamma = \frac{1}{1 + \dfrac{p_{EO}}{n_{BO}}\cdot\dfrac{D_E}{D_B}\cdot\dfrac{L_B}{L_E}\cdot\dfrac{\tanh(x_B/L_B)}{\tanh(x_E/L_E)}}$$

where

$$L_E = \sqrt{D_E\tau_{EO}} = \sqrt{(10)(5x10^{-8})} = 7.07\ \mu m$$

Then

$$0.9968 = \frac{1}{1 + \dfrac{p_{EO}}{n_{BO}}\cdot\left(\dfrac{10}{25}\right)\left(\dfrac{15.8}{7.07}\right)\dfrac{\tanh(1.26/15.8)}{\tanh(0.5/7.07)}}$$

which yields

$$\frac{p_{EO}}{n_{BO}} = 0.003186 = \frac{N_B}{N_E}$$

Finally

$$N_E = \frac{N_B}{0.003186} = \frac{10^{16}}{0.003186} \Rightarrow$$

$$\underline{N_E = 3.14x10^{18}\ cm^{-3}}$$

10.29

(a) We have $J_{rO} = 5x10^{-8}\ A/cm^2$

We find

$$n_{BO} = \frac{n_i^2}{N_B} = \frac{(1.5x10^{10})^2}{5x10^{16}} = 4.5x10^3\ cm^{-3}$$

and

$$L_B = \sqrt{D_B\tau_{BO}} = \sqrt{(25)(10^{-7})} = 15.8\ \mu m$$

Then

$$J_{sO} = \frac{eD_Bn_{BO}}{L_B\tanh(x_B/L_B)}$$

$$= \frac{(1.6x10^{-19})(25)(4.5x10^3)}{(15.8x10^{-4})\tanh(x_B/L_B)}$$

or

$$J_{sO} = \frac{1.14x10^{-11}}{\tanh(x_B/L_B)}$$

We have

$$\delta = \frac{1}{1 + \dfrac{J_{rO}}{J_{sO}}\cdot\exp\left(\dfrac{-V_{BE}}{2V_t}\right)}$$

For $T = 300K$ and $V_{BE} = 0.55\ V$,

$\delta = 0.995 =$

$$\frac{1}{1 + \left(\dfrac{5x10^{-8}}{1.14x10^{-11}}\right)\cdot\tanh\left(\dfrac{x_B}{L_B}\right)\cdot\exp\left(\dfrac{-0.55}{2(0.0259)}\right)}$$

which yields

$$\frac{x_B}{L_B} = 0.047$$

or

$$x_B = (0.047)(15.8x10^{-4}) \Rightarrow$$

$$\underline{x_B = 0.742\ \mu m}$$

(b)

For $T = 400K$ and $J_{rO} = 5x10^{-8}\ A/cm^2$,

$$\frac{n_{BO}(400)}{n_{BO}(300)} = \left(\frac{400}{300}\right)^3 \cdot \frac{\exp\left[\dfrac{-E_g}{(0.0259)(400/300)}\right]}{\exp\left[\dfrac{-E_g}{(0.0259)}\right]}$$

For $E_g = 1.12\ eV$,

$$\frac{n_{BO}(400)}{n_{BO}(300)} = 1.17x10^5$$

or

$$n_{BO}(400) = (1.17x10^5)(4.5x10^3)$$

$$= 5.27x10^8\ cm^{-3}$$

Then

$$J_{sO} = \frac{(1.6x10^{-19})(25)(5.27x10^8)}{(15.8x10^{-4})\tanh(0.742/15.8)}$$

or

$$J_{sO} = 2.84x10^{-5}\ A/cm^2$$

Finally,

$$\delta = \frac{1}{1 + \dfrac{5x10^{-8}}{2.84x10^{-5}} \cdot \exp\left[\dfrac{-0.55}{2(0.0259)(400/300)}\right]}$$

or

$$\underline{\delta = 0.9999994}$$

10.30
Computer plot

10.31
Computer plot

10.32
Computer plot

10.33
Computer plot

10.34

Metallurgical base width $= 1.2\ \mu m = x_B + x_n$

We have

$$p_{BO} = \frac{n_i^2}{N_B} = \frac{(1.5x10^{10})^2}{10^{16}} = 2.25x10^4\ cm^{-3}$$

and

$$p_B(0) = p_{BO}\exp\left(\frac{V_{EB}}{V_t}\right)$$

$$= (2.25x10^4)\exp\left(\frac{0.625}{0.0259}\right)$$

$$= 6.8x10^{14}\ cm^{-3}$$

Now

$$J_p = eD_B\frac{dp_B}{dx} = eD_B\left(\frac{p_B(0)}{x_B}\right)$$

$$= \frac{(1.6x10^{-19})(10)(6.8x10^{14})}{x_B}$$

or

$$J_p = \frac{1.09x10^{-3}}{x_B}$$

We have

$$x_n = \left\{\frac{2\in(V_{bi}+V_R)}{e}\left(\frac{N_C}{N_B}\right)\left(\frac{1}{N_C+N_B}\right)\right\}^{1/2}$$

and

$$V_{bi} = (0.0259)\ln\left[\frac{(10^{16})(10^{15})}{(1.5x10^{10})^2}\right] = 0.635\ V$$

We can write

$$x_n = \left\{\frac{2(11.7)(8.85x10^{-14})(V_{bi}+V_R)}{1.6x10^{-19}} \times\left(\frac{10^{15}}{10^{16}}\right)\left(\frac{1}{10^{15}+10^{16}}\right)\right\}^{1/2}$$

or

$$x_n = \left\{(1.177x10^{-10})(V_{bi}+V_R)\right\}^{1/2}$$

We know

$$x_B = 1.2x10^{-4} - x_n$$

For $V_R = V_{BC} = 5\ V$

$$x_n = 0.258x10^{-4}\ cm \Rightarrow x_B = 0.942x10^{-4}\ cm$$

Then

$$\underline{J_p = 11.6\ A/cm^2}$$

For $V_R = V_{BC} = 10\ V$,

$x_n = 0.354x10^{-4}\ cm \Rightarrow x_B = 0.846x10^{-4}\ cm$

Then

$$\underline{J_p = 12.9\ A/cm^2}$$

For $V_R = V_{BC} = 15\ V$,

$x_n = 0.429x10^{-4}\ cm \Rightarrow x_B = 0.771x10^{-4}\ cm$

Then

$$\underline{J_p = 14.1\ A/cm^2}$$

(b)

We can write

$$J_p = g'(V_{EC} + V_A)$$

where

$$g' = \frac{\Delta J_p}{\Delta V_{EC}} = \frac{\Delta J_p}{\Delta V_{BC}} = \frac{14.1 - 11.6}{10}$$

or

$$g' = 0.25\ mA/cm^2/V$$

Now

$J_p = 11.6\ A/cm^2$ at

$$V_{EC} = V_{BC} + V_{EB} = 5 + 0.626 = 5.626\ V$$

Then

$$11.6 = (0.25)(5.625 + V_A)$$

which yields

$$\underline{V_A = 40.8\ V}$$

10.35 We find

$$n_{BO} = \frac{n_i^2}{N_B} = \frac{\left(1.5x10^{10}\right)^2}{3x10^{16}} = 7.5x10^3\ cm^{-3}$$

and

$$n_B(0) = n_{BO}\exp\left(\frac{V_{BE}}{V_t}\right)$$

$$= \left(7.5x10^3\right)\exp\left(\frac{0.7}{0.0259}\right)$$

or

$$n_B(0) = 4.10x10^{15}\ cm^{-3}$$

We have

$$J = eD_B\frac{dn_B}{dx} = \frac{eD_B n_B(0)}{x_B}$$

$$= \frac{\left(1.6x10^{-19}\right)(20)\left(4.10x10^{15}\right)}{x_B}$$

or

$$J = \frac{1.312x10^{-2}}{x_B}$$

Neglecting the space charge width at the B-E junction, we have

$$x_B = x_{BO} - x_p$$

Now

$$V_{bi} = (0.0259)\ln\left[\frac{\left(3x10^{16}\right)\left(5x10^{15}\right)}{\left(1.5x10^{10}\right)^2}\right]$$

or

$$V_{bi} = 0.705\ V$$

and

$$x_p = \left\{\frac{2\in(V_{bi} + V_{CB})}{e}\left(\frac{N_C}{N_B}\right)\left(\frac{1}{N_C + N_B}\right)\right\}^{1/2}$$

$$= \left\{\frac{2(11.7)\left(8.85x10^{-14}\right)(V_{bi} + V_{CB})}{1.6x10^{-19}}\right.$$

$$\left.\times\left(\frac{5x10^{15}}{3x10^{16}}\right)\left(\frac{1}{5x10^{15} + 3x10^{16}}\right)\right\}^{1/2}$$

or

$$x_p = \left\{\left(6.163x10^{-11}\right)(V_{bi} + V_{CB})\right\}^{1/2}$$

Now, for $V_{CB} = 5\ V$, $x_p = 0.1875\ \mu m$, and

For $V_{CB} = 10\ V$, $x_p = 0.2569\ \mu m$

(a)

$x_{BO} = 1.0\ \mu m$

For $V_{CB} = 5\ V$, $x_B = 1.0 - 0.1875 = 0.8125\ \mu m$

Then

$$J = \frac{1.312x10^{-2}}{0.8125x10^{-4}} = 161.5\ A/cm^2$$

For $V_{CB} = 10\ V$, $x_B = 1.0 - 0.2569 = 0.7431\ \mu m$

Then

$$J = \frac{1.312x10^{-2}}{0.7431x10^{-4}} = 176.6\ A/cm^2$$

We can write

$$J = \frac{\Delta J}{\Delta V_{CE}}(V_{CE} + V_A)$$

where

$$\frac{\Delta J}{\Delta V_{CE}} = \frac{\Delta J}{\Delta V_{CB}} = \frac{176.6 - 161.5}{5}$$

$$= 3.02\ A/cm^2/V$$

Then

$$161.5 = 3.02(5.7 + V_A) \Rightarrow$$

$$\underline{V_A = 47.8\ V}$$

(b)

$x_{BO} = 0.80\ \mu m$

For $V_{CB} = 5\ V$, $x_B = 0.80 - 0.1875 = 0.6125\ \mu m$

Then

$$J = \frac{1.312x10^{-2}}{0.6125x10^{-4}} = 214.2\ A/cm^2$$

For $V_{CB} = 10\ V$, $x_B = 0.80 - 0.2569 = 0.5431\ \mu m$

Then

$$J = \frac{1.312x10^{-2}}{0.5431x10^{-4}} = 241.6\ A/cm^2$$

Now

$$\frac{\Delta J}{\Delta V_{CE}} = \frac{\Delta J}{\Delta V_{CB}} = \frac{241.6 - 214.2}{5}$$

$$= 5.48\ A/cm^2/V$$

We can write

$$J = \frac{\Delta J}{\Delta V_{CE}}(V_{CE} + V_A)$$

or

$$214.2 = 5.48(5.7 + V_A) \Rightarrow$$

$$\underline{V_A = 33.4\ V}$$

(c)

$x_{BO} = 0.60\ \mu m$

For $V_{CB} = 5\ V$, $x_B = 0.60 - 0.1875 = 0.4124\ \mu m$

Then

$$J = \frac{1.312x10^{-2}}{0.4125x10^{-4}} = 318.1\ A/cm^2$$

For $V_{CB} = 10\ V$, $x_B = 0.60 - 0.2569 = 0.3431\ \mu m$

Then

$$J = \frac{1.312x10^{-2}}{0.3431x10^{-4}} = 382.4\ A/cm^2$$

Now

$$\frac{\Delta J}{\Delta V_{CE}} = \frac{\Delta J}{\Delta V_{CB}} = \frac{382.4 - 318.1}{5}$$

$$= 12.86\ A/cm^2/V$$

We can write

$$J = \frac{\Delta J}{\Delta V_{CE}}(V_{CE} + V_A)$$

so

$$318.1 = 12.86(5.7 + V_A) \Rightarrow$$

$$\underline{V_A = 19.0\ V}$$

10.36

Neglect the B-E space charge region

$$n_{BO} = \frac{n_i^2}{N_B} = \frac{(1.5x10^{10})^2}{10^{17}} = 2.25x10^3\ cm^{-3}$$

Then

$$n_B(0) = n_{BO}\exp\left(\frac{V_{BE}}{V_t}\right)$$

$$= 2.25x10^3 \exp\left(\frac{0.60}{0.0259}\right) = 2.59x10^{13}\ cm^{-3}$$

$$J = eD_B\frac{dn_B}{dx} = \frac{eD_B n_B(0)}{x_B}$$

$$= \frac{(1.6x10^{-19})(20)(2.59x10^{13})}{x_B}$$

or

$$J = \frac{8.29x10^{-5}}{x_B}$$

(a)

Now $x_B = x_{BO} - x_p$

Now

$$V_{bi} = (0.0259)\ln\left[\frac{(10^{16})(10^{17})}{(1.5x10^{10})^2}\right] = 0.754\ V$$

Also

$$x_p = \left[\frac{2\in(V_{bi} + V_{CB})}{e}\left(\frac{N_C}{N_B}\right)\left(\frac{1}{N_C + N_B}\right)\right]^{1/2}$$

$$= \left[\frac{2(11.7)(8.85x10^{-14})(V_{bi} + V_{CB})}{1.6x10^{-19}}\right.$$

$$\left.\times\left(\frac{10^{16}}{10^{17}}\right)\left(\frac{1}{10^{16} + 10^{17}}\right)\right]^{1/2}$$

or

$$x_p = \left[(1.177x10^{-11})(V_{bi} + V_{CB})\right]^{1/2}$$

For $V_{CB} = 1\ V$, $x_p(1) = 4.544x10^{-6}\ cm$

For $V_{CB} = 5\ V$, $x_p(5) = 8.229x10^{-6}\ cm$

Now

$$x_B = x_{BO} - x_p = 1.1x10^{-4} - x_p$$

Then

For $V_{CB} = 1\ V$, $x_B(1) = 1.055\ \mu m$

For $V_{CB} = 5\ V$, $x_B(5) = 1.018\ \mu m$

So

$$\Delta x_B = 1.055 - 1.018 \Rightarrow$$

or

$$\underline{\Delta x_B = 0.037\ \mu m}$$

(b)
Now

$$J(1) = \frac{8.29x10^{-5}}{1.055x10^{-4}} = 0.7858\ A/cm^2$$

and

$$J(5) = \frac{8.29x10^{-5}}{1.018x10^{-4}} = 0.8143\ A/cm^2$$

and

$$\Delta J = 0.8143 - 0.7858$$

or

$$\underline{\Delta J = 0.0285\ A/cm^2}$$

10.37

Let $x_E = x_B$, $L_E = L_B$, $D_E = D_B$

Then the emitter injection efficiency is

$$\gamma = \frac{1}{1+\frac{p_{EO}}{n_{BO}}} = \frac{1}{1+\frac{n_{iE}^2}{N_E}\cdot\frac{N_B}{n_{iB}^2}}$$

where $n_{iB}^2 = n_i^2$

For no bandgap narrowing, $n_{iE}^2 = n_i^2$.

With bandgap narrowing, $n_{iE}^2 = n_i^2 \exp\left(\frac{\Delta E_g}{kT}\right)$,

Then

$$\gamma = \frac{1}{1+\frac{N_B}{N_E}\exp\left(\frac{\Delta E_g}{kT}\right)}$$

(a)
No bandgap narrowing, so $\Delta E_g = 0$.

$\alpha = \gamma\alpha_T\delta = \gamma(0.995)^2$. We find

N_E	γ	α	β
E17	0.5	0.495	0.980
E18	0.909	0.8999	8.99
E19	0.990	0.980	49
E20	0.9990	0.989	89.9

(b)
Taking into account bandgap narrowing, we find

N_E	$\Delta E_g(meV)$	γ	α	β
E17	0	0.5	0.495	0.98
E18	25	0.792	0.784	3.63
E19	80	0.820	0.812	4.32
E20	230	0.122	0.121	0.14

10.38

(a) We have

$$\gamma = \frac{1}{1+\frac{p_{EO}D_E L_B}{n_{BO}D_B L_E}\cdot\frac{\tanh(x_B/L_B)}{\tanh(x_E/L_E)}}$$

For $x_E = x_B$, $L_E = L_B$, $D_E = D_B$, we obtain

$$\gamma = \frac{1}{1+\frac{p_{EO}}{n_{BO}}} = \frac{1}{1+\frac{(n_i^2/N_E)\exp(\Delta E_g/kT)}{(n_i^2/N_B)}}$$

or

$$\gamma = \frac{1}{1+\frac{N_B}{N_E}\exp\left(\frac{\Delta E_g}{kT}\right)}$$

For $N_E = 10^{19}\ cm^{-3}$, we have $\Delta E_g = 80\ meV$.

Then

$$0.996 = \frac{1}{1+\frac{N_B}{10^{19}}\exp\left(\frac{0.080}{0.0259}\right)}$$

which yields

$$\underline{N_B = 1.83x10^{15}\ cm^{-3}}$$

(b)
Neglecting bandgap narrowing, we would have

$$\gamma = \frac{1}{1+\frac{N_B}{N_E}} \Rightarrow 0.996 = \frac{1}{1+\frac{N_B}{10^{19}}}$$

which yields

$$\underline{N_B = 4.02x10^{16}\ cm^{-3}}$$

10.39

(a)

$$R = \frac{\rho L}{A} = \frac{L}{\sigma A} = \frac{(S/2)}{e\mu_p N_B(Lx_B)}$$

Then

$$R = \frac{4x10^{-4}}{\left(1.6x10^{-19}\right)(400)\left(10^{16}\right)\left(100x10^{-4}\right)\left(0.7x10^{-4}\right)}$$

or

$$\underline{R = 893\ \Omega}$$

(b)

$$V = IR = \left(10x10^{-6}\right)(893) \Rightarrow$$

$$\underline{V = 8.93\ mV}$$

(c)

At $x = 0$,

$$n_p(0) = n_{pO} \exp\left(\frac{V_{BE}}{V_t}\right)$$

and at $x = \frac{S}{2}$,

$$n_p'(0) = n_{pO} \exp\left(\frac{V_{BE} - 0.00893}{V_t}\right)$$

Then

$$\frac{n_p'(0)}{n_p(0)} = \frac{n_{pO} \exp\left(\frac{V_{BE} - 0.00893}{V_t}\right)}{n_{pO} \exp\left(\frac{V_{BE}}{V_t}\right)}$$

$$= \exp\left(\frac{-0.00893}{0.0259}\right) = 0.7084$$

or

$$\underline{\frac{n_p'(0)}{n_p(0)} = 70.8\%}$$

10.40

From problem 10.39(c), we have

$$\frac{n_p'(0)}{n_p(0)} = \exp\left(\frac{-V}{V_t}\right)$$

where V is the voltage drop across the $S/2$ length. Now

$$0.90 = \exp\left(\frac{-V}{0.0259}\right)$$

which yields $V = 2.73\ mV$

We have

$$R = \frac{V}{I} = \frac{2.73x10^{-3}}{10x10^{-6}} = 273\ \Omega$$

We can also write

$$R = \frac{S/2}{e\mu_p N_B\left(Lx_B\right)}$$

Solving for S, we find

$$S = 2R\mu_p eN_B Lx_B$$

$$= 2(273)(400)\left(1.6x10^{-19}\right)\left(10^{16}\right) \times\left(100x10^{-4}\right)\left(0.7x10^{-4}\right)$$

or

$$\underline{S = 2.45\ \mu m}$$

10.41

(a)

$$N_B = N_B(0)\exp\left(\frac{-ax}{x_B}\right)$$

where

$$a = \ln\left(\frac{N_B(0)}{N_B(x_B)}\right) > 0$$

and is a constant. In thermal equilibrium

$$J_p = e\mu_p N_B \mathrm{E} - eD_p \frac{dN_B}{dx} = 0$$

so that

$$\mathrm{E} = \frac{D_p}{\mu_p}\cdot\frac{1}{N_B}\cdot\frac{dN_B}{dx} = \left(\frac{kT}{e}\right)\cdot\frac{1}{N_B}\cdot\frac{dN_B}{dx}$$

which becomes

$$\mathrm{E} = \left(\frac{kT}{e}\right)\cdot\frac{1}{N_B}\cdot N_B(0)\cdot\left(\frac{-a}{x_B}\right)\cdot\exp\left(\frac{-ax}{x_B}\right)$$

$$= \left(\frac{kT}{e}\right)\cdot\left(\frac{-a}{x_B}\right)\cdot\frac{1}{N_B}\cdot N_B$$

or

$$\underline{\mathrm{E} = -\left(\frac{a}{x_B}\right)\left(\frac{kT}{e}\right)}$$

which is a constant.

(b)

The electric field is in the negative x-direction which will aid the flow of minority carrier electrons across the base.

(c)

$$J_n = e\mu_n n\mathrm{E} + eD_n\frac{dn}{dx}$$

Assuming no recombination in the base, J_n will be a constant across the base. Then

$$\frac{dn}{dx} + \left(\frac{\mu_n}{D_n}\right)n\mathrm{E} = \frac{J_n}{eD_n} = \frac{dn}{dx} + n\left(\frac{\mathrm{E}}{V_t}\right)$$

where $V_t = \left(\frac{kT}{e}\right)$

The homogeneous solution to the differential equation is found from

$$\frac{dn_H}{dx} + An_H = 0$$

where $A = \frac{\text{E}}{V_t}$

The solution is of the form

$$n_H = n_H(0)\exp(-Ax)$$

The particular solution is found from

$$n_P \cdot A = B$$

where $B = \frac{J_n}{eD_n}$

The particular solution is then

$$n_P = \frac{B}{A} = \frac{\left(\frac{J_n}{eD_n}\right)}{\left(\frac{\text{E}}{V_t}\right)} = \frac{J_n V_t}{eD_n \text{E}} = \frac{J_n}{e\mu_n \text{E}}$$

The total solution is then

$$\underline{n = \frac{J_n}{e\mu_n \text{E}} + n_H(0)\exp(-Ax)}$$

and

$$\underline{n(0) = n_{pO}\exp\left(\frac{V_{BE}}{V_t}\right) = \frac{n_i^2}{N_B(0)}\exp\left(\frac{V_{BE}}{V_t}\right)}$$

Then

$$\underline{n_H(0) = \frac{n_i^2}{N_B(0)}\exp\left(\frac{V_{BE}}{V_t}\right) - \frac{J_n}{e\mu_n \text{E}}}$$

10.42

(a) The basic pn junction breakdown voltage from the figure for $N_C = 5x10^{15}\ cm^{-3}$ is approximately $\underline{BV_{CBO} = 90\ V}$.

(b)
We have

$$BV_{CEO} = BV_{CBO}\sqrt[n]{1-\alpha}$$

For $n = 3$ and $\alpha = 0.992$, we obtain

$$BV_{CEO} = 90 \cdot \sqrt[3]{1-0.992} = (90)(0.20)$$

or

$$\underline{BV_{CEO} = 18\ V}$$

(c)
The B-E breakdown voltage, for $N_B = 10^{17}\ cm^{-3}$, is approximately,

$$\underline{BV_{BE} = 12\ V}$$

10.43

We want $BV_{CEO} = 60\ V$

So then

$$BV_{CEO} = \frac{BV_{CBO}}{\sqrt[n]{\beta}} \Rightarrow 60 = \frac{BV_{CBO}}{\sqrt[3]{50}}$$

which yields

$$BV_{CBO} = 221\ V$$

For this breakdown voltage, we need

$$\underline{N_C \approx 1.5x10^{15}\ cm^{-3}}$$

The depletion width into the collector at this voltage is

$$x_C = x_n = \left\{\frac{2\in(V_{bi}+V_{BC})}{e}\left(\frac{N_B}{N_C}\right)\left(\frac{1}{N_B+N_C}\right)\right\}^{1/2}$$

We find

$$V_{bi} = (0.0259)\ln\left[\frac{(1.5x10^{15})(10^{16})}{(1.5x10^{10})^2}\right] = 0.646\ V$$

and $V_{BC} = BV_{CEO} = 60\ V$

so that

$$x_C = \left\{\frac{2(11.7)(8.85x10^{-14})(0.646+60)}{1.6x10^{-19}} \times \left(\frac{10^{16}}{1.5x10^{15}}\right)\left(\frac{1}{10^{16}+1.5x10^{15}}\right)\right\}^{1/2}$$

or

$$\underline{x_C = 6.75\ \mu m}$$

10.44

$$V_{bi} = (0.0259)\ln\left[\frac{(3x10^{16})(5x10^{17})}{(1.5x10^{10})^2}\right] = 0.824\ V$$

At punch-through, we have

$$x_B = 0.70x10^{-4} = x_p(V_{BC} = V_{th}) - x_p(V_{BC} = 0)$$

$$= \left\{\frac{2\in(V_{bi}+V_{pt})}{e}\left(\frac{N_C}{N_B}\right)\left(\frac{1}{N_C+N_B}\right)\right\}^{1/2} - \left\{\frac{2\in V_{bi}}{e}\left(\frac{N_C}{N_B}\right)\left(\frac{1}{N_C+N_B}\right)\right\}^{1/2}$$

which can be written as

$$0.70x10^{-4}$$
$$= \left\{ \frac{2(11.7)(8.85x10^{-14})(V_{bi}+V_{pt})}{1.6x10^{-19}} \times \left(\frac{5x10^{17}}{3x10^{16}}\right)\left(\frac{1}{5x10^{17}+3x10^{16}}\right)\right\}^{1/2}$$
$$-\left\{ \frac{2(11.7)(8.85x10^{-14})(0.824)}{1.6x10^{-19}} \times \left(\frac{5x10^{17}}{3x10^{16}}\right)\left(\frac{1}{5x10^{17}+3x10^{16}}\right)\right\}^{1/2}$$

which becomes

$$0.70x10^{-4} = (0.202x10^{-4})\sqrt{V_{bi}+V_{pt}} - (0.183x10^{-4})$$

We obtain

$$V_{bi}+V_{pt} = 19.1\ V$$

or

$$\underline{V_{pt} = 18.3\ V}$$

Considering the junction alone, avalanche breakdown would occur at approximately $BV \approx 25\ V$.

10.45

(a) Neglecting the B-E junction depletion width,

$$V_{pt} = \frac{eW_B^2}{2\in}\cdot\frac{N_B(N_C+N_B)}{N_C}$$
$$= \left\{ \frac{(1.6x10^{-19})(0.5x10^{-4})^2}{2(11.7)(8.85x10^{-14})} \cdot \frac{(10^{17})(10^{17}+7x10^{15})}{(7x10^{15})}\right\}$$

or

$$\underline{V_{pt} = 295\ V}$$

However, actual junction breakdown for these doping concentrations is $\approx 70\ V$. So punch-through will not be reached.

10.46

At punch-through,

$$x_O = \left\{ \frac{2\in(V_{bi}+V_{pt})}{e}\cdot\left(\frac{N_C}{N_B}\right)\left(\frac{1}{N_B+N_C}\right)\right\}^{1/2}$$

Since $V_{pt} = 25\ V$, we can neglect V_{bi}.

Then we have

$$(0.75x10^{-4}) = \left\{ \frac{2(11.7)(8.85x10^{-14})(25)}{1.6x10^{-19}} \times \left(\frac{10^{16}}{N_B}\right)\left(\frac{1}{10^{16}+N_B}\right)\right\}^{1/2}$$

We obtain

$$\underline{N_B = 1.95x10^{16}\ cm^{-3}}$$

10.47

$$V_{CE}(sat) = \left(\frac{kT}{e}\right)\cdot \ln\left[\frac{I_C(1-\alpha_R)+I_B}{\alpha_F I_B-(1-\alpha_F)I_C}\cdot\frac{\alpha_F}{\alpha_R}\right]$$

We can write

$$\exp\left(\frac{V_{CE}(sat)}{0.0259}\right) = \frac{(1)(1-0.2)+I_B}{(0.99)I_B-(1-0.99)(1)}\left(\frac{0.99}{0.20}\right)$$

or

$$\exp\left(\frac{V_{CE}(sat)}{0.0259}\right) = \left(\frac{0.8+I_B}{0.99I_B-0.01}\right)(4.95)$$

(a)

For $V_{CE}(sat) = 0.30\ V$, we find

$$\exp\left(\frac{0.30}{0.0259}\right) = 1.0726x10^5 = \left(\frac{0.8+I_B}{0.99I_B-0.01}\right)(4.95)$$

We find

$$\underline{I_B = 0.01014\ mA}$$

(b)

For $V_{CE}(sat) = 0.20\ V$, we find

$$\underline{I_B = 0.0119\ mA}$$

(c)

For $V_{CE}(sat) = 0.10\ V$, we find

$$\underline{I_B = 0.105\ mA}$$

10.48

For an npn in the active mode, we have $V_{BC} < 0$, so that $\exp\left(\frac{V_{BC}}{V_t}\right) \approx 0$.

Now

$$I_E + I_B + I_C = 0 \Rightarrow I_B = -(I_C+I_E)$$

Then we have

$$I_B = -\left\{\alpha_F I_{ES}\left[\exp\left(\frac{V_{BE}}{V_t}\right)-1\right]+I_{CS}\right\}$$
$$-\left\{-\alpha_R I_{CS} - I_{ES}\left[\exp\left(\frac{V_{BE}}{V_t}\right)-1\right]\right\}$$

or

$$\underline{I_B = (1-\alpha_F)I_{ES}\left[\exp\left(\frac{V_{BE}}{V_t}\right)-1\right]-(1-\alpha_R)I_{CS}}$$

10.49

We can write

$$I_{ES}\left[\exp\left(\frac{V_{BE}}{V_t}\right)-1\right]$$
$$=\alpha_R I_{CS}\left[\exp\left(\frac{V_{BC}}{V_t}\right)-1\right]-I_E$$

Substituting, we find

$$I_C = \alpha_F\left\{\alpha_R I_{CS}\left[\exp\left(\frac{V_{BC}}{V_t}\right)-1\right]-I_E\right\}$$
$$-I_{CS}\left[\exp\left(\frac{V_{BC}}{V_t}\right)-1\right]$$

From the definition of currents, we have $I_E = -I_C$ for the case when $I_B = 0$. Then

$$I_C = \alpha_F\alpha_R I_{CS}\left[\exp\left(\frac{V_{BC}}{V_t}\right)-1\right]$$
$$+\alpha_F I_C - I_{CS}\left[\exp\left(\frac{V_{BC}}{V_t}\right)-1\right]$$

When a C-E voltage is applied, then the B-C becomes reverse biased, so $\exp\left(\frac{V_{BC}}{V_t}\right)\approx 0$. Then

$$I_C = -\alpha_F\alpha_R I_{CS} + \alpha_F I_C + I_{CS}$$

We find

$$\underline{I_C = I_{CEO} = \frac{I_{CS}(1-\alpha_F\alpha_R)}{(1-\alpha_F)}}$$

10.50

We have

$$I_C = \alpha_F I_{ES}\left[\exp\left(\frac{V_{BE}}{V_t}\right)-1\right]$$
$$-I_{CS}\left[\exp\left(\frac{V_{BC}}{V_t}\right)-1\right]$$

For $V_{BC} <\approx 0.1\,V$, $\exp\left(\frac{V_{BC}}{V_t}\right)\approx 0$ and $I_C \approx$ constant. This equation does not include the base width modulation effect.

For $V_{BE} = 0.2\,V$,

$$I_C = (0.98)\left(10^{-13}\right)\exp\left(\frac{0.2}{0.0250}\right)+5x10^{-13}$$

or

$$\underline{I_C = 2.22x10^{-10}\ A}$$

For $V_{BE} = 0.4\,V$,

$$\underline{I_C = 5x10^{-7}\ A}$$

For $V_{BE} = 0.6\,V$,

$$\underline{I_C = 1.13x10^{-3}\ A}$$

10.51

Computer Plot

10.52

(a)

$$r_\pi' = \left(\frac{kT}{e}\right)\cdot\frac{1}{I_E} = \frac{0.0259}{0.5x10^{-3}} = 51.8\ \Omega$$

So

$$\tau_e = r_\pi' C_{je} = (51.8)\left(0.8x10^{-12}\right)\Rightarrow$$

or

$$\underline{\tau_e = 41.4\ ps}$$

Also

$$\tau_b = \frac{x_B^2}{2D_n} = \frac{\left(0.7x10^{-4}\right)^2}{2(25)}\Rightarrow$$

or

$$\underline{\tau_b = 98\ ps}$$

We have

$$\tau_c = r_c\left(C_\mu + C_s\right) = (30)(2)\left(0.08x10^{-12}\right)\Rightarrow$$

or

$$\underline{\tau_c = 4.8\ ps}$$

Also

$$\tau_d = \frac{x_{dc}}{v_s} = \frac{2x10^{-4}}{10^{+7}}\Rightarrow$$

or

$$\underline{\tau_d = 20\ ps}$$

(b)

$$\tau_{ec} = \tau_e + \tau_b + \tau_c + \tau_d$$
$$= 41.4 + 98 + 4.8 + 20 \Rightarrow$$

or

$$\tau_{ec} = 164.2\ ps$$

Then

$$f_T = \frac{1}{2\pi\tau_{ec}} = \frac{1}{2\pi\left(164.2x10^{-12}\right)} \Rightarrow$$

or

$$f_T = 970\ MHz$$

Also

$$f_\beta = \frac{f_T}{\beta} = \frac{970}{50} \Rightarrow$$

or

$$f_\beta = 19.4\ MHz$$

10.53

$$\tau_b = \frac{x_B^2}{2D_B} = \frac{\left(0.5x10^{-4}\right)^2}{2(20)} = 6.25x10^{-11}\ s$$

We have $\tau_b = 0.2\tau_{ec}$,
So that

$$\tau_{ec} = 3.125x10^{-10}\ s$$

Then

$$f_T = \frac{1}{2\pi\tau_{ec}} = \frac{1}{2\pi\left(3.125x10^{-10}\right)} \Rightarrow$$

or

$$f_T = 509\ MHz$$

10.54

We have

$$\tau_{ec} = \tau_e + \tau_b + \tau_d + \tau_c$$

We are given

$\tau_b = 100\ ps$ and $\tau_e = 25\ ps$

We find

$$\tau_d = \frac{x_d}{v_s} = \frac{1.2x10^{-4}}{10^7} = 1.2x10^{-11}\ s$$

or

$$\tau_d = 12\ ps$$

Also

$$\tau_c = r_c C_c = (10)\left(0.1x10^{-12}\right) = 10^{-12}\ s$$

or

$$\tau_c = 1\ ps$$

Then

$$\tau_{ec} = 25 + 100 + 12 + 1 = 138\ ps$$

We obtain

$$f_T = \frac{1}{2\pi\tau_{ec}} = \frac{1}{2\pi\left(138x10^{-12}\right)} = 1.15x10^9\ Hz$$

or

$$f_T = 1.15\ GHz$$

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Chapter 11

Problem Solutions

11.1
(a) p-type, inversion
(b) p-type, depletion
(c) p-type, accumulation
(d) n-type, inversion

11.2
(a) For $T = 300K$
Silicon:

$$\phi_p = V_t \ln\left(\frac{N_a}{n_i}\right)$$

$$= (0.0259)\ln\left(\frac{10^{16}}{1.5x10^{10}}\right) = 0.347\ V$$

Now

$$x_{dT} = \left[\frac{4 \in \phi_p}{eN_a}\right]^{1/2}$$

$$= \left[\frac{4(11.7)(8{,}85x10^{-14})(0.347)}{(1.6x10^{-19})(10^{16})}\right]^{1/2}$$

or

$$x_{dT} = 0.30\ \mu m$$

Also

$$|Q'_{SD}(\max)| = eN_a x_{dT}$$

$$= (1.6x10^{-19})(10^{16})(0.30x10^{-4})$$

or

$$|Q'_{SD}(\max)| = 4.8x10^{-8}\ C/cm^2$$

GaAs:

$$\phi_p = (0.0259)\ln\left(\frac{10^{16}}{1.8x10^{6}}\right) = 0.581\ V$$

and

$$x_{dT} = \left[\frac{4(13.1)(8.85x10^{-14})(0.581)}{(1.6x10^{-19})(10^{16})}\right]^{1/2}$$

or

$$x_{dT} = 0.410\ \mu m$$

Then

$$|Q'_{SD}(\max)| = 6.56x10^{-8}\ C/cm^2$$

Germanium:

$$\phi_p = (0.0259)\ln\left(\frac{10^{16}}{2.4x10^{13}}\right) = 0.156\ V$$

Then

$$x_{dT} = \left[\frac{4(16)(8.85x10^{-14})(0.156)}{(1.6x10^{-19})(10^{16})}\right]^{1/2}$$

or

$$x_{dT} = 0.235\ \mu m$$

Then

$$|Q'_{SD}(\max)| = 3.76x10^{-8}\ C/cm^2$$

(b)
For $T = 200K$,

$$V_t = (0.0259)\left(\frac{200}{300}\right) = 0.01727\ V$$

Silicon: $n_i = 7.68x10^4\ cm^{-3}$
We obtain $\phi_p = 0.442\ V$ and

$x_{dT} = 0.388\ \mu m$, $|Q'_{SD}(\max)| = 5.4x10^{-8}\ C/cm^2$

GaAs: $n_i = 1.38\ cm^{-3}$
We obtain $\phi_p = 0.631\ V$ and

$x_{dT} = 0.428\ \mu m$, $|Q'_{SD}(\max)| = 6.85x10^{-8}\ C/cm^2$

Germanium: $n_i = 2.16x10^{10}\ cm^{-3}$
We obtain $\phi_p = 0.225\ V$ and

$x_{dT} = 0.282\ \mu m$, $|Q'_{SD}(\max)| = 4.5x10^{-8}\ C/cm^2$

11.3
(a) We want $|Q'_{SD}(\max)| = 7.5x10^{-9}\ C/cm^2$
We have

$$|Q'_{SD}(\max)| = eN_d x_{dT}$$

where

$$x_{dT} = \left[\frac{4 \in \phi_{fn}}{eN_d}\right]^{1/2} \quad \text{and} \quad \phi_{fn} = V_t \ln\left(\frac{N_d}{n_i}\right)$$

For n-type silicon,

$$|Q'_{SD}(\max)| = 7.5x10^{-9} = \left[4eN_d \in \phi_{fn}\right]^{1/2}$$

$$= \left[4(1.6x10^{-19})(11.7)(8.85x10^{-14})N_d\phi_{fn}\right]^{1/2}$$

or

$$(7.5x10^{-9})^2 = (6.63x10^{-31})N_d\phi_{fn}$$

which yields

$$N_d\phi_{fn} = 8.48x10^{13}$$

and

$$\phi_{fn} = (0.0259)\ln\left(\frac{N_d}{1.5x10^{10}}\right)$$

By trial and error

$$\underline{N_d = 3.27x10^{14}\ cm^{-3}}$$

(b)

$$\phi_s = -2\phi_{fn}$$

where

$$\phi_{fn} = (0.0259)\ln\left(\frac{3.27x10^{14}}{1.5x10^{10}}\right) = 0.259\ V$$

Then

$$\underline{\phi_s = -0.518\ V}$$

11.4

p-type silicon

(a) Aluminum gate

$$\phi_{ms} = \left[\phi'_m - \left(\chi' + \frac{E_g}{2e} + \phi_{fp}\right)\right]$$

We have

$$\phi_{fp} = V_t \ln\left(\frac{N_a}{n_i}\right) = (0.0259)\ln\left(\frac{6x10^{15}}{1.5x10^{10}}\right)$$

Or

$$\phi_{fp} = 0.334\ V$$

Then

$$\phi_{ms} = [3.20 - (3.25 + 0.56 + 0.334)]$$

or

$$\underline{\phi_{ms} = -0.944\ V}$$

(b)

n^+ polysilicon gate:

$$\phi_{ms} = -\left(\frac{E_g}{2e} + \phi_{fp}\right) = -(0.56 + 0.334)$$

or

$$\underline{\phi_{ms} = -0.894\ V}$$

(c)

p^+ polysilicon gate:

$$\phi_{ms} = \left(\frac{E_g}{2e} - \phi_{fp}\right) = (0.56 - 0.334)$$

or

$$\underline{\phi_{ms} = +0.226\ V}$$

11.5

We want, for n-type silicon, $\phi_{ms} = -0.35\ V$.

(a) n^+ polysilicon gate:

$$\phi_{ms} = -\left(\frac{E_g}{2e} - \phi_{fn}\right) \Rightarrow -0.35 = -(0.56 - \phi_{fn})$$

or

$$\phi_{fn} = 0.21 = (0.0259)\ln\left(\frac{N_d}{1.5x10^{10}}\right)$$

which yields

$$\underline{N_d = 4.98x10^{13}\ cm^{-3}}$$

(b)

p^+ polysilicon gate:

$$\phi_{ms} = \left(\frac{E_g}{2e} + \phi_{fn}\right) \Rightarrow -0.35 = (0.56 + \phi_{fn})$$

or

$$\phi_{fn} = -0.91\ V$$

This is impossible, cannot use a p^+ polysilicon gate.

(c)

Aluminum gate:

$$\phi_{ms} = \phi'_m - \left(\chi' + \frac{E_g}{2e} - \phi_{fn}\right)$$

or

$$-0.35 = 3.20 - (3.25 + 0.56 - \phi_{fn})$$

which yields

$$\phi_{fn} = 0.26 = (0.0259)\ln\left(\frac{N_d}{1.5x10^{10}}\right)$$

or finally,

$$\underline{N_d = 3.43x10^{14}\ cm^{-3}}$$

11.6

$$V_{FB} = \phi_{ms} - \frac{Q'_{SS}}{C_{ox}} \quad \text{and} \quad C_{ox} = \frac{\epsilon_{ox}}{t_{ox}}$$

(a)

For $t_{ox} = 500\ \mathring{A}$

Then

$$C_{ox} = \frac{(3.9)(8.85x10^{-14})}{500x10^{-8}} = 6.9x10^{-8}\ F/cm^2$$

n^+ polygate-to-n type silicon,

$$\phi_{ms} = -\left(\frac{E_g}{2e} - \phi_{fn}\right)$$

where

$$\phi_{fn} = (0.0259)\ln\left(\frac{10^{15}}{1.5x10^{10}}\right) = 0.288\ V$$

Then

$$\phi_{ms} = -(0.56 - 0.288) = -0.272\ V$$

(i) For $Q'_{SS} = 10^{10}\ cm^{-2}$, we have

$$V_{FB} = -0.272 - \frac{(1.6x10^{-19})(10^{10})}{6.9x10^{-8}}$$

or

$$\underline{V_{FB} = -0.295\ V}$$

(ii) For $Q'_{SS} = 10^{11}\ cm^{-2} \Rightarrow$

$$\underline{V_{FB} = -0.504\ V}$$

(iii) For $Q'_{SS} = 5x10^{11}\ cm^{-2} \Rightarrow$

$$\underline{V_{FB} = -1.43\ V}$$

(b)

For $t_{ox} = 250\ \mathring{A}$, we find

$$C_{ox} = 1.38x10^{-7}\ F/cm^2$$

Then

(i) For $Q'_{SS} = 10^{10}\ cm^{-2} \Rightarrow \underline{V_{FB} = -0.284\ V}$

(ii) For $Q'_{SS} = 10^{11}\ cm^{-2} \Rightarrow \underline{V_{FB} = -0.388\ V}$

(iii) For $Q'_{SS} = 5x10^{11}\ cm^{-2} \Rightarrow$

$$\underline{V_{FB} = -0.852\ V}$$

11.7

$$\phi_{ms} = \phi'_m - \left(\chi' + \frac{E_g}{2e} + \phi_{fp}\right)$$

where

$$\phi_{fp} = (0.0259)\ln\left(\frac{2x10^{16}}{1.5x10^{10}}\right) = 0.365\ V$$

Then

$$\phi_{ms} = 3.20 - (3.25 + 0.56 + 0.365) = -0.975\ V$$

Now

$$V_{FB} = \phi_{ms} - \frac{Q'_{SS}}{C_{ox}} \Rightarrow Q'_{SS} = (\phi_{ms} - V_{FB})C_{ox}$$

We have

$$C_{ox} = \frac{\in_{ox}}{t_{ox}} = \frac{(3.9)(8.85x10^{-14})}{450x10^{-8}}$$

or\

$$C_{ox} = 7.67x10^{-8}\ F/cm^2$$

so now

$$Q'_{SS} = [-0.975 - (-1)] \cdot (7.67x10^{-8})$$

or

$$\underline{Q'_{SS} = 1.92x10^{-9}\ C/cm^2}$$

or

$$\underline{\frac{Q'_{SS}}{e} = 1.2x10^{10}\ cm^{-2}}$$

11.8

$$V_{TN} = \left(|Q'_{SD}(\max)| - Q'_{SS}\right)\left(\frac{t_{ox}}{\in_{ox}}\right) + \phi_{ms} + 2\phi_{fp}$$

We find

$$\phi_{fp} = (0.0259)\ln\left(\frac{2x10^{15}}{1.5x10^{10}}\right) = 0.306\ V$$

and

$$x_{dT} = \left[\frac{4(11.7)(8.85x10^{-14})(0.306)}{(1.6x10^{-19})(2x10^{15})}\right]^{1/2}$$

or

$$x_{dT} = 0.629\ \mu m$$

Also

$$|Q'_{SD}(\max)| = (1.6x10^{-19})(2x10^{15})(0.629x10^{-4})$$

or

$$|Q'_{SD}(\max)| = 2.01x10^{-8}\ C/cm^2$$

and

$$Q'_{SS} = (2x10^{11})(1.6x10^{-19})$$

or

$$Q'_{SS} = 3.2x10^{-8}\ C/cm^2$$

Then

$$V_{TN} = \frac{(2.01x10^{-8} - 3.2x10^{-8})(450x10^{-8})}{(3.9)(8.85x10^{-14})} + \phi_{ms} + 2(0.306)$$

or

$$V_{TN} = 0.457 + \phi_{ms}$$

(a)

For an aluminum gate:

$$\phi_{ms} = 3.20 - (3.25 + 0.56 + 0.306) = -0.916\ V$$

Then

$$V_{TN} = 0.457 - 0.916$$

or

$$\underline{V_{TN} = -0.459\ V}$$

(b)

For an n^+ polygate:

$$\phi_{ms} = -(0.56 + 0.306) = -0.866\ V$$

so that

$$\underline{V_{TN} = -0.409\ V}$$

(c)
For a p^+ polygate:

$$\phi_{ms} = (0.56 - 0.306) = +0.254\ V$$

so that

$$\underline{V_{TN} = +0.711\ V}$$

11.9
We find

$$\phi_{fn} = (0.0259)\ln\left(\frac{10^{15}}{1.5x10^{10}}\right) = 0.288\ V$$

So

$$x_{dT} = \left[\frac{4\in\phi_{fn}}{eN_d}\right]^{1/2}$$

$$= \left[\frac{4(11.7)(8.85x10^{-14})(0.288)}{(1.6x10^{-19})(10^{15})}\right]^{1/2}$$

or

$$x_{dT} = 0.863\ \mu m$$

Also

$$|Q'_{SD}(\max)| = eN_d x_{dT}$$

$$= (1.6x10^{-19})(10^{15})(0.863x10^{-4})$$

or

$$|Q'_{SD}(\max)| = 1.38x10^{-8}\ C/cm^2$$

We have

$$Q'_{SS} = 3.2x10^{-8}\ C/cm^2$$

Now

$$V_{TP} = -\left(|Q'_{SD}(\max)| + Q'_{SS}\right)\left(\frac{t_{ox}}{\in_{ox}}\right) + \phi_{ms} - 2\phi_{fn}$$

so

$$V_{TP} = -\frac{(1.38x10^{-8} + 3.2x10^{-8})}{(3.9)(8.85x10^{-14})}(450x10^{-8}) + \phi_{ms} - 2(0.288)$$

or

$$V_{TP} = -1.17 + \phi_{ms}$$

(a)
Aluminum gate:

$$\phi_{ms} = 3.20 - (3.25 + 0.56 - 0.288) = -0.322\ V$$

so

$$\underline{V_{TP} = -1.49\ V}$$

(b)
n^+ polygate:

$$\phi_{ms} = -(0.56 - 0.288) = -0.272\ V$$

so

$$\underline{V_{TP} = -1.44\ V}$$

(c)
p^+ polygate:

$$\phi_{ms} = +(0.56 + 0.288) = +0.848\ V$$

so

$$\underline{V_{TP} = -0.322\ V}$$

11.10

$$\phi_{fp} = (0.0259)\ln\left(\frac{5x10^{15}}{1.5x10^{10}}\right) = 0.329\ V$$

Surface potential:

$$\phi_s = 2\phi_{fp} = 2(0.329) = 0.658\ V$$

We have

$$V_{FB} = \phi_{ms} - \frac{Q'_{SS}}{C_{ox}} = -0.90\ V$$

Now

$$V_T = \frac{|Q'_{SD}(\max)|}{C_{ox}} + \phi_s + V_{FB}$$

We obtain

$$x_{dT} = \left[\frac{4\in\phi_{fp}}{eN_a}\right]^{1/2}$$

$$= \left[\frac{4(11.7)(8.85x10^{-14})(0.329)}{(1.6x10^{-19})(5x10^{15})}\right]^{1/2}$$

or

$$x_{dT} = 0.413\ \mu m$$

Then

$$|Q'_{SD}(\max)| = (1.6x10^{-19})(5x10^{15})(0.413x10^{-4})$$

or

$$|Q'_{SD}(\max)| = 3.30x10^{-8}\ C/cm^2$$

We also find

$$C_{ox} = \frac{\in_{ox}}{t_{ox}} = \frac{(3.9)(8.85x10^{-14})}{400x10^{-8}}$$

or

$$C_{ox} = 8.63x10^{-8}\ F/cm^2$$

Then

$$V_T = \frac{3.30x10^{-8}}{8.63x10^{-8}} + 0.658 - 0.90$$

or

$$\underline{V_T = +0.140\ V}$$

11.11

$$V_{TN}=\left(\left|Q'_{SD}(\max)\right|-Q'_{SS}\right)\left(\frac{t_{ox}}{\in_{ox}}\right)+\phi_{ms}+2\phi_{fp}$$

We have

$$\phi_{ms}=\phi'_m-\left(\chi'+\frac{E_g}{2e}+\phi_{fp}\right)$$

$$=3.20-\left(3.25+0.56+\phi_{fp}\right)$$

or

$$\phi_{ms}=-0.61-\phi_{fp}$$

Also

$$x_{dT}=\left[\frac{4\in\phi_{fp}}{eN_a}\right]^{1/2}$$

and

$$\left|Q'_{SD}(\max)\right|=eN_ax_{dT}=\left[4eN_a\in\phi_{fp}\right]^{1/2}$$

Then, the threshold voltage can be written as

$$+0.80$$
$$=\left\{\left[4\left(1.6x10^{-19}\right)(11.7)\left(8.85x10^{-14}\right)N_a\phi_{fp}\right]^{1/2}-1.6x10^{-8}\right\}\cdot\left[\frac{750x10^{-8}}{(3.9)\left(8.85x10^{-14}\right)}\right]-0.61-\phi_{fp}+2\phi_{fp}$$

which becomes

$$1.758=1.77x10^{-8}\left(N_a\phi_{fp}\right)^{1/2}+\phi_{fp}$$

We also have

$$\phi_{fp}=(0.0259)\ln\left(\frac{N_a}{1.5x10^{10}}\right)$$

By trial and error

$$\underline{N_a=1.71x10^{16}\ cm^{-3}}$$

11.12

We have

$$V_{TP}=-\left(\left|Q'_{SD}(\max)\right|+Q'_{SS}\right)\left(\frac{t_{ox}}{\in_{ox}}\right)+\phi_{ms}-2\phi_{fn}$$

We find

$$\phi_{ms}=\phi'_m-\left(\chi'+\frac{E_g}{2e}-\phi_{fn}\right)$$

$$=3.20-\left(3.25+0.56-\phi_{fn}\right)$$

or

$$\phi_{ms}=-0.61+\phi_{fn}$$

where

$$\phi_{fn}=(0.0259)\ln\left(\frac{N_d}{1.5x10^{10}}\right)$$

Also

$$x_{dT}=\left[\frac{4\in\phi_{fn}}{eN_d}\right]^{1/2}$$

Also

$$\left|Q'_{SD}(\max)\right|=eN_dx_{dT}=\left[4\in eN_d\phi_{fn}\right]^{1/2}$$

Now the threshold voltage can be written as

$$-1.50$$
$$=-\left\{\left[4(11.7)\left(8.85x10^{-14}\right)\left(1.6x10^{-19}\right)N_d\phi_{fn}\right]^{1/2}+1.6x10^{-8}\right\}\cdot\left[\frac{750x10^{-8}}{(3.9)\left(8.85x10^{-14}\right)}\right]-0.61+\phi_{fn}-2\phi_{fn}$$

which becomes

$$0.542=1.77x10^{-8}\left(N_d\phi_{fn}\right)^{1/2}+\phi_{fn}$$

By trial and error

$$\underline{N_d=7.7x10^{14}\ cm^{-3}}$$

11.13

(a) $V_{FB}=\phi_{ms}-\dfrac{Q'_{SS}}{C_{ox}}$

Now

$$\phi_{fp}=(0.0259)\ln\left(\frac{10^{15}}{1.5x10^{10}}\right)=0.288\ V$$

and

$$\phi_{ms}=\phi'_m-\left(\chi'+\frac{E_g}{2e}+\phi_{fp}\right)$$

$$=3.20-(3.25+0.56+0.288)=-0.898\ V$$

We find

$$C_{ox}=\frac{\in_{ox}}{t_{ox}}=\frac{(3.9)\left(8.85x10^{-14}\right)}{450x10^{-8}}$$

or

$$C_{ox}=7.67x10^{-8}\ F/cm^2$$

Then

$$V_{FB}=-0.898-\frac{\left(3x10^{11}\right)\left(1.6x10^{-19}\right)}{7.67x10^{-8}}$$

or

$$\underline{V_{FB}=-1.52\ V}$$

(b)
We have
$$V_T = \frac{|Q'_{SD}(\max)|}{C_{ox}} + 2\phi_{fp} + V_{FB}$$
We find
$$x_{dT} = \left[\frac{4(11.7)(8.85x10^{-14})(0.288)}{(1.6x10^{-19})(10^{15})}\right]^{1/2}$$
or
$$x_{dT} = 0.863\ \mu m$$
We obtain
$$|Q'_{SD}(\max)| = (1.6x10^{-19})(10^{15})(0.863x10^{-4})$$
or
$$|Q'_{SD}(\max)| = 1.38x10^{-8}\ C/cm^2$$
Then
$$V_T = \frac{1.38x10^{-8}}{7.67x10^{-8}} + 2(0.288) - 1.52$$
or
$$\underline{V_T = -0.764\ V}$$

11.14

(a) We have n-type material under the gate, so
$$x_{dT} = t_C = \left[\frac{4 \in \phi_{fn}}{eN_d}\right]^{1/2}$$
where
$$\phi_{fn} = (0.0259)\ln\left(\frac{10^{15}}{1.5x10^{10}}\right) = 0.288\ V$$
Then
$$x_{dT} = \left[\frac{4(11.7)(8.85x10^{-14})(0.288)}{(1.6x10^{-19})(10^{15})}\right]^{1/2}$$
or
$$\underline{x_{dT} = t_C = 0.863\ \mu m}$$
(b)
$$V_T = -\left(|Q'_{SD}(\max)| + Q'_{SS}\right)\left(\frac{t_{ox}}{\in_{ox}}\right) + \phi_{ms} - 2\phi_{fn}$$
For an n^+ polygate:
$$\phi_{ms} = -\left(\frac{E_g}{2e} - \phi_{fn}\right) = -(0.56 - 0.288)$$
or
$$\phi_{ms} = -0.272\ V$$
Now
$$|Q'_{SD}(\max)| = (1.6x10^{-19})(10^{15})(0.863x10^{-4})$$
or
$$|Q'_{SD}(\max)| = 1.38x10^{-8}\ C/cm^2$$
We find
$$Q'_{SS} = (1.6x10^{-19})(10^{10}) = 1.6x10^{-9}\ C/cm^2$$
We then find
$$V_T = \frac{-(1.38x10^{-8} + 1.6x10^{-9})}{(3.9)(8.85x10^{-14})}(500x10^{-8}) - 0.272 - 2(0.288)$$
or
$$\underline{V_T = -1.07\ V}$$

11.15

(b) $\phi_{ms} = \phi'_m - \left(\chi' + \frac{E_g}{2e} + \phi_{fp}\right)$

where
$$\phi'_m - \chi' = -0.20\ V$$
and
$$\phi_{fp} = (0.0259)\ln\left(\frac{10^{16}}{1.5x10^{10}}\right) = 0.347\ V$$
Then
$$\phi_{ms} = -0.20 - (0.56 + 0.347)$$
or
$$\underline{\phi_{ms} = -1.11\ V}$$
(c)
For $Q'_{SS} = 0$,
$$V_{TN} = |Q'_{SD}(\max)|\left(\frac{t_{ox}}{\in_{ox}}\right) + \phi_{ms} + 2\phi_{fp}$$
We find
$$x_{dT} = \left[\frac{4(11.7)(8.85x10^{-14})(0.347)}{(1.6x10^{-19})(10^{16})}\right]^{1/2}$$
or
$$x_{dT} = 0.30\ \mu m$$
Now
$$|Q'_{SD}(\max)| = (1.6x10^{-19})(10^{16})(0.30x10^{-4})$$
or
$$|Q'_{SD}(\max)| = 4.8x10^{-8}\ C/cm^2$$
Then
$$V_{TN} = \frac{(4.8x10^{-8})(300x10^{-8})}{(3.9)(8.85x10^{-14})} - 1.11 + 2(0.347)$$
or
$$\underline{V_{TN} = +0.0012\ V}$$

11.16
Computer plot

11.17
Computer plot

11.18
Computer plot

11.19
Computer plot

11.20
(a) For $f = 1\ Hz$
We have

$$C_{ox} = \frac{\in_{ox}}{t_{ox}} = \frac{(3.9)(8.85x10^{-14})}{400x10^{-8}}$$

or

$$\underline{C_{ox} = 8.63x10^{-8}\ F/cm^2}$$

and

$$C'_{FB} = \frac{\in_{ox}}{t_{ox} + \left(\frac{\in_{ox}}{\in_s}\right)\sqrt{\left(\frac{kT}{e}\right)\left(\frac{\in_s}{eN_a}\right)}}$$

$$= \frac{(3.9)(8.85x10^{-14})}{400x10^{-8} + \left(\frac{3.9}{11.7}\right)\sqrt{\frac{(0.0259)(11.7)(8.85x10^{-14})}{(1.6x10^{-19})(10^{16})}}}$$

or

$$\underline{C'_{FB} = 6.43x10^{-8}\ F/cm^2}$$

Also

$$C'_{\min} = \frac{\in_{ox}}{t_{ox} + \left(\frac{\in_{ox}}{\in_s}\right)\cdot x_{dT}}$$

where

$$x_{dT} = \left[\frac{4\in_s \phi_{fp}}{eN_a}\right]^{1/2}$$

Now

$$\phi_{fp} = (0.0259)\ln\left(\frac{10^{16}}{1.5x10^{10}}\right) = 0.347\ V$$

Then

$$x_{dT} = \left[\frac{4(11.7)(8.85x10^{-14})(0.347)}{(1.6x10^{-19})(10^{16})}\right]^{1/2}$$

or

$$x_{dT} = 0.30\ \mu m$$

so that

$$C'_{\min} = \frac{(3.9)(8.85x10^{-14})}{400x10^{-8} + \left(\frac{3.9}{11.7}\right)(0.30x10^{-4})}$$

or

$$\underline{C'_{\min} = 2.47x10^{-8}\ F/cm^2}$$

Also

$$\underline{C'(inv) = C_{ox} = 8.63x10^{-8}\ F/cm^2}$$

(b)
For $f = 1\ MHz$, we have

$$\underline{C_{ox} = 8.63x10^{-8}\ F/cm^2}$$

$$\underline{C'_{FB} = 6.43x10^{-8}\ F/cm^2}$$

$$\underline{C'_{\min} = 2.47x10^{-8}\ F/cm^2}$$

and

$$\underline{C'(inv) = C'_{\min} = 2.47x10^{-8}\ F/cm^2}$$

(c)

$$V_{FB} = \phi_{ms} - \frac{Q'_{SS}}{C_{ox}}$$

For the ideal MOS capacitor, $Q'_{SS} = 0$, then

$$V_{FB} = \phi_{ms} = 3.2 - (3.25 + 0.56 + 0.347)$$

or

$$\underline{V_{FB} = -0.957\ V}$$

Also

$$|Q'_{SD}(\max)| = (1.6x10^{-19})(10^{16})(0.30x10^{-4})$$

or

$$|Q'_{SD}(\max)| = 4.8x10^{-8}\ C/cm^2$$

Now

$$V_{TN} = |Q'_{SD}(\max)|\left(\frac{t_{ox}}{\in_{ox}}\right) + \phi_{ms} + 2\phi_{fp}$$

$$= \frac{(4.8x10^{-8})(400x10^{-8})}{(3.9)(8.85x10^{-14})} - 0.957 + 2(0.347)$$

or

$$\underline{V_{TN} = +0.293\ V}$$

11.21
(a) At $f = 1\ Hz$

$$C_{ox} = \frac{\in_{ox}}{t_{ox}} = \frac{(3.9)(8.85x10^{-14})}{400x10^{-8}}$$

or

$$\underline{C_{ox} = 8.63x10^{-8}\ F/cm^2}$$

Also

$$C'_{FB} = \frac{\in_{ox}}{t_{ox} + \left(\dfrac{\in_{ox}}{t_{ox}}\right)\sqrt{\left(\dfrac{kT}{e}\right)\left(\dfrac{\in_s}{eN_a}\right)}}$$

$$= \frac{(3.9)(8.85x10^{-14})}{400x10^{-8} + \left(\dfrac{3.9}{11.7}\right)\sqrt{\dfrac{(0.0259)(11.7)(8.85x10^{-14})}{(1.6x10^{-19})(5x10^{14})}}}$$

or

$$\underline{C'_{FB} = 3.42x10^{-8}\ F/cm^2}$$

Now

$$C'_{\min} = \frac{\in_{ox}}{t_{ox} + \left(\dfrac{\in_{ox}}{\in_s}\right)\cdot x_{dT}}$$

We find

$$\phi_{fn} = (0.0259)\ln\left(\frac{5x10^{14}}{1.5x10^{10}}\right) = 0.270\ V$$

and

$$x_{dT} = \left[\frac{4\in\phi_{fn}}{eN_d}\right]^{1/2}$$

$$= \left[\frac{4(11.7)(8.85x10^{-14})(0.270)}{(1.6x10^{-19})(5x10^{14})}\right]^{1/2}$$

or

$$x_{dT} = 1.18\ \mu m$$

Then

$$C'_{\min} = \frac{(3.9)(8.85x10^{-14})}{400x10^{-8} + \left(\dfrac{3.9}{11.7}\right)(1.18x10^{-4})}$$

or

$$\underline{C'_{\min} = 0.797x10^{-8}\ F/cm^2}$$

Also

$$\underline{C'(inv) = C_{ox} = 8.63x10^{-8}\ F/cm^2}$$

(b) At $f = 1\ MHz$

$$\underline{C_{ox} = 8.63x10^{-8}\ F/cm^2}$$

$$\underline{C'_{FB} = 3.42x10^{-8}\ F/cm^2}$$

$$\underline{C'_{\min} = 0.797x10^{-8}\ F/cm^2}$$

and

$$\underline{C'(inv) = C'_{\min} = 0.797x10^{-8}\ F/cm^2}$$

(c)
For the ideal oxide,

$$V_{FB} = \phi_{ms} = 3.2 - (3.25 + 0.56 - 0.27)$$

or

$$\underline{V_{FB} = -0.34\ V}$$

We find

$$|Q'_{SD}(\max)| = (1.6x10^{-19})(5x10^{14})(1.18x10^{-4})$$

or

$$|Q'_{SD}(\max)| = 0.944x10^{-8}\ C/cm^2$$

Then

$$V_{TP} = -|Q'_{SD}(\max)|\left(\frac{t_{ox}}{\in_{ox}}\right) + \phi_{ms} - 2\phi_{fn}$$

$$= \frac{-(0.944x10^{-8})(400x10^{-8})}{(3.9)(8.85x10^{-14})} - 0.34 - 2(0.27)$$

or

$$\underline{V_{TP} = -0.989\ V}$$

11.22

The amount of fixed oxide charge at x is

$$\rho(x)\Delta x \quad (C/cm^2)$$

By lever action, the effect of this oxide charge on the flatband voltage is

$$\Delta V_{FB} = -\frac{1}{C_{ox}}\left(\frac{x}{t_{ox}}\right)\rho(x)\Delta x$$

If we add the effect at each point, we must integrate so that

$$\underline{\Delta V_{FB} = -\frac{1}{C_{ox}}\int_0^{t_{ox}} \frac{x\rho(x)}{t_{ox}}dx}$$

11.23

(a) We have $\rho(x) = \dfrac{Q'_{SS}}{\Delta t}$

Then

$$\Delta V_{FB} = -\frac{1}{C_{ox}}\int_0^{t_{ox}} \frac{x\rho(x)}{t_{ox}}dx$$

$$\approx -\frac{1}{C_{ox}}\int_{(t_{ox}-\Delta t)}^{t_{ox}} \left(\frac{t_{ox}}{t_{ox}}\right)\left(\frac{Q'_{SS}}{\Delta t}\right)dx$$

$$= -\frac{1}{C_{ox}}\left(\frac{Q'_{SS}}{\Delta t}\right)\left[t_{ox} - (t_{ox} - \Delta t)\right] = -\frac{Q'_{SS}}{C_{ox}}$$

or

$$\Delta V_{FB} = -Q'_{SS}\left(\frac{t_{ox}}{\in_{ox}}\right)$$

$$= \frac{-(1.6x10^{-19})(5x10^{11})(750x10^{-8})}{(3.9)(8.85x10^{-14})}$$

or

$$\underline{\Delta V_{FB} = -1.74\ V}$$

(b)

We have $\rho(x) = \dfrac{Q'_{SS}}{t_{ox}} = \dfrac{(1.6x10^{-19})(5x10^{11})}{750x10^{-8}}$

$$= 1.067x10^{-2} = \rho_O$$

Now

$$\Delta V_{FB} = -\frac{1}{C_{ox}}\int_0^{t_{ox}} \frac{x\rho(x)}{t_{ox}}dx = -\frac{\rho_O}{C_{ox}t_{ox}}\int_0^{t_{ox}} x dx$$

or

$$\Delta V_{FB} = -\frac{\rho_O t_{ox}^2}{2 \in_{ox}}$$

$$= \frac{-(1.067x10^{-2})(750x10^{-8})^2}{2(3.9)(8.85x10^{-14})}$$

or

$$\underline{\Delta V_{FB} - 0.869\ V}$$

(c)

$$\rho(x) = \rho_O\left(\frac{x}{t_{ox}}\right)$$

We find

$$\frac{1}{2}t_{ox}\rho_O = Q'_{SS} \Rightarrow \rho_O = \frac{2(1.6x10^{-19})(5x10^{11})}{750x10^{-8}}$$

or $\rho_O = 2.13x10^{-2}$

Now

$$\Delta V_{FB} = -\frac{1}{C_{ox}}\int_0^{t_{ox}} \frac{1}{t_{ox}}\cdot x\cdot\rho_O\left(\frac{x}{t_{ox}}\right)dx$$

$$= -\frac{1}{C_{ox}}\cdot\frac{\rho_O}{(t_{ox})^2}\int_0^{t_{ox}} x^2 dx$$

which becomes

$$\Delta V_{FB} = -\frac{1}{\left(\dfrac{\in_{ox}}{t_{ox}}\right)}\cdot\frac{\rho_O}{(t_{ox})^2}\cdot\frac{x^3}{3}\Bigg|_0^{t_{ox}} = -\frac{\rho_O t_{ox}^2}{3\in_{ox}}$$

Then

$$\Delta V_{FB} = \frac{-(2.13x10^{-2})(750x10^{-8})^2}{3(3.9)(8.85x10^{-14})}$$

or $\underline{\Delta V_{FB} = -1.16\ V}$

11.24
Sketch

11.25
Sketch

11.26

(b) $V_{FB} = -V_{bi} = -V_t \ln\left(\dfrac{N_a N_d}{n_i^2}\right)$

$$= -(0.0259)\ln\left[\frac{(10^{16})(10^{16})}{(1.5x10^{10})^2}\right]$$

or

$$\underline{V_{FB} = -0.695\ V}$$

(c)

Apply $V_G = -3\ V$, $\underline{|V_{OX}| \approx 3\ V}$

For $V_G = +3\ V$,

$$\frac{d\mathrm{E}}{dx} = -\frac{\rho}{\in_s}$$

n-side: $\rho = eN_d$

$$\frac{d\mathrm{E}}{dx} = -\frac{eN_d}{\in_s} \Rightarrow E = -\frac{eN_d x}{\in_s} + C_1$$

E = 0 at $x = -x_n$, then $C_1 = -\dfrac{eN_d x_n}{\in_s}$, so

$$\mathrm{E} = -\frac{eN_d}{\in_s}(x + x_n) \quad \text{for} \quad -x_n \le x \le 0$$

Note that at $x = 0$, $\mathrm{E} = -\dfrac{eN_d x_n}{\in_s}$

In the oxide, $\rho = 0$, so

$\dfrac{d\mathrm{E}}{dx} = 0 \Rightarrow$ E = constant. From the boundary conditions. In the oxide:

$$\mathrm{E} = -\frac{eN_d x_n}{\in_s}$$

In the p-region,

$$\frac{d\mathrm{E}}{dx} = -\frac{\rho}{\in_s} = +\frac{eN_a}{\in_s} \Rightarrow \mathrm{E} = \frac{eN_a x}{\in_s} + C_2$$

E = 0 at $x = (t_{ox} + x_p)$, then

$$C_2 = -\frac{eN_a}{\in_s}\left(t_{ox} + x_p\right), \text{ then}$$

$$E = -\frac{eN_a}{\in_s}\left[\left(t_{ox} + x_p\right) - x\right]$$

At $x = t_{ox}$, $E = -\frac{eN_a x_p}{\in_s} = -\frac{eN_d x_n}{\in_s}$

So that $N_a x_p = N_d x_n$.

Since $N_a = N_d \Rightarrow x_n = x_p$

Now, the potential is

$$\phi = -\int E dx$$

For zero bias, we can write

$$V_n + V_{ox} + V_p = V_{bi}$$

where V_n, V_{ox}, V_p are the voltage drops across the n-region, the oxide, and the p-region, respectively. For the oxide:

$$V_{ox} = E \cdot t_{ox} = \frac{eN_d x_n t_{ox}}{\in_s}$$

For the n-region:

$$V_n = \frac{eN_d}{\in_s}\left(\frac{x^2}{2} + x_n \cdot x\right) + C'$$

Arbitrarily, set $V_n = 0$, at $x = -x_n$, then

$$C' = \frac{eN_d x_n^2}{2\in_s} \quad \text{so then}$$

$$V_n(x) = \frac{eN_d}{2\in_s}\left(x + x_n\right)^2$$

At $x = 0$, $V_n = \frac{eN_d x_n^2}{2\in_s}$ which is the voltage drop across the n-region. Because of symmetry, $V_n = V_p$. Then for zero bias, we have

$$2V_n + V_{ox} = V_{bi}$$

which can be written as

$$\frac{eN_d x_n^2}{\in_s} + \frac{eN_d x_n t_{ox}}{\in_s} = V_{bi}$$

or

$$x_n^2 + x_n t_{ox} - \frac{V_{bi}\in_s}{eN_d} = 0$$

Solving for x_n, we find

$$x_n = -\frac{t_{ox}}{2} + \sqrt{\left(\frac{t_{ox}}{2}\right)^2 + \frac{\in_s V_{bi}}{eN_d}}$$

If we apply a voltage V_G, then replace V_{bi} by $V_{bi} + V_G$, so

$$x_n = x_p = -\frac{t_{ox}}{2} + \sqrt{\left(\frac{t_{ox}}{2}\right)^2 + \frac{\in_s\left(V_{bi} + V_G\right)}{eN_d}}$$

We then find

$$x_n = x_p = -\frac{500x10^{-8}}{2}$$
$$+\sqrt{\left(\frac{500x10^{-8}}{2}\right)^2 + \frac{(11.7)\left(8.85x10^{-14}\right)(3.695)}{\left(1.6x10^{-19}\right)\left(10^{16}\right)}}$$

which yields

$$x_n = x_p = 4.646x10^{-5} \ cm$$

Now

$$V_{ox} = \frac{eN_d x_n t_{ox}}{\in_s}$$
$$= \frac{\left(1.6x10^{-19}\right)\left(10^{16}\right)\left(4.646x10^{-5}\right)\left(500x10^{-8}\right)}{(11.7)\left(8.85x10^{-14}\right)}$$

or

$$\underline{V_{ox} = 0.359 \ V}$$

We can also find

$$V_n = \frac{eN_d x_n^2}{2\in_s}$$
$$= \frac{\left(1.6x10^{-19}\right)\left(10^{16}\right)\left(4.646x10^{-5}\right)^2}{2(11.7)\left(8.85x10^{-14}\right)}$$

or, the voltage drop across each of the semiconductor regionsis

$$\underline{V_n = V_p = 1.67 \ V}$$

11.27

(a) n-type

(b) We have

$$C_{ox} = \frac{200x10^{-12}}{2x10^{-3}} = 1x10^{-7} \ F/cm^2$$

Also

$$C_{ox} = \frac{\in_{ox}}{t_{ox}} \Rightarrow t_{ox} = \frac{\in_{ox}}{C_{ox}} = \frac{(3.9)\left(8.85x10^{-14}\right)}{10^{-7}}$$

or

$$\underline{t_{ox} = 3.45x10^{-6} \ cm = 345 \ \overset{\circ}{A}}$$

(c)

$$V_{FB} = \phi_{ms} - \frac{Q'_{SS}}{C_{ox}}$$

or

$$-0.80 = -0.50 - \frac{Q'_{SS}}{10^{-7}}$$

or

$$\underline{Q'_{SS} = 3x10^{-8}\ C/cm^2 = 1.875x10^{11}\ cm^{-2}}$$

(d)

$$C'_{FB} = \frac{\in_{ox}}{t_{ox} + \left(\frac{\in_{ox}}{\in_s}\right)\sqrt{\left(\frac{kT}{e}\right)\left(\frac{\in_s}{eN_a}\right)}}$$

$$= \frac{(3.9)\left(8.85x10^{-14}\right)}{3.45x10^{-6} + \left(\frac{3.9}{11.7}\right)\sqrt{(0.0259)\left[\frac{(11.7)\left(8.85x10^{-14}\right)}{\left(1.6x10^{-19}\right)\left(2x10^{16}\right)}\right]}}$$

or

$$\underline{C'_{FB} = 7.82x10^{-8}\ F/cm^2}$$

and

$$\underline{C_{FB} = 156\ pF}$$

11.28

(a) Point 1: Inversion
2: Threshold
3: Depletion
4: Flat-band
5: Accumulation

11.29

We have

$$Q'_n = -C_{ox}\left[\left(V_{GS} - V_x\right) - \left(\phi_{ms} + 2\phi_{fp}\right)\right] - \left(Q'_{SS} + Q'_{SD}(\max)\right)$$

Now let $V_x = V_{DS}$, so

$$Q'_n = -C_{ox}\left\{\left(V_{GS} - V_{DS}\right) + \left[\frac{Q'_{SD}(\max) + Q'_{SS}}{C_{ox}} - \left(\phi_{ms} + 2\phi_{fp}\right)\right]\right\}$$

For a p-type substrate, $Q'_{SD}(\max)$ is really a negative value, so we can write

$$Q'_n = -C_{ox}\left\{\left(V_{GS} - V_{DS}\right) - \left[\frac{\left|Q'_{SD}(\max)\right| - Q'_{SS}}{C_{ox}} + \phi_{ms} + 2\phi_{fp}\right]\right\}$$

Using the definition of threshold voltage V_T, we have

$$Q'_n = -C_{ox}\left[\left(V_{GS} - V_{DS}\right) - V_T\right]$$

At saturation,

$$V_{DS} = V_{DS}(sat) = V_{GS} - V_T$$

which then makes Q'_n equal to zero at the drain terminal.

11.30

$$I_D(sat) = \frac{W\mu_n C_{ox}}{2L}\left(V_{GS} - V_T\right)^2$$

where

$$\frac{W\mu_n C_{ox}}{2L} = \frac{\left(30x10^{-4}\right)(450)(3.9)\left(8.85x10^{-14}\right)}{2\left(2x10^{-4}\right)\left(350x10^{-8}\right)}$$

$$= 0.333x10^{-3}\ A/V^2 = 0.333\ mA/V^2$$

We have $V_{DS}(sat) = V_{GS} - V_T$, then

(a)

V_{GS}	$V_{DS}(sat)$	$I_D(sat)\ (mA)$
1	0.2	0.0133
2	1.2	0.48
3	2.2	1.61
4	3.2	3.41
5	4.2	5.87

(b)

$$\sqrt{I_D(sat)} = \sqrt{0.333}\left(V_{GS} - V_T\right)\ (mA)^{1/2}$$

then

V_{GS}	$\sqrt{I_D(sat)}\ (mA)^{1/2}$
1	0.115
2	0.693
3	1.27
4	1.85
5	2.42

11.31

We have

$$\frac{W\mu_p C_{ox}}{2L} = \frac{\left(15x10^{-4}\right)(300)(3.9)\left(8.85x10^{-14}\right)}{2\left(1.5x10^{-4}\right)\left(350x10^{-8}\right)}$$

$$= 0.148\ mA/V^2$$

We can write

$$I_D(sat) = \frac{W\mu_p C_{ox}}{2L}\left(V_{SG} + V_T\right)^2$$

and

$$V_{SD}(sat) = V_{SG} + V_T$$

Then

V_{SG}	$V_{SD}(sat)$	$I_D(sat)\ (mA)$
1	0.2	0.00592
2	1.2	0.213
3	2.2	0.716
4	3.2	1.52
5	4.2	2.61

11.32

(a) $I_D(sat) = \dfrac{W\mu_n C_{ox}}{2L}(V_{GS} - V_T)^2$

From Problem 11.30, $\dfrac{W\mu_n C_{ox}}{2L} = 0.333\ mA/V^2$

We have

$V_{DS}(sat) = V_{GS} - V_T$

Then

V_{GS}	$V_{DS}(sat)$	$I_D(sat)\ (mA)$
-2	0	0
-1	1	0.333
0	2	1.33
+1	3	3.0
+2	4	5.33
+3	5	8.33

(b)
We have

$\sqrt{I_D(sat)} = \sqrt{0.333}(V_{GS} - V_T)\ (mA)^{1/2}$

Now

V_{GS}	$\sqrt{I_D(sat)}\ (mA)^{1/2}$
-2	0
-1	0.577
0	1.15
+1	1.73
+2	2.31
+3	2.89

11.33
Sketch

11.34
Plots

11.35
We have

$V_{DS}(sat) = V_{GS} - V_T = V_{DS} - V_T$

so that

$V_{DS} = V_{DS}(sat) + V_T$

Since $V_{DS} > V_{DS}(sat)$, the transistor is always biased in the saturation region. Then

$I_D = K_n(V_{GS} - V_T)^2$

where, from Problem 11.30,

$K_n = 0.333\ mA/V^2$.

Then

$V_{DS} = V_{GS}$	$I_D\ (mA)$
0	0
1	0.0133
2	0.48
3	1.61
4	3.41
5	5.87

11.36

$I_D = 0.148\left[2(V_{SG} + V_T)V_{SD} - V_{SD}^2\right]\ (mA)$

where $V_T = -0.8\ V$.

Now

$$g_d = \left.\frac{\partial I_D}{\partial V_{SD}}\right|_{V_{DS}=0} = 0.148\left[2(V_{SG} + V_T)\right]\ mS$$

Then

V_{SG}	$g_d\ (mS)$
1	0.0592
2	0.355
3	0.651
4	0.947
5	1.24

11.37
We find that $\underline{V_T \approx 0.2\ V}$

Now

$$\sqrt{I_D(sat)} = \sqrt{\frac{W\mu_n C_{ox}}{2L}}(V_{GS} - V_T)$$

where

$$C_{ox} = \frac{\epsilon_{ox}}{t_{ox}} = \frac{(3.9)(8.85x10^{-14})}{425x10^{-8}}$$

or

$C_{ox} = 8.12x10^{-8}\ F/cm^2$

We are given $W/L = 10$.

From the graph, for $V_{GS} = 3\ V$,

$\sqrt{I_D(sat)} \approx 0.033$, then

$$0.033 = \sqrt{\frac{W\mu_n C_{ox}}{2L}}(3 - 0.2)$$

or

$$\frac{W\mu_n C_{ox}}{2L} = 0.139x10^{-3} = \frac{1}{2}(10)\mu_n\left(8.12x10^{-8}\right)$$

which yields

$$\underline{\mu_n = 342\ cm^2 / V - s}$$

11.38

(a)

$$V_{DS}(sat) = V_{GS} - V_T$$

or

$$4 = V_{GS} - 0.8 \Rightarrow \underline{V_{GS} = 4.8\ V}$$

(b)

$$I_D(sat) = K_n(V_{GS} - V_T)^2 = K_n V_{DS}^2(sat)$$

so

$$2x10^{-4} = K_n(4)^2$$

or

$$\underline{K_n = 12.5\ \mu A / V^2}$$

(c)

$$V_{DS}(sat) = V_{GS} - V_T = 2 - 0.8 = 1.2\ V$$

so $V_{DS} > V_{DS}(sat)$

$$I_D(sat) = \left(1.25x10^{-5}\right)(2 - 0.8)^2$$

or

$$\underline{I_D(sat) = 18\ \mu A}$$

(d)

$V_{DS} < V_{DS}(sat)$

$$I_D = K_n\left[2(V_{GS} - V_T)V_{DS} - V_{DS}^2\right]$$

$$= \left(1.25x10^{-5}\right)\left[2(3 - 0.8)(1) - (1)^2\right]$$

or

$$\underline{I_D = 42.5\ \mu A}$$

11.39

(a) We have

$$I_D(sat) = \frac{W\mu_n C_{ox}}{2L}(V_{GS} - V_T)^2$$

Now

$$6x10^{-3} = \left(\frac{W}{L}\right)\left(\frac{525}{2}\right)\frac{(3.9)\left(8.85x10^{-14}\right)}{\left(400x10^{-8}\right)}(5 - 0.75)^2$$

which yields

$$\underline{\frac{W}{L} = 14.7}$$

(b)

$$I_D(sat) = \frac{W\mu_p C_{ox}}{2L}(V_{SG} + V_T)^2$$

We have

$$6x10^{-3} = \left(\frac{W}{L}\right)\left(\frac{300}{2}\right)\frac{(3.9)\left(8.85x10^{-14}\right)}{\left(400x10^{-8}\right)}(5 - 0.75)^2$$

which yields

$$\underline{\frac{W}{L} = 25.7}$$

11.40

From Problem 11.30, we have

(a) In nonsaturation

$$I_D = 0.333\left[2(V_{GS} - V_T)V_{DS} - V_{DS}^2\right]$$

Now

$$g_{mL} = \frac{\partial I_D}{\partial V_{GS}} = (0.333)(2V_{DS})$$

At $V_{DS} = 0.5\ V$, we find

$$\underline{g_{mL} = 0.333\ mS}$$

(b) In saturation

$$I_D = 0.333(V_{GS} - V_T)^2$$

so that

$$g_{mS} = \frac{\partial I_D}{\partial V_{GS}} = 2(0.333)(V_{GS} - V_T)$$

For $V_T = 0.80\ V$ and at $V_{GS} = 4\ V$,

We obtain

$$\underline{g_{mS} = 2.13\ mS}$$

11.41

From Problem 11.31, we have

(a) In nonsaturation,

$$I_D = 0.148\left[2(V_{SG} + V_T)V_{SD} - V_{SD}^2\right](mA)$$

Then

$$g_{mL} = \frac{\partial I_D}{\partial V_{SG}} = (0.148)(2V_{SD})$$

For $V_{SD} = 0.5\ V$, we obtain

$$\underline{g_{mL} = 0.148\ mS}$$

(b) In saturation

$$I_D = 0.148(V_{SG} + V_T)^2$$

so that

$$g_{mS} = \frac{\partial I_D}{\partial V_{SG}} = 2(0.148)(V_{SG} + V_T)$$

For $V_T = -0.8\ V$ and at $V_{SG} = 4\ V$,
We obtain

$$\underline{g_{mS} = 0.947\ mS}$$

11.42

We can write, for $V_{SB} = 0$,

$$V_{TO} = V_{FB} + \frac{|Q'_{SD}(\max)|}{C_{ox}} + 2\phi_{fp}$$

We find

$$\phi_{fp} = (0.0259)\ln\left(\frac{5x10^{16}}{1.5x10^{10}}\right) = 0.389\ V$$

and

$$x_{dT} = \left[\frac{4(11.7)(8.85x10^{-14})(0.389)}{(1.6x10^{-19})(5x10^{16})}\right]^{1/2}$$

or

$$x_{dT} = 0.142\ \mu m$$

Then

$$|Q'_{SD}(\max)| = (1.6x10^{-19})(5x10^{16})(0.142x10^{-4})$$

or

$$|Q'_{SD}(\max)| = 1.14x10^{-7}\ C/cm^2$$

Also

$$C_{ox} = \frac{\in_{ox}}{t_{ox}} = \frac{(3.9)(8.85x10^{-14})}{400x10^{-8}}$$

or

$$C_{ox} = 8.63x10^{-8}\ F/cm^2$$

Now

$$V_{TO} = -0.5 + \frac{1.14x10^{-7}}{8.63x10^{-8}} + 2(0.389)$$

or

$$\underline{V_{TO} = +1.60\ V}$$

Then

$$I_D(sat) = \frac{W\mu_n C_{ox}}{2L}(V_{GS} - V_T)^2$$

$$= \left(\frac{10}{2}\right)\left(\frac{450}{2}\right)(8.63x10^{-8})(V_{GS} - V_T)^2$$

or

$$I_D(sat) = 0.097(V_{GS} - V_T)^2\ (mA)$$

For $I_D(sat) = 1\ mA$, $V_{GS} - V_T = 3.21\ V$
Now with substrate voltage applied,

$$\Delta V_T = \frac{\sqrt{2e\in_s N_a}}{C_{ox}}\left[\sqrt{2\phi_{fp} + V_{SB}} - \sqrt{2\phi_{fp}}\right]$$

$$= \frac{\left[2(1.6x10^{-19})(11.7)(8.85x10^{-14})(5x10^{16})\right]^{1/2}}{8.63x10^{-8}}$$

$$\times\left[\sqrt{2(0.389) + V_{SB}} - \sqrt{2(0.389)}\right]$$

or

$$\Delta V_T = 1.49\left[\sqrt{0.778 + V_{SB}} - 0.882\right]$$

We find that

V_{SB}	ΔV_T	V_T
0	0	1.60
1	0.673	2.27
2	1.17	2.77
4	1.94	3.54

11.43

For a p-channel MOSFET,

$$\Delta V_T = -\frac{\sqrt{2e\in_s N_d}}{C_{ox}}\left[\sqrt{2\phi_{fn} + V_{BS}} - \sqrt{2\phi_{fn}}\right]$$

We find

$$\phi_{fn} = (0.0259)\ln\left(\frac{5x10^{15}}{1.5x10^{10}}\right) = 0.329\ V$$

and

$$C_{ox} = \frac{\in_{ox}}{t_{ox}} = \frac{(3.9)(8.85x10^{-14})}{600x10^{-8}}$$

or

$$C_{ox} = 5.75x10^{-8}\ F/cm^2$$

Then

$$\Delta V_T = -1.5\ V$$

$$= \frac{-\left[2(1.6x10^{-19})(11.7)(8.85x10^{-14})(5x10^{15})\right]^{1/2}}{5.75x10^{-8}}$$

$$\times\left[\sqrt{0.658 + V_{BS}} - 0.811\right]$$

or

$$1.5 = 0.708\left[\sqrt{0.658 + V_{BS}} - 0.811\right]$$

which yields

$$\underline{V_{BS} = 7.92\ V}$$

11.44

(a)

n^+ poly-to-p type $\Rightarrow \phi_{ms} \approx -1.0\ V$

$$\phi_{fp} = (0.0259)\ln\left(\frac{10^{15}}{1.5x10^{10}}\right) = 0.288\ V$$

also

$$x_{dT} = \left[\frac{4 \in \phi_{fp}}{eN_a}\right]^{1/2}$$

$$= \left[\frac{4(11.7)(8.85x10^{-14})(0.288)}{(1.6x10^{-19})(10^{15})}\right]^{1/2}$$

or

$$x_{dT} = 0.863\ \mu m$$

$$|Q'_{SD}(\max)| = (1.6x10^{-19})(10^{15})(0.863x10^{-4})$$

or

$$|Q'_{SD}(\max)| = 1.38x10^{-8}\ C/cm^2$$

also

$$C_{ox} = \frac{\in_{ox}}{t_{ox}} = \frac{(3.9)(8.85x10^{-14})}{400x10^{-8}}$$

or

$$C_{ox} = 8.63x10^{-8}\ F/cm^2$$

Now

$$Q'_{SS} = (1.6x10^{-19})(5x10^{10}) = 8x10^{-9}\ C/cm^2$$

Then

$$V_T = \frac{(|Q'_{SD}(\max)| - Q'_{SS})}{C_{ox}} + \phi_{ms} + 2\phi_{fp}$$

$$= \left(\frac{1.38x10^{-8} - 8x10^{-9}}{8.63x10^{-8}}\right) - 1.0 + 2(0.288)$$

or

$$\underline{V_T = -0.357\ V}$$

(b)

For NMOS, apply V_{SB} and V_T shifts in a positive direction, so for $V_T = 0$, we want $\Delta V_T = +0.357\ V$.

So

$$\Delta V_T = \frac{\sqrt{2e \in N_a}}{C_{ox}}\left[\sqrt{2\phi_{fp} + V_{SB}} - \sqrt{2\phi_{fp}}\right]$$

$$+0.357 = \frac{\sqrt{2(1.6x10^{-19})(11.7)(8.85x10^{-14})(10^{15})}}{8.63x10^{-8}}$$

$$\times\left[\sqrt{2(0.288) + V_{SB}} - \sqrt{2(0.288)}\right]$$

or

$$0.357 = 0.211\left[\sqrt{0.576 + V_{SB}} - 0.759\right]$$

which yields

$$\underline{V_{SB} = 5.43\ V}$$

11.45

Computer plot

11.46

(a)

$$g_{ms} = \frac{W\mu_n C_{ox}}{L}(V_{GS} - V_T)$$

$$= \frac{(10)(400)(3.9)(8.85x10^{-14})}{475x10^{-8}}(5 - 0.65)$$

or

$$g_{ms} = 1.26\ mS$$

Now

$$g'_m = \frac{g_m}{1 + g_m r_s} \Rightarrow \frac{g'_m}{g_m} = 0.8 = \frac{1}{1 + g_m r_s}$$

which yields

$$r_s = \frac{1}{g_m}\left(\frac{1}{0.8} - 1\right) = \frac{1}{1.26}\left(\frac{1}{0.8} - 1\right)$$

or

$$\underline{r_s = 0.198\ k\Omega}$$

(b)

For $V_{GS} = 3\ V \Rightarrow g_{ms} = 0.683\ mS$

Then

$$g'_m = \frac{0.683}{1 + (0.683)(0.198)} = 0.602\ mS$$

So

$$\underline{\frac{g'_m}{g_m} = \frac{0.602}{0.683} = 0.88}$$

which is a 12% reduction.

11.47

(a) The ideal cutoff frequency for no overlap capacitance,

$$f_T = \frac{g_m}{2\pi C_{gs}} = \frac{\mu_n(V_{GS} - V_T)}{2\pi L^2}$$

$$= \frac{(400)(4 - 0.75)}{2\pi(2x10^{-4})^2}$$

or

$$\underline{f_T = 5.17\ GHz}$$

(b)
Now

$$f_T = \frac{g_m}{2\pi\left(C_{gsT} + C_M\right)}$$

where

$$C_M = C_{gdT}\left(1 + g_m R_L\right)$$

We find that

$$C_{gdT} = C_{ox}\left(0.75x10^{-4}\right)\left(20x10^{-4}\right)$$

$$= \frac{(3.9)\left(8.85x10^{-14}\right)\left(0.75x10^{-4}\right)\left(20x10^{-4}\right)}{500x10^{-8}}$$

or

$$C_{gdT} = 1.04x10^{-14}\ F$$

Also

$$g_{ms} = \frac{W\mu_n C_{ox}}{L}\left(V_{GS} - V_T\right)$$

$$= \frac{\left(20x10^{-4}\right)(400)(3.9)\left(8.85x10^{-14}\right)}{\left(2x10^{-4}\right)\left(500x10^{-8}\right)}(4 - 0.75)$$

or

$$g_{ms} = 0.897x10^{-3}\ S$$

Then

$$C_M = \left(1.04x10^{-14}\right)\left[1 + \left(0.897x10^{-3}\right)\left(10x10^{3}\right)\right]$$

or

$$C_M = 1.04x10^{-13}\ F$$

Now

$$C_{gsT} = C_{ox}\left(L + 0.75x10^{-4}\right)(W)$$

$$= \frac{(3.9)\left(8.85x10^{-14}\right)}{500x10^{-8}}\left(2x10^{-4} + 0.75x10^{-4}\right)\left(20x10^{-4}\right)$$

or

$$C_{gsT} = 3.8x10^{-14}\ F$$

We now find

$$f_T = \frac{g_m}{2\pi\left(C_{gsT} + C_M\right)}$$

$$= \frac{0.897x10^{-3}}{2\pi\left(3.8x10^{-14} + 1.04x10^{-13}\right)}$$

or

$$\underline{f_T = 1.0\ MHz}$$

11.48

(a) For the ideal case

$$f_T = \frac{v_{ds}}{2\pi L} = \frac{4x10^{6}}{2\pi\left(2x10^{-4}\right)}$$

or

$$\underline{f_T = 3.18\ GHz}$$

(b)
With overlap capacitance (using the values from Problem 11.47),

$$f_T = \frac{g_m}{2\pi\left(C_{gsT} + C_M\right)}$$

We find

$$g_m = WC_{ox}v_{ds}$$

$$= \frac{\left(20x10^{-4}\right)(3.9)\left(8.85x10^{-14}\right)\left(4x10^{6}\right)}{500x10^{-8}}$$

or

$$g_m = 0.552x10^{-3}\ S$$

We have

$$C_M = C_{gdT}\left(1 + g_m R_L\right)$$

$$= \left(1.04x10^{-14}\right)\left[1 + \left(0.552x10^{-3}\right)\left(10x10^{3}\right)\right]$$

or

$$C_M = 6.78x10^{-14}\ F$$

Then

$$f_T = \frac{0.552x10^{-3}}{2\pi\left(3.8x10^{-14} + 6.78x10^{-14}\right)}$$

or

$$\underline{f_T = 0.83\ GHz}$$

Chapter 12

Problem Solutions

12.1
(a)

$$I_D = 10^{-15}\exp\left[\frac{V_{GS}}{(2.1)V_t}\right]$$

For $V_{GS} = 0.5\,V$,

$$I_D = 10^{-15}\exp\left[\frac{0.5}{(2.1)(0.0259)}\right] \Rightarrow$$

$$\underline{I_D = 9.83x10^{-12}\ A}$$

For $V_{GS} = 0.7\,V$,

$$\underline{I_D = 3.88x10^{-10}\ A}$$

For $V_{GS} = 0.9\,V$,

$$\underline{I_D = 1.54x10^{-8}\ A}$$

Then the total current is:

$$I_{Total} = I_D\left(10^6\right)$$

For $V_{GS} = 0.5\,V$, $\underline{I_{Total} = 9.83\ \mu A}$

For $V_{GS} = 0.7\,V$, $\underline{I_{Total} = 0.388\ mA}$

For $V_{GS} = 0.9\,V$, $\underline{I_{Total} = 15.4\ mA}$

(b)
Power: $P = I_{Total}\cdot V_{DD}$
Then
For $V_{GS} = 0.5\,V$, $\underline{P = 49.2\ \mu W}$

For $V_{GS} = 0.7\,V$, $\underline{P = 1.94\ mW}$

For $V_{GS} = 0.9\,V$, $\underline{P = 77\ mW}$

12.2
We have

$$\Delta L = \sqrt{\frac{2\in}{eN_a}}\cdot\left[\sqrt{\phi_{fp} + V_{DS}(sat) + \Delta V_{DS}} - \sqrt{\phi_{fp} + V_{DS}(sat)}\right]$$

where

$$\phi_{fp} = V_t \ln\left(\frac{N_a}{n_i}\right) = (0.0259)\ln\left(\frac{10^{16}}{1.5x10^{10}}\right)$$

or

$$\phi_{fp} = 0.347\,V$$

We find

$$\sqrt{\frac{2\in}{eN_a}} = \left[\frac{2(11.7)\left(8.85x10^{-14}\right)}{\left(1.6x10^{-19}\right)\left(10^{16}\right)}\right]^{1/2}$$

$$= 0.360\ \mu m / V^{1/2}$$

We have

$$V_{DS}(sat) = V_{GS} - V_T$$

(a)
For $V_{GS} = 5\,V \Rightarrow V_{DS}(sat) = 4.25\,V$
Then

$$\Delta L = 0.360\left[\sqrt{0.347+5} - \sqrt{0.347+4.25}\right]$$

or

$$\Delta L = 0.0606\ \mu m$$

If ΔL is 10%of L, then $\underline{L = 0.606\ \mu m}$

(b)
For $V_{DS} = 5\,V$, $V_{GS} = 2\,V \Rightarrow V_{DS}(sat) = 1.25\,V$
Then

$$\Delta L = 0.360\left[\sqrt{0.347+5} - \sqrt{0.347+1.25}\right]$$

or

$$\Delta L = 0.377\ \mu m$$

Now if ΔL is 10% of L, then $\underline{L = 3.77\ \mu m}$

12.3

$$\Delta L = \sqrt{\frac{2\in}{eN_a}}\cdot\left[\sqrt{\phi_{fp} + V_{DS}(sat) + \Delta V_{DS}} - \sqrt{\phi_{fp} + V_{DS}(sat)}\right]$$

where

$$\phi_{fp} = V_t \ln\left(\frac{N_a}{n_i}\right) = (0.0259)\ln\left(\frac{4x10^{16}}{1.5x10^{10}}\right)$$

or

$$\phi_{fp} = 0.383\,V$$

and

$$x_{dT} = \left[\frac{4\in\phi_{fp}}{eN_a}\right]^{1/2}$$

$$= \left[\frac{4(11.7)\left(8.85x10^{-14}\right)(0.383)}{\left(1.6x10^{-19}\right)\left(4x10^{16}\right)}\right]^{1/2}$$

or

$$x_{dT} = 0.157\ \mu m$$

Then

$$\left|Q'_{SD}(\max)\right| = eN_a x_{dT}$$

$$= (1.6x10^{-19})(4x10^{16})(0.157x10^{-4})$$

or

$$|Q'_{SD}(\max)| = 10^{-7}\ C/cm^2$$

Now

$$V_T = (|Q'_{SD}(\max)| - Q'_{SS})\left(\frac{t_{ox}}{\in_{ox}}\right) + \phi_{ms} + 2\phi_{fp}$$

so that

$$V_T = \frac{[10^{-7} - (1.6x10^{-19})(3x10^{10})](400x10^{-8})}{(3.9)(8.85x10^{-14})} + 0 + 2(0.383)$$

or

$$V_T = 1.87\ V$$

Now

$$V_{DS}(sat) = V_{GS} - V_T = 5 - 1.87 = 3.13\ V$$

We find

$$\sqrt{\frac{2\in}{eN_a}} = \left[\frac{2(11.7)(8.85x10^{-14})}{(1.6x10^{-19})(4x10^{16})}\right]^{1/2} = 1.80x10^{-5}$$

Now

$$\Delta L = 1.80x10^{-5} \cdot \left[\sqrt{0.383 + 3.13 + \Delta V_{DS}} - \sqrt{0.383 + 3.13}\right]$$

or

$$\Delta L = 1.80x10^{-5}\left[\sqrt{3.513 + \Delta V_{DS}} - \sqrt{3.513}\right]$$

We obtain

ΔV_{DS}	$\Delta L(\mu m)$
0	0
1	0.0451
2	0.0853
3	0.122
4	0.156
5	0.188

12.4
Computer plot

12.5
Plot

12.6
Plot

12.7

(a) Assume $V_{DS}(sat) = 1\ V$, We have

$$E_{sat} = \frac{V_{DS}(sat)}{L}$$

We find

$L(\mu m)$	$E_{sat}(V/cm)$
3	$3.33x10^3$
1	10^4
0.5	$2x10^4$
0.25	$4x10^4$
0.13	$7.69x10^4$

(b)

Assume $\mu_n = 500\ cm^2/V-s$, we have

$v = \mu_n E_{sat}$

Then

For $L = 3\ \mu m$, $\underline{v = 1.67x10^6\ cm/s}$

For $L = 1\ \mu m$, $\underline{v = 5x10^6\ cm/s}$

For $L \le 0.5\ \mu m$, $\underline{v \approx 10^7\ cm/s}$

12.8

We have $I'_D = L(L - \Delta L)^{-1} I_D$

We may write

$$g_O = \frac{\partial I'_D}{\partial V_{DS}} = (-1)L(L-\Delta L)^{-2} I_D\left(\frac{-\partial(\Delta L)}{\partial V_{DS}}\right)$$

$$= \frac{L}{(L-\Delta L)^2} \cdot I_D \cdot \frac{\partial(\Delta L)}{\partial V_{DS}}$$

We have

$$\Delta L = \sqrt{\frac{2\in}{eN_a}} \cdot \left[\sqrt{\phi_{fp} + V_{DS}} - \sqrt{\phi_{fp} + V_{DS}(sat)}\right]$$

We find

$$\frac{\partial(\Delta L)}{\partial V_{DS}} = \sqrt{\frac{2\in}{eN_a}} \cdot \frac{1}{2\sqrt{\phi_{fp} + V_{DS}}}$$

(a)

For $V_{GS} = 2\ V$, $\Delta V_{DS} = 1\ V$, and

$V_{DS}(sat) = V_{GS} - V_T = 2 - 0.8 = 1.2\ V$

Also

$V_{DS} = V_{DS}(sat) + \Delta V_{DS} = 1.2 + 1 = 2.2\ V$

and

$$\phi_{fp} = (0.0259)\ln\left(\frac{3x10^{16}}{1.5x10^{10}}\right) = 0.376\ V$$

Now

$$\sqrt{\frac{2\in}{eN_a}} = \left[\frac{2(11.7)(8.85x10^{-14})}{(1.6x10^{-19})(3x10^{16})}\right]^{1/2}$$
$$= 0.2077\ \mu m / V^{1/2}$$

We find

$$\Delta L = 0.2077\left[\sqrt{0.376+2.2} - \sqrt{0.376+1.2}\right]$$
$$= 0.0726\ \mu m$$

Then

$$\frac{\partial(\Delta L)}{\partial V_{DS}} = \frac{0.2077}{2}\cdot\frac{1}{\sqrt{0.376+2.2}}$$
$$= 0.0647\ \mu m / V$$

From the previous problem,
$I_D = 0.48\ mA$, $L = 2\ \mu m$
Then

$$g_O = \frac{2}{(2-0.0726)^2}(0.48x10^{-3})(0.0647)$$

or

$$g_O = 1.67x10^{-5}\ S$$

so that

$$r_O = \frac{1}{g_O} = 59.8\ k\Omega$$

(b)
If $L = 1\ \mu m$, then from the previous problem, we would have $I_D = 0.96\ mA$, so that

$$g_O = \frac{1}{(1-0.0726)^2}(0.96x10^{-3})(0.0647)$$

or

$$g_O = 7.22x10^{-5}\ S$$

so that

$$r_O = \frac{1}{g_O} = 13.8\ k\Omega$$

12.9
(a)

$$I_D(sat) = \frac{W\mu_n C_{ox}}{2L}(V_{GS}-V_T)^2$$
$$= \left(\frac{10}{2}\right)(500)(6.9x10^{-8})(V_{GS}-1)^2$$

or

$$I_D(sat) = 0.173(V_{GS}-1)^2\ (mA)$$

and

$$\sqrt{I_D(sat)} = \sqrt{0.173}(V_{GS}-1)\ (mA)^{1/2}$$

(b)
Let $\mu_{eff} = \mu_O\left(\frac{E_{eff}}{E_C}\right)^{-1/3}$

Where $\mu_O = 1000\ cm^2/V-s$ and $E_C = 2.5x10^4\ V/cm$.

Let $E_{eff} = \frac{V_{GS}}{t_{ox}}$

We find

$$C_{ox} = \frac{\in_{ox}}{t_{ox}} \Rightarrow t_{ox} = \frac{\in_{ox}}{C_{ox}} = \frac{(3.9)(8.85x10^{-14})}{6.9x10^{-8}}$$

or

$$t_{ox} = 500\ \mathring{A}$$

Then

V_{GS}	E_{eff}	μ_{eff}	$\sqrt{I_D(sat)}$
1	--	--	0
2	4E5	397	0.370
3	6E5	347	0.692
4	8E5	315	0.989
5	10E5	292	1.27

(c)
The slope of the variable mobility curve is not constant, but is continually decreasing.

12.10
Plot

12.11

$$V_T = V_{FB} + \frac{|Q'_{SD}(\max)|}{C_{ox}} + 2\phi_{fp}$$

We find

$$\phi_{fp} = V_t \ln\left(\frac{N_a}{n_i}\right) = (0.0259)\ln\left(\frac{5x10^{16}}{1.5x10^{10}}\right)$$

or

$$\phi_{fp} = 0.389\ V$$

and

$$x_{dT} = \left[\frac{4\in\phi_{fp}}{eN_a}\right]^{1/2}$$
$$= \left[\frac{4(11.7)(8.85x10^{-14})(0.389)}{(1.6x10^{-19})(5x10^{16})}\right]^{1/2}$$

or

$$x_{dT} = 0.142\ \mu m$$

Now

$$|Q'_{SD}(\max)| = eN_a x_{dT}$$
$$= (1.6x10^{-19})(5x10^{16})(0.142x10^{-4})$$

or

$$|Q'_{SD}(\max)| = 1.14x10^{-7}\ C/cm^2$$

Also

$$C_{ox} = \frac{\in_{ox}}{t_{ox}} = \frac{(3.9)(8.85x10^{-14})}{400x10^{-8}} = 8.63x10^{-8}\ F/cm^2$$

Then

$$V_T = -1.12 + \frac{1.14x10^{-7}}{8.63x10^{-8}} + 2(0.389)$$

or

$$\underline{V_T = +0.90\ V}$$

(a)

$$I_D = \frac{W\mu_n C_{ox}}{2L}\left[2(V_{GS} - V_T)V_{DS} - V_{DS}^2\right]$$

and

$$V_{DS}(sat) = V_{GS} - V_T$$

We have

$$I_D = \left(\frac{20}{2}\right)\left(\frac{1}{2}\right)(400)(8.63x10^{-8})$$
$$\times\left[2(V_{GS} - V_T)V_{DS} - V_{DS}^2\right]$$

or

$$I_D = 0.173\left[2(V_{GS} - V_T)V_{DS} - V_{DS}^2\right](mA)$$

For $V_{DS} = V_{DS}(sat) = V_{GS} - V_T = 1\ V$,

$$\underline{I_D(sat) = 0.173\ mA}$$

For $V_{DS} = V_{DS}(sat) = V_{GS} - V_T = 2\ V$,

$$\underline{I_D(sat) = 0.692\ mA}$$

(b)

For $V_{DS} \le 1.25\ V$, $\mu = \mu_n = 400\ cm^2/V-s$.
The curve for $V_{GS} - V_T = 1\ V$ is unchanged. For $V_{GS} - V_T = 2\ V$ and $0 \le V_{DS} \le 1.25\ V$, the curve in unchanged. For $V_{DS} \ge 1.25\ V$, the current is constant at

$$I_D = 0.173\left[2(2)(1.25) - (1.25)^2\right] = 0.595\ mA$$

When velocity saturation occurs, $V_{DS}(sat) = 1.25\ V$ for the case of $V_{GS} - V_T = 2\ V$.

12.12

Plot

12.13

(a) Non-saturation region

$$I_D = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)\left[2(V_{GS} - V_T)V_{DS} - V_{DS}^2\right]$$

We have

$$C_{ox} = \frac{\in_{ox}}{t_{ox}} \Rightarrow \frac{C_{ox}}{k}$$

and

$$W \Rightarrow kW,\ L \Rightarrow kL$$

also

$$V_{GS} \Rightarrow kV_{GS},\ V_{DS} \Rightarrow kV_{DS}$$

So

$$I_D = \frac{1}{2}\mu_n\left(\frac{C_{ox}}{k}\right)\left(\frac{kW}{kL}\right)\left[2(kV_{GS} - V_T)kV_{DS} - (kV_{DS})^2\right]$$

Then

$$\underline{I_D \Rightarrow\ \approx kI_D}$$

In the saturation region

$$I_D = \frac{1}{2}\mu_n\left(\frac{C_{ox}}{k}\right)\left(\frac{kW}{kL}\right)\left[kV_{GS} - V_T\right]^2$$

Then

$$\underline{I_D \Rightarrow\ \approx kI_D}$$

(b)

$$\underline{P = I_D V_{DD} \Rightarrow (kI_D)(kV_{DD}) \Rightarrow k^2 P}$$

12.14

$$I_D(sat) = WC_{ox}(V_{GS} - V_T)v_{sat}$$
$$\Rightarrow (kW)\left(\frac{C_{ox}}{k}\right)(kV_{GS} - V_T)v_{sat}$$

or

$$\underline{I_D(sat) \approx kI_D(sat)}$$

12.15

(a)

(i) $I_D = K_n(V_{GS} - V_T)^2 = (0.1)(5 - 0.8)^2$

or

$$\underline{I_D = 1.764\ mA}$$

(ii)

$$I_D = \left(\frac{0.1}{0.6}\right)\left[(0.6)(5) - 0.8\right]^2$$

or

$$\underline{I_D = 0.807\ mA}$$

(b)

(i) $P = (1.764)(5) \Rightarrow \underline{P = 8.82\ mW}$

(ii) $P = (0.807)(0.6)(5) \Rightarrow \underline{P = 2.42\ mW}$

(c)

Current: Ratio $= \dfrac{0.807}{1.764} = 0.457$

Power: Ratio $= \dfrac{2.42}{8.82} = 0.274$

12.16

$$\Delta V_T = -\frac{eN_a x_{dT}}{C_{ox}}\left\{\frac{r_j}{L}\left[\sqrt{1+\frac{2x_{dT}}{r_j}}-1\right]\right\}$$

Now

$$\phi_{fp} = V_t \ln\left(\frac{N_a}{n_i}\right) = (0.0259)\ln\left(\frac{10^{16}}{1.5x10^{10}}\right)$$

or

$$\phi_{fp} = 0.347\ V$$

and

$$x_{dT} = \left[\frac{4\in\phi_{fp}}{eN_a}\right]^{1/2}$$

$$= \left[\frac{4(11.7)(8.85x10^{-14})(0.347)}{(1.6x10^{-19})(10^{16})}\right]^{1/2}$$

or

$$x_{dT} = 0.30\ \mu m$$

Also

$$C_{ox} = \frac{\in_{ox}}{t_{ox}} = \frac{(3.9)(8.85x10^{-14})}{450x10^{-8}}$$

$$= 7.67x10^{-8}\ F/cm^2$$

Then

$$\Delta V_T = -\frac{(1.6x10^{-19})(10^{16})(0.3x10^{-4})}{7.67x10^{-8}} \times\left\{\frac{0.3}{1}\left[\sqrt{1+\frac{2(0.3)}{0.3}}-1\right]\right\}$$

or

$$\underline{\Delta V_T = -0.137\ V}$$

12.17

$$\Delta V_T = -\frac{eN_a x_{dT}}{C_{ox}}\left\{\frac{r_j}{L}\left[\sqrt{1+\frac{2x_{dT}}{r_j}}-1\right]\right\}$$

Now

$$\phi_{fp} = (0.0259)\ln\left(\frac{3x10^{16}}{1.5x10^{10}}\right) = 0.376\ V$$

and

$$x_{dT} = \left[\frac{4\in\phi_{fp}}{eN_a}\right]^{1/2}$$

$$= \left[\frac{4(11.7)(8.85x10^{-14})(0.376)}{(1.6x10^{-19})(3x10^{16})}\right]^{1/2}$$

or

$$x_{dT} = 0.180\ \mu m$$

Also

$$C_{ox} = \frac{\in_{ox}}{t_{ox}} = \frac{(3.9)(8.85x10^{-14})}{800x10^{-8}}$$

or

$$C_{ox} = 4.31x10^{-8}\ F/cm^2$$

Then

$$\Delta V_T = -0.20 = -\frac{(1.6x10^{19})(3x10^{16})(0.18x10^{-4})}{4.31x10^{-8}} \times\left\{\frac{0.6}{L}\left[\sqrt{1+\frac{2(0.18)}{0.6}}-1\right]\right\}$$

or

$$= -0.20 = -\frac{0.319}{L}$$

which yields

$$\underline{L = 1.59\ \mu m}$$

12.18

We have

$$L' = L-(a+b)$$

and from the geometry

(1) $(a+r_j)^2 + x_{dT}^2 = (r_j + x_{dS})^2$

and

(2) $(b+r_j)^2 + x_{dT}^2 = (r_j + x_{dD})^2$

From (1),

$$(a+r_j)^2 = (r_j + x_{dS})^2 - x_{dT}^2$$

so that

$$a = \sqrt{(r_j + x_{dS})^2 - x_{dT}^2} - r_j$$

which can be written as

$$a = r_j\left[\sqrt{\left(1+\frac{x_{dS}}{r_j}\right)^2 - \left(\frac{x_{dT}}{r_j}\right)^2} - 1\right]$$

or

$$a = r_j\left[\sqrt{1+\frac{2x_{dS}}{r_j}+\left(\frac{x_{dS}}{r_j}\right)^2-\left(\frac{x_{dT}}{r_j}\right)^2}-1\right]$$

Define

$$\alpha^2 = \frac{x_{dS}^2 - x_{dT}^2}{r_j^2}$$

We can then write

$$a = r_j\left[\sqrt{1+\frac{2x_{dS}}{r_j}+\alpha^2}-1\right]$$

Similarly from (2), we will have

$$b = r_j\left[\sqrt{1+\frac{2x_{dD}}{r_j}+\beta^2}-1\right]$$

where

$$\beta^2 = \frac{x_{dD}^2 - x_{dT}^2}{r_j^2}$$

The average bulk charge in the trapezoid (per unit area) is

$$|Q'_B|\cdot L = eN_a x_{dT}\left(\frac{L+L'}{2}\right)$$

or

$$|Q'_B| = eN_a x_{dT}\left(\frac{L+L'}{2L}\right)$$

We can write

$$\frac{L+L'}{2L} = \frac{1}{2}+\frac{L'}{2L} = \frac{1}{2}+\frac{1}{2L}\left[L-(a+b)\right]$$

which is

$$= 1-\frac{(a+b)}{2L}$$

Then

$$|Q'_B| = eN_a x_{dT}\left[1-\frac{(a+b)}{2L}\right]$$

Now $|Q'_B|$ replaces $|Q'_{SD}(\max)|$ in the threshold equation. Then

$$\Delta V_T = \frac{|Q'_B|}{C_{ox}} - \frac{|Q'_{SD}(\max)|}{C_{ox}}$$

$$= \frac{eN_a x_{dT}}{C_{ox}}\left[1-\frac{(a+b)}{2L}\right] - \frac{eN_a x_{dT}}{C_{ox}}$$

or

$$\Delta V_T = -\frac{eN_a x_{dT}}{C_{ox}}\cdot\frac{(a+b)}{2L}$$

Then substituting, we obtain

$$\Delta V_T = -\frac{eN_a x_{dT}}{C_{ox}}\cdot\frac{r_j}{2L}\left\{\left[\sqrt{1+\frac{2x_{dS}}{r_j}+\alpha^2}-1\right] + \left[\sqrt{1+\frac{2x_{dD}}{r_j}+\beta^2}-1\right]\right\}$$

Note that if $x_{dS} = x_{dD} = x_{dT}$, then $\alpha = \beta = 0$ and the expression for ΔV_T reduces to that given in the text.

12.19

We have $L' = 0$, so Equation (12.25) becomes

$$\frac{L+L'}{2L} \Rightarrow \frac{L}{2L} = \frac{1}{2} = \left\{1-\frac{r_j}{L}\left[\sqrt{1+\frac{2x_{dT}}{r_j}}-1\right]\right\}$$

or

$$\frac{r_j}{L}\left[\sqrt{1+\frac{2x_{dT}}{r_j}}-1\right] = \frac{1}{2}$$

Then Equation (12.26) is

$$|Q'_B| = eN_a x_{dT}\left(\frac{1}{2}\right)$$

The change in the threshold voltage is

$$\Delta V_T = \frac{|Q'_B|}{C_{ox}} - \frac{|Q'_{SD}(\max)|}{C_{ox}}$$

or

$$\Delta V_T = \frac{(1/2)(eN_a x_{dT})}{C_{ox}} - \frac{(eN_a x_{dT})}{C_{ox}}$$

or

$$\underline{\Delta V_T = -\left(\frac{1}{2}\right)\frac{(eN_a x_{dT})}{C_{ox}}}$$

12.20

Computer plot

12.21

Computer plot

12.22

$$\Delta V_T = -\frac{eN_a x_{dT}}{C_{ox}}\left\{\frac{r_j}{L}\left[\sqrt{1+\frac{2x_{dT}}{r_j}}-1\right]\right\}$$

$$\Rightarrow -\frac{e\left(\frac{N_a}{k}\right)(kx_{dT})}{\left(\frac{C_{ox}}{k}\right)}\left\{\frac{kr_j}{kL}\left[\sqrt{1+\frac{2kx_{dT}}{kr_j}}-1\right]\right\}$$

or

$$\underline{\Delta V_T = k\Delta V_T}$$

12.23

$$\Delta V_T = \frac{eN_a x_{dT}}{C_{ox}}\left(\frac{\xi x_{dT}}{W}\right)$$

We find

$$\phi_{fp} = (0.0259)\ln\left(\frac{10^{16}}{1.5x10^{10}}\right) = 0.347\ V$$

and

$$x_{dT} = \left[\frac{4\in\phi_{fp}}{eN_a}\right]^{1/2}$$

$$= \left[\frac{4(11.7)(8.85x10^{-14})(0.347)}{(1.6x10^{-19})(10^{16})}\right]^{1/2}$$

or

$$x_{dT} = 0.30\ \mu m$$

Also

$$C_{ox} = \frac{\in_{ox}}{t_{ox}} = \frac{(3.9)(8.85x10^{-14})}{450x10^{-8}}$$

or

$$C_{ox} = 7.67x10^{-8}\ F/cm^2$$

Then

$$\Delta V_T = \frac{(1.6x10^{-19})(10^{16})(0.3x10^{-4})}{7.67x10^{-8}} \times\left[\frac{(\pi/2)(0.3x10^{-4})}{2.5x10^{-4}}\right]$$

or

$$\underline{\Delta V_T = +0.118\ V}$$

12.24

Additional bulk charge due to the ends:

$$\Delta Q_B = eN_a L\left(\frac{1}{2}x_{dT}^2\right)\cdot 2 = eN_a Lx_{dT}(\xi x_{dT})$$

where $\xi = 1$.

Then

$$\Delta V_T = \frac{eN_a x_{dT}^2}{C_{ox}W}$$

We find

$$\phi_{fp} = (0.0259)\ln\left(\frac{3x10^{16}}{1.5x10^{10}}\right) = 0.376\ V$$

and

$$x_{dT} = \left[\frac{4\in\phi_{fp}}{eN_a}\right]^{1/2}$$

$$= \left[\frac{4(11.7)(8.85x10^{-14})(0.376)}{(1.6x10^{-19})(3x10^{16})}\right]^{1/2}$$

or

$$x_{dT} = 0.180\ \mu m$$

Also

$$C_{ox} = \frac{\in_{ox}}{t_{ox}} = \frac{(3.9)(8.85x10^{-14})}{800x10^{-8}}$$

or

$$C_{ox} = 4.31x10^{-8}\ F/cm^2$$

Now, we can write

$$W = \frac{eN_a x_{dT}^2}{C_{ox}(\Delta V_T)}$$

$$= \frac{(1.6x10^{-19})(3x10^{16})(0.18x10^{-4})^2}{(4.31x10^{-8})(0.25)}$$

or

$$\underline{W = 1.44\ \mu m}$$

12.25

Computer plot

12.26

$$\Delta V_T = \frac{eN_a x_{dT}}{C_{ox}}\left(\frac{\xi x_{dT}}{W}\right)$$

Assume that ξ is a constant

$$\Rightarrow \frac{e\left(\frac{N_a}{k}\right)(kx_{dT})}{\left(\frac{C_{ox}}{k}\right)}\left(\frac{\xi\cdot kx_{dT}}{kW}\right)$$

or

$$\underline{\Delta V_T = k\Delta V_T}$$

12.27

(a)

$$V_{BD} = (6x10^6)t_{ox} = (6x10^6)(250x10^{-8})$$

or

$$\underline{V_{BD} = 15\ V}$$

(b)
With a safety factor of 3,

$$V_{BD} = \frac{1}{3} \cdot 15 \Rightarrow \underline{V_{BD} = 5\ V}$$

12.28
We want $V_G = 20\ V$. With a safety factor of 3, then $V_{BD} = 60\ V$, so that

$$60 = \left(6x10^{6}\right)t_{ox} \Rightarrow \underline{t_{ox} = 1000\ \overset{\circ}{A}}$$

12.29
Snapback breakdown means $\alpha M = 1$, where

$$\alpha = (0.18)\log_{10}\left(\frac{I_D}{3x10^{-9}}\right)$$

and

$$M = \frac{1}{1 - \left(\dfrac{V_{CE}}{V_{BD}}\right)^{m}}$$

Let $V_{BD} = 15\ V$, $m = 3$. Now when

$$\alpha M = 1 = \frac{\alpha}{1 - \left(\dfrac{V_{CE}}{15}\right)^{3}}$$

we can write this as

$$1 - \left(\frac{V_{CE}}{15}\right)^{3} = \alpha \Rightarrow V_{CE} = 15\sqrt[3]{1-\alpha}$$

Now

$\underline{I_D}$	$\underline{\alpha}$	$\underline{V_{CE}}$
E-8	0.0941	14.5
E-7	0.274	13.5
E-6	0.454	12.3
E-5	0.634	10.7
E-4	0.814	8.6
E-3	0.994	2.7

12.30
One Debye length is

$$L_D = \left[\frac{\in (kT/e)}{eN_a}\right]^{1/2}$$

$$= \left[\frac{(11.7)\left(8.85x10^{-14}\right)(0.0259)}{\left(1.6x10^{-19}\right)\left(10^{16}\right)}\right]^{1/2}$$

or

$$L_D = 4.09x10^{-6}\ cm$$

Six Debye lengths:

$$6\left(4.09x10^{-6}\right) = 0.246\ \mu m$$

From Example 12.4, we have $x_{dO} = 0.336\ \mu m$, which is the zero-biased source-substrate junction width.
At near punch-through, we will have

$$x_{dO} + 6L_D + x_d = L$$

where x_d is the reverse-biased drain-substrate junction width. Now

$$0.336 + 0.246 + x_d = 1.2 \Rightarrow x_d = 0.618\ \mu m$$ at near punch-through.

We have

$$x_d = \left[\frac{2 \in \left(V_{bi} + V_{DS}\right)}{eN_a}\right]^{1/2}$$

or

$$V_{bi} + V_{DS} = \frac{x_d^2 eN_a}{2 \in}$$

$$= \frac{\left(0.618x10^{-4}\right)^2\left(1.6x10^{-19}\right)\left(10^{16}\right)}{2(11.7)\left(8.85x10^{-14}\right)}$$

which yields

$$V_{bi} + V_{DS} = 2.95\ V$$

From Example 12.4, we have $V_{bi} = 0.874\ V$, so that

$$\underline{V_{DS} = 2.08\ V}$$

which is the near punch-through voltage. The ideal punch-through voltage was

$$\underline{V_{DS} = 4.9\ V}$$

12.31

$$V_{bi} = (0.0259)\ln\left[\frac{\left(10^{19}\right)\left(3x10^{16}\right)}{\left(1.5x10^{10}\right)^2}\right] = 0.902\ V$$

The zero-biased source-substrate junction width:

$$x_{dO} = \left[\frac{2 \in V_{bi}}{eN_a}\right]^{1/2}$$

$$= \left[\frac{2(11.7)\left(8.85x10^{-14}\right)(0.902)}{\left(1.6x10^{-19}\right)\left(3x10^{16}\right)}\right]^{1/2}$$

or

$$x_{dO} = 0.197\ \mu m$$

The Debye length is

$$L_D = \left[\frac{\in (kT/e)}{eN_a}\right]^{1/2}$$

$$= \left[\frac{(11.7)(8.85x10^{-14})(0.0259)}{(1.6x10^{-19})(3x10^{16})}\right]^{1/2}$$

or

$$L_D = 2.36x10^{-6}\ cm$$

so that

$$6L_D = 6(2.36x10^{-6}) = 0.142\ \mu m$$

Now

$$x_{dO} + 6L_D + x_d = L$$

We have for $V_{DS} = 5\ V$,

$$x_d = \left[\frac{2 \in (V_{bi} + V_{DS})}{eN_a}\right]^{1/2}$$

$$= \left[\frac{2(11.7)(8.85x10^{-14})(0.902+5)}{(1.6x10^{-19})(3x10^{16})}\right]^{1/2}$$

or

$$x_d = 0.505\ \mu m$$

Then

$$L = 0.197 + 0.142 + 0.505$$

or

$$\underline{L = 0.844\ \mu m}$$

12.32

With a source-to-substrate voltage of 2 volts,

$$x_{dO} = \left[\frac{2 \in (V_{bi} + V_{SB})}{eN_a}\right]^{1/2}$$

$$= \left[\frac{2(11.7)(8.85x10^{-14})(0.902+2)}{(1.6x10^{-19})(3x10^{16})}\right]^{1/2}$$

or

$$x_{dO} = 0.354\ \mu m$$

We have $6L_D = 0.142\ \mu m$ from the previous problem.

Now

$$x_d = \left[\frac{2 \in (V_{bi} + V_{DS} + V_{SB})}{eN_a}\right]^{1/2}$$

$$= \left[\frac{2(11.7)(8.85x10^{-14})(0.902+5+2)}{(1.6x10^{-19})(3x10^{16})}\right]^{1/2}$$

or

$$x_d = 0.584\ \mu m$$

Then

$$L = x_{dO} + 6L_D + x_d$$

$$= 0.354 + 0.142 + 0.584$$

or

$$\underline{L = 1.08\ \mu m}$$

12.33

(a) $$\phi_{fp} = (0.0259)\ln\left(\frac{2x10^{15}}{1.5x10^{10}}\right) = 0.306\ V$$

and

$$\phi_{ms} = -\left(\frac{E_g}{2e} + \phi_{fp}\right) = -\left(\frac{1.12}{2} + 0.306\right)$$

or

$$\phi_{ms} = -0.866\ V$$

Also

$$x_{dT} = \left[\frac{4 \in \phi_{fp}}{eN_a}\right]^{1/2}$$

$$= \left[\frac{4(11.7)(8.85x10^{-14})(0.306)}{(1.6x10^{-19})(2x10^{15})}\right]^{1/2}$$

or

$$x_{dT} = 0.629\ \mu m$$

Now

$$|Q'_{SD}(\max)| = eN_a x_{dT}$$

$$= (1.6x10^{-19})(2x10^{15})(0.629x10^{-4})$$

or

$$|Q'_{SD}(\max)| = 2.01x10^{-8}\ C/cm^2$$

We have

$$Q'_{SS} = (2x10^{11})(1.6x10^{-19}) = 3.2x10^{-8}\ C/cm^2$$

Then

$$V_T = \left(|Q'_{SD}(\max)| - Q'_{SS}\right)\left(\frac{t_{ox}}{\in_{ox}}\right) + \phi_{ms} + 2\phi_{fp}$$

$$= \frac{(2.01x10^{-8} - 3.2x10^{-8})(650x10^{-8})}{(3.9)(8.85x10^{-14})} - 0.866 + 2(0.306)$$

which yields

$$\underline{V_T = -0.478\ V}$$

(b) We need a shift in threshold voltage in the positive direction, which means we must add acceptor atoms. We need

$$\Delta V_T = +0.80 - (-0.478) = 1.28\ V$$

Then

$$D_I = \frac{(\Delta V_T)C_{ox}}{e} = \frac{(1.28)(3.9)\left(8.85x10^{-14}\right)}{\left(1.6x10^{-19}\right)\left(650x10^{-8}\right)}$$

or

$$\underline{D_I = 4.25x10^{11}\ cm^{-2}}$$

12.34

(a) $\phi_{fn} = (0.0259)\ln\left(\dfrac{10^{16}}{1.5x10^{10}}\right) = 0.347\ V$

and

$$\phi_{ms} = \phi'_{ms} - \left(\chi' + \frac{E_g}{2e} - \phi_{fn}\right)$$

$$= 3.2 - (3.25 + 0.56 - 0.347)$$

or

$$\phi_{ms} = -0.263\ V$$

Also

$$x_{dT} = \left[\frac{4\in\phi_{fn}}{eN_d}\right]^{1/2}$$

$$= \left[\frac{4(11.7)\left(8.85x10^{-14}\right)(0.347)}{\left(1.6x10^{-19}\right)\left(10^{16}\right)}\right]^{1/2}$$

or

$$x_{dT} = 0.30\ \mu m$$

Now

$$\left|Q'_{SD}(\max)\right| = eN_d x_{dT}$$

$$= \left(1.6x10^{-19}\right)\left(10^{16}\right)\left(0.30x10^{-4}\right)$$

or

$$\left|Q'_{SD}(\max)\right| = 4.8x10^{-8}\ C/cm^2$$

We have

$$Q'_{SS} = \left(5x10^{11}\right)\left(1.6x10^{-19}\right) = 8x10^{-8}\ C/cm^2$$

Now

$$V_T = -\left(\left|Q'_{SD}(\max)\right| + Q'_{SS}\right)\left(\frac{t_{ox}}{\in_{ox}}\right) + \phi_{ms} - 2\phi_{fn}$$

$$= \frac{-\left(4.8x10^{-8} + 8x10^{-8}\right)\left(750x10^{-8}\right)}{(3.9)\left(8.85x10^{-14}\right)}$$

$$-0.263 - 2(0.347)$$

which becomes

$$\underline{V_T = -3.74\ V}$$

(b)

We want $V_T = -0.50\ V$. Need to shift V_T in the positive direction which means we need to add acceptor atoms.

So

$$\Delta V_T = -0.50 - (-3.74) = 3.24\ V$$

Now

$$D_I = \frac{(\Delta V_T)C_{ox}}{e} = \frac{(3.24)(3.9)\left(8.85x10^{-14}\right)}{\left(1.6x10^{-19}\right)\left(750x10^{-8}\right)}$$

or

$$\underline{D_I = 9.32x10^{11}\ cm^{-2}}$$

12.35

(a) $\phi_{fp} = (0.0259)\ln\left(\dfrac{10^{15}}{1.5x10^{10}}\right) = 0.288\ V$

and

$$x_{dT} = \left[\frac{4\in\phi_{fp}}{eN_a}\right]^{1/2}$$

$$= \left[\frac{4(11.7)\left(8.85x10^{-14}\right)(0.288)}{\left(1.6x10^{-19}\right)\left(10^{15}\right)}\right]^{1/2}$$

or

$$x_{dT} = 0.863\ \mu m$$

Now

$$\left|Q'_{SD}(\max)\right| = eN_a x_{dT}$$

$$= \left(1.6x10^{-19}\right)\left(10^{15}\right)\left(0.863x10^{-4}\right)$$

or

$$\left|Q'_{SD}(\max)\right| = 1.38x10^{-8}\ C/cm^2$$

Also

$$C_{ox} = \frac{\in_{ox}}{t_{ox}} = \frac{(3.9)\left(8.85x10^{-14}\right)}{750x10^{-8}}$$

or

$$C_{ox} = 4.6x10^{-8}\ F/cm^2$$

Then

$$V_T = V_{FB} + 2\phi_{fp} + \frac{\left|Q'_{SD}(\max)\right|}{C_{ox}}$$

$$= -1.50 + 2(0.288) + \frac{1.38x10^{-8}}{4.6x10^{-8}}$$

or

$$\underline{V_T = -0.624\ V}$$

(b)

Want $V_T = +0.90\ V$, which is a positive shift and we must add acceptor atoms.

$$\Delta V_T = 0.90 - (-0.624) = 1.52\ V$$

Then

$$D_I = \frac{(\Delta V_T)C_{ox}}{e} = \frac{(1.52)\left(4.6x10^{-8}\right)}{1.6x10^{-19}}$$

or

$$D_I = 4.37x10^{11}\ cm^{-2}$$

(c)
With an applied substrate voltage,

$$\Delta V_T = \frac{\sqrt{2e \in N_a}}{C_{ox}}\left[\sqrt{2\phi_{fp} + V_{SB}} - \sqrt{2\phi_{fp}}\right]$$

$$= \frac{\left[2\left(1.6x10^{-19}\right)(11.7)\left(8.85x10^{-14}\right)\left(10^{15}\right)\right]^{1/2}}{4.6x10^{-8}}$$

$$\times\left[\sqrt{2(0.288)+2} - \sqrt{2(0.288)}\right]$$

or

$$\Delta V_T = +0.335\ V$$

Then the threshold voltage is

$$V_T = +0.90 + 0.335$$

or

$$V_T = 1.24\ V$$

12.36

The total space charge width is greater than x_I, so from chapter 11,

$$\Delta V_T = \frac{\sqrt{2e \in N_a}}{C_{ox}}\left[\sqrt{2\phi_{fp} + V_{SB}} - \sqrt{2\phi_{fp}}\right]$$

Now

$$\phi_{fp} = (0.0259)\ln\left(\frac{10^{14}}{1.5x10^{10}}\right) = 0.228\ V$$

and

$$C_{ox} = \frac{\in_{ox}}{t_{ox}} = \frac{(3.9)\left(8.85x10^{-14}\right)}{500x10^{-8}}$$

or

$$C_{ox} = 6.90x10^{-8}\ F/cm^2$$

Then

$$\Delta V_T = \frac{\left[2\left(1.6x10^{-19}\right)(11.7)\left(8.85x10^{-14}\right)\left(10^{14}\right)\right]^{1/2}}{6.90x10^{-8}}$$

$$\times\left[\sqrt{2(0.228)+V_{SB}} - \sqrt{2(0.228)}\right]$$

or

$$\Delta V_T = 0.0834\left[\sqrt{0.456+V_{SB}} - \sqrt{0.456}\right]$$

Then

$V_{SB}(V)$	$\Delta V_T(V)$
1	0.0443
3	0.0987
5	0.399

11.37

(a) $\phi_{fn} = (0.0259)\ln\left(\frac{10^{17}}{1.5x10^{10}}\right) = 0.407\ V$

and

$$x_{dT} = \left[\frac{4(11.7)\left(8.85x10^{-14}\right)(0.407)}{\left(1.6x10^{-19}\right)\left(10^{17}\right)}\right]^{1/2}$$

or

$$x_{dT} = 1.026x10^{-5}\ cm$$

n^+ poly on $n \Rightarrow \phi_{ms} = -0.32\ V$

We have

$$\left|Q'_{SD}(\max)\right| = \left(1.6x10^{-19}\right)\left(10^{17}\right)\left(1.026x10^{-5}\right)$$

or

$$\left|Q'_{SD}(\max)\right| = 1.64x10^{-7}\ C/cm^2$$

Now

$$V_{TP} = \left[-1.64x10^{-7} - \left(1.6x10^{-19}\right)\left(5x10^{10}\right)\right]$$

$$\times\frac{\left(80x10^{-8}\right)}{(3.9)\left(8.85x10^{-14}\right)} - 0.32 - 2(0.407)$$

or

$V_{TP} = -1.53\ V$, Enhancement PMOS

(b)
For $V_T = 0$, shift threshold voltage in positive direction, so implant acceptor ions

$$\Delta V_T = \frac{eD_I}{C_{ox}} \Rightarrow D_I = \frac{(\Delta V_T)C_{ox}}{e}$$

so

$$D_I = \frac{(1.53)(3.9)\left(8.85x10^{-14}\right)}{\left(80x10^{-8}\right)\left(1.6x10^{-19}\right)}$$

or

$$D_I = 4.13x10^{12}\ cm^{-2}$$

12.38

Shift in negative direction means implanting donor ions. We have

$$\Delta V_T = \frac{eD_I}{C_{ox}}$$

where

$$C_{ox} = \frac{\in_{ox}}{t_{ox}} = \frac{(3.9)\left(8.85x10^{-14}\right)}{400x10^{-8}}$$

or

$$C_{ox} = 8.63x10^{-8}\ F/cm^2$$

Now

$$D_I = \frac{C_{ox}(\Delta V_T)}{e} = \frac{(8.63x10^{-8})(1.4)}{1.6x10^{-19}}$$

or

$$\underline{D_I = 7.55x10^{11}\ cm^{-2}}$$

12.39

The areal density of generated holes is

$$= (8x10^{12})(10^5)(750x10^{-8}) = 6x10^{12}\ cm^{-2}$$

The equivalent surface charge trapped is

$$= (0.10)(6x10^{12}) = 6x10^{11}\ cm^{-2}$$

Then

$$\Delta V_T = -\frac{Q'_{SS}}{C_{ox}} = -\frac{(6x10^{11})(1.6x10^{-19})}{(3.9)(8.85x10^{-14})}(750x10^{-8})$$

or

$$\underline{\Delta V_T = -2.09\ V}$$

12.40

The areal density of generated holes is $6x10^{12}\ cm^{-2}$. Now

$$C_{ox} = \frac{\in_{ox}}{t_{ox}} = \frac{(3.9)(8.85x10^{-14})}{750x10^{-8}}$$

or

$$C_{ox} = 4.6x10^{-8}\ F/cm^2$$

Then

$$\Delta V_T = -\frac{Q'_{SS}}{C_{ox}} = -\frac{(6x10^{12})(x)(1.6x10^{-19})}{4.6x10^{-8}}$$

For $\Delta V_T = -0.50\ V$

Where the parameter x is the maximum fraction of holes that can be trapped. Then we find

$$\underline{x = 0.024 \Rightarrow 2.4\%}$$

12.41

We have the areal density of generated holes as $= (g)(\gamma)(t_{ox})$ where g is the generation rate and γ is the dose. The equivalent charge trapped is $= xg\gamma t_{ox}$.

Then

$$\Delta V_T = -\frac{Q'_{SS}}{C_{ox}} = -\frac{exg\gamma t_{ox}}{(\in_{ox}/t_{ox})} = -exg\gamma(t_{ox})^2$$

so that

$$\underline{\Delta V_T \propto -(t_{ox})^2}$$

Chapter 13

Problem Solutions

13.1
Sketch

13.2
Sketch

13.3
p-channel JFET – Silicon
(a)

$$V_{PO} = \frac{ea^2 N_a}{2 \in} = \frac{(1.6x10^{-19})(0.5x10^{-4})^2(3x10^{16})}{2(11.7)(8.85x10^{-14})}$$

or

$$\underline{V_{PO} = 5.79\ V}$$

Now

$$V_{bi} = (0.0259)\ln\left[\frac{(5x10^{18})(3x10^{16})}{(1.5x10^{10})^2}\right]$$

or

$$V_{bi} = 0.884\ V$$

so

$$V_P = V_{PO} - V_{bi} = 5.79 - 0.884$$

or

$$\underline{V_P = 4.91\ V}$$

(b)

$$a - h = a - \left[\frac{2 \in (V_{bi} - V_{DS} + V_{GS})}{eN_a}\right]^{1/2}$$

(i)
For $V_{GS} = 1\ V$, $V_{DS} = 0$
Then

$$a - h = 0.5x10^{-4} - \left[\frac{2(11.7)(8.85x10^{-14})(0.884 + 1 - V_{DS})}{(1.6x10^{-19})(3x10^{16})}\right]^{1/2}$$

or

$$a - h = 0.5x10^{-4} - [(4.31x10^{-10})(1.884 - V_{DS})]^{1/2}$$

or

$$\underline{a - h = 0.215\ \mu m}$$

(ii) For $V_{GS} = 1\ V$, $V_{DS} = -2.5\ V$

$$\underline{a - h = 0.0653\ \mu m}$$

(iii) For $V_{GS} = 1\ V$, $V_{DS} = -5\ V$

$$\underline{a - h = -0.045\ \mu m}$$

which implies no undepleted region.

13.4
p-channel JFET – GaAs
(a)

$$V_{PO} = \frac{2a^2 N_a}{2 \in} = \frac{2(0.5x10^{-4})^2(3x10^{16})}{2(13.1)(8.85x10^{-14})}$$

or

$$\underline{V_{PO} = 5.18\ V}$$

Now

$$V_{bi} = (0.0259)\ln\left[\frac{(5x10^{18})(3x10^{16})}{(1.8x10^{6})^2}\right]$$

or

$$V_{bi} = 1.35\ V$$

so

$$V_P = V_{PO} - V_{bi} = 5.18 - 1.35$$

or

$$\underline{V_P = 3.83\ V}$$

(b)

$$a - h = a - \left[\frac{2 \in (V_{bi} - V_{DS} + V_{GS})}{eN_a}\right]^{1/2}$$

(i) For $V_{GS} = 1\ V$, $V_{DS} = 0$
Then

$$a - h = 0.5x10^{-4} - \left[\frac{2(13.1)(8.85x10^{-14})(1.35 + 1 - V_{DS})}{(1.6x10^{-19})(3x10^{16})}\right]^{1/2}$$

or

$$a - h = 0.5x10^{-4} - [(4.83x10^{-10})(2.35 - V_{DS})]^{1/2}$$

which yields

$$\underline{a - h = 0.163\ \mu m}$$

(ii) For $V_{GS} = 1\ V$, $V_{DS} = -2.5\ V$

$$\underline{a - h = 0.016\ \mu m}$$

(iii) For $V_{GS} = 1\ V$, $V_{DS} = -5\ V$

$$\underline{a - h = -0.096\ \mu m}$$

which implies no undepleted region.

13.5

(a) $V_{PO} = \dfrac{ea^2 N_d}{2\in}$

$= \dfrac{(1.6x10^{-19})(0.5x10^{-4})^2(8x10^{16})}{2(11.7)(8.85x10^{-14})}$

or

$\underline{V_{PO} = 15.5\ V}$

(b)

$$a - h = a - \left[\frac{2\in(V_{bi} - V_{GS})}{eN_d}\right]^{1/2}$$

so

$$0.2x10^{-4} = 0.5x10^{-4} - \left[\frac{2(11.7)(8.85x10^{-14})(V_{bi} - V_{GS})}{(1.6x10^{-19})(8x10^{16})}\right]^{1/2}$$

or

$9x10^{-10} = 1.618x10^{-10}(V_{bi} - V_{GS})$

which yields

$V_{bi} - V_{GS} = 5.56\ V$

Now

$$V_{bi} = (0.0259)\ln\left[\frac{(3x10^{18})(8x10^{16})}{(1.5x10^{10})^2}\right]$$

or

$V_{bi} = 0.896\ V$

Then

$V_{GS} = 0.897 - 5.56 \Rightarrow \underline{V_{GS} = -4.66\ V}$

13.6

For GaAs:

(a)

$V_{PO} = \dfrac{ea^2 N_d}{2\in}$

$= \dfrac{(1.6x10^{-19})(0.5x10^{-4})^2(8x10^{16})}{2(13.1)(8.85x10^{-14})}$

or

$\underline{V_{PO} = 13.8\ V}$

(b)

$$a - h = a - \left[\frac{2\in(V_{bi} - V_{GS})}{eN_d}\right]^{1/2}$$

$$0.2x10^{-4} = 0.5x10^{-4} - \left[\frac{2(13.1)(8.85x10^{-14})(V_{bi} - V_{GS})}{(1.6x10^{-19})(8x10^{16})}\right]^{1/2}$$

which can be written as

$9x10^{-10} = 1.811x10^{-10}(V_{bi} - V_{GS})$

or

$V_{bi} - V_{GS} = 4.97\ V$

Now

$$V_{bi} = (0.0259)\ln\left[\frac{(3x10^{18})(8x10^{16})}{(1.8x10^{6})^2}\right]$$

or

$V_{bi} = 1.36\ V$

Then

$V_{GS} = V_{bi} - 4.97 = 1.36 - 4.97$

or

$\underline{V_{GS} = -3.61\ V}$

13.7

(a) $V_{PO} = \dfrac{ea^2 N_a}{2\in}$

$= \dfrac{(1.6x10^{-19})(0.3x10^{-4})^2(3x10^{16})}{2(13.1)(8.85x10^{-14})}$

or

$\underline{V_{PO} = 1.863\ V}$

$$V_{bi} = (0.0259)\ln\left[\frac{(5x10^{18})(3x10^{16})}{(1.8x10^{6})^2}\right]$$

or

$V_{bi} = 1.352\ V$

Then

$V_P = V_{PO} - V_{bi} = 1.863 - 1.352$

or

$\underline{V_P = 0.511\ V}$

(b) (i)

$$a - h = a - \left[\frac{2\in(V_{bi} + V_{GS})}{eN_a}\right]^{1/2}$$

or

$$a - h = (0.3x10^{-4}) - \left[\frac{2(13.1)(8.85x10^{-14})(1.352)}{(1.6x10^{-19})(3x10^{16})}\right]^{1/2}$$

which yields

$$\underline{a-h=4.45x10^{-6}\ cm}$$

(ii)

$$a-h=\left(0.3x10^{-4}\right)-\left[\frac{2(13.1)\left(8.85x10^{-14}\right)(1.351+1)}{\left(1.6x10^{-19}\right)\left(3x10^{16}\right)}\right]^{1/2}$$

which yields

$$\underline{a-h=-3.7x10^{-6}\ cm}$$

which implies no undepleted region.

13.8

(a) n-channel JFET – Silicon

$$V_{PO}=\frac{ea^2N_d}{2\in}=\frac{\left(1.6x10^{-19}\right)\left(0.35x10^{-4}\right)^2\left(4x10^{16}\right)}{2(11.7)\left(8.85x10^{-14}\right)}$$

or

$$\underline{V_{PO}=3.79\ V}$$

Now

$$V_{bi}=(0.0259)\ln\left[\frac{\left(5x10^{18}\right)\left(4x10^{16}\right)}{\left(1.5x10^{10}\right)^2}\right]$$

or

$$V_{bi}=0.892\ V$$

so that

$$V_P=V_{bi}-V_{PO}=0.892-3.79$$

or

$$\underline{V_P=-2.90\ V}$$

(b)

$$a-h=a-\left[\frac{2\in\left(V_{bi}+V_{DS}-V_{GS}\right)}{eN_d}\right]^{1/2}$$

We have

$$a-h=0.35x10^{-4}-\left[\frac{2(11.7)\left(8.85x10^{-14}\right)\left(0.892+V_{DS}-V_{GS}\right)}{\left(1.6x10^{-19}\right)\left(4x10^{16}\right)}\right]^{1/2}$$

or

$$a-h=0.35x10^{-4}-\left[\left(3.24x10^{-10}\right)\left(0.892+V_{DS}-V_{GS}\right)\right]^{1/2}$$

(i) For $V_{GS}=0$, $V_{DS}=1\ V$,

$$\underline{a-h=0.102\ \mu m}$$

(ii) For $V_{GS}=-1\ V$, $V_{DS}=1\ V$,

$$\underline{a-h=0.044\ \mu m}$$

(iii) For $V_{GS}=-1\ V$, $V_{DS}=2\ V$,

$$\underline{a-h=-0.0051\ \mu m}$$

which implies no undepleted region

13.9

$$V_{bi}=(0.0259)\ln\left(\frac{\left(5x10^{18}\right)\left(4x10^{16}\right)}{\left(1.8x10^{6}\right)^2}\right)$$

or

$$V_{bi}=1.359\ V$$

$$a-h=a-\left[\frac{2\in\left(V_{bi}+V_{DS}-V_{GS}\right)}{eN_D}\right]^{1/2}$$

or

$$a-h=0.35x10^{-4}-\left[\frac{2(13.1)\left(8.85x10^{-14}\right)\left(1.359+V_{DS}-V_{GS}\right)}{\left(1.6x10^{-19}\right)\left(4x10^{16}\right)}\right]^{1/2}$$

We want $a-h=0.05x10^{-4}\ cm$,

Then

$$0.05x10^{-4}=0.35x10^{-4}-\left[\left(3.623x10^{-10}\right)\left(1.359+V_{DS}-V_{GS}\right)\right]^{1/2}$$

(a)

For $V_{DS}=0$, we find

$$\underline{V_{GS}=-1.125\ V}$$

(b)

For $V_{DS}=1\ V$, we find

$$\underline{V_{GS}=-0.125\ V}$$

13.10

(a)

$$I_{P1}=\frac{\mu_n\left(eN_d\right)^2Wa^3}{6\in L}$$

$$=\frac{(1000)\left[\left(1.6x10^{-19}\right)\left(10^{16}\right)\right]^2}{6(11.7)\left(8.85x10^{-14}\right)}\times\frac{\left(400x10^{-4}\right)\left(0.5x10^{-4}\right)^3}{\left(20x10^{-4}\right)}$$

or

$$\underline{I_{P1}=1.03\ mA}$$

(b)

$$V_{PO}=\frac{ea^2N_d}{2\in}$$

$$= \left[\frac{(1.6x10^{-19})(0.5x10^{-4})^2(10^{16})}{2(11.7)(8.85x10^{-14})}\right]$$

or

$$\underline{V_{PO} = 1.93\ V}$$

Also

$$V_{bi} = (0.0259)\ln\left[\frac{(10^{19})(10^{16})}{(1.5x10^{10})^2}\right]$$

or

$$V_{bi} = 0.874\ V$$

Now

$$V_{DS}(sat) = V_{PO} - (V_{bi} - V_{GS})$$
$$= 1.93 - 0.874 + V_{GS}$$

or

$$V_{DS}(sat) = 1.06 + V_{GS}$$

We have

$$V_P = V_{bi} - V_{PO} = 0.874 - 1.93$$

or

$$\underline{V_P = -1.06\ V}$$

Then

(i) $V_{GS} = 0 \Rightarrow \underline{V_{DS}(sat) = 1.06\ V}$

(ii) $V_{GS} = \frac{1}{4}V_P = -0.265\ V \Rightarrow$
$\underline{V_{DS}(sat) = 0.795\ V}$

(iii) $V_{GS} = \frac{1}{2}V_P = -0.53\ V \Rightarrow$
$\underline{V_{DS}(sat) = 0.53\ V}$

(iv) $V_{GS} = \frac{3}{4}V_P = -0.795\ V \Rightarrow$
$\underline{V_{DS}(sat) = 0.265\ V}$

(c)

$$I_{D1}(sat)$$
$$= I_{P1}\left[1 - 3\left(\frac{V_{bi} - V_{GS}}{V_{PO}}\right)\left(1 - \frac{2}{3}\sqrt{\frac{V_{bi} - V_{GS}}{V_{PO}}}\right)\right]$$
$$= 1.03\left[1 - 3\left(\frac{0.874 - V_{GS}}{1.93}\right) \times \left(1 - \frac{2}{3}\sqrt{\frac{0.874 - V_{GS}}{1.93}}\right)\right]$$

(i) For $V_{GS} = 0 \Rightarrow \underline{I_{D1}(sat) = 0.258\ mA}$

(ii) For $V_{GS} = -0.265\ V \Rightarrow$
$\underline{I_{D1}(sat) = 0.140\ mA}$

(iii) For $V_{GS} = -0.53\ V \Rightarrow$
$\underline{I_{D1}(sat) = 0.061\ mA}$

(iv) For $V_{GS} = -0.795\ V \Rightarrow$
$\underline{I_{D1}(sat) = 0.0145\ mA}$

13.11

$$g_d = G_{O1}\left[1 - \left(\frac{V_{bi} - V_{GS}}{V_{PO}}\right)^{1/2}\right]$$

where

$$G_{O1} = \frac{3I_{P1}}{V_{PO}} = \frac{3(1.03x10^{-3})}{1.93} = 1.60x10^{-3}$$

or

$$G_{O1} = 1.60\ mS$$

Then

$\underline{V_{GS}}$	$\underline{[(V_{bi} - V_{GS})/V_{PO}]}$	$\underline{g_d(mS)}$
0	0.453	0.523
-0.265	0.590	0.371
-0.53	0.727	0.236
-0.795	0.945	0.112
-1.06	1.0	0

13.12

n-channel JFET – GaAs

(a)

$$G_{O1} = \frac{e\mu_n N_d Wa}{L}$$
$$= \frac{(1.6x10^{-19})(8000)(2x10^{16})(30x10^{-4})(0.35x10^{-4})}{10x10^{-4}}$$

or

$$\underline{G_{O1} = 2.69x10^{-3}\ S}$$

(b)

$$V_{DS}(sat) = V_{PO} - (V_{bi} - V_{GS})$$

We have

$$V_{PO} = \frac{ea^2 N_d}{2\in}$$
$$= \frac{(1.6x10^{-19})(0.35x10^{-4})^2(2x10^{16})}{2(13.1)(8.85x10^{-14})}$$

or

$$\underline{V_{PO} = 1.69\ V}$$

We find

$$V_{bi} = (0.0259)\ln\left[\frac{(5x10^{18})(2x10^{16})}{(1.8x10^{6})^{2}}\right]$$

or

$$V_{bi} = 1.34\ V$$

Then

$$V_P = V_{bi} - V_{PO} = 1.34 - 1.69$$

or

$$\underline{V_P = -0.35\ V}$$

We then obtain

$$V_{DS}(sat) = 1.69 - (1.34 - V_{GS}) = 0.35 + V_{GS}$$

For $V_{GS} = 0 \Rightarrow$ $\underline{V_{DS}(sat) = 0.35\ V}$

For $V_{GS} = \frac{1}{2}V_P = -0.175\ V \Rightarrow$

$$\underline{V_{DS}(sat) = 0.175\ V}$$

(c)

$$I_{D1}(sat)$$

$$= I_{P1}\left[1 - 3\left(\frac{V_{bi} - V_{GS}}{V_{PO}}\right)\left(1 - \frac{2}{3}\sqrt{\frac{V_{bi} - V_{GS}}{V_{PO}}}\right)\right]$$

where

$$I_{P1} = \frac{\mu_n (eN_d)^2 Wa^3}{6 \in L}$$

$$= \frac{(8000)\left[(1.6x10^{-19})(2x10^{16})\right]^2}{6(13.1)(8.85x10^{-14})} \times \frac{(30x10^{-4})(0.35x10^{-4})^3}{(10x10^{-4})}$$

or

$$\underline{I_{P1} = 1.51\ mA}$$

Then

$$I_{D1}(sat) = 1.51\left[1 - 3\left(\frac{1.34 - V_{GS}}{1.69}\right) \times \left(1 - \frac{2}{3}\sqrt{\frac{1.34 - V_{GS}}{1.69}}\right)\right](mA)$$

For

$V_{GS} = 0 \Rightarrow$ $\underline{I_{D1}(sat) = 0.0504\ mA}$

and for

$V_{GS} = -0.175\ V \Rightarrow$ $\underline{I_{D1}(sat) = 0.0123\ mA}$

13.13

$$g_{mS} = \frac{3I_{P1}}{V_{PO}}\left(1 - \sqrt{\frac{V_{bi} - V_{GS}}{V_{PO}}}\right)$$

We have

$$I_{P1} = 1.03\ mA\ , V_{PO} = 1.93\ V\ , V_{bi} = 0.874\ V$$

The maximum transconductance occurs when $V_{GS} = 0$

Then

$$g_{mS}(\max) = \frac{3(1.03)}{1.93}\left(1 - \sqrt{\frac{0.874}{1.93}}\right)$$

or

$$g_{mS} = 0.524\ mS$$

For $W = 400\ \mu m$,

We have

$$g_{mS}(\max) = \frac{0.524\ mS}{400x10^{-4}\ cm}$$

or

$$\underline{g_{mS} = 13.1\ mS/cm = 1.31\ mS/mm}$$

13.14

The maximum transconductance occurs for $V_{GS} = 0$, so we have

(a)

$$g_{mS}(\max) = \frac{3I_{P1}}{V_{PO}}\left(1 - \sqrt{\frac{V_{bi}}{V_{PO}}}\right) = G_{O1}\left(1 - \sqrt{\frac{V_{bi}}{V_{PO}}}\right)$$

We found

$$G_{O1} = 2.69\ mS\ , V_{bi} = 1.34\ V\ , V_{PO} = 1.69\ V$$

Then

$$g_{mS}(\max) = (2.69)\left(1 - \sqrt{\frac{1.34}{1.69}}\right)$$

or

$$\underline{g_{mS}(\max) = 0.295\ mS}$$

This is for a channel length of $L = 10\ \mu m$.

(b)

If the channel length is reduced to $L = 2\ \mu m$, then

$$g_{mS}(\max) = (0.295)\left(\frac{10}{2}\right) \Rightarrow$$

$$\underline{g_{mS}(\max) = 1.48\ mS}$$

13.15
n-channel MESFET – GaAs
(a)

$$V_{PO} = \frac{ea^2 N_d}{2\in}$$

$$= \frac{(1.6x10^{-19})(0.5x10^{-4})^2(1.5x10^{16})}{2(13.1)(8.85x10^{-14})}$$

or

$$V_{PO} = 2.59\ V$$

Now

$$V_{bi} = \phi_{Bn} - \phi_n$$

where

$$\phi_n = V_t \ln\left(\frac{N_C}{N_d}\right) = (0.0259)\ln\left(\frac{4.7x10^{17}}{1.5x10^{16}}\right)$$

or

$$\phi_n = 0.0892\ V$$

so that

$$V_{bi} = 0.90 - 0.0892 = 0.811\ V$$

Then

$$V_T = V_{bi} - V_{PO} = 0.811 - 2.59$$

or

$$\underline{V_T = -1.78\ V}$$

(b)
If $V_T < 0$ for an n-channel device, the device is a depletion mode MESFET.

13.16
n-channel MESFET – GaAs
(a)
We want $V_T = +0.10\ V$
Then

$$V_T = V_{bi} - V_{PO} = \phi_{Bn} - \phi_n - V_{PO}$$

so

$$V_T = 0.10 = 0.89 - V_t \ln\left(\frac{N_C}{N_d}\right) - \frac{ea^2 N_d}{2\in}$$

which can be written as

$$(0.0259)\ln\left(\frac{4.7x10^{17}}{N_d}\right) + \frac{(1.6x10^{-19})(0.35x10^{-4})^2 N_d}{2(13.1)(8.85x10^{-14})} = 0.89 - 0.10$$

or

$$(0.0259)\ln\left(\frac{4.7x10^{17}}{N_d}\right) + (8.45x10^{17})N_d = 0.79$$

By trial and error

$$\underline{N_d = 8.1x10^{15}\ cm^{-3}}$$

(b)
At $T = 400K$,

$$N_C(400) = N_C(300)\cdot\left(\frac{400}{300}\right)^{3/2}$$

$$= (4.7x10^{17})(1.54)$$

or

$$N_C(400) = 7.24x10^{17}\ cm^{-3}$$

Also

$$V_t = (0.0259)\left(\frac{400}{300}\right) = 0.03453$$

Then

$$V_T = 0.89 - (0.03453)\ln\left(\frac{7.24x10^{17}}{8.1x10^{15}}\right) - (8.45x10^{-17})(8.1x10^{15})$$

which becomes

$$\underline{V_T = +0.051\ V}$$

13.17
We have

$$a - h = a - \left[\frac{2\in(V_{bi} + V_{DS} - V_{GS})}{eN_d}\right]^{1/2}$$

where

$$V_{bi} = \phi_{Bn} - \phi_n$$

Now

$$\phi_n = (0.0259)\ln\left(\frac{4.7x10^{17}}{5x10^{16}}\right) = 0.058\ V$$

Then

$$V_{bi} = 0.80 - 0.058 = 0.742\ V$$

For $V_{GS} = 0.5\ V$,

$$a - h = (0.8x10^{-4}) - \left[\frac{2(13.1)(8.85x10^{-14})(0.742 + V_{DS} - 0.5)}{(1.6x10^{-19})(5x10^{16})}\right]^{1/2}$$

or

$$a - h = (0.80x10^{-4}) - \left[(2.898x10^{-10})(0.242 + V_{DS})\right]^{1/2}$$

Then

$V_{DS}(V)$	$a-h\ (\mu m)$
0	0.716
1	0.610
2	0.545
5	0.410

13.18

$V_T = V_{bi} - V_{PO} = \phi_{Bn} - \phi_n - V_{PO}$

We want

$V_T = 0 \Rightarrow \phi_n + V_{PO} = \phi_{Bn}$

Device 1: $N_d = 3x10^{16}\ cm^{-3}$

Then

$$\phi_n = (0.0259)\ln\left(\frac{4.7x10^{17}}{3x10^{16}}\right) = 0.0713\ V$$

so that

$V_{PO} = 0.89 - 0.0713 = 0.8187\ V$

Now

$$V_{PO} = \frac{ea^2 N_d}{2\in} \Rightarrow a = \left[\frac{2\in V_{PO}}{eN_d}\right]^{1/2}$$

$$= \left[\frac{2(13.1)(8.85x10^{-14})(0.8187)}{(1.6x10^{-19})(3x10^{16})}\right]^{1/2}$$

or

$\underline{a = 0.199\ \mu m}$

Device 2: $N_d = 3x10^{17}\ cm^{-3}$

Then

$$\phi_n = (0.0259)\ln\left(\frac{4.7x10^{17}}{3x10^{17}}\right) = 0.0116\ V$$

so that

$V_{PO} = 0.89 - 0.0116 = 0.8784\ V$

Now

$$a = \left[\frac{2\in V_{PO}}{eN_d}\right]^{1/2}$$

$$= \left[\frac{2(13.1)(8.85x10^{-14})(0.8784)}{(1.6x10^{-19})(3x10^{17})}\right]^{1/2}$$

or

$\underline{a = 0.0651\ \mu m}$

13.19

$V_T = V_{bi} - V_{PO} = \phi_{Bn} - \phi_n - V_{PO}$

We want $V_T = 0.5\ V$, so

$0.5 = 0.85 - \phi_n - V_{PO}$

Now

$$\phi_n = (0.0259)\ln\left(\frac{4.7x10^{17}}{N_d}\right)$$

and

$$V_{PO} = \frac{ea^2 N_d}{2\in}$$

$$= \frac{(1.6x10^{-19})(0.25x10^{-4})^2 N_d}{2(13.1)(8.85x10^{-14})}$$

or

$V_{PO} = (4.31x10^{-17})N_d$

Then

$$0.5 = 0.85 - (0.0259)\ln\left(\frac{4.7x10^{17}}{N_d}\right) - (4.31x10^{-17})N_d$$

By trial and error, we find

$\underline{N_d = 5.45x10^{15}\ cm^{-3}}$

13.20

n-channel MESFET – silicon

(a) For a gold contact, $\phi_{Bn} = 0.82\ V$.

We find

$$\phi_n = (0.0259)\ln\left(\frac{2.8x10^{19}}{10^{16}}\right) = 0.206\ V$$

and

$V_{bi} = \phi_{Bn} - \phi_n = 0.82 - 0.206 = 0.614\ V$

With $V_{DS} = 0$, $V_{GS} = 0.35\ V$

We find

$$a - h = 0.075x10^{-4} = a - \left[\frac{2\in(V_{bi} - V_{GS})}{eN_d}\right]^{1/2}$$

so that

$$a = 0.075x10^{-4} + \left[\frac{2(11.7)(8.85x10^{-14})(0.614-0.35)}{(1.6x10^{-19})(10^{16})}\right]^{1/2}$$

or

$a = 0.26\ \mu m$

Now

$$V_T = V_{bi} - V_{PO} = 0.614 - \frac{ea^2 N_d}{2\in}$$

or

$$V_T = 0.614 - \frac{\left(1.6x10^{-19}\right)\left(0.26x10^{-4}\right)^2\left(10^{16}\right)}{2(11.7)\left(8.85x10^{-14}\right)}$$

We obtain

$$\underline{V_T = 0.092\ V}$$

(b)

$$V_{DS}(sat) = V_{PO} - \left(V_{bi} - V_{GS}\right)$$
$$= \left(V_{bi} - V_T\right) - \left(V_{bi} - V_{GS}\right) = V_{GS} - V_T$$

Now

$$V_{DS}(sat) = 0.35 - 0.092$$

or

$$\underline{V_{DS}(sat) = 0.258\ V}$$

13.21

(a) n-channel MESFET - silicon

$$V_{bi} = \phi_{Bn} - \phi_n$$

and

$$\phi_n = (0.0259)\ln\left(\frac{2.8x10^{19}}{2x10^{16}}\right) = 0.188\ V$$

so

$$V_{bi} = 0.80 - 0.188 \Rightarrow \underline{V_{bi} = 0.612\ V}$$

Now

$$V_{PO} = \frac{ea^2 N_d}{2\in}$$
$$= \frac{\left(1.6x10^{-19}\right)\left(0.4x10^{-4}\right)^2\left(2x10^{16}\right)}{2(11.7)\left(8.85x10^{-14}\right)}$$

or

$$\underline{V_{PO} = 2.47\ V}$$

We find

$$V_T = V_{bi} - V_{PO} = 0.612 - 2.47$$

or

$$\underline{V_T = -1.86\ V}$$

and

$$V_{DS}(sat) = V_{PO} - \left(V_{bi} - V_{GS}\right)$$
$$= 2.47 - (0.612 - (-1))$$

or

$$\underline{V_{DS}(sat) = 0.858\ V}$$

(b)

For $V_{PO} = 4.5\ V$, additional donor atoms must be added.

We have

$$V_{PO} = \frac{ea^2 N_d}{2\in} \Rightarrow N_d = \frac{2\in V_{PO}}{ea^2}$$

so that

$$N_d = \frac{2(11.7)\left(8.85x10^{-14}\right)(4.5)}{\left(1.6x10^{-19}\right)\left(0.4x10^{-4}\right)^2}$$

or

$$\underline{N_d = 3.64x10^{16}\ cm^{-3}}$$

which means that

$$\Delta N_d = 3.64x10^{16} - 2x10^{16}$$

or

$$\underline{\Delta N_d = 1.64x10^{16}\ cm^{-3}}$$

Donors must be added

Then

$$\phi_n = (0.0259)\ln\left(\frac{2.8x10^{19}}{3.64x10^{16}}\right) = 0.172\ V$$

so that

$$V_{bi} = 0.80 - 0.172 = 0.628\ V$$

We find

$$V_T = V_{bi} - V_{PO} = 0.628 - 4.5$$

or

$$\underline{V_T = -3.87\ V}$$

Also

$$V_{DS}(sat) = V_{PO} - \left(V_{bi} - V_{GS}\right)$$
$$= 4.5 - (0.628 - (-1))$$

or

$$\underline{V_{DS}(sat) = 2.87\ V}$$

13.22

(a) $$k_n = \frac{\mu_n \in W}{2aL}$$
$$= \frac{(7800)(13.1)\left(8.85x10^{-14}\right)\left(20x10^{-4}\right)}{2\left(0.30x10^{-4}\right)\left(1.2x10^{-4}\right)}$$

or

$$\underline{k_n = 2.51\ mA/V^2}$$

(b)

$$V_{DS}(sat) = V_{PO} - \left(V_{bi} - V_{GS}\right) = V_{GS} - V_T$$

So for $V_{GS} = 1.5V_T \Rightarrow V_{DS}(sat) = (0.5)(0.12)$

Or

$$\underline{V_{DS}(sat) = 0.06\ V}$$

and for $V_{GS} = 2V_T \Rightarrow V_{DS}(sat) = (1)(0.12)$

or

$$\underline{V_{DS}(sat) = 0.12\ V}$$

(c)

$$I_{D1}(sat) = k_n\left(V_{GS} - V_T\right)^2$$

For $V_{GS} = 1.5V_T \Rightarrow I_{D1}(sat) = (2.51)(0.06)^2$
Or
$$I_{D1}(sat) = 9.04\ \mu A$$
and for $V_{GS} = 2V_T \Rightarrow I_{D1}(sat) = (2.51)(0.12)^2$
or
$$I_{D1}(sat) = 36.1\ \mu A$$

13.23
(a) We have
$$g_m = 2k_n(V_{GS} - V_T)$$
so that
$$1.75x10^{-3} = 2k_n(0.50 - 0.25)$$
which gives
$$k_n = 3.5x10^{-3}\ A/V^2 = \frac{\mu_n \in W}{2aL}$$
We obtain
$$W = \frac{(3.5x10^{-3})(2)(0.35x10^{-4})(10^{-4})}{(8000)(13.1)(8.85x10^{-14})}$$
or
$$W = 26.4\ \mu m$$
(b)
$$I_{D1}(sat) = k_n(V_{GS} - V_T)^2$$
For $V_{GS} = 0.4\ V$,
$$I_{D1}(sat) = (3.5x10^{-3})(0.4 - 0.25)^2$$
or
$$I_{D1}(sat) = 78.8\ \mu A$$
For $V_{GS} = 0.65\ V$,
$$I_{D1}(sat) = (3.5x10^{-3})(0.65 - 0.25)^2$$
or
$$I_{D1}(sat) = 0.56\ mA$$

13.24
Computer plot

13.25
Computer plot

13.26

We have $L' = L - \frac{1}{2}\Delta L$
Or
$$\frac{L'}{L} = 0.90 = 1 - \frac{1}{2}\cdot\frac{\Delta L}{L}$$
We have
$$\Delta L = \left[\frac{2\in(V_{DS} - V_{DS}(sat))}{eN_d}\right]^{1/2}$$
and
For $V_{GS} = 0$, $V_{DS}(sat) = V_{PO} - V_{bi}$
We find
$$V_{PO} = \frac{ea^2N_d}{2\in}$$
$$= \frac{(1.6x10^{-19})(0.4x10^{-4})^2(3x10^{16})}{2(11.7)(8.85x10^{-14})}$$
or
$$V_{PO} = 3.71\ V$$
and
$$V_{bi} = (0.0259)\ln\left[\frac{(10^{19})(3x10^{16})}{(1.5x10^{10})^2}\right]$$
or
$$V_{bi} = 0.902\ V$$
so that
$$V_{DS}(sat) = 3.71 - 0.902 = 2.81\ V$$
Then
$$\Delta L = \left[\frac{2(11.7)(8.85x10^{-14})(5 - 2.81)}{(1.6x10^{-19})(3x10^{16})}\right]^{1/2}$$
or
$$\Delta L = 0.307\ \mu m$$
Now
$$\frac{L'}{L} = 0.90 = 1 - \frac{1}{2}\cdot\frac{\Delta L}{L}$$
or
$$\frac{1}{2}\cdot\frac{\Delta L}{L} = 1 - 0.9 = 0.10$$
so
$$L = \frac{\Delta L}{2(0.10)} = \frac{0.307x10^{-4}}{2(0.10)}$$
or
$$L = 1.54\ \mu m$$

13.27

We have that $I'_{D1} = I_{D1}\left(\frac{L}{L-(1/2)\Delta L}\right)$

Assuming that we are in the saturation region, then $I'_{D1} = I'_{D1}(sat)$ and $I_{D1} = I_{D1}(sat)$. We can write

$$I'_{D1}(sat) = I_{D1}(sat)\cdot\frac{1}{1-\frac{1}{2}\cdot\frac{\Delta L}{L}}$$

If $\Delta L << L$, then

$$I'_{D1}(sat) = I_{D1}(sat)\left[1+\frac{1}{2}\cdot\frac{\Delta L}{L}\right]$$

We have that

$$\Delta L = \left[\frac{2\in(V_{DS}-V_{DS}(sat))}{eN_d}\right]^{1/2}$$

$$= \left[\frac{2\in V_{DS}}{eN_d}\left(1-\frac{V_{DS}(sat)}{V_{DS}}\right)\right]^{1/2}$$

which can be written as

$$\Delta L = V_{DS}\left[\frac{2\in}{eN_dV_{DS}}\left(1-\frac{V_{DS}(sat)}{V_{DS}}\right)\right]^{1/2}$$

If we write

$$I'_{D1}(sat) = I_{D1}(sat)(1+\lambda V_{DS})$$

then by comparing equations, we have

$$\lambda = \frac{1}{2L}\left[\frac{2\in}{eN_dV_{DS}}\left(1-\frac{V_{DS}(sat)}{V_{DS}}\right)\right]^{1/2}$$

The parameter is not independent of V_{DS}. Define

$x = \frac{V_{DS}}{V_{DS}(sat)}$ and consider the function

$f = \frac{1}{x}\left(1-\frac{1}{x}\right)$ which is directly proportional to λ. We find that

x	$f(x)$
1..5	0.222
1.75	0.245
2.0	0.250
2.25	0.247
2.50	0.240
2.75	0.231
3.0	0.222

So that λ is nearly a constant.

13.28

(a) Saturation occurs when $\mathrm{E} = 1x10^4\ V/cm$

As a first approximation, let

$$\mathrm{E} = \frac{V_{DS}}{L}$$

Then

$$V_{DS} = \mathrm{E}\cdot L = (1x10^4)(2x10^{-4})$$

or

$$\underline{V_{DS} = 2\ V}$$

(b)

We have that

$$h_2 = h_{sat} = \left[\frac{2\in(V_{bi}+V_{DS}-V_{GS})}{eN_d}\right]^{1/2}$$

and

$$V_{bi} = (0.0259)\ln\left[\frac{(5x10^{18})(4x10^{16})}{(1.5x10^{10})^2}\right]$$

or

$$V_{bi} = 0.892\ V$$

For $V_{GS} = 0$, we obtain

$$h_{sat} = \left[\frac{2(11.7)(8.85x10^{-14})(0.892+2)}{(1.6x10^{-19})(4x10^{16})}\right]^{1/2}$$

or

$$\underline{h_{sat} = 0.306\ \mu m}$$

(c)

We then find

$$I_{D1}(sat) = eN_dv_{sat}(a-h_{sat})W$$

$$= (1.6x10^{-19})(4x10^{16})(10^7)(0.50-0.306)$$

$$\times(10^{-4})(30x10^{-4})$$

or

$$\underline{I_{D1}(sat) = 3.72\ mA}$$

(d)

For $V_{GS} = 0$, we have

$$I_{D1}(sat) = I_{P1}\left[1-3\left(\frac{V_{bi}}{V_{PO}}\right)\left(1-\frac{2}{3}\sqrt{\frac{V_{bi}}{V_{PO}}}\right)\right]$$

Now

$$I_{P1} = \frac{\mu_n(eN_d)^2Wa^3}{6\in L}$$

$$= \frac{(1000)\left[\left(1.6x10^{-19}\right)\left(4x10^{16}\right)\right]^2}{6(11.7)\left(8.85x10^{-14}\right)}$$

$$\times \frac{\left(30x10^{-4}\right)\left(0.5x10^{-4}\right)^3}{\left(2x10^{-4}\right)}$$

or

$$I_{P1} = 12.4\ mA$$

Also

$$V_{PO} = \frac{ea^2 N_d}{2\in}$$

$$= \frac{\left(1.6x10^{-19}\right)\left(0.5x10^{-4}\right)^2\left(4x10^{16}\right)}{2(11.7)\left(8.85x10^{-14}\right)}$$

or

$$V_{PO} = 7.73\ V$$

Then

$$I_{D1}(sat) = 12.4\left[1 - 3\left(\frac{0.892}{7.73}\right)\left(1 - \frac{2}{3}\sqrt{\frac{0.892}{7.73}}\right)\right]$$

or

$$\underline{I_{D1}(sat) = 9.08\ mA}$$

13.29

(a) If $L = 1\ \mu m$, then saturation will occur when

$$V_{DS} = \mathrm{E} \cdot L = \left(10^4\right)\left(1x10^{-4}\right) = 1\ V$$

We find

$$h_2 = h_{sat} = \left[\frac{2\in\left(V_{bi} + V_{DS} - V_{GS}\right)}{eN_d}\right]^{1/2}$$

We have $V_{bi} = 0.892\ V$ and for $V_{GS} = 0$, we obtain

$$h_{sat} = \left[\frac{2(11.7)\left(8.85x10^{-14}\right)(0.892+1)}{\left(1.6x10^{-19}\right)\left(4x10^{16}\right)}\right]^{1/2}$$

or

$$h_{sat} = 0.247\ \mu m$$

Then

$$I_{D1}(sat) = eN_d v_{sat}\left(a - h_{sat}\right)W$$

$$= \left(1.6x10^{-19}\right)\left(4x10^{16}\right)\left(10^7\right)(0.50 - 0.247)$$

$$\times\left(10^{-4}\right)\left(30x10^{-4}\right)$$

or

$$\underline{I_{D1}(sat) = 4.86\ mA}$$

If velocity saturation did not occur, then from the previous problem, we would have

$$I_{D1}(sat) = 9.08\left(\frac{2}{1}\right) \Rightarrow \underline{I_{D1}(sat) = 18.2\ mA}$$

(b)
If velocity saturation occurs, then the relation $I_{D1}(sat) \propto (1/L)$ does not apply.

13.30

(a)

$$v = \mu_n \mathrm{E} = (8000)\left(5x10^3\right) = 4x10^7\ cm/s$$

Then

$$t_d = \frac{L}{v} = \frac{2x10^{-4}}{4x10^7} \Rightarrow$$

or

$$\underline{t_d = 5\ ps}$$

(b)
Assume $v_{sat} = 10^7\ cm/s$
Then

$$t_d = \frac{L}{v_{sat}} = \frac{2x10^{-4}}{10^7} \Rightarrow$$

$$\underline{t_d = 20\ ps}$$

13.31

(a) $v = \mu_n \mathrm{E} = (1000)\left(10^4\right) = 10^7\ cm/s$

$$t_d = \frac{L}{v} = \frac{2x10^{-4}}{10^7} \Rightarrow \underline{t_d = 20\ ps}$$

(b)
For $v_{sat} = 10^7\ cm/s$,

$$t_d = \frac{L}{v_{sat}} = \frac{2x10^{-4}}{10^7} \Rightarrow \underline{t_d = 20\ ps}$$

13.32

The reverse-bias current is dominated by the generation current. We have

$$V_P = V_{bi} - V_{PO}$$

We find

$$V_{bi} = (0.0259)\ln\left[\frac{\left(5x10^{18}\right)\left(3x10^{16}\right)}{\left(1.5x10^{10}\right)^2}\right]$$

or

$$V_{bi} = 0.884\ V$$

and

$$V_{PO} = \frac{ea^2 N_d}{2\in}$$

$$=\frac{\left(1.6x10^{-19}\right)\left(0.3x10^{-4}\right)^2\left(3x10^{16}\right)}{2(11.7)\left(8.85x10^{-14}\right)}$$

or

$$V_{PO}=2.09\ V$$

Then

$$V_P=0.884-2.09=-1.21=V_{GS}$$

Now

$$x_n=\left[\frac{2\in\left(V_{bi}+V_{DS}-V_{GS}\right)}{eN_d}\right]^{1/2}$$

$$=\left[\frac{2(11.7)\left(8.85x10^{-14}\right)\left(0.884+V_{DS}-(-1.21)\right)}{\left(1.6x10^{-19}\right)\left(3x10^{16}\right)}\right]^{1/2}$$

or

$$x_n=\left[\left(4.31x10^{-10}\right)\left(2.09+V_{DS}\right)\right]^{1/2}$$

(a)
For $V_{DS}=0\Rightarrow$, $x_n=0.30\ \mu m$
(b)
For $V_{DS}=1\ V\Rightarrow$, $x_n=0.365\ \mu m$
(c)
For $V_{DS}=5\ V\Rightarrow$ $x_n=0.553\ \mu m$

The depletion region volume is

$$Vol=(a)\left(\frac{L}{2}\right)(W)+\left(x_n\right)(2a)(W)$$

$$=\left(0.3x10^{-4}\right)\left(\frac{2.4x10^{-4}}{2}\right)\left(30x10^{-4}\right)$$
$$+\left(x_n\right)\left(0.6x10^{-4}\right)\left(30x10^{-4}\right)$$

or

$$Vol=10.8x10^{-12}+x_n\left(18x10^{-8}\right)$$

(a)
For $V_{DS}=0\Rightarrow$ $Vol=1.62x10^{-11}\ cm^3$
(b)
For $V_{DS}=1\ V\Rightarrow$ $Vol=1.74x10^{-11}\ cm^3$
(c)
For $V_{DS}=5\ V\Rightarrow$ $Vol=2.08x10^{-11}\ cm^3$

The generation current is

$$I_{DG}=e\left(\frac{n_i}{2\tau_O}\right)\cdot Vol=\frac{\left(1.6x10^{-19}\right)\left(1.5x10^{10}\right)}{2\left(5x10^{-8}\right)}\cdot Vol$$

or

$$I_{DG}=\left(2.4x10^{-2}\right)\cdot Vol$$

(a)
For $V_{DS}=0\Rightarrow$ $\underline{I_{DG}=0.39\ pA}$
(b)
For $V_{DS}=1\ V\Rightarrow$ $\underline{I_{DG}=0.42\ pA}$
(c)
For $V_{DS}=5\ V\Rightarrow$ $\underline{I_{DG}=0.50\ pA}$

13.33

(a) The ideal transconductance for $V_{GS}=0$ is

$$g_{mS}=G_{O1}\left(1-\sqrt{\frac{V_{bi}}{V_{PO}}}\right)$$

where

$$G_{O1}=\frac{e\mu_n N_d Wa}{L}$$

$$=\frac{\left(1.6x10^{-19}\right)(4500)\left(7x10^{16}\right)}{1.5x10^{-4}}$$
$$\times\left(5x10^{-4}\right)\left(0.3x10^{-4}\right)$$

or

$$G_{O1}=5.04\ mS$$

We find

$$V_{PO}=\frac{ea^2N_d}{2\in}$$

$$=\frac{\left(1.6x10^{-19}\right)\left(0.3x10^{-4}\right)^2\left(7x10^{16}\right)}{2(13.1)\left(8.85x10^{-14}\right)}$$

or

$$V_{PO}=4.35\ V$$

We have

$$\phi_n=(0.0259)\ln\left(\frac{4.7x10^{17}}{7x10^{16}}\right)=0.049\ V$$

so that

$$V_{bi}=\phi_{Bn}-\phi_n=0.89-0.049=0.841\ V$$

Then

$$g_{mS}=5.04\left(1-\sqrt{\frac{0.841}{4.35}}\right)$$

or

$$\underline{g_{mS}=2.82\ mS}$$

(b)
With a source resistance

$$g_m'=\frac{g_m}{1+g_mr_s}\Rightarrow\frac{g_m'}{g_m}=\frac{1}{1+g_mr_s}$$

For

$$\frac{g_m'}{g_m}=0.80=\frac{1}{1+(2.82)r_s}$$

which yields

$$\underline{r_s = 88.7\ \Omega}$$

(c)

$$r_s = \frac{\rho L}{A} = \frac{L}{\sigma A} = \frac{L}{(e\mu_n n)\left(0.3x10^{-4}\right)\left(5x10^{-4}\right)}$$

so

$$L = (88.7)\left(1.6x10^{-19}\right)(4500)\left(7x10^{16}\right) \times\left(0.3x10^{-4}\right)\left(5x10^{-4}\right)$$

or

$$\underline{L = 0.67\ \mu m}$$

13.34

$$f_T = \frac{g_m}{2\pi C_G}$$

where

$$C_G = \frac{\in WL}{a}$$

$$= \frac{(13.1)\left(8.85x10^{-14}\right)\left(5x10^{-4}\right)\left(1.5x10^{-4}\right)}{0.3x10^{-4}}$$

or

$$C_G = 2.9x10^{-15}\ F$$

We must use g_m', so we obtain

$$f_T = \frac{\left(2.82x10^{-3}\right)(0.80)}{2\pi\left(2.9x10^{-15}\right)} = 124\ GHz$$

We have

$$f_T = \frac{1}{2\pi\tau_C} \Rightarrow \tau_C = \frac{1}{2\pi f_T} = \frac{1}{2\pi\left(124x10^9\right)}$$

or

$$\tau_C = 1.28x10^{-12}\ s$$

The channel transit time is

$$t_t = \frac{1.5x10^{-4}}{10^7} = 1.5x10^{-11}\ s$$

The total time constant is

$$\tau = 1.5x10^{-11} + 1.28x10^{-12} = 1.63x10^{-11}\ s$$

so that

$$f_T = \frac{1}{2\pi\tau} = \frac{1}{2\pi\left(1.63x10^{-11}\right)}$$

or

$$\underline{f_T = 9.76\ GHz}$$

13.35

(a) For a constant mobility

$$f_T = \frac{e\mu_n N_d a^2}{2\pi \in L^2}$$

$$= \frac{\left(1.6x10^{-19}\right)(5500)\left(10^{17}\right)\left(0.25x10^{-4}\right)^2}{2\pi(13.1)\left(8.85x10^{-14}\right)\left(10^{-4}\right)^2}$$

or

$$\underline{f_T = 755\ GHz}$$

(b)

Saturation velocity model:

$$f_T = \frac{v_{sat}}{2\pi L}$$

Assuming $v_{sat} = 10^7\ cm/s$, we find

$$f_T = \frac{10^7}{2\pi\left(10^{-4}\right)}$$

or

$$\underline{f_T = 15.9\ GHz}$$

13.36

(a) $V_{off} = \phi_B - \dfrac{\Delta E_C}{e} - V_{P2}$

where

$$V_{P2} = \frac{eN_d d_d^2}{2 \in_N}$$

$$= \frac{\left(1.6x10^{-19}\right)\left(3x10^{18}\right)\left(350x10^{-8}\right)^2}{2(12.2)\left(8.85x10^{-14}\right)}$$

or

$$V_{P2} = 2.72\ V$$

Then

$$V_{off} = 0.89 - 0.24 - 2.72$$

or

$$\underline{V_{off} = -2.07\ V}$$

(b)

$$n_S = \frac{\in_N}{e(d + \Delta d)}\left(V_g - V_{off}\right)$$

For $V_g = 0$, we have

$$n_S = \frac{(12.2)\left(8.85x10^{-14}\right)}{\left(1.6x10^{-19}\right)(350 + 80)\cdot 10^{-8}}(2.07)$$

or

$$\underline{n_S = 3.25x10^{12}\ cm^{-2}}$$

13.37

(a) We have

$$I_D(sat) = \frac{\in_N W}{(d+\Delta d)}\left(V_g - V_{off} - V_O\right)v_s$$

We find

$$\left(\frac{g_{mS}}{W}\right) = \frac{\partial}{\partial V_g}\left[\frac{I_D(sat)}{W}\right] = \frac{\in_N v_s}{(d+\Delta d)}$$

$$= \frac{(12.2)\left(8.85x10^{-14}\right)\left(2x10^{7}\right)}{(350+80)\cdot 10^{-8}} = 5.02\ \frac{S}{cm}$$

or

$$\frac{g_{mS}}{W} = 502\ \frac{mS}{mm}$$

(b)

At $V_g = 0$, we obtain

$$\frac{I_D(sat)}{W} = \frac{\in_N}{(d+\Delta d)}\left(-V_{off} - V_O\right)v_S$$

$$= \frac{(12.2)\left(8.85x10^{-14}\right)}{(350+80)\cdot 10^{-8}}(2.07-1)\left(2x10^{7}\right)$$

or

$$\frac{I_D(sat)}{W} = 5.37\ A/cm = 537\ mA/mm$$

13.38

$$V_{off} = \phi_B - \frac{\Delta E_C}{e} - V_{P2}$$

We want $V_{off} = -0.3\ V$, so

$$-0.30 = 0.85 - 0.22 - V_{P2}$$

or

$$V_{P2} = 0.93\ V = \frac{eN_d d_d^2}{2\in_N}$$

We can then write

$$d_d^2 = \frac{2\in_N V_{P2}}{eN_d}$$

$$= \frac{2(12.2)\left(8.85x10^{-14}\right)(0.93)}{\left(1.6x10^{-19}\right)\left(2x10^{18}\right)}$$

We then obtain

$$d_d = 2.51x10^{-6}\ cm = 251\ \mathring{A}$$

Chapter 14

Problem Solutions

14.1

(a) $\lambda = \dfrac{1.24}{E}\ \mu m$

Then

Ge: $E_g = 0.66\ eV \Rightarrow \underline{\lambda = 1.88\ \mu m}$

Si: $E_g = 1.12\ eV \Rightarrow \underline{\lambda = 1.11\ \mu m}$

GaAs: $E_g = 1.42\ eV \Rightarrow \underline{\lambda = 0.873\ \mu m}$

(b)

$$E = \frac{1.24}{\lambda}$$

For $\lambda = 570\ nm \Rightarrow \underline{E = 2.18\ eV}$

For $\lambda = 700\ nm \Rightarrow \underline{E = 1.77\ eV}$

14.2

(a) GaAs

$h\nu = 2\ eV \Rightarrow \lambda = 0.62\ \mu m$

so

$$\underline{\alpha \approx 1.5x10^4\ cm^{-1}}$$

Then

$$\frac{I(x)}{I_o} = \exp(-\alpha x) = \exp\left[-\left(1.5x10^4\right)\left(0.35x10^{-4}\right)\right]$$

or

$$\underline{\frac{I(x)}{I_o} = 0.59}$$

so the percent absorbed is (1-0.59), or
<u>41%</u>

(b) Silicon

Again $h\nu = 2\ eV \Rightarrow \lambda = 0.62\ \mu m$

So

$$\underline{\alpha \approx 4x10^3\ cm^{-1}}$$

Then

$$\frac{I(x)}{I_o} = \exp(-\alpha x) = \exp\left[-\left(4x10^3\right)\left(0.35x10^{-4}\right)\right]$$

or

$$\underline{\frac{I(x)}{I_o} = 0.87}$$

so the percent absorbed is (1-0.87), or
<u>13%</u>

14.3

$$g' = \frac{\alpha I(x)}{h\nu}$$

For $h\nu = 1.3\ eV \Rightarrow \lambda = \dfrac{1.24}{1.3} = 0.95\ \mu m$

For silicon, $\alpha \approx 3x10^2\ cm^{-1}$,

Then for

$I(x) = 10^{-2}\ W/cm^2$

we obtain

$$g' = \frac{\left(3x10^2\right)\left(10^{-2}\right)}{\left(1.6x10^{-19}\right)(1.3)} \Rightarrow$$

$$\underline{g' = 1.44x10^{19}\ cm^{-3}s^{-1}}$$

The excess concentration is

$$\delta n = g'\tau = \left(1.44x10^{19}\right)\left(10^{-6}\right) \Rightarrow$$

$$\underline{\delta n = 1.44x10^{13}\ cm^{-3}}$$

14.4

n-type GaAs, $\tau = 10^{-7}\ s$

(a)

We want

$$\delta n = \delta p = 10^{15}\ cm^{-3} = g'\tau = g'\left(10^{-7}\right)$$

or

$$g' = \frac{10^{15}}{10^{-7}} = 10^{22}\ cm^{-3}s^{-1}$$

We have

$$h\nu = 1.9\ eV \Rightarrow \lambda = \frac{1.24}{1.9} = 0.65\ \mu m$$

so that

$\alpha \approx 1.3x10^4\ cm^{-1}$

Then

$$g' = \frac{\alpha I(x)}{h\nu} \Rightarrow I(x) = \frac{(g')(h\nu)}{\alpha}$$

$$= \frac{\left(10^{22}\right)\left(1.6x10^{-19}\right)(1.9)}{1.3x10^4}$$

or

$$\underline{I(0) = 0.234\ W/cm^2 = I_o}$$

(b)

$$\frac{I(x)}{I_o} = 0.20 = \exp\left[-\left(1.3x10^4\right)x\right]$$

We obtain $\underline{x = 1.24\ \mu m}$

14.5
GaAs
(a)
For $h\nu = 1.65\ eV \Rightarrow \lambda = 0.75\ \mu m$
So
$$\alpha \approx 0.7x10^4\ cm^{-1}$$
For 75% obsorbed,
$$\frac{I(x)}{I_O} = 0.25 = \exp(-\alpha x)$$
Then
$$\alpha x = \ln\left(\frac{1}{0.25}\right) \Rightarrow x = \frac{1}{0.7x10^4}\ln\left(\frac{1}{0.25}\right)$$
or
$$\underline{x = 1.98\ \mu m}$$
(b)
For 75% transmitted,
$$\frac{I(x)}{I_O} = 0.75 = \exp\left[-\left(0.7x10^4\right)x\right]$$
we obtain
$$\underline{x = 0.41\ \mu m}$$

14.6
GaAs
For $x = 1\ \mu m = 10^{-4}\ cm$, we have 50% absorbed or 50% transmitted, then
$$\frac{I(x)}{I_O} = 0.50 = \exp(-\alpha x)$$
We can write
$$\alpha = \left(\frac{1}{x}\right)\cdot\ln\left(\frac{1}{0.5}\right) = \left(\frac{1}{10^{-4}}\right)\cdot\ln(2)$$
or
$$\alpha = 0.69x10^4\ cm^{-1}$$
This value corresponds to
$$\underline{\lambda = 0.75\ \mu m\ ,\ E = 1.65\ eV}$$

14.7
The ambipolar transport equation for minority carrier holes in steady state is
$$D_p\frac{d^2(\delta p_n)}{dx^2} + G_L - \frac{\delta p_n}{\tau_p} = 0$$
or
$$\frac{d^2(\delta p_n)}{dx^2} - \frac{\delta p_n}{L_p^2} = -\frac{G_L}{D_p}$$
where $L_p^2 = D_p\tau_p$
The photon flux in the semiconductor is
$$\Phi(x) = \Phi_O\exp(-\alpha x)$$
and the generation rate is
$$G_L = \alpha\Phi(x) = \alpha\Phi_O\exp(-\alpha x)$$
so we have
$$\frac{d^2(\delta p_n)}{dx^2} - \frac{\delta p_n}{L_p^2} = -\frac{\alpha\Phi_O}{D_p}\exp(-\alpha x)$$
The general solution is of the form
$$\delta p_n = A\exp\left(\frac{-x}{L_p}\right) + B\exp\left(\frac{+x}{L_p}\right) - \frac{\alpha\Phi_O\tau_p}{\alpha^2L_p^2 - 1}\exp(-\alpha x)$$
At $x \to \infty$, $\delta p_n = 0$
So that $B = 0$, then
$$\delta p_n = A\exp\left(\frac{-x}{L_p}\right) - \frac{\alpha\Phi_O\tau_p}{\alpha^2L_p^2 - 1}\exp(-\alpha x)$$
At $x = 0$, we have
$$D_p\frac{d(\delta p_n)}{dx}\bigg|_{x=0} = s\delta p_n\big|_{x=0}$$
so we can write
$$\delta p_n\big|_{x=0} = A - \frac{\alpha\Phi_O\tau_p}{\alpha^2L_p^2 - 1}$$
and
$$\frac{d(\delta p_n)}{dx}\bigg|_{x=0} = -\frac{A}{L_p} + \frac{\alpha^2\Phi_O\tau_p}{\alpha^2L_p^2 - 1}$$
Then we have
$$-\frac{AD_p}{L_p} + \frac{\alpha^2\Phi_O\tau_pD_p}{\alpha^2L_p^2 - 1} = sA - \frac{s\alpha\Phi_O\tau_p}{\alpha^2L_p^2 - 1}$$
Solving for A, we find
$$A = \frac{\alpha\Phi_O\tau_p}{\alpha^2L_p^2 - 1}\cdot\left[\frac{s + \alpha D_p}{s + \left(D_p/L_p\right)}\right]$$
The solution can now be written as
$$\delta p_n = \frac{\alpha\Phi_O\tau_p}{\alpha^2L_p^2 - 1}\cdot\left\{\frac{s + \alpha D_p}{s + \left(D_p/L_p\right)}\cdot\exp\left(\frac{-x}{L_p}\right) - \exp(-\alpha x)\right\}$$

14.8
We have

$$D_n \frac{d^2(\delta n_p)}{dx^2} + G_L - \frac{\delta n_p}{\tau_n} = 0$$

or

$$\frac{d^2(\delta n_p)}{dx^2} - \frac{\delta n_p}{L_n^2} = -\frac{G_L}{D_n}$$

where $L_n^2 = D_n \tau_n$

The general solution can be written in the form

$$\delta n_p = A\cosh\left(\frac{x}{L_n}\right) + B\sinh\left(\frac{x}{L_n}\right) + G_L\tau_n$$

For $s = \infty$ at $x = 0$ means that $\delta n_p(0) = 0$,
Then

$$0 = A + G_L\tau_n \Rightarrow A = -G_L\tau_n$$

At $x = W$,

$$-D_n \frac{d(\delta n_p)}{dx}\Big|_{x=W} = s_o \delta n_p\Big|_{x=W}$$

Now

$$\delta n_p(W) = -G_L\tau_n \cosh\left(\frac{W}{L_n}\right) + B\sinh\left(\frac{W}{L_n}\right) + G_L\tau_n$$

and

$$\frac{d(\delta n_p)}{dx}\Big|_{x=W} = -\frac{G_L\tau_n}{L_n}\sinh\left(\frac{W}{L_n}\right) + \frac{B}{L_n}\cosh\left(\frac{W}{L_n}\right)$$

so we can write

$$\frac{G_L\tau_n D_n}{L_n}\sinh\left(\frac{W}{L_n}\right) - \frac{BD_n}{L_n}\cosh\left(\frac{W}{L_n}\right)$$
$$= s_o\left[-G_L\tau_n\cosh\left(\frac{W}{L_n}\right) + B\sinh\left(\frac{W}{L_n}\right) + G_L\tau_n\right]$$

Solving for B, we find

$$B = \frac{G_L\left[L_n\sinh\left(\frac{W}{L_n}\right) + s_o\tau_n\cosh\left(\frac{W}{L_n}\right) - s_o\tau_n\right]}{\frac{D_n}{L_n}\cosh\left(\frac{W}{L_n}\right) + s_o\sinh\left(\frac{W}{L_n}\right)}$$

The solution is then

$$\delta n_p = G_L\tau_n\left[1 - \cosh\left(\frac{x}{L_n}\right)\right] + B\sinh\left(\frac{x}{L_n}\right)$$

where B was just given.

14.9

$$V_{OC} = V_t \ln\left(1 + \frac{J_L}{J_S}\right)$$
$$= (0.0259)\ln\left(1 + \frac{30x10^{-3}}{J_S}\right)$$

where

$$J_S = en_i^2\left[\frac{1}{N_a}\cdot\sqrt{\frac{D_n}{\tau_n}} + \frac{1}{N_d}\cdot\sqrt{\frac{D_p}{\tau_p}}\right]$$

which becomes

$$J_S = (1.6x10^{-19})(1.8x10^6)^2 \times\left[\frac{1}{N_a}\cdot\sqrt{\frac{225}{5x10^{-8}}} + \frac{1}{10^{19}}\cdot\sqrt{\frac{7}{5x10^{-8}}}\right]$$

or

$$J_S = (5.18x10^{-7})\left[\frac{6.7x10^4}{N_a} + 1.18x10^{-15}\right]$$

Then

N_a	$J_S(A/cm^2)$	$V_{OC}(V)$
1E15	3.47E-17	0.891
1E16	3.47E-18	0.950
1E17	3.48E-19	1.01
1E18	3.53E-20	1.07

14.10
(a)

$$I_L = J_L \cdot A = (25x10^{-3})(2) = 50\ mA$$

We have

$$J_S = en_i^2\left[\frac{1}{N_a}\cdot\sqrt{\frac{D_n}{\tau_n}} + \frac{1}{N_d}\cdot\sqrt{\frac{D_p}{\tau_p}}\right]$$

or

$$J_S = (1.6x10^{-19})(1.5x10^{10})^2 \times\left[\frac{1}{3x10^{16}}\cdot\sqrt{\frac{18}{5x10^{-6}}} + \frac{1}{10^{19}}\cdot\sqrt{\frac{6}{5x10^{-7}}}\right]$$

which becomes

$$J_S = 2.29x10^{-12}\ A/cm^2$$

or

$$I_S = 4.58x10^{-12}\ A$$

We have

$$I = I_L - I_S\left[\exp\left(\frac{V}{V}\right) - 1\right]$$

or

$$I = 50x10^{-3} - 4.58x10^{-12}\left[\exp\left(\frac{V}{V_t}\right) - 1\right]$$

We see that when $I = 0$, $\underline{V = V_{OC} = 0.599\ V}$.

We find

$V(V)$	$I(mA)$
0	50
0.1	50
0.2	50
0.3	50
0.4	49.9
0.45	49.8
0.50	48.9
0.55	42.4
0.57	33.5
0.59	14.2

(b)
The voltage at the maximum power point is found from

$$\left[1 + \frac{V_m}{V_t}\right] \cdot \exp\left(\frac{V_m}{V_t}\right) = 1 + \frac{I_L}{I_S}$$

$$= 1 + \frac{50x10^{-3}}{4.58x10^{-12}} = 1.092x10^{10}$$

By trial and error,

$$\underline{V_m = 0.520\ V}$$

At this point, we find

$$\underline{I_m = 47.6\ mA}$$

so the maximum power is

$$P_m = I_m V_m = (47.6)(0.520)$$

or

$$\underline{P_m = 24.8\ mW}$$

(c)
We have

$$V = IR \Rightarrow R = \frac{V}{I} = \frac{V_m}{I_m} = \frac{0.520}{47.6x10^{-3}}$$

or

$$\underline{R = 10.9\ \Omega}$$

14.11

If the solar intensity increases by a factor of 10, then I_L increases by a factor of 10 so that $I_L = 500\ mA$. Then

$$I = 500x10^{-3} - 4.58x10^{-12}\left[\exp\left(\frac{V}{V_t}\right) - 1\right]$$

At the maximum power point

$$\left[1 + \frac{V_m}{V_t}\right] \cdot \exp\left(\frac{V_m}{V_t}\right) = 1 + \frac{I_L}{I_S}$$

$$= 1 + \frac{500x10^{-3}}{4.58x10^{-12}} = 1.092x10^{11}$$

By trial and error, we find

$$\underline{V_m = 0.577\ V}$$

and the current at the maximum power point is

$$\underline{I_m = 478.3\ mA}$$

The maximum power is then

$$\underline{P_m = I_m V_m = 276\ mW}$$

The maximum power has increased by a factor of 11.1 compared to the previous problem, which means that the efficiency has increased slightly.

14.12

Let $x = 0$ correspond to the edge of the space charge region in the p-type material. Then

$$D_n \frac{d^2(\delta n_p)}{dx^2} - \frac{\delta n_p}{\tau_n} = -G_L$$

or

$$\frac{d^2(\delta n_p)}{dx^2} - \frac{\delta n_p}{L_n^2} = -\frac{G_L}{D_n}$$

where

$$G_L = \alpha\Phi(x) = \alpha\Phi_O \exp(-\alpha x)$$

Then we have

$$\frac{d^2(\delta n_p)}{dx^2} - \frac{\delta n_p}{L_n^2} = -\frac{\alpha\Phi_O}{D_n}\exp(-\alpha x)$$

The general solution is of the form

$$\delta n_p = A\exp\left(\frac{-x}{L_n}\right) + B\exp\left(\frac{+x}{L_p}\right) - \frac{\alpha\Phi_O\tau_n}{\alpha^2 L_n^2 - 1}\exp(-\alpha x)$$

At $x \to \infty$, $\delta n_p = 0$ so that $B = 0$, then

$$\delta n_p = A\exp\left(\frac{-x}{L_n}\right) - \frac{\alpha\Phi_O\tau_n}{\alpha^2 L_n^2 - 1} \cdot \exp(-\alpha x)$$

We also have $\delta n_p(0) = 0 = A - \dfrac{\alpha\Phi_O\tau_n}{\alpha^2 L_n^2 - 1}$,

which yields

$$A = \frac{\alpha\Phi_O\tau_n}{\alpha^2 L_n^2 - 1}$$

We then obtain

$$\delta n_p = \frac{\alpha \Phi_O \tau_n}{\alpha^2 L_n^2 - 1}\left[\exp\left(\frac{-x}{L_n}\right) - \exp(-\alpha x)\right]$$

where Φ_O is the incident flux at $x = 0$.

14.13

For 90% absorption, we have

$$\frac{\Phi(x)}{\Phi_O} = \exp(-\alpha x) = 0.10$$

Then

$$\exp(+\alpha x) = \frac{1}{0.1} = 10$$

or

$$x = \left(\frac{1}{\alpha}\right) \cdot \ln(10)$$

For $h\nu = 1.7\ eV$, $\alpha \approx 10^4\ cm^{-1}$

Then

$$x = \left(\frac{1}{10^4}\right) \cdot \ln(10) \Rightarrow \underline{x = 2.3\ \mu m}$$

and for $h\nu = 2.0\ eV$, $\alpha \approx 10^5\ cm^{-1}$, so that

$$\underline{x = 0.23\ \mu m}$$

14.14

$G_L = 10^{20}\ cm^{-3}s^{-1}$ and $N_d > N_a$ so holes are the minority carrier.

(a)

$$\delta p = g'\tau = G_L \tau_p$$

so that

$$\delta p = \delta n = \left(10^{20}\right)\left(10^{-7}\right)$$

or

$$\underline{\delta p = \delta n = 10^{13}\ cm^{-3}}$$

(b)

$$\Delta\sigma = e(\delta p)\left(\mu_n + \mu_p\right)$$

$$= \left(1.6x10^{-19}\right)\left(10^{13}\right)(1000 + 430)$$

or

$$\underline{\Delta\sigma = 2.29x10^{-3}\ (\Omega - cm)^{-1}}$$

(c)

$$I_L = J_L \cdot A = \frac{(\Delta\sigma)AV}{L}$$

$$= \frac{\left(2.29x10^{-3}\right)\left(10^{-3}\right)(5)}{100x10^{-4}}$$

or

$$\underline{I_L = 1.15\ mA}$$

(d)

The photoconductor gain is

$$\Gamma_{ph} = \frac{\tau_p}{t_n}\left(1 + \frac{\mu_p}{\mu_n}\right)$$

where

$$t_n = \frac{L}{\mu_n \mathrm{E}} = \frac{L^2}{\mu_n V}$$

Then

$$\Gamma_{ph} = \frac{\tau_p \mu_n V}{L^2}\left(1 + \frac{\mu_p}{\mu_n}\right) = \frac{\tau_p V}{L^2}\left(\mu_n + \mu_p\right)$$

or

$$\Gamma_{ph} = \frac{\left(10^{-7}\right)(5)}{\left(100x10^{-4}\right)^2}(1000 + 430)$$

or

$$\underline{\Gamma_{ph} = 7.15}$$

14.15

n-type, so holes are the minority carrier

(a)

$$\delta p = G_L \tau_p = \left(10^{21}\right)\left(10^{-8}\right)$$

so that

$$\underline{\delta p = \delta n = 10^{13}\ cm^{-3}}$$

(b)

$$\Delta\sigma = e(\delta p)\left(\mu_n + \mu_p\right)$$

$$= \left(1.6x10^{-19}\right)\left(10^{13}\right)(8000 + 250)$$

or

$$\underline{\Delta\sigma = 1.32x10^{-2}\ (\Omega - cm)^{-1}}$$

(c)

$$I_L = J_L \cdot A = (\Delta\sigma)A\mathrm{E} = \frac{(\Delta\sigma)AV}{L}$$

$$= \frac{\left(1.32x10^{-2}\right)\left(10^{-4}\right)(5)}{100x10^{-4}}$$

or $\underline{I_L = 0.66\ mA}$

(d)

$$\Gamma_{ph} = \frac{\tau_p}{t_n}\left(1 + \frac{\mu_p}{\mu_n}\right) = \frac{\tau_p V}{L^2}\left(\mu_n + \mu_p\right)$$

$$= \frac{\left(10^{-8}\right)(5)}{\left(100x10^{-4}\right)^2}(8000 + 250)$$

or $\underline{\Gamma_{ph} = 4.13}$

14.16

$\Phi(x) = \Phi_O \exp(-\alpha x)$

The electron-hole generation rate is

$g' = \alpha\Phi(x) = \alpha\Phi_O \exp(-\alpha x)$

and the excess carrier concentration is

$\delta p = \tau_p \alpha\Phi(x)$

Now

$\Delta\sigma = e(\delta p)(\mu_n + \mu_p)$

and

$J_L = \Delta\sigma \mathrm{E}$

The photocurrent is now found from

$$I_L = \iint \Delta\sigma \mathrm{E} \cdot dA = \int_0^W dy \int_0^{x_O} \Delta\sigma \mathrm{E} \cdot dx$$

$$= We(\mu_n + \mu_n)\mathrm{E}\int_0^{x_O} \delta p \cdot dx$$

Then

$$I_L = We(\mu_n + \mu_p)\mathrm{E}\alpha\Phi_O\tau_p \int_0^{x_O} \exp(-\alpha x)dx$$

$$= We(\mu_n + \mu_p)\mathrm{E}\alpha\Phi_O\tau_p \left[-\frac{1}{\alpha}\exp(-\alpha x)\right]_0^{x_O}$$

which becomes

$$I_L = We(\mu_n + \mu_p)\mathrm{E}\Phi_O\tau_p[1 - \exp(-\alpha x_O)]$$

Now

$$I_L = (50x10^{-4})(1.6x10^{-19})(1200 + 450)(50)$$
$$\times(10^{16})(2x10^{-7})[1 - \exp(-(5x10^4)(10^{-4}))]$$

or

$$\underline{I_L = 0.131\ \mu A}$$

14.17

(a)

$$V_{bi} = (0.0259)\ln\left[\frac{(2x10^{16})(10^{18})}{(1.5x10^{10})^2}\right] = 0.832\ V$$

The space charge width is

$$W = \left[\frac{2\in(V_{bi} + V_R)}{e}\left(\frac{N_a + N_d}{N_a N_d}\right)\right]^{1/2}$$

$$= \left[\frac{2(11.7)(8.85x10^{-14})(0.832 + 5)}{1.6x10^{-19}} \times\left(\frac{2x10^{16} + 10^{18}}{(2x10^{16})(10^{18})}\right)\right]^{1/2}$$

or

$W = 0.620\ \mu m$

The prompt photocurrent density is

$$J_{L1} = eG_L W = (1.6x10^{-19})(10^{21})(0.620x10^{-4})$$

or

$$\underline{J_{L1} = 9.92\ mA/cm^2}$$

(b)

The total steady-state photocurrent density is

$$J_L = e(W + L_n + L_p)G_L$$

We find

$$L_n = \sqrt{D_n\tau_n} = \sqrt{(25)(2x10^{-7})} = 22.4\ \mu m$$

and

$$L_p = \sqrt{D_p\tau_p} = \sqrt{(10)(10^{-7})} = 10.0\ \mu m$$

Then

$$J_L = (1.6x10^{-19})(0.62 + 22.4 + 10.0)(10^{-4})(10^{21})$$

or

$$\underline{J_L = 0.528\ A/cm^2}$$

14.18

In the n-region under steady state and for $\mathrm{E} = 0$, we have

$$D_p\frac{d^2(\delta p_n)}{dx'^2} + G_L - \frac{\delta p_n}{\tau_p} = 0$$

or

$$\frac{d^2(\delta p_n)}{dx'^2} - \frac{\delta p_n}{L_p^2} = -\frac{G_L}{D_p}$$

where $L_p^2 = D_p\tau_p$ and where x' is positive in the negative x direction. The homogeneous solution is found from

$$\frac{d^2(\delta p_{nh})}{dx'^2} - \frac{\delta p_{nh}}{L_p^2} = 0$$

The general solution is found to be

$$\delta p_{nh} = A\exp\left(\frac{-x'}{L_p}\right) + B\exp\left(\frac{+x'}{L_p}\right)$$

The particular solution is found from

$$\frac{-\delta p_{np}}{L_p^2} = \frac{-G_L}{D_p}$$

which yields

$$\delta p_{np} = \frac{G_L L_p^2}{D_p} = G_L\tau_p$$

The total solution is the sum of the homogeneous and particular solutions, so we have

$$\delta p_n = A\exp\left(\frac{-x'}{L_p}\right) + B\exp\left(\frac{+x'}{L_p}\right) + G_L\tau_p$$

One boundary condition is that δp_n remains finite as $x' \to \infty$ which means that $B = 0$. Then At $x' = 0$, $p_n(0) = 0 = \delta p_n(0) + p_{nO}$, so that

$$\delta p_n(0) = -p_{nO}$$

We find that

$$A = -(p_{nO} + G_L\tau_p)$$

The solution is then written as

$$\delta p_n = G_L\tau_p - (G_L\tau_p + p_{nO})\exp\left(\frac{-x'}{L_p}\right)$$

The diffusion current density is found as

$$J_p = -eD_p \frac{d(\delta p_n)}{dx}\Big|_{x'=0}$$

But

$$\frac{d(\delta p_n)}{dx} = -\frac{d(\delta p_n)}{dx'}$$

since x and x' are in opposite directions.
So

$$J_p = +eD_p \frac{d(\delta p_n)}{dx'}\Big|_{x'=0}$$

$$= eD_p\left[-(G_L\tau_p + p_{nO})\right]\left(\frac{-1}{L_p}\right)\exp\left(\frac{-x'}{L_p}\right)\Big|_{x'=0}$$

Then

$$\underline{J_p = eG_L L_p + \frac{eD_p p_{nO}}{L_p}}$$

14.19
We have

$$J_L = e\Phi_O\left[1 - \exp(-\alpha W)\right]$$

$$= (1.6x10^{-19})(10^{17})\left[1 - \exp\left(-(3x10^3)W\right)\right]$$

or

$$J_L = 16\left[1 - \exp\left(-(3x10^3)W\right)\right]\,(mA)$$

Then for $W = 1\ \mu m = 10^{-4}\ cm$, we find

$\underline{J_L = 4.15\ mA}$

For $W = 10\ \mu m \Rightarrow$ $\underline{J_L = 15.2\ mA}$

For $W = 100\ \mu m \Rightarrow$ $\underline{J_L = 16\ mA}$

14.20
The minimum α occurs when $\lambda = 1\ \mu m$ which gives $\alpha = 10^2\ cm^{-1}$. We want

$$\frac{\Phi(x)}{\Phi_O} = \exp(-\alpha x) = 0.10$$

which can be written as

$$\exp(+\alpha x) = \frac{1}{0.1} = 10$$

Then

$$x = \frac{1}{\alpha}\ln(10) = \frac{1}{10^2}\ln(10)$$

or

$\underline{x = 230\ \mu m}$

14.21
For the $Al_xGa_{1-x}As$ system, a direct bandgap for $0 \le x \le 0.45$, we have

$$E_g = 1.424 + 1.247x$$

At $x = 0.45$, $E_g = 1.985\ eV$, so for the direct bandgap

$\underline{1.424 \le E_g \le 1.985\ eV}$

which yields

$\underline{0.625 \le \lambda \le 0.871\ \mu m}$

14.22
For $x = 0.35$ in $GaAs_{1-x}P_x$, we find
(a) $\underline{E_g = 1.85\ eV}$ and (b) $\underline{\lambda = 0.670\ \mu m}$

14.23
(a)
For GaAs, $\bar{n}_2 = 3.66$ and for air, $\bar{n}_1 = 1.0$.
The critical angle is

$$\theta_C = \sin^{-1}\left(\frac{\bar{n}_1}{\bar{n}_2}\right) = \sin^{-1}\left(\frac{1}{3.66}\right) = 15.9°$$

The fraction of photons that will not experience total internal reflection is

$$\frac{2\theta_C}{360} = \frac{2(15.9)}{360} \Rightarrow \underline{8.83\%}$$

(b)
Fresnel loss:

$$R = \left(\frac{\bar{n}_2 - \bar{n}_1}{\bar{n}_2 + \bar{n}_1}\right)^2 = \left(\frac{3.66 - 1}{3.66 + 1}\right)^2 = 0.326$$

The fraction of photons emitted is then

$$(0.0883)(1 - 0.326) = 0.0595 \Rightarrow \underline{5.95\%}$$

14.24

We can write the external quantum efficiency as

$\eta_{ext} = T_1 \cdot T_2$

where $T_1 = 1 - R_1$ with R_1 is the reflection coefficient (Fresnel loss), and the factor T_2 is the fraction of photons that do not experience total internal reflection. We have

$$R_1 = \left(\frac{\bar{n}_2 - \bar{n}_1}{\bar{n}_2 + \bar{n}_1}\right)^2$$

so that

$$T_1 = 1 - R_1 = 1 - \left(\frac{\bar{n}_2 - \bar{n}_1}{\bar{n}_2 + \bar{n}_1}\right)^2$$

which reduces to

$$T_1 = \frac{4\bar{n}_1\bar{n}_2}{\left(\bar{n}_1 + \bar{n}_2\right)^2}$$

Now consider a solid angle from the source point. The surface area described by the solid angle is πp^2. The factor T_1 is given by

$$T_1 = \frac{\pi p^2}{4\pi R^2}$$

From the geometry, we have

$$\sin\left(\frac{\theta_C}{2}\right) = \frac{p/2}{R} \Rightarrow p = 2R\sin\left(\frac{\theta_C}{2}\right)$$

Then the area is

$$A = \pi p^2 = 4R^2\pi\sin^2\left(\frac{\theta_C}{2}\right)$$

Now

$$T_1 = \frac{\pi p^2}{4\pi R^2} = \sin^2\left(\frac{\theta_C}{2}\right)$$

From a trig identity, we have

$$\sin^2\left(\frac{\theta_C}{2}\right) = \frac{1}{2}\left(1 - \cos\theta_C\right)$$

Then

$$T_1 = \frac{1}{2}\left(1 - \cos\theta_C\right)$$

The external quantum efficiency is now

$$\eta_{ext} = T_1 \cdot T_2 = \frac{4\bar{n}_1\bar{n}_2}{\left(\bar{n}_1 + \bar{n}_2\right)^2} \cdot \frac{1}{2}\left(1 - \cos\theta_C\right)$$

or

$$\underline{\eta_{ext} = \frac{2\bar{n}_1\bar{n}_2}{\left(\bar{n}_1 + \bar{n}_2\right)^2}\left(1 - \cos\theta_C\right)}$$

14.25

For an optical cavity, we have

$$N\left(\frac{\lambda}{2}\right) = L$$

If λ changes slightly, then N changes slightly also. We can write

$$\frac{N_1\lambda_1}{2} = \frac{\left(N_1 + 1\right)\lambda_2}{2}$$

Rearranging terms, we find

$$\frac{N_1\lambda_1}{2} - \frac{\left(N_1 + 1\right)\lambda_2}{2} = \frac{N_1\lambda_1}{2} - \frac{N_1\lambda_2}{2} - \frac{\lambda_2}{2} = 0$$

If we define $\Delta\lambda = \lambda_1 - \lambda_2$, then we have

$$\frac{N_1}{2}\Delta\lambda = \frac{\lambda_2}{2}$$

We can approximate $\lambda_2 = \lambda$, then

$$\frac{N_1\lambda}{2} = L \Rightarrow N_1 = \frac{2L}{\lambda}$$

Then

$$\frac{1}{2}\cdot\frac{2L}{\lambda}\Delta\lambda = \frac{\lambda}{2}$$

which yields

$$\underline{\Delta\lambda = \frac{\lambda^2}{2L}}$$

14.26

For GaAs,

$$h\nu = 1.42\ eV \Rightarrow \lambda = \frac{1.24}{E} = \frac{1.24}{1.42}$$

or

$$\lambda = 0.873\ \mu m$$

Then

$$\Delta\lambda = \frac{\lambda^2}{2L} = \frac{\left(0.873x10^{-4}\right)^2}{2\left(0.75x10^{-4}\right)} = 5.08x10^{-7}\ cm$$

or

$$\underline{\Delta\lambda = 5.08x10^{-3}\ \mu m}$$

Chapter 15

Problem Solutions

15.1
The limit of low injection means that

$$n_B(0) = (0.1)N_B = (0.1)(10^{16}) = 10^{15}\ cm^{-3}$$

Now

$$I_C = \frac{AeD_B n_B(0)}{x_B} = \frac{(0.5)(1.6x10^{-19})(20)(10^{15})}{3x10^{-4}}$$

or

$$\underline{I_C = 5.33\ A}$$

15.2
From the junction breakdown curve, for $BV_{CBO} = 1000\ V$, we need the collector doping concentration to be $\underline{N_C \approx 2x10^{14}\ cm^{-3}}$

Depletion width into the base (neglect V_{bi})

$$x_p = \left[\frac{2 \in V_{BC}}{e}\left(\frac{N_C}{N_B}\right)\left(\frac{1}{N_C + N_B}\right)\right]^{1/2}$$

$$= \left[\frac{2(11.7)(8.85x10^{-14})(1000)}{1.6x10^{-19}} \times\left(\frac{2x10^{14}}{5x10^{15}}\right)\left(\frac{1}{5x10^{15} + 2x10^{14}}\right)\right]^{1/2}$$

or

$\underline{x_p = 3.16\ \mu m}$ (Minimum base width)

Depletion width into the collector

$$x_n = \left[\frac{2(11.7)(8.85x10^{-14})(1000)}{1.6x10^{-19}} \times\left(\frac{5x10^{15}}{2x10^{14}}\right)\left(\frac{1}{5x10^{15} + 2x10^{14}}\right)\right]^{1/2}$$

or

$\underline{x_n = 78.9\ \mu m}$ (Minimum collector width)

15.3
Compute plot

15.4
(a)
We have $\beta_{eff} = \beta_A\beta_B + \beta_A + \beta_B$

Then

$$180 = 25\beta_B + 25 + \beta_B$$

or

$$155 = 26\beta_B$$

which yields

$$\underline{\beta_B = 5.96}$$

(b)
We have

$$\beta_B i_{EA} = i_{CB}$$

or

$$\beta_B\left(\frac{1+\beta_A}{\beta_A}\right)i_{CA} = i_{CB}$$

so

$$(5.96)\left(\frac{1+25}{25}\right)i_{CA} = 20$$

which yields

$$\underline{i_{CA} = 3.23\ A}$$

15.5
Sketch

15.6
We want

$$P_T = \frac{1}{2}I_{C,rated}\cdot\frac{V_{CC}}{2} \Rightarrow \frac{1}{2}I_{C,rated}\left(\frac{24}{2}\right) = 20$$

which yields

$$\underline{I_{C,rated} = 3.33\ A}$$

Then

$$R_L = \frac{V_{CC}}{I_{C,rated}} = \frac{24}{3.33}$$

or

$$\underline{R_L = 7.2\ \Omega}$$

15.7
If $V_{CC} = 25\ V$, then

$$I_C(\max) = \frac{V_{CC}}{R_L} = \frac{25}{100} = 0.25\ A < I_{C,rated}$$

The power

$$P = I_C V_{CE} = I_C(V_{CC} - I_C R_L)$$

To find the maximum power point, set

$$\frac{dP}{dI_C} = 0 = V_{CC} - 2I_C R_L = 25 - I_C(2)100$$

which yields $I_C = 0.125\ A$

So

$$P(\max) = (0.125)[25 - (0.125)(100)]$$

or

$$P(\max) = 1.56\ W < P_T$$

So, maximum V_{CC} is $\underline{V_{CC} = 25\ V}$

15.8

Now $R_{on} = \dfrac{V_{DS}}{I_D}$

Power dissipated in transistor

$$P = I_D V_{DS} = \frac{V_{DS}^2}{R_{on}}$$

We have

$$I_D = \frac{200 - V_{DS}}{100}$$

so we can write

$$P = \left(\frac{200 - V_{DS}}{100}\right) \cdot V_{DS} = \frac{V_{DS}^2}{R_{on}}$$

For $T = 25C$, $R_{on} = 2\ \Omega$,

Then

$$\left(\frac{200 - V_{DS}}{100}\right) \cdot V_{DS} = \frac{V_{DS}^2}{2}$$

which yields

$V_{DS} = 3.92\ V$ and

$$P = \left(\frac{200 - 3.92}{100}\right)(3.92) = 7.69\ W$$

We then have

T	R_{on}	V_{DS}	P
25	2.0	3.92	7.69
50	2.33	4.55	8.89
75	2.67	5.20	10.1
100	3	5.83	11.3

15.9

(a)

We have, for three devices in parallel,

$$\frac{V}{1.8} + \frac{V}{2} + \frac{V}{2.2} = 5 \Rightarrow V(1.51) = 5$$

or

$$V = 3.31\ V$$

Then, $I = \dfrac{V}{R}$, so that

$\underline{I_1 = 1.839\ A}$

$\underline{I_2 = 1.655\ A}$

$\underline{I_3 = 1.505\ A}$

Now, $P = IV$, so

$\underline{P_1 = 6.09\ W}$

$\underline{P_2 = 5.48\ W}$

$\underline{P_3 = 4.98\ W}$

(b)

Now

$$V\left(\frac{1}{1.8} + \frac{1}{3.6} + \frac{1}{2.2}\right) = 5 = V(1.288)$$

or

$$V = 3.88\ V$$

Then

$\underline{I_1 = 2.16\ A,\ P_1 = 8.38\ W}$

$\underline{I_2 = 1.08\ A,\ P_2 = 4.19\ W}$

$\underline{I_3 = 1.77\ A,\ P_3 = 6.85\ W}$

15.10

For $BV = 200\ V$, from the junction breakdown curve, we need the drain doping concentration to be $\underline{N_D \approx 1.5x10^{15}\ cm^{-3}}$

For the channel length (neglect V_{bi})

$$L(\min) = \left[\frac{2\in(V_D)}{e}\left(\frac{N_D}{N_B}\right)\left(\frac{1}{N_D + N_B}\right)\right]^{1/2}$$

$$= \left[\frac{2(11.7)(8.85x10^{-14})(200)}{1.6x10^{-19}} \times \left(\frac{1.5x10^{15}}{10^{16}}\right)\left(\frac{1}{1.5x10^{15} + 10^{16}}\right)\right]^{1/2}$$

or $\underline{L(\min) = 1.84\ \mu m}$

For the drift region

$$W(\min) = \left[\frac{2(11.7)(8.85x10^{-14})(200)}{1.6x10^{-19}} \times \left(\frac{10^{16}}{1.5x10^{15}}\right)\left(\frac{1}{1.5x10^{15} + 10^{16}}\right)\right]^{1/2}$$

or $\underline{W(\min) = 12.3\ \mu m}$

15.11

(b) In saturation region,

$$I_D = K_n(V_{GS} - V_T)^2 = 0.25(V_{GS} - 4)^2 \text{ and}$$

$$V_{DS} = V_{DD} - I_D R = 40 - I_D(10)$$

We find

$V_{GS} = 5\ V$, $I_D = 0.25\ A$, $V_{DS} = 37.5 > V_{DS}(sat)$

$V_{GS} = 6\ V$, $I_D = 1\ A$, $V_{DS} = 30 > V_{DS}(sat)$

$V_{GS} = 7\ V$, $I_D = 2.25\ A$, $V_{DS} = 17.5 > V_{DS}(sat)$

For $V_{GS} = 8\ V$ and $V_{GS} = 9\ V$, transistor is biased in the nonsaturation region. For $V_{GS} = 8\ V$.

$$I_D = \frac{40 - V_{DS}}{10} = 0.25\left[2(8-4)V_{DS} - V_{DS}^2\right]$$

We find

$V_{DS} = 2.92\ V$, $I_D = 3.71\ A$

For $V_{GS} = 9\ V$,

$$I_D = \frac{40 - V_{DS}}{10} = 0.25\left[2(9-4)V_{DS} - V_{DS}^2\right]$$

and we find

$V_{DS} = 1.88\ V$, $I_D = 3.81\ A$

Power dissipated in the transistor is $P_T = I_D V_{DS}$. We find

$\underline{V_{GS} = 5\ V\ ,\ P_T = 9.375\ W}$

$\underline{V_{GS} = 6\ V\ ,\ P_T = 30\ W}$

$\underline{V_{GS} = 7\ V\ ,\ P_T = 39.4\ W}$

$\underline{V_{GS} = 8\ V\ ,\ P_T = 10.8\ W}$

$\underline{V_{GS} = 9\ V\ ,\ P_T = 7.16\ W}$

15.12

$$T_{dev} - T_{amb} = P_D(\theta_{dev-case} - \theta_{case-amb})$$

which can be written as

$$\theta_{dev-case} = \frac{T_{dev} - T_{amb}}{P_D} - \theta_{case-amb}$$

$$= \frac{175 - 25}{10} - 6 = 9°\ C/W$$

Now

$$P_{D,rated} = \frac{T_{j,\max} - T_{amb}}{\theta_{dev-case}} = \frac{175 - 25}{9}$$

or

$$\underline{P_{D,rated} = 16.7\ W}$$

15.13

$$P_{D,rated} = \frac{T_{j,\max} - T_{amb}}{\theta_{dev-case}}$$

or

$$\theta_{dev-case} = \frac{T_{j,\max} - T_{amb}}{P_{D,rated}}$$

$$= \frac{150 - 25}{50} = 2.5°\ C/W$$

Then

$$T_{dev} - T_{amb} = P_D(\theta_{dev-case} + \theta_{case-amb})$$

so

$$150 - 25 = P_D(2.5 + \theta_{case-amb})$$

or

$$\underline{125 = P_D(2.5 + \theta_{case-amb})}$$

15.14

We have

$$P_D = I_D \cdot V_{DS} = (4)(5) = 20\ W$$

Now

$$T_{dev} - T_{amb} = P_D(\theta_{dev-case} + \theta_{case-snk} + \theta_{snk-amb})$$

or

$$T_{dev} - 25 = 20(1.75 + 0.8 + 3) = 111$$

which yields

$$\underline{T_{dev} = 136°\ C}$$

Also

$$T_{dev} - T_{case} = P_D \cdot \theta_{dev-case} = (20)(1.75) = 35$$

so

$$T_{case} = T_{dev} - 35 = 136 - 35$$

or

$$\underline{T_{case} = 101°\ C}$$

And

$$T_{case} - T_{snk} = P_D \cdot \theta_{case-snk} = (20)(0.8) = 16°\ C$$

so

$$T_{snk} = T_{case} - 16 = 101 - 16$$

or

$$\underline{T_{snk} = 85°\ C}$$

15.15

We have

$$T_{dev} - T_{amb} = P_D(\theta_{dev-case} + \theta_{case-amb})$$

so

$$200 - 25 = 25(3 + \theta_{case-amb})$$

or

$$\underline{\theta_{case-amb} = 4°\ C/W}$$

15.16
We have

$$\theta_{dev-case} = \frac{T_{j,\max} - T_{amb}}{P_{D,rated}} = \frac{175-25}{15} = 10^\circ\, C / W$$

Now

$$P_D = \frac{T_{j,\max} - T_{amb}}{\theta_{dev-case} + \theta_{case-snk} + \theta_{snk-amb}}$$

$$= \frac{175-25}{10+1+4}$$

or

$$\underline{P_D = 10\, W}$$

15.17
We have $\alpha_1 + \alpha_2 = 1$. Now

$$\alpha_1 = \frac{\beta_1}{1+\beta_1} \quad \text{and} \quad \alpha_2 = \frac{\beta_2}{1+\beta_2}$$

so

$$\alpha_1 + \alpha_2 = \frac{\beta_1}{1+\beta_1} + \frac{\beta_2}{1+\beta_2} = 1$$

which can be written as

$$1 = \frac{\beta_1(1+\beta_2) + \beta_2(1+\beta_1)}{(1+\beta_1)(1+\beta_2)}$$

or

$$(1+\beta_1)(1+\beta_2) = \beta_1(1+\beta_2) + \beta_2(1+\beta_1)$$

Expanding, we find

$$1 + \beta_1 + \beta_2 + \beta_1\beta_2$$
$$= \beta_1 + \beta_1\beta_2 + \beta_2 + \beta_1\beta_2$$

which yields

$$\underline{\beta_1\beta_2 = 1}$$

15.18
The reverse-biased p-well to substrate junction corresponds to the J_2 junction in an SCR. The photocurrent generated in this junction will be similar to the avalanche generated current in an SCR, which can trigger the device.

15.19
Case 1: Terminal 1(+), terminal 2(-), and I_G negative. This triggering was discussed in the text.
Case 2: Terminal 1(+), terminal 2(-), and I_G positive. Gate current enters the P2 region directly so that J3 becomes forward biased. Electrons are injected from N2 and diffuse into N1, lowering the potential of N1. The junction J2 becomes more forward biased, and the increased current triggers the SCR so that P2N1P1N4 turns on.
Case 3: Terminal 1(-), terminal 2(+), and I_G positive. Gate current enters the P2 region directly so that the J3 junction becomes more forward biased. More electrons are injected from N2 into N1 so that J1 also becomes more forward biased. The increased current triggers the P1N1P2N2 device into its conducting state.
Case 4: Terminal 1(-), terminal 2(+), and I_G negative. In this case, the J4 junction becomes forward biased. Electrons are injected from N3 and diffuse into N1. The potential of N1 is lowered which increases the forward biased potential of J1. This increased current then triggers the P1N1P2N2 device into its conducting state.