VE320 HW2 Solution

1.

For $T = 100$ K,

$$E_g = 1.170 - \frac{(4.73\times10^{-4})(100)^2}{636+100} \Rightarrow$$

$$E_g = 1.164 \text{ eV}$$

$T = 200$ K, $E_g = 1.147$ eV

$T = 300$ K, $E_g = 1.125$ eV

$T = 400$ K, $E_g = 1.097$ eV

$T = 500$ K, $E_g = 1.066$ eV

$T = 600$ K, $E_g = 1.032$ eV

2.

Points A,B: $\frac{dE}{dk} < 0 \Rightarrow$ velocity in -x direction

Points C,D: $\frac{dE}{dk} > 0 \Rightarrow$ velocity in +x direction

Points A,D: $\frac{d^2E}{dk^2} < 0 \Rightarrow$ negative effective mass

Points B,C: $\frac{d^2E}{dk^2} > 0 \Rightarrow$ positive effective mass

3.

For A: $E = C_i k^2$

At $k = 0.08\times10^{+10}\ \text{m}^{-1}$, $E = 0.05$ eV

Or $E = (0.05)(1.6\times10^{-19}) = 8\times10^{-21}$ J

So $8\times10^{-21} = C_1(0.08\times10^{10})^2$

$\Rightarrow C_1 = 1.25\times10^{-38}$

Now $m^* = \frac{\hbar^2}{2C_1} = \frac{(1.054\times10^{-34})^2}{2(1.25\times10^{-38})}$

$= 4.44\times10^{-31}$ kg

or $m^* = \frac{4.4437\times10^{-31}}{9.11\times10^{-31}}\cdot m_o$

$m^* = 0.488\, m_o$

For B: $E = C_i k^2$

At $k = 0.08\times10^{+10}\ \text{m}^{-1}$, $E = 0.5$ eV

Or $E = (0.5)(1.6\times10^{-19}) = 8\times10^{-20}$ J

So $8\times10^{-20} = C_1(0.08\times10^{10})^2$

$\Rightarrow C_1 = 1.25\times10^{-37}$

Now $m^* = \frac{\hbar^2}{2C_1} = \frac{(1.054\times10^{-34})^2}{2(1.25\times10^{-37})}$

$= 4.44\times10^{-32}$ kg

or $m^* = \frac{4.4437\times10^{-32}}{9.11\times10^{-31}}\cdot m_o$

$m^* = 0.0488\, m_o$

4.

(a) (i) $E = h\nu$

or $\nu = \frac{E}{h} = \frac{(1.42)(1.6\times10^{-19})}{6.625\times10^{-34}}$

$= 3.429\times10^{14}$ Hz

(ii) $\lambda = \frac{hc}{E} = \frac{c}{\nu} = \frac{3\times10^{10}}{3.429\times10^{14}}$

$= 8.75\times10^{-5}$ cm $= 875$ nm

(b) (i) $\nu = \frac{E}{h} = \frac{(1.12)(1.6\times10^{-19})}{6.625\times10^{-34}}$

$= 2.705\times10^{14}$ Hz

(ii) $\lambda = \frac{c}{\nu} = \frac{3\times10^{10}}{2.705\times10^{14}}$

$= 1.109\times10^{-4}$ cm $= 1109$ nm

5.

$$E = E_O - E_1 \cos\left[\alpha\left(k - k_O\right)\right]$$

Then

$$\frac{dE}{dk} = \left(-E_1\right)\left(-\alpha\right)\sin\left[\alpha\left(k - k_O\right)\right]$$

$$= +E_1\alpha\sin\left[\alpha\left(k - k_O\right)\right]$$

and

$$\frac{d^2E}{dk^2} = E_1\alpha^2\cos\left[\alpha\left(k - k_O\right)\right]$$

Then

$$\frac{1}{m^*} = \frac{1}{\hbar^2}\cdot\left.\frac{d^2E}{dk^2}\right|_{k=k_o} = \frac{E_1\alpha^2}{\hbar^2}$$

or

$$m^* = \frac{\hbar^2}{E_1\alpha^2}$$