Hw5 Solution

1.

(a) $p_o = N_a = 10^{16}\ \text{cm}^{-3}$

$$n_o = \frac{n_i^2}{p_o} = \frac{\left(1.5\times10^{10}\right)^2}{10^{16}} = 2.25\times10^4\ \text{cm}^{-3}$$

$$\sigma = e\mu_n\left(n_o + \delta n\right) + e\mu_p\left(p_o + \delta p\right)$$

$$\cong e\mu_p p_o + e\left(\mu_n + \mu_p\right)\delta n$$

Now $\delta n = \delta p = g'\tau_{n0}\left(1 - e^{-t/\tau_{n0}}\right)$

$$= \left(8\times10^{20}\right)\left(5\times10^{-7}\right)\left(1 - e^{-t/\tau_{n0}}\right)$$

$$= 4\times10^{14}\left(1 - e^{-t/\tau_{n0}}\right)\ \text{cm}^{-3}$$

Then $\sigma = \left(1.6\times10^{-19}\right)(380)\left(10^{16}\right)$

$$+\left(1.6\times10^{-19}\right)(900 + 380)$$

$$\times\left(4\times10^{14}\right)\left(1 - e^{-t/\tau_{n0}}\right)$$

$$\sigma = 0.608 + 0.0819\left(1 - e^{-t/\tau_{n0}}\right)\ (\Omega\text{-cm})^{-1}$$

(b) (i) $\sigma(0) = 0.608\ (\Omega\text{-cm})^{-1}$

(ii) $\sigma(\infty) = 0.690\ (\Omega\text{-cm})^{-1}$

2.

$$I = \frac{V}{R};\quad R = \frac{L}{\sigma A}$$

$$\Rightarrow I = \frac{\sigma A}{L}\cdot V$$

For $N_I = N_d + N_a = 8\times10^{15} + 2\times10^{15}$

$$= 10^{16}\ \text{cm}^{-3}$$

Then, $\mu_n \cong 1300\ \text{cm}^2/\text{V-s}$

$\mu_p \cong 400\ \text{cm}^2/\text{V-s}$

$$\sigma \cong e\mu_n n_o + e\left(\mu_n + \mu_p\right)\delta p$$

where $\delta p = g'\tau_{p0}e^{-t/\tau_{p0}}$

$$= \left(8\times10^{20}\right)\left(5\times10^{-7}\right)e^{-t/\tau_{p0}}$$

$$= 4\times10^{14}e^{-t/\tau_{p0}}\ \text{cm}^{-3}$$

$$\sigma = \left(1.6\times10^{-19}\right)(1300)\left(8\times10^{15} - 2\times10^{15}\right)$$

$$+\left(1.6\times10^{-19}\right)(1300 + 400)$$

$$\times\left(4\times10^{14}\right)e^{-t/\tau_{p0}}$$

$$\sigma = 1.248 + 0.109e^{-t/\tau_{p0}}$$

$$I = \frac{\left[1.248 + 0.109e^{-t/\tau_{p0}}\right]\left(10^{-5}\right)(10)}{0.05}$$

$$= 2.496\times10^{-3} + 2.18\times10^{-4}e^{-t/\tau_{p0}}\ \text{A}$$

or $I = 2.496 + 0.218e^{-t/\tau_{p0}}\ \text{mA}$

3.

(a) For $0 \le t \le 2\times10^{-6}$ s

$$\delta n(t) = g'\tau_{n0}e^{-t/\tau_{n0}}$$

$$= \left(10^{21}\right)\left(5\times10^{-7}\right)e^{-t/\tau_{n0}}$$

$$= 5\times10^{14}e^{-t/\tau_{n0}}\ \text{cm}^{-3}$$

At $t = 2\times10^{-6}$ s,

$$\delta n_1 = 5\times10^{14}e^{-\left(2\times10^{-6}\right)/\left(5\times10^{-7}\right)}$$

$$= 9.16\times10^{12}\ \text{cm}^{-3}$$

For $t \ge 2\times10^{-6}$ s

$$\delta n = \left(5\times10^{14} - 9.16\times10^{12}\right)\left(1 - e^{-(t-2\times10^{-6})/\tau_{n0}}\right) + 9.16\times10^{12}$$

$$= 4.908\times10^{14}\left(1 - e^{-(t-2\times10^{-6})/\tau_{n0}}\right) + 9.16\times10^{12}\ \text{cm}^{-3}$$

(b) (i) $\delta n(0) = 5\times10^{14}\ \text{cm}^{-3}$

(ii) $\delta n\left(2\times10^{-6}\right) = 9.16\times10^{12}\ \text{cm}^{-3}$

(iii) $\delta n(\infty) = 5\times10^{14}\ \text{cm}^{-3}$

4.

(a) p-type; $p_{pO} = 10^{14}\ \text{cm}^{-3}$

and

$$n_{pO} = \frac{n_i^2}{p_{pO}} = \frac{\left(1.5\times10^{10}\right)^2}{10^{14}} = 2.25\times10^6\ \text{cm}^{-3}$$

(b) Excess minority carrier concentration

$$\delta n = n_p - n_{pO}$$

At $x = 0$, $n_p = 0$ so that

$$\delta n(0) = 0 - n_{pO} = -2.25\times10^6\ \text{cm}^{-3}$$

(c) For the one-dimensional case,

$$D_n\frac{d^2(\delta n)}{dx^2} - \frac{\delta n}{\tau_{nO}} = 0$$

or

$$\frac{d^2(\delta n)}{dx^2} - \frac{\delta n}{L_n^2} = 0 \quad \text{where } L_n^2 = D_n\tau_{nO}$$

The general solution is of the form

$$\delta n = A\exp\left(\frac{-x}{L_n}\right) + B\exp\left(\frac{+x}{L_n}\right)$$

For $x \to \infty$, δn remains finite, so $B = 0$.
Then the solution is

$$\delta n = -n_{pO} \exp\left(\frac{-x}{L_n}\right)$$

5.

n-type, so minority carriers are holes and

$$D_p \nabla^2(\delta p) - \mu_p \mathrm{E} \bullet \nabla(\delta p) + g' - \frac{\delta p}{\tau_{pO}} = \frac{\partial(\delta p)}{\partial t}$$

We have $\tau_{pO} = \infty$, $\mathrm{E} = 0$, and

$\frac{\partial(\delta p)}{\partial t} = 0$ (steady-state). Then we have

$$D_p \frac{d^2(\delta p)}{dx^2} + g' = 0 \text{ or } \frac{d^2(\delta p)}{dx^2} = -\frac{g'}{D_p}$$

For $-L < x < +L$, $g' = G'_o$ = constant. Then

$$\frac{d(\delta p)}{dx} = -\frac{G'_o}{D_p} x + C_1$$

and

$$\delta p = -\frac{G'_o}{2D_p} x^2 + C_1 x + C_2$$

For $L < x < 3L$, $g' = 0$ so we have

$\frac{d^2(\delta p)}{dx^2} = 0$ so that $\frac{d(\delta p)}{dx} = C_3$ and

$\delta p = C_3 x + C_4$

For $-3L < x < -L$, $g' = 0$ so that

$\frac{d^2(\delta p)}{dx^2} = 0$ so that $\frac{d(\delta p)}{dx} = C_5$ and

$\delta p = C_5 x + C_6$

The boundary conditions are:

(1) $\delta p = 0$ at $x = +3L$

(2) $\delta p = 0$ at $x = -3L$

(3) δp continuous at $x = L$

(4) δp continuous at $x = -L$

(5) $\frac{d(\delta p)}{dx}$ continuous at $x = L$

(6) $\frac{d(\delta p)}{dx}$ continuous at $x = -L$

Applying the boundary conditions, we find

$$\delta p = \frac{G'_o}{2D_p}\left(5L^2 - x^2\right) \text{ for } -L < x < +L$$

$$\delta p = \frac{G'_o L}{D_p}(3L - x) \text{ for } L < x < 3L$$

$$\delta p = \frac{G'_o L}{D_p}(3L + x) \text{ for } -3L < x < -L$$