VE320 HW4 Solution

1.

(a) $p_o = N_a = 10^{16}\ \text{cm}^{-3}$

$$n_o = \frac{n_i^2}{p_o} = \frac{(1.5\times10^{10})^2}{10^{16}} = 2.25\times10^4\ \text{cm}^{-3}$$

$$\sigma = e\mu_n(n_o+\delta n) + e\mu_p(p_o+\delta p)$$

$$\cong e\mu_p p_o + e(\mu_n+\mu_p)\delta n$$

Now $\delta n = \delta p = g'\tau_{n0}\left(1-e^{-t/\tau_{n0}}\right)$

$$= (8\times10^{20})(5\times10^{-7})\left(1-e^{-t/\tau_{n0}}\right)$$

$$= 4\times10^{14}\left(1-e^{-t/\tau_{n0}}\right)\ \text{cm}^{-3}$$

Then $\sigma = (1.6\times10^{-19})(380)(10^{16})$

$$+(1.6\times10^{-19})(900+380)$$

$$\times(4\times10^{14})\left(1-e^{-t/\tau_{n0}}\right)$$

$$\sigma = 0.608 + 0.0819\left(1-e^{-t/\tau_{n0}}\right)\ (\Omega\text{-cm})^{-1}$$

(b) (i) $\sigma(0) = 0.608\ (\Omega\text{-cm})^{-1}$

(ii) $\sigma(\infty) = 0.690\ (\Omega\text{-cm})^{-1}$

2.

(a) For $0 \le t \le 2\times10^{-6}$ s

$$\delta n(t) = g'\tau_{n0}e^{-t/\tau_{n0}}$$

$$= (10^{21})(5\times10^{-7})e^{-t/\tau_{n0}}$$

$$= 5\times10^{14}e^{-t/\tau_{n0}}\ \text{cm}^{-3}$$

At $t = 2\times10^{-6}$ s,

$$\delta n_1 = 5\times10^{14}e^{-(2\times10^{-6})/(5\times10^{-7})}$$

$$= 9.16\times10^{12}\ \text{cm}^{-3}$$

For $t \ge 2\times10^{-6}$ s

$$\delta n = (5\times10^{14} - 9.16\times10^{12})\left(1-e^{-(t-2\times10^{-6})/\tau_{n0}}\right) + 9.16\times10^{12}$$

$$= 4.908\times10^{14}\left(1-e^{-(t-2\times10^{-6})/\tau_{n0}}\right) + 9.16\times10^{12}\ \text{cm}^{-3}$$

(b) (i) $\delta n(0) = 5\times10^{14}\ \text{cm}^{-3}$

(ii) $\delta n(2\times10^{-6}) = 9.16\times10^{12}\ \text{cm}^{-3}$

(iii) $\delta n(\infty) = 5\times10^{14}\ \text{cm}^{-3}$

3.

n-type, so minority carriers are holes and

$$D_p\nabla^2(\delta p) - \mu_p \mathrm{E}\bullet\nabla(\delta p) + g' - \frac{\delta p}{\tau_{pO}} = \frac{\partial(\delta p)}{\partial t}$$

We have $\tau_{pO} = \infty$, $\mathrm{E} = 0$, and $\frac{\partial(\delta p)}{\partial t} = 0$ (steady-state). Then we have

$$D_p\frac{d^2(\delta p)}{dx^2} + g' = 0 \quad\text{or}\quad \frac{d^2(\delta p)}{dx^2} = -\frac{g'}{D_p}$$

For $-L < x < +L$, $g' = G'_o$ = constant. Then

$$\frac{d(\delta p)}{dx} = -\frac{G'_o}{D_p}x + C_1$$

and

$$\delta p = -\frac{G'_o}{2D_p}x^2 + C_1x + C_2$$

For $L < x < 3L$, $g' = 0$ so we have

$$\frac{d^2(\delta p)}{dx^2} = 0 \text{ so that } \frac{d(\delta p)}{dx} = C_3 \text{ and}$$

$$\delta p = C_3x + C_4$$

For $-3L < x < -L$, $g' = 0$ so that

$$\frac{d^2(\delta p)}{dx^2} = 0 \text{ so that } \frac{d(\delta p)}{dx} = C_5 \text{ and}$$

$$\delta p = C_5x + C_6$$

The boundary conditions are:

(1) $\delta p = 0$ at $x = +3L$

(2) $\delta p = 0$ at $x = -3L$

(3) δp continuous at $x = L$

(4) δp continuous at $x = -L$

(5) $\frac{d(\delta p)}{dx}$ continuous at $x = L$

(6) $\frac{d(\delta p)}{dx}$ continuous at $x = -L$

Applying the boundary conditions, we find

$$\delta p = \frac{G'_o}{2D_p}\left(5L^2 - x^2\right) \text{ for } -L < x < +L$$

$$\delta p = \frac{G'_oL}{D_p}(3L - x) \text{ for } L < x < 3L$$

$$\delta p = \frac{G'_oL}{D_p}(3L + x) \text{ for } -3L < x < -L$$

4.

(b) $N_d = n_i \exp\left(\frac{E_F - E_{Fi}}{kT}\right)$

$$= \left(1.5\times10^{10}\right)\exp\left(\frac{0.365}{0.0259}\right)$$

or $N_d = 1.98\times10^{16}\ \text{cm}^{-3}$

$$N_a = n_i \exp\left(\frac{E_{Fi} - E_F}{kT}\right)$$

$$= \left(1.5\times10^{10}\right)\exp\left(\frac{0.330}{0.0259}\right)$$

or $N_a = 5.12\times10^{15}\ \text{cm}^{-3}$

(c)

$$V_{bi} = (0.0259)\ln\left[\frac{\left(5.12\times10^{15}\right)\left(1.98\times10^{16}\right)}{\left(1.5\times10^{10}\right)^2}\right]$$

$$= 0.695\ \text{V}$$

5.

$$x_n = 0.25W = 0.25\left(x_n + x_p\right)$$

$$0.75x_n = 0.25x_p \Rightarrow \frac{x_p}{x_n} = 3$$

$$x_n N_d = x_p N_a \Rightarrow \frac{N_d}{N_a} = \frac{x_p}{x_n} = 3$$

So $N_d = 3N_a$

(a) $V_{bi} = (0.0259)\ln\left[\frac{N_a N_d}{\left(1.5\times10^{10}\right)^2}\right]$

$$0.710 = (0.0259)\ln\left[\frac{3N_a^2}{\left(1.5\times10^{10}\right)^2}\right]$$

or $3N_a^2 = \left(1.5\times10^{10}\right)^2 \exp\left(\frac{0.710}{0.0259}\right)$

which yields $N_a = 7.766\times10^{15}\ \text{cm}^{-3}$

$N_d = 2.33\times10^{16}\ \text{cm}^{-3}$

$$x_n = \left\{\frac{2\epsilon_s V_{bi}}{e}\left(\frac{N_a}{N_d}\right)\left(\frac{1}{N_a + N_d}\right)\right\}^{1/2}$$

$$= \left\{\frac{2(11.7)\left(8.85\times10^{-14}\right)(0.710)}{1.6\times10^{-19}} \times\left(\frac{1}{3}\right)\left[\frac{1}{4\left(7.766\times10^{15}\right)}\right]\right\}^{1/2}$$

$\Rightarrow x_n = 9.93\times10^{-6}\ \text{cm}$

or $x_n = 0.0993\ \mu\text{m}$

$$x_p = \left\{\frac{2(11.7)\left(8.85\times10^{-14}\right)(0.710)}{1.6\times10^{-19}} \times\left(\frac{3}{1}\right)\left[\frac{1}{4\left(7.766\times10^{15}\right)}\right]\right\}^{1/2}$$

$= 2.979\times10^{-5}\ \text{cm}$

or $x_p = 0.2979\ \mu\text{m}$

Now

$$\left|\mathrm{E}_{\max}\right| = \frac{eN_d x_n}{\epsilon_s}$$

$$= \frac{\left(1.6\times10^{-19}\right)\left(2.33\times10^{16}\right)\left(0.0993\times10^{-4}\right)}{(11.7)\left(8.85\times10^{-14}\right)}$$

$= 3.58\times10^4\ \text{V/cm}$

(b) From part (a), we can write

$$3N_a^2 = \left(1.8\times10^6\right)^2 \exp\left(\frac{1.180}{0.0259}\right)$$

which yields $N_a = 8.127\times10^{15}\ \text{cm}^{-3}$

$N_d = 2.438\times10^{16}\ \text{cm}^{-3}$

$$x_n = \left\{\frac{2(13.1)\left(8.85\times10^{-14}\right)(1.180)}{1.6\times10^{-19}} \times\left(\frac{1}{3}\right)\left[\frac{1}{4\left(8.127\times10^{15}\right)}\right]\right\}^{1/2}$$

$= 1.324\times10^{-5}\ \text{cm}$

or $x_n = 0.1324\ \mu\text{m}$

$$x_p = \left\{\frac{2(13.1)\left(8.85\times10^{-14}\right)(1.180)}{1.6\times10^{-19}} \times\left(\frac{3}{1}\right)\left[\frac{1}{4\left(8.127\times10^{15}\right)}\right]\right\}^{1/2}$$

$= 3.973\times10^{-5}\ \text{cm}$

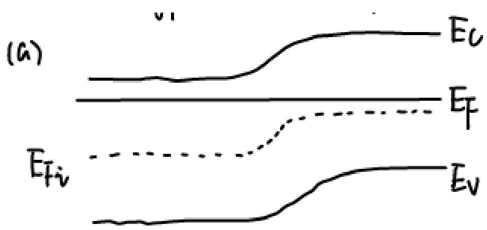
or $x_p = 0.3973\ \mu\text{m}$

$$\left|\mathrm{E}_{\max}\right| = \frac{eN_d x_n}{\epsilon_s}$$

$$= \frac{\left(1.6\times10^{-19}\right)\left(2.438\times10^{16}\right)\left(0.1324\times10^{-4}\right)}{(13.1)\left(8.85\times10^{-14}\right)}$$

$= 4.45\times10^4\ \text{V/cm}$

6.



(a) For $N_d = 10^{16}\ \text{cm}^{-3}$,

$$E_F - E_{Fi} = kT \ln\left(\frac{N_d}{n_i}\right)$$

$$= (0.0259)\ln\left(\frac{10^{16}}{1.5\times 10^{10}}\right)$$

or

$$E_F - E_{Fi} = 0.3473\ \text{eV}$$

For $N_d = 10^{15}\ \text{cm}^{-3}$

$$E_F - E_{Fi} = (0.0259)\ln\left(\frac{10^{15}}{1.5\times 10^{10}}\right)$$

or

$$E_F - E_{Fi} = 0.2877\ \text{eV}$$

Then

$$V_{bi} = 0.34732 - 0.28768$$

or

$$V_{bi} = 0.0596\ \text{V}$$

7.

(a) $V_{bi} = V_t \ln\left(\frac{N_a N_d}{n_i^2}\right)$

$$= V_t \ln\left(\frac{80 N_d^2}{n_i^2}\right)$$

We find

$$80 N_d^2 = n_i^2 \exp\left(\frac{V_{bi}}{V_t}\right)$$

$$= \left(1.5\times 10^{10}\right)^2 \exp\left(\frac{0.740}{0.0259}\right)$$

$$= 5.762\times 10^{32}$$

$$\Rightarrow N_d = 2.684\times 10^{15}\ \text{cm}^{-3}$$

$$N_a = 2.147\times 10^{17}\ \text{cm}^{-3}$$

(b)

$$x_n = \left\{\frac{2\in_s (V_{bi}+V_R)}{e}\left(\frac{N_a}{N_d}\right)\left(\frac{1}{N_a+N_d}\right)\right\}^{1/2}$$

$$= \left\{\frac{2(11.7)(8.85\times 10^{-14})(0.740+10)}{1.6\times 10^{-19}}\right.$$

$$\left.\times\left(\frac{80}{1}\right)\left(\frac{1}{2.147\times 10^{17}+2.684\times 10^{15}}\right)\right\}^{1/2}$$

$$= 2.262\times 10^{-4}\ \text{cm}$$

or $x_n = 2.262\ \mu\text{m}$

$$x_p = \left\{\frac{2\in_s (V_{bi}+V_R)}{e}\left(\frac{N_d}{N_a}\right)\left(\frac{1}{N_a+N_d}\right)\right\}^{1/2}$$

$$= \left\{\frac{2(11.7)(8.85\times 10^{-14})(0.740+10)}{1.6\times 10^{-19}}\right.$$

$$\left.\times\left(\frac{1}{80}\right)\left(\frac{1}{2.147\times 10^{17}+2.684\times 10^{15}}\right)\right\}^{1/2}$$

$$= 2.83\times 10^{-6}\ \text{cm}$$

or $x_p = 0.0283\ \mu\text{m}$

(c) $\left|\text{E}_{max}\right| = \frac{2(V_{bi}+V_R)}{W}$

$$= \frac{2(0.740+10)}{(2.262+0.0283)\times 10^{-4}}$$

$$= 9.38\times 10^{4}\ \text{V/cm}$$

(d) $C' = \left\{\frac{e\in_s N_a N_d}{2(V_{bi}+V_R)(N_a+N_d)}\right\}^{1/2}$

$$= \left\{\frac{(1.6\times 10^{-19})(11.7)(8.85\times 10^{-14})}{2(0.740+10)}\right.$$

$$\left.\times\left[\frac{(2.147\times 10^{17})(2.684\times 10^{15})}{2.147\times 10^{17}+2.684\times 10^{15}}\right]\right\}^{1/2}$$

$$C' = 4.52\times 10^{-9}\ \text{F/cm}^2$$

8.

(a) $V_{bi} = (0.0259)\ln\left[\frac{(5\times 10^{15})(10^{14})}{(1.5\times 10^{10})^2}\right]$

or

$$V_{bi} = 0.5574\ \text{V}$$

(b)

$$x_p = \left[\frac{2\in_s V_{bi}}{e}\left(\frac{N_d}{N_a}\right)\left(\frac{1}{N_a+N_d}\right)\right]^{1/2}$$

$$= \left[\frac{2(11.7)(8.85\times 10^{-14})(0.5574)}{1.6\times 10^{-19}}\right.$$

$$\left.\times\left(\frac{10^{14}}{5\times 10^{15}}\right)\left(\frac{1}{10^{14}+5\times 10^{15}}\right)\right]^{1/2}$$

or

$$x_p = 5.32\times 10^{-6}\ \text{cm}$$

Also

$$x_n = \left[\frac{2\in_s V_{bi}}{e}\left(\frac{N_a}{N_d}\right)\left(\frac{1}{N_a+N_d}\right)\right]^{1/2}$$

$$= \left[\frac{2(11.7)(8.85\times 10^{-14})(0.5574)}{1.6\times 10^{-19}}\right.$$

$$\times\left(\frac{5\times10^{15}}{10^{14}}\right)\left(\frac{1}{10^{14}+5\times10^{15}}\right)\Bigg]^{1/2}$$

or

$$x_n = 2.66\times10^{-4}\text{ cm}$$

(c) For $x_n = 30\ \mu$ m, we have

$$30\times10^{-4} = \left[\frac{2(11.7)(8.85\times10^{-14})(V_{bi}+V_R)}{1.6\times10^{-19}}\right.$$

$$\left.\times\left(\frac{5\times10^{15}}{10^{14}}\right)\left(\frac{1}{10^{14}+5\times10^{15}}\right)\right]^{1/2}$$

which becomes

$$9\times10^{-6} = 1.269\times10^{-7}(V_{bi}+V_R)$$

We find

$$V_R = 70.4\text{ V}$$