

1.

$$(a) \quad I_D = \frac{k'_p}{2} \cdot \frac{W}{L} \left[ 2(V_{SG} + V_T)V_{SD} - V_{SD}^2 \right]$$

$$= \left( \frac{0.10}{2} \right) (15) \left[ 2(0.8 - 0.4)(0.25) - (0.25)^2 \right]$$

$$I_D = 0.103 \text{ mA}$$

$$(b) \quad I_D = \frac{k'_p}{2} \cdot \frac{W}{L} (V_{SG} + V_T)^2$$

$$= \left( \frac{0.10}{2} \right) (15) (0.8 - 0.4)^2$$

$$= 0.12 \text{ mA}$$

$$(c) \quad I_D = \frac{k'_p}{2} \cdot \frac{W}{L} (V_{SG} + V_T)^2$$

$$= \left( \frac{0.10}{2} \right) (15) (1.2 - 0.4)^2$$

$$= 0.48 \text{ mA}$$

$$(d) \quad \text{Same as (c), } I_D = 0.48 \text{ mA}$$

2.

(a) Assume biased in saturation region

$$I_D = \frac{k'_p}{2} \cdot \frac{W}{L} (V_{SG} + V_T)^2$$

$$0.10 = \left( \frac{0.12}{2} \right) (20) (0 + V_T)^2$$

$$\Rightarrow V_T = +0.289 \text{ V}$$

Note:  $V_{SD} = 1.0 \text{ V} > V_{SG} + V_T = 0 + 0.289 \text{ V}$

So the transistor is biased in the saturation region.

$$(b) \quad I_D = \left( \frac{0.12}{2} \right) (20) (0.4 + 0.289)^2$$

$$= 0.570 \text{ mA}$$

$$(c) \quad I_D = \left( \frac{0.12}{2} \right) (20) \left[ 2(0.6 + 0.289)(0.15) \right. \\ \left. - (0.15)^2 \right]$$

or

$$I_D = 0.293 \text{ mA}$$

3.

(a)  $n^+$  poly-to-p-type  $\Rightarrow \phi_{ms} = -1.0 \text{ V}$

$$\phi_{fp} = (0.0259) \ln \left( \frac{10^{15}}{1.5 \times 10^{10}} \right) = 0.288 \text{ V}$$

also

$$x_{dT} = \left[ \frac{4 \epsilon_s \phi_{fp}}{e N_a} \right]^{1/2} = \left[ \frac{4(11.7)(8.85 \times 10^{-14})(0.288)}{(1.6 \times 10^{-19})(10^{15})} \right]^{1/2}$$

or

$$x_{dT} = 0.863 \times 10^{-4} \text{ cm}$$

Now

$$|Q'_{SD}(\max)| = (1.6 \times 10^{-19})(10^{15})(0.863 \times 10^{-4})$$

or

$$|Q'_{SD}(\max)| = 1.38 \times 10^{-8} \text{ C/cm}^2$$

Also

$$C_{ox} = \frac{\epsilon_{ox}}{t_{ox}} = \frac{(3.9)(8.85 \times 10^{-14})}{400 \times 10^{-8}}$$

or

$$C_{ox} = 8.63 \times 10^{-8} \text{ F/cm}^2$$

We find

$$Q'_{ss} = (1.6 \times 10^{-19})(5 \times 10^{10}) = 8 \times 10^{-9} \text{ C/cm}^2$$

Then

$$V_T = \frac{|Q'_{SD}(\max)| - Q'_{ss}}{C_{ox}} + \phi_{ms} + 2\phi_{fp} = \left( \frac{1.38 \times 10^{-8} - 8 \times 10^{-9}}{8.63 \times 10^{-8}} \right) - 1.0 + 2(0.288)$$

or

$$V_T = -0.357 \text{ V}$$

(b) For NMOS, apply  $V_{SB}$  and  $V_T$  shifts in a positive direction, so for  $V_T = 0$ , we want

$$\Delta V_T = +0.357 \text{ V.}$$

So

$$\Delta V_T = \frac{\sqrt{2e \epsilon_s N_a}}{C_{ox}} \left[ \sqrt{2\phi_{fp} + V_{SB}} - \sqrt{2\phi_{fp}} \right]$$

or

$$+0.357 = \frac{\sqrt{2(1.6 \times 10^{-19})(11.7)(8.85 \times 10^{-14})(10^{15})}}{8.63 \times 10^{-8}} \times \left[ \sqrt{2(0.288) + V_{SB}} - \sqrt{2(0.288)} \right]$$

or

$$0.357 = 0.211 \left[ \sqrt{0.576 + V_{SB}} - \sqrt{0.576} \right]$$

which yields

$$V_{SB} = 5.43 \text{ V}$$

4.

(a)

$$\begin{aligned} g_m &= \frac{W \mu_n C_{ox}}{L} (V_{GS} - V_T) \\ &= \frac{W \mu_n \epsilon_{ox}}{L t_{ox}} (V_{GS} - V_T) \\ &= \frac{(10)(400)(3.9)(8.85 \times 10^{-14})}{475 \times 10^{-8}} (5 - 0.65) \end{aligned}$$

or

$$g_m = 1.26 \text{ mS}$$

Now

$$g'_m = \frac{g_m}{1 + g_m r_s} \Rightarrow \frac{g'_m}{g_m} = 0.8 = \frac{1}{1 + g_m r_s}$$

which yields

$$r_s = \frac{1}{g_m} \left( \frac{1}{0.8} - 1 \right) = \frac{1}{1.26} \left( \frac{1}{0.8} - 1 \right)$$

or

$$r_s = 0.198 \text{ k}\Omega$$

(b) For  $V_{GS} = 3 \text{ V}$ ,  $g_m = 0.683 \text{ mS}$

Then

$$g'_m = \frac{0.683}{1 + (0.683)(0.198)} = 0.602 \text{ mS}$$

or

$$\frac{g'_m}{g_m} = \frac{0.602}{0.683} = 0.88$$

which is a 12% reduction.

5.

(a)

$$I_D = 10^{-15} \exp\left(\frac{V_{GS}}{(2.1)V_t}\right)$$

For  $V_{GS} = 0.5 \text{ V}$ ,

$$I_D = 10^{-15} \exp\left[\frac{0.5}{(2.1)(0.0259)}\right] \Rightarrow$$

$$I_D = 9.83 \times 10^{-12} \text{ A}$$

For  $V_{GS} = 0.7 \text{ V}$ ,

$$I_D = 3.88 \times 10^{-10} \text{ A}$$

For  $V_{GS} = 0.9 \text{ V}$ ,

$$I_D = 1.54 \times 10^{-8} \text{ A}$$

Then the total current is:

$$I_T = I_D (10^6)$$

For  $V_{GS} = 0.5 \text{ V}$ ,  $I_T = 9.83 \mu\text{A}$

For  $V_{GS} = 0.7 \text{ V}$ ,  $I_T = 0.388 \text{ mA}$

For  $V_{GS} = 0.9 \text{ V}$ ,  $I_T = 15.4 \text{ mA}$

(b)

$$\text{Power: } P = I_T \cdot V_{DD}$$

Then

For  $V_{GS} = 0.5 \text{ V}$ ,  $P = 49.2 \mu\text{W}$

For  $V_{GS} = 0.7 \text{ V}$ ,  $P = 1.94 \text{ mW}$

For  $V_{GS} = 0.9 \text{ V}$ ,  $P = 77 \text{ mW}$

6.

$$C_{ox} = \frac{\epsilon_{ox}}{t_{ox}} = \frac{(3.9)(8.85 \times 10^{-14})}{120 \times 10^{-8}} \\ = 2.876 \times 10^{-7} \text{ F/cm}^2$$

$$V_{FB} = \phi_{ms} - \frac{Q'_{ss}}{C_{ox}} \\ = -0.5 - \frac{(4 \times 10^{10})(1.6 \times 10^{-19})}{2.876 \times 10^{-7}}$$

$$V_{FB} = -0.5223 \text{ V}$$

Now

$$V_T = \frac{|Q'_{SD}(\max)|}{C_{ox}} + V_{FB} + 2\phi_{fp}$$

We find

$$\phi_{fp} = (0.0259) \ln \left( \frac{4 \times 10^{16}}{1.5 \times 10^{10}} \right) = 0.3832 \text{ V}$$

$$x_{dT} = \left[ \frac{4(11.7)(8.85 \times 10^{-14})(0.3832)}{(1.6 \times 10^{-19})(4 \times 10^{16})} \right]^{1/2} \\ = 1.575 \times 10^{-5} \text{ cm}$$

$$|Q'_{SD}(\max)| \\ = (1.6 \times 10^{-19})(4 \times 10^{16})(1.575 \times 10^{-5}) \\ = 1.008 \times 10^{-7} \text{ C/cm}^2$$

So

$$V_T = \frac{1.008 \times 10^{-7}}{2.876 \times 10^{-7}} - 0.5223 + 2(0.3832) \\ = 0.595 \text{ V}$$

$$V_{DS}(\text{sat}) = V_{GS} - V_T = 1.25 - 0.595 = 0.655 \text{ V}$$

$$\sqrt{\frac{2\epsilon_s}{eN_a}} = \sqrt{\frac{2(11.7)(8.85 \times 10^{-14})}{(1.6 \times 10^{-19})(4 \times 10^{16})}} \\ = 1.799 \times 10^{-5} \text{ cm/V}^{1/2}$$

$$(a) \Delta L = \sqrt{\frac{2\epsilon_s}{eN_a}} \left[ \sqrt{\phi_{fp} + V_{DS}(\text{sat}) + \Delta V_{DS}} - \sqrt{\phi_{fp} + V_{DS}(\text{sat})} \right]$$

$$(i) \Delta L = (1.799 \times 10^{-5}) \\ \times \left[ \sqrt{0.3832 + 0.655 + 1} - \sqrt{0.3832 + 0.655} \right] \\ \Delta L = 7.35 \times 10^{-5} \text{ cm} = 0.0735 \mu\text{m}$$

$$(ii) \Delta L = (1.799 \times 10^{-5}) \\ \times \left[ \sqrt{0.3832 + 0.655 + 2} - \sqrt{0.3832 + 0.655} \right] \\ \Delta L = 1.303 \times 10^{-5} \text{ cm} = 0.1303 \mu\text{m}$$

$$(iii) \Delta L = (1.799 \times 10^{-5}) \\ \times \left[ \sqrt{0.3832 + 0.655 + 4} - \sqrt{0.3832 + 0.655} \right] \\ \Delta L = 2.205 \times 10^{-5} \text{ cm} = 0.2205 \mu\text{m}$$

$$(b) \frac{\Delta L}{L} = 0.12 = \frac{0.2205}{L} \\ L = 1.84 \mu\text{m}$$

7.

(a)

$$\begin{aligned} \text{(i)} \quad I_D &= \frac{k'_n}{2} \cdot \frac{W}{L} (V_{GS} - V_T)^2 \\ &= \left( \frac{0.075}{2} \right) (10) (0.8 - 0.35)^2 \\ &= 0.07594 \text{ mA} = 75.94 \mu\text{A} \end{aligned}$$

$$\begin{aligned} \text{(ii)} \quad I'_D &= I_D (1 + \lambda V_{DS}) \\ &= (75.9375) [1 + (0.02)(1.5)] \\ &= 78.22 \mu\text{A} \end{aligned}$$

$$\begin{aligned} \text{(iii)} \quad r_o &= \frac{1}{\lambda I_D} = \frac{1}{(0.02)(75.94)} \\ &= 0.658 \text{ M}\Omega = 658 \text{ k}\Omega \end{aligned}$$

(b)

$$\begin{aligned} \text{(i)} \quad I_D &= \left( \frac{0.075}{2} \right) (10) (1.25 - 0.35)^2 \\ &= 0.30375 \text{ mA} \end{aligned}$$

$$\begin{aligned} \text{(ii)} \quad I'_D &= (0.30375) [1 + (0.02)(1.5)] \\ &= 0.3129 \text{ mA} \end{aligned}$$

$$\text{(iii)} \quad r_o = \frac{1}{(0.02)(0.30375)} = 165 \text{ k}\Omega$$

8.

(a)

$$\begin{aligned} \text{(i)} \quad I_D(\text{max}) &= \frac{k'_n}{2} \cdot \frac{W}{L} (V_{GS} - V_T)^2 \\ &= \left( \frac{0.15}{2} \right) \left( \frac{6}{1.2} \right) (3 - 0.45)^2 \\ &= 2.438 \text{ mA} \end{aligned}$$

(ii) Scaled device:

$$V_D = V_{GS} = k(3) = (0.65)(3) = 1.95 \text{ V}$$

$$k'_n = \left( \frac{0.15}{k} \right) = \left( \frac{0.15}{0.65} \right) = 0.2308 \text{ mA/V}^2$$

$$L = k(1.2) = (0.65)(1.2) = 0.78 \mu\text{m}$$

$$W = k(6) = (0.65)(6) = 3.90 \mu\text{m}$$

Then

$$\begin{aligned} I_D(\text{max}) &= \left( \frac{0.2308}{2} \right) \left( \frac{3.9}{0.78} \right) (1.95 - 0.45)^2 \\ &= 1.298 \text{ mA} \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad \text{(i)} \quad P(\text{max}) &= I_D(\text{max}) V_D = (2.438)(3) \\ &= 7.314 \text{ mW} \end{aligned}$$

$$\begin{aligned} \text{(ii)} \quad P(\text{max}) &= (1.298)(1.95) \\ &= 2.531 \text{ mW} \end{aligned}$$

9.

$$\begin{aligned}
 C_{ox} &= \frac{(3.9)(8.85 \times 10^{-14})}{80 \times 10^{-8}} \\
 &= 4.314 \times 10^{-7} \text{ F/cm}^2 \\
 \phi_{\text{fb}} &= (0.0259) \ln \left( \frac{2 \times 10^{16}}{1.5 \times 10^{10}} \right) = 0.3653 \text{ V} \\
 x_{dT} &= \left[ \frac{4(11.7)(8.85 \times 10^{-14})(0.3653)}{(1.6 \times 10^{-19})(2 \times 10^{16})} \right]^{1/2} \\
 &= 2.174 \times 10^{-5} \text{ cm} \\
 \Delta V_T &= - \frac{(1.6 \times 10^{-19})(2 \times 10^{16})(2.174 \times 10^{-5})}{4.314 \times 10^{-7}} \\
 &\quad \times \left[ \frac{0.30}{0.70} \left( \sqrt{1 + \frac{2(0.2174)}{0.30}} - 1 \right) \right]
 \end{aligned}$$

$$\Delta V_T = -0.0391 \text{ V}$$

$$V_T = V_{TO} + \Delta V_T$$

$$0.35 = V_{TO} - 0.0391$$

$$\Rightarrow V_{TO} = 0.389 \text{ V}$$



$$\Delta V_T = \frac{eN_a x_{dT}}{C_{ox}} \left( \frac{\xi x_{dT}}{W} \right)$$

We find

$$\phi_{\text{fb}} = (0.0259) \ln \left( \frac{10^{16}}{1.5 \times 10^{10}} \right) = 0.347 \text{ V}$$

and

$$\begin{aligned}
 x_{dT} &= \left[ \frac{4 \epsilon \phi_{\text{fb}}}{eN_a} \right]^{1/2} \\
 &= \left[ \frac{4(11.7)(8.85 \times 10^{-14})(0.347)}{(1.6 \times 10^{-19})(10^{16})} \right]^{1/2}
 \end{aligned}$$

or

$$x_{dT} = 0.30 \text{ } \mu\text{m}$$

Also

$$C_{ox} = \frac{\epsilon_{ox}}{t_{ox}} = \frac{(3.9)(8.85 \times 10^{-14})}{450 \times 10^{-8}}$$

or

$$C_{ox} = 7.67 \times 10^{-8} \text{ F/cm}^2$$

Then

$$\begin{aligned}
 \Delta V_T &= \frac{(1.6 \times 10^{-19})(10^{16})(0.3 \times 10^{-4})}{7.67 \times 10^{-8}} \\
 &\quad \times \left[ \frac{(\pi/2)(0.3 \times 10^{-4})}{2.5 \times 10^{-4}} \right]
 \end{aligned}$$

or

$$\Delta V_T = +0.118 \text{ V}$$