# VE320 HW1 Solution

1.

$$E = h\nu = \frac{hc}{\lambda} \Rightarrow \lambda = \frac{hc}{E}$$

Gold: $E = 4.90 \text{ eV} = (4.90)(1.6\times10^{-19})$ J

So,

$$\lambda = \frac{(6.625\times10^{-34})(3\times10^{10})}{(4.90)(1.6\times10^{-19})} = 2.54\times10^{-5} \text{ cm}$$

or

$$\lambda = 0.254\ \mu\text{m}$$

Cesium: $E = 1.90 \text{ eV} = (1.90)(1.6\times10^{-19})$ J

So,

$$\lambda = \frac{(6.625\times10^{-34})(3\times10^{10})}{(1.90)(1.6\times10^{-19})} = 6.54\times10^{-5} \text{ cm}$$

or

$$\lambda = 0.654\ \mu\text{m}$$

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2.

(a) $p = \dfrac{h}{\lambda} = \dfrac{6.625\times10^{-34}}{550\times10^{-9}}$

$= 1.205\times10^{-27}$ kg-m/s

$\upsilon = \dfrac{p}{m} = \dfrac{1.2045\times10^{-27}}{9.11\times10^{-31}} = 1.32\times10^{3}$ m/s

or $\upsilon = 1.32\times10^{5}$ cm/s

(b) $p = \dfrac{h}{\lambda} = \dfrac{6.625\times10^{-34}}{440\times10^{-9}}$

$= 1.506\times10^{-27}$ kg-m/s

$\upsilon = \dfrac{p}{m} = \dfrac{1.5057\times10^{-27}}{9.11\times10^{-31}} = 1.65\times10^{3}$ m/s

or $\upsilon = 1.65\times10^{5}$ cm/s

(c) Yes

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3.

(a)

Simple cubic: $a = 4.50\ \mathring{A}$

(i) (100) plane, surface density,

$$= \frac{1\ atom}{(4.50x10^{-8})^2} \Rightarrow \underline{4.94x10^{14}\ cm^{-2}}$$

(ii) (110) plane, surface density,

$$= \frac{1\ atom}{\sqrt{2}(4.50x10^{-8})^2} \Rightarrow \underline{3.49x10^{14}\ cm^{-2}}$$

(iii) (111) plane, surface density,

$$= \frac{3\left(\frac{1}{6}\right) atoms}{\frac{1}{2}(a\sqrt{2})(x)} = \frac{\frac{1}{2}}{\frac{1}{2}\cdot a\sqrt{2}\cdot\frac{a\sqrt{3}}{\sqrt{2}}} = \frac{1}{\sqrt{3}a^2}$$

$$= \frac{1}{\sqrt{3}(4.50x10^{-8})^2} \Rightarrow \underline{2.85x10^{14}\ cm^{-2}}$$

(b)
Body-centered cubic

(i) (100) plane, surface density,

Same as (a),(i); surface density $\underline{4.94x10^{14}\ cm^{-2}}$

(ii) (110) plane, surface density,

$$= \frac{2\ atoms}{\sqrt{2}(4.50x10^{-8})^2} \Rightarrow \underline{6.99x10^{14}\ cm^{-2}}$$

(iii) (111) plane, surface density,

Same as (a),(iii), surface density $\underline{2.85x10^{14}\ cm^{-2}}$

(c)
Face centered cubic

(i) (100) plane, surface density

$$= \frac{2\ atoms}{(4.50x10^{-8})^2} \Rightarrow \underline{9.88x10^{14}\ cm^{-2}}$$

(ii) (110) plane, surface density,

$$= \frac{2\ atoms}{\sqrt{2}(4.50x10^{-8})^2} \Rightarrow \underline{6.99x10^{14}\ cm^{-2}}$$

(iii) (111) plane, surface density,

$$= \frac{\left(3\cdot\frac{1}{6}+3\cdot\frac{1}{2}\right)}{\frac{\sqrt{3}}{2}a^2} = \frac{4}{\sqrt{3}(4.50x10^{-8})^2}$$

or $\underline{1.14x10^{15}\ cm^{-2}}$

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4.

$$E_{avg} = \frac{3}{2}kT = \left(\frac{3}{2}\right)(0.0259) = 0.03885 \text{ eV}$$

Now

$$p_{avg} = \sqrt{2mE_{avg}}$$

$$= \sqrt{2(9.11\times10^{-31})(0.03885)(1.6\times10^{-19})}$$

or

$$p_{avg} = 1.064\times10^{-25}\text{ kg-m/s}$$

Now

$$\lambda = \frac{h}{p} = \frac{6.625\times10^{-34}}{1.064\times10^{-25}} = 6.225\times10^{-9}\text{ m}$$

or

$$\lambda = 62.25\ \overset{o}{A}$$

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5.

(a) $$E_n = \frac{\hbar^2 n^2 \pi^2}{2ma^2} = \frac{n^2(1.054\times10^{-34})^2\pi^2}{2(9.11\times10^{-31})(10\times10^{-10})^2}$$

$$= n^2(6.018\times10^{-20})\text{ J}$$

or $$E_n = \frac{n^2(6.018\times10^{-20})}{1.6\times10^{-19}} = n^2(0.3761)\text{ eV}$$

Then

$$E_1 = 0.376\text{ eV}$$
$$E_2 = 1.504\text{ eV}$$
$$E_3 = 3.385\text{ eV}$$

(b) $$\lambda = \frac{hc}{\Delta E}$$

$$\Delta E = (3.385 - 1.504)(1.6\times10^{-19})$$

$$= 3.01\times10^{-19}\text{ J}$$

$$\lambda = \frac{(6.625\times10^{-34})(3\times10^{8})}{3.01\times10^{-19}}$$

$$= 6.604\times10^{-7}\text{ m}$$

or $$\lambda = 660.4\text{ nm}$$

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6.

(a) $$p = \frac{h}{\lambda} = \frac{6.625\times10^{-34}}{85\times10^{-10}}$$

$$= 7.794\times10^{-26}\text{ kg-m/s}$$

$$\upsilon = \frac{p}{m} = \frac{7.794\times10^{-26}}{9.11\times10^{-31}} = 8.56\times10^{4}\text{ m/s}$$

or $$\upsilon = 8.56\times10^{6}\text{ cm/s}$$

$$E = \frac{1}{2}m\upsilon^2 = \frac{1}{2}(9.11\times10^{-31})(8.56\times10^{4})^2$$

$$= 3.33\times10^{-21}\text{ J}$$

or $$E = \frac{3.334\times10^{-21}}{1.6\times10^{-19}} = 2.08\times10^{-2}\text{ eV}$$

(b) $$E = \frac{1}{2}(9.11\times10^{-31})(8\times10^{3})^2$$

$$= 2.915\times10^{-23}\text{ J}$$

or $$E = \frac{2.915\times10^{-23}}{1.6\times10^{-19}} = 1.82\times10^{-4}\text{ eV}$$

$$p = m\upsilon = (9.11\times10^{-31})(8\times10^{3})$$

$$= 7.288\times10^{-27}\text{ kg-m/s}$$

$$\lambda = \frac{h}{p} = \frac{6.625\times10^{-35}}{7.288\times10^{-27}} - 9.09\times10^{-8}\text{ m}$$

or $$\lambda = 909\ \overset{o}{A}$$

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7.

$$\int_{-1}^{+3} A^2\cos^2\left(\frac{\pi x}{2}\right)dx = 1$$

$$A^2\left[\frac{x}{2} + \frac{\sin(\pi x)}{2\pi}\right]\Bigg|_{-1}^{+3} = 1$$

$$A^2\left[\frac{3}{2} - \left(\frac{-1}{2}\right)\right] = 1$$

so $$A^2 = \frac{1}{2}$$

or $$|A| = \frac{1}{\sqrt{2}}$$

8.

(a) Region I: Since $V_O > E$, we can write

$$\frac{\partial^2 \psi_1(x)}{\partial x^2} - \frac{2m(V_O - E)}{\hbar^2}\psi_1(x) = 0$$

Region II: $V = 0$, so

$$\frac{\partial^2 \psi_2(x)}{\partial x^2} + \frac{2mE}{\hbar^2}\psi_2(x) = 0$$

Region III: $V \to \infty \Rightarrow \psi_3 = 0$

The general solutions can be written, keeping in mind that $\psi_1$ must remain finite for $x < 0$, as

$$\psi_1(x) = B_1 \exp(k_1 x)$$
$$\psi_2(x) = A_2 \sin(k_2 x) + B_2 \cos(k_2 x)$$
$$\psi_3(x) = 0$$

where

$$k_1 = \sqrt{\frac{2m(V_O - E)}{\hbar^2}} \text{ and } k_2 = \sqrt{\frac{2mE}{\hbar^2}}$$

(b) Boundary conditions

At $x = 0$: $\psi_1 = \psi_2 \Rightarrow B_1 = B_2$

$$\frac{\partial \psi_1}{\partial x} = \frac{\partial \psi_2}{\partial x} \Rightarrow k_1 B_1 = k_2 A_2$$

At $x = a$: $\psi_2 = \psi_3 \Rightarrow$

At $x = a$: $\psi_2 = \psi_3 \Rightarrow$

$$A_2 \sin(k_2 a) + B_2 \cos(k_2 a) = 0$$

or

$$B_2 = -A_2 \tan(k_2 a)$$

(c)

$$k_1 B_1 = k_2 A_2 \Rightarrow A_2 = \left(\frac{k_1}{k_2}\right) B_1$$

and since $B_1 = B_2$, then

$$A_2 = \left(\frac{k_1}{k_2}\right) B_2$$

From $B_2 = -A_2 \tan(k_2 a)$, we can write

$$B_2 = -\left(\frac{k_1}{k_2}\right) B_2 \tan(k_2 a)$$

or

$$1 = -\left(\frac{k_1}{k_2}\right) \tan(k_2 a)$$

This equation can be written as

$$1 = -\sqrt{\frac{V_O - E}{E}} \cdot \tan\left[\sqrt{\frac{2mE}{\hbar^2}} \cdot a\right]$$

or

$$\sqrt{\frac{E}{V_O - E}} = -\tan\left[\sqrt{\frac{2mE}{\hbar^2}} \cdot a\right]$$

This last equation is valid only for specific values of the total energy $E$. The energy levels are quantized.