VE320 HW3 Solution

1.

(a) $$\frac{g_c}{g_\upsilon} = \frac{(m_n^*)^{3/2}}{(m_p^*)^{3/2}} = \left(\frac{1.08}{0.56}\right)^{3/2} = 2.68$$

(b) $$\frac{g_c}{g_\upsilon} = \frac{(m_n^*)^{3/2}}{(m_p^*)^{3/2}} = \left(\frac{0.067}{0.48}\right)^{3/2} = 0.0521$$

2.

(a) $$f_F \cong \exp\left[\frac{-(E-E_F)}{kT}\right]$$

$$E = E_c;\ f_F = \exp\left[\frac{-0.30}{0.0259}\right] = 9.32\times10^{-6}$$

$$E_c + \frac{kT}{2};\ f_F = \exp\left[\frac{-(0.30+0.0259/2)}{0.0259}\right] = 5.66\times10^{-6}$$

$$E_c + kT;\ f_F = \exp\left[\frac{-(0.30+0.0259)}{0.0259}\right] = 3.43\times10^{-6}$$

$$E_c + \frac{3kT}{2};\ f_F = \exp\left[\frac{-(0.30+3(0.0259/2))}{0.0259}\right] = 2.08\times10^{-6}$$

$$E_c + 2kT;\ f_F = \exp\left[\frac{-(0.30+2(0.0259))}{0.0259}\right] = 1.26\times10^{-6}$$

(b) $$1 - f_F = 1 - \frac{1}{1+\exp\left[\frac{E-E_F}{kT}\right]} \cong \exp\left[\frac{-(E_F-E)}{kT}\right]$$

$$E = E_\upsilon;\ 1-f_F = \exp\left[\frac{-0.25}{0.0259}\right] = 6.43\times10^{-5}$$

$$E_\upsilon - \frac{kT}{2};\ 1-f_F = \exp\left[\frac{-(0.25+0.0259/2)}{0.0259}\right] = 3.90\times10^{-5}$$

$$E_\upsilon - kT;\ 1-f_F = \exp\left[\frac{-(0.25+0.0259)}{0.0259}\right] = 2.36\times10^{-5}$$

$$E_\upsilon - \frac{3kT}{2};\ 1-f_F = \exp\left[\frac{-(0.25+3(0.0259/2))}{0.0259}\right] = 1.43\times10^{-5}$$

$$E_\upsilon - 2kT;\ 1-f_F = \exp\left[\frac{-(0.25+2(0.0259))}{0.0259}\right] = 8.70\times10^{-6}$$

3.

(a) $$f_F = \exp\left[\frac{-(E-E_F)}{kT}\right]$$

$$10^{-8} = \exp\left[\frac{-0.60}{kT}\right]$$

or $$\frac{0.60}{kT} = \ln(10^{+8})$$

$$kT = \frac{0.60}{\ln(10^8)} = 0.032572 \text{ eV}$$

$$0.032572 = (0.0259)\left(\frac{T}{300}\right)$$

so $T = 377$ K

(b) $$10^{-6} = \exp\left[\frac{-0.60}{kT}\right]$$

$$\frac{0.60}{kT} = \ln(10^{+6})$$

$$kT = \frac{0.60}{\ln(10^6)} = 0.043429$$

$$0.043429 = (0.0259)\left(\frac{T}{300}\right)$$

or $T = 503$ K

4.

(a) $$E_{Fi} - E_{midgap} = \frac{3}{4}kT\ln\left(\frac{m_p^*}{m_n^*}\right) = \frac{3}{4}(0.0259)\ln\left(\frac{0.70}{1.21}\right) \Rightarrow -10.63 \text{ meV}$$

(b) $$E_{Fi} - E_{midgap} = \frac{3}{4}(0.0259)\ln\left(\frac{0.75}{0.080}\right) \Rightarrow +43.47 \text{ meV}$$

5.

(a) $E_F - E_\upsilon = kT \ln\left(\dfrac{N_\upsilon}{p_o}\right)$

$$= (0.0259)\ln\left(\frac{1.04\times10^{19}}{5\times10^{15}}\right)$$

$$= 0.1979 \text{ eV}$$

(b) $E_c - E_F = E_g - (E_F - E_\upsilon)$

$$= 1.12 - 0.19788 = 0.92212 \text{ eV}$$

(c) $n_o = (2.8\times10^{19})\exp\left[\dfrac{-0.92212}{0.0259}\right]$

$$= 9.66\times10^3 \text{ cm}^{-3}$$

(d) Holes

(e) $E_{Fi} - E_F = kT \ln\left(\dfrac{p_o}{n_i}\right)$

$$= (0.0259)\ln\left(\frac{5\times10^{15}}{1.5\times10^{10}}\right)$$

$$= 0.3294 \text{ eV}$$

6.

(a) Ge: $n_i = 2.4\times10^{13} \text{ cm}^{-3}$

(i) $n_o = \dfrac{N_d}{2} + \sqrt{\left(\dfrac{N_d}{2}\right)^2 + n_i^2}$

$$= \frac{2\times10^{15}}{2} + \sqrt{\left(\frac{2\times10^{15}}{2}\right)^2 + (2.4\times10^{13})^2}$$

or

$$n_o \cong N_d = 2\times10^{15} \text{ cm}^{-3}$$

$$p_o = \frac{n_i^2}{n_o} = \frac{(2.4\times10^{13})^2}{2\times10^{15}}$$

$$= 2.88\times10^{11} \text{ cm}^{-3}$$

(ii) $p_o \cong N_a - N_d = 10^{16} - 7\times10^{15}$

$$= 3\times10^{15} \text{ cm}^{-3}$$

$$n_o = \frac{n_i^2}{p_o} = \frac{(2.4\times10^{13})^2}{3\times10^{15}}$$

$$= 1.92\times10^{11} \text{ cm}^{-3}$$

(b) GaAs: $n_i = 1.8\times10^6 \text{ cm}^{-3}$

(i) $n_o \cong N_d = 2\times10^{15}$ cm

$$p_o = \frac{(1.8\times10^6)^2}{2\times10^{15}} = 1.62\times10^{-3} \text{ cm}^{-3}$$

(ii) $p_o \cong N_a - N_d = 3\times10^{15} \text{ cm}^{-3}$

$$n_o = \frac{(1.8\times10^6)^2}{3\times10^{15}} = 1.08\times10^{-3} \text{ cm}^{-3}$$

(c) The result implies that there is only one minority carrier in a volume of 10^3 cm^3.

7.

(a) $N_a > N_d \Rightarrow$ p-type

Majority carriers are holes

$$p_o = N_a - N_d = 3\times10^{16} - 1.5\times10^{16}$$

$$= 1.5\times10^{16} \text{ cm}^{-3}$$

Minority carriers are electrons

$$n_o = \frac{n_i^2}{p_o} = \frac{(1.5\times10^{10})^2}{1.5\times10^{16}} = 1.5\times10^4 \text{ cm}^{-3}$$

(b) Boron atoms must be added

$$p_o = N_a' + N_a - N_d$$

$$5\times10^{16} = N_a' + 3\times10^{16} - 1.5\times10^{16}$$

So $N_a' = 3.5\times10^{16} \text{ cm}^{-3}$

$$n_o = \frac{(1.5\times10^{10})^2}{5\times10^{16}} = 4.5\times10^3 \text{ cm}^{-3}$$

8.

(a) $n_o = \dfrac{N_d}{2} + \sqrt{\left(\dfrac{N_d}{2}\right)^2 + n_i^2}$

$$n_o = 1.05N_d = 1.05\times10^{15} \text{ cm}^{-3}$$

$$(1.05\times10^{15} - 0.5\times10^{15})^2 = (0.5\times10^{15})^2 + n_i^2$$

so $n_i^2 = 5.25\times10^{28}$

Now

$$n_i^2 = (2.8\times10^{19})(1.04\times10^{19})\left(\frac{T}{300}\right)^3 \times \exp\left[\frac{-1.12}{(0.0259)(T/300)}\right]$$

$$5.25\times10^{28} = (2.912\times10^{38})\left(\frac{T}{300}\right)^3 \times \exp\left[\frac{-12972.973}{T}\right]$$

By trial and error, $T = 536.5$ K

(b) At $T = 300$ K,

$$E_c - E_F = kT\ln\left(\frac{N_c}{n_o}\right)$$

$$E_c - E_F = (0.0259)\ln\left(\frac{2.8\times10^{19}}{10^{15}}\right)$$

$$= 0.2652 \text{ eV}$$

At $T = 536.5$ K,

$$kT = (0.0259)\left(\frac{536.5}{300}\right) = 0.046318 \text{ eV}$$

$$N_c = (2.8\times10^{19})\left(\frac{536.5}{300}\right)^{3/2}$$

$$= 6.696\times10^{19} \text{ cm}^{-3}$$

$$E_c - E_F = kT\ln\left(\frac{N_c}{n_o}\right)$$

$$E_c - E_F = (0.046318)\ln\left(\frac{6.696\times10^{19}}{1.05\times10^{15}}\right)$$

$$= 0.5124 \text{ eV}$$

then $\Delta(E_c - E_F) = 0.2472$ eV

(c) Closer to the intrinsic energy level.

9.

(a) $E_{Fi} - E_{midgap} = \frac{3}{4} kT \ln\left(\frac{m_p^*}{m_n^*}\right)$

$= \frac{3}{4}(0.0259)\ln(10)$

or

$E_{Fi} - E_{midgap} = +0.0447$ eV

(b) Impurity atoms to be added so

$E_{midgap} - E_F = 0.45$ eV

(i) p-type, so add acceptor atoms

(ii) $E_{Fi} - E_F = 0.0447 + 0.45 = 0.4947$ eV

Then

$p_o = n_i \exp\left(\frac{E_{Fi} - E_F}{kT}\right)$

$= (10^5)\exp\left(\frac{0.4947}{0.0259}\right)$

or

$p_o = N_a = 1.97 \times 10^{13}$ cm^{-3}

10.

(a) Replace Ga atoms $\Rightarrow$ Silicon acts as a donor

$N_d = (0.05)(7 \times 10^{15}) = 3.5 \times 10^{14}$ cm^{-3}

Replace As atoms $\Rightarrow$ Silicon acts as an acceptor

$N_a = (0.95)(7 \times 10^{15}) = 6.65 \times 10^{15}$ cm^{-3}

(b) $N_a > N_d \Rightarrow$ p-type

(c) $p_o = N_a - N_d = 6.65 \times 10^{15} - 3.5 \times 10^{14}$

$= 6.3 \times 10^{15}$ cm^{-3}

$n_o = \frac{n_i^2}{p_o} = \frac{(1.8 \times 10^6)^2}{6.3 \times 10^{15}} = 5.14 \times 10^{-4}$ cm^{-3}

(a) $E_{Fi} - E_F = kT \ln\left(\frac{p_o}{n_i}\right)$

$= (0.0259)\ln\left(\frac{6.3 \times 10^{15}}{1.8 \times 10^6}\right) = 0.5692$ eV

11.

(a) $n_i^2 = N_c N_\upsilon \exp\left(\frac{-E_g}{kT}\right)$

$= (2 \times 10^{19})(1 \times 10^{19})\exp\left(\frac{-1.10}{0.0259}\right)$

$= 7.18 \times 10^{19}$

or

$n_i = 8.47 \times 10^9$ cm^{-3}

For $N_d = 10^{14}$ cm$^{-3} >> n_i \Rightarrow n_o = 10^{14}$ cm^{-3}

Then

$J = \sigma \mathrm{E} = e\mu_n n_o \mathrm{E}$

$= (1.6 \times 10^{-19})(1000)(10^{14})(100)$

or

$J = 1.60$ A/cm^2

(b) A 5% increase is due to a 5% increase in electron concentration, so

$n_o = 1.05 \times 10^{14} = \frac{N_d}{2} + \sqrt{\left(\frac{N_d}{2}\right)^2 + n_i^2}$

which becomes

$(1.05 \times 10^{14} - 5 \times 10^{13})^2 = (5 \times 10^{13})^2 + n_i^2$

and yields

$n_i^2 = 5.25 \times 10^{26}$

$= (2 \times 10^{19})(1 \times 10^{19})\left(\frac{T}{300}\right)^3 \exp\left(\frac{-E_g}{kT}\right)$

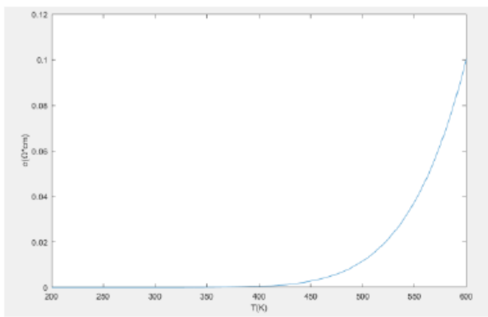
or

$2.625 \times 10^{-12} = \left(\frac{T}{300}\right)^3 \exp\left[\frac{-1.10}{(0.0259)(T/300)}\right]$

By trial and error, we find

$T = 456$ K

12.



13.

(a) $$E_X = -\left(\frac{kT}{e}\right)\cdot\frac{1}{N_d(x)}\cdot\frac{dN_d(x)}{dx}$$

$$= \frac{-(0.0259)}{N_{do}e^{-x/L}}\cdot\frac{d}{dx}\left[N_{do}e^{-x/L}\right]$$

$$= \frac{-(0.0259)}{N_{do}e^{-x/L}}\cdot\left(\frac{-1}{L}\right)N_{do}e^{-x/L}$$

$$= \frac{0.0259}{L} = \frac{0.0259}{10\times10^{-4}}$$

or $E_X = 25.9$ V/cm

(b) $$\phi = -\int_0^L E_X\,dx = -(25.9)(L-0)$$

$$= -(25.9)(10\times10^{-4}) = -0.0259 \text{ V}$$

or $\phi = -25.9$ mV

14.

(a) (i) $D_n = (0.0259)(1150) = 29.8 \text{ cm}^2/\text{s}$

(ii) $D_n = (0.0259)(6200) = 160.6 \text{ cm}^2/\text{s}$

(b) (i) $$\mu_p = \frac{8}{0.0259} = 308.9 \text{ cm}^2/\text{V-s}$$

(ii) $$\mu_p = \frac{35}{0.0259} = 1351 \text{ cm}^2/\text{V-s}$$

15.

$$J_n = eD_n\frac{dn}{dx} = eD_n\frac{\Delta n}{\Delta x}$$

$$J_n = (1.6\times10^{-19})(27)\left[\frac{2\times10^{16} - 5\times10^{15}}{0-0.012}\right]$$

$$J_n = -5.4 \text{ A/cm}^2$$