Hw6 solution

1.

(b) $N_d = n_i \exp\left(\frac{E_F - E_{Fi}}{kT}\right)$

$$= \left(1.5\times10^{10}\right)\exp\left(\frac{0.365}{0.0259}\right)$$

or $N_d = 1.98\times10^{16}\ \text{cm}^{-3}$

$$N_a = n_i \exp\left(\frac{E_{Fi} - E_F}{kT}\right)$$

$$= \left(1.5\times10^{10}\right)\exp\left(\frac{0.330}{0.0259}\right)$$

or $N_a = 5.12\times10^{15}\ \text{cm}^{-3}$

(c)

$$V_{bi} = (0.0259)\ln\left[\frac{\left(5.12\times10^{15}\right)\left(1.98\times10^{16}\right)}{\left(1.5\times10^{10}\right)^2}\right]$$

$$= 0.695\ \text{V}$$

2.

$$x_n = 0.25W = 0.25\left(x_n + x_p\right)$$

$$0.75x_n = 0.25x_p \Rightarrow \frac{x_p}{x_n} = 3$$

$$x_n N_d = x_p N_a \Rightarrow \frac{N_d}{N_a} = \frac{x_p}{x_n} = 3$$

So $N_d = 3N_a$

(a) $V_{bi} = (0.0259)\ln\left[\frac{N_a N_d}{\left(1.5\times10^{10}\right)^2}\right]$

$$0.710 = (0.0259)\ln\left[\frac{3N_a^2}{\left(1.5\times10^{10}\right)^2}\right]$$

or $3N_a^2 = \left(1.5\times10^{10}\right)^2 \exp\left(\frac{0.710}{0.0259}\right)$

which yields $N_a = 7.766\times10^{15}\ \text{cm}^{-3}$

$N_d = 2.33\times10^{16}\ \text{cm}^{-3}$

$$x_n = \left\{\frac{2\in_s V_{bi}}{e}\left(\frac{N_a}{N_d}\right)\left(\frac{1}{N_a + N_d}\right)\right\}^{1/2}$$

$$= \left\{\frac{2(11.7)\left(8.85\times10^{-14}\right)(0.710)}{1.6\times10^{-19}} \times\left(\frac{1}{3}\right)\left[\frac{1}{4\left(7.766\times10^{15}\right)}\right]\right\}^{1/2}$$

$$\Rightarrow x_n = 9.93\times10^{-6}\ \text{cm}$$

or $x_n = 0.0993\ \mu\text{m}$

$$x_p = \left\{\frac{2(11.7)\left(8.85\times10^{-14}\right)(0.710)}{1.6\times10^{-19}} \times\left(\frac{3}{1}\right)\left[\frac{1}{4\left(7.766\times10^{15}\right)}\right]\right\}^{1/2}$$

$$= 2.979\times10^{-5}\ \text{cm}$$

or $x_p = 0.2979\ \mu\text{m}$

Now

$$\left|\mathrm{E}_{max}\right| = \frac{eN_d x_n}{\in_s}$$

$$= \frac{\left(1.6\times10^{-19}\right)\left(2.33\times10^{16}\right)\left(0.0993\times10^{-4}\right)}{(11.7)\left(8.85\times10^{-14}\right)}$$

$$= 3.58\times10^{4}\ \text{V/cm}$$

(b) From part (a), we can write

$$3N_a^2 = \left(1.8\times10^{6}\right)^2 \exp\left(\frac{1.180}{0.0259}\right)$$

which yields $N_a = 8.127\times10^{15}\ \text{cm}^{-3}$

$N_d = 2.438\times10^{16}\ \text{cm}^{-3}$

$$x_n = \left\{\frac{2(13.1)\left(8.85\times10^{-14}\right)(1.180)}{1.6\times10^{-19}} \times\left(\frac{1}{3}\right)\left[\frac{1}{4\left(8.127\times10^{15}\right)}\right]\right\}^{1/2}$$

$$= 1.324\times10^{-5}\ \text{cm}$$

or $x_n = 0.1324\ \mu\text{m}$

$$x_p = \left\{\frac{2(13.1)\left(8.85\times10^{-14}\right)(1.180)}{1.6\times10^{-19}} \times\left(\frac{3}{1}\right)\left[\frac{1}{4\left(8.127\times10^{15}\right)}\right]\right\}^{1/2}$$

$$= 3.973\times10^{-5}\ \text{cm}$$

or $x_p = 0.3973\ \mu\text{m}$

$$\left|\mathrm{E}_{max}\right| = \frac{eN_d x_n}{\in_s}$$

$$= \frac{\left(1.6\times10^{-19}\right)\left(2.438\times10^{16}\right)\left(0.1324\times10^{-4}\right)}{(13.1)\left(8.85\times10^{-14}\right)}$$

$$= 4.45\times10^{4}\ \text{V/cm}$$

3.

(a) For $N_d = 10^{16}\ \mathrm{cm}^{-3}$,

$$E_F - E_{Fi} = kT \ln\left(\frac{N_d}{n_i}\right)$$

$$= (0.0259)\ln\left(\frac{10^{16}}{1.5\times 10^{10}}\right)$$

or

$$E_F - E_{Fi} = 0.3473\ \mathrm{eV}$$

For $N_d = 10^{15}\ \mathrm{cm}^{-3}$

$$E_F - E_{Fi} = (0.0259)\ln\left(\frac{10^{15}}{1.5\times 10^{10}}\right)$$

or

$$E_F - E_{Fi} = 0.2877\ \mathrm{eV}$$

Then

$$V_{bi} = 0.34732 - 0.28768$$

or

$$V_{bi} = 0.0596\ \mathrm{V}$$

4.

(a) $$V_{bi} = (0.0259)\ln\left[\frac{(5\times 10^{16})(10^{15})}{(1.5\times 10^{10})^2}\right]$$

$$= 0.6767\ \mathrm{V}$$

(b) $$W = \left\{\frac{2\in_s (V_{bi}+V_R)}{e}\left(\frac{N_a+N_d}{N_a N_d}\right)\right\}^{1/2}$$

(i) For $V_R = 0$,

$$W = \left\{\frac{2(11.7)(8.85\times 10^{-14})(0.6767)}{1.6\times 10^{-19}} \times \left[\frac{5\times 10^{16}+10^{15}}{(5\times 10^{16})(10^{15})}\right]\right\}^{1/2}$$

$$= 9.452\times 10^{-5}\ \mathrm{cm}$$

or $W = 0.9452\ \mu\mathrm{m}$

(ii) For $V_R = 5\ \mathrm{V}$,

$$W = \left\{\frac{2(11.7)(8.85\times 10^{-14})(0.6767+5)}{1.6\times 10^{-19}} \times \left[\frac{5\times 10^{16}+10^{15}}{(5\times 10^{16})(10^{15})}\right]\right\}^{1/2}$$

$$= 2.738\times 10^{-4}\ \mathrm{cm}$$

or $W = 2.738\ \mu\mathrm{m}$

(c) $$|\mathrm{E}_{max}| = \frac{2(V_{bi}+V_R)}{W}$$

(i)For $V_R = 0$,

$$|\mathrm{E}_{max}| = \frac{2(0.6767)}{0.9452\times 10^{-4}} = 1.43\times 10^4\ \mathrm{V/cm}$$

(ii)For $V_R = 5\ \mathrm{V}$,

$$|\mathrm{E}_{max}| = \frac{2(0.6767+5)}{2.738\times 10^{-4}} = 4.15\times 10^4\ \mathrm{V/cm}$$

5.

(a) $$V_{bi} = V_t \ln\left(\frac{N_a N_d}{n_i^2}\right)$$

$$= V_t \ln\left(\frac{80 N_d^2}{n_i^2}\right)$$

We find

$$80 N_d^2 = n_i^2 \exp\left(\frac{V_{bi}}{V_t}\right)$$

$$= (1.5\times 10^{10})^2 \exp\left(\frac{0.740}{0.0259}\right)$$

$$= 5.762\times 10^{32}$$

$$\Rightarrow N_d = 2.684\times 10^{15}\ \mathrm{cm}^{-3}$$

$$N_a = 2.147\times 10^{17}\ \mathrm{cm}^{-3}$$

(b)

$$x_n = \left\{\frac{2\in_s (V_{bi}+V_R)}{e}\left(\frac{N_a}{N_d}\right)\left(\frac{1}{N_a+N_d}\right)\right\}^{1/2}$$

$$= \left\{\frac{2(11.7)(8.85\times 10^{-14})(0.740+10)}{1.6\times 10^{-19}} \times \left(\frac{80}{1}\right)\left(\frac{1}{2.147\times 10^{17}+2.684\times 10^{15}}\right)\right\}^{1/2}$$

$$= 2.262\times 10^{-4}\ \mathrm{cm}$$

or $x_n = 2.262\ \mu\mathrm{m}$

$$x_p = \left\{\frac{2\in_s (V_{bi}+V_R)}{e}\left(\frac{N_d}{N_a}\right)\left(\frac{1}{N_a+N_d}\right)\right\}^{1/2}$$

$$= \left\{\frac{2(11.7)(8.85\times 10^{-14})(0.740+10)}{1.6\times 10^{-19}} \times \left(\frac{1}{80}\right)\left(\frac{1}{2.147\times 10^{17}+2.684\times 10^{15}}\right)\right\}^{1/2}$$

$$= 2.83\times 10^{-6}\ \mathrm{cm}$$

or $x_p = 0.0283\ \mu\text{m}$

(c) $$\left|E_{max}\right| = \frac{2(V_{bi}+V_R)}{W}$$

$$= \frac{2(0.740+10)}{(2.262+0.0283)\times 10^{-4}}$$

$$= 9.38\times 10^{4}\ \text{V/cm}$$

(d) $$C' = \left\{\frac{e\in_s N_a N_d}{2(V_{bi}+V_R)(N_a+N_d)}\right\}^{1/2}$$

$$= \left\{\frac{(1.6\times 10^{-19})(11.7)(8.85\times 10^{-14})}{2(0.740+10)} \times\left[\frac{(2.147\times 10^{17})(2.684\times 10^{15})}{2.147\times 10^{17}+2.684\times 10^{15}}\right]\right\}^{1/2}$$

$$C' = 4.52\times 10^{-9}\ \text{F/cm}^2$$

6.

(a) $$V_{bi} = (0.0259)\ln\left[\frac{(5\times 10^{15})(10^{14})}{(1.5\times 10^{10})^2}\right]$$

or

$$V_{bi} = 0.5574\ \text{V}$$

(b)

$$x_p = \left[\frac{2\in_s V_{bi}}{e}\left(\frac{N_d}{N_a}\right)\left(\frac{1}{N_a+N_d}\right)\right]^{1/2}$$

$$= \left[\frac{2(11.7)(8.85\times 10^{-14})(0.5574)}{1.6\times 10^{-19}} \times\left(\frac{10^{14}}{5\times 10^{15}}\right)\left(\frac{1}{10^{14}+5\times 10^{15}}\right)\right]^{1/2}$$

or

$$x_p = 5.32\times 10^{-6}\ \text{cm}$$

Also

$$x_n = \left[\frac{2\in_s V_{bi}}{e}\left(\frac{N_a}{N_d}\right)\left(\frac{1}{N_a+N_d}\right)\right]^{1/2}$$

$$= \left[\frac{2(11.7)(8.85\times 10^{-14})(0.5574)}{1.6\times 10^{-19}}\right.$$

$$\left.\times\left(\frac{5\times 10^{15}}{10^{14}}\right)\left(\frac{1}{10^{14}+5\times 10^{15}}\right)\right]^{1/2}$$

or

$$x_n = 2.66\times 10^{-4}\ \text{cm}$$

(c) For $x_n = 30\ \mu\text{m}$, we have

$$30\times 10^{-4} = \left[\frac{2(11.7)(8.85\times 10^{-14})(V_{bi}+V_R)}{1.6\times 10^{-19}} \times\left(\frac{5\times 10^{15}}{10^{14}}\right)\left(\frac{1}{10^{14}+5\times 10^{15}}\right)\right]^{1/2}$$

which becomes

$$9\times 10^{-6} = 1.269\times 10^{-7}(V_{bi}+V_R)$$

We find

$$V_R = 70.4\ \text{V}$$

7.

(a) $$V_B = \frac{\in_s E_{crit}^2}{2eN_B}$$

or

$$N_B = \frac{\in_s E_{crit}^2}{2eV_B} = \frac{(11.7)(8.85\times 10^{-14})(4\times 10^{5})^2}{2(1.6\times 10^{-19})(40)}$$

Then $N_B = N_a = 1.294\times 10^{16}\ \text{cm}^{-3}$

(b) $$N_B = \frac{(11.7)(8.85\times 10^{-14})(4\times 10^{5})^2}{2(1.6\times 10^{-19})(20)}$$

Or $N_B = N_a = 2.59\times 10^{16}\ \text{cm}^{-3}$

8.

(a) $$n_{po} = \frac{n_i^2}{N_a} = \frac{(1.5\times 10^{10})^2}{5\times 10^{16}}$$

$$= 4.5\times 10^{3}\ \text{cm}^{-3}$$

$$p_{no} = \frac{n_i^2}{N_d} = \frac{(1.5\times 10^{10})^2}{5\times 10^{15}}$$

$$= 4.5\times 10^{4}\ \text{cm}^{-3}$$

(i) $$p_n(x_n) = p_{no}\exp\left(\frac{V_a}{V_t}\right)$$

or $$V_a = V_t\ln\left[\frac{p_n(x_n)}{p_{no}}\right]$$

$$= (0.0259)\ln\left[\frac{(0.1)(5\times 10^{15})}{4.5\times 10^{4}}\right]$$

$$= 0.599\ \text{V}$$

(ii) n-region - lower doped side

(b) $n_{po} = \dfrac{n_i^2}{N_a} = \dfrac{\left(1.5\times10^{10}\right)^2}{7\times10^{15}}$

$= 3.214\times10^4\ \text{cm}^{-3}$

$p_{no} = \dfrac{n_i^2}{N_d} = \dfrac{\left(1.5\times10^{10}\right)^2}{3\times10^{16}}$

$= 7.5\times10^3\ \text{cm}^{-3}$

(i) $V_a = V_t \ln\left[\dfrac{(0.1)N_a}{n_{po}}\right]$

$= (0.0259)\ln\left[\dfrac{(0.1)\left(7\times10^{15}\right)}{3.214\times10^4}\right]$

$= 0.6165\ \text{V}$

(ii) p-region - lower doped side

9.

(a) $J_s = en_i^2\left[\dfrac{1}{N_a}\sqrt{\dfrac{D_n}{\tau_{no}}} + \dfrac{1}{N_d}\sqrt{\dfrac{D_p}{\tau_{p0}}}\right]$

$= \left(1.6\times10^{-19}\right)\left(1.5\times10^{10}\right)^2$

$\times\left[\dfrac{1}{5\times10^{17}}\sqrt{\dfrac{25}{10^{-7}}} + \dfrac{1}{8\times10^{15}}\sqrt{\dfrac{10}{8\times10^{-8}}}\right]$

$J_s = 5.145\times10^{-11}\ \text{A/cm}^2$

$I_s = AJ_s = \left(2\times10^{-4}\right)\left(5.145\times10^{-11}\right)$

$= 1.029\times10^{-14}\ \text{A}$

(b) $I = I_s \exp\left(\dfrac{V_a}{V_t}\right)$

(i) $I = \left(1.029\times10^{-14}\right)\exp\left(\dfrac{0.45}{0.0259}\right)$

$= 3.61\times10^{-7}\ \text{A}$

(ii) $I = \left(1.029\times10^{-14}\right)\exp\left(\dfrac{0.55}{0.0259}\right)$

$= 1.72\times10^{-5}\ \text{A}$

(iii) $I = \left(1.029\times10^{-14}\right)\exp\left(\dfrac{0.65}{0.0259}\right)$

$= 8.16\times10^{-4}\ \text{A}$

10.

Initially, at the metallurgical junction, there is a very large density gradient in both electron and hole concentrations. Majority carrier electrons in the n region will begin diffusing into the p region, and majority carrier holes in the p region will begin diffusing into the n region. If we assume there are no external connections to the semiconductor, then this diffusion process cannot continue indefinitely. As electrons diffuse from the n region, positively charged donor atoms are left behind. Similarly, as holes diffuse from the p region, they uncover negatively charged acceptor atoms. The net positive and negative charges in the n and p regions induce an electric field in the region near the metallurgical junction, in the direction from the positive to the negative charge, or from the n to the p region.

The net positively and negatively charged regions are referred to as the space charge region. Essentially all electrons and holes are swept out of the space charge region by the electric field. Since the space charge region is depleted of any mobile charge, this region is also referred to as the depletion region.

11.

(a) V_{bi}.

$$V_{bi} = kT\ln\left(\frac{N_a N_d}{n_i^2}\right) = 0.731V.$$

(b) the junction capacitance at reverse bias V_R

(i) $V_R = 1V$,
(ii) $V_R = 3V$,
(iii) $V_R = 5V$,

$$C = A\sqrt{\frac{q\varepsilon}{2(V_{bi}+V_R)}\frac{N_a N_d}{N_a+N_d}}\ (F), \varepsilon = \varepsilon_0\varepsilon_r = 11.7\times8.85\times10^{-14}$$

(c) plot $1/C^2$ versus V_R and identify how the slope and intercept at the voltage axis are related to N_d and V_{bi}, respectively.

$$\frac{1}{C^2} = \frac{2(V_{bi}+V_R)}{q\varepsilon A^2}\frac{N_a+N_d}{N_a N_d}$$

$$slope \approx \frac{2}{q\varepsilon A^2 N_d}$$

$$intercept = -V_{bi}$$