Homework 1 solution

1

$$E = h\nu = \frac{hc}{\lambda} \Rightarrow \lambda = \frac{hc}{E}$$

Gold: $E = 4.90\ \text{eV} = (4.90)(1.6\times 10^{-19})\ \text{J}$

So,

$$\lambda = \frac{(6.625\times 10^{-34})(3\times 10^{10})}{(4.90)(1.6\times 10^{-19})} = 2.54\times 10^{-5}\ \text{cm}$$

or

$$\lambda = 0.254\ \mu\text{m}$$

Cesium: $E = 1.90\ \text{eV} = (1.90)(1.6\times 10^{-19})\ \text{J}$

So,

$$\lambda = \frac{(6.625\times 10^{-34})(3\times 10^{10})}{(1.90)(1.6\times 10^{-19})} = 6.54\times 10^{-5}\ \text{cm}$$

or

$$\lambda = 0.654\ \mu\text{m}$$

2

(a) $p = \frac{h}{\lambda} = \frac{6.625\times 10^{-34}}{550\times 10^{-9}}$

$$= 1.205\times 10^{-27}\ \text{kg-m/s}$$

$$\upsilon = \frac{p}{m} = \frac{1.2045\times 10^{-27}}{9.11\times 10^{-31}} = 1.32\times 10^{3}\ \text{m/s}$$

or $\upsilon = 1.32\times 10^{5}$ cm/s

(b) $p = \frac{h}{\lambda} = \frac{6.625\times 10^{-34}}{440\times 10^{-9}}$

$$= 1.506\times 10^{-27}\ \text{kg-m/s}$$

$$\upsilon = \frac{p}{m} = \frac{1.5057\times 10^{-27}}{9.11\times 10^{-31}} = 1.65\times 10^{3}\ \text{m/s}$$

or $\upsilon = 1.65\times 10^{5}$ cm/s

(c) Yes

3

$$E_{avg} = \frac{3}{2}kT = \left(\frac{3}{2}\right)(0.0259) = 0.03885\ \text{eV}$$

Now

$$p_{avg} = \sqrt{2mE_{avg}}$$

$$= \sqrt{2(9.11\times 10^{-31})(0.03885)(1.6\times 10^{-19})}$$

or

$$p_{avg} = 1.064\times 10^{-25}\ \text{kg-m/s}$$

Now

$$\lambda = \frac{h}{p} = \frac{6.625\times 10^{-34}}{1.064\times 10^{-25}} = 6.225\times 10^{-9}\ \text{m}$$

or

$$\lambda = 62.25\ \mathring{A}$$

4

(a) $p = \frac{h}{\lambda} = \frac{6.625\times 10^{-34}}{85\times 10^{-10}}$

$$= 7.794\times 10^{-26}\ \text{kg-m/s}$$

$$\upsilon = \frac{p}{m} = \frac{7.794\times 10^{-26}}{9.11\times 10^{-31}} = 8.56\times 10^{4}\ \text{m/s}$$

or $\upsilon = 8.56\times 10^{6}$ cm/s

$$E = \frac{1}{2}m\upsilon^2 = \frac{1}{2}(9.11\times 10^{-31})(8.56\times 10^{4})^2$$

$$= 3.33\times 10^{-21}\ \text{J}$$

or $E = \frac{3.334\times 10^{-21}}{1.6\times 10^{-19}} = 2.08\times 10^{-2}\ \text{eV}$

(b) $E = \frac{1}{2}(9.11\times 10^{-31})(8\times 10^{3})^2$

$$= 2.915\times 10^{-23}\ \text{J}$$

or $E = \frac{2.915\times 10^{-23}}{1.6\times 10^{-19}} = 1.82\times 10^{-4}\ \text{eV}$

$$p = m\upsilon = (9.11\times 10^{-31})(8\times 10^{3})$$

$$= 7.288\times 10^{-27}\ \text{kg-m/s}$$

$$\lambda = \frac{h}{p} = \frac{6.625\times 10^{-35}}{7.288\times 10^{-27}} - 9.09\times 10^{-8}\ \text{m}$$

or $\lambda = 909\ \mathring{A}$

5

$$\int_{-1/2}^{+1/2} A^2 \cos^2(n\pi x)dx = 1$$

$$A^2\left[\frac{x}{2} + \frac{\sin(2n\pi x)}{4n\pi}\right]\Bigg|_{-1/2}^{+1/2} = 1$$

$$A^2\left[\frac{1}{4} - \left(-\frac{1}{4}\right)\right] = 1 = A^2\left(\frac{1}{2}\right)$$

or $|A| = \sqrt{2}$

6

(a) $E_n = \dfrac{\hbar^2 n^2 \pi^2}{2ma^2} = \dfrac{n^2\left(1.054\times10^{-34}\right)^2\pi^2}{2\left(9.11\times10^{-31}\right)\left(10\times10^{-10}\right)^2}$

$= n^2\left(6.018\times10^{-20}\right)$ J

or $E_n = \dfrac{n^2\left(6.018\times10^{-20}\right)}{1.6\times10^{-19}} = n^2(0.3761)$ eV

Then

$E_1 = 0.376$ eV

$E_2 = 1.504$ eV

$E_3 = 3.385$ eV

(b) $\lambda = \dfrac{hc}{\Delta E}$

$\Delta E = (3.385 - 1.504)\left(1.6\times10^{-19}\right)$

$= 3.01\times10^{-19}$ J

$\lambda = \dfrac{\left(6.625\times10^{-34}\right)\left(3\times10^{8}\right)}{3.01\times10^{-19}}$

$= 6.604\times10^{-7}$ m

or $\lambda = 660.4$ nm

7

$\psi_2(x) = A_2\exp(-k_2 x)$

$P = \dfrac{|\psi(x)|^2}{A_2 A_2^*} = \exp(-2k_2 x)$

where $k_2 = \sqrt{\dfrac{2m(V_o - E)}{\hbar^2}}$

$= \dfrac{\sqrt{2\left(9.11\times10^{-31}\right)(3.5-2.8)\left(1.6\times10^{-19}\right)}}{1.054\times10^{-34}}$

$k_2 = 4.286\times10^{9}\ \text{m}^{-1}$

(a) For $x = 5\ \overset{o}{A} = 5\times10^{-10}$ m

$P = \exp(-2k_2 x)$

$= \exp\left[-2\left(4.2859\times10^{9}\right)\left(5\times10^{-10}\right)\right]$

$= 0.0138$

(b) For $x = 15\ \overset{o}{A} = 15\times10^{-10}$ m

$P = \exp\left[-2\left(4.2859\times10^{9}\right)\left(15\times10^{-10}\right)\right]$

$= 2.61\times10^{-6}$

(c) For $x = 40\ \overset{o}{A} = 40\times10^{-10}$ m

$P = \exp\left[-2\left(4.2859\times10^{9}\right)\left(40\times10^{-10}\right)\right]$

$= 1.29\times10^{-15}$

8

(a) Region I: Since $V_O > E$, we can write

$$\frac{\partial^2\psi_1(x)}{\partial x^2} - \frac{2m(V_O - E)}{\hbar^2}\psi_1(x) = 0$$

Region II: $V = 0$, so

$$\frac{\partial^2\psi_2(x)}{\partial x^2} + \frac{2mE}{\hbar^2}\psi_2(x) = 0$$

Region III: $V \to \infty \Rightarrow \psi_3 = 0$

The general solutions can be written, keeping in mind that ψ_1 must remain finite for $x < 0$, as

$\psi_1(x) = B_1\exp(k_1 x)$

$\psi_2(x) = A_2\sin(k_2 x) + B_2\cos(k_2 x)$

$\psi_3(x) = 0$

where

$$k_1 = \sqrt{\frac{2m(V_O - E)}{\hbar^2}} \quad \text{and} \quad k_2 = \sqrt{\frac{2mE}{\hbar^2}}$$

(b) Boundary conditions

At $x = 0$: $\psi_1 = \psi_2 \Rightarrow B_1 = B_2$

$$\frac{\partial\psi_1}{\partial x} = \frac{\partial\psi_2}{\partial x} \Rightarrow k_1 B_1 = k_2 A_2$$

At $x = a$: $\psi_2 = \psi_3 \Rightarrow$

$$A_2\sin(k_2 a) + B_2\cos(k_2 a) = 0$$

or

$B_2 = -A_2\tan(k_2 a)$

(c)

$$k_1 B_1 = k_2 A_2 \Rightarrow A_2 = \left(\frac{k_1}{k_2}\right)B_1$$

and since $B_1 = B_2$, then

$$A_2 = \left(\frac{k_1}{k_2}\right)B_2$$

From $B_2 = -A_2\tan(k_2 a)$, we can write

$$B_2 = -\left(\frac{k_1}{k_2}\right)B_2\tan(k_2 a)$$

or

$$1 = -\left(\frac{k_1}{k_2}\right)\tan(k_2 a)$$

This equation can be written as

$$1 = -\sqrt{\frac{V_O - E}{E}}\cdot\tan\left[\sqrt{\frac{2mE}{\hbar^2}}\cdot a\right]$$

or

$$\sqrt{\frac{E}{V_O - E}} = -\tan\left[\sqrt{\frac{2mE}{\hbar^2}} \cdot a\right]$$

This last equation is valid only for specific values of the total energy E. The energy levels are quantized.