Problem 3.

Gibbs sampling is a special kind of MH algorithm when it's single updated or blocked. And the proposal distribution is the conditional distribution. acceptance pr = 1. and proposal and target distributions are the same

Problem 4.

a). Exchangeable.

Independent

Yes. independent.

b). Exchangeable.

Not independent

No. not independent.

c). Exchangeable.

Not Independent.

Yes. independent.

Problem 5.

Let $u(\phi) = E(\theta_i | \phi)$.

$$\Rightarrow \quad cov(\theta_i, \theta_j) = E(cov(\theta_i, \theta_j | \phi)) + cov(E(\theta_i|\phi), E(\theta_j|\phi))$$

$$= 0 + cov(u(\phi), u(\phi))$$

$$= var(u(\phi))$$

$$\geq 0$$