Welcome to VE 414

Bayesian Analysis

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Lecture 1: Probability & Bayes' Theorem

- Meet your instructor
- Course Structure
- Course Logistics

群聊

该二维码7天内(9月20日前)有效，重新进入将更新

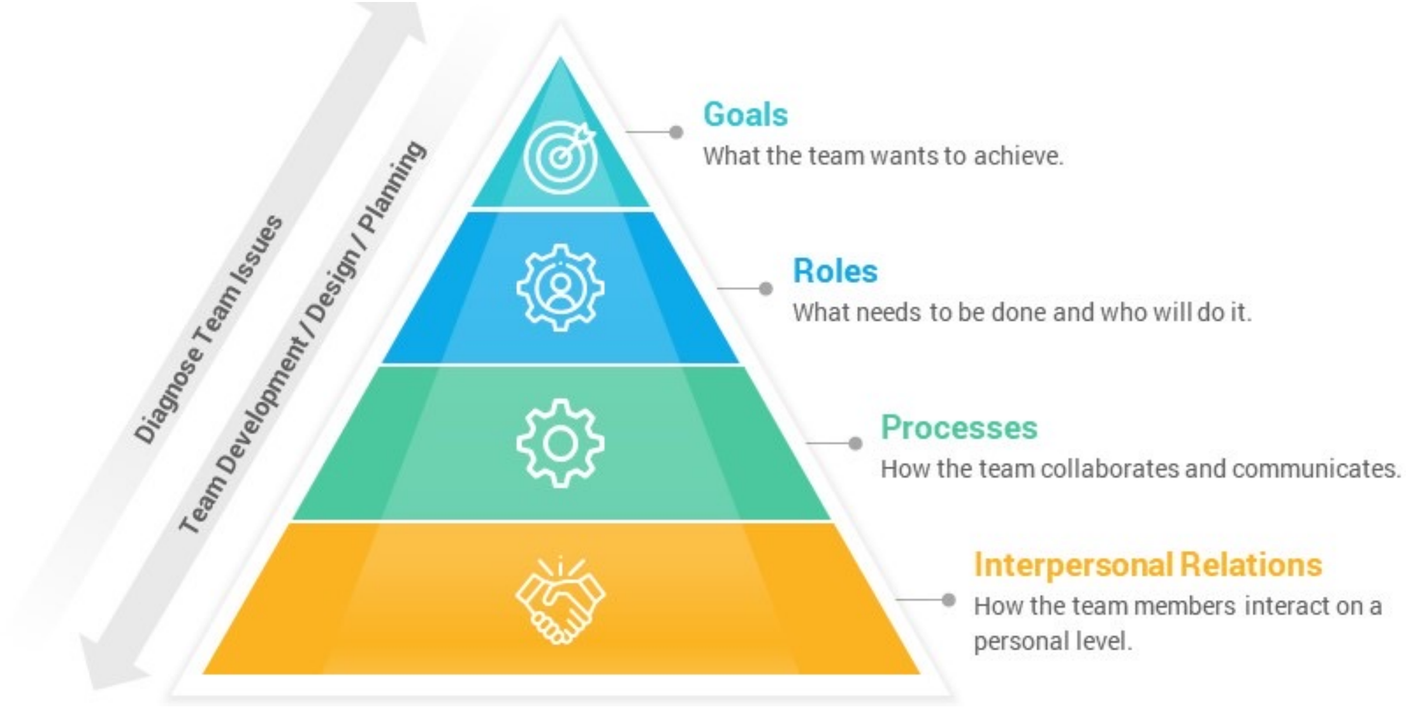
VE414 Bayesian Ana...

该二维码 7 天内 (9/20前)有效

Meet your instructor

- New JI faculty, just joined in 2021.9
- Background: PhD in Geophysics from UCLA (2019). Formerly data scientist @ ExxonMobil
- Research Interests: Data-driven solutions to geoscience problems: earthquake rupture, oil and gas exploration, seismic signal processing.

GRPI team effectiveness model



Course goals, desired results, expectations.

Role clarity, responsibility, and boundaries

Class norms, styles, agreement on course content and reward systems

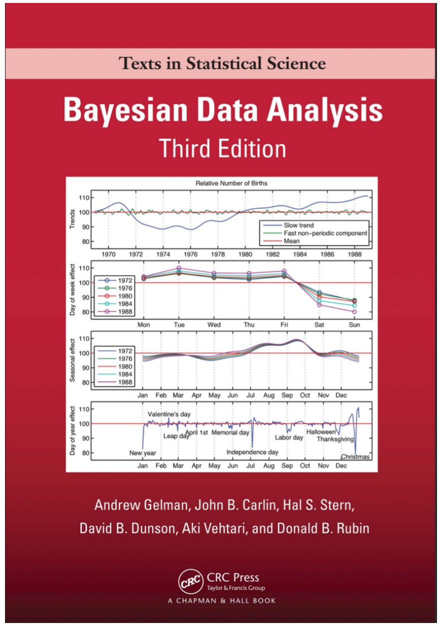
Trust, Psychological Safety, Flexibility, Courage of conviction

Course Structure

Tentative structure of the course:
1. Fundamentals of Bayesian inference: prior and posterior distributions, decision theory, point and interval estimations, single-layer models.
2. Hierarchical models: hyperparameters, hyperprior, shrinkage, Stein estimators.
3. Bayesian missing data problems: missing data, data augmentation, Gibbs sampler, incomplete normal data, mixture models.
4. Bayesian nonparametrics: Dirichlet process models, Gaussian process models, density estimation.

- Textbook: Gelman et al., Bayesian Data Analysis (3rd edition, 2013).

- Statistics (60%) + Computing with Python/R (40%)

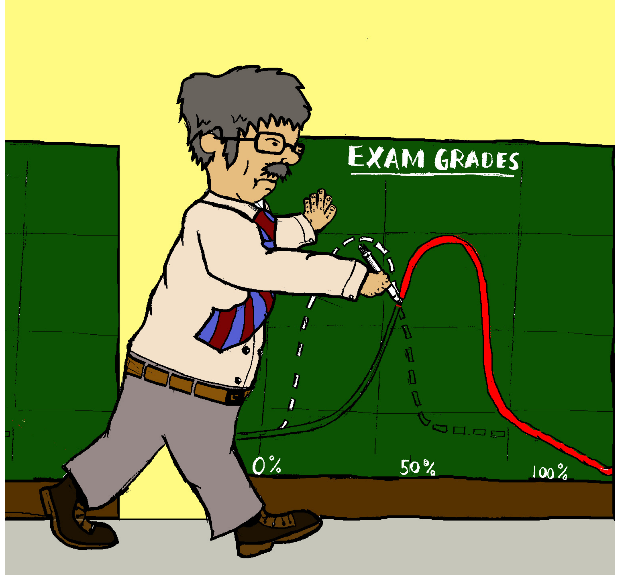


Grading

Grades breakdown:

- Homework Assignment 30% (Problem sets from BDA3 + computing)
- Project 20%
- Midterm 25%
- Final Exam 25%

- I reserve the right to curve the scale if there are less than 30% of students with grades ≥ A.



Event and Probability

"The true logic of this world is in the calculus of probabilities."
----James Clerk **Maxwell**

- Sample Space (S): S is the collection of all possible outcomes of the random experiment.

- Event (denoted with capital letters A,B,C): Event is a subset of the sample space S. That is, for example $A \subset S$, where "$\subset$" denotes "is a subset of."
  - Follow the algebra of sets (null set, union, intersection, complement)

- Probability: Probability is a (real-valued) set function P that assigns to each event A in the sample space S a number P(A), called the **probability of the event** A.
  - Axioms:
    - [Non-negative] For any event A, $P(A) \geq 0$.
    - For the sample space Ω, $P(\Omega)=1$.
    - [additivity] If $A \cap B = \phi$, then $P(A \cup B) = P(A) + P(B)$.

Classical, Frequentist, and Bayesian Probability

1. Roll a die, P(X=4)
2. P(Die is fair)
3. P(rain)
4. P(rain) given that you know the typhoon is coming.

Classical: Outcomes are equal likely
Frequentist: Hypothetical infinite sequence, consider relative frequency (Objective)
Bayesian: Personal (subjective) perspective, rigorous equation.

Bayes Theorem

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(B|A)P(A)}{P(B|A)P(A) + P(B|A^c)P(A^c)}$$

$$P(A_i|B) = \frac{P(B|A_i)P(A_i)}{\sum_j P\left(B\middle|A_j\right)P(A_j)}$$

It is named after Thomas Bayes, however, he actually has very little to do with this formula, it was Laplace that worked out the mathematical form

EX 1. 30 students, 9 females, 12 ECE major of which 4 are female.
P(F), P(ECE)
P(F|ECE), P(ECE|F)

EX2. Early test for HIV antibodies:
P (+ |HIV) = 0.977. P (- | no HIV) = 0.926.
A study found that among North American's, P(HIV) = 0.0026
P(HIV|+)

Monty Hall Problem

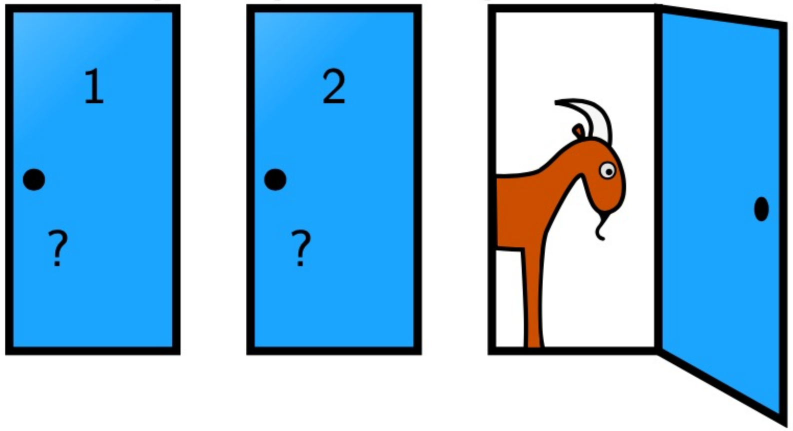
- The famous Monty Hall problem arises from a popular television game show

  Let's Make a Deal

  is often used to illustrate the Bayes' theorem and Bayesian inference.

Monty Hall Problem

Suppose you are on a game show, and you are given the choice of three doors: Behind one door is a car; behind the others, goats. You pick a door, say No. 2, and the host, who knows what's behind the doors, opens another door, say No. 3, which has a goat. He then says to you, "Do you want to switch to door No. 1?"



Is it to your advantage to switch your choice?

A typical solution:

Let $\mathcal{C}_1$, $\mathcal{C}_2$ and $\mathcal{C}_3$ denote the events in which the car is placed behind door No.1, No.2 and No.3, respectively. Without loss of generality, suppose we select door No.2 and Monty opens No.3.

$$\Pr(\mathcal{C}_2 \mid \mathcal{O}_3) = \frac{\Pr(\mathcal{O}_3 \mid \mathcal{C}_2)\Pr(\mathcal{C}_2)}{\sum_{j=1}^{3} \Pr(\mathcal{O}_3 \mid \mathcal{C}_j)\Pr(\mathcal{C}_j)} = \frac{1/2 \cdot 1/3}{1 \cdot 1/3 + 1/6 + 0 \cdot 1/3} = \frac{1}{3}$$

No switching

$$\Pr(\mathcal{C}_1 \mid \mathcal{O}_3) = \frac{\Pr(\mathcal{O}_3 \mid \mathcal{C}_1)\Pr(\mathcal{C}_1)}{\sum_{j=1}^{3} \Pr(\mathcal{O}_3 \mid \mathcal{C}_j)\Pr(\mathcal{C}_j)} = \frac{1 \cdot 1/3}{1 \cdot 1/3 + 1/6 + 0 \cdot 1/3} = \frac{2}{3}$$

Switching

where $\mathcal{O}_1$, $\mathcal{O}_2$ and $\mathcal{O}_3$ denote the events in which Monty opens door No.1, No.2 and No.3, respectively.

uses Bayes' theorem to see switching is the right choice. But it often fails to attract one's attention to the reasoning behind this solution thus this choice.

Basic probability distribution

Name	Probability	Expectation	Variance	Notes
Bernoulli B(p)				
Binomial Bin(n,p)				Normal Approximation
Geometric (p)				
Poisson				