上 海 交 通 大 学 试 卷

（ 2019~ 2020~2 Academic Year/Summer Semester ）

Class No.____VG441______ _ Name in English or Pinyin: ____________________

Student ID No.______________ Name in Hanzi(if applicable): ____________________

VG441 Supply Chain Management

Midterm II

July 22, 1pm

The exam paper has 4 pages in total.

You are to abide by the University of Michigan-Shanghai Jiao Tong University Joint Institute (UM-SJTU JI) honor code. Please sign below to signify that you have kept the honor code pledge.

THE UM-SJTU JI HONOR CODE

I accept the letter and spirit of the honor code:

I have neither given nor received unauthorized aid on this examination, nor have I concealed any violations of the Honor Code by myself or others.

Signature:____________

Please enter grades here:

Exercises No. 题号	Points 得分	Grader's Signature 流水批阅人签名
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		
Total 总分		

VG441 Take-Home Midterm II

You are encouraged to type your answer using LaTeX, but scanning document is acceptable. The due date of this midterm is on Canvas. No late submission is allowable. It is also not allowed to ask any questions to **peers or Professor or TA** *since this is an exam. But you can post clarification questions on Piazza. Bonus problem is optional to submit.*

Problem 1 (Traveling Salesman Problem)

There are 6 cities that a salesman must visit (as a tour). The distance matrix is given by

City/City	**1**	**2**	**3**	**4**	**5**	**6**
1	0	10	100	50	33	66
2	10	0	22	86	952	3
3	100	22	0	6	86	2
4	50	86	6	0	5	4
5	33	952	86	5	0	9
6	66	3	2	4	9	0

Task 1: Run double tree algorithm on paper.

Task 2: Run Christofides' algorithm on paper. (Try eyeballing minimum cost matching solution.)

Problem 2 (Knapsack)

Consider the following simple example of $n = 4$ items (each with a value and a size):

$$\begin{aligned} \text{Sizes:} \quad & s_1 = 3, s_2 = 3, s_3 = 8, s_4 = 5 \\ \text{Values:} \quad & v_1 = 4, v_2 = 4, v_3 = 6, v_4 = 5 \end{aligned}$$

Finally, the bag size is $B = 8$.

Task 1: Run the exact dynamic program (ExactKS) on paper to solve this problem.

Task 2: Run the simple greedy algorithm (ranking via v_i/s_i) and show that it is not optimal.

Problem 3 (Minimum Cost Set Cover)

The ground set (to be covered) is $V = \{e_1, e_2, \ldots, e_{11}, e_{12}\}$. You are given 3 (overlapping) sets

$$\begin{aligned} S_1 &= \{e_1, e_2, e_3, e_4, e_5\} \\ S_2 &= \{e_1, e_2, e_3, e_6, e_7\} \\ S_3 &= \{e_4, e_5, e_8, e_9, e_{10}, e_{11}, e_{12}\} \end{aligned}$$

The cost of using S_1 is 6. The cost of using S_2 is 15. The cost of using S_3 is 7. We want to cover the ground set using minimum cost. A greedy algorithm would be as follows. In each iteration, for each unselected set, see how many uncovered elements can be covered using this set, and compute the ratio of cost to this number. Then rank these ratios and select the set with the smallest ratio.

Task 1: Run this greedy algorithm on paper.

Task 2: Is your greedy solution optimal? Can you eyeball a better solution?

Bonus Problem* (Facility Location)

We consider the following metric uncapacitated facility location problem. The **input** is given by

- Set D of demands
- Set F of facilities
- Metric distance function d_{ij} for every $i \in F$ and $j \in D$
- Facility setup costs f_i for every $i \in F$

The **output** should be $S \subseteq F$ that minimizes $\sum_{i \in S} f_i + \sum_{j \in D} \min_{i \in S} d_{ij}$.

In class, we have formulated this problem as a MILP. If we relax the binary decision variables to continuous ones, we obtain the following linear programming relaxation, denoted by **(P)**:

$$
\begin{aligned}
\textbf{(P)} \quad \min \quad & \sum_{i \in F} f_i y_i + \sum_{i \in F, j \in D} d_{ij} x_{ij} \\
\text{s.t.} \quad & \sum_{i \in F} x_{ij} \geq 1 \quad \forall j \in D \\
& x_{ij} \leq y_i \quad \forall i \in F, j \in D \\
& x_{ij}, y_i \geq 0
\end{aligned}
$$

Here x_{ij} is the fraction of demand j that is served by facility i, and y_i is the (continuous) decision of whether facility i should be opened. Note that y_i should be binary $\{0, 1\}$ but we relax it to be $y_i \geq 0$.

Task 1: Assign dual variables α_j for every demand $j \in D$, and dual variables β_{ij} for every (facility, demand) pair. Derive the dual linear program **(D)** in terms of these α's and β's.

Task 2: Think of α_j as the amount of money demand j is willing to contribute to the solution, and β_{ij} as the amount of money demand j's contributes towards opening facility i. Try to interpret the dual linear program **(D)**.