Problem 1.

Please check "Problem 1. ipynb"

Problem 2:

$\lambda = 50$ #/month $\quad K = 50 \quad i = C_j = \frac{200}{12}$

$Q_j^* = \sqrt{\frac{2K\lambda}{h}} = \sqrt{\frac{2 \times 50 \times 50}{200/12}} = 17.3.$

Only Q_1^* is reasonable, and the cost is:

$g(17) = 510 \times 50 + \frac{50 \times 50}{17} + \frac{\frac{200}{12} \times 17}{2} = 25788.7$

$g_2(65) = 495 \times 50 + \frac{50 \times 50}{65} + \frac{\frac{200}{12} \times 65}{2} = 25330.1$

$g_3(129) = 485 \times 50 + \frac{50 \times 50}{129} + \frac{\frac{200}{12} \times 129}{2} = 25344.4$

Therefore, the optimal order quantity is $Q = 65$.

the cost per month is: \$25330.1

(b) $g(65) = 520 \times 50 + \frac{0.50}{65} + \frac{0.65}{2} = 26000 > 25330.1$

$\Rightarrow$ Zens should not accept the offer.

Problem 3

(a) $K = 1000, \quad h = 1.2$

$Q8 = 0$

$Q7 = K + h(0 \cdot d_7) + \theta_8 = 1000. \qquad [S(7) = 8]$

$Q6 = \min\{K + h(0 \cdot d_6) + \theta_7,\ K + h(0 \cdot d_6 + 1 \cdot d_7) + \theta_8\}$

$= \min\{2000, 1348\}$

$= 1348 \qquad [S(6) = 8]$

$Q5 = \min\{K + h(0 \cdot d_5) + \theta_6,\ K + h(0 \cdot d_5 + 1 \cdot d_6) + \theta_7,\ K + h(0 \cdot d_5 + 1 \cdot d_6 + 2d_7) + \theta_8\}$

$= \min\{2348, 2252, 1948\}$

$= 1948 \qquad [S(5) = 8]$

$Q4 = \min\{K + h(0 \cdot d_4) + \theta_5,\ K + h(0 \cdot d_4 + 1 \cdot d_5) + \theta_6,\ K + h(0 \cdot d_4 + 1 \cdot d_5 + 2 \cdot d_6) + \theta_7$

$K + h(0 \cdot d_4 + 1 \cdot d_5 + 2 \cdot d_6 + 3 \cdot d_7) + \theta_8\}$

$= \min\{2948, 2552, 2708, 2752\}$

$= 2552 \qquad [S(4) = 6]$

$Q3 = \min\{K + h(0 \cdot d_3) + \theta_4,\ K + h(0 \cdot d_3 + 1 \cdot d_4) + \theta_5.\ K + h(0 \cdot d_3 + 1 \cdot d_4 + 2 \cdot d_5) + \theta_6,$

$K + h(0 \cdot d_3 + 1 \cdot d_4 + 2 \cdot d_5 + 3 \cdot d_6) + \theta_7,\ K + h(0 \cdot d_3 + 1 \cdot d_4 + 2 \cdot d_5 + 3 \cdot d_6 + 4 \cdot d_7) + \theta_8\}$

$= \min\{3552, 3056, 2864, 3272, 3664\}$

$= 2864 \qquad [S(3) = 6]$

$Q2 = \min\{K + h(0 \cdot d_2) + \theta_3,\ K + h(0 \cdot d_2 + 1 \cdot d_3) + \theta_4,\ K + h(0 \cdot d_2 + 1 \cdot d_3 + 2 \cdot d_4) + \theta_5$

$K + h(0 \cdot d_2 + 1 \cdot d_3 + 2 \cdot d_4 + 3 \cdot d_5) + \theta_6,\ K + h(0 \cdot d_2 + 1 \cdot d_3 + 2 \cdot d_4 + 3 \cdot d_5 + 4 \cdot d_6) + \theta_7,$

$K + h(0 \cdot d_2 + 1 \cdot d_3 + 2 \cdot d_4 + 3 \cdot d_5 + 4 \cdot d_6 + 5 \cdot d_7) + \theta_8\}$

$= \min\{3864, 3678, 3290, 3302, 3962, 4702\}$

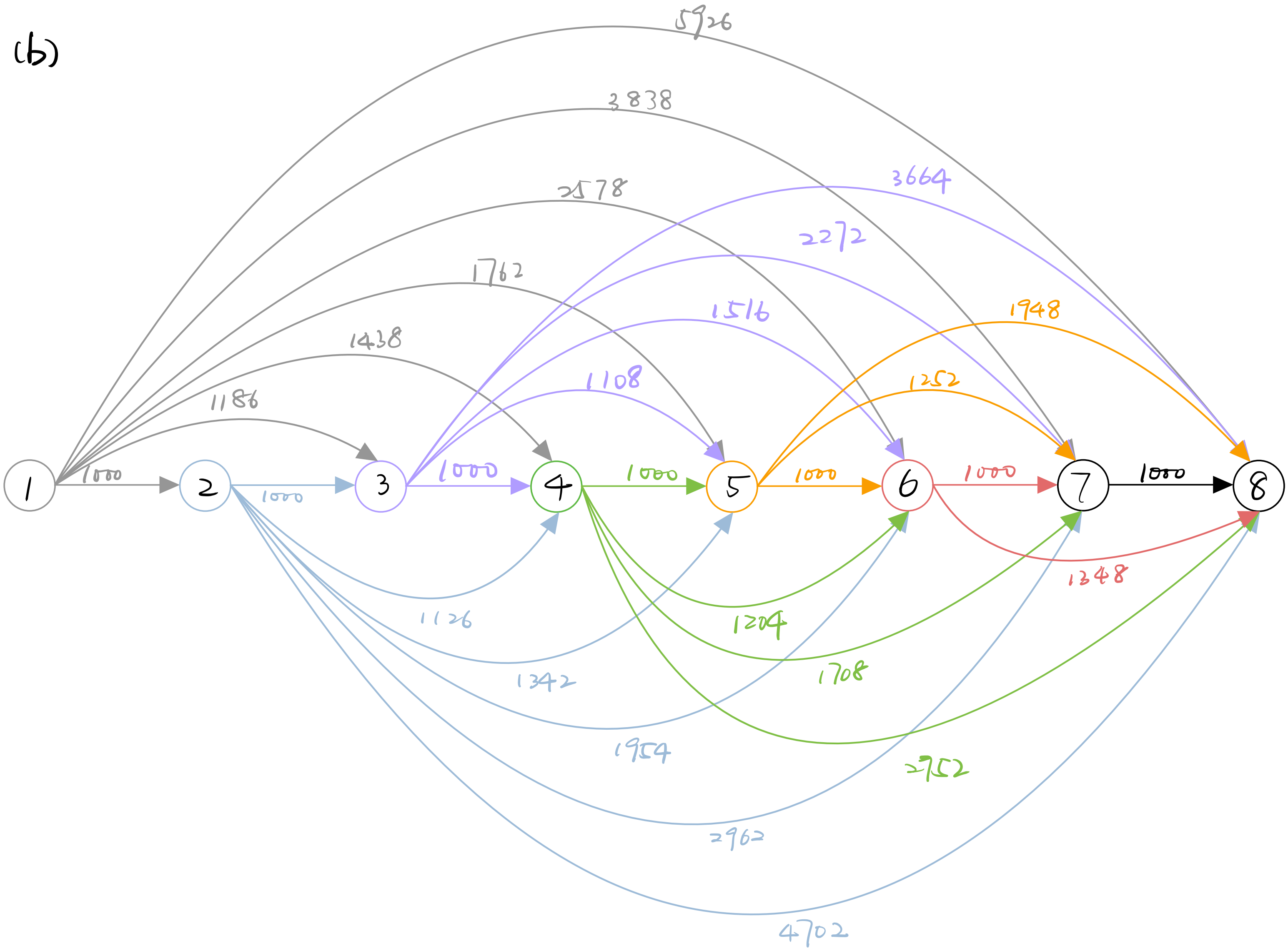
$= 3290 \qquad [S(2) = 5]$

$$Q_1 = \min \{ k + h(0\cdot d_1) + \theta_2,\ k + h(0\cdot d_1 + 1\cdot d_2) + \theta_3,\ k + h(0\cdot d_1 + 1\cdot d_2 + 2\cdot d_3) + \theta_4,$$
$$k + h(0\cdot d_1 + 1\cdot d_2 + 2\cdot d_3 + 3\cdot d_4) + \theta_5,\ k + h(0\cdot d_1 + 1\cdot d_2 + 2\cdot d_3 + 3\cdot d_4 + 4\cdot d_5) + \theta_6,$$
$$k + h(0\cdot d_1 + 1\cdot d_2 + 2\cdot d_3 + 3\cdot d_4 + 4\cdot d_5 + 5\cdot d_6) + \theta_7,$$
$$k + h(0\cdot d_1 + 1\cdot d_2 + 2\cdot d_3 + 3\cdot d_4 + 4\cdot d_5 + 5\cdot d_6 + 6\cdot d_7) + \theta_8 \}$$
$$= \min \{4290, 4050, 3990, 3710, 3926, 4838, 5926\}$$
$$= 3710 \qquad [S(1) = 5]$$

Therefore, we should order 570 on Day #1,
and order 670 on Day #5.

The total cost is \$3710.

(b)



path	∅	1→2	1→3	1→4	1→5	1→6	1→5→7	1→5→8
cost	0	1000	1186	1438	1762	2578	3014	3710

Therefore, we should order 570 on Day #1,
and order 670 on Day #5.

The total cost is \$3710.

(c) Denote q_t: number of units ordered in period t.

y_t: 1 for order in period t, 0 otherwise

x_t: the inventory level at the end of period t.

$$\min \sum_{t=1}^{T} (1000 y_t + 1.2 x_t)$$

s.t. $x_t = x_{t-1} + q_t - d_t$

$q_t \leq 100000 y_t$

$x_t \geq 0$ for $\forall t \in \{1, \dots, 7\}$

$q_t \geq 10$

$y_t \in \{0, 1\}$

The code is in 'Problem 3c.ipynb'

Problem 4

(a) $A = \begin{pmatrix} 3 & 2 & 1 \\ 1 & 2 & 4 \end{pmatrix}$ $\quad x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$. $\quad C = \begin{pmatrix} 6 \\ 14 \\ 13 \end{pmatrix}$

$A^T = \begin{pmatrix} 3 & 1 \\ 2 & 2 \\ 1 & 4 \end{pmatrix}$ $\quad b^T = \begin{pmatrix} 24 & 60 \end{pmatrix}$

$$\min \quad (24y_1 + 60y_2)$$

$$\begin{aligned} \text{s.t} \quad & 3y_1 + y_2 \geqslant 6 \\ & 2y_1 + 2y_2 \leqslant 14 \\ & y_1 + 4y_2 = 13 \\ & y_1 \geqslant 0 \\ & y_2 \leqslant 0 \end{aligned}$$

(b) $C = \begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix}$ $\quad$ C.size() $= |I| \times |J|$ $\quad b = \begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix}$ $\quad$ b.size() $= |I| + |J|$

$$A = \begin{pmatrix} 1 & 1 & 1 & \cdots & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & & 1 & 1 & & \vdots \\ \vdots & & & & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & & 0 & 0 & & \vdots \\ 1 & 0 & 0 & & 1 & 0 & & 0 \\ 0 & 1 & 0 & & 0 & 1 & & \vdots \\ \vdots & \vdots & \vdots & & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & & 0 & 0 & & 1 \end{pmatrix}$$

$$A^T = \begin{array}{c} \begin{array}{cc} |I| & |J| \end{array} \\ \begin{pmatrix} 1 & 0 & \cdots & 0 & 1 & 0 & \cdots & 0 \\ 1 & 0 & \cdots & 0 & 0 & 1 & \cdots & 0 \\ 1 & 0 & \cdots & 0 & 0 & 0 & 1 \cdots & 0 \\ \vdots & & & \vdots & & & & \\ 0 & 1 & \cdots & 0 & 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 & 0 & 1 & \cdots & 0 \\ \vdots & & & \vdots & & & & \vdots \\ 0 & \cdots & 0 & 1 & 0 & 0 & \cdots & 1 \end{pmatrix} \end{array}$$

For A^T, we can find that there're two 1 in each row. one in $|I|$. one in $|J|$.

The daul is:

$$\min \sum_{i \in I} u_i + \sum_{j \in J} v_j$$

$$\begin{aligned} \text{s.t} \quad & u_i + v_j \geq 1 && \text{for } \forall i \in I, \forall j \in J. \\ & u_i \geq 0 && \text{for } \forall i \in I \\ & v_j \geq 0 && \text{for } \forall j \in J. \end{aligned}$$