VG 441 HW1
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## Problem 1.

$$\frac{\partial \theta^T X^T X \theta}{\partial \theta} = 2X^T X \theta$$

two steps:

① $X^T X$ is symmetric? ✓ assume $X^T X = B$

assume $x$ is $n \times n$, $X_{ij}$ $(1 \le i, j \le n)$, $X^T = X^T_{ij}$

$$B_{ij} = \sum_{k=1}^{n} X^T_{ik} \cdot X_{kj} = \sum_{k=1}^{n} X_{ki} \cdot X_{kj}$$

$$= \sum_{k=1}^{n} X_{kj} \cdot X_{ki} = \sum_{k=1}^{n} X^T_{jk} \cdot X_{ki} = B_{ji}$$

② $$\frac{\partial \theta^T B \theta}{\partial \theta} = 2B\theta \quad (\text{where } B_{ij} = B_{ji})$$

(mentioned in class)

assume $A = B\theta$ $(n \times 1)$

$$\therefore A_{k1} = \sum_{j=1}^{n} B_{kj} \cdot \theta_{j1}$$
$(1 \le k \le n)$

assume $C = \theta^T B \theta = \theta^T A$

$$C_{11} = \sum_{k=1}^{n} A_{k1} \cdot \theta^T_{1k} = \sum_{k=1}^{n} A_{k1} \cdot \theta_{k1}$$

assume $\frac{\partial C}{\partial \theta} = D$ ($D$ is $n \times 1$)

$$D_{k1} = \frac{\partial C}{\partial \theta_{k1}}$$

$$= \frac{\partial \sum A_{j1} \theta_{j1}}{\partial \theta_{k1}} \quad (j \le n)$$

$$= \frac{\partial (A_{j1} \theta_{j1})}{\partial \theta_{k1}} (j=k) + \cdots (j \ne k)$$

$$= A_{k1} + \theta_{k1} \cdot \frac{\partial A_{k1}}{\partial \theta_{k1}} + \sum \theta_{j1} \cdot \frac{\partial A_{j1}}{\partial \theta_{k1}} (j \ne k)$$

$$= A_{k1} + \sum \theta_{j1} \cdot \frac{\partial A_{j1}}{\partial \theta_{k1}} (1 \le j \le n)$$

$$= A_{k1} + \sum \theta_{j1} \cdot B_{jk} = A_{k1} + \sum \theta_{j1} \cdot B_{kj} \ (\because \text{symmetric})$$

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$\therefore D_{k1} = 2A_{k1} \ (1 \le k \le n)$
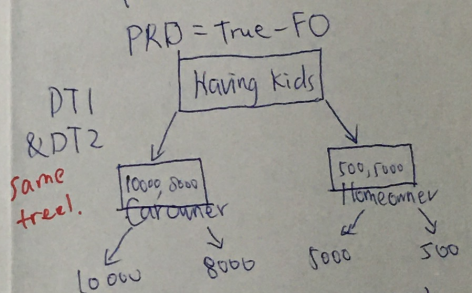
$\Rightarrow D = A$

## Problem 2.

① GBM

age < 25 | homeowner, age < 35, | having kids,
age < 45 / Car owner /
in total, 4 possible choices.

$$\overline{Salary} = \frac{23500}{4} = 5875 \ \$$$

| FO | PRO | F1 | PR1 | F2 | PR2 |
|---|---|---|---|---|---|
| 5875 | 4125 | 6287.5 | 3712.5 | 6658.75 | 3341.25 |
| 5875 | -5375 | 5337.5 | -4837.5 | 4853.75 | -4353.75 |
| 5875 | 2125 | 6087.5 | 1912.5 | 6278.75 | 1721.25 |
| 5875 | -875 | 5787.5 | -787.5 | 5708.75 | -108.75 |

PRD = True - FO

DT1 & DT2
same tree!.



$\gamma = 1$

$F_1 = F_0 + 0.1 \times (PRO)$
$F_2 = F_1 + 0.1 \times (PR1)$

we can always divide 4 people evenly,
and get deviance = 0 **-10**

② $\lambda = 1, \gamma = 0, \mu = 0.1,$

$$SS = \frac{\sum^2}{n+1}.$$

since $\lambda = 1$
we need to compare
$x^2 + y$ and $4xy$

| FO | PRO | F1 | PR1 | F2 | PR2 |
|---|---|---|---|---|---|
| Same as ① | Same as ① | 6081.25 | 3918.75 | 6277.12 | 3722.81 |
| | | 5666.67 | -5166.67 | 5472.23 | -4972.23 |
| | | 5981.25 | 2012.75 | 6081.89 | 1918.11 |
| | | 5666.67 | -666.67 | 5472.23 | -472.23 |

same tree! pruned Tree 1&2



(prune)
stop here