

VG 441 HW1

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Problem 1.

$$\frac{\partial \theta^T X^T X \theta}{\partial \theta} = 2 X^T X \theta$$

two steps:

① $X^T X$ is symmetric? \checkmark , assume $X^T X = B$

assume X is $n \times n$, X_{ij} ($1 \leq i, j \leq n$), $X^T = X^T$

$$B_{ij} = \sum_{k=1}^n X_{ik}^T \cdot X_{kj} = \sum_{k=1}^n X_{ki} \cdot X_{kj}$$

$$= \sum_{k=1}^n X_{kj} \cdot X_{ki} = \sum_{k=1}^n X_{jk}^T \cdot X_{ki} = B_{ji}$$

$$\textcircled{2} \frac{\partial \theta^T B \theta}{\partial \theta} = 2 B \theta \quad (\text{where } B_{ij} = B_{ji})$$

(mentioned in class)

assume $A = B \theta$ ($n \times 1$)

$$\therefore A_{ki} = \sum_{j=1}^n B_{kj} \cdot \theta_{ji}$$

assume $C = \theta^T B \theta = \theta^T A$

$$C_{ii} = \sum_{k=1}^n A_{ki} \cdot \theta_{ik}^T = \sum_{k=1}^n A_{ki} \cdot \theta_{ki}$$

$$\text{assume } \frac{\partial C}{\partial \theta} = D \quad (D \text{ is } n \times 1)$$

$$D_{ki} = \frac{\partial C}{\partial \theta_{ki}}$$

$$= \frac{\partial \sum_{j=1}^n A_{ji} \cdot \theta_{ji}}{\partial \theta_{ki}} \quad (1 \leq i \leq n)$$

$$= \frac{\partial (A_{ji} \theta_{ji})}{\partial \theta_{ki}} \quad (j=k) + \dots \quad (j \neq k)$$

$$= A_{ki} + \theta_{ki} \cdot \frac{\partial A_{ki}}{\partial \theta_{ki}} + \sum_{j \neq k} \theta_{ji} \cdot \frac{\partial A_{ji}}{\partial \theta_{ki}} \quad (j \neq k)$$

$$= A_{ki} + \sum_{j=1}^n \theta_{ji} \cdot \frac{\partial A_{ji}}{\partial \theta_{ki}} \quad (1 \leq j \leq n)$$

$$= A_{ki} + \sum_{j=1}^n \theta_{ji} \cdot B_{jk} = A_{ki} + \sum_{j=1}^n \theta_{ji} B_{kj} \quad (\text{symmetric})$$

$$\therefore D_{ki} = 2 A_{ki} \quad (1 \leq k \leq n)$$

$$\Rightarrow D = A$$

Problem 2.

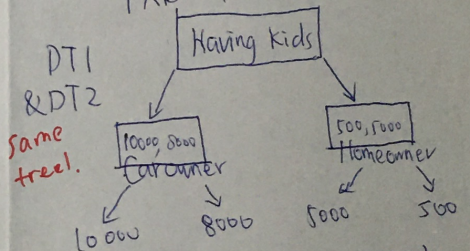
① GBM

age < 25 | home owner, age < 35, | having kids,
age < 45 | Car owner,
in total, 4 possible choices.

$$\text{Salary} = \frac{23500}{4} = 5875 \$$$

FO	PRO	F1	PR1	F2	PR2
5875	4125	6287.5	3712.5	6658.75	3341.25
5875	-5375	5337.5	-4837.5	4853.75	-4353.75
5875	2125	6087.5	1912.5	6278.75	1721.25
5875	-875	5787.5	-787.5	5708.75	-708.75

PRO = True - FO



$$F_1 = F_0 + 0.1 \times (\text{PRO})$$

$$F_2 = F_1 + 0.1 \times (\text{PR1})$$

We can always divide 4 people evenly, and get deviance = 0

$$\textcircled{2} \lambda = 1, \gamma = 0, M = 0.1$$

$$SS = \frac{\sum^2}{n+1}$$

since $\lambda = 1$
we need to compare $x^2 + y$ and $4xy$

FO	PRO	F1	PR1	F2	PR2
Same as ①	Same as ①	6081.25	3918.75	6277.12	3722.81
		5666.67	-5166.67	5472.23	-4972.23
		5981.25	2012.75	6081.89	1918.11
		5666.67	-666.67	5472.23	-472.23

same tree!

