Problem 1.

① Denote $X^TX = A$.

To prove A is symmetric, we need to show that $A_{ij} = A_{ji}$

Suppose X is a $n \times n$ matrix.

$$A_{ij} = \sum_{k=1}^{n} X_{k,i} \cdot \sum_{k=1}^{n} X_{k,j}$$
$$A_{ji} = \sum_{k=1}^{n} X_{k,j} \cdot \sum_{k=1}^{n} X_{n,i}$$
$$\Rightarrow A_{ij} = A_{ji}$$

A is symmetric.

② For $\frac{\partial \theta^T A \theta}{\partial \theta}$

$$\frac{\partial \theta^T A \theta}{\partial \theta} = \left(\frac{\partial \theta^T}{\partial \theta}\right) \cdot A\theta + \left(\frac{\partial (A\theta)^T}{\partial \theta}\right)\theta = A\theta + A^T\theta$$

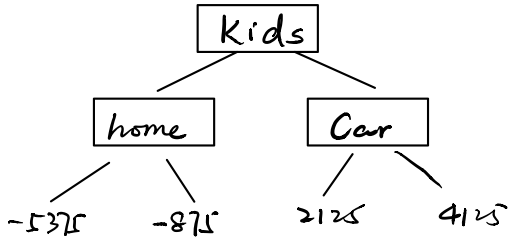
Since we've proved $A = X^TX$ is symmetric.

thus. $\frac{\partial \theta^T X^T X \theta}{\partial \theta} = 2A\theta = 2X^TX\theta$

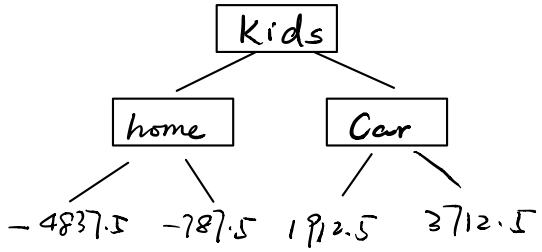
Problem 2.

① GBM:

DT for F1



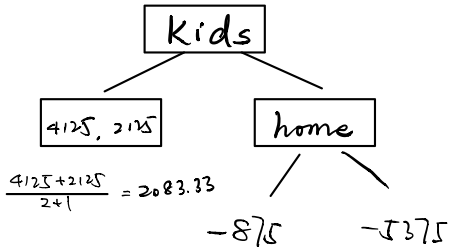
DT for F2



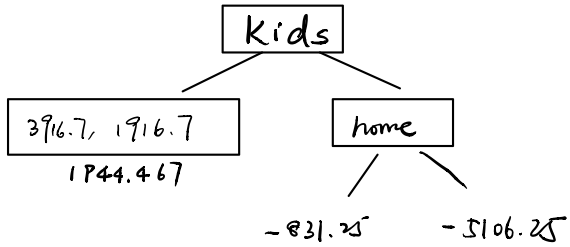
Age	F0	PR0	F1	PR1	F2	PR2
40	5875	4125	6287.5	3712.5	66587.5	3341.25
20	5875	-5375	5337.5	-4837.5	4853.755	-4353.7
50	5875	2125	6087.5	1912.5	6278.75	1721.25
30	5875	-875	5787.5	-787.5	5708.75	-708.75

② XGBoost

DT for F1



DT for F2



Age	F0	PR0	F1	PR1	F2	PR2
40	5875	4125	6083.33	3916.67	6277.78	3722.22
20	5875	-5375	5606.25	-5016.25	5350.94	-4850.94
50	5875	2125	6083.33	1916.67	6277.75	1722.25
30	5875	-875	5831.25	-831.25	5789.69	-789.69