VG441

Final

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Problem 1.

Task 1. Double Tree.

① MST (greedy & Kruskal Algo.)

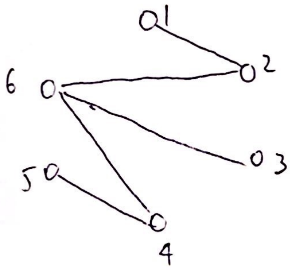
$2 \le 3 \le 4 \le 5 \le 6 \le 9 \le 10 \le 22 \le 33 \le 50$

$\le 66 \le 86 \le 86 \le 100 \le 952$

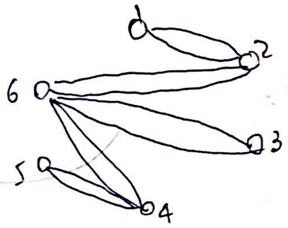
finally choose

2 (3&6), 3 (2&6), 4 (4&6)

5 (4&5), 10 (1&2)



② double it



③ path (Eulerian)

$1 \to 2 \to 6 \to 3 \to 6 \to 4 \to 5 \to 4 \to 6 \to 2 \to 1$

④ shortcut.

$1 \to 2 \to 6 \to 3 \to 4 \to 5 \to 1$

and get cost as

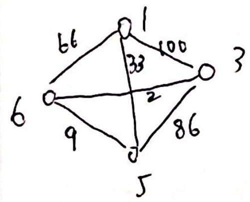
$10+3+2+6+5+33=59$.

Task 2. Christofide's

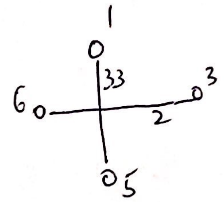
(just improve performance on nodes that has odd degree)

① MST (same as task 1)

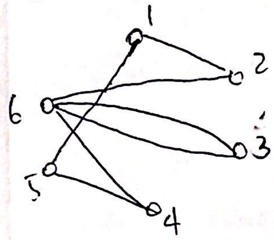
② set of odd degree $\{1, 6, 3, 5\}$



∴ min-weight matching is (eyeball)



③ Add them up



④ Eulerian path (walk)

$1 \to 2 \to 6 \to 3 \to 6 \to 4 \to 5 \to 1$

⑤ shortcut

$1 \to 2 \to 6 \to 3 \to 4 \to 5 \to 1$

and get cost as

$10+3+2+6+5+33=59$

Problem 2

Task 2. (Greedy)

we rank the $\frac{V_i}{S_i}$ and get

$\frac{6}{8} \le \frac{5}{5} \le \frac{4}{3} \le \frac{4}{3}$

$(0, \frac{2}{5}, 1, 1)$

$\downarrow$

$(0, 0, 1, 1)$

Running greedy algo. we will choose item 1 and 2, the total value is 8.

Not optimal:

↳ By choosing item 1&4, total value is $4+5=9$ which is more than 8.

Problem 2

Task 1 (KS) (* blank grids are ∞) $\checkmark$

no need for nv_{max}, 19 is enough

$i \backslash w$	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
i=1	0				3															
i=2	0				3				6											
i=3	0				3		8		6		11				14					
i=4	0					5				8		13		11		16				19

$\binom{v}{s} = \binom{4}{3}, \binom{4}{3}, \binom{6}{8}\binom{5}{5}$

we can read from the table that

$\arg\max_{w}\{T[n,w] : T[n,w] \le B\} = 9$.

Problem 3

Task 1 Greedy

first iteration:

$S_1 = \frac{6}{5} = 1.2$ $\quad \because 3 > 1.2 > 1$

$S_2 = \frac{15}{5} = 3$ $\quad$ we choose S_3.

$S_3 = \frac{7}{7} = 1$

second iteration:

$S_1 = \frac{6}{3} = 2$ $\quad \because 3 > 2$

$S_2 = \frac{15}{5} = 3$ $\quad$ we choose S_1

third iteration:

we still need to choose.

However, only S_2 is left, choose S_2.

Luckily, after all iterations,

$V = S_1 \cup S_2 \cup S_3$,

we call it a day. Total cost is

$6 + 15 + 7 = 28$ (S_1, S_2, S_3 are chosen)

Problem 3

Task 2.

this greedy solution is not optional,

By choosing S_2 & S_3,

$V = S_2 \cup S_3$ with cost $15 + 7 = 22 < 28$

which is a better solution.

Q.E.D.

Also, I will prove $S_2 \cup S_3$ is optimal.

Since V should be covered,

we have to cover e_{12} and e_6,

which means at least S_2 and S_3 should be chosen. Thus optimal.

Bonus.

Task 1.

Dual (D) is

object: maximize $\sum_{j \in D} \alpha_j$ $\quad$ ☆

constrain: $\alpha_j - \beta_{ij} \le d_{ij}$, $\forall i \in F, j \in D$. $\quad$ (*)

$\sum_{j \in D} \beta_{ij} \le f_i$, $\forall i \in F$ $\quad$ (*)(*)

$\alpha_j \ge 0$, $\forall j \in D$

$\beta_{ij} \ge 0$ $\quad \forall i \in F, j \in D$

(By plugging in general form in Lec 011) detailed process: 令 $i=|F|, j=|D|$.

obj. $f_1 \cdot y_1 + \cdots + f_i y_i + d_{11} x_{11} + \cdots + d_{ij} x_{ij}$ min

$$\text{s.t.}\quad \begin{cases} x_{11} + x_{21} + \cdots + x_{i1} \geq 1 & \alpha_1 \\ \vdots \\ x_{1j} + \cdots + x_{ij} \geq 1 & \alpha_j \end{cases}$$

$$\begin{cases} 1 \cdot x_{11} \leq y_1 & \beta_{11} \\ \vdots & \vdots \\ 1 \cdot x_{1j} \leq y_1 & \beta_{1j} \\ \vdots & \vdots \\ 1 \cdot x_{i1} \leq y_i & \beta_{i1} \\ \vdots & \vdots \\ 1 \cdot x_{ij} \leq y_i & \beta_{ij} \end{cases}$$

$\Rightarrow$ coefficient comparison.

$$\therefore (\beta_{11} + \cdots + \beta_{1j}) \cdot y_1 + \cdots + (\beta_{i1} + \cdots + \beta_{ij}) y_i + (\alpha_1 - \beta_{11}) x_{11} + \cdots + (\alpha_j - \beta_{ij}) x_{ij} \geq \alpha_1 + \cdots + \alpha_j$$

Task 2, Bonus.

it is easy to understand that $\alpha_j \geq 0$ and $\beta_{ij} \geq 0$.

⊛⊛ for each facility, it should be fully paid for by all the demands, which is $\sum_{j \in D} \beta_{ij} = f_i$ when optimal.

⊛ for every demand, α_j is the total amount of money j is willing to pay, we want to maximize $\sum \alpha_j$ to get most profit (purchasing potential) I guess

⊛ $\alpha_j = \min_i (\beta_{ij} + d_{ij})$ when optimal. the RHS is the cost to open facility + cost of travel, should be covered by the demand j

⊛ the demand side should be willing to pay for the total cost when optimal.

Honor Code

I accept the letter and spirit of the honor code:

I have neither given nor received unauthorized aid on this examination, nor have I concealed any violations of the Honor Code by myself or others.

Wang Yichao