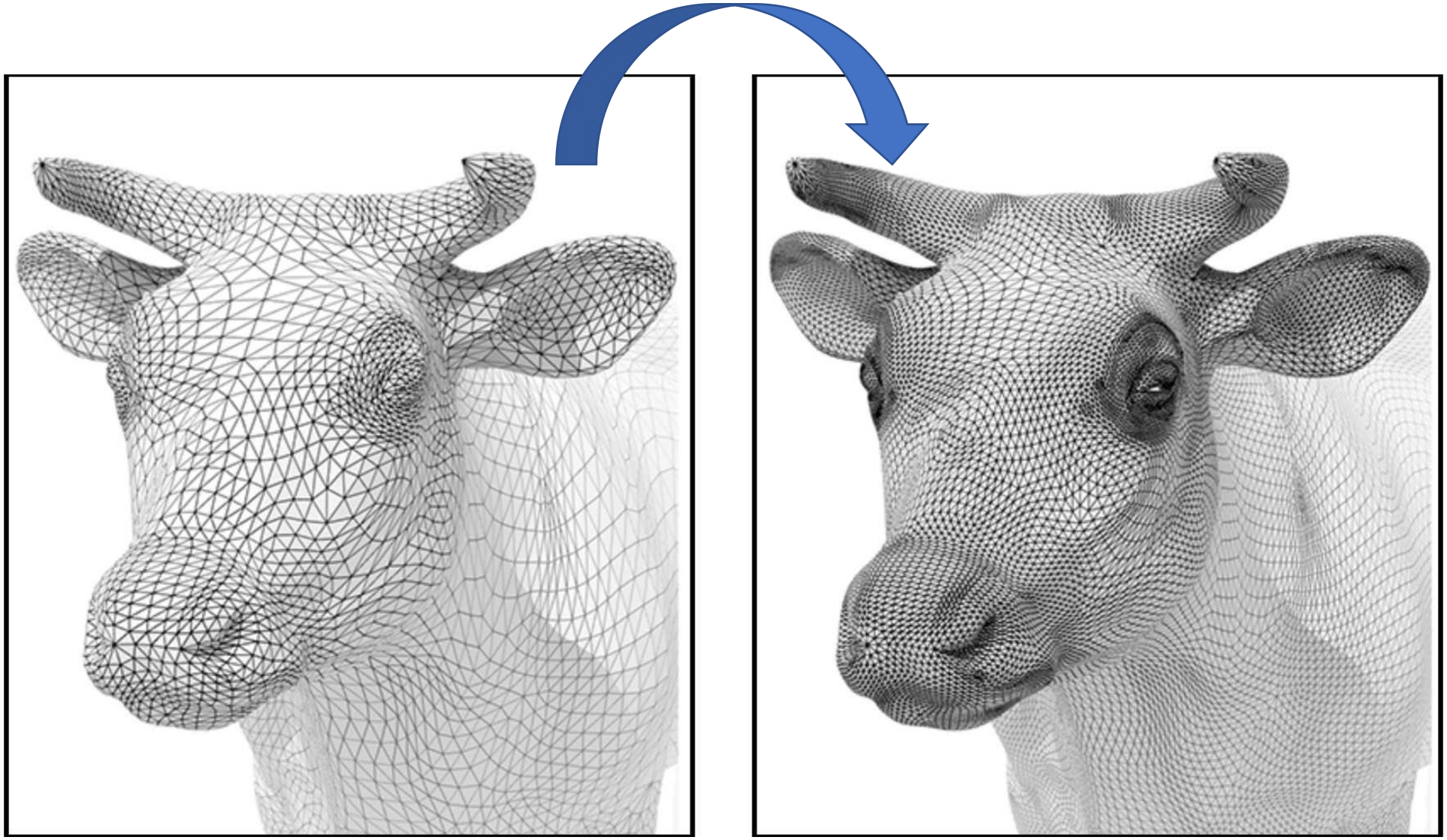
Mesh Processing

Mesh Subdivision

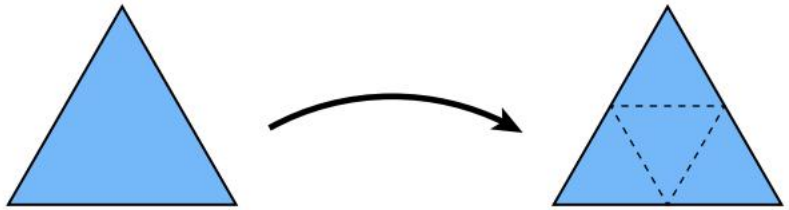
Mesh Subdivision

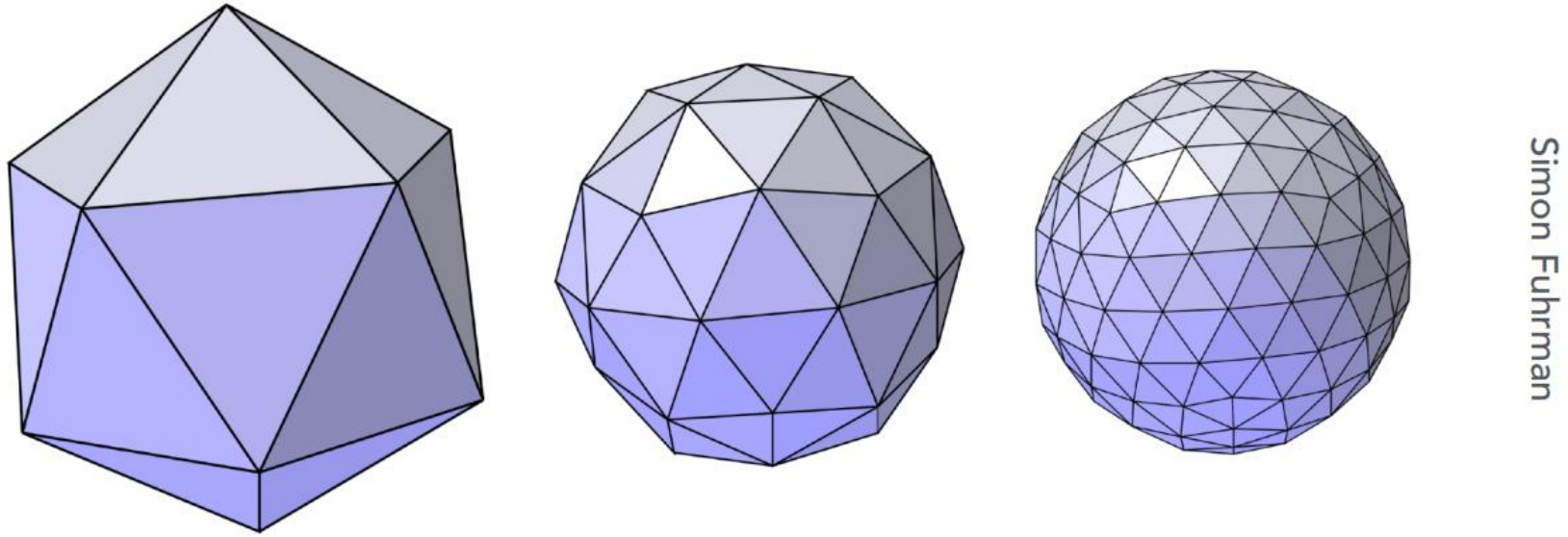


Loop Subdivision

Triangular Mesh Subdivision

- One-to-four split





Simon Fuhrman

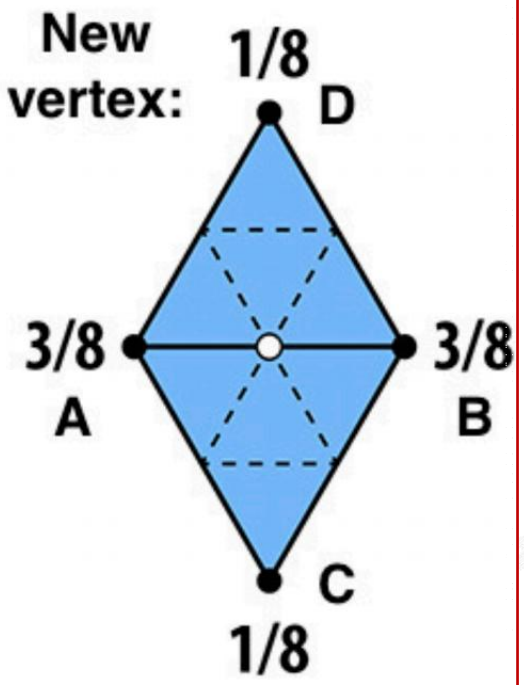
Loop Subdivision

Triangular Mesh Subdivision

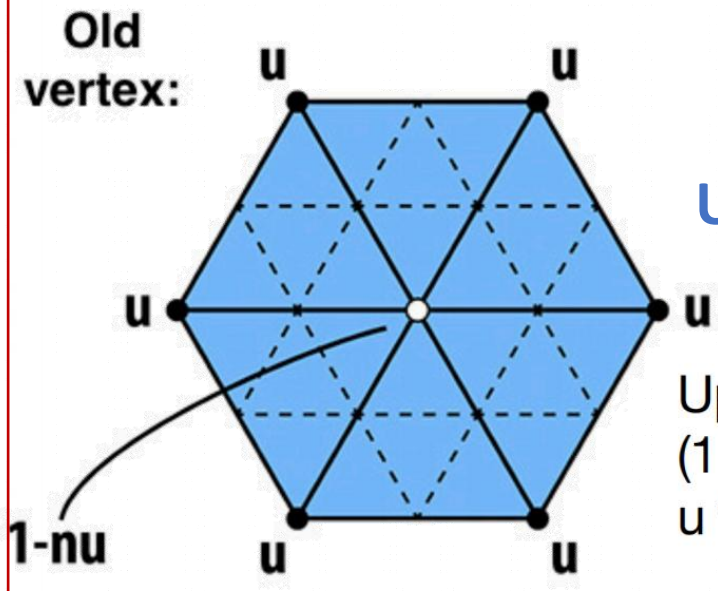
- One-to-four split
- Update vertex position

Update new vertex

Update to:
3/8 * (A + B) + 1/8 * (C + D)



Update old vertex

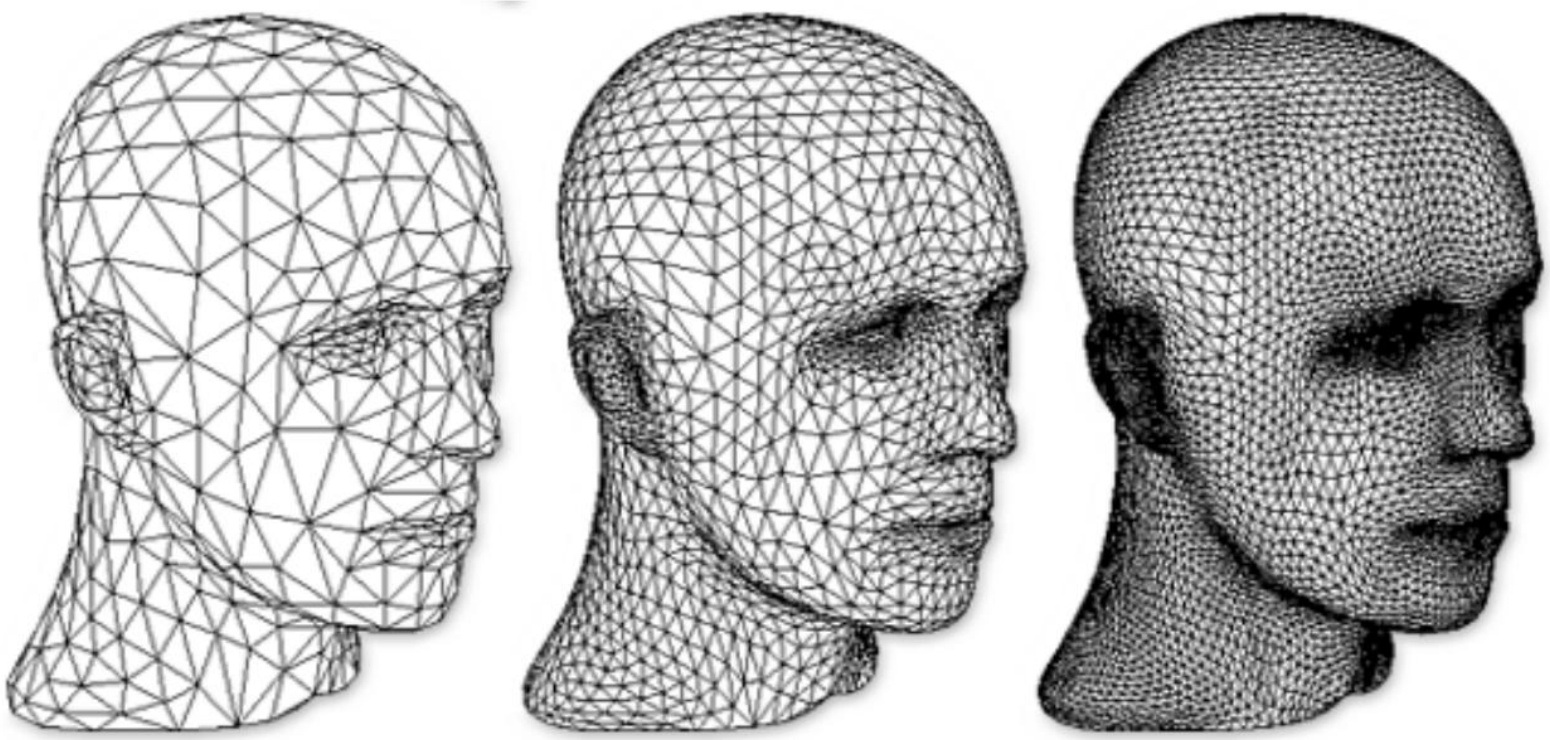


Update to:
(1 - n*u) * original_position +
u * neighbor_position_sum

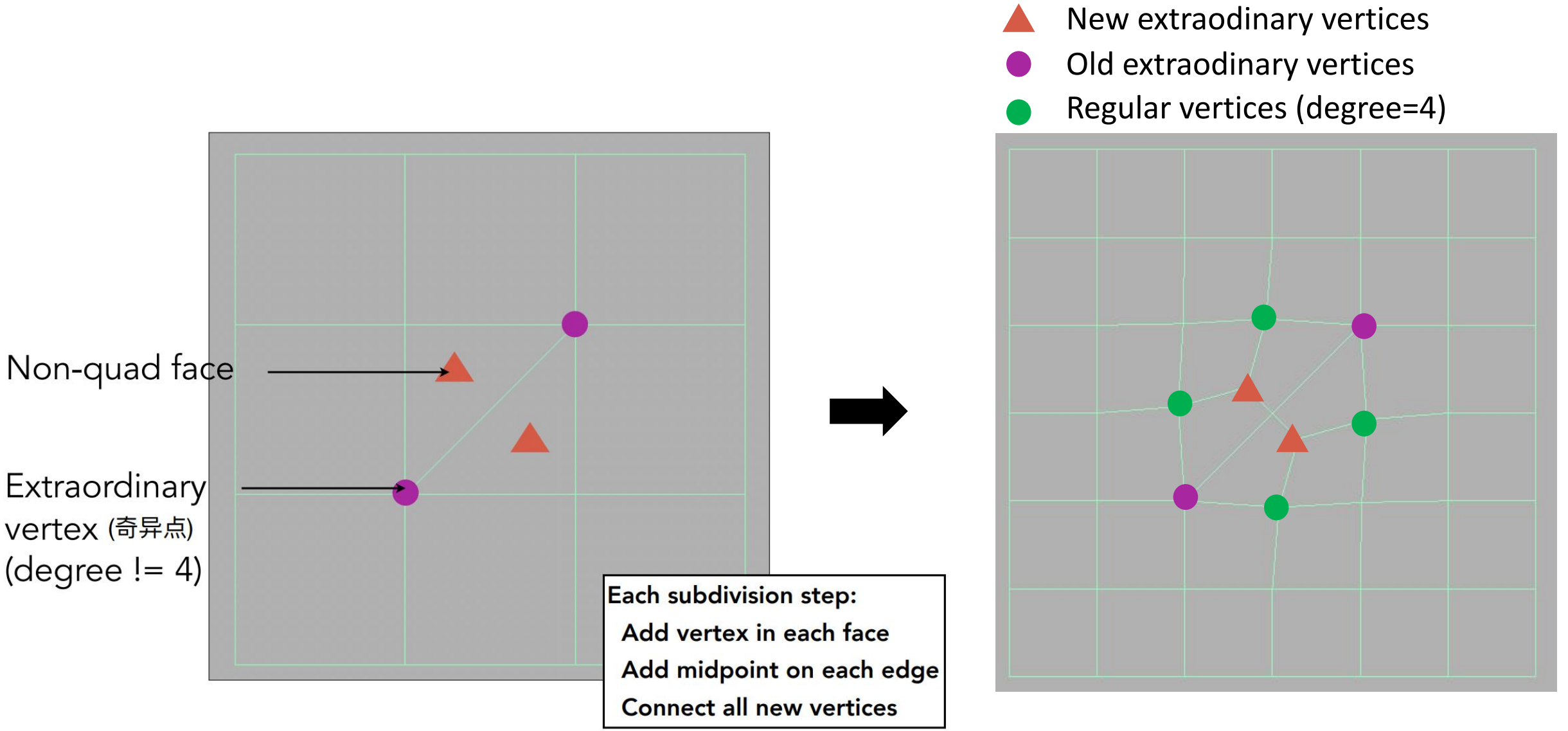
Loop Subdivision

Triangular Mesh Subdivision

- One-to-four split
- Update vertex position



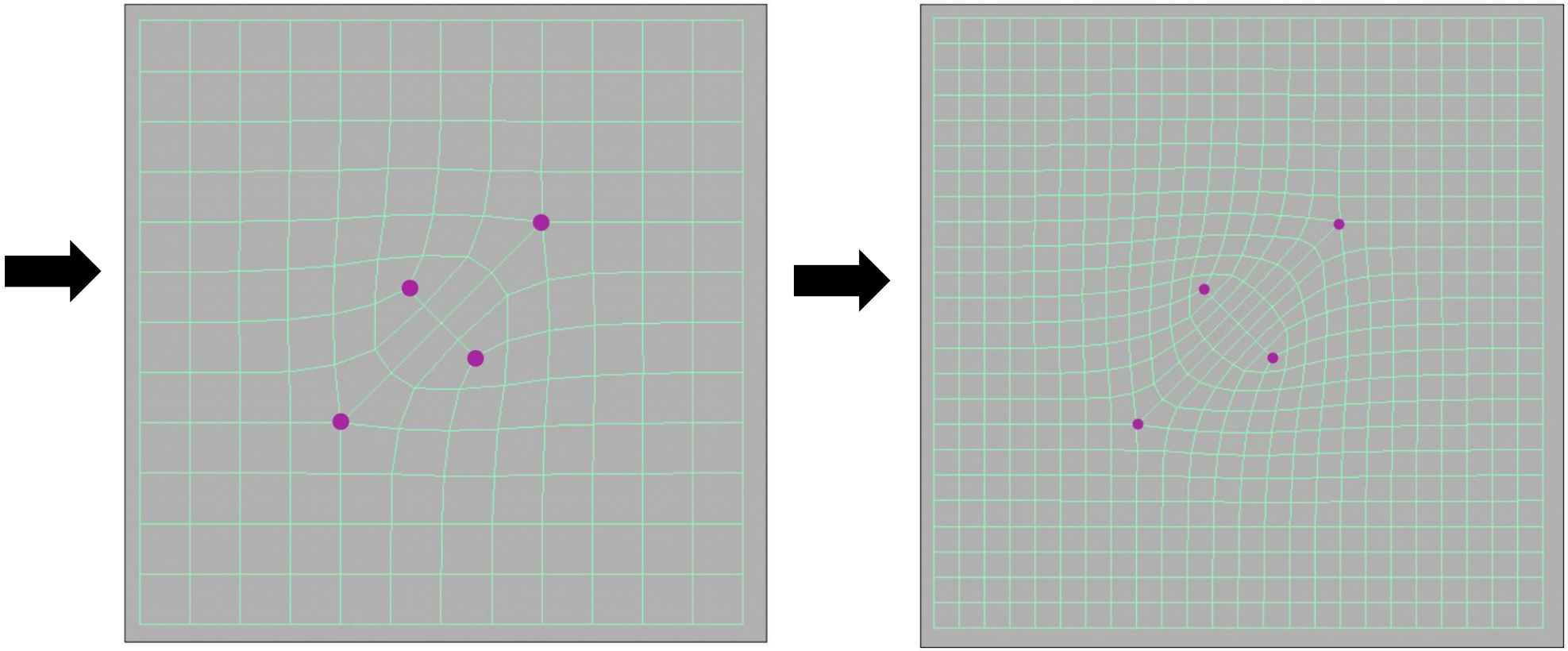
Catmull-Clark Subdivision (General Mesh)



Catmull-Clark Subdivision (General Mesh)

No more extraordinary vertices!

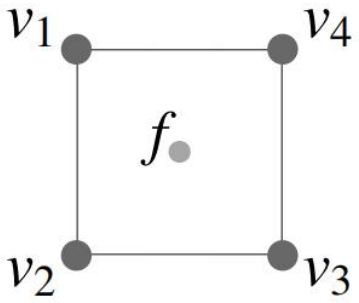
All quad faces!



Catmull-Clark Subdivision (General Mesh)

Three types of vertices

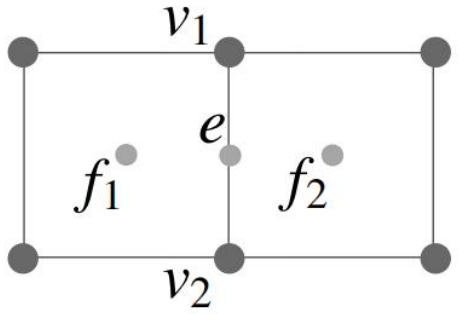
Face point

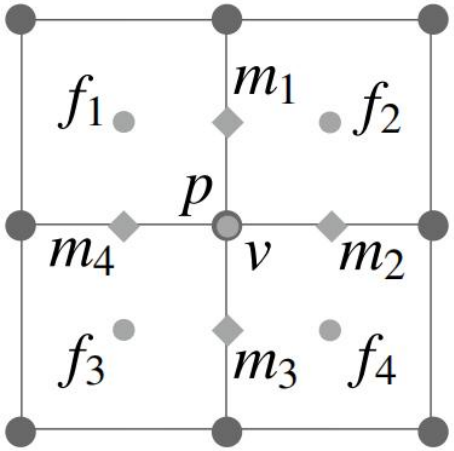


$$f = \frac{v_1 + v_2 + v_3 + v_4}{4}$$

$$e = \frac{v_1 + v_2 + f_1 + f_2}{4}$$

Edge point





Vertex point

$$v = \frac{f_1 + f_2 + f_3 + f_4 + 2(m_1 + m_2 + m_3 + m_4) + 4p}{16}$$

m midpoint of edge

p old "vertex point"

Convergence: Overall Shape and Creases

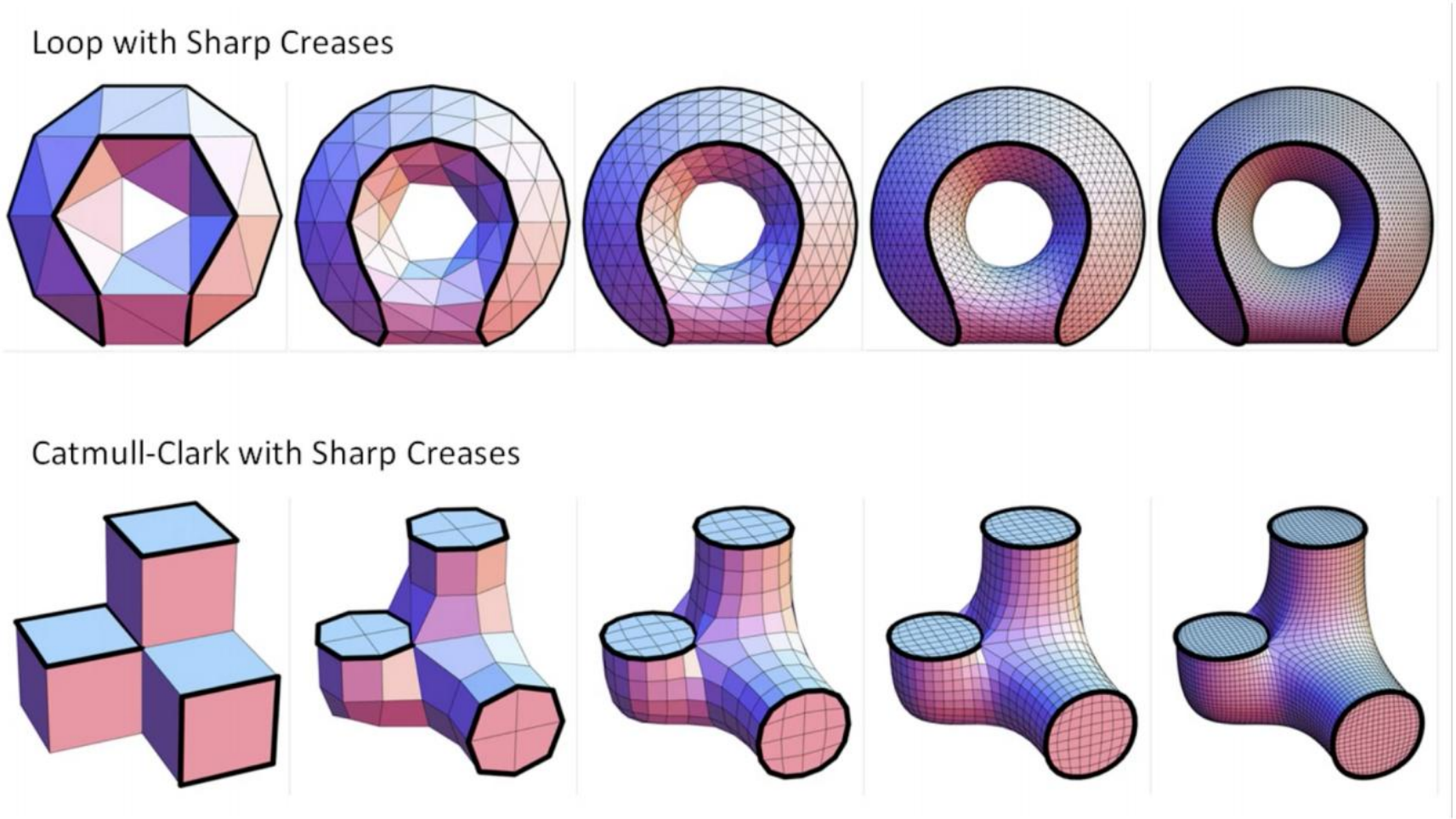
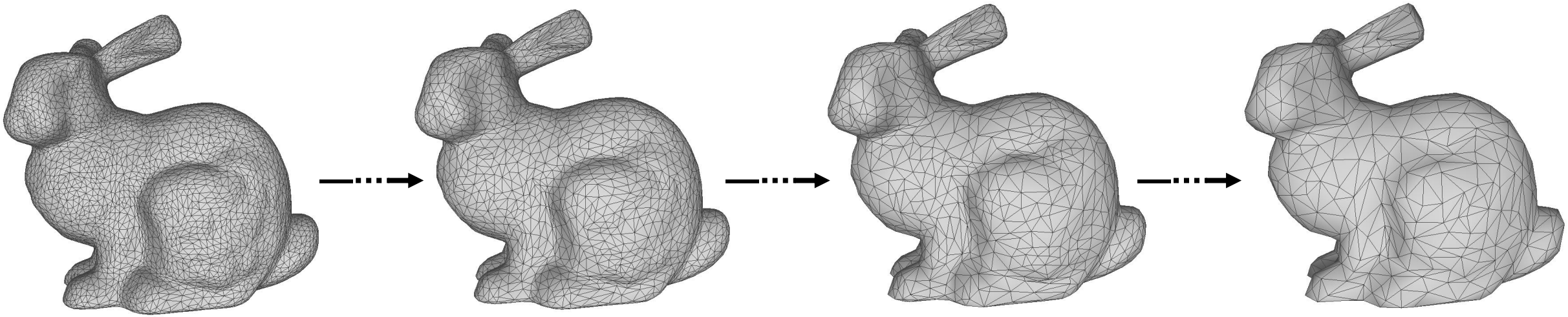


Figure from: Hakenberg et al. Volume Enclosed by Subdivision Surfaces with Sharp Creases

Mesh Simplification

Mesh Simplification

- Iteratively remove one vertex/edge per step



|V|=12000 |V|=6000 |V|=3000 |V|=1500

Mesh Simplification

- Initialization
- Repeat:
- select a vertex/edge to remove
- single-simplification step

When to stop?

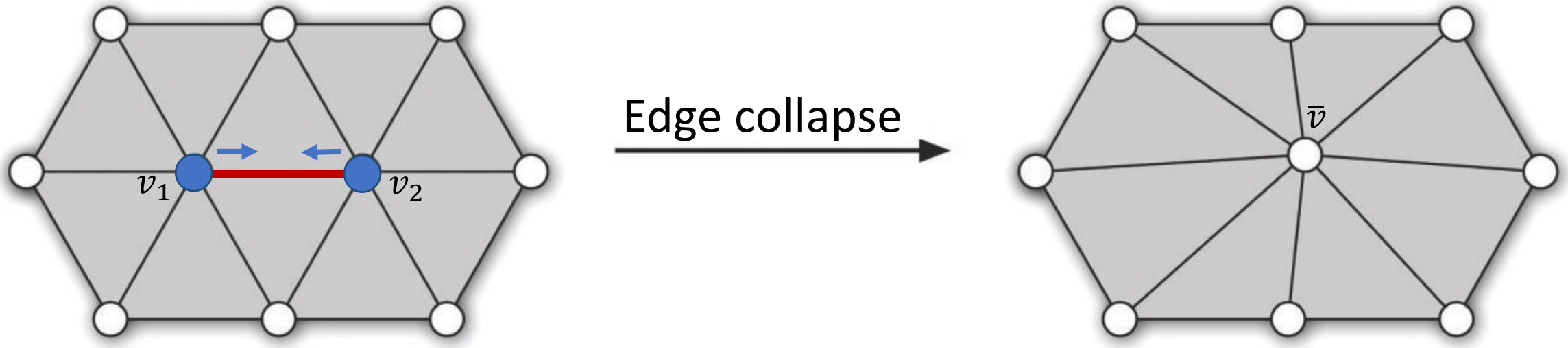
How to select?

How to simplify?

QEM(Quadric Error Metrics) Simplification

How to simplify?

Edge Collapse: one edge contracted to one vertex

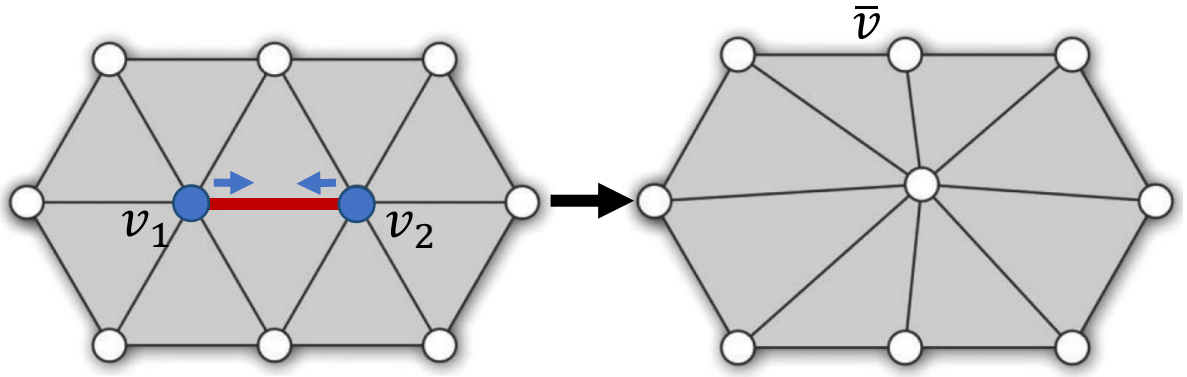


QEM(Quadric Error Metrics) Simplification

How to simplify?

Edge Collapse: one edge contracted to one vertex

Update position of new vertex: Quadric Error Metrics



优化收缩点$\bar{v}$的位置，使之尽可能接近原模型。
优化目标函数：

$$\bar{v} = \underset{v}{\operatorname{argmin}} \sum_{P \in plane(v_1) \cup plane(v_2)} distance(v, P)^2$$

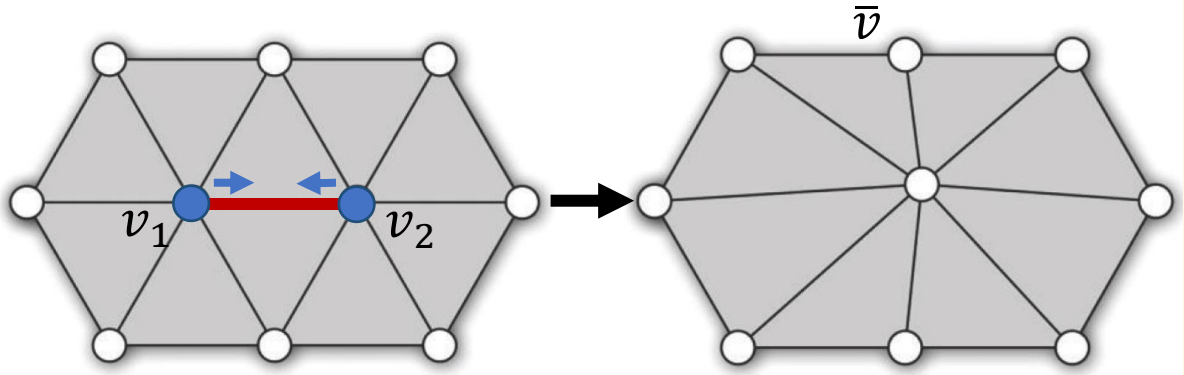
$plane(v)$指顶点v邻接的三角面片集合

QEM(Quadric Error Metrics) Simplification

How to simplify?

dge Collapse: one edge contracted to one vertex

Update position of new vertex: Quadric Error Metrics



优化收缩点$\bar{v}$的位置，使之尽可能接近原模型。
优化目标函数：

$$\bar{v} = \underset{v}{\operatorname{argmin}} \sum_{P \in plane(v_1) \cup plane(v_2)} distance(v, P)^2$$

$plane(v)$指顶点v邻接的三角面片集合

令平面P为$ax + by + cz + d = 0$，其中$a^2 + b^2 + c^2 = 1$。
记$\boldsymbol{p} = [a, b, c, d]^T$，$v = [x, y, z, 1]^T$，
则有：

$$distance(v, P)^2 = (v^T p)^2 = v^T p p^T v = v^T K_p v$$

因此，设$plane(v)$为顶点v邻接的三角面片，收缩点的位置可写为：

$$\bar{v} = \underset{v}{\operatorname{argmin}}\, v^T \left(\sum_{P \in plane(v_1) \cup plane(v_2)} K_p \right) v$$

又因：

$$\bar{v} \approx \underset{v}{argmin}\, v^T \left(\sum_{p \in plane(v_1)} K_p + \sum_{p \in plane(v_2)} K_p \right) v$$

所以，最终可写成：

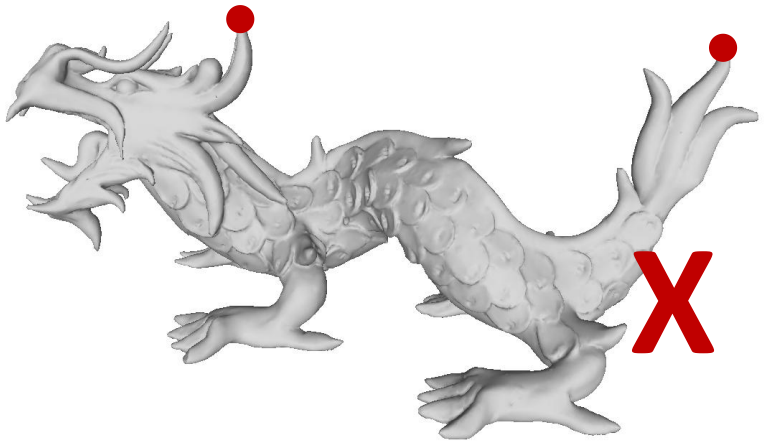
$$\bar{v} = \underset{v}{\operatorname{argmin}}\, v^T (Q_1 + Q_2) v$$

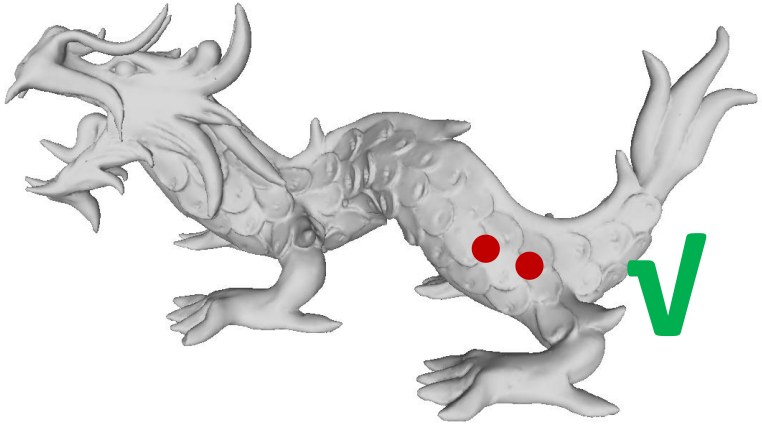
由v_1和v_2的邻接面计算得到

QEM(Quadric Error Metrics) Simplification

How to select?

- Select the connected nearby vertices

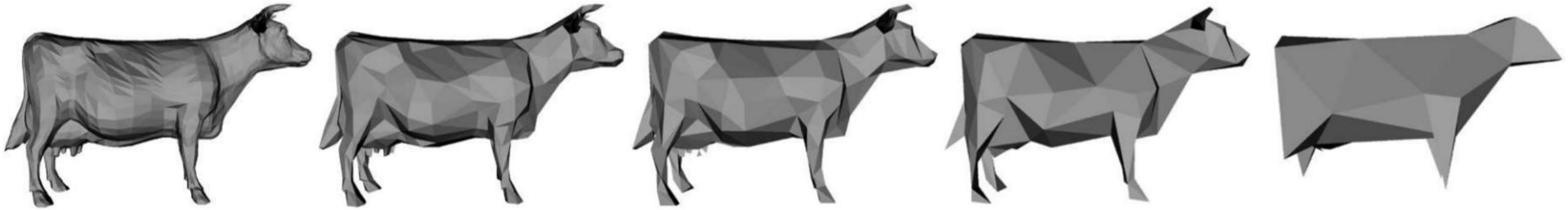




QEM(Quadric Error Metrics) Simplification

QEM Algorithm Outline

1. Compute the Q_v matrices for all the initial vertices.
2. Select all valid vertex pairs.
3. Compute the optimal $\overline{v}$ for each pair, $\Delta(\overline{v})$ becomes the cost of contracting that pair.
4. Place all the pairs in a heap keyed on cost with the minimum cost pair at the top.
5. Iteratively remove the pair (v_1, v_2) of least cost from the heap, contract this pair, and update the costs of all valid pairs involving the new vertex $\overline{v}$.



Simplification Results

Thank you