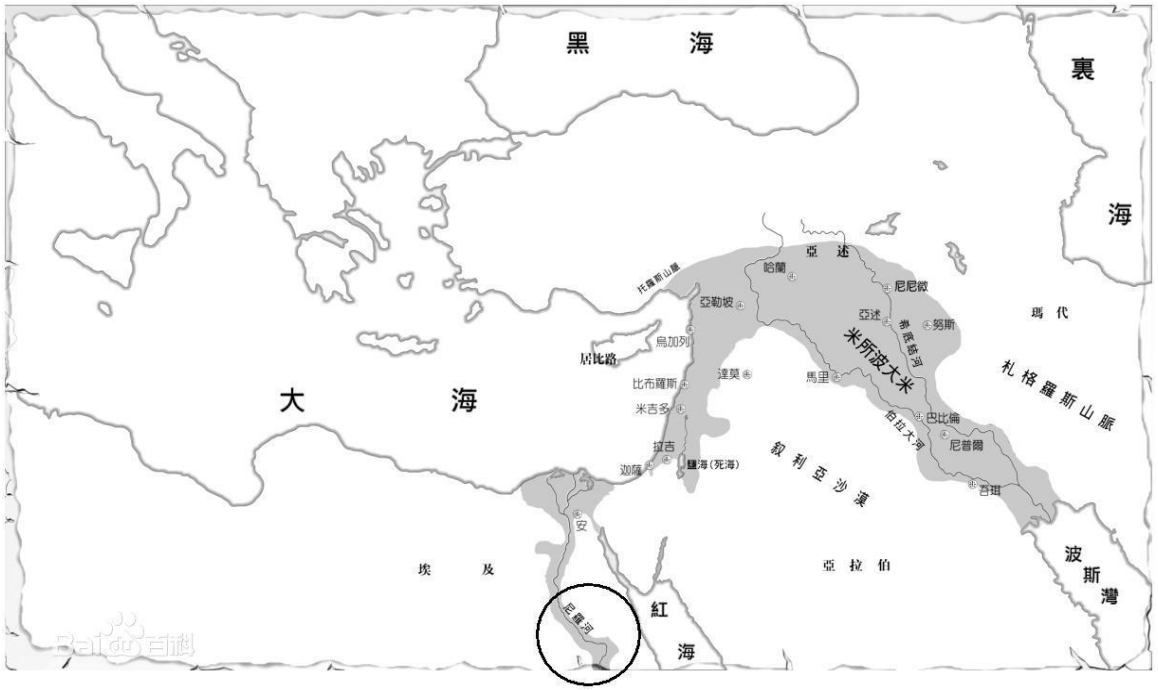
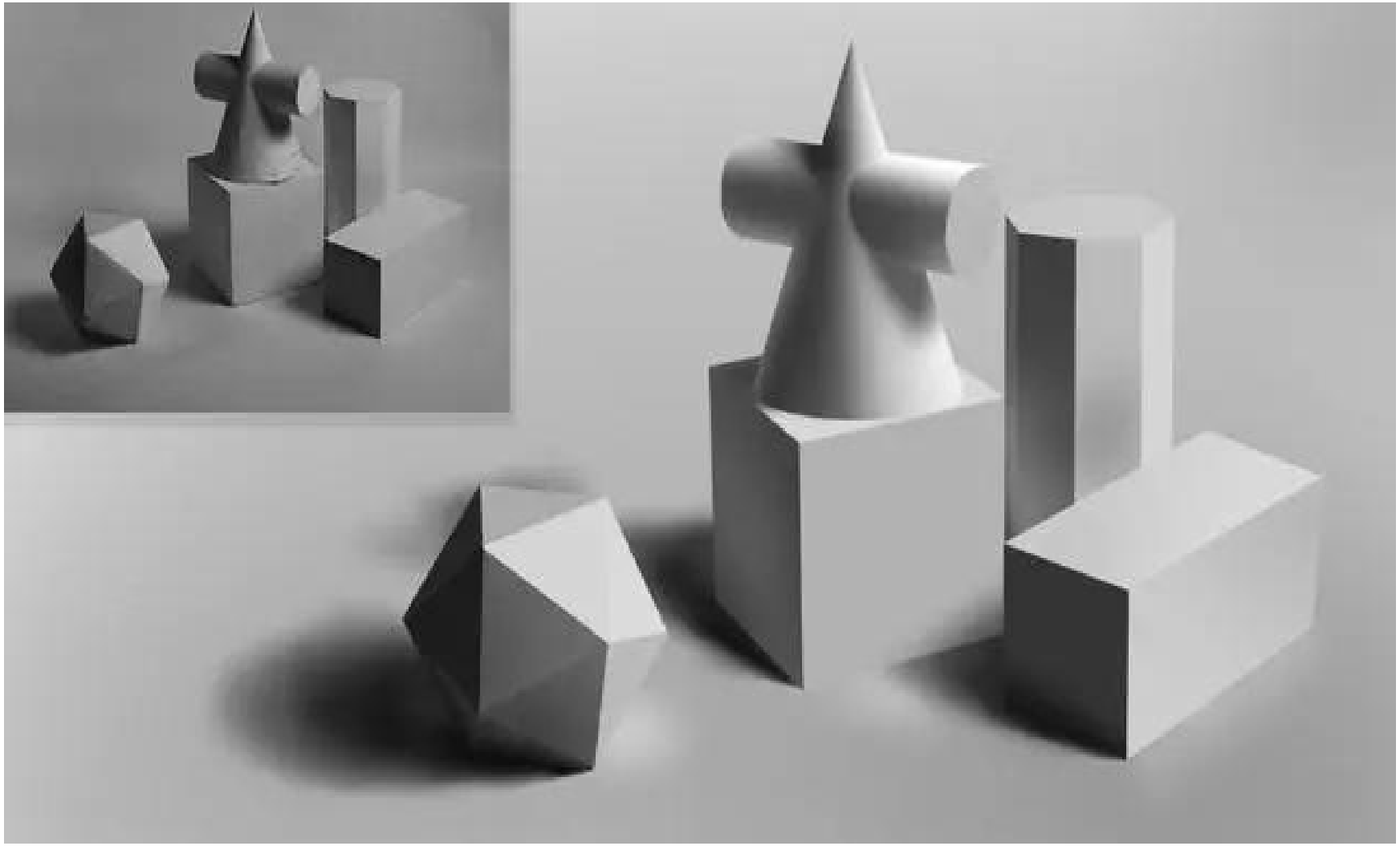
Geometry

Geometry

- 几何学
- 阿拉伯语，指土地的测量，即测地术。
- 《几何原本》,公元前338年
  - 欧几里得 著
  - 明 徐光启、利玛窦 译
- 《墨经》、《九章算术》



Geometry



Geometry

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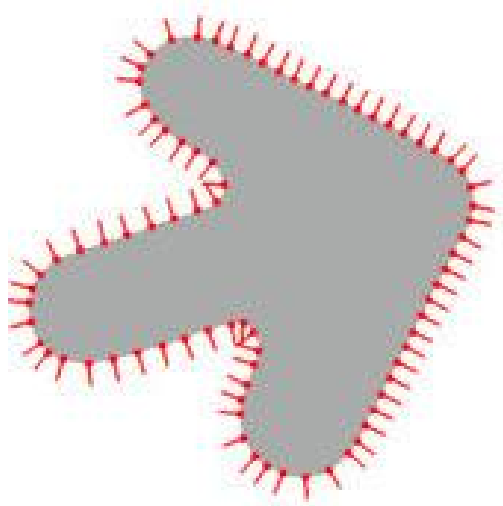
Points and Meshes

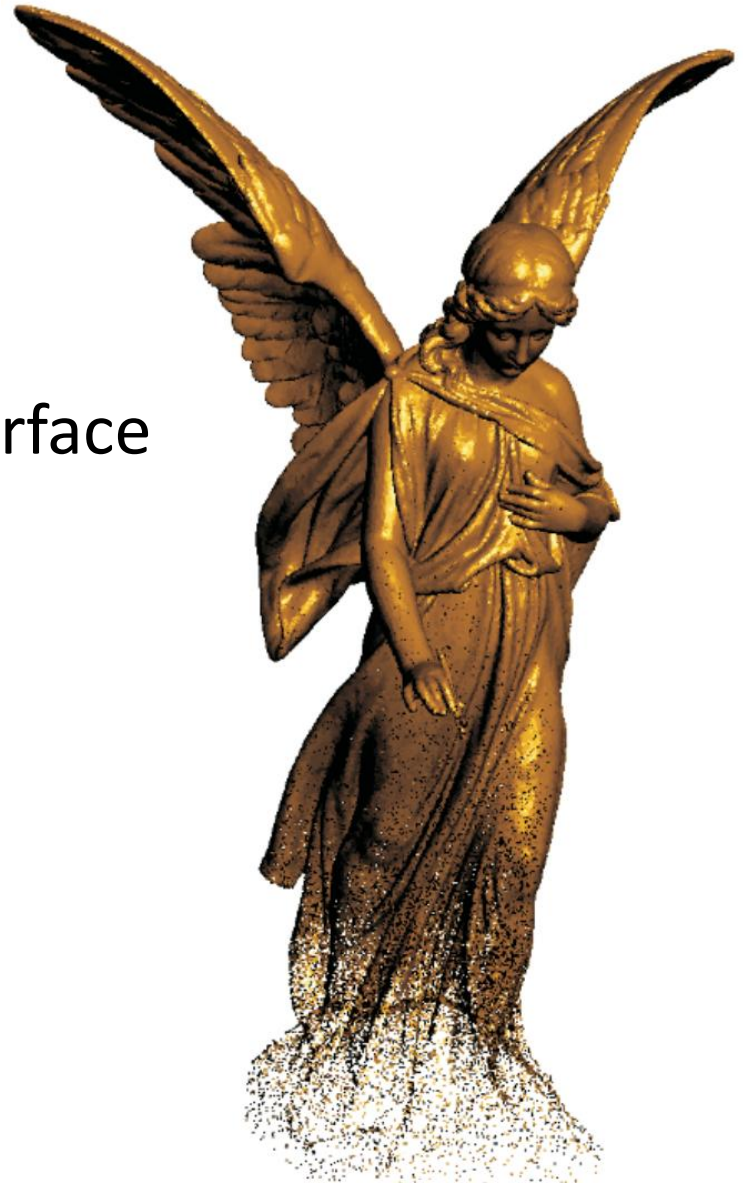
Point Cloud

- A set of datapoints sampled from the underlying surface
- Attributes: color, normal, etc.

2D point cloud



Oriented point cloud

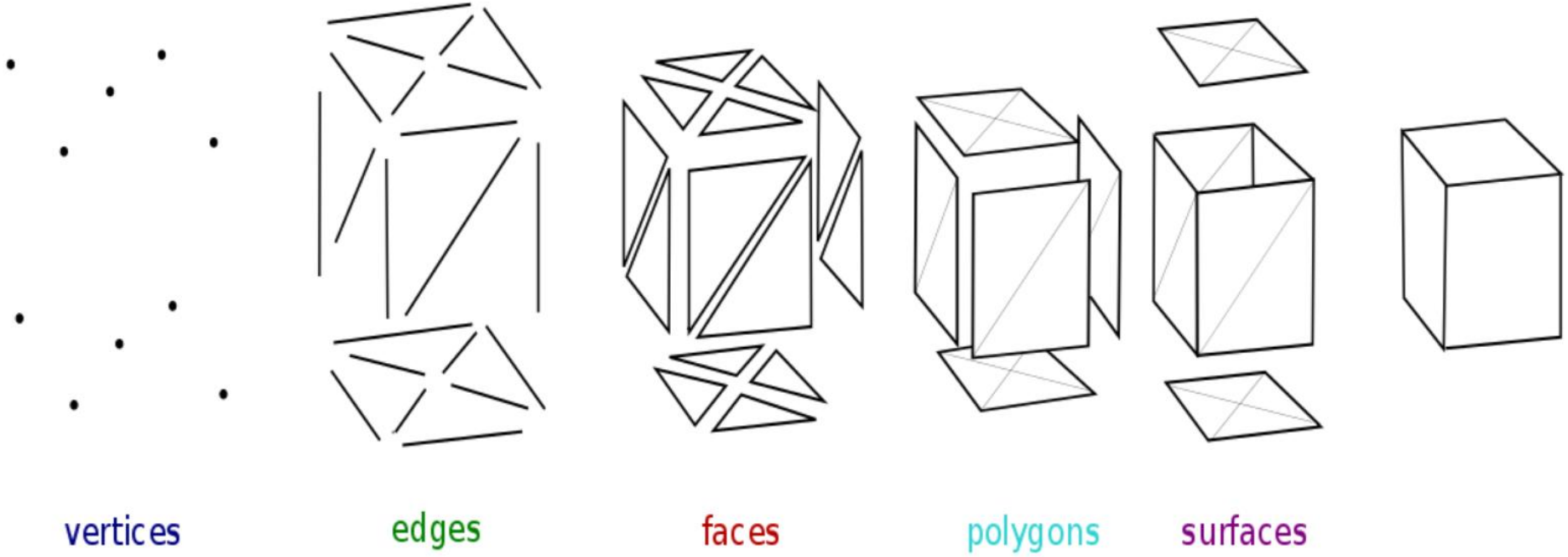


Point Set Surface

Points and Meshes

Polygon Meshes

- A collection of vertices, edges and faces
- Triangular mesh, quad mesh, etc.



Points and Meshes

Polygon Meshes

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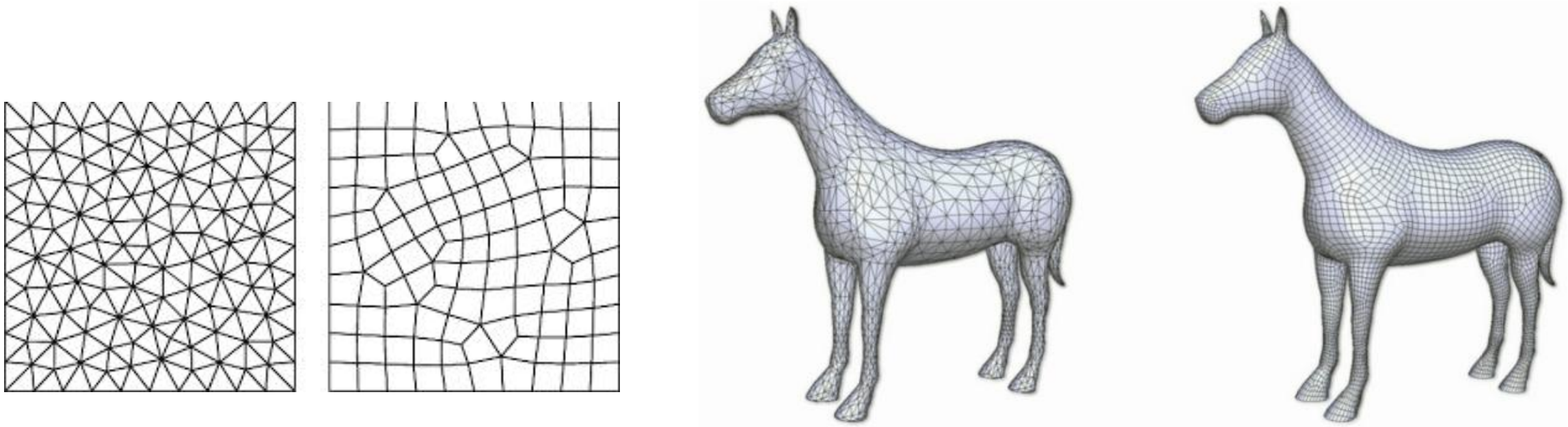


Triangular and quadrilateral meshes

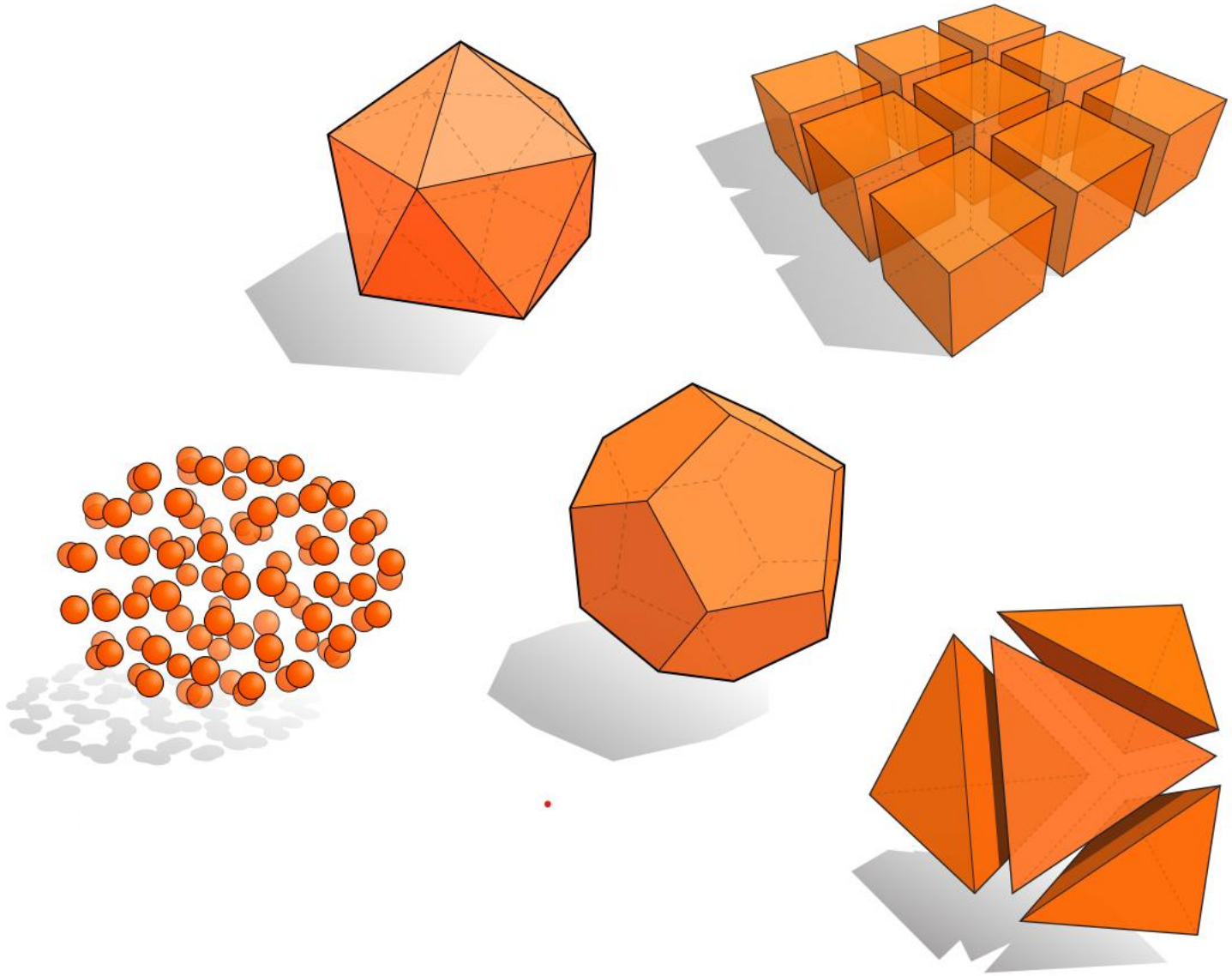
Many Representations of Geometry

Explicit

- point cloud
- polygon mesh

Implicit

- algebraic surface
- level sets
- distance function

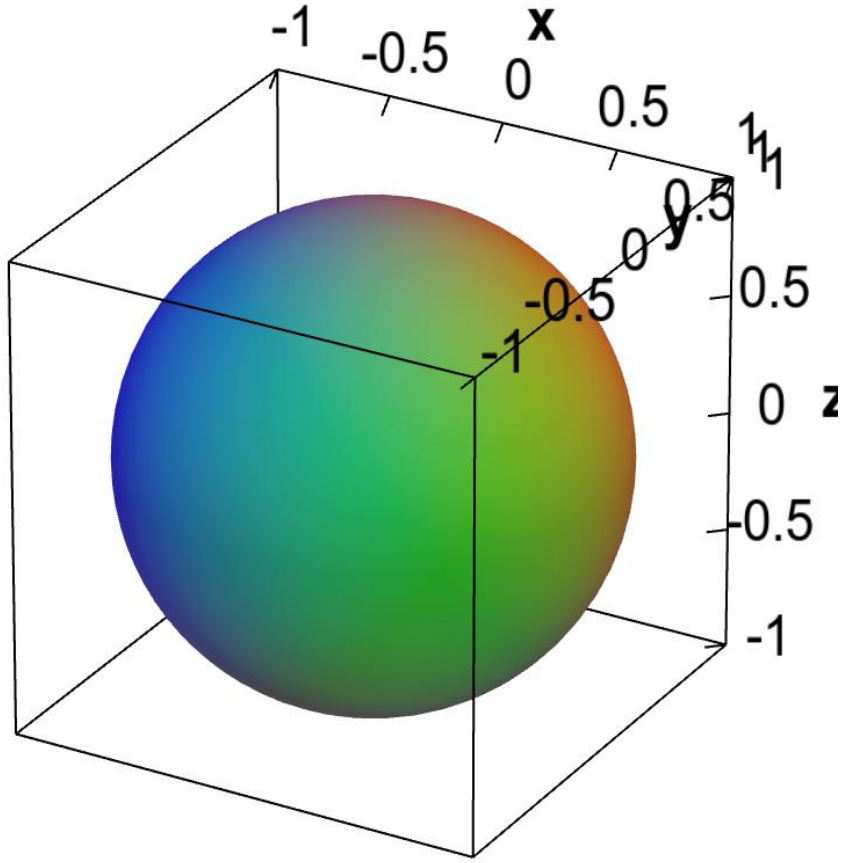


“Implicit” Representations of Geometry

Classifying points with implicit function f(x,y,z) = 0

- Points satisfy some specified relationship

E.g. sphere: all points in 3D, where $x^2+y^2+z^2-1=0$

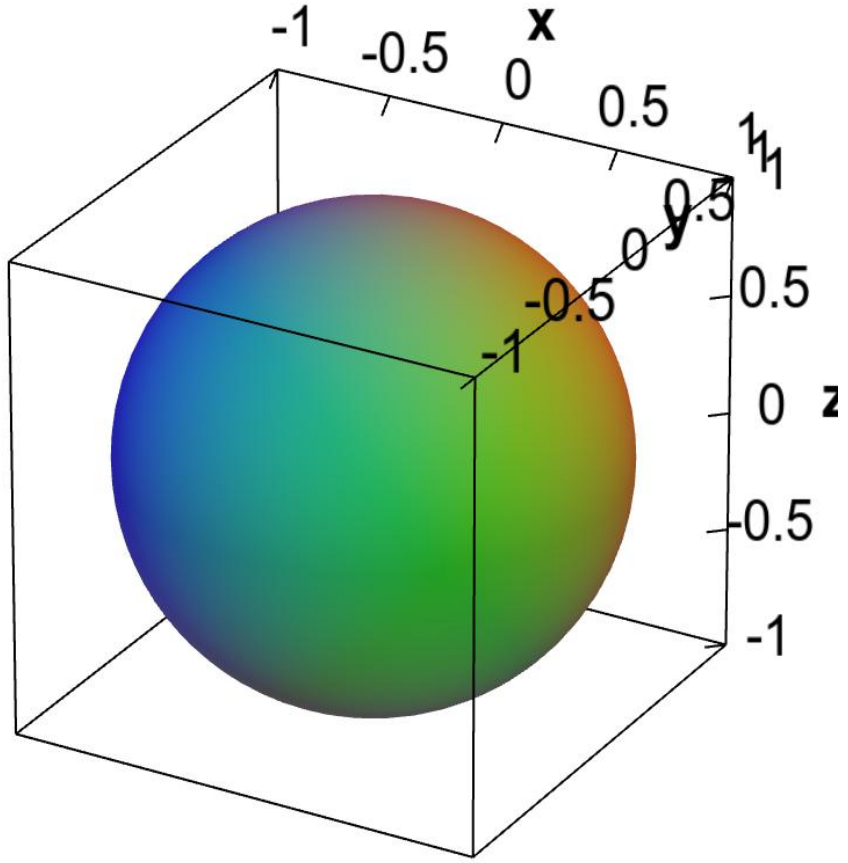


“Implicit” Representations of Geometry

Is (0.3, 0.3, 0.5) inside or outside the sphere?

- $0.3^2+0.3^2+0.3^2-1 = -0.73 < 0$
- Inside!

Inside/Outside Test Easy!

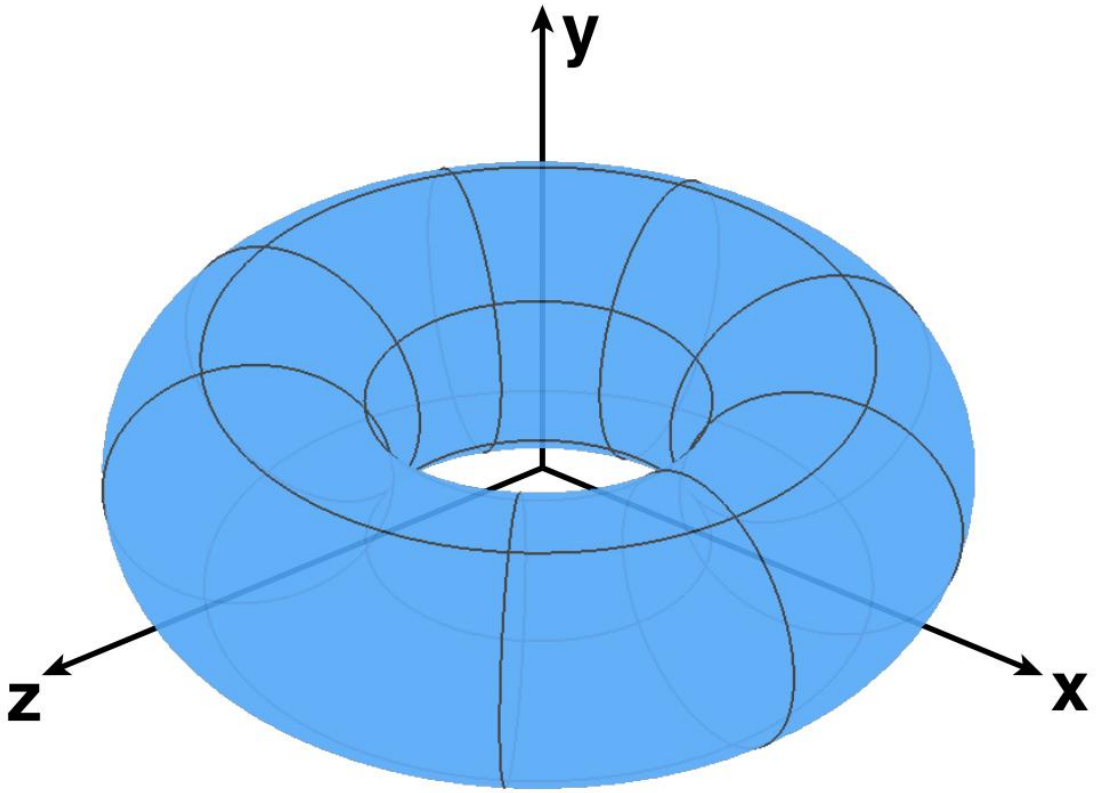


“Implicit” Representations of Geometry

$$f(x, y, z) = (2 - \sqrt{x^2 + y^2})^2 + z^2 - 1$$

What points lie on f(x,y,z) = 0?

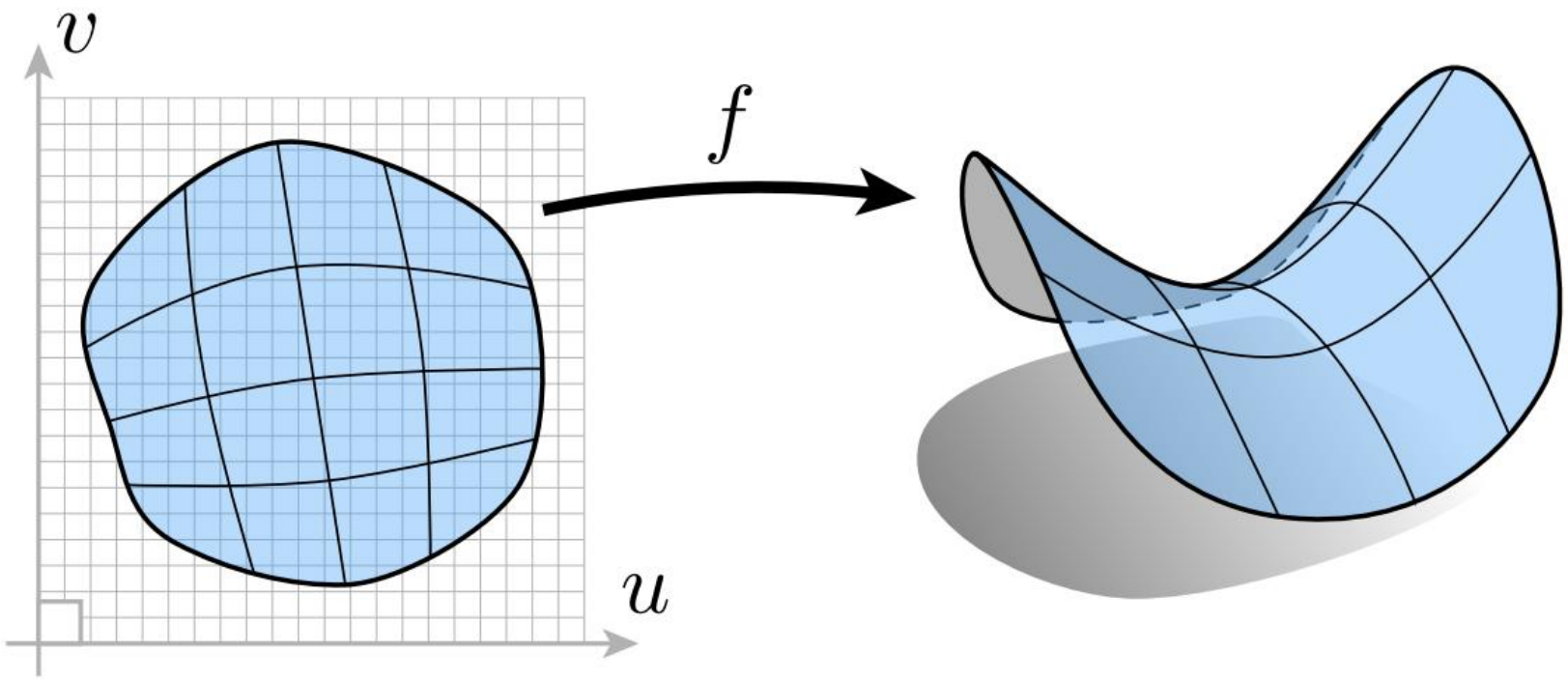
Point Sampling Hard!



“Explicit” Representations of Geometry

All points are given directly or via parameter mapping

Generally: $f : \mathbb{R}^2 \to \mathbb{R}^3; (u, v) \mapsto (x, y, z)$

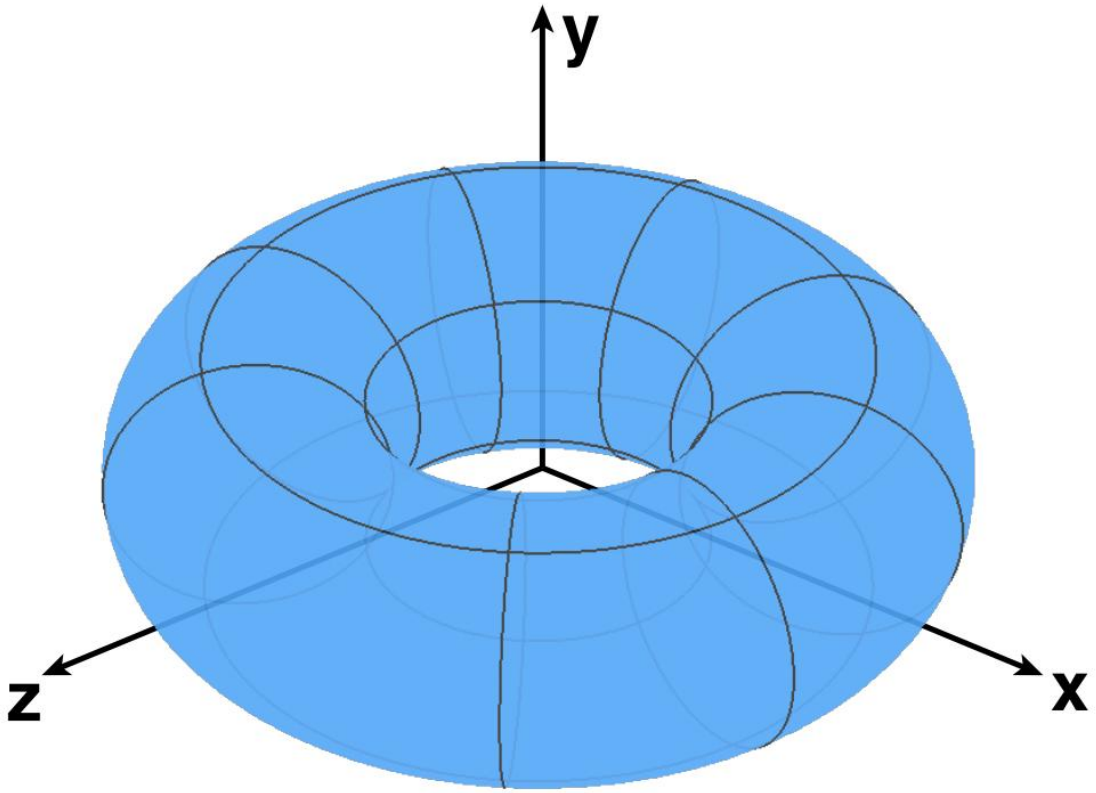


“Explicit” Representations of Geometry

$$f(u,v) = ((2+\cos u)\cos v, (2+\cos u)\sin v, \sin u)$$

What points lie on this surface?

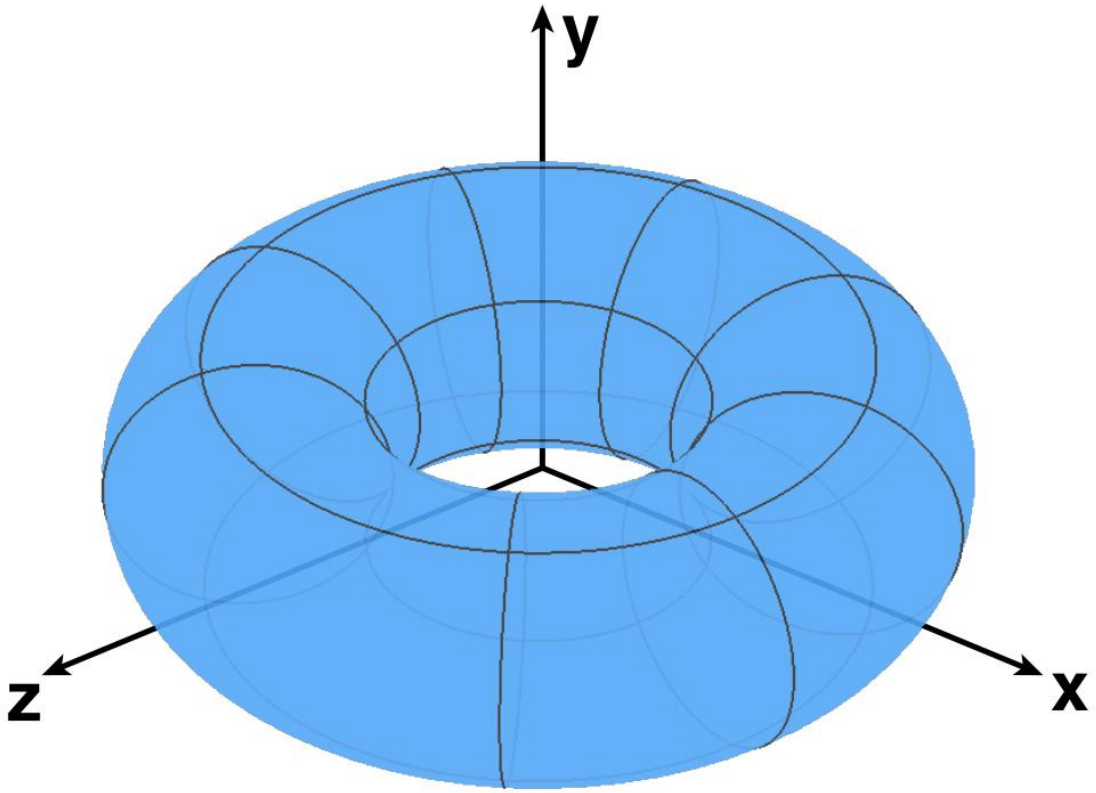
Just plug in (u,v) values!



“Explicit” Representations of Geometry

$$f(u,v) = ((2+\cos u)\cos v, (2+\cos u)\sin v, \sin u)$$

Is (0.3,0.3,0.5) inside or outside this shape?



No “Best” Representation – Geometry is Hard!

“I hate meshes.

I cannot believe how hard this is.

Geometry is hard.”

— David Baraff

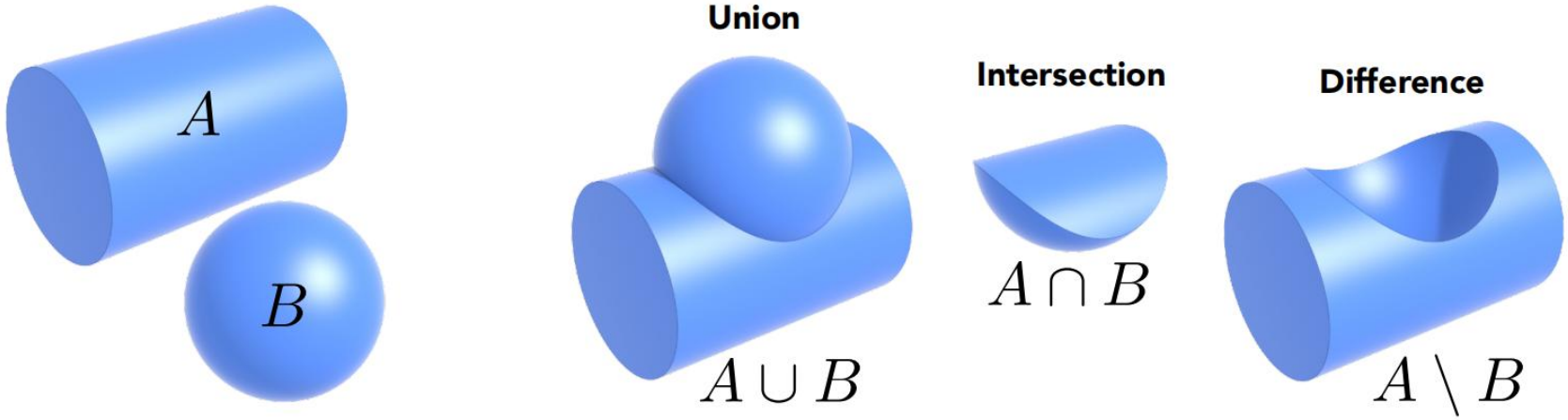
Senior Research Scientist

Pixar Animation Studios

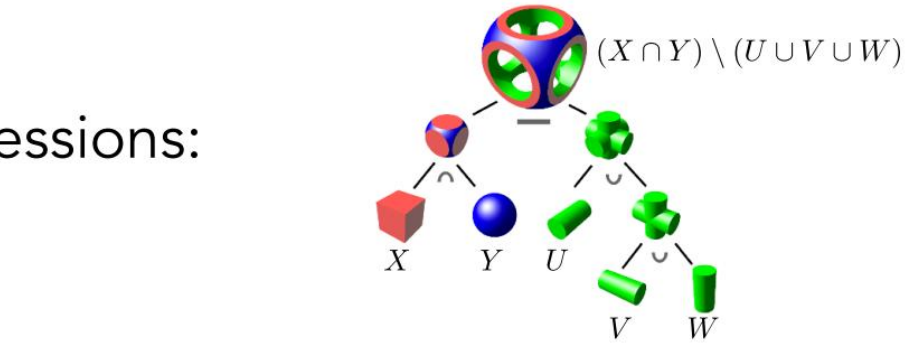
More implicit representations

Constructive Solid Geometry

- Combine implicit geometry via Boolean operations



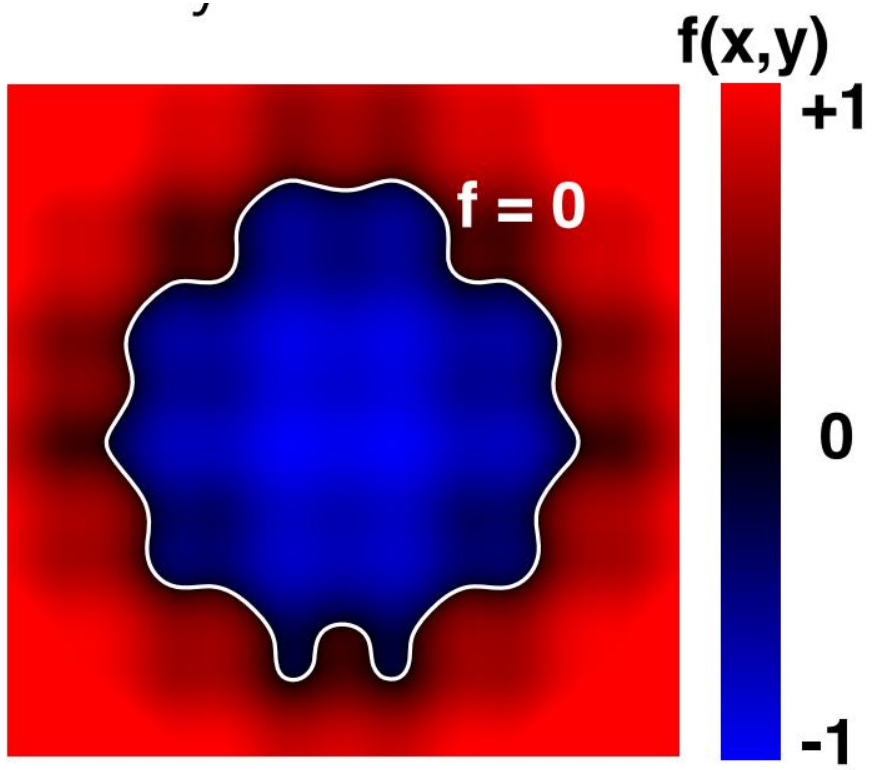
Boolean expressions:



More implicit representations

Signed Distance Function

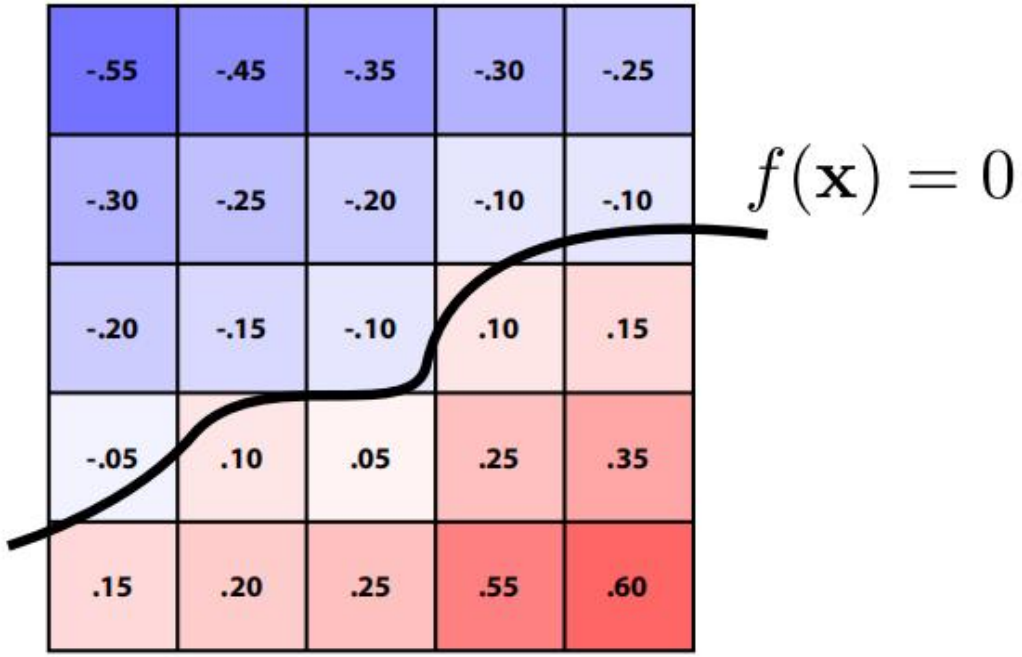
- Distance of a given point x to the surface boundary
- Sign: whether or not x is in the interior of surface



More implicit representations

Level-Set

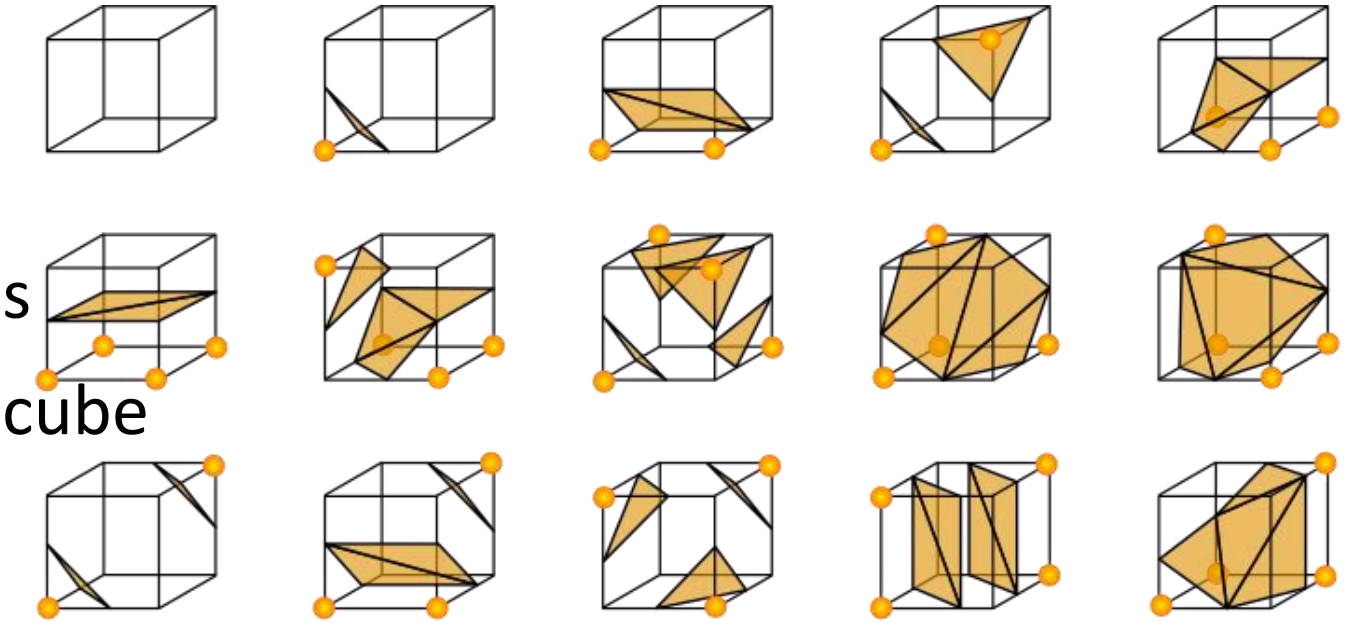
- A grid of values approximating function
- Surface is found where interpolated values equal zero



Mesh Extraction From Implicit Representation

MarchingCube Algorithm, 1987

- Assume f(x,y,z)=0
- Estimate f(x,y,z) at the grid points
- Determine the polygon for each cube
- 2^8=256 cases!

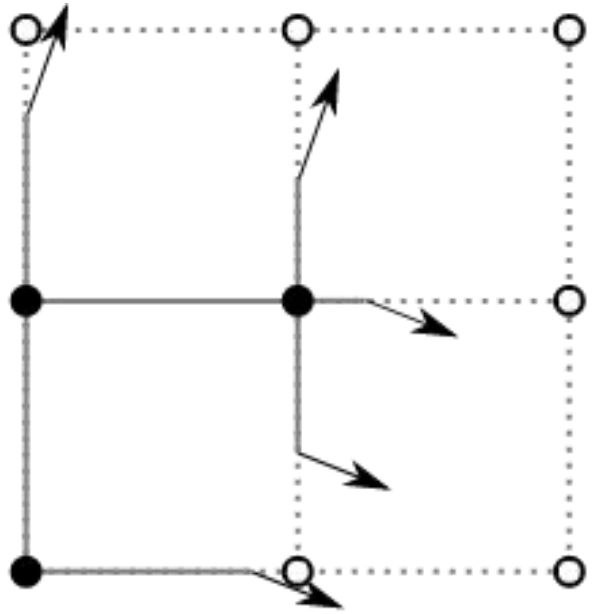


MarchingCube15, 1987
(Then MarchingCube33 in 1995)

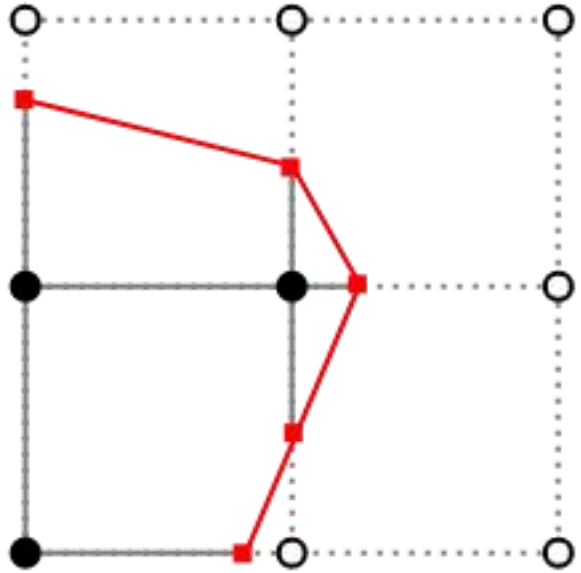
Mesh Extraction From Implicit Representation

MarchingCube cannot do sharp edges and corners

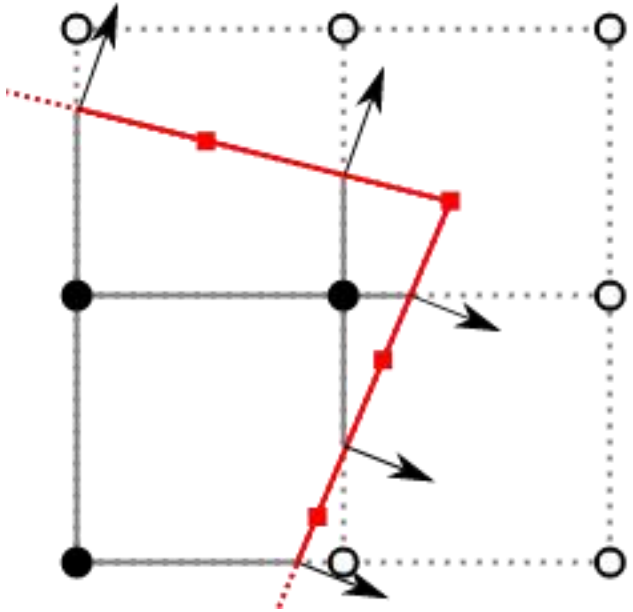
Dual Contour Algorithm: not only f(x), but also f'(x)



sample values and normals



MarchingCube



Dual Contour

Thank you