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DIU_Curious_Trio

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Team Reference Document

Contents

1 Code

1.1 1_Build System Linux

```
{
  "cmd" : ["ulimit -s 268435456;g++ -std=c++20 $file_name -o $file_base_name && timeout 4s ./$file_base_name<input.txt>output.txt"],
  "selector" : "source.c",
  "shell": true,
  "working_dir" : "$file_path"
}
```

1.2 2_Build System Windows

```
{
  "cmd": ["g++.exe","-std=c++20", "${file}", "-o", "${file_base_name}.exe", "&&" , "${file_base_name}.exe<input.txt>output.txt"],
  "selector":"source.cpp",
  "shell":true,
  "working_dir":"$file_path"
}
```

1.3 3_Stress Testing(check.sh)

```
// chmod u+x check.sh
// ./check.sh
set -e
g++ gen.cpp -o gen
g++ code.cpp -o code
g++ brute.cpp -o brute
for ((i = 1; ; ++i)); do
  echo "Passed on TestCase: " $i
  ./gen $i > in
  ./code < in > out1
  ./brute < in > out2
  diff -Z out1 out2 || break
done
echo -e "WA on the following test:"
cat in
echo -e "\nYour Answer is:"
cat out1
echo -e "\nCorrect answer is:"
cat out2
```

1.4 3_Stress Testing(gen.cpp)

```
#include <bits/stdc++.h>
using namespace std;
using ll = long long;
mt19937_64 rng(chrono::steady_clock::now().time_since_epoch().count());
inline ll gen_random(ll l, ll r) {
  return uniform_int_distribution<ll>(l, r)(rng);
}
inline double gen_random_real(double l, double r) {
  return uniform_real_distribution<double>(l, r)(rng);
}
int main(int argc, char* args[]) {
  int _ = atoi(args[1]);
  mt19937 mt(_);
  int n = gen_random(1, 5);
  cout << n << '\n';
  vector<int> per;
  string s;
  for (int i = 0; i < n; ++i) {
    per.push_back(i + 1);
    cout << gen_random(-50, 50) << " \n"[i == n - 1];
    char c = 'a' + gen_random(0, 25);
    s += c;
  }
  shuffle(per.begin(), per.end(), rng);//generates permutation
  return 0;
}
```

1.5 Aho Corasick

```
const int N = 1e6 + 3, A = 26;
int trie[N][A], node[N], dp[N];
int total = 0;
void add(string& s, int i) {
  int u = 0;
  for (char c: s) {
    int k = c - 'a';
    if (!trie[u][k]) {
      trie[u][k] = ++total;
    }
    u = trie[u][k];
  }
  node[i] = u;
}
vector<int> ord;
int slink[N];
void build() {
  queue<int> q;
  q.push(0);
  while (q.size()) {
    int p = q.front();
    q.pop();
    ord.push_back(p);
    for (int c = 0; c < A; ++c) {
      int u = trie[p][c];
      if (!u) continue;
      q.push(u);
      if (!p) continue;
      int v = slink[p];
      while (v and !trie[v][c]) v = slink[v];
      if (trie[v][c]) slink[u] = trie[v][c];
    }
  }
}
```

```
void solve() {
  build();
  int u = 0;
  for (char c: text) {
    c -= 'a';
    while (u and !trie[u][c]) u = slink[u];
    u = trie[u][c];
    dp[u]++;
  }
  reverse(ord.begin(), ord.end());
  for (int u: ord) {
    dp[slink[u]] += dp[u];
  }
}
```

1.6 Articulation Point and Bridges

```
// Articulation point
vector<vector<int>> adj;
vector<int> tin, low;
vector<bool> vis;
int timer;
void is_cutpoint(int v) {
  // process the cutpoint
}
void dfs(int v, int p = -1) {
  vis[v] = true;
  tin[v] = low[v] = timer++;
  int children = 0;
  for (int u : adj[v]) {
    if (u == p) continue;
    if (vis[u]) {
      low[v] = min(low[v], tin[u]);
    } else {
      dfs(u, v);
      low[v] = min(low[v], low[u]);
      if (low[u] >= tin[v] && p != -1) {
        is_cutpoint[v] = true;
      }
      ++children;
    }
  }
  if(p == -1 && children > 1) {
    is_cutpoint[v] = true;
  }
}
void find_cutpoints(int n) {
  timer = 0;
  vis.assign(n + 1, false);
  is_cutpoint.assign(n + 1, false);
  tin.assign(n + 1, -1);
  low.assign(n + 1, -1);
  for (int i = 1; i <= n; ++i) {
    if (!vis[i]) {
      dfs (i);
    }
  }
}
// Bridges
vector<vector<int>> adj;
vector<int> tin, low;
vector<bool> vis;
int timer;
void is_bridge(int v,int to) {
  //process the found bridge
}
void dfs(int v, int p = -1) {
  vis[v] = true;
  tin[v] = low[v] = timer++;
  bool parent_skipped = false;
  for (int u : adj[v]) {
    if (u == p && !parent_skipped) {
      parent_skipped = true;
      continue;
    }
    if (vis[u]) {
      low[v] = min(low[v], tin[u]);
    } else {
      dfs(u, v);
      low[v] = min(low[v], low[u]);
      if (low[u] > tin[v]) {
        is_bridge(v, u);
      }
    }
  }
}
void find_bridges() {
  timer = 0;
  vis.assign(n, false);
  tin.assign(n, -1);
  low.assign(n, -1);
  for (int i = 0; i < n; ++i) {
    if (!vis[i]) {
      dfs(i);
    }
  }
}
```

1.7 Bellman Ford

```
const int INF = 1e9;
struct Edge {
  int u, v, w;
};
void solve() {
  int n, m;
  cin >> n >> m;
  vector<Edge> e(m);
  for (int i = 0; i < m; ++i) {
    cin >> e[i].u >> e[i].v >> e[i].w;
  }
  vector<int> d(n + 1, INF);
  d[1] = 0;    // distance of source node
  vector<int> p(n + 1, -1);   // parent vector
  int x;
  for (int i = 1; i <= n; ++i) {
    x = -1;
    for (auto [u, v, w]: e)
      if (d[u] < INF and d[u] + w < d[v]) {
        d[v] = d[u] + w;
        p[v] = u;
        x = v;
      }
  }
  if (x == -1) cout << "No negative cycle found\n";
  else {
    int y = x;
    for (int i = 0; i < n; ++i) y = p[y];

    vector<int> path;
    for (int cur = y; ; cur = p[cur]) {
      path.push_back(cur);
      if (cur == y && path.size() > 1) break;
    }
    reverse(path.begin(), path.end());

    cout << "Negative cycle: ";
    for (int u : path) cout << u << " ";
    cout << "\n";
  }
}
```

1.8 Big Integer

```
class BIG_INT {
private:
  string result;
public:
  string bigfinder(string a, string b){
    if(a.size() < b.size()) swap(a, b);
    string d = b;
    reverse(full(b));
    while(b.size() < a.size()) b.pb('0');
    reverse(full(b));
    int i = 0;
    while(a[i]){
      if(a[i] > b[i]) return a;
      else if(a[i] < b[i]) return d;
      i++;
    }
    return "same";
  }
  llu stringtonumber(string a){
    llu n = 0;
    for(llu i = 0; a[i]; i++) n = ( n*10 ) + (a[i]-48);
    return n;
  }
  string add(string a,string b){
    result.clear();
    reverse(full(a));
    reverse(full(b));
    if(a.size() < b.size()) swap(a, b);
    while(b.size() < a.size()) b.pb('0');
    llu i = 0, carry = 0;
    while(a[i]){
      carry = carry + a[i]-48 + b[i]-48;
      result.pb((carry %10) + 48);
      carry = carry / 10;
      i++;
    }
    while(carry > 9){
      result.pb((carry % 10) + 48);
      carry = carry / 10;
    }
    if(carry != 0) result.pb(carry + 48);
    reverse(full(result));
    return result;
  }
  string subtraction(string a, string b){
    result.clear();
    bool flag = true;
    if(bigfinder(a, b) == b){
      swap(a, b);
      flag = false;
    }
    reverse(full(a));
    reverse(full(b));
    while(b.size() < a.size()) b.pb('0');
    int i = 0, carry = 0, x = 0;
    while(a[i]){
      if(b[i] > a[i]) x = (a[i]-48) + 10;
      else x = a[i]-48;
      carry = x-(carry + (b[i]-48));
      result.pb(carry+48);
      carry = x / 10;
      i++;
    }
    while(result[result.size()-1] == '0' and
    ↪ result.size() > 1)
        result.erase(result.size()-1, 1);
    if(!flag) result.pb('-');
    reverse(full(result));
    return result;
  }
  string multiplication(string a, string b){
    if(b.size() > a.size()) swap(a, b);
    reverse(full(a));
    reverse(full(b));
    while(a.size() > b.size()) b.pb('0');
    vector < string > x;
    for(llu i = 0; b[i]; i++){
      llu carry = 0;
```

```cpp
      string str;
      for(llu j = 0; a[j]; j++){
        str += ((((b[i]-48)*(a[j]-48))+carry)%10)+48;
        carry = (((b[i]-48)*(a[j]-48))+carry)/10;
      }
      if(carry > 0) str += carry + 48;
      reverse(full(str));
      llu zero = i;
      while(zero--) str += '0';
      x.pb(str); }
    llu len = x.size();
    if(len == 1) result = x[0];
    else{
      for(llu i = 0; i < len-1; i++){
        x[i+1] = add(x[i], x[i+1]);
      }
    }
    result = x[len-1];
    while(result[0] == '0' and result.size() > 1)
    result.erase(result.begin() + 0);
    return result;
  }
};
// Big Integer Division
void bigDivision() {
  string a = "50";
  ll b = 6;
  ll len = a.length(), mod = 0, d = Digit(b), lowest =
  0, i = 0;
  while (i < d or lowest < b) {
    lowest = (lowest * 10) + (a[i] - 48);
    i++;
  }
  while (i < len + 1) {
    mod = lowest % b;
    lowest = (mod * 10) + (a[i] - 48);
    if (b > lowest) {
      lowest = (lowest * 10) + (a[i] - 48);
      i++;
    }
    i++;
  }
  cout << mod << endl;
}
```

1.9 Centroid Decomposition

```cpp
const int N = 2e5+5;
int n, k;
vector<int> adj[N];
int sz[N], cen[N];
ll ans = 0;
void dfs_sz(int u, int p) {
  sz[u] = 1;
  for (auto v: adj[u]) {
    if (v != p and !cen[v]) {
      dfs_sz(v, u);
      sz[u] += sz[v];
    }
  }
}
int get_cen(int u, int p, int s) {
  for (auto v: adj[u]) {
    if (v != p and !cen[v] and 2 * sz[v] > s)  return
    get_cen(v, u, s);
  }
  return u;
}
int t, tin[N], tout[N], nodes[N], dep[N];
void dfs(int u, int p) {
  nodes[t] = u;
  tin[u] = t++;
  for (auto v: adj[u]) {
    if (v != p and !cen[v]) {
      dep[v] = dep[u] + 1;
      dfs(v, u);
    }
  }
  tout[u] = t - 1;
}
void go(int u) {
  dfs_sz(u, u);
  int c = get_cen(u, u, sz[u]);
  cen[c] = 1;
  t = 0;
  dep[c] = 0;
  dfs(c, c);
  int cnt[t]{1};
  for (auto v: adj[c]) {
    if (!cen[v]) {
      for (int i = tin[v]; i <= tout[v]; ++i) {
        int w = nodes[i];
        int req = k - dep[w];
        if (req >= 0 and req < t) {
          ans += cnt[req];
        }
      }
      for (int i = tin[v]; i <= tout[v]; ++i) {
        int w = nodes[i];
        cnt[dep[w]]++;
      }
    }
  }
  for (auto v: adj[c]) {
    if (!cen[v]) {
      go(v);
    }
  }
}
void solve () {
  cin >> n >> k;
  for (int e = 0; e < n - 1; ++e) {
    int u, v;  cin >> u >> v;  u--, v--;
    adj[u].push_back(v);
    adj[v].push_back(u);
  }
  go(0);
  cout << ans << "\n";
}
```

1.10 Chinese Remainder Theorem

```cpp
struct Congruence {
  long long a, m;
};
long long chinese_remainder_theorem(vector<Congruence>
 const& congruences) {
  long long M = 1;
  for (auto const& congruence : congruences) {
    M *= congruence.m;
  }
  long long solution = 0;
  for (auto const& congruence : congruences) {
    long long a_i = congruence.a;
    long long M_i = M / congruence.m;
    long long N_i = mod_inv(M_i, congruence.m);
    solution = (solution + a_i * M_i % M * N_i) % M;
  }
  return solution;
}
```

1.11 Closest Pair of Points

```cpp
const int N = 3e5 + 9;
#define x first
#define y second
long long dist2(pair<int, int> a, pair<int, int> b) {
  return 1LL * (a.x - b.x) * (a.x - b.x) + 1LL * (a.y
  - b.y) * (a.y - b.y);
}
pair<int, int> closest_pair(vector<pair<int, int>> a) {
  int n = a.size();
  assert(n >= 2);
  vector<pair<pair<int, int>, int>> p(n);
  for (int i = 0; i < n; i++) p[i] = {a[i], i};
  sort(p.begin(), p.end());
  int l = 0, r = 2;
  long long ans = dist2(p[0].x, p[1].x);
  pair<int, int> ret = {p[0].y, p[1].y};
  while (r < n) {
    while (l < r && 1LL * (p[r].x.x - p[l].x.x) *
    (p[r].x.x - p[l].x.x) >= ans) l++;
    for (int i = l; i < r; i++) {
      long long nw = dist2(p[i].x, p[r].x);
      if (nw < ans) {
        ans = nw;
        ret = {p[i].y, p[r].y};
      }
    }
    r++;
  }
  return ret;
}
int32_t main() {
  ios_base::sync_with_stdio(0);
  cin.tie(0);
  int n; cin >> n;
  vector<pair<int, int>> p(n);
  for (int i = 0; i < n; i++) cin >> p[i].x >> p[i].y;
  pair<int, int> z = closest_pair(p);
  if (z.x > z.y) swap(z.x, z.y);
  cout << z.x << ' ' << z.y << ' ' << fixed <<
    setprecision(6) << sqrtl(dist2(p[z.x], p[z.y])) <<
    '\n';
  return 0;
}
```

1.12 Convex Hull

```cpp
struct Point {
  int x, y;
  Point () {
    this->x = 0;
    this->y = 0;
  }
  Point (int x, int y) {
    this->x = x;
    this->y = y;
  }
  bool operator ==(const Point& p) {
    return (this->x == p.x and this->y == p.y);
  }
  bool operator <(const Point& p) {
    return make_pair(this->x, this->y) <
    make_pair(p.x, p.y); // with respect to x-axis
    // // with respect to angle from (0, 0)
    // if (*this * p == 0) {
    //    return dis() < p.dis();
    // }
    // return (*this * p < 0);
  }
  void operator -=(const Point& p) {
    this->x -= p.x;
    this->y -= p.y;
  }
  Point operator -(const Point& p) const {
    Point q;
    q.x = this->x - p.x;
    q.y = this->y - p.y;
```

```cpp
    return q;
  }
  long long operator *(const Point& p) const {
    return 1LL * x * p.y - 1LL * y * p.x;
  }
  bool isInside(Point& a, Point& b) const { // if p is inside segment a-b
    if ((a - *this) * (b - *this) != 0) return false;
    bool d1 = this->x >= min(a.x, b.x) and this->x <= max(a.x, b.x);
    bool d2 = this->y >= min(a.y, b.y) and this->y <= max(a.y, b.y);
    return d1 and d2;
  }
  bool rayIntersect(Point a, Point b) {
    Point q(this->x, INT32_MAX);  // if p-q ray intersects segment a-b
    for (int rep = 0; rep < 2; ++rep) {
      if ((a - *this) * (q - *this) <= 0 and (b - *this) * (q - *this) > 0 and (a - *this) * (b - *this) < 0) {
        return true;
      }
      swap(a, b);
    }
    return false;
  }
  friend istream& operator >>(istream& cin, Point& p) {
    cin >> p.x >> p.y;
    return cin;
  }
  friend ostream& operator <<(ostream& cout, const Point& p) {
    cout << p.x << " " << p.y;
    return cout;
  }
};
// upper and lower part
void solve() {
  int n;
  cin >> n;
  vector<Point> v(n);
  for (int i = 0; i < n; ++i) {
    cin >> v[i];
  }
  sort(v.begin(), v.end());
  vector<Point> hull;
  for (int rep = 0; rep < 2; ++rep) {
    const int sz = hull.size();
    for (auto C: v) {
      while (hull.size() >= sz + 2) {
        Point A = hull.end()[-2];
        Point B = hull.end()[-1];
        if (((B - A) * (C - A)) <= 0) {
          break;
        }
        hull.pop_back();
      }
      hull.push_back(C);
    }
    hull.pop_back();
    reverse(v.begin(), v.end());
  }
  cout << hull.size() << "\n";
  for (auto p: hull) {
    cout << p << "\n";
  }
}
// sorting by angle
void solve() {
  int n;
  cin >> n;
  vector<Point> v(n);
  for (int i = 0; i < n; ++i) {
    cin >> v[i];
    if (make_pair(v[i].x, v[i].y) < make_pair(v[0].x, v[0].y)) {
      swap(v[i], v[0]);
    }
  }
  for (int i = 1; i < n; ++i) {
    v[i] -= v[0];
  }
  sort(v.begin() + 1, v.end());
  int j = n - 1;
  while (j >= 2 and v[j] * v[j - 1] == 0) {
    --j;
  }
  reverse(v.begin() + j, v.end());
  vector<Point> hull;
  hull.push_back(Point{0, 0});
  for (int i = 1; i < n; ++i) {
    auto C = v[i];
    while (hull.size() >= 2) {
      Point A = hull.end()[-2];
      Point B = hull.end()[-1];
      if (((B - A) * (C - A)) <= 0) {
        break;
      }
      hull.pop_back();
    }
    hull.push_back(C);
  }
  cout << hull.size() << "\n";
  for (auto& p: hull) {
    p += v[0];
    cout << p << "\n";
  }
}
```

1.13 Custom Hash

```cpp
struct custom_hash {
  static uint64_t splitmix64(uint64_t x) {
    x += 0x9e3779b97f4a7c15;
    x = (x ^ (x >> 30)) * 0xbf58476d1ce4e5b9;
    x = (x ^ (x >> 27)) * 0x94d049bb133111eb;
    return x ^ (x >> 31);
  }
  size_t operator()(uint64_t x) const {
    static const uint64_t FIXED_RANDOM = chrono::steady_clock::now().time_since_epoch().count();
    return splitmix64(x + FIXED_RANDOM);
  }
};
unordered_map<long long int, int, custom_hash> mp; // this will work when the key is an int or long long int
```

1.14 Custom Map(Pair Query)

```cpp
// a1 <= a2 <= a3 <= a4......
// b1 >= b2 >= b3 >= b4......
map<ll, ll> mp;
void insert(ll a, ll b) {
  auto it = mp.lower_bound(a);
  if (it != mp.end() and it->second >= b) return;
  it = mp.insert(it, {a, b});
  it->second = b;
  while (it != mp.begin() and prev(it)->second <= b) {
    mp.erase(prev(it));
  }
}
// returns the largest b among the a's that are greater than or equal to x
ll query(ll x) {
  auto it = mp.lower_bound(x);
  if (it == mp.end()) return 0;
  return it->second;
}
```

1.15 DP Group Sum

```cpp
// How many nCm ways have sum divisible by D?
ll n, q, d, m;
ll a[210],dp[210][22][22];
ll rec(ll i, ll cnt, ll sum) {
  if (cnt < 0) return 0;
  if (i < 1) {
    if (cnt == 0 and sum == 0) return 1;
    return 0;
  }
  if (dp[i][cnt][sum] != -1) return dp[i][cnt][sum];
  ll ans = rec(i - 1, cnt - 1, (sum + ((a[i] % d) + d) % d) % (d));
  ans += rec(i - 1, cnt, sum);
  return dp[i][cnt][sum] = ans;
}
ll cs = 0;
void dracarys() {
  cin >> n >> q;
  for (ll i = 1; i <= n; i++) {cin >> a[i];}
  cout << "Case " << ++cs << ":\n";
  while (q--) {
    cin >> d >> m;
    memset(dp, 0, sizeof dp);
    cout << rec(n,m,0) << endl;
  }
}
```

1.16 DSU

```cpp
const int N = 1e5 + 9;
int parent[N], sz[N];
void make_set(int v) {
  parent[v] = v;
  sz[v] = 1;
}
int find_set(int v) {
  if (v == parent[v]) return v;
  return parent[v] = find_set(parent[v]);
}
void union_sets(int a, int b) {
  a = find_set(a);
  b = find_set(b);
  if (a != b) {
    if (sz[a] < sz[b]) swap(a, b);
    parent[b] = a;
    sz[a] += sz[b];
  }
}
```

1.17 Digit DP

```cpp
int dp[10][90][90][2];
int fun(int pos, int digSum, int dig, int smaint){
  if(pos==num.size()){
    if(!dig and !digSum) return 1;
    return 0;
  }
  if(dp[pos][digSum][dig][smaint] != -1) return dp[pos][digSum][dig][smaint];
  int ans = 0;
  int limit = num[pos];
  if(smaint == 1) limit = 9;
  for(int i=0; i<=limit; i++){
    int nsm = (i < num[pos] || smaint);
    int ndigSum = (digSum + i) % c;
```

```
    int ndig = (dig* 10 + i) % c;
    ans += fun(pos+1, ndigSum, ndig, nsm);
  }
  return dp[pos][digSum][dig][smaint] = ans;
}
```

1.18 Dijkstra

```
#define inf (ll)(1e12)
#define pi pair < int, int >
vector<pi> graph[maxx];
priority_queue < pi, vector< pi >, greater < pi > > pq;
ll dis[maxx]; int parent[maxx];
void solve() {
  int n, m; cin >> n >> m;
  for (int i = 0; i < m; i++) {
    int a, b, w; cin >> a >> b >> w;
    graph[a].pb({b, w}); graph[b].pb({a, w});
  }
  for (int i = 1; i <= n; i++) dis[i] = inf;
  for (int i = 1; i <= n; i++) parent[i] = i;
  dis[1] = 0;
  pq.push({0, 1});
  while (pq.size()) {
    int v = pq.top().second;
    pq.pop();
    for (int i = 0; i < graph[v].size(); i++) {
      int u = graph[v][i].first;
      int ucost = graph[v][i].second;
      if (dis[u] > dis[v] + ucost) {
        dis[u] = dis[v] + ucost;
        parent[u] = v;
        pq.push({dis[u], u});
      }
    }
  }
  vector < ll > v; int at = n;
  while (at != 1) {
    if (parent[at] == at) {
      cout << -1 << endl;
      return; }
    v.pb(at);
    at = parent[at];
  }
  v.pb(at);
  reverse(full(v));
  for (int i = 0; i < v.size(); i++) cout << v[i]  <<
  ↪ ' ';
  cout << endl;
}
```

1.19 Euler Phi

```
1. phi(n) = n * (p1 - 1) / p1 * (p2 - 1) / p2 . . . .
2. gcd d: phi(n / d)
3. Sum of coprime numbers of an integer = phi(n) * n /
↪ 2
4. N = phi(d) where, d | N
5. Code:
vector<int> phi(n + 1);
void prec(int n) {  //nlogn
  phi[1] = 1;
  for (int i = 2; i <= n; i++)
    phi[i] = i - 1;
  for (int i = 2; i <= n; i++)
    for (int j = 2 * i; j <= n; j += i)
      phi[j] -= phi[i];
}
int phi(int n) { //sqrt(n)
  int result = n;
  for (int i = 2; i * i <= n; i++) {
    if (n % i == 0) {
      while (n % i == 0) n /= i;
      result -= result / i;
    }
  }
  if (n > 1) result -= result / n;
  return result;
}
```

1.20 Extended GCD

```
ll egcd(ll a, ll b, ll &x, ll &y) {
  if (b == 0) {
    x = 1; y = 0;
    return a;
  }
  ll x1, y1;
  ll d = egcd(b, a % b, x1, y1);
  x = y1; y = x1 - y1 * (a / b);
  return d;
}
```

1.21 Factorial Prime Factorrization

```
ll factorialPrimePower (ll n, ll p ) {
  ll freq = 0; ll cur = p;
  while (n / cur) { freq += n / cur; cur *= p; }
  return freq;
}
void factFactorize (ll n) {
  for ( ll i = 0; i < primes.size() && prime[i] <= n;
  ↪  i++ ) {
    ll p = prime[i];
    ll freq = 0;
    while (n / p) { freq += n / p; p *= prime[i]; }
    cout << prime[i] << ' ' << freq << endl;
  }
}
```

1.22 Floyd Warshall

```
const int N = 100, inf = 1e9 + 9;
int d[N][N], nextof[N][N];
int n;
void init() {
  for (int i = 1; i <= n; ++i) {
    for (int j = 1; j <= n; ++j) {
      nextof[i][j] = j;
      d[i][j] = inf;
      if (i == j) d[i][j] = 0;
    }
  }
}
void cal() {
  for (int k = 1; k <= n; ++k) {
    for (int i = 1; i <= n; ++i) {
      for (int j = 1; j <= n; ++j) {
        if (d[i][k] + d[k][j] < d[i][j]) {
          d[i][j] = d[i][k] + d[k][j];
          nextof[i][j] = nextof[i][k];
        }
      }
    }
  }
}
vector<int> findPath(int i, int j) {
  vector<int> path = {i};
  while(i != j) {
    i = nextof[i][j];
    path.push_back(i);
  }
  return path;
}
```

1.23 GCD and LCM Notes

```
gcd(a,gcd(b,c))=gcd(gcd(a,b),c)
gcd(a,b,c)=gcd(gcd(a,b),c)
gcd(a,b)=gcd(ab,b)
lcm(a, gcd(b,c)) = gcd(lcm(a, b), lcm(a, c))
gcd(a, lcm(b, c)) = lcm(gcd(a, b), gcd(a, c))
```

1.24 GP Hash Table

```
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace __gnu_pbds;
const int RANDOM = chrono::high_resolution_clock::now(
↪  ).time_since_epoch().count();
struct custom_hash {
  int operator()(int x) const { return x ^ RANDOM; }
};
gp_hash_table<int, int, custom_hash> mp;
```

1.25 Geometric Sum

```
ll geometric_Sum() {
  ll a, r, n; cin >> a >> r >> n; //a = first value r
  ↪  = ratio, n = n-term
  ll res = BigMod(r, n);
  ll numara = (a * (1 - res)) % MOD;
  numara = (numara + MOD) % MOD;
  ll deno = ((1 - r) % MOD + MOD) % MOD;
  ll ans = (numara * BigMod(deno, MOD - 2)) % MOD;
  return ans;
}
```

1.26 KMP

```
vector<ll> build_lps(string &p) {
  ll sz = p.size();
  vector < ll > lps(sz, 0);
  ll j = 0;
  lps[0] = 0;
  for (ll i = 1; i < sz; i++) {
    while (j >= 0 && p[i] != p[j]) {
      if (j >= 1) j = lps[j - 1];
      else j = -1;
    }
    j++; lps[i] = j;
  }
  return lps;
}
vector<ll> ans;
void kmp(vector<ll> &lps, string &s, string &p) {
  ll psz = p.size(), ssz = s.size();
  ll j = 0;
  for (ll i = 0; i < ssz; i++) {
    while (j >= 0 && p[j] != s[i]) {
      if (j >= 1) j = lps[j - 1];
      else j = -1;
    }
    j++;
    if (j == psz) {
      j = lps[j - 1];
      ans.push_back(i - psz + 1); // pattern found at
  ↪  position i-psz+1
    }
  }
}
```

1.27 LCA

```
const int N = 1e6 + 9, LOG = 21;
int up[N][LOG], depth[N];
vector<int> children[N];
void dfs(int a) {
  for (auto b: children[a]) {
    depth[b] = depth[a] + 1;
    up[b][0] = a;  // a is parent of b
    for (int i = 1; i < LOG; ++i) {
      up[b][i] = up[up[b][i-1]][i-1];
    }
    dfs(b);
  }
}
int getKthAncestor(int node, int k) {
  if (depth[node] < k) return -1;
  for (int i = 0; i < LOG; ++i) {
    if (k & (1 << i)) {
      node = up[node][i];
    }
  }
  return node;
}
int getLCA(int u, int v) {
  if (depth[u] < depth[v]) swap(u, v);
  u = getKthAncestor(u, depth[u] - depth[v]);
  if (u == v) return v;
  for (int i = LOG - 1; i >= 0; --i) {
    if (up[u][i] != up[v][i]) {
      u = up[u][i];
      v = up[v][i];
    }
  }
  return up[v][0];
}
```

1.28 LIS Generation

```
vector<int> generateLIS(const vector<int>& a) {
  int n = a.size();
  if (n == 0) {
    return {};
  }
  vector<int> piles;
  vector<int> indices(n);
  for (int i = 0; i < n; ++i) {
    auto it = lower_bound(piles.begin(), piles.end(),
    a[i]);
    auto index = it - piles.begin();
    if (it == piles.end()) {
      piles.push_back(a[i]);
    } else {
      *it = a[i];
    }
    indices[i] = index;
  }
  // Find the length of the LIS
  int lisLength = *max_element(indices.begin(),
    indices.end()) + 1;
  // Reconstruct the LIS
  vector<int> lis(lisLength);
  for (int i = n - 1; i >= 0; --i) {
    if (indices[i] == lisLength - 1) {
      lis[--lisLength] = a[i];
    }
  }
  return lis;
}
```

1.29 Linear Diophantine Equation

```
int gcd(int a, int b, int& x, int& y) {
  if (b == 0) {
    x = 1;
    y = 0;
    return a;
  }
  int x1, y1;
  int d = gcd(b, a % b, x1, y1);
  x = y1;
  y = x1 - y1 * (a / b);
  return d;
}
bool find_any_solution(int a, int b, int c, int &x0,
  int &y0, int &g) {
  g = gcd(abs(a), abs(b), x0, y0);
  if (c % g) {
    return false;
  }
  x0 *= c / g;
  y0 *= c / g;
  if (a < 0) x0 = -x0;
  if (b < 0) y0 = -y0;
  return true;
}
void shift_solution(int & x, int & y, int a, int b,
  int cnt) {
  x += cnt * b;
  y -= cnt * a;
}
int find_all_solutions(int a, int b, int c, int minx,
  int maxx, int miny, int maxy) {
  int x, y, g;
  if (!find_any_solution(a, b, c, x, y, g))
    return 0;
  a /= g;
  b /= g;
  int sign_a = a > 0 ? +1 : -1;
  int sign_b = b > 0 ? +1 : -1;
  shift_solution(x, y, a, b, (minx - x) / b);
  if (x < minx)
    shift_solution(x, y, a, b, sign_b);
  if (x > maxx)
    return 0;
  int lx1 = x;
  shift_solution(x, y, a, b, (maxx - x) / b);
  if (x > maxx)
    shift_solution(x, y, a, b, -sign_b);
  int rx1 = x;
  shift_solution(x, y, a, b, -(miny - y) / a);
  if (y < miny)
    shift_solution(x, y, a, b, -sign_a);
  if (y > maxy)
    return 0;
  int lx2 = x;
  shift_solution(x, y, a, b, -(maxy - y) / a);
  if (y > maxy)
    shift_solution(x, y, a, b, sign_a);
  int rx2 = x;
  if (lx2 > rx2)
    swap(lx2, rx2);
  int lx = max(lx1, lx2);
  int rx = min(rx1, rx2);
  if (lx > rx)
    return 0;
  return (rx - lx) / abs(b) + 1;
}
```

1.30 MEX of All Subarray

```
const int N = 1e5 + 9, inf = 1e9;
struct ST {
  int t[4 * N];
  ST() {}
  void build(int n, int b, int e) {
    t[n] = 0;
    if (b == e) {
      return;
    }
    int mid = (b + e) >> 1, l = n << 1, r = l | 1;
    build(l, b, mid);
    build(r, mid + 1, e);
    t[n] = min(t[l], t[r]);
  }
  void upd(int n, int b, int e, int i, int x) {
    if (b > i || e < i) return;
    if (b == e && b == i) {
      t[n] = x;
      return;
    }
    int mid = (b + e) >> 1, l = n << 1, r = l | 1;
    upd(l, b, mid, i, x);
    upd(r, mid + 1, e, i, x);
    t[n] = min(t[l], t[r]);
  }
  int get_min(int n, int b, int e, int i, int j) {
    if (b > j || e < i) return inf;
    if (b >= i && e <= j) return t[n];
    int mid = (b + e) >> 1, l = n << 1, r = l | 1;
    int L = get_min(l, b, mid, i, j);
    int R = get_min(r, mid + 1, e, i, j);
    return min(L, R);
  }
  int get_mex(int n, int b, int e, int i) { // mex of
    [i... cur_id]
    if (b == e) return b;
    int mid = (b + e) >> 1, l = n << 1, r = l | 1;
    if (t[l] >= i) return get_mex(r, mid + 1, e, i);
    return get_mex(l, b, mid, i);
  }
}t;
int a[N], f[N];
int32_t main() {
  ios_base::sync_with_stdio(0);
  cin.tie(0);
  int n; cin >> n;
  for (int i = 1; i <= n; i++) {
    cin >> a[i];
    --a[i];
  }
  t.build(1, 0, n);
  set<array<int, 3>> seg; // for cur_id = i,
    [x[0]...i], [x[0] + 1...i], ... [x[1]...i] has mex
    x[2]
  for (int i = 1; i <= n; i++) {
    int x = a[i];
    int r = min(i - 1, t.get_min(1, 0, n, 0, x - 1));
    int l = t.get_min(1, 0, n, 0, x) + 1;
    if (l <= r) {
      auto it = seg.lower_bound({l, -1, -1});
      while (it != seg.end() && (*it)[1] <= r) {
        auto x = *it;
        it = seg.erase(it);
      }
    }
    t.upd(1, 0, n, x, i);
    for (int j = r; j >= l; ) {
      int m = t.get_mex(1, 0, n, j);
      int L = max(l, t.get_min(1, 0, n, 0, m) + 1);
      f[m] = 1;
      seg.insert({L, j, m});
      j = L - 1;}
    int m = !a[i];
    seg.insert({i, i, m});
    f[m] = 1;
  }
  int ans = 0;
```

```cpp
  while (f[ans]) ++ans;
  cout << ans + 1 << '\n';
  return 0;
}
```

1.31 Manacher

Description: pal[1][i] = longest odd (half rounded down) palindrome around pos i and starts at i - pal[1][i] and ends at i + pal[1][i]
pal[0][i] = half length of longest even palindrome around pos i, i + 1 and starts at i - par[0][i] + 1 and ends at i + pal[0][i]

```cpp
int pal[2][N];
void manacher(string &s) {
  int n = s.size(), idx = 2;
  while (idx--) {
    for (int l=-1, r=-1, i=0; i<n-1; ++i){
      if (i > r) l = r = i;
      else {
        int k = min(r-i, pal[idx][l+r-i]);
        l = i - k,   r = i + k;
      }
      while (l - idx >= 0 and r + 1 < n and s[l - idx] == s[r + 1]) l--, r++;
      pal[idx][i] = r - i;
      // [l - 1 + idx : r] palindrome
    }
  }
}
```

1.32 Matrix Exponentiation

```cpp
const int mod = 1e9 + 7;
struct Mat {
  int sz;
  vector<vector<int>> val;
  Mat(int sz) {
    this->sz = sz;
    val.resize(sz, vector<int>(sz, 0));
  }
  Mat(int sz, int v) {
    this->sz = sz;
    val.resize(sz, vector<int>(sz, 0));
    for (int i = 0; i < sz; ++i) {
      val[i][i] = v; // diagonal values
    }
  }
  Mat operator * (Mat m2) {
    Mat ans(sz);
    for (int i = 0; i < sz; ++i) {
      for (int j = 0; j < sz; ++j) {
        for (int k = 0; k < sz; ++k) {
          ans.val[i][j] = (ans.val[i][j] + (1LL * val[i][k] * m2.val[k][j]) % mod) % mod;
        }
      }
    }
    return ans;
  }
};
Mat Mat_Expo(Mat a, long long n) {
  Mat ans(a.sz, 1);  // identity matrix
  while (n) {
    if (n & 1) {
      ans = ans * a;
    }
    a = a * a;
    n >>= 1;
  }
  return ans;
}
```

1.33 Merge Sort Tree

Description: A tree is given, with the value of every node.
Find the number of element greater than k-1 of asub-tree v for every query
Input :
3 2
1 2
5 6 7
1 3
1 6
Output :
3
2

```cpp
const ll MAXN = 1e6 + 10;
ll a[MAXN], val[MAXN], FT[MAXN * 2], Start[MAXN], End[MAXN];
vector<ll>g[MAXN * 4], gp[MAXN];
void build (ll node, ll b, ll e) {
  if (b == e) {
    g[node].pb(val[b]);
    return;}
  ll left_node = 2 * node;
  ll right_node = 2 * node + 1;
  ll mid = (b + e) / 2;
  build(left_node, b, mid);
  build (right_node, mid + 1, e);
  g[node].resize(g[left_node].size() + g[right_node].size());
  merge(g[left_node].begin(), g[left_node].end(),
    g[right_node].begin(), g[right_node].end(), g[node].begin());
}
ll query(ll node, ll b, ll e, ll i , ll j, ll k) {
  if (e<i or b>j) return 0;
  if (b >= i and e <= j) {
    // returning the number of values which is greater than k-1
    ll ans = g[node].end() - lower_bound(g[node].begin(), g[node].end(), k);
    return ans;
  }
  ll mid = (b + e) / 2;
  ll left_node = 2 * node;
  ll right_node = left_node + 1;
  return query(left_node , b , mid , i, j, k ) + query (right_node, mid + 1, e, i, j, k);
}
ll timer = 0;
## Tree Flattening: After flattening, every node will
    have a starting index and ending index like - 1 2 2 3 4 4 3 1
## Now I can make operation on any subtree of a node
 void dfs(ll node, ll par) {
  Start[node] = timer;
  FT[timer] = node;
  timer++;
  for (auto child : gp[node])
    if (child != par) dfs(child, node);
  End[node] = timer;
  FT[timer] = node;
  timer++;
}
void solve() {
  ll n, q; cin >> n >> q;
  for (ll i = 2; i <= n; i++) {
    ll x; cin >> x;
    gp[x].pb(i);
    gp[i].pb(x);
  }
  for (ll i = 1; i <= n; i++)
    cin >> a[i];
  dfs(1, -1);
  for (ll i = 0; i < timer; i++) {
    val[i + 1] = a[FT[i]];
  }
  build(1, 1, timer);
  while (q--) {
    ll in, k; cin >> in >> k;
    cout << query(1, 1, timer, Start[in] + 1, End[in] + 1, k) / 2 << '\n';
  }
}
```

1.34 Mobius Function

```cpp
const int N = 1E6 + 5;
int mu[N];
void pre() {
  mu[1] = 1;
  for (int i = 1; i < N; ++i)
    for (int j = i + i; j < N; j += i)
      mu[j] -= mu[i];
}
```

1.35 N-th Permutation

```cpp
vector<ll> fact(21, 1);
//does not handle if given ff-th permutation does not exist
string n_th_Permutation(string s, ll ff){
  int n = s.size();
  for(int i=0; i<n; i++){
    sort(s.begin()+i, s.end());
    int pos = i+ff/fact[n-1-i];
    ff %= fact[n-1-i];
    swap(s[i], s[pos]);
  }
  return s;
}
```

1.36 NOD_SOD

```cpp
pair<ll, ll> nod_sod(ll n) {
  ll divisor = 1; ll sum = 1;
  for (ll i = 0; primes[i]*primes[i] <= n; i++) {
    if (n % primes[i] == 0) {
      ll cnt = 1;
      while (n % primes[i] == 0) {
        n /= primes[i];
        cnt++;
      }
      divisor *= cnt;
      sum *= (pow(primes[i], cnt) - 1) / (primes[i] - 1);
    }
  }
  if (n > 1) divisor *= 2, sum *= (pow(n, 2) - 1) / (n - 1);
  return {divisor, sum};
}
```

1.37 Ordered Set - Custom Compare

```cpp
struct custom_compare {
  bool operator()(const tuple<int, int, int>& a, const tuple<int, int, int>& b) const {
    // Compare in decreasing order of the first element
    if (get<0>(a) != get<0>(b)) {
      return get<0>(a) > get<0>(b);
    }
    // If the first element is equal, compare in increasing order of the second element
    if (get<1>(a) != get<1>(b))
      return get<1>(a) < get<1>(b);
    return get<2>(a) < get<2>(b);
```

```
  }
};
#include <ext/pb_ds/assoc_container.hpp>
using namespace __gnu_pbds;
template <typename T> using indexed_set = tree<T,
    null_type, custom_compare, rb_tree_tag,
    tree_order_statistics_node_update>;
```

1.38 Ordered Set

Description: *x.find_by_order(k) : iterator to the k-th index
x.order_of_key(k) : number of items smaller than k

```
#include <ext/pb_ds/assoc_container.hpp>
using namespace __gnu_pbds;
template <typename T> using ordered_set = tree<T,
    null_type, less_equal<T>, rb_tree_tag,
    tree_order_statistics_node_update>;
```

1.39 Polynomial Interpolation

```
// P(x) = a0 + a1x + a2x^2 + ... + anx^n
// y[i] = P(i)
int inv(ll a) {
   a = (a + mod) % mod;
   return power(a, -1);
}
ll eval (vector<ll> y, ll k) {
  int n = y.size() - 1;
  if (k <= n) {
    return y[k];
  }
  vector<ll> L(n + 1, 1);
  for (int x = 1; x <= n; ++x) {
    L[0] = L[0] * (k - x) % mod;
    L[0] = L[0] * inv(-x) % mod;
  }
  for (int x = 1; x <= n; ++x) {
    L[x] = L[x - 1] * inv(k - x) % mod * (k - (x - 1)) % mod;
    L[x] = L[x] * ((x - 1) - n + mod) % mod * inv(x) % mod;
  }
  ll yk = 0;
  for (int x = 0; x <= n; ++x) {
    yk = (yk + L[x] * y[x] % mod) % mod;
  }
  return yk;
}
```

1.40 Prefix Sum 3D

```
pref[x][y][z] = pref[x - 1][y][z] + pref[x][y - 1][z]
    + pref[x][y][z - 1] - pref[x - 1][y - 1][z] -
    pref[x - 1][y][z - 1] - pref[x][y - 1][z - 1] +
    pref[x - 1][y - 1][z - 1] + arr[x][y][z];
// from x1 to x2, y1 to y2, z1 to z2
ans = pref[x2][y2][z2] - pref[x1 - 1][y2][z2] -
    pref[x2][y1 - 1][z2] - pref[x2][y2][z1 - 1] +
    pref[x1 - 1][y1 - 1][z] + pref[x1 - 1][y][z1 - 1]
    + pref[x2][y1 - 1][z1 - 1] - pref[x1 - 1][y1 -
    1][z1 - 1];
```

1.41 SCC

```
int vis[N], id[N];
vector<int> adj[N], adj_t[N];
vector<int> order;
void dfs(int v) {
  vis[v] = 1;
  for (int u: adj[v]) {
    if (!vis[u]) {
      dfs(u);
    }
  }
  order.push_back(v);
}
void dfs2(int v, int cnt) {
  vis[v] = 1;
  for (int u: adj_t[v]) {
    if (!vis[u]) {
      dfs2(u, cnt);
    }
  }
  id[v] = cnt;
}
void solve() {
  int n, m;
  cin >> n >> m;
  for (int i = 0; i < m; ++i) {
    int u, v;
    cin >> u >> v;
    adj[u].push_back(v);
    adj_t[v].push_back(u);
  }
  for (int i = 1; i <= n; ++i) {
    if (!vis[i]) {
      dfs(i);
    }
  }
  int cnt = 0;
  memset(vis, 0, sizeof(vis));
  reverse(order.begin(), order.end());
  for (auto v: order) {
    if (!vis[v]) {
      dfs2(v, cnt);
      ++cnt;
    }
  }
  cout << cnt << "\n";
  for (int i = 1; i <= n; ++i) {
    cout << id[i] << "\n";
  }
}
```

1.42 Segment Tree

```
struct ST {
  #define lc (n << 1)
  #define rc ((n << 1) + 1)
  long long t[4 * N], lazy[4 * N];
  ST() {
    memset(t, 0, sizeof t);
    memset(lazy, 0, sizeof lazy);
  }
  inline void push(int n, int b, int e) { // change this
    if (lazy[n] == 0) return;
    t[n] = t[n] + lazy[n] * (e - b + 1);
    if (b != e) {
      lazy[lc] = lazy[lc] + lazy[n];
      lazy[rc] = lazy[rc] + lazy[n];
    }
    lazy[n] = 0;
  }
  inline long long combine(long long a,long long b) {
    // change this
    return a + b;
  }
  inline void pull(int n) { // change this
    t[n] = t[lc] + t[rc];
  }
  void build(int n, int b, int e) {
    lazy[n] = 0; // change this
    if (b == e) {
      t[n] = a[b];
      return;
    }
    int mid = (b + e) >> 1;
    build(lc, b, mid);
    build(rc, mid + 1, e);
    pull(n);
  }
  void upd(int n, int b, int e, int i, int j, long long v) {
    push(n, b, e);
    if (j < b || e < i) return;
    if (i <= b && e <= j) {
      lazy[n] = v; //set lazy
      push(n, b, e);
      return;
    }
    int mid = (b + e) >> 1;
    upd(lc, b, mid, i, j, v);
    upd(rc, mid + 1, e, i, j, v);
    pull(n);
  }
  long long query(int n, int b, int e, int i, int j) {
    push(n, b, e);
    if (i > e || b > j) return 0; //return null
    if (i <= b && e <= j) return t[n];
    int mid = (b + e) >> 1;
    return combine(query(lc, b, mid, i, j), query(rc, mid + 1, e, i, j));
  }
}t;
```

1.43 Segmented Seive

```
void segSeive(ll low, ll high) {
  vector < bool > area((high - low) + 1, true);
  for (ll i = 0; primes[i]*primes[i] <= high; i++) {
    ll start = ((low / primes[i]) * primes[i]);
    if (start < low) start += primes[i];
    for (ll j = start; j <= high; j += primes[i]) {
      if (j == primes[i]) continue;
      area[j - low] = false;
    }
  }
  for (ll i = 0; i < (high - low) + 1; i++) {
    if (area[i]) {
      if (i + low != 1 and i + low != 0) {
          cout << i + low << endl;
      }
    }
  }
}
```

1.44 Seive upto 1e9

```
// credit: min_25
// takes 0.5s for n = 1e9
vector<int> sieve(const int N, const int Q = 17, const int L = 1 << 15) {
  static const int rs[] = {1, 7, 11, 13, 17, 19, 23, 29};
  struct P {
    P(int p) : p(p) {}
    int p; int pos[8];
  };
  auto approx_prime_count = [] (const int N) -> int {
    return N > 60184 ? N / (log(N) - 1.1) : max(1., N / (log(N) - 1.11)) + 1;
  };
  const int v = sqrt(N), vv = sqrt(v);
  vector<bool> isp(v + 1, true);
  for (int i = 2; i <= vv; ++i) if (isp[i]) {
    for (int j = i * i; j <= v; j += i) isp[j] = false;
  }
```

```cpp
  const int rsize = approx_prime_count(N + 30);
  vector<int> primes = {2, 3, 5}; int psize = 3;
  primes.resize(rsize);
  vector<P> sprimes; size_t pbeg = 0;
  int prod = 1;
  for (int p = 7; p <= v; ++p) {
    if (!isp[p]) continue;
    if (p <= Q) prod *= p, ++pbeg, primes[psize++] = p;
    auto pp = P(p);
    for (int t = 0; t < 8; ++t) {
      int j = (p <= Q) ? p : p * p;
      while (j % 30 != rs[t]) j += p << 1;
      pp.pos[t] = j / 30;
    }
    sprimes.push_back(pp);
  }
  vector<unsigned char> pre(prod, 0xFF);
  for (size_t pi = 0; pi < pbeg; ++pi) {
    auto pp = sprimes[pi]; const int p = pp.p;
    for (int t = 0; t < 8; ++t) {
      const unsigned char m = ~(1 << t);
      for (int i = pp.pos[t]; i < prod; i += p) pre[i] &= m;
    }
  }
  const int block_size = (L + prod - 1) / prod * prod;
  vector<unsigned char> block(block_size); unsigned char* pblock = block.data();
  const int M = (N + 29) / 30;
  for (int beg = 0; beg < M; beg += block_size, pblock -= block_size) {
    int end = min(M, beg + block_size);
    for (int i = beg; i < end; i += prod) {
      copy(pre.begin(), pre.end(), pblock + i);
    }
    if (beg == 0) pblock[0] &= 0xFE;
    for (size_t pi = pbeg; pi < sprimes.size(); ++pi) {
      auto& pp = sprimes[pi];
      const int p = pp.p;
      for (int t = 0; t < 8; ++t) {
        int i = pp.pos[t]; const unsigned char m = ~(1 << t);
        for (; i < end; i += p) pblock[i] &= m;
        pp.pos[t] = i;
      }
    }
    for (int i = beg; i < end; ++i) {
      for (int m = pblock[i]; m > 0; m &= m - 1) {
        primes[psize++] = i * 30 + rs[__builtin_ctz(m)];
      }
    }
  }
  assert(psize <= rsize);
  while (psize > 0 && primes[psize - 1] > N) --psize;
  primes.resize(psize);
  return primes;
}
int32_t main() {
  ios_base::sync_with_stdio(0);
  cin.tie(0);
  int n, a, b; cin >> n >> a >> b;
  auto primes = sieve(n);
  vector<int> ans;
  for (int i = b; i < primes.size() && primes[i] <= n; i += a) ans.push_back(primes[i]);
  cout << primes.size() << ' ' << ans.size() << '\n';
  for (auto x: ans) cout << x << ' '; cout << '\n';
  return 0;
}
```

1.45 Seive

```cpp
const int N = 1e6 + 3;
bitset<N> isPrime;
vector<int> prime;
void seive() {
  isPrime[2] = 1;
  for (int i = 3; i <= N; i+=2) {
    isPrime[i] = 1;
  }
  for (int i = 3; i * i <= N; i += 2) {
    if(isPrime[i]) {
      for (int j = i * i; j <= N; j += (i + i)) {
        isPrime[j] = 0;
      }
    }
  }
  prime.push_back(2);
  for (int i = 3; i <= N; i+=2) {
    if(isPrime[i]) {
      prime.push_back(i);
    }
  }
}
```

1.46 Sparse Table

```cpp
const int N = 2e5 + 3, M = __bit_width(N) + 1;
int maxTable[N][M], a[N];
void buildTable(int n) {
  for (int i = 0; i < n; ++i) {
    maxTable[i][0] = a[i];
  }
  for (int k = 1; k < M; ++k) {
    for (int i = 0; i + (1 << k) <= n; ++i) {
      maxTable[i][k] = max(maxTable[i][k - 1], maxTable[i + (1 << (k - 1))][k - 1]);
    }
  }
}
int maxQuery(int i, int j, int n) {
  if (j < 0 or i >= n) return INT32_MIN;
  int k = __bit_width(j - i + 1) - 1;
  return max(maxTable[i][k], maxTable[j - (1 << k) + 1][k]);
}
```

1.47 String Hashing

```cpp
const int p1 = 137, mod1 = 127657753, p2 = 277, mod2 = 987654319; // 911382323, 972663749
const int N = 1e6 + 3;
array<int, 2> pref[N], rev[N];
int pw1[N], pw2[N], ipw1[N], ipw2[N];
int power(int a, int n, int mod) {
  int ans = 1 % mod;
  while (n) {
    if (n & 1) ans = 1LL * ans * a % mod;
    a = 1LL * a * a % mod;
    n >>= 1;
  }
  return ans;
}
void prec() {
  pw1[0] = pw2[0] = ipw1[0] = ipw2[0] = 1;
  int ip1 = power(p1, mod1 - 2, mod1);
  int ip2 = power(p2, mod2 - 2, mod2);
  for (int i = 1; i < N; ++i) {
    pw1[i] = 1LL * pw1[i - 1] * p1 % mod1;
    pw2[i] = 1LL * pw2[i - 1] * p2 % mod2;
    ipw1[i] = 1LL * ipw1[i - 1] * ip1 % mod1;
    ipw2[i] = 1LL * ipw2[i - 1] * ip2 % mod2;
  }
}
void build(string& s) {
  int n = s.size();
  for (int i = 0; i < n; ++i) {
    pref[i][0] = 1LL * s[i] * pw1[i] % mod1;
    if (i) pref[i][0] = (pref[i][0] + pref[i - 1][0]) % mod1;
    pref[i][1] = 1LL * s[i] * pw2[i] % mod2;
    if (i) pref[i][1] = (pref[i][1] + pref[i - 1][1]) % mod2;
    rev[i][0] = 1LL * s[i] * ipw1[i] % mod1;
    if (i) rev[i][0] = (rev[i][0] + rev[i - 1][0]) % mod1;
    rev[i][1] = 1LL * s[i] * ipw2[i] % mod2;
    if (i) rev[i][1] = (rev[i][1] + rev[i - 1][1]) % mod2;
  }
}
array<int, 2> get_hash(int i, int j) {
  array<int, 2> ans = {0, 0};
  ans[0] = pref[j][0];
  if (i) ans[0] = (pref[j][0] - pref[i - 1][0] + mod1) % mod1;
  ans[1] = pref[j][1];
  if (i) ans[1] = (pref[j][1] - pref[i - 1][1] + mod2) % mod2;
  ans[0] = 1LL * ans[0] * ipw1[i] % mod1;
  ans[1] = 1LL * ans[1] * ipw2[i] % mod2;
  return ans;
}
array<int, 2> get_rev_hash(int i, int j) {
  array<int, 2> ans = {0, 0};
  ans[0] = rev[j][0];
  if (i) ans[0] = (rev[j][0] - rev[i - 1][0] + mod1) % mod1;
  ans[1] = rev[j][1];
  if (i) ans[1] = (rev[j][1] - rev[i - 1][1] + mod2) % mod2;
  ans[0] = 1LL * ans[0] * pw1[j] % mod1;
  ans[1] = 1LL * ans[1] * pw2[j] % mod2;
  return ans;
}
```

1.48 Suffix Array

Description: This funtion return two vectors (first vector is sorted suffix array position , second vector is longest common prefix with previous string)

```cpp
array<vector<int>, 2> get_sa(string& s, int lim=128) {
    // for integer, just change string to vector<int> and minimum value of vector must be >= 1
  int n = s.size() + 1, k = 0, a, b;
  vector<int> x(begin(s), end(s)+1), y(n), sa(n), lcp(n), ws(max(n, lim)), rank(n);
  x.back() = 0;
  iota(begin(sa), end(sa), 0);
  for (int j = 0, p = 0; p < n; j = max(1, j * 2), lim = p) {
    p = j, iota(begin(y), end(y), n - j);
    for (int i = 0; i < n; ++i) if (sa[i] >= j) y[p++] = sa[i] - j;
    fill(begin(ws), end(ws), 0);
    for (int i = 0; i < n; ++i) ws[x[i]]++;
    for (int i = 1; i < lim; ++i) ws[i] += ws[i - 1];
    for (int i = n; i--;) sa[--ws[x[y[i]]]] = y[i];
    swap(x, y), p = 1, x[sa[0]] = 0;
    for (int i = 1; i < n; ++i) a = sa[i - 1], b = sa[i], x[b] =
      (y[a] == y[b] && y[a + j] == y[b + j]) ? p - 1 : p++;
```

```
  }
  for (int i = 1; i < n; ++i) rank[sa[i]] = i;
  for (int i = 0, j; i < n - 1; lcp[rank[i++]] = k)
    for (k && k--, j = sa[rank[i] - 1]; s[i + k] ==
    s[j + k]; k++);
  sa.erase(sa.begin()), lcp.erase(lcp.begin());
  return {sa, lcp};
}
## Comparing Two Substrings
auto query = [&] (int l1, int r1, int l2, int r2) {
  int len1 = r1 - l1 + 1, len2 = r2 - l2 + 1;
  int len = min(len1, len2);
  int i = pos[l1], j = pos[l2], x;
  if (l1 != l2) x = st.query(i, j);
  else x = len;
  if (x >= len) {
    if (len1 == len2) return 0;
    if (len1 < len2) return -1;
    return 1;
  }
  if (s[l1 + x] < s[l2 + x]) return -1;
  return 1;
};
## Kth Unique Substring
auto kth = [&] (ll k) {
  int i = 0;
  while (i + 1 < n and k > n - sa[i] - lcp[i]) {
    k -= n - sa[i] - lcp[i];
    i++;
  }
  k = min(k, 0ll + n - sa[i] - lcp[i]);
  array<int, 2> ret = {sa[i], k + lcp[i]};
  return ret;
};
## Several Consecutive Identical Substrings
for (int i = 1; i < n; ++i) {
  for (int j = i; j < n; j += i) {
    // Block = [j-i...j-1]
    int e1 = rmq(0, pos[j - i], pos[j]), e2 = 0;
    if (i < j) {
      e2 = rmq(1, rev_pos[j - i - 1], rev_pos[j - 1]);
    }
    int k = (e1 + e2) / i + 1;
    // [j-i-e2 ... j-1+e1] is periodic with period
    length = i
  }
}
```

1.49 Suffix Automaton

```
int len[2 * N], lnk[2 * N], last, sz = 1;
unordered_map<char, int> to[2 * N]; // Use map during
  finding kth substring
int deg[2 * N], focc[2 * N];  // First Occurrence
ll cnt[2 * N], dp[2 * N];
void init(int n) {
  fill(deg, deg + sz, 0);
  fill(cnt, cnt + sz, 0);
  while (sz) to[--sz].clear();
  lnk[0] = -1, last = 0, sz = 1;
}
void add (char c, int i) {
  int cur = sz++;
  len[cur] = len[last] + 1;
  cnt[cur] = 1; dp[cur] = i;
  focc[cur] = i;
  int u = last;
  last = cur;
  while (u != -1 and !to[u].count(c)) {
    to[u][c] = cur;
    u = lnk[u];
  }
  if (u == -1) {
    lnk[cur] = 0;
  }
  else {
    int v = to[u][c];
    if (len[u] + 1 == len[v]) {
      lnk[cur] = v;
    }
    else {
      int w = sz++;
      len[w] = len[u] + 1, lnk[w] = lnk[v], to[w] =
      to[v];
      focc[w] = focc[v];
      while (u != -1 and to[u][c] == v) {
        to[u][c] = w, u = lnk[u];
      }
      lnk[cur] = lnk[v] = w;
    }
  }
}
bool exist (string &p) {
  int u = 0;
  for (auto c: p) {
    if (!to[u].count(c)) return false;
    u = to[u][c];
  }
  return true;
}
void build() {
  deg[0] = 1;
  for (int u = 1; u < sz; ++u) {
    deg[lnk[u]]++;
  }
  queue<int> q;
  for (int u = 0; u < sz; ++u) {
    if (!deg[u]) q.push(u);
  }
  while (!q.empty()) {
    int u = q.front(); q.pop();
    int v = lnk[u];
    cnt[v] += cnt[u]; // DP on suffix link tree
    for (auto [c, v]: to[u]) { // DP on DAG
      dp[u] = max(dp[u], dp[v]);
    }
    deg[v]--;
    if (!deg[v]) q.push(v);
  }
}
## Count number of occurrence for each k length
  substring of s in SA
ll count (string s, int k) {
  ll ret = 0;
  int u = 0, L = 0;
  for (auto c: s) {
    while (u and !to[u].count(c)) u = lnk[u], L =
    len[u];
    if (!to[u].count(c)) continue;
    u = to[u][c], L++;
    while (len[lnk[u]] >= k) u = lnk[u], L = len[u];
    if (L >= k) ret += cnt[u];
  }
  return ret;
}
## Kth substring (not distinct)
ll dp[2 * N];
ll dfs (int u) {
  if (dp[u] != -1) return dp[u];
  dp[u] = cnt[u]; // For distinct dp[u] = 1
  for (auto [c, v]: to[u]) {
    dp[u] += dfs(v);
  }
  return dp[u];
}
void yo (int u, ll k, string &s) {
  if (k <= 0) return ;
  for (auto [c, v]: to[u]) {
    if (k > dfs(v)) k -= dfs(v);
    else {
      s += c;
      k -= cnt[v];  // For distinct k -= 1
      yo(v, k, s);
      return ;
    }
  }
}
```

1.50 Ternary Search

```
double ternary_search(double l, double r) {
  double eps = 1e-9;      //set the error limit here
  while (r - l > eps) {
    double m1 = l + (r - l) / 3;
    double m2 = r - (r - l) / 3;
    double f1 = f(m1); //value of function at m1
    double f2 = f(m2); //value of function at m2
    if (f1 < f2)
      l = m1;
    else
      r = m2;
  }
  return f(l)      //return the maximum of f(x) in [l,
  r]
}
```

1.51 Topological Sorting

```
const int N = 1e5 + 9;
vector<int> g[N];
bool vis[N];
vector<int> ord;
void dfs(int u) {
  vis[u] = true;
  for (auto v: g[u]) {
    if (!vis[v]) {
      dfs(v);
    }
  }
  ord.push_back(u);
}
int32_t main() {
  ios_base::sync_with_stdio(0);
  cin.tie(0);
  int n, m; cin >> n >> m;
  while (m--) {
    int u, v; cin >> u >> v;
    g[u].push_back(v);
  }
  for (int i = 1; i <= n; i++) {
    if (!vis[i]) {
      dfs(i);
    }
  }
  reverse(ord.begin(), ord.end());
  // check if feasible
  vector<int> pos(n + 1);
  for (int i = 0; i < (int) ord.size(); i++) {
    pos[ord[i]] = i;
  }
  for (int u = 1; u <= n; u++) {
    for (auto v: g[u]) {
      if (pos[u] > pos[v]) {
        cout << "IMPOSSIBLE\n";
        return 0;
      }
    }
  }
  // print the order
  for (auto u: ord) cout << u << ' ';
  cout << '\n';
```

```
  return 0;
}
```

1.52 Tricks

```
//Maximum Subarray Sum (Kadane's algo)
ll max_so_far = -inf, max_end_here = 0;
for (ll i = 1; i <= n; i++) {
    max_end_here += a[i];
    if (max_end_here > max_so_far) max_so_far =
    max_end_here;
    if (max_end_here < 0) max_end_here = 0;
}
return max_so_far;
// Maximum Subarray Size Thats Sum = K
ll n, k; cin >> n >> k;
ll total_sum = 0;
vector < ll > pre(n + 7, 0);
for (ll i = 1; i <= n; i++) {
  ll temp; cin >> temp;
  total_sum += temp;
  if (i == 1) pre[i] = temp;
  else pre[i] = pre[i - 1] + temp;
}
if (total_sum < k) { cout << "-1" << endl; return; }
if (total_sum == k) { cout << "0" << endl; return; }
ll maximum_subSize = 0;
gp_hash_table < ll, ll, customHash> table;
for (ll i = 1; i <= n; i++) {
  if (pre[i] >= k) {
    ll subSUM = pre[i] - k;
    if (subSUM == 0) maximum_subSize =
    max(maximum_subSize, i);
    else if (table[subSUM]) {
      ll left = table[subSUM];
      ll right = i;
      ll subSize = right - left;
      maximum_subSize = max(subSize, maximum_subSize);
    }
  }
  if (!table[pre[i]]) table[pre[i]] = i;
}
cout << maximum_subSize << endl;
// Number of Subarray Sum Equal to K
ll n, k; cin >> n >> k;
ll total_sum = 0;
vector < ll > pre(n + 7, 0);
for (ll i = 1; i <= n; i++) {
  ll temp; cin >> temp;
  total_sum += temp;
  if (i == 1) pre[i] = temp;
  else pre[i] = pre[i - 1] + temp;
}
ll cnt_subarry = 0;
gp_hash_table < ll, ll, customHash> table;
table[0] = 1;
for (ll i = 1; i <= n; i++) {
  cnt_subarry += table[pre[i] - k];
  table[pre[i]]++;
}
cout << cnt_subarry << endl;
// How Many Pairs Of The Array Have GCD g, For All
   1<=g<=n
/*a[i] <= 1e6
for all 1<=g<=n, how many pairs exist such that g =
   gcd(a[i], a[j]);
complexity : nlogn
*/
ll n; cin >> n;
ll a[n + 1];
ll cnt[n + 1]; memset(cnt, 0, sizeof cnt);
for (ll i = 1; i <= n; i++) {cin >> a[i]; cnt[a[i]]++;}
ll gcd[n + 1]; memset(gcd, 0, sizeof gcd);
for (ll i = n; i >= 1; i--) {
  ll pair = 0, invalid_pair = 0;
  for (ll j = i; j <= n; j += i) {
    pair += cnt[j];
    invalid_pair += gcd[j];
  }
  pair = (pair * (pair - 1)) / 2;
  gcd[i] = pair - invalid_pair;
  // how many pairs exist whose gcd is i
}
```

1.53 Trie

```
const int N = 1e6 + 3;
int nextof[N][26], cnt[N];
int tot = 1;
void add(string& s) {
  int u = 1;
  ++cnt[u];
  for (auto c: s) {
    int v = c - 'a';
    if (!nextof[u][v]) {
      nextof[u][v] = ++tot;
    }
    u = nextof[u][v];
    ++cnt[u];
  }
}
int countPref(string& s) {
  int u = 1;
  for (auto c: s) {
    int v = c - 'a';
    if (!nextof[u][v]) return 0;
    u = nextof[u][v];
  }
  return cnt[u];
}
```

1.54 int128

```
istream& operator >>(istream& cin, __int128& x) {
  string s;
  cin >> s;
  x = 0;
  for (int i = 0; i < s.size(); ++i) {
    x = x * 10 + (s[i] - '0');
  }
  return cin;
}
ostream& operator <<(ostream& cout, __int128 x) {
  string s;
  while (x) {
    s += (x % 10) + '0';
    x /= 10;
  }
  reverse(s.begin(), s.end());
  cout << s;
  return cout;
}
```

1.55 nCr and nPr-1

```
int fact[N], ifact[N];
void prec() { // O(n)
  fact[0] = 1;
  for (int i = 1; i < N; i++) {
    fact[i] = 1LL * fact[i - 1] * i % mod;
  }
  ifact[N - 1] = power(fact[N - 1], -1);
  for (int i = N - 2; i >= 0; i--) {
    ifact[i] = 1LL * ifact[i + 1] * (i + 1) % mod; //
    1 / i! = (1 / (i + 1)!) * (i + 1)
  }
}
int nPr(int n, int r) { // O(1)
  if (n < r) return 0;
  return 1LL * fact[n] * ifact[n - r] % mod;
}
int nCr(int n, int r) { // O(1)
  if (n < r) return 0;
  return 1LL * fact[n] * ifact[r] % mod * ifact[n - r]
    % mod;
}
```

1.56 nCr and nPr-2

```
const int N = 2005, mod = 1e9 + 7;
int C[N][N], fact[N];
void prec() { // O(n^2)
  for (int i = 0; i < N; i++) {
    C[i][0] = C[i][i] = 1;
    for (int j = 1; j < i; j++) {
      C[i][j] = (C[i - 1][j - 1] + C[i - 1][j]) % mod;
    }
  }
  fact[0] = 1;
  for (int i = 1; i < N; i++) {
    fact[i] = 1LL * fact[i - 1] * i % mod;
  }
}
int nCr(int n, int r) { // O(1)
  if (n < r) return 0;
  return C[n][r];
}
int nPr(int n, int r) { // O(1)
  if (n < r) return 0;
  return 1LL * nCr(n, r) * fact[r] % mod;
}
```

2 Geometry

2.1 Angular Sort

```
inline bool up (point p) {
  return p.y > 0 or (p.y == 0 and p.x >= 0);
}
sort(v.begin(), v.end(), [] (point a, point b) {
  return up(a) == up(b) ? a.x * b.y > a.y * b.x :
    up(a) < up(b);
});
inline int quad (point p) {
  if (p.y >= 0)  return p.x < 0;
  return 2 + (p.x >= 0);
}
sort(pt.begin(), pt.end(), [] (point a, point b) {
  return quad(a) == quad(b) ? a.x * b.y > a.y * b.x :
    quad(a) < quad(b);
});
```

2.2 CircleCircleIntersection

Description: compute intersection of circle centered at a with radius r with circle centered at b with radius R.

```
vector<PT> CircleCircleIntersection(PT a, PT b, double
  r, double R) {
  vector<PT> ret;
  double d = sqrt(dist2(a, b));
  if (d > r+R || d+min(r, R) < max(r, R)) return ret;
  double x = (d*d-R*R+r*r)/(2*d);
  double y = sqrt(r*r-x*x);
  PT v = (b-a)/d;
  ret.push_back(a+v*x + RotateCCW90(v)*y);
  if (y > 0)
    ret.push_back(a+v*x - RotateCCW90(v)*y);
  return ret;
}
```

2.3 CircleLineIntersection

Description: Compute intersection of line through points a and b with circle centered at c with radius $r > 0$.

```
vector<PT> CircleLineIntersection(PT a, PT b, PT c, double r) {
  vector<PT> ret;
  b = b-a; a = a-c;
  double A = dot(b, b); double B = dot(a, b);
  double C = dot(a, a) - r*r;
  double D = B*B - A*C;
  if (D < -EPS) return ret;
  ret.push_back(c+a+b*(-B+sqrt(D+EPS))/A);
  if (D > EPS)
    ret.push_back(c+a+b*(-B-sqrt(D))/A);
  return ret;
}
```

2.4 Closest Pair of Points

```
ll min_dis(vector<array<int, 2>> &pts, int l, int r) {
  if (l + 1 >= r)  return LLONG_MAX;
  int m = (l + r) / 2;
  ll my = pts[m-1][1];
  ll d = min(min_dis(pts, l, m), min_dis(pts, m, r));
  inplace_merge(pts.begin()+l, pts.begin()+m, pts.begin()+r);
  for (int i = l; i < r; ++i) {
    if ((pts[i][1] - my) * (pts[i][1] - my) < d) {
      for (int j = i + 1; j < r and (pts[i][0] - pts[j][0]) * (pts[i][0] - pts[j][0]) < d; ++j) {
        ll dx = pts[i][0] - pts[j][0], dy = pts[i][1] - pts[j][1];
        d = min(d, dx * dx + dy * dy);
      }
    }
  }
  return d;
}
vector<array<int, 2>> pts(n);
sort(pts.begin(), pts.end(), [&] (array<int, 2> a, array<int, 2> b){
  return make_pair(a[1], a[0]) < make_pair(b[1], b[0]);
});
```

2.5 ComputeCentroid

```
// centroid of a (possibly nonconvex) polygon.
PT ComputeCentroid(const vector<PT> &p) {
  PT c(0,0);
  double scale = 6.0 * ComputeSignedArea(p);
  for (int i = 0; i < p.size(); i++){
    int j = (i+1) % p.size();
    c = c + (p[i]+p[j])*(p[i].x*p[j].y - p[j].x*p[i].y);
  }
  return c / scale;
}
```

2.6 ComputeCircleCenter

```
// compute center of circle passing through three points
PT ComputeCircleCenter(PT a, PT b, PT c) {
  b=(a+b)/2;
  c=(a+c)/2;
  return ComputeLineIntersection(b, b+RotateCW90(a-b), c, c+RotateCW90(a-c));
}
```

2.7 ComputeLineIntersection

Description: compute intersection of line passing through a and b with line passing through c and d, assuming that unique intersection exists; for segment intersection, check if segments intersect first.

```
PT ComputeLineIntersection(PT a, PT b, PT c, PT d) {
  b=b-a; d=c-d; c=c-a;
  assert(dot(b, b) > EPS && dot(d, d) > EPS);
  return a + b*cross(c, d)/cross(b, d);
}
```

2.8 ComputeSignedArea

Description: Computes the area of a (possibly nonconvex) polygon, assuming that the coordinates are listed in a clockwise or counterclockwise fashion.

```
double ComputeSignedArea(const vector<PT> &p) {
  double area = 0;
  for(int i = 0; i < p.size(); i++) {
    int j = (i+1) % p.size();
    area += p[i].x*p[j].y - p[j].x*p[i].y;
  }
  return area / 2.0;
}
double ComputeArea(const vector<PT> &p) {
  return fabs(ComputeSignedArea(p));
}
```

2.9 Convex Hull

```
vector <PT> convexHull (vector <PT> p) {
  int n = p.size(), m = 0;
  if (n < 3) return p;
  vector <PT> hull(n + n);
  sort(p.begin(), p.end(), [&] (PT a, PT b) {
    return (a.x==b.x? a.y<b.y: a.x<b.x);
  });
  for (int i = 0; i < n; ++i) {
    while (m > 1 and cross(hull[m - 2] - p[i], hull[m - 1] - p[i]) <= 0) --m;
    hull[m++] = p[i];
  }
  for (int i = n - 2, j = m + 1; i >= 0; --i) {
    while (m >= j and cross(hull[m - 2] - p[i], hull[m - 1] - p[i]) <= 0) --m;
    hull[m++] = p[i];
  }
  hull.resize(m - 1); return hull;
}
```

2.10 DistancePointPlane

Description: compute distance between point (x, y, z) and plane $ax + by + cz = d$

```
double DistancePointPlane(double x, double y, double z, double a, double b, double c, double d) {
  return fabs(a*x+b*y+c*z-d)/sqrt(a*a+b*b+c*c);
}
```

2.11 DistancePointSegment

```
// compute distance from c to segment between a and b
double DistancePointSegment(PT a, PT b, PT c) {
  return sqrt(dist2(c, ProjectPointSegment(a, b, c)));
}
```

2.12 Half Plane Intersection

Description: Calculates the intersection of halfplanes, assuming every half-plane allows the region to the left of its line.

```
struct Halfplane {
  PT p, pq; ld angle;
  Halfplane() {}
  // Two points on line
  Halfplane(const PT& a, const PT& b) : p(a), pq(b - a) {
    angle = atan2l(pq.y, pq.x);
  }
  bool out(const PT& r) {
    return cross(pq, r - p) < -EPS;
  }
  bool operator < (const Halfplane& e) const {
    return angle < e.angle;
  }
  friend PT inter(const Halfplane& s, const Halfplane& t) {
    ld alpha = cross((t.p - s.p), t.pq) / cross(s.pq, t.pq);
    return s.p + (s.pq * alpha);
  }
};
vector<PT> hp_intersect(vector<Halfplane>& H) {
  PT box[4] = {  // Bounding box in CCW order
    PT(INF, INF),  PT(-INF, INF),
    PT(-INF, -INF), PT(INF, -INF)
  };
  for(int i = 0; i<4; i++) { // Add bounding box half-planes.
      Halfplane aux(box[i], box[(i+1) % 4]);
      H.push_back(aux);
  }
  sort(H.begin(), H.end());
  deque<Halfplane> dq; int len = 0;
  for(int i = 0; i < int(H.size()); i++) {
    while (len > 1 && H[i].out(inter(dq[len-1], dq[len-2]))) {
      dq.pop_back(); --len;
    }
    while (len > 1 && H[i].out(inter(dq[0], dq[1]))) {
      dq.pop_front(); --len;
    }
    if (len > 0 && fabsl(cross(H[i].pq, dq[len-1].pq)) < EPS) {
      if (dot(H[i].pq, dq[len-1].pq) < 0.0)
        return vector<PT>();
      if (H[i].out(dq[len-1].p)) {
        dq.pop_back(); --len;
      }
      else continue;
    }
    dq.push_back(H[i]); ++len;
  }
  while (len > 2 && dq[0].out(inter(dq[len-1], dq[len-2]))) {
    dq.pop_back(); --len;
  }
  while (len > 2 && dq[len-1].out(inter(dq[0], dq[1]))) {
    dq.pop_front(); --len;
  }
  // Report empty intersection if necessary
  if (len < 3) return vector<PT>();
  // Reconstruct the convex polygon from the remaining half-planes.
```

```
  vector<PT> ret(len);
  for(int i = 0; i+1 < len; i++) {
    ret[i] = inter(dq[i], dq[i+1]);
  }
  ret.back() = inter(dq[len-1], dq[0]);
  return ret;
}
```

2.13 IsSimple

```
// tests whether or not a given polygon (in CW or CCW order) is simple
bool IsSimple(const vector<PT> &p) {
  for (int i = 0; i < p.size(); i++) {
    for (int k = i+1; k < p.size(); k++) {
      int j = (i+1) % p.size();
      int l = (k+1) % p.size();
      if (i == l || j == k) continue;
      if (SegmentsIntersect(p[i], p[j], p[k], p[l]))
        return false;
    }
  }
  return true;
}
```

2.14 LinesCollinear

```
bool LinesCollinear(PT a, PT b, PT c, PT d) {
  return LinesParallel(a, b, c, d)
      && fabs(cross(a-b, a-c)) < EPS
      && fabs(cross(c-d, c-a)) < EPS;
}
```

2.15 LinesParallel

```
// determine if lines from a to b and c to d are parallel or collinear
bool LinesParallel(PT a, PT b, PT c, PT d) {
  return fabs(cross(b-a, c-d)) < EPS;
}
```

2.16 Point

```
double INF = 1e100;
double EPS = 1e-12;
struct PT {
  double x, y;
  PT() {}
  PT(double x, double y) : x(x), y(y) {}
  PT(const PT &p) : x(p.x), y(p.y)    {}
  PT operator + (const PT &p)  const { return PT(x+p.x, y+p.y); }
  PT operator - (const PT &p)  const { return PT(x-p.x, y-p.y); }
  PT operator * (double c)     const { return PT(x*c,  y*c  ); }
  PT operator / (double c)     const { return PT(x/c,  y/c  ); }
};
double dot(PT p, PT q)     { return p.x*q.x+p.y*q.y; }
double dist2(PT p, PT q)   { return dot(p-q,p-q); }
double abs(PT p) { return sqrt(p.x*p.x + p.y*p.y); }
double cross(PT p, PT q)   { return p.x*q.y-p.y*q.x; }
ostream &operator<<(ostream &os, const PT &p) {
  return os << "(" << p.x << "," << p.y << ")";
}
// rotate a point CCW or CW around the origin
PT RotateCCW90(PT p)   { return PT(-p.y,p.x); }
PT RotateCW90(PT p)    { return PT(p.y,-p.x); }
PT RotateCCW(PT p, double t) {
  return PT(p.x*cos(t)-p.y*sin(t), p.x*sin(t)+p.y*cos(t));
}
// angle (range [0, pi]) between two vectors
double angle(PT v, PT w) {
  return acos(clamp(dot(v,w) / abs(v) / abs(w), -1.0, 1.0));
}
```

2.17 PointInPolygon

Description: -1 = strictly inside, 0 = on, 1 = strictly outside.

```
int PointInPolygon(vector<PT> &P, PT a) {
  int cnt = 0, n = P.size();
  for(int i = 0; i < n; ++i) {
    PT q = P[(i + 1) % n];
    if (onSegment(P[i], q, a)) return 0;
    cnt ^= ((a.y < P[i].y) - (a.y < q.y)) * cross(P[i] - a, q - a) > 0;
  } return cnt > 0 ? -1 : 1;
}
int PointInConvexPolygon(vector<PT> &P, const PT& q) { // O(log n)
  int n = P.size();
  ll a = cross(P[0] - q, P[1] - q), b = cross(P[0] - q, P[n - 1] - q);
  if (a < 0 or b > 0) return 1;
  int l = 1, r = n - 1;
  while (l + 1 < r) {
    int mid = l + r >> 1;
    if (cross(P[0] - q, P[mid] - q) >= 0) l = mid;
    else r = mid;
  }
  ll k = cross(P[l] - q, P[r] - q);
  if (k <= 0) return k < 0 ? 1 : 0;
  if (l == 1 and a == 0) return 0;
  if (r == n - 1 and b == 0) return 0;
  return -1;
}
```

2.18 ProjectPointLine

```
// project point c onto line through a and b, assuming a != b
PT ProjectPointLine(PT a, PT b, PT c) {
  return a + (b-a)*dot(c-a, b-a)/dot(b-a, b-a);
}
```

2.19 ProjectPointSegment

```
// project point c onto line segment through a and b
PT ProjectPointSegment(PT a, PT b, PT c) {
  double r = dot(b-a,b-a);
  if (fabs(r) < EPS) return a;
  r = dot(c-a, b-a)/r;
  if (r < 0) return a;
  if (r > 1) return b;
  return a + (b-a)*r;
}
```

2.20 SegmentsIntersect

```
// determine if line segment from a to b intersects with line segment from c to d
bool SegmentsIntersect(PT a, PT b, PT c, PT d) {
  if (LinesCollinear(a, b, c, d)) {
    if (dist2(a, c) < EPS || dist2(a, d) < EPS ||
      dist2(b, c) < EPS || dist2(b, d) < EPS) return true;
    if (dot(c-a, c-b) > 0 && dot(d-a, d-b) > 0 && dot(c-b, d-b) > 0)
      return false;
    return true;
  }
  if (cross(d-a, b-a) * cross(c-a, b-a) > 0) return false;
  if (cross(a-c, d-c) * cross(b-c, d-c) > 0) return false;
  return true;
}
```

3 Notes

3.1 Geometry

3.1.1 Triangles

Circumradius: $R = \dfrac{abc}{4A}$, Inradius: $r = \dfrac{A}{s}$

Length of median (divides triangle into two equal-area triangles): $m_a = \frac{1}{2}\sqrt{2b^2 + 2c^2 - a^2}$

Length of bisector (divides angles in two): $s_a = \sqrt{bc\left[1 - \left(\frac{a}{b+c}\right)^2\right]}$

Law of tangents: $\dfrac{a+b}{a-b} = \dfrac{\tan\frac{\alpha+\beta}{2}}{\tan\frac{\alpha-\beta}{2}}$

3.1.2 Quadrilaterals

With side lengths a, b, c, d, diagonals e, f, diagonals angle θ, area A and magic flux $F = b^2 + d^2 - a^2 - c^2$:

$$4A = 2ef \cdot \sin\theta = F \tan\theta = \sqrt{4e^2f^2 - F^2}$$

For cyclic quadrilaterals the sum of opposite angles is 180°, $ef = ac + bd$, and $A = \sqrt{(p-a)(p-b)(p-c)(p-d)}$.

3.1.3 Spherical coordinates

$$\begin{aligned} x &= r\sin\theta\cos\phi & r &= \sqrt{x^2+y^2+z^2} \\ y &= r\sin\theta\sin\phi & \theta &= \operatorname{acos}(z/\sqrt{x^2+y^2+z^2}) \\ z &= r\cos\theta & \phi &= \operatorname{atan2}(y,x) \end{aligned}$$

3.2 Binomial Coefficent

- Factoring in: $\binom{n}{k} = \frac{n}{k}\binom{n-1}{k-1}$
- Sum over k: $\sum_{k=0}^{n}\binom{n}{k} = 2^n$
- Alternating sum: $\sum_{k=0}^{n}(-1)^k\binom{n}{k} = 0$
- Even and odd sum: $\sum_{k=0}^{n}\binom{n}{2k} = \sum_{k=0}^{n}\binom{n}{2k+1}2^{n-1}$
- The Hockey Stick Identity

  - (Left to right) Sum over n and k: $\sum_{k=0}^{m}\binom{n+k}{k} = \binom{n+m-1}{m}$

  - (Right to left) Sum over n: $\sum_{m=0}^{n}\binom{m}{k} = \binom{n+1}{k+1}$
- Sum of the squares: $\sum_{k=0}^{n}(\binom{n}{k})^2 = \binom{2n}{n}$
- Weighted sum: $\sum_{k=1}^{n} k\binom{n}{k} = n2^{n-1}$
- Connection with the fibonacci numbers: $\sum_{k=0}\binom{n-k}{k} = F_{n+1}$
- Vandermonde's Identity: $\sum_{i=0}^{k}\binom{m}{i}\binom{n}{k-i} = \binom{m+n}{k}$
- If $f(n,k) = C(n,0) + C(n,1) + \ldots + C(n,k)$, Then $f(n+1,k) = 2 * f(n,k) - C(n,k)$ [For multiple $f(n,k)$ queries, use Mo's algo]

Lucas Theorem

$$\binom{m}{n} \equiv \prod_{i=0}^{k} \binom{m_i}{n_i} (\mod p)$$

- $\binom{m}{n}$ is divisible by p if and only if at least one of the base-p digits of n is greater than the corresponding base-p digit of m.
- The number of entries in the nth row of Pascal's triangle that are not divisible by $p = \prod_{i=0}^{k}(n_i+1)$
- All entries in the $(p^k-1)th$ row are not divisible by p.
- $\binom{n}{m} \equiv \lfloor \frac{n}{p} \rfloor (\mod p)$

3.3 Fibonacci Number

1. $k = A-B, F_A F_B = F_{k+1}F_A^2 + F_k F_A F_{A-1}$

2. $\sum_{i=0}^{n} F_i^2 = F_{n+1}F_n$ **3.** $\sum_{i=0}^{n} F_i F_{i+1} = F_{n+1}^2 - (-1)^n$

4. $\sum_{i=0}^{n} F_i F_{i+1} = F_{n+1}^2 - (-1)^n$ **5.** $\sum_{i=0}^{n} F_i F_{i-1} = \sum_{i=0}^{n-1} F_i F_{i+1}$

6. $\gcd(F_m, F_n) = F_{\gcd(m,n)}$ **7.** $\sum_{0 \le k \le n} \left(\binom{n-k}{k}\right) = F_{n+1}$

8. $\gcd(F_n, F_{n+1}) = \gcd(F_n, F_{n+2}) = \gcd(F_{n+1}, F_{n+2}) = 1$

3.4 Sums

$1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(2n+1)(n+1)}{6}$

$1^3 + 2^3 + 3^3 + \cdots + n^3 = \frac{n^2(n+1)^2}{4}$

$1^4 + 2^4 + 3^4 + \cdots + n^4 = \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30}$

$\sum_{i=1}^{n} i^m = \frac{1}{m+1}\left[(n+1)^{m+1} - 1 - \sum_{i=1}^{n}\left((i+1)^{m+1} - i^{m+1} - (m+1)i^m\right)\right]$

$\sum_{i=1}^{n-1} i^m = \frac{1}{m+1}\sum_{k=0}^{m}\binom{m+1}{k}B_k n^{m+1-k}$

$\sum_{k=0}^{n} kx^k = (x-(n+1)x^{n+1} + nx^{n+2})/(x-1)^2$

3.5 Series

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots, (-\infty < x < \infty)$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots, (-1 < x \le 1)$$

$$\sqrt{1+x} = 1 + \frac{x}{2} - \frac{x^2}{8} + \frac{2x^3}{32} - \frac{5x^4}{128} + \ldots, (-1 \le x \le 1)$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots, (-\infty < x < \infty)$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots, (-\infty < x < \infty)$$

$$(x+a)^{-n} = \sum_{k=0}^{\infty}(-1)^k \binom{n+k-1}{k} x^k a^{-n-k}$$

Generating Function

$$1/(1-x) = 1 + x + x^2 + x^3 + \ldots$$

$$1/(1-ax) = 1 + ax + (ax)^2 + (ax)^3 + \ldots$$

$$1/(1-x)^2 = 1 + 2x + 3x^2 + 4x^3 + \ldots$$

$$1/(1-x)^3 = C(2,2) + C(3,2)x + C(4,2)x^2 + C(5,2)x^3 + \ldots$$

$$1/(1-ax)^(k+1) = 1 + C(1+k,k)(ax) + C(2+k,k)(ax)^2 + C(3+k,k)(ax)^3 + \ldots$$

$$x(x+1)(1-x)^-3 = 1 + x + 4x^2 + 9x^3 + 16x^4 + 25x^5 + \ldots$$

$$e^x = 1 + x + (x^2)/2! + (x^3)/3! + (x^4)/4! + \ldots$$

3.6 Pythagorean Triples

The Pythagorean triples are uniquely generated by

$$a = k\cdot(m^2-n^2),\ b = k\cdot(2mn),\ c = k\cdot(m^2+n^2),$$

with $m > n > 0$, $k > 0$, $m \perp n$, and either m or n even.

3.7 Number Theory

- HCN: 1e6(240), 1e9(1344), 1e12(6720), 1e14(17280), 1e15(26880), 1e16(41472)
- $gcd(a,b,c,d,\ldots) = gcd(a, b-a, c-b, d-c, \ldots)$
- $gcd(a+k, b+k, c+k, d+k, \ldots) = gcd(a+k, b-a, c-b, d-c, \ldots)$
- Primitive root exists iff $n = 1, 2, 4, p^k, 2\times p^k$, where p is an odd prime.
- If primtive root exists, there are $\phi(\phi(n))$ primtive roots of n.
- The numbers from 1 to n have in total $O(n\log\log n)$ unique prime factors.
- $x \equiv r_1 \mod m1$ and $x \equiv r_2 \mod m2$ has a solution iff $\gcd(m_1, m_2) | (r_1 - r_2)$ Solution of $x^2 \equiv a(\mod p)$
- $ca \equiv cb \pmod m \iff a \equiv b \pmod{\frac{n}{\gcd(n,c)}}$
- $ax \equiv b \pmod m$ has a solution $\iff \gcd(a,m)|b$
- If $ax \equiv b \pmod m$ has a solution, then it has $gcd(a,m)$ solutions and they are separated by $\frac{m}{\gcd(a,m)}$
- $ax \equiv 1 \pmod m$ has a solution or a is invertible $\pmod m$ $\iff$ $\gcd(a,m) = 1$
- $x^2 \equiv 1 \pmod p$ then $x \equiv \pm 1 \pmod p$
- There are $\frac{p-1}{2}$ has no solution.
- There are $\frac{p-1}{2}$ has exaclty two solutions.
- When $p\%4 = 3$, $x \equiv \pm a^{\frac{p+1}{4}}$
- When $p\%8 = 5$, $x \equiv a^{\frac{p+3}{8}}$ or $x \equiv 2^{\frac{p-1}{4}} a^{\frac{p+3}{8}}$

3.7.1 Primes

$p = 962592769$ is such that $2^{21} \mid p-1$, which may be useful. For hashing use 970592641 (31-bit number), 31443539979727 (45-bit), 3006703054056749 (52-bit). There are 78498 primes less than 1 000 000.

Primitive roots exist modulo any prime power p^a, except for $p = 2, a > 2$, and there are $\phi(\phi(p^a))$ many. For $p = 2, a > 2$, the group $\mathbb{Z}_{2^a}^{\times}$ is instead isomorphic to $\mathbb{Z}_2 \times \mathbb{Z}_{2^{a-2}}$.

3.7.2 Estimates

$\sum_{d|n} d = O(n\log\log n)$.

The number of divisors of n is at most around 100 for $n < 5e4$, 500 for $n < 1e7$, 2000 for $n < 1e10$, 200 000 for $n < 1e19$.

3.7.3 Perfect numbers

$n > 1$ is called perfect if it equals sum of its proper divisors and 1. Even n is perfect iff $n = 2^{p-1}(2^p - 1)$ and $2^p - 1$ is prime (Mersenne's). No odd perfect numbers are yet found.

3.7.4 Carmichael numbers

A positive composite n is a Carmichael number ($a^{n-1} \equiv 1 \pmod n$ for all a: $\gcd(a,n) = 1$), iff n is square-free, and for all prime divisors p of n, $p-1$ divides $n-1$.

3.7.5 Totient

- If p is a prime $(p^k) = p^k - p^{k-1}$
- If a b are relatively prime, $\phi(ab) = \phi(a)\phi(b)$
- $\phi(n) = n(1-\frac{1}{p_1})(1-\frac{1}{p_2})(1-\frac{1}{p_3})\ldots(1-\frac{1}{p_k})$
- Sum of coprime to $n = n * \frac{\phi(n)}{2}$
- If $n = 2^k, \phi(n) = 2^{k-1} = \frac{n}{2}$
- For a b, $\phi(ab) = \phi(a)\phi(b)\frac{d}{\phi(d)}$
- $\phi(ip) = p\phi(i)$ whenever p is a prime and it divides i
- The number of $a(1 <= a <= N)$ such that $gcd(a,N) = d$ is $\phi(\frac{n}{d})$
- If $n > 2$, $\phi(n)$ is always even
- Sum of gcd, $\sum_{i=1}^{n} gcd(i,n) = \sum_{d|n} d\phi(\frac{n}{d})$
- Sum of lcm, $\sum_{i=1} nlcm(i,n) = \frac{n}{2}(\sum_{d|n}(d\phi(d)) + 1)$
- $\phi(1) = 1$ and $\phi(2) = 1$ which two are only odd ϕ
- $\phi(3) = 2$ and $\phi(4) = 2$ and $\phi(6) = 2$ which three are only prime ϕ
- Find minimum n such that $\frac{\phi(n)}{n}$ is maximum- Multiple of small primes- $2*3*5*7*11*13*\ldots$

3.7.6 Mobius function

$\mu(1) = 1$. $\mu(n) = 0$, if n is not squarefree. $\mu(n) = (-1)^s$, if n is the product of s distinct primes. Let f, F be functions on positive integers. If for all $n \in N$, $F(n) = \sum_{d|n} f(d)$, then $f(n) = \sum_{d|n}\mu(d)F(\frac{n}{d})$, and vice versa. $\phi(n) = \sum_{d|n}\mu(d)\frac{n}{d}$. $\sum_{d|n}\mu(d) = 1$.
If f is multiplicative, then $\sum_{d|n}\mu(d)f(d) = \prod_{p|n}(1-f(p))$, $\sum_{d|n}\mu(d)^2 f(d) = \prod_{p|n}(1+f(p))$.

$$\sum_{i=1}^{n}\sum_{j=1}^{n}[gcd(i,j) = 1] = \sum_{k=1}^{n}\mu(k)\lfloor\frac{n}{k}\rfloor^2$$

$$\sum_{i=1}^{n}\sum_{j=1}^{n} gcd(i,j) = \sum_{k=1}^{n} k \sum_{l=1}^{\lfloor\frac{n}{k}\rfloor}\mu(l)\lfloor\frac{n}{kl}\rfloor^2$$

$$\sum_{i=1}^{n}\sum_{j=1}^{n} gcd(i,j) = \sum_{k=1}^{n}\left(\frac{\lfloor\frac{n}{k}\rfloor(1+\lfloor\frac{n}{k}\rfloor)}{2}\right)^2\sum_{d|k}\mu(d)kd$$

3.7.7 Legendre symbol

If p is an odd prime, $a \in \mathbb{Z}$, then $\left(\frac{a}{p}\right)$ equals 0, if $p|a$; 1 if a is a quadratic residue modulo p; and -1 otherwise. Euler's criterion: $\left(\frac{a}{p}\right) = a^{\left(\frac{p-1}{2}\right)} \pmod p$.

3.7.8 Jacobi symbol

If $n = p_1^{a_1}\cdots p_k^{a_k}$ is odd, then $\left(\frac{a}{n}\right) = \prod_{i=1}^{k}\left(\frac{a}{p_i}\right)^{k_i}$.

3.7.9 Primitive roots

If the order of g modulo m (min $n > 0$: $g^n \equiv 1 \pmod m$) is $\phi(m)$, then g is called a primitive root. If Z_m has a primitive root, then it has $\phi(\phi(m))$ distinct primitive roots. Z_m has a primitive root iff m is one of 2, 4, p^k, $2p^k$, where p is an odd prime. If Z_m has a primitive root g, then for all a coprime to m, there exists unique integer $i = \text{ind}_g(a)$ modulo $\phi(m)$, such that $g^i \equiv a \pmod m$. $\text{ind}_g(a)$ has logarithm-like properties: $\text{ind}(1) = 0$, $\text{ind}(ab) = \text{ind}(a) + \text{ind}(b)$.

If p is prime and a is not divisible by p, then congruence $x^n \equiv a \pmod p$ has $\gcd(n, p-1)$ solutions if $a^{(p-1)/\gcd(n,p-1)} \equiv 1 \pmod p$, and no solutions otherwise. (Proof sketch: let g be a primitive root, and $g^i \equiv a \pmod p$, $g^u \equiv x \pmod p$. $x^n \equiv a \pmod p$ iff $g^{nu} \equiv g^i \pmod p$ iff $nu \equiv i \pmod p$.)

3.7.10 Discrete logarithm problem

Find x from $a^x \equiv b \pmod m$. Can be solved in $O(\sqrt{m})$ time and space with a meet-in-the-middle trick. Let $n = \lceil \sqrt{m} \rceil$, and $x = ny - z$. Equation becomes $a^{ny} \equiv ba^z \pmod m$. Precompute all values that the RHS can take for $z = 0, 1, \ldots, n-1$, and brute force y on the LHS, each time checking whether there's a corresponding value for RHS.

3.7.11 Pythagorean triples

Integer solutions of $x^2 + y^2 = z^2$ All relatively prime triples are given by: $x = 2mn, y = m^2 - n^2, z = m^2 + n^2$ where $m > n, \gcd(m, n) = 1$ and $m \not\equiv n \pmod 2$. All other triples are multiples of these. Equation $x^2 + y^2 = 2z^2$ is equivalent to $(\frac{x+y}{2})^2 + (\frac{x-y}{2})^2 = z^2$.

3.7.12 Postage stamps/McNuggets problem

Let a, b be relatively-prime integers. There are exactly $\frac{1}{2}(a-1)(b-1)$ numbers *not* of form $ax + by$ ($x, y \ge 0$), and the largest is $(a-1)(b-1) - 1 = ab - a - b$.

3.7.13 Fermat's two-squares theorem

Odd prime p can be represented as a sum of two squares iff $p \equiv 1 \pmod 4$. A product of two sums of two squares is a sum of two squares. Thus, n is a sum of two squares iff every prime of form $p = 4k + 3$ occurs an even number of times in n's factorization.

3.8 Permutations

3.8.1 Factorial

n	1	2	3	4	5	6	7	8	9	10
$n!$	1	2	6	24	120	720	5040	40320	362880	3628800

n	11	12	13	14	15	16	17
$n!$	4.0e7	4.8e8	6.2e9	8.7e10	1.3e12	2.1e13	3.6e14

n	20	25	30	40	50	100	150	171
$n!$	2e18	2e25	3e32	8e47	3e64	9e157	6e262	>DBL_MAX

3.8.2 Cycles

Let $g_S(n)$ be the number of n-permutations whose cycle lengths all belong to the set S. Then

$$\sum_{n=0}^{\infty} g_S(n) \frac{x^n}{n!} = \exp\left(\sum_{n \in S} \frac{x^n}{n}\right)$$

3.8.3 Derangements

Permutations of a set such that none of the elements appear in their original position.

$$D(n) = (n-1)(D(n-1) + D(n-2)) = nD(n-1) + (-1)^n = \left\lfloor \frac{n!}{e} \right\rceil$$

3.8.4 Burnside's lemma

Given a group G of symmetries and a set X, the number of elements of X *up to symmetry* equals

$$\frac{1}{|G|} \sum_{g \in G} |X^g|,$$

where X^g are the elements fixed by g ($g.x = x$).

If $f(n)$ counts "configurations" (of some sort) of length n, we can ignore rotational symmetry using $G = \mathbb{Z}_n$ to get

$$g(n) = \frac{1}{n} \sum_{k=0}^{n-1} f(\gcd(n,k)) = \frac{1}{n} \sum_{k|n} f(k)\phi(n/k)$$

3.9 Partitions and subsets

3.9.1 Partition function

Number of ways of writing n as a sum of positive integers, disregarding the order of the summands.

$$p(0) = 1,\ p(n) = \sum_{k \in \mathbb{Z} \setminus \{0\}} (-1)^{k+1} p(n - k(3k-1)/2)$$

$$p(n) \sim 0.145/n \cdot \exp(2.56\sqrt{n})$$

n	0	1	2	3	4	5	6	7	8	9	20	50	100
$p(n)$	1	1	2	3	5	7	11	15	22	30	627	~2e5	~2e8

3.9.2 Partition Number

- Time Complexity: $O(n\sqrt{n})$

```
for (int i = 1; i <= n; ++i) {
  pent[2 * i - 1] = i * (3 * i - 1) / 2;
  pent[2 * i] = i * (3 * i + 1) / 2;
}
p[0] = 1;
for (int i = 1; i <= n; ++i) {
  p[i] = 0;
  for (int j = 1, k = 0; pent[j] <= i; ++j) {
    if (k < 2) p[i] = add(p[i], p[i - pent[j]]);
    else p[i] = sub(p[i], p[i - pent[j]]); ++k, k &= 3;
  }
}
```

- The number of partitions of a positive integer n into exactly k parts equals the number of partitions of n whose largest part equals k

$$p_k(n) = p_k(n-k) + p_{k-1}(n-1)$$

3.9.3 2nd Kaplansky's Lemma

The number of ways of selecting k objects, no two consecutive,from n labelled objects arrayed in a circle is $\frac{n}{k}\binom{n-k-1}{k-1} = \frac{n}{n-k}\binom{n-k}{k}$

3.9.4 Distinct Objects into Distinct Bins

- n distinct objects into r distinct bins = r^n
- Among n distinct objects, exactly k of them into r distincts bins = $\binom{n}{k} r^k$
- n distinct objects into r distinct bins such that each bin contains at least one object = $\sum_{i=0}^{r} (-1)^i \binom{r}{i} (r-i)^n$

3.10 Coloring

The number of labeled undirected graphs with n vertices, $G_n = 2^{\binom{n}{2}}$

The number of labeled directed graphs with n vertices, $G_n = 2^{n(n1)}$

The number of connected labeled undirected graphs with n vertices, $C_n = 2^{\binom{n}{2}} - \frac{1}{n}\sum_{k=1}^{n-1} k\binom{n}{k} 2^{\binom{n-k}{2}} C_k = 2^{\binom{n}{2}} - \sum_{k=1}^{n-1}\binom{n-1}{k-1} 2^{\binom{n-k}{2}} C_k$

The number of k-connected labeled undirected graphs with n vertices, $D[n][k] = \sum_{s=1}^{n} \binom{n-1}{s-1} C_s D[n-s][k-1]$

Cayley's formula: the number of trees on n labeled vertices = the number of spanning trees of a complete graph with n labeled vertices = n^{n-2}

Number of ways to color a graph using k color such that no two adjacent nodes have same color

Complete graph = $k(k-1)(k-2)...(k-n+1)$

Tree = $k(k-1)^{n-1}$

Cycle = $(k-1)^n + (-1)^n(k-1)$

Number of trees with n labeled nodes: n^{n-2}

3.11 General purpose numbers

3.11.1 Eulerian numbers

Number of permutations $\pi \in S_n$ in which exactly k elements are greater than the previous element. k j:s s.t. $\pi(j) > \pi(j+1)$, $k+1$ j:s s.t. $\pi(j) \ge j$, k j:s s.t. $\pi(j) > j$.

$$E(n,k) = (n-k)E(n-1,k-1) + (k+1)E(n-1,k)$$

$$E(n,0) = E(n,n-1) = 1$$

$$E(n,k) = \sum_{j=0}^{k} (-1)^j \binom{n+1}{j} (k+1-j)^n$$

3.11.2 Bell numbers

Total number of partitions of n distinct elements. $B(n) = 1, 1, 2, 5, 15, 52, 203, 877, 4140, 21147, \ldots$. For p prime,

$$B(p^m + n) \equiv mB(n) + B(n+1) \pmod p$$

3.11.3 Bernoulli numbers

$\sum_{j=0}^{m} \binom{m+1}{j} B_j = 0$. $B_0 = 1, B_1 = -\frac{1}{2}$. $B_n = 0$, for all odd $n \neq 1$.

3.11.4 Catalan numbers

$$C_n = \frac{1}{n+1}\binom{2n}{n} = \binom{2n}{n} - \binom{2n}{n+1} = \frac{(2n)!}{(n+1)!n!}$$

$$C_0 = 1,\ C_{n+1} = \frac{2(2n+1)}{n+2} C_n,\ C_{n+1} = \sum C_i C_{n-i}$$

- $C_n = 1, 1, 2, 5, 14, 42, 132, 429, 1430, 4862, 16796, 58786, \ldots$
- sub-diagonal monotone paths in an $n \times n$ grid.
- strings with n pairs of parenthesis, correctly nested.
- binary trees with with $n+1$ leaves (0 or 2 children).
- ordered trees with $n+1$ vertices.
- ways a convex polygon with $n+2$ sides can be cut into triangles by connecting vertices with straight lines.
- permutations of $[n]$ with no 3-term increasing subseq.
- Find the count of balanced parentheses sequences consisting of $n+k$ pairs of parentheses where the first k symbols are open brackets.

$$C_n^{(k)} = \frac{k+1}{n+k+1}\binom{2n+k}{n}$$

- Recursive formula of Catalan Numbers:

$$C_n^{(k)} = \frac{(2n+k-1)\cdot(2n+k)}{n\cdot(n+k+1)} C_{n-1}^{(k)}$$

3.11.5 Lucas Number

Number of edge cover of a cycle graph C_n is L_n

$$L(n) = L(n-1) + L(n-2); L(0) = 2, L(1) = 1$$

3.12 Ballot Theorem

Suppose that in an election, candidate A receives a votes and candidate B receives b votes, where a kb for some positive integer k. Compute the number of ways the ballots can be ordered so that A maintains more than k times as many votes as B throughout the counting of the ballots.

The solution to the ballot problem is $\frac{a-kb}{a+b} \times C(a+b, a)$

3.13 Classical Problem

$F(n,k)$ = number of ways to color n objects using exactly k colors
Let $G(n,k)$ be the number of ways to color n objects using no more than k colors.
Then, $F(n,k) = G(n,k) - C(k,1)*G(n,k-1) + C(k,2)*G(n,k-2) - C(k,3)*G(n,k-3)...$

Determining G(n, k) :
Suppose, we are given a 1 * n grid. Any two adjacent cells can not have same color. Then, $G(n,k) = k*((k-1)^(n-1))$
If no such condition on adjacent cells. Then, $G(n,k) = k^n$

3.14 Matching Formula

3.14.1 Normal Graph

MM + MEC = n (exculding vertex), IS + VC = G, MIS + MVC = G

3.14.2 Bipartite Graph

MIS = n - MBM, MVC = MBM, MEC = n - MBM

3.15 Inequalities

3.15.1 Titu's Lemma

For positive reals $a_1, a_2, \ldots, a_n$ and $b_1, b_2, \ldots, b_n$,

$$\frac{a_1{}^2}{b_1} + \frac{a_2{}^2}{b_2} + \ldots + \frac{a_n{}^2}{b_n} \geq \frac{a_1 + a_2 + \ldots + a_n{}^2}{b_1 + b_2 + \ldots + b_n}$$

Equality holds if and only if $a_i = kb_i$ for a non-zero real constant k.

3.16 Games

3.16.1 Grundy numbers

For a two-player, normal-play (last to move wins) game on a graph (V,E): $G(x) = \text{mex}(\{G(y) : (x,y) \in E\})$, where $\text{mex}(S) = \min\{n \geq 0 : n \notin S\}$. x is losing iff $G(x) = 0$.

3.16.2 Sums of games

- *Player chooses a game and makes a move in it* Grundy number of a position is xor of grundy numbers of positions in summed games.
- *Player chooses a non-empty subset of games (possibly, all) and makes moves in all of them* A position is losing iff each game is in a losing position.
- *Player chooses a proper subset of games (not empty and not all), and makes moves in all chosen ones.* A position is losing iff grundy numbers of all games are equal.
- *Player must move in all games, and loses if can't move in some game* A position is losing if any of the games is in a losing position.

3.16.3 Misère Nim

A position with pile sizes $a_1, a_2, \ldots, a_n \geq 1$, not all equal to 1, is losing iff $a_1 \oplus a_2 \oplus \cdots \oplus a_n = 0$ (like in normal nim.) A position with n piles of size 1 is losing iff n is *odd*.

3.17 Tree Hashing

$f(u) = sz[u] * \sum_{i=0} f(v) * p^i$; $f(v)$ are sorted $f(child) = 1$

3.18 Permutation

To maximize the sum of adjacent differences of a permuation, it is necessary and sufficient to place the smallest half numbers in odd position and the greatest half numbers in even position. Or, vice versa.

3.19 String

- If the sum of length of some strings is N, there can be at most $\sqrt{N}$ distinct length.
- A Text can have at most $O(N \times \sqrt{N})$ distinct substrings that match with given patterns where the sum of the length of the given patterns is N.
- Period = n % (n - pi.back() == 0)? n - pi.back(): n
- The first $(period)$ cyclic rotations of a string are distinct. Further cyclic rotations repeat the previous strings.
- S is a palindrome if and only if it's period is a palindrome.
- If S and T are palindromes, then the periods of S T are same if and only if $S+T$ is a palindrome.

3.20 Bit

- (a xor b) and (a + b) has the same parity
- (a + b) = (a xor b) + 2 (a b)
- gcd(a, b) <= a - b <= xor(a, b)

3.21 Convolution

- Hamming Distance: Replace 0 with -1 - SQRT Decomposition: Find block size, B = sqrt(8 * n)